COMPUTER VISION
ASSIGNMENT II

1. 2D Point (x,y) described by.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = R \begin{bmatrix} x \\ y \end{bmatrix} \quad \text{where } R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

x' and y' are the points after Rotation
& the θ is the counterclockwise direction.

i) To rotate points other than the origin (0,0
we need to move the whole matrix to the point of rotation
ii) Perform Rotation.
iii) move back to the Origin (0,0).

i) $x = x - x_0 \qquad y = y - y_0$

ii) $x' = (x - x_0)\cos\theta - (y - y_0)\sin\theta$.

$y' = (x - x_0)\sin\theta + (y - y_0)\cos\theta$

iii) $x' = x' + x_0$

$y' = y' + y_0$

1 b $\theta = 15^\circ \qquad (x_0, y_0) = (200, 150) \qquad (x, y) = (0, 0)$

$x' = \cancel{\cos} (0-200)\cos 15^\circ - (0-150)\sin 15^\circ$

$y' = (0-200)\sin 15^\circ + (0-150)\sin 15^\circ$

$x' = -193.18 + 38.82 = -154.36$
$y = -51.76 - 144.89 = -196.65$

$x' = -154.36 + 200 = 45.64$
$y' = -196.65 + 150 = -46.65$

$(x', y') = (45.64, -46.65)$

ii) $(x, y) = (200, 150)$

$x' = (200-200)\cos 15^\circ - (150-150)\sin 15^\circ$
$= 0$

$y' = (200-200)\sin 15 + (150-150)\cos 15^\circ$
$= 0$

$x' = 0 + 200 = 200$
$y' = 0 + 150 = 150$
$(x', y') = (200, 150)$

iii) $(x, y) = (210, 150)$

$x' = (210-200)\cos 15 - (150-150)\sin 15$
$= 9.66$

$y' = (210-200)\sin 15 + (150-150)\cos 15$
$= 2.59$

$x' = 9.66 + 200 = 209.66$
$y' = 2.59 + 150 = 152.59$
$(x', y') = (209.66, 152.59)$

$(x, y) = (400, 300)$

$x' = (400-200)\cos 15° - (300-150)\sin 15°$

$= 154.3$

$y' = (400-200)\sin 15° + (300-150)\cos 15°$

$= 196.65$

$x' = 154.3 + 200 = 354.3$

$y' = 196.65 + 150 = 346.65$

$(x', y') = (354.3, 346.65)$.

(2) $g = \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix}$ $h = \begin{bmatrix} \boxed{1} & 3 \\ 2 & 4 \end{bmatrix}$

$(I * g) * h \Rightarrow I * f$

where $f = g * h$.

$$f = \begin{bmatrix} -1 - 12 - 4 + 20 & -4 + 21 + 10 + 32 & 7 + 16 \\ -2 + 15 + 0 + 24 & +5 + 24 + 12 + 0 & 8 + 0 \\ 0 + 18 & 6 + 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 59 & 23 \\ 37 & 41 & 8 \\ 18 & 6 & 0 \end{bmatrix}$$

Problem3

Approximate using Laplacian operation.

Laplacian of Gaussian, the second order filter.

$$\nabla^2 = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}$$

First order detects the edge at local maxima and minima while laplacian detects at zero-crossing

In x direction

$$\frac{\partial f}{\partial x} = f(x+1,y) - f(x,y)$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1,y) + f(x-1),y) - 2f(x,y)$$

Simillaarly in y direction.

$$\frac{\partial f}{\partial y} = f(x,y-1) - f(x,y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x,y+1) + f(x,y-1) - 2f(x,y)$$

Adding $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = f(x,y-1) + f(x,y+1) + f(x-1,y)$
$f(x+1,y) - 4f(x,y)$.

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Considering the previous proof as a diagnoal form we get

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

This is a stable Approximation the ∇^2

we can rewrite as

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & -8 & 1 \\ 1 & 1 & 1 \end{bmatrix} + 3 \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

So we can assume its a reasonable approximation.

(4) To prove $f(x+u, y+v) \approx f(x,y) + \nabla f(x,y) \cdot \begin{bmatrix} u \\ v \end{bmatrix}$

$f(x+u, y+v)$ can be written as the following

$$f(x+u, y+v) = f(x,y) + f_x(x,y).(x+u-x)$$
$$+ f_y(x,y)(y+v-y) + f_{xy}(x,y)(y+v-y)(x+u-x)$$
$$+ \frac{f_{xx}}{2}(x,y)(x+u-x)^2 + \frac{f_{yy}}{2}(x,y)(y-v-y)^2$$

$$= f(x,y) + f_x(x,y).u + f_y(x,y) \cdot v$$
$$+ f_{xy}(x,y)\, u.v + \frac{f_{xx}}{2}(x,y)\, u^2 + \frac{f_{yy}}{2}(x,y) v^2$$

as u & v are small values

$u^2 \approx 0 \quad v^2 \approx 0 \quad u.v \approx 0$

$$\therefore \quad f(x+u, y+v) = f(x,y) + f_x(x,y) \cdot u + f_y(x,y) \cdot v$$
$$= f(x,y) + \nabla f(x,y) \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$