Report for Assignment

October 17, 2020

0.0.1 Programming a simulator of a level control system in Python; Course: FM1220-120H Automatic Control; Dhanush Wagle, M.Sc EPE, 238810

0.1 Wood Chip Tank

The starting point of the following tasks is this simulator of the chip tank (no controller is included in the simulator): sim_chiptank.py. The process model is as presented in Ch. 36.1 in the document Process ModelsPreview the document, but with the following difference: The control signal u [kg/s] sets the flow through feed screw (no feed screw gain is included in the simulator).
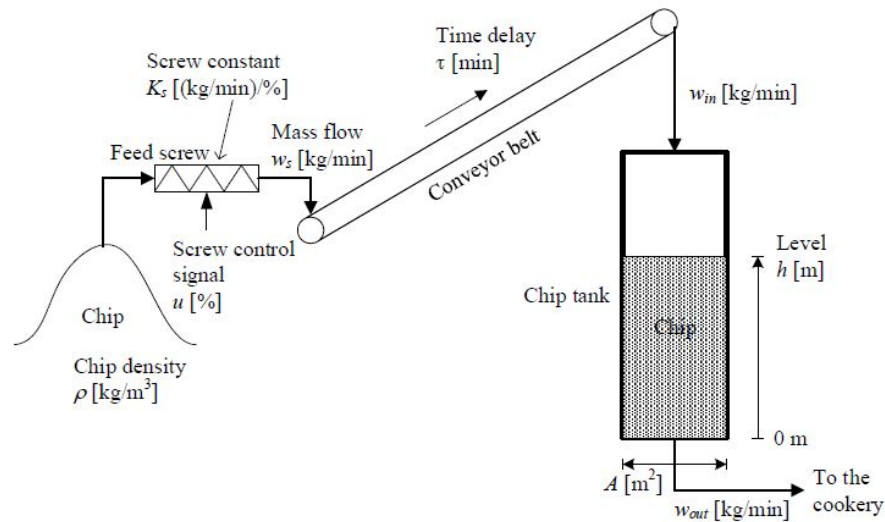

Figure 36.1: Wood chip tank.

The nominal wood-chip level setpoint is 10 m. The nominal wood-chip outflow (disturbance) is 25 kg/s. The maximum control signal is 50 kg/s, and the minimum is 0 kg/s (the control signal should should . The maximum level is 15 m, and the minimum is 0 m.

Table 36.1: Wood chip tank: Variables and parameters

Symbol	Value (default)	Unit	Description
h	10	m	Wood chip level
-	[0, 15]	m	Range of level
u	50	%	Control signal to feed screw
w_s	1500	kg/min	Feed screw flow (flow into conveyor belt)
w_{in}	1500	kg/min	Wood chip flow into tank (from belt)
w_{out}	1500	kg/min	Wood chip outflow from tank
ρ	145	kg/m^3	Wood chip density
A	13.4	m^2	Tank cross sectional area
K_s	30	(kg/min)/%	Feed screw gain (capacity)
τ	250 s	s	Transport time (time delay) on conveyor belt

Figure 36.2 shows a block diagram of the wood chip tank.
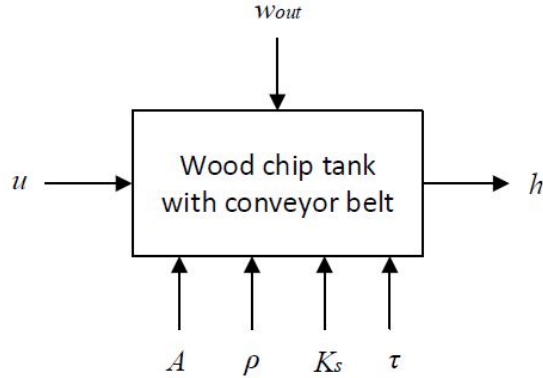

Figure 36.2: Block diagram of the wood chip tank.

A mathematical model of the tank based on material balance is

$$\rho A h'(t) = w_{\text{in}}(t) - w_{\text{out}}(t) \tag{36.1}$$
$$= w_s(t - \tau) - w_{\text{out}}(t) \tag{36.2}$$
$$= K_s u(t - \tau) - w_{\text{out}}(t) \tag{36.3}$$

where w_{in} is the chip flow through the feed screw assumed being proportional to the control signal, u. w_s is the inflow to the conveyor belt, and it arrives time delayed to the tank.

