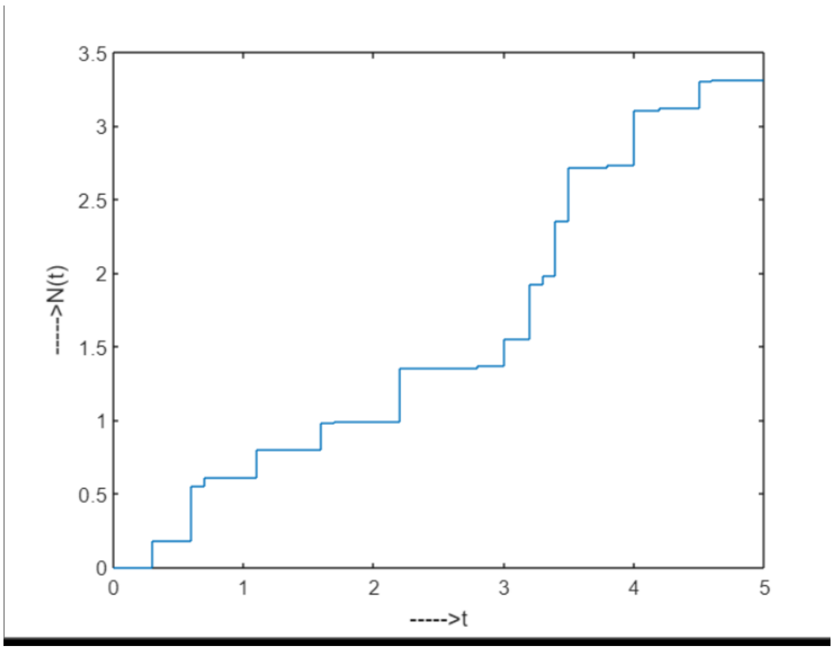
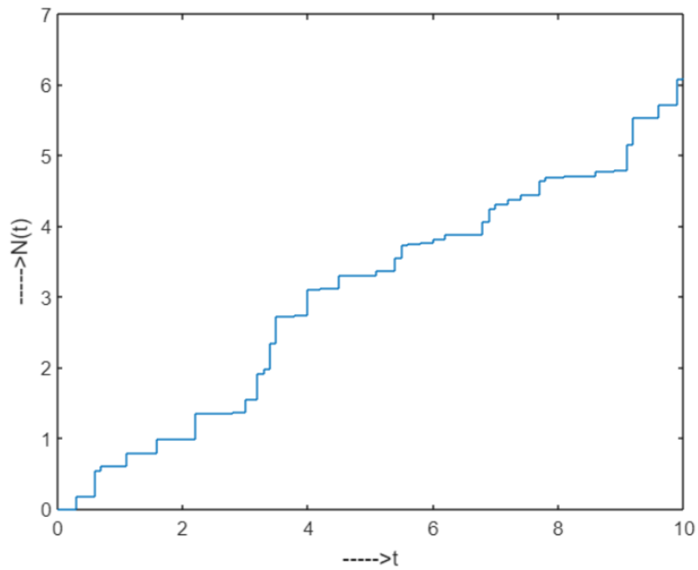
Q.1

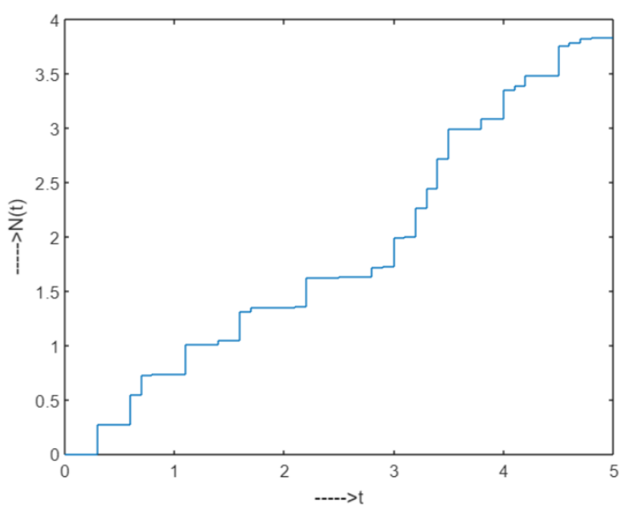
```
lambda = 1;
n = 50; % Periods
dt = 0.1; % total time steps
T = n.*dt;
t = 0:0.1:T;
rng('default')
k=randi([1,10],1,n);
f = (lambda.^k).*exp(-lambda)./factorial(k); % Poission Distribution.
Nd = cumsum(f);
N = [0 Nd(1:end) ]; % N(0)=0.
stairs(t,N)
xlabel('----->t');
ylabel('----->N(t)');
```



