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UG

CIE QUESTION PAPER

Global Academy of Technology, Bengaluru

Department of Electronics and Communication Engineering

First Internal Assessment – NOV 2024		Semester	V						
Subject Name	Digital Communication Systems	Subject Code	2	2	E	C	E	5	2
Time: 75 mins	*Note:* Answer any THREE Full Questions selecting TWO Full Questions from MODULE - 1 and ONE Full Question from MODULE – 2.	Max. Marks: 40							

Q. No.	MODULE-1	CO's	Marks
1a)	Define Hilbert Transform. List and Prove the Properties of the Hilbert Transform.	CO1	9
b)	Determine the Hilbert Transform of $g(t) = m(t)\sin(2\pi f_c t)$.		4
2a)	Discuss Complex envelope of band pass signal. Draw its spectra.		5
b)	Explain the schemes used for extracting the in-phase and quadrature components of the band pass signal s(t) and construction of band pass signal s(t) from in-phase and quadrature components.		8
3a)	For the binary stream 10010100 sketch the following line codes: i) Unipolar NRZ ii) Unipolar RZ iii) Bipolar RZ iv) Manchester		8
b)	Determine the pre-envelope and complex envelope of the RF pulse defined by $x(t)=A\,\mathrm{rect}(\frac{t}{T})\cos(2\pi f_c t)$.		5
	MODULE-2		
4a)	Explain geometric representation of set of M energy signals as linear combination of N orthonormal basis functions. Illustrate for case N=2 and M=3 with necessary diagrams and expressions.	CO2	8
b)	Show that the energy of a signal is equal to the squared length of the signal vector.		6
5)	Using the Gram Schmidt Orthogonalization procedure, find a set of orthonormal basis functions to represent the three signals $S_1(t)$, $S_2(t)$ and $S_3(t)$ shown in Figure Q5 below. Also express each of these signals in terms of the set of basis functions.		14

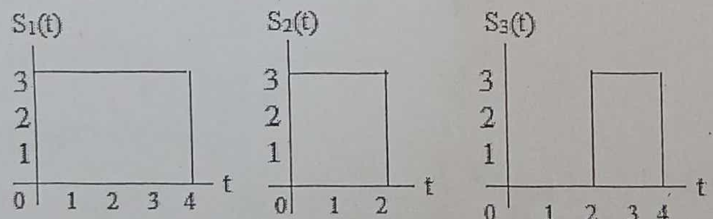


Figure Q5

Course Outcomes: At the end of the course the students are able to

CO1	Represent the signals in various forms.
CO2	Analyze Signals over AWGN channel.
CO3	Generate and detect various Digital Modulation techniques.
CO4	Compute performance parameters of band limited channels.
CO5	Explain the concept of Spread spectrum communication system.

