☆ A : Set

P(A) : Power set of A = Set containing all possible subsets of set A.

→ $A = \{1, 2\}$

$\{1\} \subseteq A$

$\{2\} \subseteq A$

$\{1, 2\} \subseteq A$

ϕ or $\{\} \subseteq A$

$P(A) = \{\{1\}, \{2\}, \{1, 2\}, \{\}\}$

→ $|A| = 2$; $|P(A)| = 4$

→ $|A| = n$; $|P(A)| = 2^n$

→ $A = \{1\} \Rightarrow |A| = 1$

$P(A) = \{\{1\}, \phi\} \Rightarrow |P(A)| = 2$

→ $A = \{2, 3, 4\} \Rightarrow |A| = 3$

$P(A) = \{\{2\}, \{3\}, \{4\}, \{2, 3\}, \{3, 4\}, \{2, 4\}, \{2, 3, 4\}, \phi\} \Rightarrow |P(A)| = 2^3 = 8.$

☆ Multiset : A set having repeated elements.

Ex. $A = \{2, 2, 3, 4\}$

☆ $\subset$: Proper set [$A \subset B$: A is a proper subset of B]

Not all elements of B are present in A.

Ex. $A = \{1, 3, 5\}$

$B = \{1, 3, 5, 7\}$

Q. Find.

a. $\{a, b\} \cup \{a, c\}$

→ $\{a, b, c\}$

b. $\{a, b\} \cup \{\{a, b\}\}$

→ $\{a, b, \{a, b\}\}$

[redacted]

c. $\{a,b\} \cup \emptyset$

$\rightarrow$ $\{a,b,\emptyset\}$ ✗ $\quad \{a,b\}$

d. $\{a,b\} \cap \{a,c\}$

$\rightarrow$ $\{a\}$

e. $\{a,b\} \cap \{c,d\}$

$\rightarrow$ $\{\emptyset\}$

f. $\{a,b\} \cap \emptyset$

$\rightarrow$ $\emptyset$

g. $\{a,b,c\} - \{a\}$

$\rightarrow$ $\{b,c\}$

h. $\{a,b,c\} - \{a,d\}$

$\rightarrow$ $\{b,c\}$

i. $\{a,b,c\} - \{d,e,f\}$

$\rightarrow$ $\{a,b,c\}$

j. $\{a,b\} \oplus \{a,c\}$

$\rightarrow$ $\{b,c\}$

k. $\{a,b\} \oplus \{a,b\}$

$\rightarrow$ $\{\}$

l. $\{a,b\} \oplus \emptyset$

$\rightarrow$ $\{a,b\}$

m. $\{a,b\} \oplus \{a,c\} \oplus \{c\}$

$\rightarrow$ $\{b\}$ $\quad [\{b,c\} \oplus \{c\}]$

* $A = \{1, 2, 3\}$

$1 \in A$, $2 \in A$, $3 \in A$, $\{2\} \notin A$

* $\{a, b, \{c, d\}, e\}$

$\{c, d\} \subset A \rightarrow$ False

$\{c, d\} \in A \rightarrow$ True

Q. $A = \{a, b, \{a, c\}, \phi\}$

a. $A - \{a, b\}$

$\rightarrow \{\{a, c\}, \phi\}$

b. $\{a, c\} - A$

$\rightarrow \{c\}$

Q. $A = \{\phi\}$, $B = \{a, \phi, \{\phi\}\}$. Find $A \oplus B$

$\rightarrow A \oplus B = \{a, \{\phi\}\}$

* Complement of a set (A' or $\bar{A}$ or A^c)

$\rightarrow \bar{A} = U - A$

* Cardinality of Finite Set. ($n(A)$ or $|A|$).

$\rightarrow$ Number of distinct element in a set.