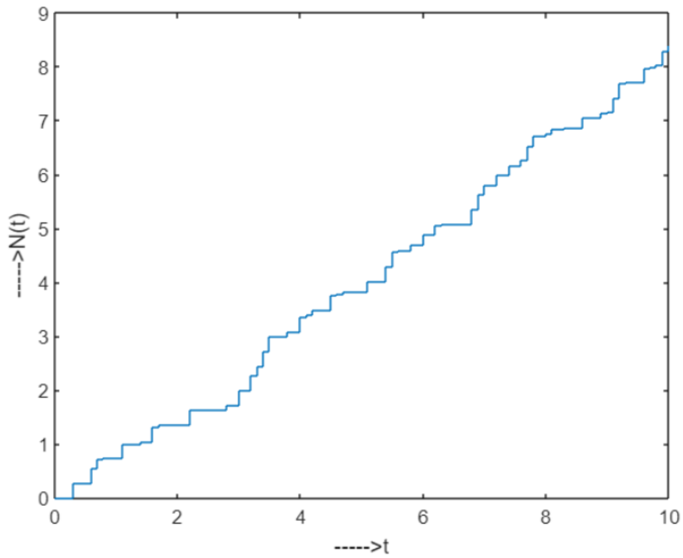

```
lambda = 1;
n = 100; % Periods
dt = 0.1; % total time steps
T = n.*dt;
t = 0:0.1:T;
rng('default')
k=randi([1,10],1,n);
f = (lambda.^k).*exp(-lambda)./factorial(k); % Poission Distribution.
Nd = cumsum(f);
N = [0 Nd(1:end) ]; % N(0)=0.
stairs(t,N)
xlabel('----->t');
ylabel('----->N(t)');
```




```
lambda = 2;
n = 50; % Periods
dt = 0.1; % total time steps
T = n.*dt;
t = 0:0.1:T;
rng('default')
k=randi([1,10],1,n);
f = (lambda.^k).*exp(-lambda)./factorial(k); % Poission Distribution.
Nd = cumsum(f);
N = [0 Nd(1:end) ]; % N(0)=0.
stairs(t,N)
xlabel('----->t');
ylabel('----->N(t)');
```



```
lambda = 2;
n = 100; % Periods
dt = 0.1; % total time steps
T = n.*dt;
t = 0:0.1:T;
rng('default')
k=randi([1,10],1,n);
f = (lambda.^k).*exp(-lambda)./factorial(k); % Poission Distribution.
Nd = cumsum(f);
N = [0 Nd(1:end) ]; % N(0)=0.
stairs(t,N)
xlabel('----->t');
ylabel('----->N(t)');
```



Q. 2

Since N(t) is a Poisson process, N(0) = 1 and it takes the values 0,1,2,3,...

(a) N(2t)
N(2t) statisfies all the properties of a Poisson process.
(b) N(t/2)
N(t/2) statisfies all the properties of a Poisson process.
(c) 2N(t)
At t=0, 2N(t)= 0. But 2N(t) takes the values 0,2,4,6,8,....
So it is not a Poisson process
(d) N(t)/2
At t=0, N(t)/2= 0. But N(t)/2 takes the values 0,1/2,1,3/2,2,....
So it is not a Poisson process
(e) N(t+ 2)
At t=0, N(t+2) = N(2) is not equal to zero.So it is not a Poisson process
(f) N(t) - N(t-1)
At t=0, N(t) - N(t-1) = N(0) - N(-1)= N(-1) is not equal to zero. So It is not a Poisson process

Q. 3. It matters whether $t \geq 2$ min. If $t \leq 2$, then any customers that have arrived must still be in service. Since a Poisson number of arrivals occur during $(0, t]$.

$$P_{N(t)}(n) = \begin{cases} (\lambda t)^n e^{-\lambda t}/n! & n = 0, 1, 2 \ldots \\ 0 & \text{o.w.} \end{cases} \quad (0 \leq t \leq 2) \quad (1)$$