0.2 TASK

1. Implementation of a P (proportional) level controller:

    a. Enhance the simulator with a P controller including a manual control term, u_man, with an appriorpate value. (You may put the controller before (above) the process simulator in the simulation loop.) The setpoint should be plotted together with the process variable (level).

```
[4]:  # %% Import

      import matplotlib.pyplot as plt
      import numpy as np

      # %% Time settings:

      ts = 1   # Time-step [s]
      t_start = 0.0   # [s]
      t_stop = 5000.0   # [s]
      N_sim = int((t_stop-t_start)/ts) + 1

      # %% Process params:

      rho = 145   # [kg/m^3]
      A = 13.4   # [m^2]
      t_delay = 250.0   # [s]
      h_min = 0   # [m]
      h_max = 15   # [m]
      u_min = 0 # [kg/s]
      u_max = 50 # [kg/s]

      # %% Initialization of time delay:

      u_delayed_init = 25   # [kg/s]
      N_delay = int(round(t_delay/ts)) + 1
      delay_array = np.zeros(N_delay) + u_delayed_init

      # %% Arrays for plotting:

      t_array = np.zeros(N_sim)
      h_array = np.zeros(N_sim)
      u_array = np.zeros(N_sim)
      h_sp_array = np.zeros(N_sim)

      # %% Initial state:

      h_k = 10.0   # m setpoint
      h_sp = h_k # just for graph
      h_kp1 = 0.0 # m initializing only
```

```python
# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    k_p = 12 # try from 1 to 15 to find constant oscillation and substitue to↵
↪k_pu
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_k = u_man + u_p_k # total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:
```

```
plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

The control error is: -0.7760253425257435
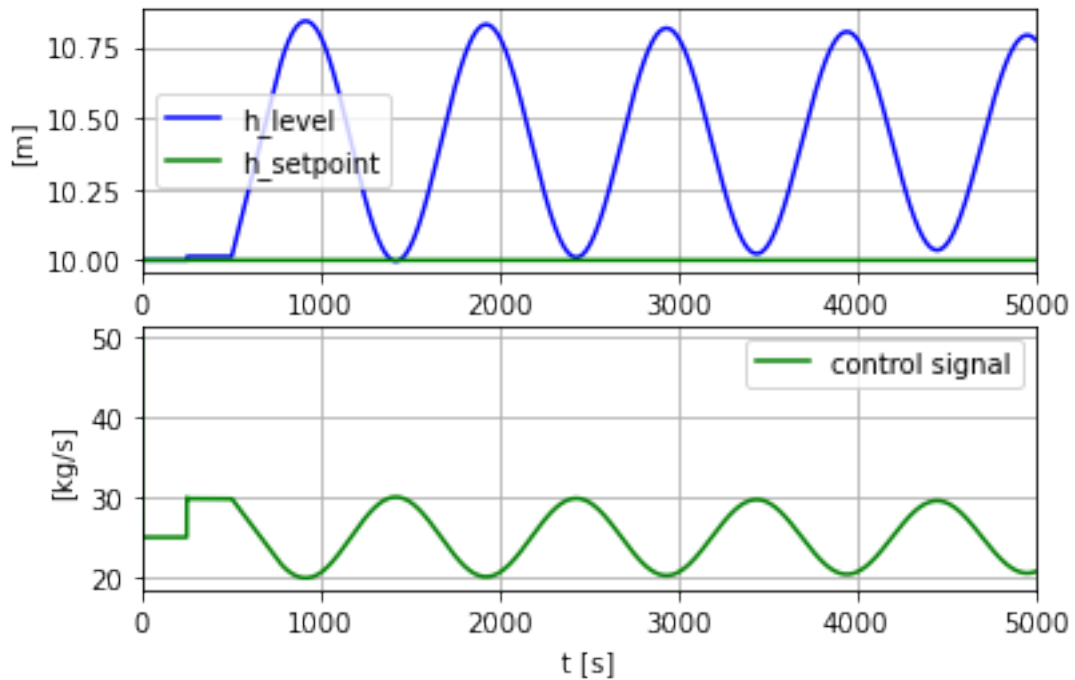
b. Tune the P controller with the Ziegler-Nichols method. Is the stability of the control system

acceptable with the P controller?

