CSE 579 - Knowledge Representation and Reasoning

Week 2

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To Do in Module 1

Quiz 1- Due Jan 22, 11:59 PM.

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Extra examples + lectures subject

Q1

What is the aim of KRR?

a) To think abstractly as measured by objective criteria
b) To make the computer reason with knowledge and behave like humans
c) To calculate over symbols standing for propositions rather than numbers
d) To apply knowledge to manipulate one’s environment

Q1- Answer

What is the aim of KRR?

a) To think abstractly as measured by objective criteria
b) To make the computer reason with knowledge and behave like humans
c) To calculate over symbols standing for propositions rather than numbers
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The aim of Knowledge Representation and Reasoning is to enable computers to reason like humans by first understanding the nature of intelligence and cognition and then simulate them in computers.

Declarative vs. Non-Declarative

Declarative sentences (we can con- sider whether they're true or not):

The sum of the numbers 3 and 5 equals 8.

Every even natural number is the sum of two prime numbers.

Non-declarative sentences (can't tell whether they're true or not):

Ready, steady, go.

May fortune come your way.

We want to turn declarative sen- tences into formulas and create a formalism to manipulate such for- mulas.

Logic Formulas

- **Declarative sentences**
  - The sum of the numbers 3 and 5 equals 8
  - A dog has four legs
- **Symbols**

  p

  q
- **Connectives**

  $\wedge$ (and)

  $\vee$ (or)

  $\rightarrow$ (imply)

  $\neg$ (not)
- **Formula**

  $p \wedge q \rightarrow p \vee q$

Example

- If the train arrives late and there are no taxis at the station then John is late for his meeting. John is not late for his meeting. The train did arrive late.

- True or False: there was a taxi at the station (and prove)

- We know about:
  - $p \wedge \neg q \rightarrow r$
  - $\neg r$
  - p

p	The train is late
q	There are taxis at the station
r	John is late for his meeting

Therefore q

Types of reasoning

- **Default reasoning**

Default reasoning is that, in the case when we do not have enough evidence, we reach a conclusion that is likely to be true.

- **Model finding**

Model finding is that, given some constraints, we check all possible solutions and find a solution that satisfies all the constraints.

- **Deductive reasoning**

Deductive reasoning is that, given some conditions, we derive some conclusions using inference rules.

- **Abductive reasoning**

Abductive reasoning is that, given a background theory, a set of explanations and an observation, we find the most likely explanation.

Q2

Which kind of reasoning is used in the following proof?

Given: x is an even number.
Prove: $x \times x$ is an even number.
Proof: Since x is an even number, by the definition of even number, x equals to 2k for some integer k. Then $x \times x$ equals to $4 \times k \times k = 2 \times (2 \times k \times k)$. Since $2 \times k \times k$ is an integer, $x \times x$ is an even number.

a) Default reasoning
b) Model finding
c) Deductive reasoning
d) Abductive reasoning

Q2-Answer

Which kind of reasoning is used in the following proof?

Given: x is an even number.

Prove: $x \times x$ is an even number.

Proof: Since x is an even number, by the definition of even number, x equals to 2k for some integer k. Then $x \times x$ equals to $4 \times k \times k = 2 \times (2 \times k \times k)$. Since $2 \times k \times k$ is an integer, $x \times x$ is an even number.

a) Deductive reasoning

Deductive reasoning is that, given some conditions, we derive some conclusions using inference rules. This is exactly what is done in the proof: given x is an even number, we derive x*x is an even number using some mathematical knowledge.

a) Model finding
b) Default reasoning
c) Abductive reasoning

Q3

Which statement explains abductive reasoning?

a) Given some conditions, derive some conclusions using inference rules.
b) Given some constraints, check all possible solutions and find a solution that satisfies all the constraints.
c) Without enough evidence, reach a conclusion that is likely to be true.
d) Given a background theory, a set of explanations and an observation, find the most likely explanation.

