ECE 4554 / 5554: Computer Vision: Homework 3

Fall 2021

Instructions

- The assignment is due at Canvas on Nov. 6 before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after midnight will cost an additional token.
- Please review the Honor Code statement in the syllabus. This is an "individual" assignment, not a "team" assignment. The work that you submit for a grade must be your own.
- This assignment has a slightly different format than previous assignments.
  - Problems 1 through 3 are analytical in nature, and are worth 10 points each.
  - Problem 4 is a programming assignment, and is worth 20 points.
  - Problem 5 requires programming and discussion. It is worth 10 points, and is required for 5554 students but is optional (extra credit) for 4554 students.
- For the first 3 problems, please submit separate PDF files that contain your answers. Use the following file names, but with your own VT Username: `Homework3_P1_USERNAME.pdf`, `Homework3_P2_USERNAME.pdf`, and `Homework3_P3_USERNAME.pdf`. Handwritten solutions are permitted, but they must be easily legible to the grader.
- Problems 4 and 5 should be submitted together as one Jupyter notebook file. Details are provided at the end of this assignment. For any coding problem, the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for the coding problems.
- After you have submitted to Canvas, please take to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. (10 points.) We have discussed the 2D *planar perspective transformation*, also known as 2D *homography*, which maps a point (x, y) to a new location (x', y') in the plane. This transformation can be represented using the following equation.

$$\begin{bmatrix} s\,x' \\ s\,y' \\ s \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{1}$$

When using homogeneous coordinate representation as shown here, recall that s is simply a scalar term that is to be eliminated when solving for (x', y'). As discussed in the textbook near equations (2.20)-(2.21), there are only 8 degrees of freedom in this equation. For this reason, many formulations set a_9 to 1. It may help you when working on coding problems later if you do <u>not</u> constrain a_9 to be 1.

a) Consider the case that you are given n corresponding pairs of points, where $n \geq 4$, and you want to use those points to determine the parameters a_1 through a_9. For example, assume that the following correspondences are known:

$$\begin{aligned} (x_1', y_1') &\leftrightarrow (x_1, y_1) \\ (x_2', y_2') &\leftrightarrow (x_2, y_2) \\ (x_3', y_3') &\leftrightarrow (x_3, y_3) \\ (x_4', y_4') &\leftrightarrow (x_4, y_4) \\ &\;\;\ldots \\ (x_n', y_n') &\leftrightarrow (x_n, y_n) \end{aligned}$$

Show how to derive one matrix equation that represents the relationship between these all of these scalar values (not including s). The form of the equation should be $\boldsymbol{Q}\,\boldsymbol{a} = \boldsymbol{0}$, where $\boldsymbol{a}$ is a 9x1 vector that contains the individual homography parameters only; $\boldsymbol{Q}$ is a matrix of size $2n$x9 that you specify containing known values; and $\boldsymbol{0}$ represents the $2n$x1 vector containing only values of 0. For this part of the problem, you do not need to solve for the parameter vector $\boldsymbol{a}$. *Hint*: you may find some inspiration in the derivation near the end of packet 5, although those lecture slides are discussing a problem that is different from 2D homography.

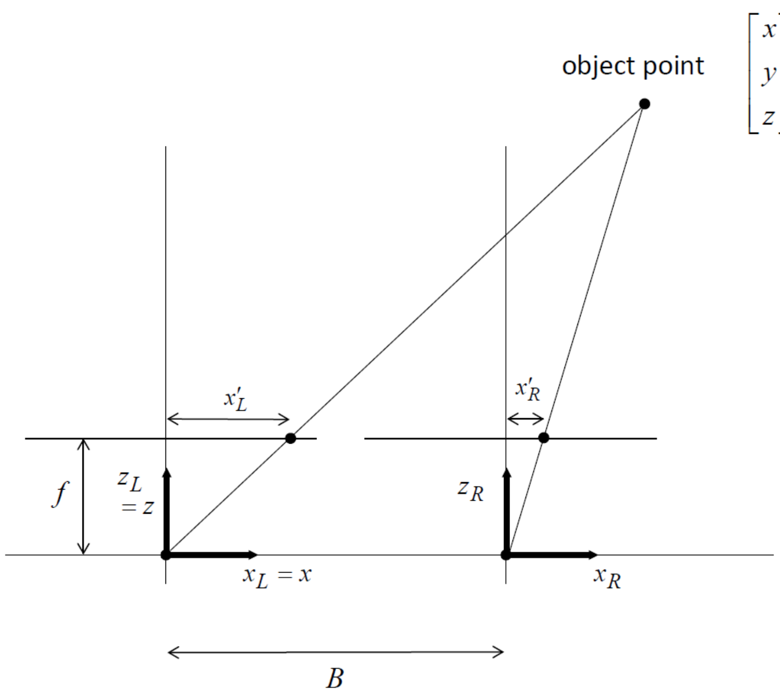
b) Continuing from part (a), a least-squares solution to parameter vector $\boldsymbol{a}$ is the eigenvector associated with the smallest eigenvalue of the matrix $\boldsymbol{Q}^{\mathrm{T}}\boldsymbol{Q}$. Use this approach to find numerical solutions for homography parameters a_1 through a_9 for the following point correspondences:

$$
\begin{array}{ccc}
(x_i', y_i') & \leftrightarrow & (x_i, y_i) \\
(5, 4) & \leftrightarrow & (0, 0) \\
(7, 4) & \leftrightarrow & (1, 0) \\
(7, 5) & \leftrightarrow & (1, 1) \\
(6, 6) & \leftrightarrow & (0, 1)
\end{array}
$$

To help with the grading, please <u>normalize your numerical solution</u> by dividing all parameters a_1 through a_9 by the value of a_9.

You may use any matrix solver to find the numerical values. For example, the NumPy functions `np.linalg.eig()` or `np.linalg.eigh()` might be used. If you use a matrix solver, cut and paste your code as part of your solution.

Problem 2. (10 points.) Consider the simple stereo imaging geometry that was introduced in class, as shown below. Both optical axes are parallel, and both cameras have the same focal length. In this view from above, the overall coordinate reference frame (*x, y, z*) is centered at the left camera.



Assume that all distances are given in units of meters. Let $f = 0.035$ and $B = 0.15$. Suppose that you are given the following corresponding pair of points from the two images:

$$(x_L', y_L') = (0.0047, 0.0) \quad \text{and} \quad (x_R', y_R') = (0.0029, 0.0).$$

Solve for the 3D point (*x, y, z*) that is associated with these two image points.

Problem 3. (10 points.) Consider again the stereo imaging geometry from the previous problem. Now assume that you have implemented a stereo matching procedure that has produced an incorrect horizontal disparity value, $d + \Delta d$, where d is the correct disparity and Δd is an unknown error. When your system solves for the corresponding 3D location, the computed depth will be $z + \Delta z$, where z is the correct value and Δz is the error that results from Δd. Try to find an expression for Δz that is a function of z, d, and Δd only. Collect terms and give a simplifed expression. (Do not plug in numerical values for B, f, z, d, etc.)

Problem 4. (20 points.) You have been given some image files and a Jupyter notebook file named `Homework3_USERNAME.ipynb`. Replace "`USERNAME`" with your Virginia Tech Username. Then upload all of these files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

Problem 5. (10 points. For <u>5554</u> students, this problem is <u>required</u>. For <u>4554</u> students, this problem is optional and can be submitted for <u>extra credit</u>.)

Near the end of your Jupyter notebook file for the previous problem, append new code blocks and text blocks in which you make comparisons between SIFT-based and ORB-based keypoints. <u>Do not change your answers for Problem 4</u>, but instead <u>add new blocks at the end</u> of the notebook file in which you write code that detects both types of keypoints. You may use images that have been provided to you, or you may upload images of your own. Add text block(s) in which you discuss the relative merits of these two types of keypoints. Try to find cases in which SIFT performs better than ORB, and vice versa. Clearly indicate those cases to the grader. You may discuss differences the accuracy of matches that are reported, differences in computation time, or other differences that you find to be interesting.

You do not need to provide a lengthy report for this problem. Roughly 1 or 2 pages of discussion might be expected, in addition to interesting displays of images/figures that illustrate your findings.

What to hand in: After you have finished your work, please upload your solutions to Canvas. In a change from previous assignments, this homework has been set up as a Canvas "quiz". You will need to upload your answers in different files for some of the problems. The purpose is to help the graders work more efficiently, with fewer errors in grading and with better feedback to you.

- Problem 1: upload your answer in file `Homework3_P1_USERNAME.pdf`.
- Problem 2: upload your answer in file `Homework3_P2_USERNAME.pdf`.
- Problem 3: upload your answer in file `Homework3_P3_USERNAME.pdf`.
- Problem 4 and (possibly) Problem 5: Create the following 2 files and upload both of them to Problem 4 of the Canvas "quiz". (Later, a separate Canvas gradebook entry will be created to hold grades for Problem 5.)
  - Upload `Homework3_Code_USERNAME.zip`, which contains your one Jupyter notebook file, <u>along with the image files that you provided</u> for Problem 4 and (possibly) for Problem 5.
  - Upload `Homework3_Notebook_USERNAME.pdf`, which contains a PDF version of your Colab session for Problem 4 and (possibly) Problem 5.