```python
[5]: # %% Import

import matplotlib.pyplot as plt
import numpy as np

# %% Time settings:

ts = 1   # Time-step [s]
t_start = 0.0   # [s]
t_stop = 5000.0   # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145   # [kg/m^3]
A = 13.4   # [m^2]
t_delay = 250.0   # [s]
h_min = 0   # [m]
h_max = 15   # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25   # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)

# %% Initial state:

h_k = 10.0   # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
k_pu = 12

# %% Simulation for-loop:

for k in range(0, N_sim):
```

```python
    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 0.5*k_pu # try from 1 to 15 to find constant oscillation and␣
 ↪substitue to k_pu
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_k = u_man + u_p_k # total control signal
    #u_k = u_man
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
```
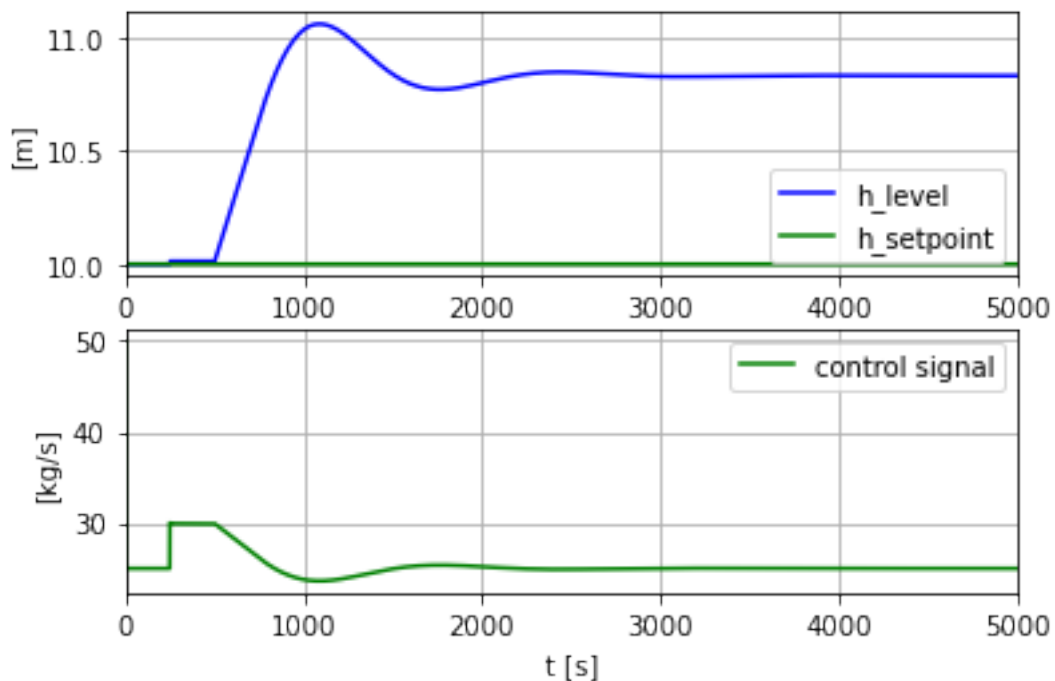
```
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

The control error is: -0.8333944397990685

From the figure above it is clear that the stability of the control system is hardly acceptable with the P controller but the error is large.