For $t \geq 2$, customer in service are precisely those customers that arrived in interval $(t-2, t]$. Number of such customers has a Poisson PMF with mean $\lambda[t-(t-2)] = 2\lambda$. The resulting PMF of $N(t)$ is.

$$P_{N(t)}(n) = \begin{cases} (2\lambda)^n e^{-2\lambda}/n! & n = 0, 1, 2 \ldots \\ 0 & \text{o.w} \end{cases} \quad (t \geq 2) \quad (2)$$

Q. 4 Customer entering (or not entering) casino is a Bernoulli decomposition of poisson process of arrivals at casino doors. By Theorem 10.6, customers entering casino are a poisson process of rate $100/2 = 50$ customers/hour. Thus in two hours from 5 to 7 PM, the number, N, of customer entering casino is a Poisson random variable with expected value $\alpha = 2 \cdot 50 = 100$.

The PMF of N is

$$P_N(n) = \begin{cases} 100^n e^{-100}/n! & n = 0, 1, 2 \ldots \\ 0 & \text{o.w.} \end{cases} \quad (1)$$

Q5 Number Y_k of failures between success $k-1$ and k is exactly $y \geq 0$ iff after success $k-1$, there are y failures followed by success. Sience the Bernoulli trials are independent, probability of event is $(1-p)^y p$.

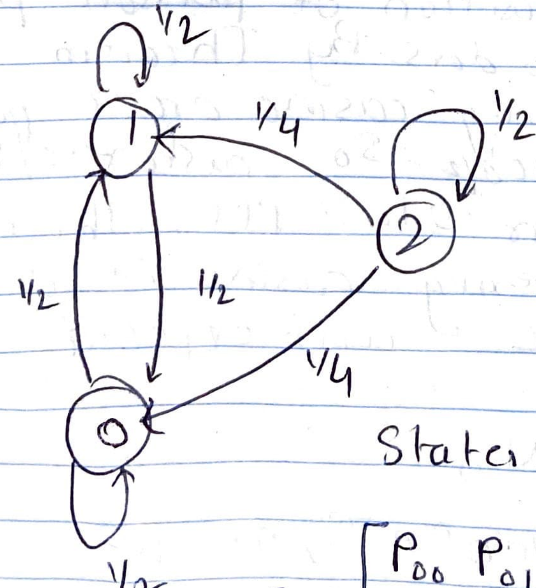
$$\therefore \quad P_{Y_k}(y) = \begin{cases} (1-p)^y p & y = 0, 1, \ldots \\ 0 & o.w \end{cases} \qquad (1)$$

this argument is valid for all k includiny $k=1$, we can conclude that $Y_1, Y_2, \ldots$ are identically distributed.

Sience the trials are independent, the failure between sucess $k-1$ and k and number of failure between success $k'-1$ and k' are independent.

Hence, Y_1, Y_2 is an iid sequence.

Q. 6



from given Markov chain

State transition matrix is,

$$P = \begin{bmatrix} P_{00} & P_{01} & P_{02} \\ P_{10} & P_{11} & P_{12} \\ P_{20} & P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

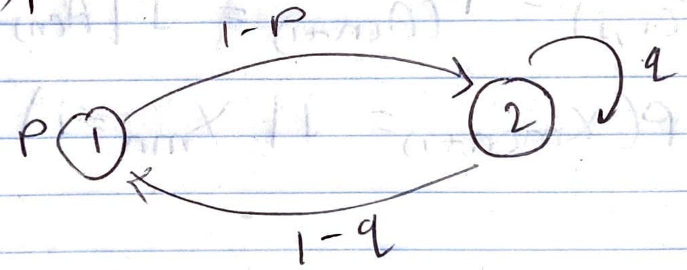
Q.7 If we keep in mind that p_{ij} is probability that we transition from stat i to stat j.

