Design and Analysis of Algorithms Assignment - 6

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INTRODUCTION

We are given an un-directed weighted graph.our goal is to find build minimum spanning tree using kruskal's algorithm.

Minimum Spanning Tree- A sub graph(basically a tree)made from set of edges of input graph G(m,n) such that the graph still remains connected and sum of weights of all edges is minimum. MST's have a lot of practical applications in real world.

ALGORITHM DESIGN

Given graph is, **G(m,n)**
m -set of edges with correspong weighrs
n -set of vertices

Kruskal's Algorithm is based on **cyclic property of graphs**.
Firstly,we sort all edges with respect to weights in **ascending order**.Then we consider each edge sequentially and check if those 2 vertices forming that edge are in same set or not.
If they are in same set,then including that edge creates a cycle which we don't want, since those 2 vertices are already connected,including that edge is useless.Hence we exclude that edge.
If they belong to 2 different sets ,we include that edge in our MST and combine those 2 sets.
We use **disjoint-set-union(DSU) data structure** to work with sets.

Code Implementation

```
#include<bits/stdc++.h>
using namespace std;
#define IOS ios_base::sync_with_stdio(false),cin.tie(NULL)
#define mod 1000000007
#define  pb push_back
typedef long long ll;
typedef unsigned long long ull;
typedef  long double ld;
typedef vector<int> vi;
typedef vector<bool> vb;
typedef vector<vi >vvi;
typedef vector<ll> vll;
typedef vector<vll >vvll;


struct edge_ {
        ll v1, v2, wt;
};

ll max_vertices = 25, max_weights = 10;
ll n, m, u, v, w, cost;
ll _rank[1001], par[1001];
vector<edge_>edges, result;


void create_random_graph() {

        n = 50 + rand() % max_vertices;
```

```
        ll max_edges = n * (n - 1) / 2;
        m = 1 + rand() % max_edges;


        ll edge_count = m;
        set<pair<ll, ll> >s;

        while (edge_count) {
                ll v1 = 1 + rand() % n, v2 = 1 + rand() % n;
                if (v1 == v2)continue;
                if (s.find({v1, v2}) != s.end() || s.find({v2, v1}) != s.end())continue;
                ll wt = 1 + rand() % max_weights;
                edges.pb({v1, v2, wt});
                s.insert({v1, v2});
                edge_count--;
        }

}


bool comp(const edge_ &e1, const edge_ &e2) {
        return (e1.wt < e2.wt);
}

ll find(ll v1) {

        if (par[v1] == -1)return v1;

        return par[v1] = find(par[v1]);
}

void combine(ll v1, ll v2) {

        ll a = find(v1), b = find(v2);

        if (_rank[a] == _rank[b])_rank[b]++, par[a] = b;
        if (_rank[a] < _rank[b])par[a] = b;
        else par[b] = a;
}


int main() {

        srand(time(0));

        create_random_graph();

        cout << "n  ---> " << n << "\n";
        cout << "m  ---> " << m << "\n";
        cout << "\ninput graph--->\n";
        for (auto ele : edges)cout << ele.v1 << " " << ele.v2 << " " << ele.wt << "\n";


        for (ll i = 0; i <= n; i++)par[i] = -1;

        sort(edges.begin(), edges.end(), comp);


        for (ll i = 0; i < m; i++) {
                ll v1 = edges[i].v1, v2 = edges[i].v2;
                ll a = find(v1), b = find(v2);
                if (a != b) {
                        combine(a, b);
                        result.pb({v1, v2, edges[i].wt});
                        cost += edges[i].wt;
                }
        }

        cout << "\ncost ---> " << cost << "\n";
        cout << "\nminimum spanning forest --->\n";

        for (auto ele : result)cout << ele.v1 << " " << ele.v2 << " " << ele.wt << "\n";
```

```
}
```

ALGORITHM ANALYSIS

Time-
sorting step -$O(m * log(m))$.
For each check if both vertices belong to same set or not(find() in dsu) and combine 2 sets (union() in dsu) if they belong to different sets.

In our implementation of DSU,we used both **rank-heursitic** and **path compression technique**.
Hence find() and union() works in $\alpha(m, n)$ time,where $\alpha(m, n)$ is inverse ackermann function.
It doesn't cross '5' for any practically large real input.
Hence **time taken=**$O(m * log(m) + m * \alpha(m, n))$.

Space-
We used rank[] and parent[] arrays to work with sets ,which are linear.
Hence,**Auxillary space** =$O(m + n)$

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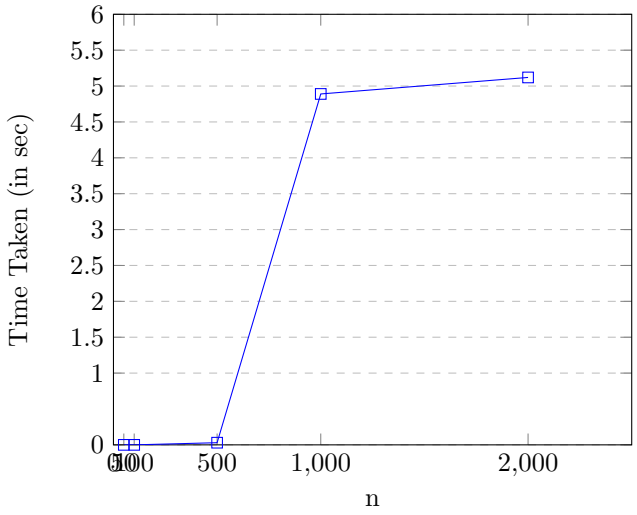
1 Experimental Analysis

In the following table some cases are plotted for the first algorithm,

m	n	Time Taken (in ms)
50	662	0.0000
100	4011	0.0000
500	22238	0.03
1000	497566	4.89
2000	900937	5.12

Algorithm 1



Discussion on Parallelising Kruskal's Algorithm

Kruskal's algorithm involves
1. sorting step.

2. determing if 2 vertices belong to same set or not and combine if belong to different sets.
Second task is difficult to parallelise .Because ,say 2 edges are being checked for that condition and calling a union() function by 2 different processors simultaneously,we could get incorrect results for find() and there is possibility that we combine 2 incorrect sets.
This is due to concurrency issues when working with multi processor systems.

However,**sorting step can be parallelised.**
Sorting can be done in **linear time** if worked with **O(log(m)) processors**.

Hence **Time taken** = O(n+m*$\alpha(m,n)$) = $O(m * \alpha(m,n))$.

This is better than usual serial kruskal's algorithm.

CONCLUSION

We can conclude that the given problem of finding minimum spanning tree(MST) of a given graph works in $O(m * log(m) + m * \alpha(m,n))$ time and consumes $O(m+n)$ space.

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REFERENCES

1.https://en.wikipedia.org/wiki/Kruskal%27s_$algorithm$