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Find Fourier integral representation of the following function.

① $f(x) = \begin{cases} 1 & ; x > 0 \\ 0 & ; x < 0 \end{cases}$ $\qquad F(\lambda) = \dfrac{1}{i\lambda}$

② $f(x) = \begin{cases} \cos x & ; x > 0 \\ 0 & ; x < 0 \end{cases}$ $\qquad F(\lambda) = \dfrac{i\lambda}{1-\lambda^2}$

③ $f(x) = \begin{cases} e^{-x} & ; x > 0 \\ 0 & ; x < 0 \end{cases}$ $\qquad F(\lambda) = \dfrac{1-i\lambda}{1+\lambda^2}$

④ $f(x) = \begin{cases} x & ; 0 < x < a \\ 0 & ; x > a \ \& -\infty < x < 0 \end{cases}$ $\qquad F(\lambda) = \dfrac{\sin\lambda}{\lambda} + \dfrac{\cos\lambda - 1}{\lambda^2}$

⑤ $f(x) = \begin{cases} 2 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ $\qquad F_c(\lambda) = \dfrac{4\sin\lambda}{\lambda}$

⑥ $f(x) = \begin{cases} x & ; |x| < 1 \\ 0 & ; \text{otherwise} \end{cases}$ $\qquad F_s(\lambda) = \dfrac{\sin\lambda}{\lambda^2} - \dfrac{\cos\lambda}{\lambda}$

⑦ $f(x) = \begin{cases} 1 & ; |x| < 1 \\ 0 & ; |x| > 1 \end{cases}$ $\qquad F_c(\lambda) = \dfrac{\sin\lambda}{\lambda}$

& hence i) evaluate $\displaystyle\int_0^\infty \frac{\sin\lambda}{\lambda} \cos\lambda x \, d\lambda$ $\qquad$ Ans: $\frac{\pi}{2} f(x)$

ii) value of $\displaystyle\int_0^\infty \frac{\sin\lambda}{\lambda} \, d\lambda$ $\qquad$ Ans: $\frac{\pi}{2}$

⑧ $f(x) = \begin{cases} 1 - x^2 ; & |x| < 1 \\ 0 ; & |x| > 1 \end{cases}$

& hence evaluate $\displaystyle\int_0^\infty \left(\frac{x\cos x - \sin x}{x^3}\right) \cos\frac{x}{2} \, dx$

$F_c(\lambda) = 2\left(\dfrac{\sin\lambda - \lambda\cos\lambda}{\lambda^3}\right)$ $\qquad$ Ans: $\dfrac{-3\pi}{16}$

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(9) Find Fourier sine & cosine transform of $f(x) = e^{-x}$
& hence show that

$$\int_0^\infty \frac{\cos mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m}$$

$$\int_0^\infty \frac{x \sin mx}{1+x^2}\,dx = \frac{\pi}{2} e^{-m}$$

(10) Find the Fourier cosine transform of

$$f(x) = \begin{cases} x ; & 0 \le x \le 1 \\ 2-x ; & 1 \le x \le 2 \\ 0 ; & x > 2 \end{cases}$$

Ans: $F_c(\lambda) = \dfrac{2\cos\lambda(1-\cos\lambda)}{\lambda^2}$

(11) Using Fourier integral representation show that

$$\int_0^\infty \frac{1-\cos\pi\lambda}{\lambda} \sin\lambda x \, d\lambda = \begin{cases} \frac{\pi}{2} ; & 0 < x < \pi \\ 0 ; & x > \pi \end{cases}$$

S.T.

(12) $$\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda = \begin{cases} \frac{\pi}{2} ; & 0 < x < 1 \\ 0 ; & x > 1 \end{cases}$$

(13) $$\frac{1}{2}\int_{-\infty}^{\infty} \left(\frac{1-i\lambda}{1+\lambda^2}\right) e^{i\lambda x}\, d\lambda = \begin{cases} \pi e^{-x} ; & x > 0 \\ 0 ; & x < 0 \end{cases}$$

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Solve the following integral equations:

(1) $\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda} ; \quad \lambda > 0$

Ans: $\frac{2}{\pi} \left(\frac{1}{1+x^2} \right)$

(2) $\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1-\lambda ; & 0 \le \lambda \le 1 \\ 0 ; & \lambda > 1 \end{cases}$

Ans: $\frac{2}{\pi} \left(\frac{1}{x} - \frac{\sin x}{x^2} \right)$

(3) $\int_0^\infty f(x) \cos \lambda x \, dx = \begin{cases} 1-\lambda & ; \quad 0 \le \lambda \le 1 \\ 0 & ; \quad \lambda > 1 \end{cases}$

Ans: $\frac{2}{\pi} \frac{(1-\cos x)}{x^2}$

(4) $\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1 ; & 0 \le \lambda \le 1 \\ 0 ; & \lambda \ge 1 \end{cases}$