* Algebra of Set Operations:

1. Commutativity:

$\rightarrow A \cup B = B \cup A$

$\rightarrow A \cap B = B \cap A$

2. De Morgan's Law.

$\rightarrow \overline{A \cup B} = \bar{A} \cap \bar{B}$

$\rightarrow \overline{A \cap B} = \bar{A} \cup \bar{B}$

3. Associativity:

→ $A \cup (B \cup C) = (A \cup B) \cup C$

→ $A \cap (B \cap C) = (A \cap B) \cap C$

4. Distributivity:

→ $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

→ $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. Idempotent Law:

→ $A \cup A = A$

→ $A \cap A = A$

6. Absorption Law:

→ $A \cup (A \cap B) = A$

→ $A \cap (A \cup B) = A$

7. Double Complement

→ $\overline{\overline{A}} = A$

★ Theorems.

→ For two disjoint finite set:

$|A \cup B| = |A| + |B|$

→ $\boxed{|A - B| = |A| - |A \cap B|}$

★ ★ Mutual Inclusion-Exclusion Principle.

1. For two sets:

→ $|A \cup B| = |A| + |B| - |A \cap B|$

2. For three sets.

→ $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$

★ Single comb. − ~~+~~ Double comb. + Triple comb. − Quad. comb. + ...

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Q. How many positive integers, not exceeding 1000, are divisible by 7 or 11?

→ Let A be a set of integers divisible by 7.

Let B be a set of integers divisible by 11.

$|A| = \cancel{\left(\frac{1000}{7}\right)}$ (int) $\left|\frac{1000}{7}\right| = 142$

$|B| = \left|\frac{1000}{11}\right|$ (int) $= 90$

$|A \cap B| = \left|\frac{1000}{7 \times 11}\right|$ (int) $= 12$

According to inclusion-exclusion principle:

$|A \cup B| = |A| + |B| - |A \cap B|$

$= 142 + 90 - 12$

$\therefore$ $\underline{\underline{|A \cup B| = 220}}$

Q. Among integers 1 to 300, how many are not divisible by 3, nor by 5? Also find how many are divisible by 3 but not by 7.

→ Let A be a set of integers divisible by 3.

Let B be a set of integers divisible by 5.

Let C be a set of integers divisible by 7.

$|A| =$ (int) $\left|\frac{300}{3}\right| = 100$

$|B| =$ (int) $\left|\frac{300}{5}\right| = 60$

~~|AUB|~~ Using inclusion-exclusion principle:

$|A \cup B| = |A| + |B| - |A \cap B|$

$|A \cap B| = \left|\frac{300}{3 \times 5}\right| = 20$

$|A \cup B| = 100 + 60 - \cancel{50}\ 20$

$= \cancel{110}\ \underline{140}$

$|\bar{A} \cap \bar{B}| = |\overline{A \cup B}|$

$|\bar{A} \cap \bar{B}| = 300 - 140$

$= \underline{\underline{160}}$

$|C| = (int) \left| \frac{300}{7} \right| = 42$

$|A \cap C| = (int) \left| \frac{300}{3 \times 7} \right| = 14$

$|A - C| = A - |A \cap C|$

$= 100 - 14$

$\therefore = \underline{\underline{86}}$

Q. Among integers 1 to 1000 :

a. How many of them are not divisible by 3, ~~nor~~ by 5, nor by 7?

b. How many are ~~d~~ not divisible by 5 ~~and~~ or 7, but divisible by 3.

→ A is divisible by 3, B is divisible by 5, C is divisible by 7.

$|A| = (int) \left| \frac{1000}{3} \right| = 333$

$|B| = (int) \left| \frac{1000}{5} \right| = 200$

$|C| = (int) \left| \frac{1000}{7} \right| = 142$

a.