Q3-Answer

Which statement explains abductive reasoning?

a) Given some conditions, derive some conclusions using inference rules.
b) Given some constraints, check all possible solutions and find a solution that satisfies all the constraints.
c) Without enough evidence, reach a conclusion that is likely to be true.
d) Given a background theory, a set of explanations and an observation, find the most likely explanation.

Abductive reasoning starts with an observation or set of observations then seeks to find the simplest and most likely explanation for the observations.

Definition of Propositional Formulas

A propositional formula of signature σ is defined recursively as follows:

- Every atom is a formula
- Both 0-place connectives are formulas
- If F is a formula then $\neg$ F is a formula
- For any binary connective $\odot$, if F and G are formulas then $(F \odot G)$ is a formula

Q4

Which option is a propositional formula according to the definition of propositional formula ?

a) $\bot$
b) $q \longrightarrow p$
c) $(\neg(q) \vee r)$
d) $\bot\neg T$

Q4-Answer

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Q5

How many of the six strings below are propositional formulas?

String 1: $\bot$
String 2: p
String 3:$((\neg q \lor r) \land (\neg q \lor p))$
String 4: $\bot\neg T$
String 5: $:\neg q$
String 6: $q \land r \land \neg q \lor p$

a) 3
b) 4
c) 6
d) 1

Q5-Answer

How many of the six strings below are propositional formulas?

String 1: $\bot$
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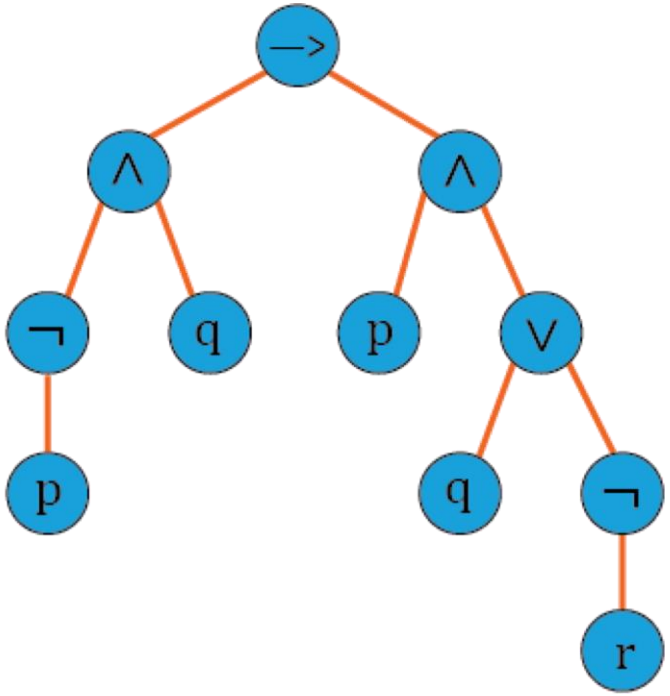
String 3: $((\neg q \lor r) \land (\neg q \lor p))$
String 4: $\bot \neg T$
String 5: $:\neg q$
String 6: $q \land r \land \neg q \lor p$

a) 3
b) 4
c) 6
d) 1

Strings 1,2,3,5 are propositional formula

Parse Tree

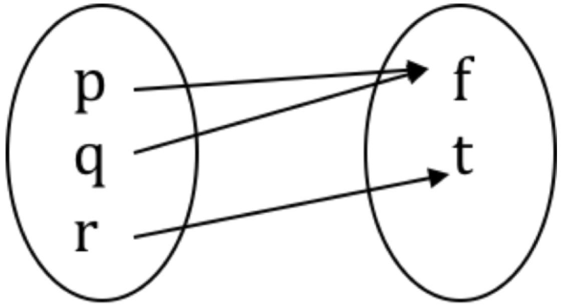
- $((\neg p \wedge q) \rightarrow (p \wedge (q \vee \neg r)))$



- **Subformulas** are the formulas corresponding to the subtrees of the parse tree
- Priority of connectives: $\neg, \wedge, \vee, \rightarrow$

Interpretation

- An **interpretation** of a propositional signature σ is a function from σ into $\{f, t\}$
  - A propositional signature is a set of symbols called atoms, such as p, q, r
  - The symbols f and t are called truth values.
  - If σ is finite, an interpretation can be defined by a truth table



p	q	r
f	f	t

How many interpretations are for {P,Q,R}?

