Dhanraj Babasaheb Bhosale.
dbhosale1@asu.edu
ASU ID: 1225506620

$$J(\theta) = \frac{-1}{m}\left\{\sum_{i=1}^{m} y^{(i)} \log\left[h_{(\theta)}(x^{(i)})\right] + (1-y^{(i)})\log\left[1-h_\theta(x^{(i)})\right]\right\}$$

where,

$$h_\theta(x) = \frac{1}{1+e^{-\theta x}}$$

$$\frac{\partial J(\theta)}{\partial \theta_j} = \frac{\partial}{\partial \theta_j}\left[\frac{-1}{m}\sum_{i=1}^{m} y^i \log(h_\theta(x^{(i)})) + (1-y^i)\log(1-h_\theta(x^i))\right]$$

$$\text{I} \rightarrow = \frac{-1}{m}\sum_{i=1}^{m} y^i \underbrace{\frac{\partial}{\partial \theta_j}\left[\log(h_\theta(x^i))\right]}_{1} + (1-y^i)\underbrace{\frac{\partial}{\partial \theta_j}\left[\log(1-h_\theta(x^i))\right]}_{2}$$

= we solve 1 & 2

$$\Rightarrow \frac{\partial}{\partial \theta_j}\left[\log(h_\theta(x^i))\right] = \frac{1}{h(x^i)}\frac{\partial}{\partial \theta_j} h_\theta(x^i) \quad \text{from 1}$$

$$\Rightarrow \frac{\partial}{\partial \theta_j}\left[\log(1-h_\theta(x^i))\right] = \frac{1}{1-h_\theta(x^i)}\frac{\partial}{\partial \theta_j}(1-h_\theta(x^i))$$

$$= \frac{-1}{1-h_\theta(x^i)}\frac{\partial}{\partial \theta_j} h_\theta(x^i)$$

from 2

Substitute in A

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{-1}{m} \sum^{m} \Big[y^i \frac{1}{h_\theta(x^i)} \frac{\partial}{\partial \theta_j} h_\theta(x^i) +$$

$$(1-y^i) \frac{-1}{1-h_\theta(x^i)} \frac{\partial}{\partial \theta_j} h_\theta(x^i) \Big]$$

$$= \frac{-1}{m} \sum_{i=1}^{m} \left[\frac{y^i}{h_\theta(x^i)} - \frac{(1-y^i)}{1-h_\theta(x^i)} \right] * \frac{\partial}{\partial \theta_j} h_\theta(x^i)$$

from the hint identity:

$$\frac{\partial}{\partial \theta_j} h_\theta(x) = \frac{1+e^{-\theta^T x} - 1}{(1+e^{-\theta^T x})^2} = h_\theta(x) \left[1 - h_\theta(x) \right] x_j$$

Therefore,

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{-1}{m} \sum_{i=1}^{m} \left[\frac{y^i}{h_\theta(x^i)} - \frac{1-y^i}{1-h_\theta(x^i)} \right]$$

$$\times \left[h_\theta(x^i) \cdot (1-h_\theta(x^i)) x_j^i \right]$$

$$= \frac{-1}{m} \sum_{i=1}^{m} \left[\frac{y^i(1-h_\theta x^i) - h_\theta(1-y^i)}{\cancel{h_\theta(x^i)(1-h_\theta(x^i))}} \right]$$

$$\times \left[\cancel{h_\theta(x^i)(1-h_\theta(x^i))} x_j^i \right]$$

$$= \frac{-1}{m} \sum_{i=1}^{m} \left[y^i - h_\theta(x^i) \right] x_j^i$$

$$\# \quad \frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \left[(h_\theta(x^i) - y^i) x_j^i \right] \quad *$$