21/11/24

12

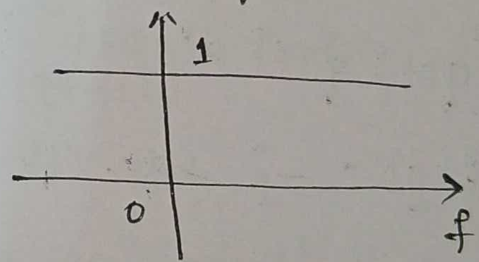
21/11/23

22ECE52

Global Academy of Technology, Bengaluru-98

Scheme & Solution

Sub with code: Digital communication system Dept: EC and E IA: I/II/III

Solution	Marks Allocated
Definition: Phase selectivity uses shift between pertinent signals to achive desire separation i.e phase angles of all frequency contents of given signals are shifted by $\mp 90°$. The resulting function of time is Hilbert Transform of signal. It is also called as quadrature filter. $$\hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{g(\tau)}{t-\tau}\,d\tau$$	-2-
Properties 1. Signal $g(t)$ and its H.T $\hat{g}(t)$ have same magnitude spectrum. $\lvert H(f)\rvert = \lvert -j\,sgn(f)\rvert$  $\lvert G(f)\rvert = \lvert\hat{G}(f)\rvert$ W.K.T $\hat{G}(f) = -j\,sgn(f)\,G(f)$ $\lvert\hat{G}(f)\rvert = \lvert -j\,sgn(f)\,G(f)\rvert$ $\quad\because \lvert -j\,sgn(f)\rvert = 1$ $\lvert\hat{G}(f)\rvert = \lvert G(f)\rvert$	-2-
2. If $\hat{g}(t)$ is H.T of $g(t)$ then H.T of $\hat{g}(t)$ is $-g(t)$ $g(t) \rightarrow \boxed{H.T} \rightarrow \boxed{H.T} \rightarrow -g(t)$ $H.T \times H.T \Rightarrow -j\,sgn(f) \cdot -j\,sgn(f)$ $= j^2\,sgn^2(f)$ $\quad \because sgn^2(f) = 1,\ j^2 = -1$ $= -1$	-2-

Subject Title: Subject code:

Solution

Q No.

Proof $\hat{\hat{g}}(f) = -j\,\mathrm{sgn}(f) \cdot \hat{G}(f)$

$= -j\,\mathrm{sgn}(f)\,[-j\,\mathrm{sgn}(f) \cdot G(f)]$

$= j^2\,\mathrm{sgn}^2(f) \cdot G(f)$

$\hat{\hat{G}}(f) = -G(f)$

$\hat{\hat{G}}(f) \rightarrow \boxed{IFT} \rightarrow -g(t)$

IFT of $\hat{\hat{G}}(f)$ is $-g(t)$

3. signal $g(t)$ and its HT $\hat{g}(t)$ are orthogonal to each other over entire time interval $(-\infty, \infty)$

i.e $\int_{-\infty}^{\infty} g(t)\,\hat{g}(t)\,dt = 0$ -3-

$\int_{-\infty}^{\infty} g(t)\,\hat{g}(t)\,dt = \int_{-\infty}^{\infty} G(f) \cdot \hat{G}(f)\,df$

$\hat{G}(f) = -j\,\mathrm{sgn}(f)\,G(f)$

$\hat{G}(-f) = -j\,\mathrm{sgn}(-f)\,G(-f)$

$= j\,\mathrm{sgn}(f)\,G(f)$

$\int_{-\infty}^{\infty} g(t)\,\hat{g}(t)\,dt = \int_{-\infty}^{\infty} G(f)\,j\,\mathrm{sgn}(f)\,G(-f)\,df$

$= \int_{-\infty}^{\infty} j\,\mathrm{sgn}(f)\,|G(f)|^2\,df$

$= \int_{-\infty}^{\infty} j\,\mathrm{sgn}(f)\,|G(f)|^2\,df$

$\mathrm{sgn}(f) \rightarrow$ odd function

$|G(f)|^2 \rightarrow$ even function

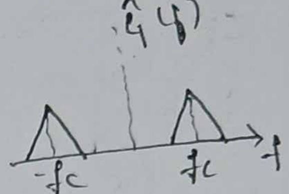
Integration of an odd function over the range $-\infty$ to ∞ will be <u>zero</u>

$$\therefore \int_{-\infty}^{\infty} g(t) \cdot \hat{g}(t)\, dt = 0$$

1 b.

$$g(t) = m(t) \sin 2\pi f_c t$$

$$G(f) = \frac{1}{2j}\left[M(f-f_c) - M(f+f_c)\right]$$ — 2 —



W.K.T spectrum of H.T is

$$\hat{G}(f) = -j\, \mathrm{sgn}(f)\, G(f)$$

$$= -j\, \mathrm{sgn}(f) \left\{ \frac{1}{2j}\left[M(f-f_c) - M(f+f_c)\right]\right\}$$

$$= -\frac{1}{2}\left[\mathrm{sgn}(f)\, M(f-f_c) - \mathrm{sgn}(f)\, M(f+f_c)\right]$$