c. What is the steady-state control error if the outflow changes from 25 to 30 kg/s?

```
[6]: for k in range(0, N_sim):

         t_k = k*ts

         if t_k <= 250:
             u_man = 25    # kg/s
             w_out_k = 30   # kg/s
         else:
             u_man = 30    # kg/s
             w_out_k = 30   # kg/s

         # PID Tuning with the Ziegler-Nichols

         k_p = 0.5*k_pu
         e_k = h_sp - h_kp1 # control error
         u_p_k = k_p*e_k # p-term
         u_k = u_man + u_p_k # total control signal
         u_k = np.clip(u_k, u_min, u_max)

         # Time delay:
         u_delayed_k = delay_array[-1]
         delay_array[1:] = delay_array[0:-1]
         delay_array[0] = u_k

         # Euler-forward integration (Euler step):
         w_in_k = u_delayed_k   # kg/s
         dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
         h_kp1 = h_k + ts*dh_dt_k
         h_kp1 = np.clip(h_kp1, h_min, h_max)

         # Storage for plotting:
         t_array[k] = t_k
         u_array[k] = u_k
         h_array[k] = h_k
         h_sp_array[k] = h_sp

         # Time shift:
         h_k = h_kp1

     # %% Printing control error

     print('The control error is: ', e_k)
```

The control error is: 1.920716737480177e-05

2. Implementation of a PI level controller:

a. Implement a PI controller instead of the P controller. The controller should have anti windup (you can limit the integral term between u_max and u_min using the numpy clip() function).

[7]:
```python
# %% Import

import matplotlib.pyplot as plt
import numpy as np

# %% Time settings:

ts = 1   # Time-step [s]
t_start = 0.0   # [s]
t_stop = 5000.0   # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145   # [kg/m^3]
A = 13.4   # [m^2]
t_delay = 250   # [s]
h_min = 0   # [m]
h_max = 15   # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25   # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0   # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:
```

10

```python
for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    u_k = u_man
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
```

```
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()

slope = (h_array[2000] - h_array[1000]) / (t_array[2000] - t_array[1000])
U = 5
ki = slope/U
kp = 1/(2*ki*t_delay)
print('kp: ',kp)
```
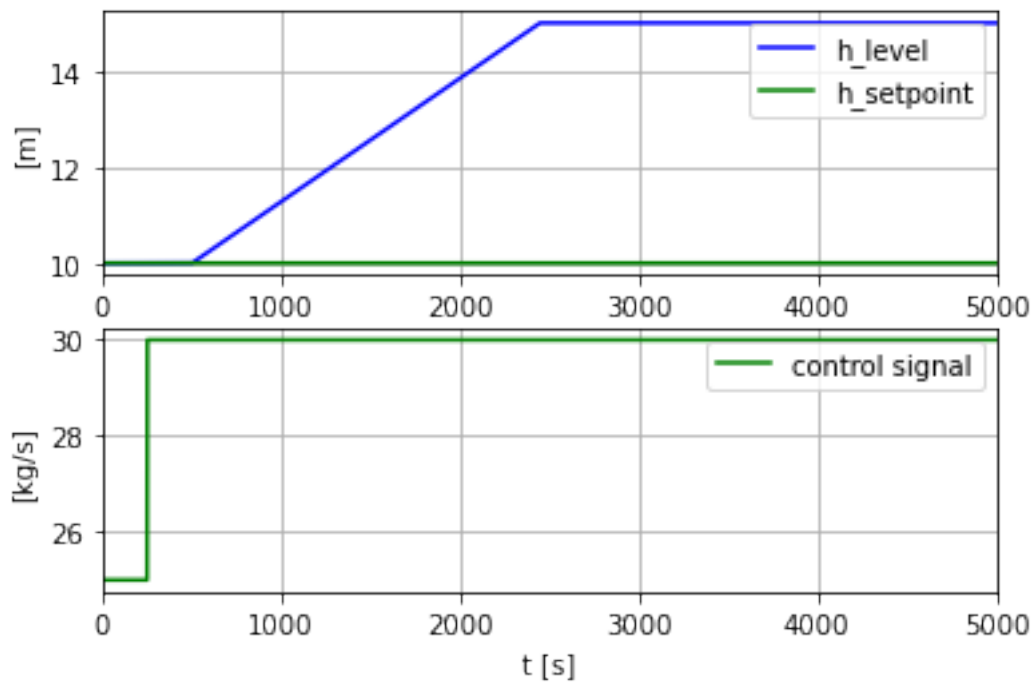
The control error is: 1.920716737480177e-05

kp: 3.8860000000013186

b. Tune the PI controller with the Skogestad method. Is the stability of the control system acceptable with the PI controller?