The state transition matrix is,

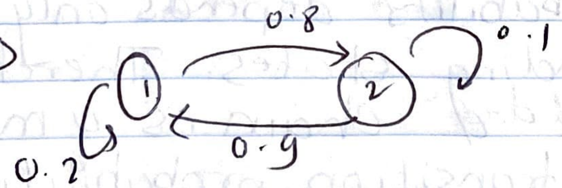
$$P = \begin{bmatrix} P_{00} & P_{01} \\ P_{10} & P_{11} \end{bmatrix} = \begin{bmatrix} 1-p & p \\ q & 1-q \end{bmatrix}$$

~~Q8~~

$P_{00} = 0.2$
$P_{11} = 0.1$



$\Rightarrow$



$$\therefore P = \begin{bmatrix} 0.2 & 0.8 \\ 0.9 & 0.1 \end{bmatrix}$$

Q.8. Given,

discrete-time markov chain with transition matrix P.

X_n : System state after n seconds observation $\hat{X}_0, \hat{X}_1, \ldots$ where

$$\hat{X}_n = X_{mn}$$

Q. ~~#2 Since N(t) is a poisson~~

Yes, observation sequence $\vec{X_0}, \vec{X_1}, \dots$ is a markov chain, known as "sampled" markov chain.

To find the transition matrix P for this Markov chain, we need to compute probability of transitioning from one state to another in one observation interval of m seconds,

$$P(i,j) = P(A(n+1) = j \mid A(n) = i)$$

$$P(X_{m(n+1)} = j \mid X_{mn} = i) = P(X_{(m)} = j \mid X_{(0)} = i)$$

The probability depends only on starting and ending states. Therefore the embedded chain is a markov chain with transition probabilities is given by

$$P(i,j) = P(X_{(m)} = j \mid X_{(0)} = i)$$

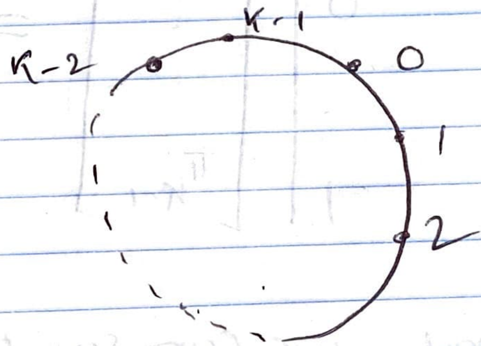
To simply put it, let's say a markov chain system has a transition matrix, P samples are taken after "m" switches, since markov chain systems are memoryless, the system provides sar transition. matrix is raised to the power "m".

Therefore,

The state transition matrix is P^m.

Q.9

Given a circular board with K spaces numbered from 0 to $K-1$



Given

$$X_{n+1} = (X_n + R_n) \bmod K$$

R_n → result of n^{th} roll.

X_n: Position of player during n^{th} roll

The state transition matrix P can be given as follows:

$$P = \begin{array}{c|cccccc|} & 0 & 1 & 2 & 3 & \cdots & K-1 \\ \hline 0 & 0 & 1/6 & 1/6 & 1/6 & \cdots & 0 \\ 1 & 0 & 0 & 1/6 & 1/6 & \cdots & 0 \\ 2 & 0 & 0 & 0 & 1/6 & \cdots & 0 \\ 3 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \vdots \\ & 1/6 & 1/6 & 1/6 & 1/6 & \cdots & 0 \; 0 \end{array}$$

In order to find the eigen vector :

$(P - I_{k \times k}) * X = 0$

$$\begin{bmatrix} -1 & 1/6 & 1/6 & & & 0 \\ 0 & -1 & 1/6 & & & 0 \\ 0 & 0 & -1 & 1/6 & & 0 \\ \vdots & & & & & \vdots \\ \frac{1}{6} & 1/6 & 1/6 & & & -1 \end{bmatrix} \begin{bmatrix} \pi_0 \\ \pi_1 \\ \vdots \\ \pi_{k-1} \end{bmatrix} = 0$$

from this analogy, we can say that

$\pi_0 = \pi_1 = \pi_2 \cdots = \pi_{k-1}$ — (a)

we know that, in a stationary probability vector i,

$\pi_0 + \pi_1 + \pi_2 + \cdots + \pi_{k-1} = 1$ — (b)

therefore from (a) and (b)

$\pi_0 = \pi_1 = \pi_2 = \cdots = \pi_{k-1} = 1/k$

∴ The stationary probability vector is

$$\boxed{\pi = \underbrace{\begin{bmatrix} 1/k & 1/k & \cdots & 1/k \end{bmatrix}'}_{\text{there are 'k' entries in this vector.}}}$$