$$= -\frac{1}{2}\left[(1)\, M(f-f_c) - (-1)\, M(f+f_c)\right]$$

$$= -\frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$

= ~~H cos m(t) cos~~

Taking IFT of the above equation we get — 2 —

$$\hat{g}(t) = -m(t) \cos 2\pi f_c t$$

$$\therefore HT(g(t)) = \hat{g}(t) = HT[m(t) \sin 2\pi f_c t]$$

$$\hat{g}(t) = -m(t) \cos 2\pi f_c t$$

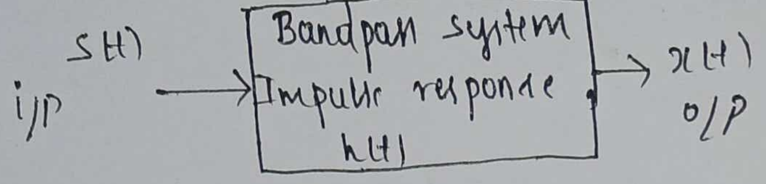
2 a. Complex envelope of bandpass signal

$s(t) \rightarrow$ modulated signal (Narrowband signal)

signal modulated on high frequency carrier signal. signal bandwidth restricted to band pass system

Subject Title: Subject code:

Q No.	Solution



$s(t) \xrightarrow{FT} s(f)$

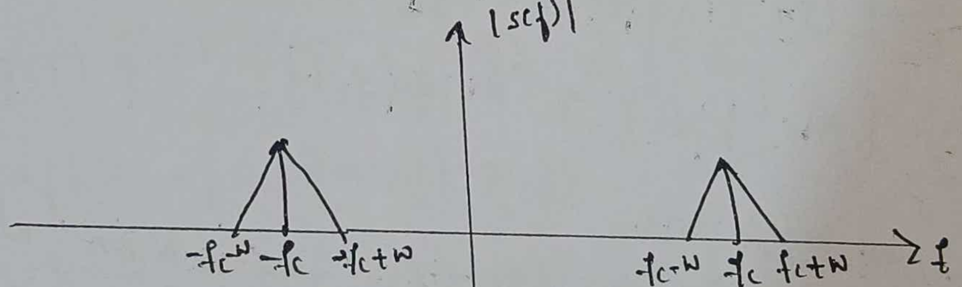
B.W of $s(f) = \underline{2W}$ centered around frequency $\pm f_c$