[8]:
```python
# %% Import

import matplotlib.pyplot as plt
import numpy as np

# %% Time settings:

ts = 1   # Time-step [s]
t_start = 0.0   # [s]
t_stop = 5000.0   # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145   # [kg/m^3]
A = 13.4   # [m^2]
t_delay = 250   # [s]
h_min = 0   # [m]
h_max = 15   # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25   # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0   # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:
```

```python
for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    # PID Tuning with the Skogested Method

    k_p = 3.886
    ti = 4*t_delay
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k # i-term
    u_k = u_man + u_p_k + u_i_k # total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:
```

```
plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```
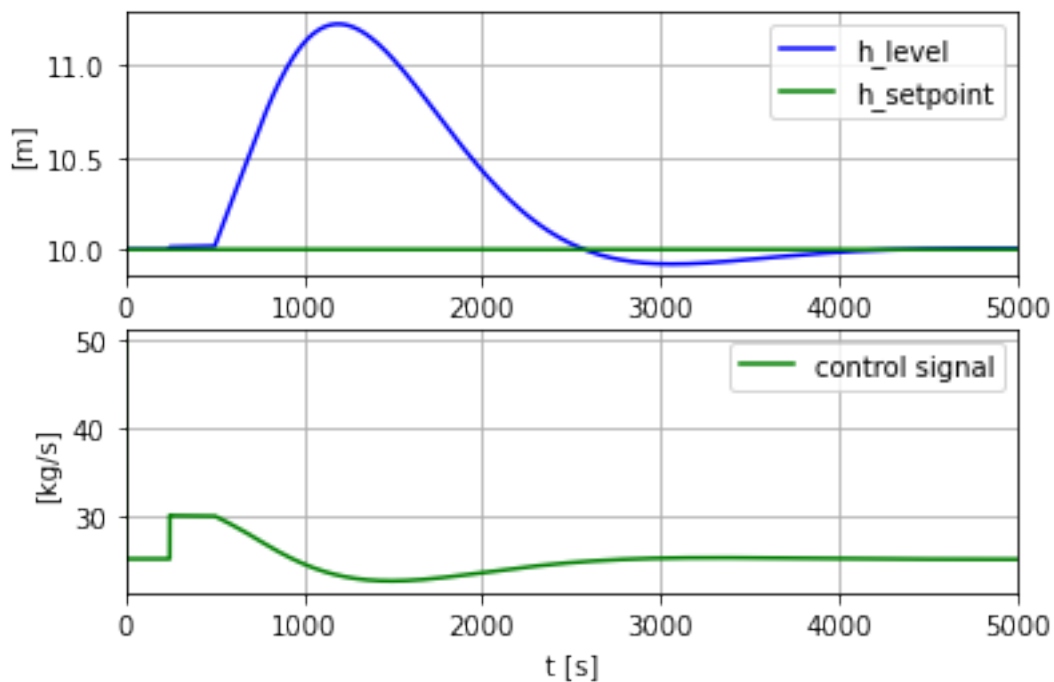
The control error is: -0.005476450241953756

From the figure above it is clear that the stability of the control system is acceptable with the PI controller.

   c. What is the steady-state control error if the outflow changes from 25 to 30 kg/s?

[9]:
```python
for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 30   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 30   # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 3.886
    ti = 4*t_delay
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error
```

```
print('The control error is: ', e_k)
```

```
The control error is:   0.0010859729140140217
```

3. Stability of the control system: Assume PI control.

a. Demonstrate that the control system becomes unstable if the time-delay in the control loop is too large (which may be due to a reduction of the conveyor belt speed). Specifically: Which time-delay makes the control system marginally stable (oscillatory with no damping)?

[14]:
```python
# %% Time settings:

ts = 1   # Time-step [s]
t_start = 0.0   # [s]
t_stop = 50000.0   # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145   # [kg/m^3]
A = 13.4   # [m^2]
t_delay = 2500.0   # [s]
h_min = 0   # [m]
h_max = 15   # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25   # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0   # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0
```

```python
# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 3.886
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)
```