→ $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - \cancel{|A \cap} |B \cap C| + |A \cap B \cap C|$

$|A \cap B| = \left| \frac{1000}{15} \right| = 66$

$|A \cap C| = \left| \frac{1000}{21} \right| = 47$

$|B \cap C| = \left| \frac{1000}{35} \right| = 28$

$|A\cap B\cap C| = \left|\frac{1000}{15\times 7}\right| = 9$

$|A\cup B\cup C| = 333+200+142-66-47-28+9$

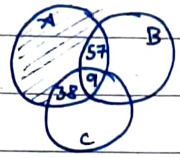
$\therefore$ $|A\cup B\cup C| = \underline{543}$

$|\overline{A\cup B\cup C}| = 1000-543$

$\therefore$ $|\bar{A}\cap\bar{B}\cap\bar{C}| = \underline{\underline{457}}$

b.

→



Only by 3: $|A|-|A\cap B|-|A\cap C|+|A\cap B\cap C|$

$= 333-66-47+9$ or $333-57-38-9$

$\therefore$ $= \underline{\underline{229}}$

Q. An investigator interviewed 100 students. Determine their ~~diff~~ preferences for 3 drinks : milk, coffee and tea. He reported the following:

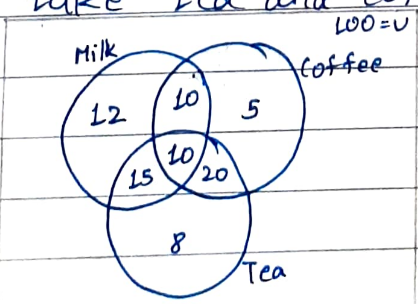
10 students had all 3 drinks, 20 had milk and coffee, 30 had coffee and tea, 25 had milk and tea, 12 had milk only, 5 had coffee only and 8 had tea only.

a. How many did not take any of the 3 drinks?

b. How many take milk, but not coffee?

c. How many take tea and coffee but not milk?

→



a. → $100-(12+10+5+15+10+20+8)$: $100-|M\cup C\cup T|$

$= \underline{\underline{20 \text{ students}}}$

b. → $12 + 15$ $\quad |M - C| = |M| - |M \cap C| = 47 - 20 = 27$

$= \underline{27 \text{ students}}$

c. → ~~5+20+8~~ $\quad$ 20 students $\quad |(T \cap C) - M|$

= ~~33 students~~ $\quad = |T \cap C| - |(T \cap C \cap M)|$

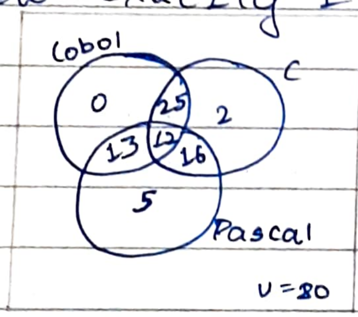
Q. It was found that in the First Year CS of 80 students, 50 know Cobol language, 55 know C, 46 know Pascal. It was also known that 37 ~~kow~~ know C and Cobol, 28 know C and Pascal, 25 know Pascal and Cobol. 7 students however, know none of the languages.

a. How many know all 3 languages?

b. How many know exactly 2 languages?

c. How many know exactly 1 language?

→



Students knowing at least 1 language $= |A \cup B \cup C| = 73$

a. → $|A \cup B \cup C| = |A| + |B| + |C| + |A \cap B \cap C| - |A \cap B| - |B \cap C| - |A \cap C|$

$73 = 50 + 55 + 46 - 37 - 28 - 25 + |A \cap B \cap C|$

$\therefore \underline{|A \cap B \cap C| = 12 \text{ students}}$

b. → $|A \cap B| + |B \cap C| + |A \cap C| - 3|A \cap B \cap C| = 13 + 16 + 25$

$\therefore \quad = \underline{54 \text{ students}}$

c. → $0 + 2 + 5 = \underline{7 \text{ students}}$

Students knowing only Cobol = 0

Students knowing only C = 2

Students knowing only Pascal = 5

★ Logic

1. Proposition
→ A declarative statement that can be true or false.

2. Tautology
→ When all propositions are true.

3. Contradiction.
→ When all propositions are false.

4. Contingency
→ When some propositions are true and some are false

5. Logical Connectives.
→ Disjunction : $p \vee q$. (OR)
→ Conjunction : $p \wedge q$. (AND)
→ Disjunctions and Conjunction Truth table :