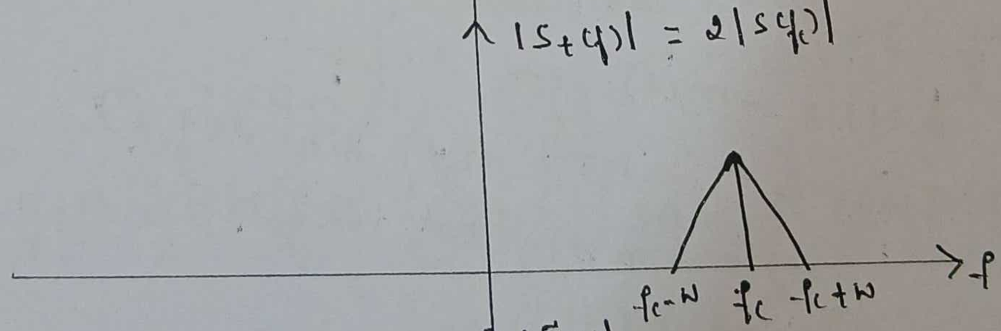
Pre envelope

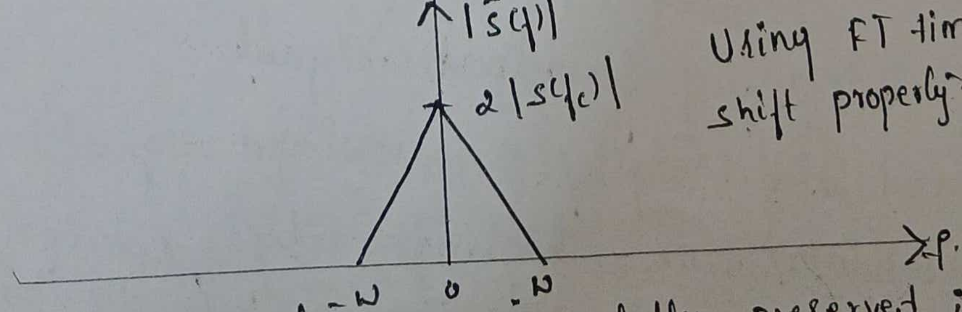
$s_+(t) = \tilde{s}(t)\exp(j2\pi f_c t)$

$\tilde{s}(t) \rightarrow$ complex envelope of BP signal $s(t)$

$s_+(t) \rightarrow$ pre envelope for positive frequency band







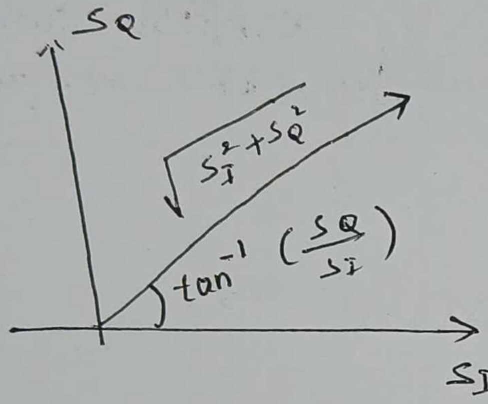
Using FT time shift property.

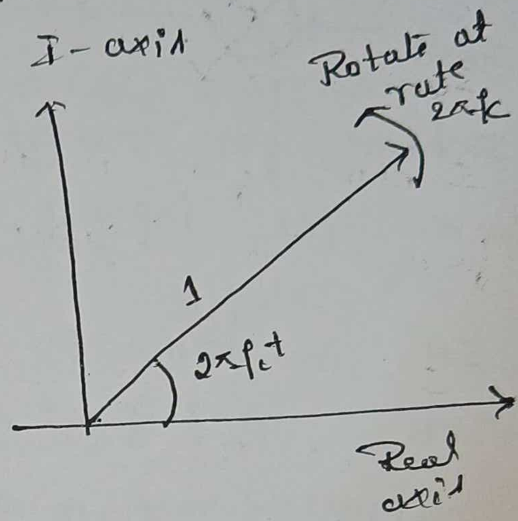
Modulated signal $s(f)$ is fully preserved in the complex envelope $\tilde{s}(f)$.

Analysis of bandpass signal s(t) is complicated by the presence of the carrier frequency 'f_c'

* complex envelope $\tilde{s}(t)$ dispenses with f_c making its analysis simpler to deal with

2 b





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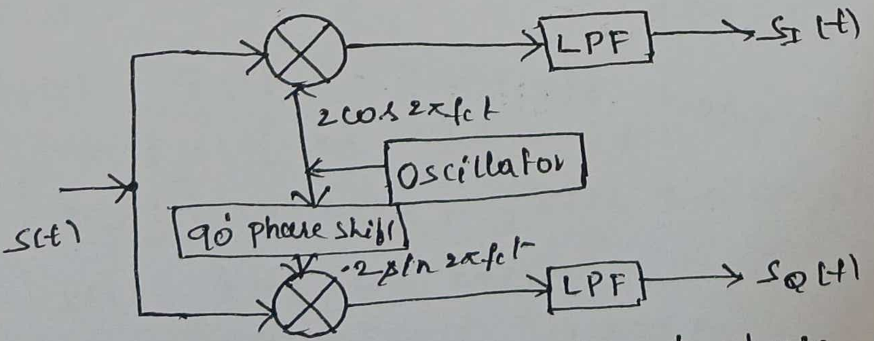