```
# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```
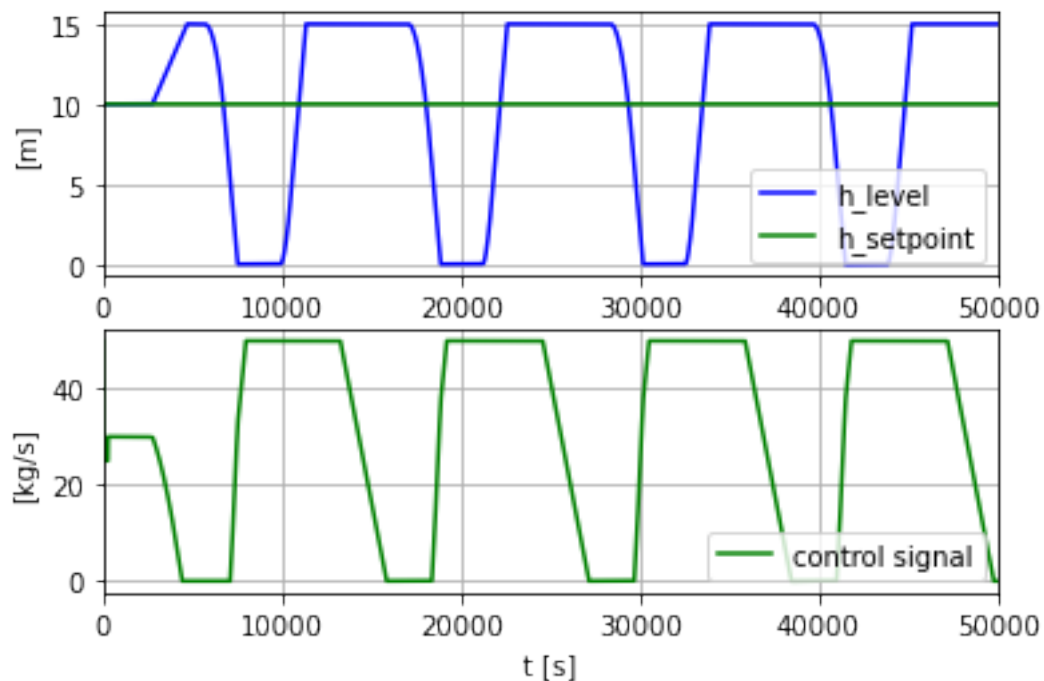
The control error is: -5.0

The figure above clerly represents that the control erros is too big which means the system is unstable or there is no damping in the system.

If we set time delay to 685s, it makes the control system marginally stable (oscillatory with no damping).

[13]:
```python
# %% Time settings:

ts = 1  # Time-step [s]
t_start = 0.0  # [s]
t_stop = 50000.0  # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145  # [kg/m^3]
A = 13.4  # [m^2]
t_delay = 685  # [s]
h_min = 0  # [m]
h_max = 15  # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25  # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0  # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:

for k in range(0, N_sim):
```

```python
    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 3.886
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k   # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)
```
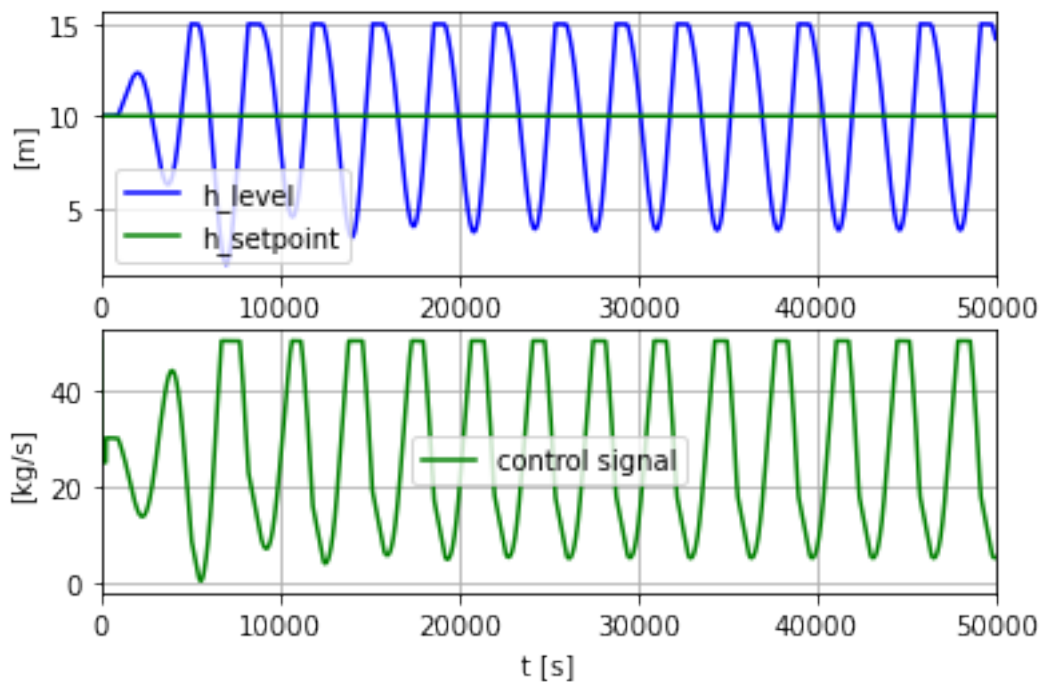
```
plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

The control error is: -4.191844563900755

b. Demonstrate that the control system becomes unstable if the cross-sectional area of the tank is too small (in general, the area may decrease if the walls are not straight). Specifically: Which area value makes the control system marginally stable (oscillatory with no damping)?