p	q	$p \vee q$	$p \wedge q$
T	T	T	T
T	F	T	F
F	T	T	F
F	F	F	F

6. Related Implications
→ $p \to q$: If p, then q or p implies q. (Conditional statement or implication).
p : Antecedent ; q : consequent.
* → ~~p~~ Converse : $p \to q$
* → Contrapositive : neg $q \to$ neg p.
* → Inverse : neg $p \to$ neg q
→ Biconditional: $p \leftrightarrow q$: , p if and only if q or p is necessary and sufficient for q.

→ Implication Truth table:

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

→ Biconditional Truth table:

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

→ Only contrapositive has the same truth values as $p \to q$.

Q. Write inverse, converse and contrapositive for the following statements:

a. if $(3<b)$ and $(1+1)=2$, then $\sin \pi/3 = 1/2$.

→ $p: 3<b$

$q: (1+1)=2$

$r: \sin \pi/3 = 1/2$

$(p \wedge q) \to r$

∴ Converse: $r \to (p \wedge q)$

∴ If $\sin \pi/3 = 1/2$, then $(3<b)$ and $(1+1)=2$

∴ Contrapositive: $\sim r \to \sim(p \wedge q) = \sim r \to \sim p \vee \sim q$

∴ If $\sin \pi/3 \neq 1/2$, then $(3 \geq b)$ ~~and~~ or $(1+1) \neq 2$.

∴ Inverse: ~~$\sim p$~~ $\sim(p \wedge q) \to \sim r = \sim p \vee \sim q \to \sim r$

If $(3>b)$ or $(1+1) \neq 2$, then $\sin \pi/3 \neq 1/2$

Q. With help of truth table, determine which of the following is tautology, contradiction or contingency.

a. $p \to (q \to p)$
b. $(p \wedge q) \to \sim(p \vee q)$
c. $p \wedge \sim(p \vee q) \wedge \sim q$

a. $p \to (q \to p)$

→

p	q	$q \to p$	$p \to (q \to p)$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

∴ ~~$p \to (q \to p)$~~ : Tautology.

b. $(p \wedge q) \to \sim(p \vee q)$

→

p	q	$p \wedge q$	$p \vee q$	$\sim(p \vee q)$	$(p \wedge q) \to \sim(p \vee q)$
T	T	T	T	F	F
T	F	F	T	F	T
F	T	F	T	F	T
F	F	F	F	T	T

∴ Contingency.

c. $(p \wedge \sim(p \vee q)) \wedge \sim q$

→

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim q$	$p \wedge \sim(p \vee q)$	$p \wedge \sim(p \vee q) \wedge \sim q$
T	T	T	F	F	F	F
T	F	T	F	T	F	F
F	T	T	F	F	F	F
F	F	F	T	T	F	F

∴ Contradiction.

☆ Logical Identities

→ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

→ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

→ $\sim(p \vee q) \equiv \sim p \wedge \sim q$

→ $\sim(p \wedge q) \equiv \sim p \vee \sim q$

→ $p \rightarrow q \equiv \sim p \vee q$

Q. Translate the following statements into symbolic form.

a. Atul and Ram going to movie.

→ ∴ $p \wedge q$

p: Atul is going to movie.

q: Ram is going to movie.

b. He ran fast while he went to the ground.

→ p: He ran fast.

q: He went to the ground.

∴ $p \wedge q$

c. It is false that Ragini is not a hard-worker or intelligent.

→ p: Ragini is a hard worker.

q: Ragini is intelligent.

∴ ~~$\sim p$~~ $\sim(\sim p \vee q)$

d. It is not the case that both food is good and the rating is 3-star.

→ p: Food is ~~be~~ good.

p: Rating is 3-star.

∴ $\sim(p \wedge q)$