

1.13

a) 17 is  $O(1)$

17 is a constant therefore it is  $O(k)$  where  $k \geq 1$

b)  $\frac{n(n-1)}{2}$  is  $O(n^2)$

the highest polynomial is  $n^2$   
therefore it is  $O(n^2)$

c)  $\max(n^3, 10n^2)$  is  $O(n^3)$

$$n^3 \leq cn^3 \text{ where } c \geq 1$$

$$10n^2 \leq cn^3 \text{ where } c \geq 1$$

therefore it is  $O(n^3)$

~~$$n^3 \leq cn^2 \text{ where } c \geq 10$$~~

it is  $O(n^3)$

d)  $\sum_{i=1}^n i^k$  is  $O(n^{k+1})$  and  $\Omega(n^{k+1})$  for  $k$  int

$$T(n) \leq cn^{k+1} \text{ where } c \geq 1$$

when  $i \geq 8$

$$1^k + 2^k + 3^k + 4^k + 5^k + 6^k + 7^k + 8^k \leq c \cdot 8^{k+1}$$

therefore

therefore it is  $O(n^{k+1})$

when  $i \geq 8$

$$1^k + 2^k + 3^k + 4^k + 5^k + 6^k + 7^k + 8^k \geq c \cdot 8^{k+1} \text{ where } c \geq 1$$

therefore it is  $\Omega(n^{k+1})$

e)  $P(x)$   $k^{\text{th}}$  degree polynomial  $c \neq 0$

$$P(x) = ax^k + bx^{k-1} + cx^{k-2} + \dots + d$$

$$P(n) = an^k + bn^{k-1} + cn^{k-2} + \dots + d$$

$$P(n) \leq cn^k \text{ for some value of } c$$

$$an^k + bn^{k-1} + cn^{k-2} + \dots + d \leq cn^k$$

therefore it is  $O(n^k) \Rightarrow O(n^3)$

for some reason

$$an^k + bn^{k-1} + cn^{k-2} + \dots + d \leq cn^k$$

it is  $\Omega(n^k) \Rightarrow \Omega(n^3)$



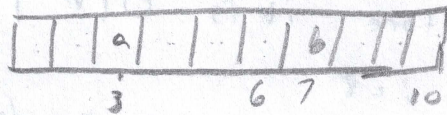
1.16

- a)  $n$   
 b)  $\sqrt{n}$   
 c)  $\log n$   
 d)  $\log \log(n)$   
 e)  $\log^2 n$   
 f)  $\frac{n}{\log n}$   
 g)  $\sqrt{n} \log^2 n$   
 h)  $\left(\frac{1}{3}\right)^n$   
 i)  $\left(\frac{3}{2}\right)^n$   
 j) 17
- Answers:  $\frac{n}{\log n} < \log^2 n < \left(\frac{1}{3}\right)^n < 17 < \log \log(n) < \log n < n^{1/2} < n < \left(\frac{3}{2}\right)^n$

1.18)  $\max(i, n)$   
 returns largest element in position  $i$  through  $i+n-1$  of an int array  
 $n$  is a power of 2

$\max(3, 8)$

$3, 3+8-1$



$m1 = \max(3, 4) \Rightarrow (3, 6) \Rightarrow \max(3, 2) \Rightarrow \max(3, 1)$   
 $m2 = \max(\underbrace{3+4}_7, 4) \Rightarrow 7, 10 \Rightarrow \max(7, 2) \Rightarrow \max(7, 1)$

$T(n), T(b)$

$T(n) = \max(i, n)$

$T(n) = c$  (index at  $A[i]$ )

$T(2) = T(n/2) + T(n/2) + b$

$= 2T(n/2) + b$

$T(n) = 2T(n/2) + b$

$T(n) = nc + \frac{n}{2}b \geq kn$   
 $\Omega(b)$   
 $T(n) = nc + \frac{n}{2}b \geq kn$

$\Theta(n)$

$T(n) = c$

$T(2) = 2T(1) + b$

$= 2c + b$

$T(4) = 4c + 2b$

$T(8) = 8c + 4b$

is  $O(n)$  and  $\Omega(n)$



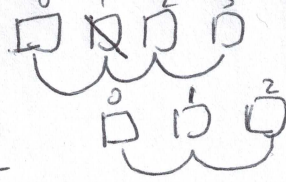
2.9)

while  $p \neq \text{end}(L)$

if ~~element~~  $\text{element}(p) = x$

delete( $p, L$ )

$p := \text{next}(p, L)$



The following ~~procedure~~ procedure skips the next node if it finds a match. For example if position  $p$  and  $p+1$  has  $x$  it will only remove  $x$  from position  $p$

to fix this

if ~~DELETE~~  $\text{DELETE}(p, L) = x$  then

~~DELETE~~  $(p, L)$

else

$p := \text{NEXT}(p, L)$

2.11)  $L$  is a list

$p, q, r$  positions

$n \rightarrow$  length of the list

$p := \text{first}(L) \Rightarrow 1$

while  $p \neq \text{END}(L)$  do begin

$q := p$

while  $q \neq \text{END}(L)$  do begin

$q := \text{next}(q, L)$

$r := \text{first}(L)$

while  $r \neq q$  do

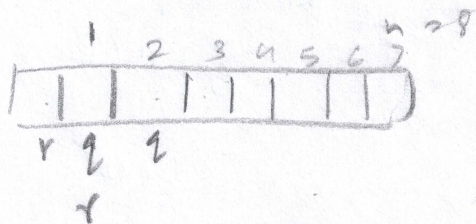
$r := \text{next}(r, L)$

end

end

$p := \text{next}(p, L)$

end



$\text{END}$  will execute  $\approx 2n$

~~first and next will~~

first will  $\approx n^2$

next will  $\approx n^3$