```
[16]:  # %% Time settings:

       ts = 1   # Time-step [s]
       t_start = 0.0   # [s]
       t_stop = 5000.0   # [s]
       N_sim = int((t_stop-t_start)/ts) + 1

       # %% Process params:

       rho = 145   # [kg/m^3]
       A = 1.34   # [m^2]
       t_delay = 250.0   # [s]
       h_min = 0   # [m]
       h_max = 15   # [m]
       u_min = 0 # [kg/s]
       u_max = 50 # [kg/s]

       # %% Initialization of time delay:

       u_delayed_init = 25   # [kg/s]
       N_delay = int(round(t_delay/ts)) + 1
       delay_array = np.zeros(N_delay) + u_delayed_init

       # %% Arrays for plotting:

       t_array = np.zeros(N_sim)
       h_array = np.zeros(N_sim)
       u_array = np.zeros(N_sim)
       h_sp_array = np.zeros(N_sim)


       # %% Initial state:

       h_k = 10.0   # m setpoint
       h_sp = h_k # just for graph
       h_kp1 = 0.0 # m initializing only
       u_i_k = 0

       # %% Simulation for-loop:

       for k in range(0, N_sim):

           t_k = k*ts

           if t_k <= 250:
               u_man = 25   # kg/s
               w_out_k = 25   # kg/s
```

```python
    else:
        u_man = 30   # kg/s
        w_out_k = 25  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 3.886
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
```
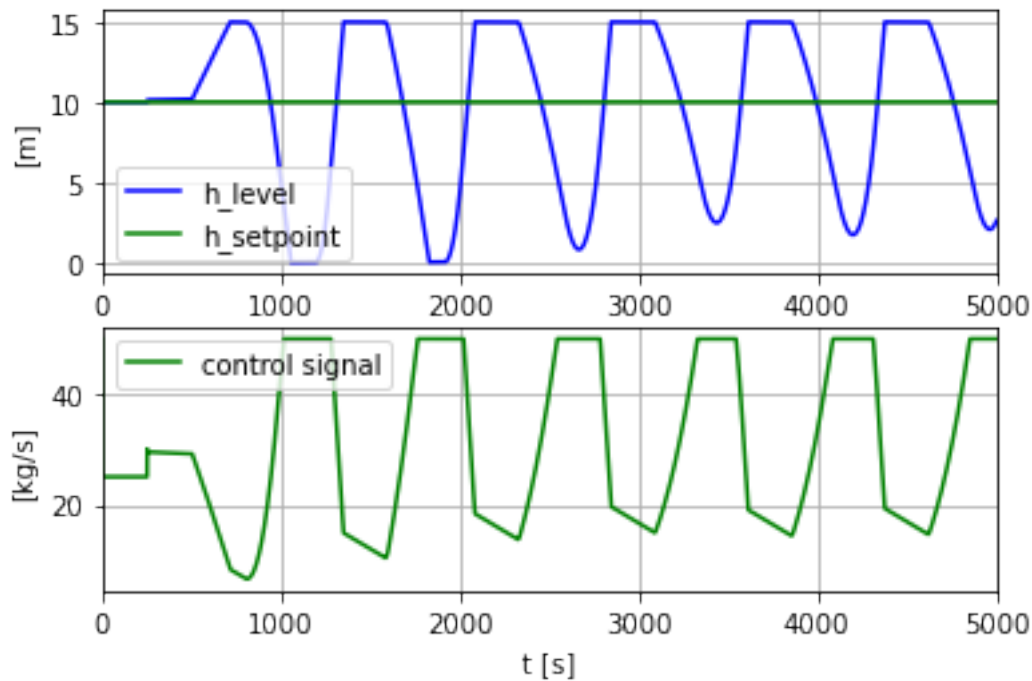
```
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

