ECE 4554 / 5554: Computer Vision: Homework 2

Fall 2021

Instructions

- The assignment is due at Canvas on Sept. 26 before 11:59 PM. Late submissions are allowed at the cost of 1 token per 24-hour period. A submission received only a minute after midnight will cost an additional token.
- Please review the Honor Code statement in the syllabus. The work that you submit for a grade must be your own.
- Each problem is worth 10 points. One of the problems is required for 5554 students, but is optional (extra credit) for 4554 students.
- Prepare an answer sheet that contains all of your written answers in a single file named `Homework2_Problems1-4_USERNAME.pdf`. (Use your own VT Username.) Handwritten solutions are permitted, but they must be easily legible to the grader. In addition, 2 more files related to Python coding must be uploaded to Canvas. Details are provided at the end of this assignment.
- For problems 5 and 6 (the coding problems), the notebook file that you submit must be compatible with Google Colab. Your code should execute after the grader makes only 1 change to your file, which is the location of the working directory. If the notebook file does not execute, then the grader will be tempted to assign a grade of 0 for problems 5 and 6.
- After you have submitted to Canvas, it is your responsibility to download the files that you submitted and verify that they are correct and complete. *The files that you submit to Canvas are the files that will be graded.*

Problem 1. Recall that rotation of a 2D point (x, y) about the origin can be described by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = R \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \tag{1}$$

where

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2}$$

and where (x', y') is the location of the point after rotation. A positive value of θ causes rotation in the counterclockwise direction.

a) Suppose that you want to rotate a point about an arbitrary reference location (x_0, y_0), instead of rotating about the origin. Show how this new operation can be expressed as a sequence of matrix-multiplication operations using homogeneous coordinate representation: translation, followed by rotation matrix R, followed by translation.

b) Compute some numerical results using your solution to part (a). Assume that $\theta = 15$ degrees and $(x_0, y_0) = (200, 150)$. Find (x', y') for the following cases:

$(x, y) = (0, 0)$
$(x, y) = (200, 150)$
$(x, y) = (210, 150)$
$(x, y) = (400, 300)$

Problem 2. Consider two kernels g and h, which are shown below. Let I represent an arbitrary image, and let "*" represent 2D convolution.

$$g = \begin{bmatrix} -1 & -4 & 7 \\ -2 & 5 & 8 \\ 0 & 6 & 0 \end{bmatrix} \qquad\qquad h = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

Instead of computing $(I * g) * h$, we wish to obtain the same result using a single filter f and the computation $I * f$. What is f?

Problem 3. In a recent lecture, we stated that the following two kernels can be used to approximate the Laplacian operation:

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad\qquad \begin{bmatrix} 1 & 4 & 1 \\ 4 & -20 & 4 \\ 1 & 4 & 1 \end{bmatrix}$$

Provide analytical derivations for both kernels to explain why these are reasonable approximations to the Laplacian.

Problem 4. (For 5554 students, this problem is required. For 4554 students, this problem is optional and can be submitted for extra credit.)

Let f be a function of 2 variables, such as x and y, in the continuous domain. Let u and v represent small displacements from some location of interest (x, y). Use a Taylor-series expansion to show that the following is a reasonable 1st-order approximation. Here, the operator "·" represents the inner product of two vectors.

$$f(x + u, y + v) \approx f(x, y) + \nabla f(x, y) \cdot \begin{bmatrix} u \\ v \end{bmatrix}$$

Problems 5 and 6.
You have been given a Jupyter notebook file `Homework2_USERNAME.ipynb` and an image file `wheel.png`. Replace "`USERNAME`" with your Virginia Tech Username. Then upload both files to Google Drive. Open the `ipynb` file in Google Colab. Follow the instructions that you will find inside the notebook file.

What to hand in: After you have finished, you will have created the following 3 files. Upload these 3 files to Canvas before the deadline. Do not combine them in a single ZIP file.

`Homework2_Problems1-4_USERNAME.pdf` ← Your solutions to problems 1 through 4

`Homework2_Code_USERNAME.zip` ← Your zipped Jupyter notebook file

`Homework2_Notebook_USERNAME.pdf` ← A PDF version of your Colab session