Truth tables

Φ	Ψ	$\neg\Psi$	$\Phi \wedge \Psi$	$\Phi \vee \Psi$	$\Phi \rightarrow \Psi$	$\top$	$\bot$
F	F	T	F	F	T	T	F
F	T	F	F	T	T		
T	F		F	T	F		
T	T		T	T	T		

Truth tables are means of exploring all possible interpretations for a given formula.

ϕ ↔ φ

F↔F T
F↔T F
T↔F F
T↔T T

Valuation

- A **valuation** of a formula *F* is an assignment of each propositional atom in *F* to a truth value
  - $F=(p \rightarrow \neg q) \rightarrow (q \vee \neg p)$
  - List all valuations of its subformulas

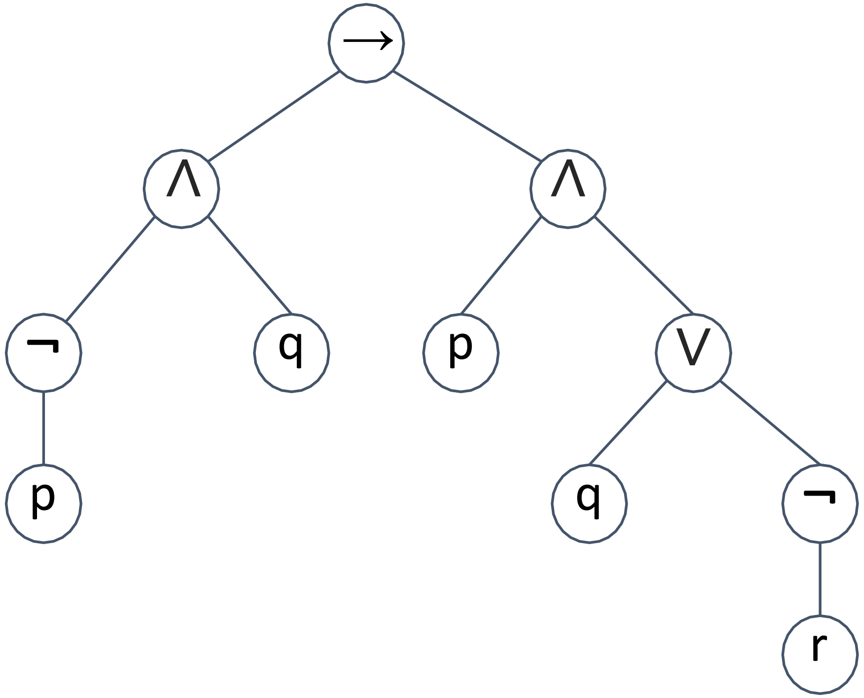
p	q	$\neg p$	$\neg q$	$p \longrightarrow \neg q$	$q \vee \neg p$	$(p \longrightarrow \neg q) \longrightarrow (q \vee \neg p)$
t	t	f	f	f	t	t
t	f	f	t	t	f	f
f	t	t	f	t	t	t
f	f	t	t	t	t	t

Valuation with Parse Tree

- **Example:** F= $\neg p \wedge q \rightarrow p \wedge (q \vee \neg r)$

I=

p	q	r
f	f	t



Q: Truth value of the formula
if I(p)=f, I(q)=t, I(r)=f ?

- If $F^I = t$ then we say that the interpretation I satisfies F (symbolically $I \models F$)