The control error is: 7.418252393775059

From figure above it is clear that if the area is too small then the oscillations are too high.

```
[17]:   # %% Time settings:

        ts = 1   # Time-step [s]
        t_start = 0.0   # [s]
        t_stop = 5000.0   # [s]
        N_sim = int((t_stop-t_start)/ts) + 1
```

```python
# %% Process params:

rho = 145   # [kg/m^3]
A = 4.9   # [m^2]
t_delay = 250.0   # [s]
h_min = 0   # [m]
h_max = 15   # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25   # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0   # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25   # kg/s
        w_out_k = 25   # kg/s
    else:
        u_man = 30   # kg/s
        w_out_k = 25   # kg/s

    # PID Tuning with the Ziegler-Nichols
```

```python
    k_p = 3.886
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
```
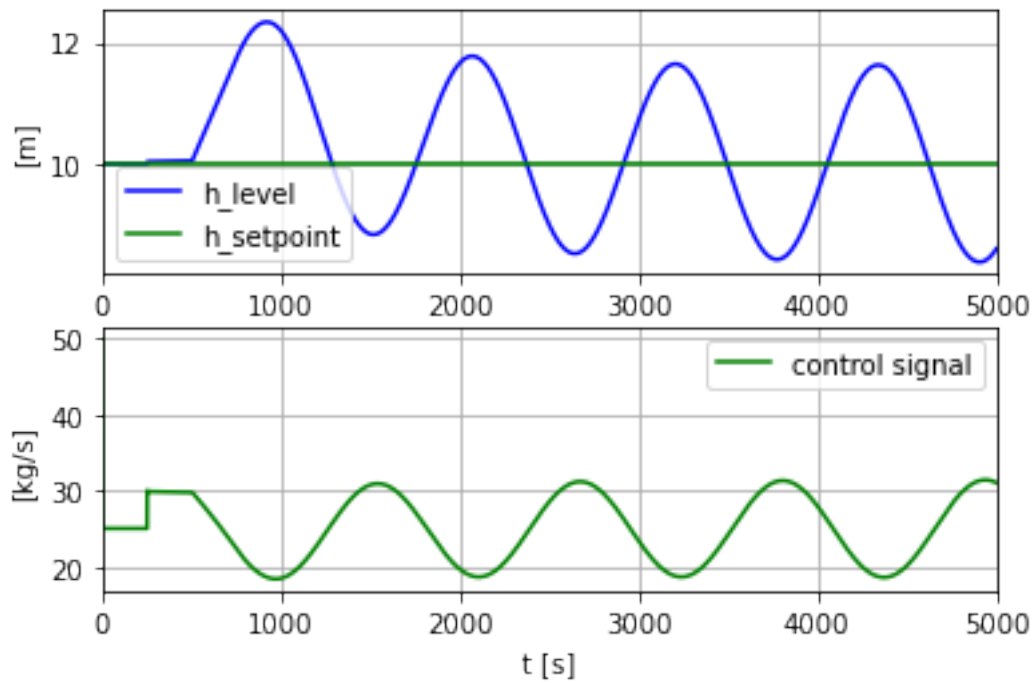
```
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

The control error is: 1.4012724684749873

If the area is 4.9 meter square then it gives oscillation with no damping.

[ ]: