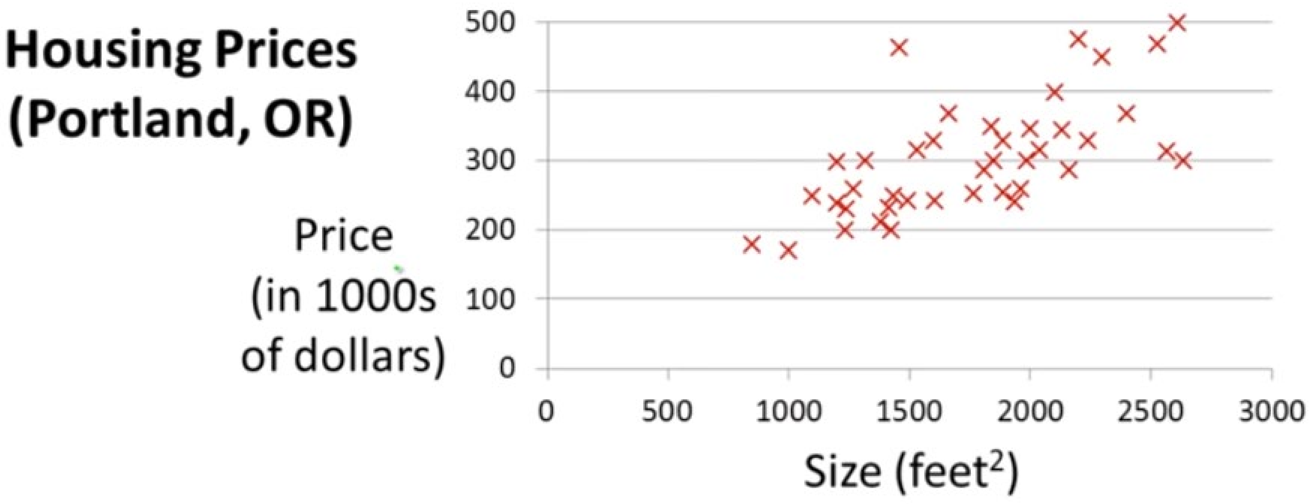
# Univariate Linear Regression

# Regression and machine learning

- Regression is  a process of estimating the relationship between a dependent variable (or outcome variable) and independent variables (or features, predictors…)
- Linear regression – the dependent variable relates to the independent variables linearly.
- In the context of machine learning, regression is used for prediction/forecasting.



# Supervised learning

Given data pairs (size, price), create parameterized models by supervised learning

Simplest such model, univariate linear regression:

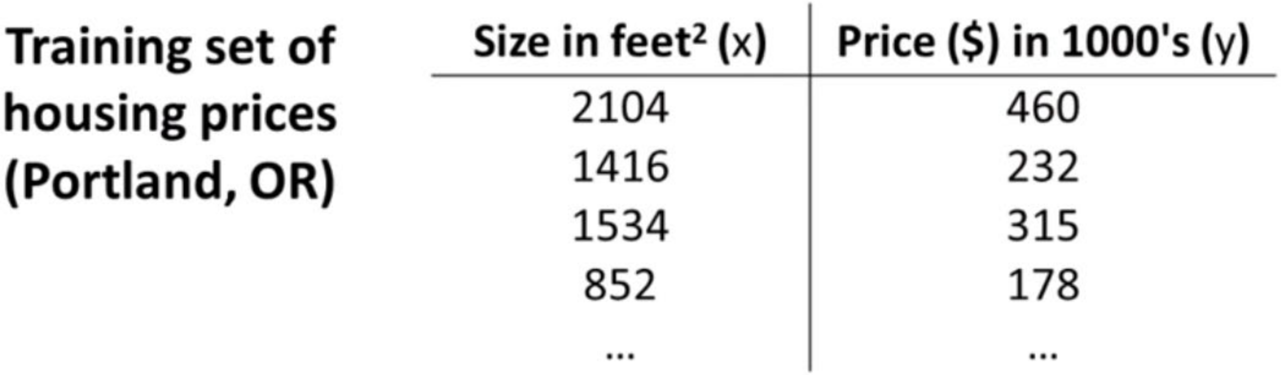
$$h_\theta(x) = \theta_0 + \theta_1 x$$

## Regression problem

Predict real valued output (price) given the size of a house just put on market

## Classification problem

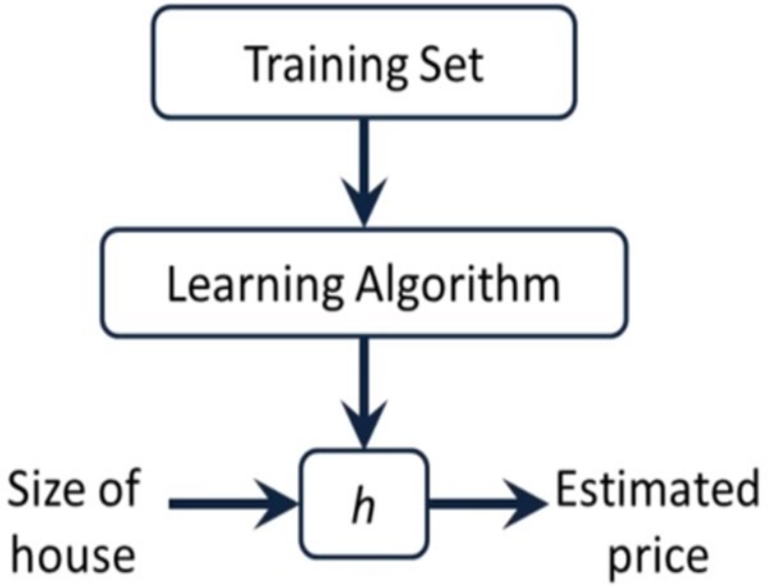
Predict discrete valued output (top moon/bottom moon) given the coordinates of a data point



**Training set of housing prices (Portland, OR)**

| Size in feet$^2$ (x) | Price ($) in 1000's (y) |
|---|---|
| 2104 | 460 |
| 1416 | 232 |
| 1534 | 315 |
| 852 | 178 |
| … | … |

Notation:
m: number of training examples
x: input variable or input feature or feature
y: output variable



Hypothesis / predictive model

To represent h – need a structure/model & model parameters

Univariate linear regression:

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$x$: size of house

θ: parameter of model

How to choose model parameters $\theta_0$ and $\theta_1$?

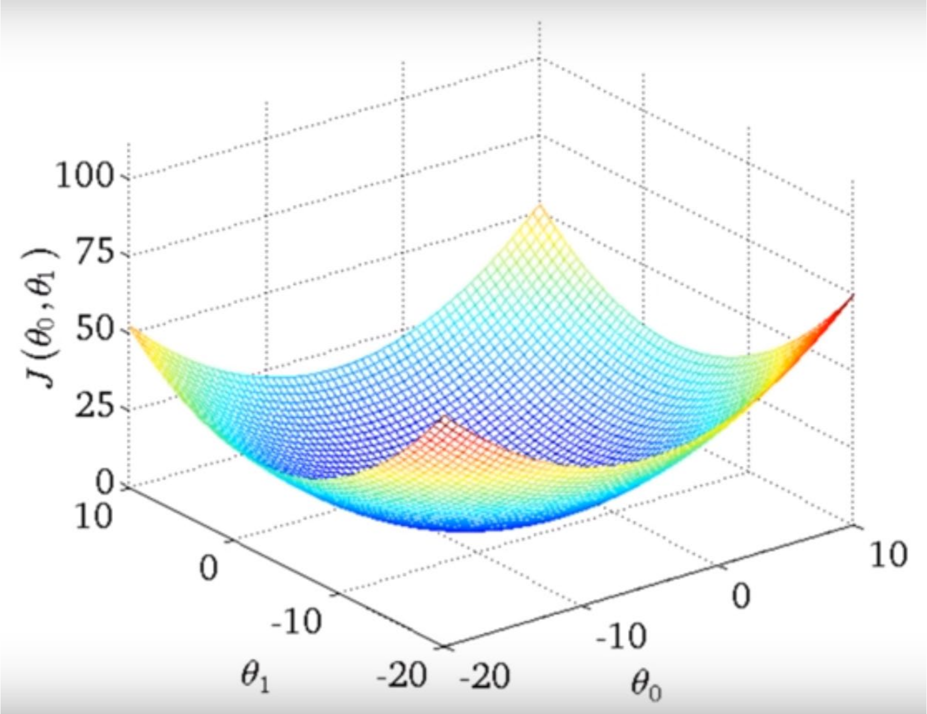
Goal: to make $h_\theta(x)$ as close to real house price $(y)$ as possible (using the training data)

Problem formulation by first defining a cost function, $J(\theta_0, \theta_1)$:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$$

Choose $\theta_0$ and $\theta_1$ so that is $J(\theta_0, \theta_1)$ minimized

Let's use examples to gain some intuition about the cost function $J(\theta_0, \theta_1)$
As a function of the parameters $\theta_0$ and $\theta_1$


100
75
50
25
0
$J(\theta_0, \theta_1)$
10
0
-10
-20
$\theta_1$
-20
-10
0
10
$\theta_0$

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

Parameters: $\theta_0$ , $\theta_1$

Cost function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta\left(x^{(i)}\right) - y^{(i)}\right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}} \; J(\theta_0, \theta_1)$

Outline of learning procedure:

- Start with some $\theta_0$ , $\theta_1$
- Keep changing $\theta_0$ , $\theta_1$ to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum

Update parameters $\theta_0$ and $\theta_1$ using gradient descent until convergence

Simultaneously update $\theta_0$ and $\theta_1$ according to

$$\theta_0 \coloneqq \theta_0 - \eta \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_1 \coloneqq \theta_1 - \eta \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$

$\eta$ is a positive number, which is called the learning rate
$-$ too small, slow learning; too large, divergence

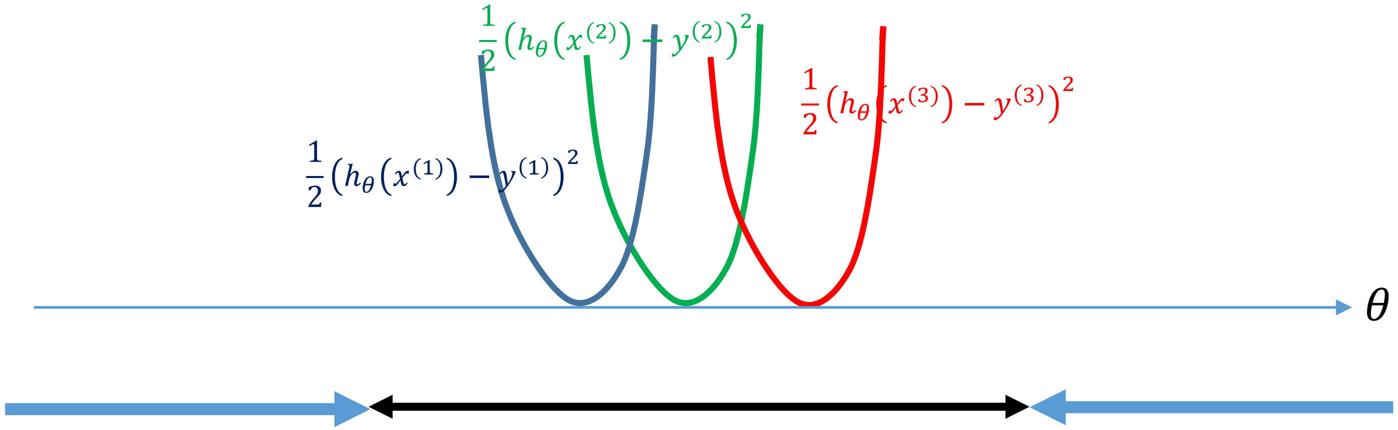
Exercise – derive the two gradients

- notice this derivation is a “batch” learning algorithm
- “online” learning – the LMS algorithm

How to optimize (minimize) a cost/loss function by tuning neural network weights for a predicted value to approach an actual value?

- Stochastic gradient descent (such as the LMS by Widrow and Hoff)
- Batch gradient descent as in linear regression, quadratic problem with least squares solution in closed form (may not be the best for large data set)
- Mini-batches (as in many deep networks)

# Insights on SGD (using Widrow-Hoff LMS setting)


$\frac{1}{2}\left(h_\theta\left(x^{(2)}\right) - y^{(2)}\right)^2$
$\frac{1}{2}\left(h_\theta\left(x^{(3)}\right) - y^{(3)}\right)^2$
$\frac{1}{2}\left(h_\theta\left(x^{(1)}\right) - y^{(1)}\right)^2$
$\theta$

Near convergence to solution of optimal filter, entering a region of confusion

- When far from convergence, behaves similarly as ordinary GD but less computation (1/m, m is the # of samples)
- When near convergence, gradient decent for one learning sample may be an ascent for another sample. A diminishing step size is required, e.g., the Robbins-Monro condition (e.g., η=1/n, n is # of learning iteration)