COMPUTER VISION
ASSIGNMENT-3

Q1) 2D homography.
The Planar homography relates the transformation between 2 plains.

$$s\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = H \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

So lets consider a matrix

$$\begin{bmatrix} x \\ y \\ 0 \\ w \end{bmatrix} = \underbrace{\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix}}_{H} \begin{bmatrix} x \\ y \\ 0 \\ w \end{bmatrix} \rightarrow \text{z is zero as there is no hight}$$

4×4 4×1

∴ we get

$$\begin{bmatrix} x_2 \\ y_2 \\ w_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ w_1 \end{bmatrix}$$

$$\begin{bmatrix} x_2' \\ y_2' \\ 1 \end{bmatrix} =$$

by normilazation: $x_2' = \dfrac{x_2}{w_2} = \dfrac{H_{11}x_1 + H_{12}y_1 + H_{13}w_1}{H_{31}x_1 + H_{32}y_1 + H_{33}w_1}$

$\therefore y_2' = \dfrac{y_2}{w_2} = \dfrac{H_{21}x_1 + H_{22}y_1 + H_{23}w_1}{H_{31}x_1 + H_{32}y_1 + H_{33}w_1}$

$$x_2'\{x_1 H_{31} + x_2 H_{31} y_1 + x_2 H_{33}\} - H_{11} x_1 - H_{12} y_1 - H_{13} w_1 = 0$$

$$y_2'\{y_2' H_{31} x_1 + y_2' y_1 H_{31} + y_2' H_{33}\} - H_{21} x_1 - H_{22} y_1 - H_{23} = 0$$

$$w = 1$$

dest

$$\begin{bmatrix} -x_1 & -y_1 & -w_1 & 0 & 0 & 0 & x_1 x_1' & y_1 x_1' & x_1' \\ 0 & 0 & 0 & -x_1 & -y_1 & -1 & x_1 y_1' & y_1' y_1 & +y_1' \\ -x_2 & -y_2 & -1 & 0 & 0 & 0 & x_2 x_2' & y_2 x_2' & x_2' \\ 0 & 0 & 0 & -x_2 & -y_2 & -1 & x_2 y_2' & y_2 y_2' & y_2' \\ -x_3 & -y_3 & 1 & 0 & 0 & 0 & x_3 x_3' & y_3 x_3' & x_3' \\ 0 & 0 & 0 & -x_3 & -y_3 & -1 & x_3 y_3' & y_2 y_3' & y_3' \\ -x_4 & -y_4 & -1 & 0 & 0 & 0 & x_4 x_4' & y_4 x_4' & x_4' \\ 0 & 0 & 0 & -x_4 & -y_4 & -1 & x_4 y_4' & y_4 y_4' & y_4' \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = 0$$

b) $(x_i', y_i') \longleftrightarrow (x_i, y_i)$

$(5,4) \longleftrightarrow (0,0)$

$(7,4) \longleftrightarrow (1,0)$

$(7,5) \longleftrightarrow (0,1)$

$(6,6) \longleftrightarrow (1,1)$

Creation of H matrix

$$\begin{bmatrix} 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 4 \\ -1 & 0 & -1 & 0 & 0 & 0 & 7 & 0 & 7 \\ 0 & 0 & 0 & -1 & 0 & -1 & 4 & 0 & 4 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 7 & 7 \\ 0 & 0 & 0 & 0 & -1 & -1 & 0 & 5 & 5 \\ -1 & -1 & -1 & 0 & 0 & 0 & 6 & 6 & 6 \\ 0 & 0 & 0 & -1 & -1 & -1 & 6 & 6 & 6 \end{bmatrix} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{13} \\ H_{21} \\ H_{22} \\ H_{23} \\ H_{31} \\ H_{32} \\ H_{33} \end{bmatrix} = 0$$

Using the code we could find the eigen value and eigen vectors

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{bmatrix} = \begin{bmatrix} 16 & 7 & 5 \\ 8 & 8 & 4 \\ 2 & 1 & 1 \end{bmatrix}$$

The code used to determine the eigen value is pasted in the following page.

```python
def compute_homography(src, dst):
  Q = np.empty([src.shape[0]*2,9])
  for i in range(src.shape[0]):
    row = np.array([-src[i,0],-src[i,1],-1,0,0,0,src[i,0]*dst[i,0],src[i,1]*dst[i,0],dst[i,0]])
    Q[i*2,0:9] = row
    row = np.array([0,0,0,-src[i,0],-src[i,1],-1,src[i,0]*dst[i,1],src[i,1]*dst[i,1],dst[i,1]])
    Q[(i*2)+1,0:9] = row
  u, s, v = np.linalg.svd(Q)
  H = np.reshape(v[8],(3,3))
  H = (1/H.item(8)) * H
  return H
def apply_homography(src, H):
  dst = np.zeros([src.shape[0], 2])
  for i in range(src.shape[0]):
    sour = np.reshape([src[i,0],src[i,1],1],(3,1))
    mul = np.dot(H,sour)
    mul = (1/mul.item(2))*mul
    dst[i,0]= mul[0,0]
    dst[i,1]=mul[1,0]
  return dst
def test_homography():
  src_pts = np.matrix('0, 0; 1, 0; 1, 1; 0, 1')
  dst_pts = np.matrix('5, 4; 7, 4; 7, 5; 6, 6')
  H = compute_homography(src_pts, dst_pts)
  print(H)

test_homography()
```

```
[[16.  7.  5.]
 [ 8.  8.  4.]
 [ 2.  1.  1.]]
```