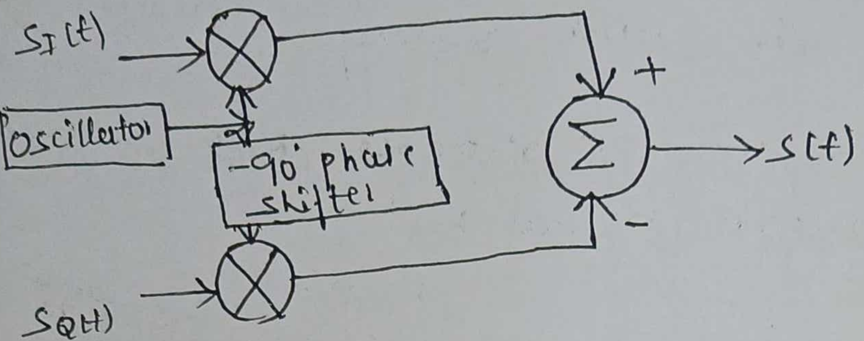
2

Scheme for derive the inphase and quadrature phase components

Subject Title: Subject code:

Solution

Q No.	



scheme for reconstruction of bandpass signal from its inphase and quadrature phase component

$s(t)$ in terms of $\tilde{s}(t)$

$s(t) = R_e\{\tilde{s}(t)\exp(j2\pi f_c t)\} \longrightarrow 1$

cartesian form of $\tilde{s}(t)$

$\tilde{s}(t) = s_I(t) + j s_Q(t) \longrightarrow 2$

equation 2 in 1

$s(t) = [s_I(t) + j s_Q(t)]\, e^{j2\pi f_c t}$

$= [s_I(t) + j s_Q(t)][\cos 2\pi f_c t + j \sin 2\pi f_c t]$

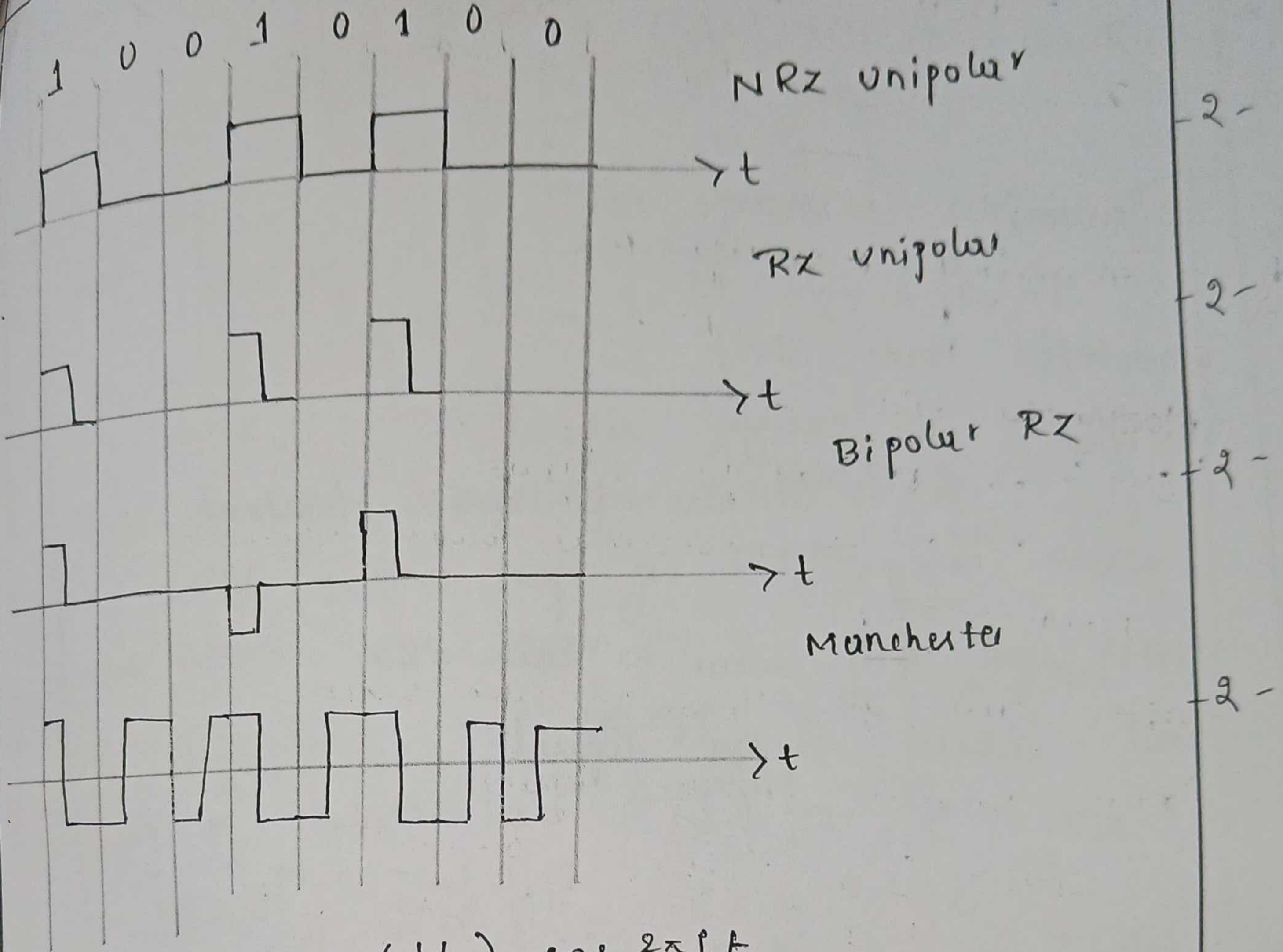
$= s_I(t)\cos 2\pi f_c t + j \underbrace{s_I(t)\sin 2\pi f_c t}_{I-part} + \underbrace{j s_Q(t)\cos 2\pi f_c t}_{I-part}$ $\quad j^2 = -1$

$+ j^2 s_Q(t)\sin 2\pi f_c t$

$= s_I(t)\cos 2\pi f_c t - s_Q(t)\sin 2\pi f_c t.$

Title: Digital communication system | Subject code: 22ECE52

Solution	Marks Allocated



3b

$x(t) = A\, \mathrm{rect}\,(t/T) \cos 2\pi f_c t$

$x_+(t) = x(t) + j\hat{x}(t) \longrightarrow ①$

$x_+(t) = \tilde{x}(t)\, e^{j2\pi f_c t}$

$= \tilde{x}(t)\,[\cos 2\pi f_c t + j \sin 2\pi f_c t]$

$= \tilde{x}(t) \cos 2\pi f_c t + j\tilde{x}(t) \sin 2\pi f_c t \longrightarrow ②$ — 2

$x(t) = \tilde{x}(t) \cos 2\pi f_c t$

$= A\, \mathrm{rect}\,[t/T] \cos 2\pi f_c t$

$\tilde{x}(t) = A\, \mathrm{rect}\,(t/T) \longrightarrow ③$

$\hat{x}(t) = A\, \mathrm{rect}\,[t/T] \sin 2\pi f_c t \longrightarrow (4)$

$x_+(t) = A\, \mathrm{rect}\,[t/T] \cos 2\pi f_c t + j A\, \mathrm{rect}\,[t/T] \sin 2\pi f_c t$ — 2

Subject Title:

Subject code:

Q No.	Solution

$$= A \, \text{rect}[t/T] \{ \cos 2\pi f_c t + j \sin 2\pi f_c t \}$$

$$x_+(t) = A \, \text{rect}[t/T] \, e^{j 2\pi f_c t}$$

4 a. Incoming Message m_i

$i = 1, 2, \ldots M;$

Modulated wave $s_i(t)$

$\{s_i(t)\} \rightarrow M$ energy signal

$N \rightarrow$ orthonormal basis function

Where $\underline{\underline{N \leq M}}$

$s_i(t)$ and $\phi_i(t)$ linear to each other

Real valued energy signal

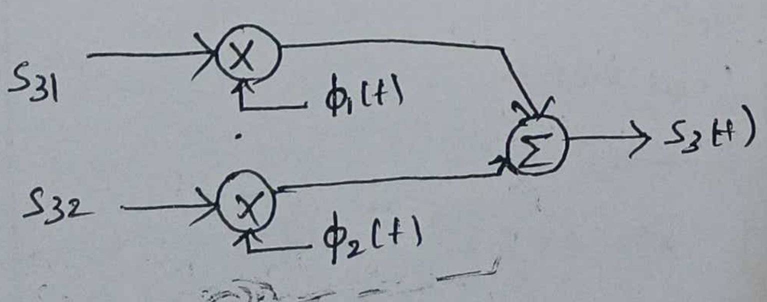
$$s_i(t) = \sum_{j=1}^{N} s_{ij} \phi_j(t) \qquad \begin{array}{l} 0 \leq t \leq T \\ i = 1, 2, \ldots M \\ j = 1, 2, \ldots N \end{array}$$ -2-

$s_{ij} \rightarrow$ coefficient

$$s_{ij} = \int_0^T s_i(t) \phi_j(t) \, dt$$ -2-

$$\int_0^T \phi_i(t) \phi_j(t) \, dt = \begin{cases} 1 \rightarrow i = j \\ 0 \rightarrow i \neq j \end{cases}$$ -1-

$M = 3 \quad N = 2$



$$= \sum_{j=1}^{2} s_{3j} \phi_j(t)$$ -2-

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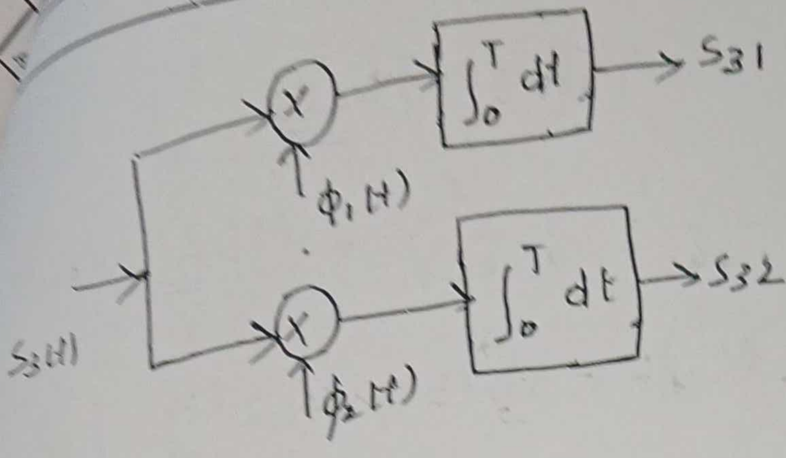
Scheme & Solution

Subject: Digital communication sy^n Dept: ECE IA: I/II/III

Solution	Marks Allocated
	2

There is an interesting relationship between the energy content of a signal and its representations are vector.

By definition, the energy of a signal $s_i(t)$ of duration T seconds is

$$E_i = \int_0^T s_i^2(t)\, dt \qquad i = 1, 2, \ldots M$$

$$E_i = \int_0^T \sum_{j=1}^{N} [s_{ij}\, \phi_j(t)] \left[\sum_{k=1}^{N} s_{ik}\, \phi_k(t)\right] dt$$

$$E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\, s_{ik} \int_0^T \phi_j(t)\, \phi_k(t)\, dt$$

$$\therefore E_i = \sum_{j=1}^{N} \sum_{k=1}^{N} s_{ij}\, s_{ik} = \sum_{j=1}^{N} s_{ij}^2 \rightarrow$$

$$= \|s_i\|^2$$

Subject code:

Solution	Ma Alloca

$s_1(t) = 3 \quad 0 \le t \le 4.$

$E_1 = \int_0^4 (3)^2 dt = 9[4] = 36 J$

$\phi_1(t) = \frac{s_1(t)}{\sqrt{E_1}} = \frac{3}{\sqrt{36}} = \frac{3}{6} = 0.5. \quad 0 \le t \le 4$

$s_{11} = \int_0^4 s_1(t)\,\phi_1(t)\,dt = \int_0^4 3. \times 0.5\, dt$

$= 1.5[4] = 6$

$\boxed{s_{11} = 6}$

$s_2(t) = 3 \;; \quad 0 \le t \le 2$

$E_2 = \int_0^2 3^2 dt = 9[2] = 18$

$g_2(t) = s_2(t) - s_{21}\,\phi_1(t)$

$s_{21} = \int_0^T s_2(t)\,\phi_1(t)\,dt$

$= \int_0^2 3 \times 0.5\, dt$

$= 1.5[2]$

$= 3$

$g_2(t) = 3 - 1.5 = 1.5 \quad 0 \le t \le 2$

$\quad\quad\;\; 0 - 1.5 = -1.5 \quad 2 \le t \le 4$

$E_{g_2} = E_2 - s_{21}^2 = 18 - 9 = 9$

$E_{g_2} = \int_0^2 (1.5)^2\, dt + \int_2^4 (1.5)^2\, dt$

$= (1.5)^2 \times 2 + (1.5)^2 \times 2 = 9$

Subject code:

DC

Solution	Marks Allocated

$$\phi_2(t) = \frac{g_2(t)}{\sqrt{E_{g_2}}} = \frac{g_2(t)}{\sqrt{9}} ; \quad 0 \le t \le 4$$

$$= \begin{cases} \frac{1.5}{3} & 0 \le t \le 2 \\ -\frac{1.5}{3} & 2 \le t \le 4 \end{cases}$$

$$S_{22} = \int_0^2 3 \times \frac{1.5}{3} dt$$

$$= 1.5 [2]$$

$$S_{22} = 3$$

$$\phi_2(t) = 3\phi_1(t) + 3\phi_2(t)$$

$$s_3(t) = 3 \quad 2 \le t \le 4$$

$$q_3(t) = s_3(t) - S_{31}\phi_1(t) - S_{32}\phi_2(t)$$

$$q_3(t) = s_3(t) - S_{31}\phi_1(t) - S_{32}\phi_2(t)$$

$$S_{31} = \int_2^4 3 \times 0.5\, dt = 1.5 \times 2 = 3$$

$$S_{32} = \int_2^4 3 \times -\frac{1.5}{3} dt = (-1.5)(2)$$

$$\boxed{S_{32} = -3}$$

Subject code:

Subject Title:

Q No.	Solution	Marks Allocated
	$q_3(t) = s_3(t) - 3\phi_1(t) + 3\phi_2(t)$ $= 0 - 3\times 0.5 + \cancel{3} \times \frac{1.5}{\cancel{3}}$ $= -1.5 + 1.5 = 0 \quad 0 \le t \le 2$ $q_3(t) = 3 - 1.5 + \cancel{3}\times\left(\frac{-1.5}{\cancel{3}}\right) \quad 2 \le t \le 4$ $q_3(t) = 3 - 3 = 0 \quad \phi_3(t) = 0$ $s_{33} = 0$ $s_3(t) = s_{31}\phi_1(t) + s_{32}\phi_2(t)$ $= 3\times 0.5 + (-\cancel{3})\left[\frac{1.5}{\cancel{3}}\right]$ $= 0 \quad 0 < t < 2$ $= 3\times 0.5 + (-\cancel{3})\times\frac{-1.5}{\cancel{3}}$ $= 1.5 + 1.5 = 3 \quad 2 < t < 4$	