MATLAB PROJECT for ATW
EUREKOM
January, 2023

- Read carefully the following questions, and using matlab, provide the answers/plots in the form of a report.
- The report should include a title page, and should be properly labeled and named. *The report should be in the form of a pdf.*
- Graphs should include labels, titles, and captions.
- Each graph should be accompanied with pertinent comments.
- Use optimal (maximum likelihood) decoders, unless stated otherwise.
- To compare the empirical results with the corresponding theoretical result, you should superimpose the two corresponding graphs, and should provide comments and intuition on the comparison.
- For each plot, describe the theoretical background that guides the proper choice of parameters for simulations (i.e., power constraint).
- You can work in groups of four.
- Regarding grading:
  - 25% of total grade
  - All questions are weighted equally.
  - Submit your report (labeled and named), via email, to elia@eurecom.fr
  - Submission deadline is February 16th, 2023.
- Enjoy!

Problem 1 (Space-Time Codes, Diversity Techniques, ML decoding)

Consider communication over the 3×1 quasi-static fading MISO channel, using a diagonal code (see below for details) such that the channel model is given by

$$\overbrace{\begin{pmatrix} y_1 & y_2 & y_3 \end{pmatrix}}^{\underline{y}} = \theta \overbrace{\begin{pmatrix} h_1 & h_2 & h_3 \end{pmatrix}}^{\underline{h}} \overbrace{\begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}}^{X_{tr}} + \overbrace{\begin{pmatrix} w_1 & w_2 & w_3 \end{pmatrix}}^{\underline{w}}$$

where $h_i \sim \mathbb{C}N(0,1)$ and $w_i \sim \mathbb{C}N(0,1)$, and where θ is the power normalization factor that lets you regulate SNR.

Here, you are supposed to do a simulation of the action of decoding. Provide the details of how you simulated. Tell us which variables you change in each iteration: h_i, codewords, noise, and tell us how you power normalize (emphasis on θ). Naturally, in each iteration, you decode, using the ML rule

$$\hat{X} = argmin_{X \in \mathcal{X}_{tr}} \|\underline{y} - \theta \underline{h} X\|^2$$

going over all choices of X in the code $\mathcal{X}_{tr}$.
NOTE: Do many iterations so that your plots are 'smooth'. In all the above, the y-axis is the probability of error, in log scale ($\log_{10}$(Prob)), and the x-axis is the SNR, in dB.

1. Plot the probability of error on a logarithmic scale as a function of SNR(dB) by performing Monte-Carlo simulations for when x_1, x_2, x_3 are independently chosen from 16-PAM
2. when $\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} = \begin{pmatrix} s_1 & s_2 & s_3 \end{pmatrix} \cdot R$, where s_1, s_2, s_3 are independently chosen from a QAM constellation such that the rate is 4 bits per channel use, and where R is a nice rotation (orthogonal) matrix of your choice. Justify your choice.
   - Compare and explain the two plots in the above.
   - Comment on each plot, and specifically comment on
     (a) the value of the minimum SNR that gives a non-zero slope in the probability of error in your plot, as well as
     (b) comment on the diversity (slope) that your results suggest.

For the above, use ML decoder, and plot for SNR values - in steps of 4dB - up to an SNR value for which your probability of error drops below 10^{-4}. Again, clearly explain how you calculate θ in each case.

Problem 2 - Receiver Beamforming with CSIR, MRC, SIMO

Recall that when multiple antennas are used at the receiver side, the channel between the transmitter and the receiver becomes a vector. Therefore, the channel model becomes

$$\mathbf{y} = \mathbf{h}x + \mathbf{z}$$

where the channel coefficients are represented by a vector $\mathbf{h}$ of N_r entries (one for each receive antenna). So are the received signal $\mathbf{y}$ and noise $\mathbf{z}$. In particular, we assume that the noises at different antennas are i.i.d. circularly symmetric Gaussian distributed. We also consider i.i.d. Rayleigh fading for the channel coefficients with the following normalization $\mathbb{E}\{\|\mathbf{h}\|^2\} = 1$. For simplicity, we order the antennas in such a way that $|h_1| \geq |h_2| \geq \cdots \geq |h_{N_r}|$.

Also recall that Maximum ratio combining (MRC) is optimal, in the sense that the resulting equivalent channel $\mathbf{h}^H\mathbf{y} = \|\mathbf{h}\|^2 x + \mathbf{h}^H\mathbf{z}$ has the same capacity as the original SIMO channel for a given channel realization $\mathbf{h}$. In the following, you will calculate the outage probability $P_{out} = P(log(1 + SNR||\mathbf{h}||^2) < R)$ for different values of SNR and for different values of rate of your choice.

In particular, use simulations to establish the probability of *deep fade*

$$P(|\mathbf{h}||^2) < SNR^{-1}$$

for the 1×4 SIMO model

$$\mathbf{y} = \mathbf{h} \cdot x + \mathbf{w}$$

(in the presence of the optimal MRC) for the following three cases (recall $\mathbf{h} = [h_1 \;\; h_2 \;\; h_3 \;\; h_4]$)

1. $h_i \sim \mathbb{C}N(0, 1)$ i.i.d
2. $h_2 = h_4 = h_1 \times h_3$ where $h_1, h_3 \sim i.i.d.\ \mathbb{C}N(0, 1)$.
3. $h_4 = \frac{1}{3}(h_1 + h_2 + h_3)$ where now $h_1, h_2, h_3 \sim i.i.d.\mathbb{C}N(0, 1)$.

In all the above, the y-axis is the probability of deep fade, in log scale ($\log_{10}$(Prob)), and the x-axis is the SNR, in dB.

Problem 3 - Fading statistics and Ergodic Capacity

Consider a SISO fast fading channel. Let us assume that at each instance, the fading coefficient h is circularly symmetric Gaussian distributed (fast Rayleigh fading).

- Plot the instantaneous capacity

$$C(h) = \log(1 + SNR|h|^2)$$

  for different values of SNR, for 100 fading realizations.

- Then plot the ergodic capacity versus SNR from -10 to 30 dB and the outage probability versus SNR for the same SNR range and a target rate $R = 2$ bits per channel use. Comment.

Problem 4 - MISO, Transmit Diversity, and Transmit Beamforming

Recall that in the single-user MISO case with CSIT (no rate adaptation here), the optimal transmit beamformer is indeed matched filtering (MF), in which case, under the channel model

$$y = \mathbf{h}^T \cdot \mathbf{x} + w$$

the optimal (instantaneous) capacity, under a given SNR ρ, takes the form

$$C(\mathbf{h}) = \log(1 + \rho\|\mathbf{h}\|^2).$$

Use simulations to establish — under the assumption of the optimal MF beamforming — the probability of *deep fade*

$$P(|\mathbf{h}\|^2) < \rho^{-1}$$

as well as the probability of outage

$$P(C(\mathbf{h}) < R)$$

for $R = 4$ bits per channel use, both for the scenarios:

- 2×1 MISO quasi-static channel with i.i.d channel coefficients
- 3×1 MISO quasi-static channel with i.i.d channel coefficients
- 5×1 MISO quasi-static channel with i.i.d channel coefficients
- 5×1 MISO quasi-static channel when all the fading coefficients are the same.

Do so for increasing SNR (dB), where *again the fading coefficients are drawn randomly i.i.d. from a Gaussian distribution* $\mathbb{C}N(0, 1)$).

Problem 5 - Channel and Noise Statistics

Create different experiments to check the validity of each of the following assumptions

- For Gaussian random variables $h_r \sim \mathcal{N}(0,1)$, the far tail $P(|\|h_r\| > \alpha)$ is approximated by the Q-function $Q(\alpha)$. (See https://en.wikipedia.org/wiki/Q-function)
- For $h \sim \mathbb{C}\mathcal{N}(0,1)$ (complex Gaussian), the near-zero behavior is approximated as follows

  $$P(|\|h\|^2 < \epsilon) \approx \epsilon.$$

  (i.e., validate with equations that $P(|\|h\|^2 < \epsilon) \approx \epsilon$.)
- Same as the above, but for $h \sim \mathbb{C}\mathcal{N}(0,10)$. Show how the near-zero behavior is approximated.
- For $\mathbf{h} = [h_1 \;\; h_2 \;\; h_3 \;\; \cdots \;\; h_k]$ with $h_i \sim \mathbb{C}\mathcal{N}(0,1)$, simulate the near zero behavior of a chi-squared random variable $\|\mathbf{h}\|^2$ with $2k$ degrees of freedom. In particular show how the near zero behavior is approximated - for small values of ϵ - as

  $$P(|\|\mathbf{h}\|^2 < \epsilon) \approx \epsilon^k.$$

  (do this for smaller values of k.)

NOTE: In the above exercise, describe how you perform the different experiments.
NOTE: We need statistical experiments, i.e., experiments that involve the generation of random variables, and the measuring of their behavior using - if you wish - histograms.

Problem 6 - Basic Capacity Approximations

Check, using matlab, the validity of the following assumptions

- $\log_2(1 + SNR) \approx \log_2(SNR)$ for high SNR
- $\log_2(1 + SNR) \approx SNR \cdot \log_2 e$ for low values of SNR

Note, this is not a statistical experiment.