Wood Chip Tank

October 2, 2020

0.1 Report for assignment: Programming a simulator of a level control system in Python

0.1.1 Course: FM1220-120H Automatic Control

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0.2 Wood Chip Tank

The starting point of the following tasks is this simulator of the chip tank (no controller is included in the simulator): sim_chiptank.py. The process model is as presented in Ch. 36.1 in the document Process ModelsPreview the document, but with the following difference: The control signal u [kg/s] sets the flow through feed screw (no feed screw gain is included in the simulator).

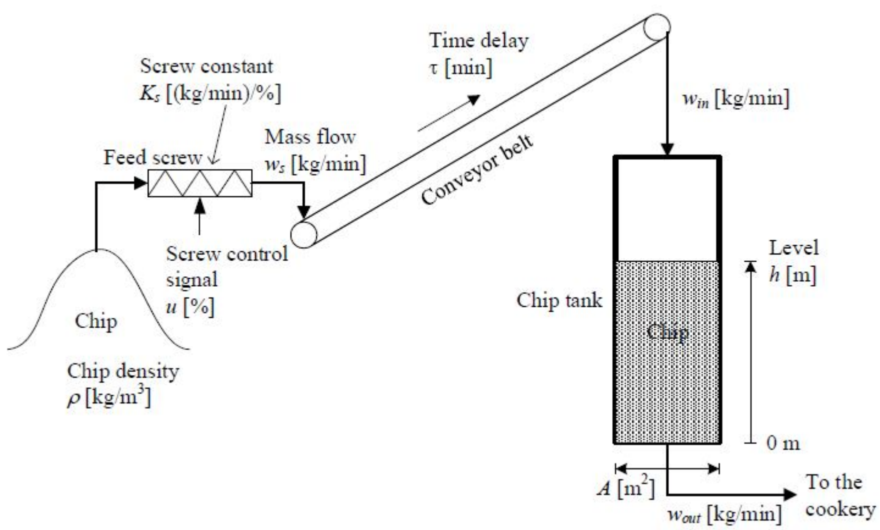


Figure 36.1: Wood chip tank.

The nominal wood-chip level setpoint is 10 m. The nominal wood-chip outflow (disturbance) is 25 kg/s. The maximum control signal is 50 kg/s, and the minimum is 0 kg/s (the control signal should should . The maximum level is 15 m, and the minimum is 0 m.

Table 36.1: Wood chip tank: Variables and parameters

Symbol	Value (default)	Unit	Description
h	10	m	Wood chip level
-	[0, 15]	m	Range of level
u	50	%	Control signal to feed screw
w_s	1500	kg/min	Feed screw flow (flow into conveyor belt)
w_{in}	1500	kg/min	Wood chip flow into tank (from belt)
w_{out}	1500	kg/min	Wood chip outflow from tank
ρ	145	kg/m^3	Wood chip density
A	13.4	m^2	Tank cross sectional area
K_s	30	(kg/min)/%	Feed screw gain (capacity)
τ	250 s	s	Transport time (time delay) on conveyor belt

Figure 36.2 shows a block diagram of the wood chip tank.

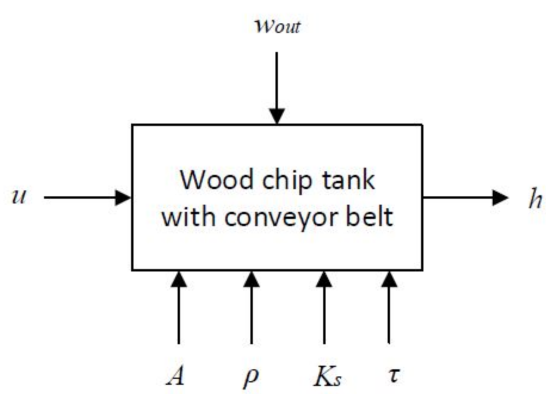


Figure 36.2: Block diagram of the wood chip tank.

A mathematical model of the tank based on material balance is

$$\begin{aligned}
\rho A h'(t) &= w_{\text{in}}(t) - w_{\text{out}}(t) && (36.1)\\
&= w_s(t-\tau) - w_{\text{out}}(t) && (36.2)\\
&= K_s u(t-\tau) - w_{\text{out}}(t) && (36.3)
\end{aligned}$$

where w_{in} is the chip flow through the feed screw assumed being proportional to the control signal, u. w_s is the inflow to the conveyor belt, and it arrives time delayed to the tank.

0.3 TASK

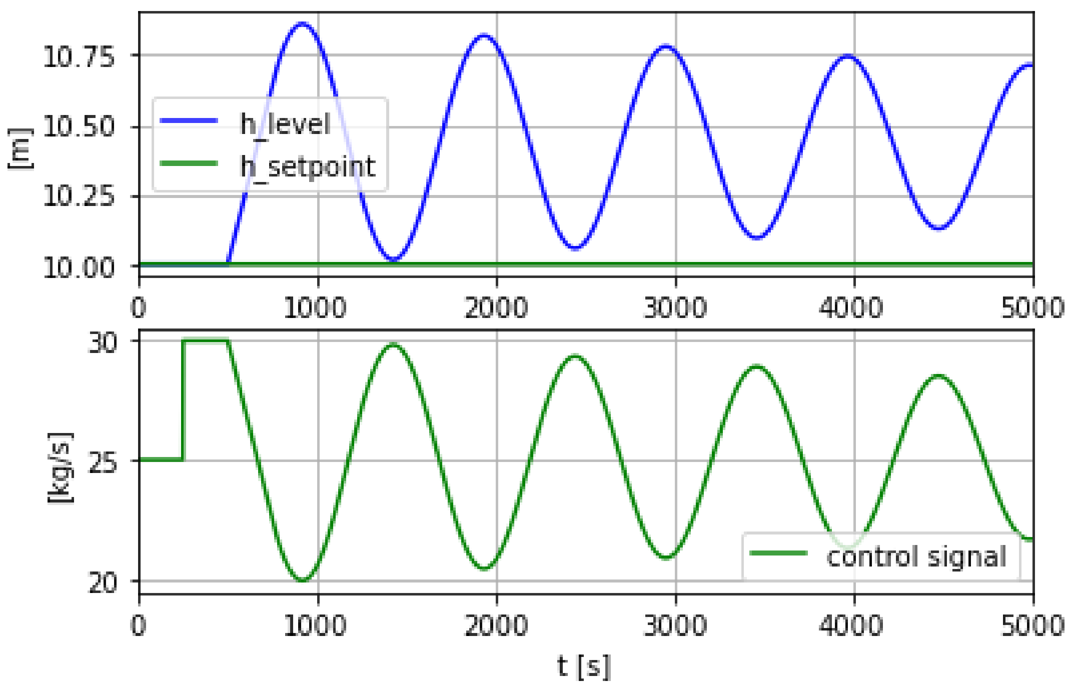
1. Implementation of a P (proportional) level controller:

a. Enhance the simulator with a P controller including a manual control term, u_man, with an appriorpate value. (You may put the controller before (above) the process simulator in the simulation loop.) The setpoint should be plotted together with the process variable (level).

[10]:
```python
# %% Import

import matplotlib.pyplot as plt
import numpy as np

# %% Time settings:

ts = 1  # Time-step [s]
t_start = 0.0  # [s]
t_stop = 5000.0  # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145  # [kg/m^3]
A = 13.4  # [m^2]
t_delay = 250.0  # [s]
h_min = 0  # [m]
h_max = 15  # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25  # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)

# %% Initial state:

h_k = 10.0  # m setpoint
h_sp = h_k # just for graph
h_kp1 = 10.0 # m initializing only
```

```python
k_pu = 26 # from labview simulation

# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 25  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 25  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 0.45*k_pu
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_k = u_man + u_p_k # total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Plotting:

plt.close('all')
plt.figure(1)
```

```
plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```



b. Tune the P controller with the Ziegler-Nichols method. Is the stability of the control system acceptable with the P controller?

Ans: The model was tuned with the P controller with the Ziegler-Nichols method as:

```
k_p = 0.45*k_pu
```

```
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_k = u_man + u_p_k # total control signal
    u_k = np.clip(u_k, u_min, u_max)
```

From the figure above it is clear that the stability of the control system is not acceptable with the P controller.

c. What is the steady-state control error if the outflow changes from 25 to 30 kg/s?

[7]:
```
for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 30  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 30  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 0.45*k_pu
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_k = u_man + u_p_k # total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1
```

```python
# %% Printing control error

print('The control error is: ', e_k)
```

```
The control error is:  -0.5753475930068248
```

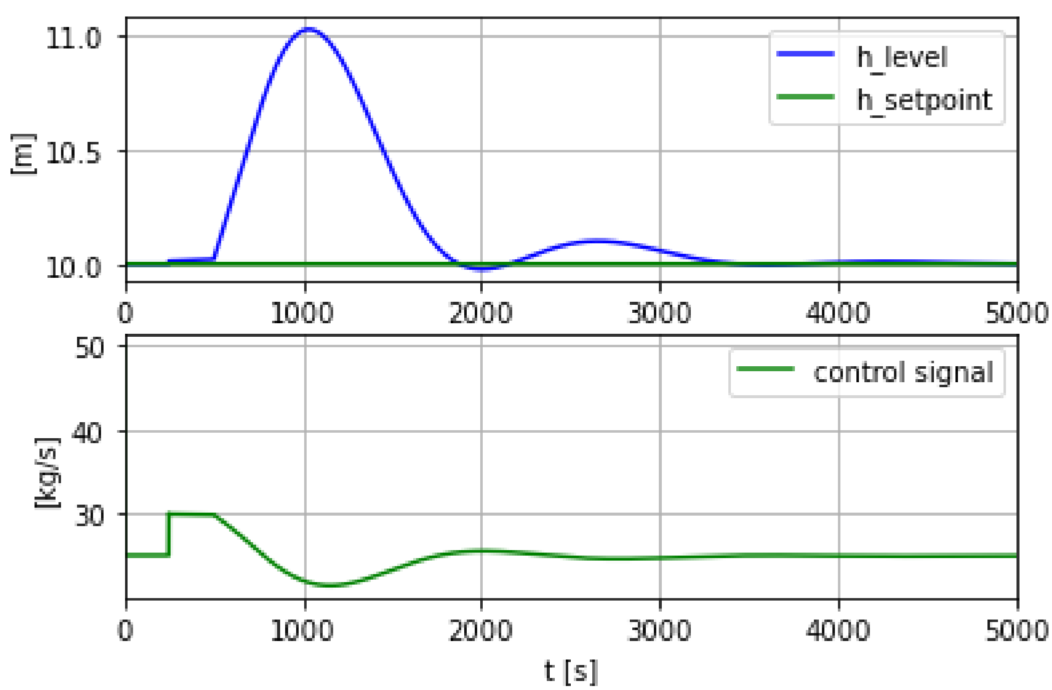
2. Implementation of a PI level controller:

a. Implement a PI controller instead of the P controller. The controller should have anti windup (you can limit the integral term between u_max and u_min using the numpy clip() function).

[13]:
```python
# %% Import

import matplotlib.pyplot as plt
import numpy as np

# %% Time settings:

ts = 1  # Time-step [s]
t_start = 0.0  # [s]
t_stop = 5000.0  # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145  # [kg/m^3]
A = 13.4  # [m^2]
t_delay = 250.0  # [s]
h_min = 0  # [m]
h_max = 15  # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25  # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)
```

```python
# %% Initial state:

h_k = 10.0  # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 25  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 25  # kg/s

    # PID Tuning with the Skogested Method

    k_p = 6
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp
```

```
    # Time shift:
    h_k = h_kp1

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```



b. Tune the PI controller with the Skogestad method. Is the stability of the control system acceptable with the PI controller?

Ans: The model was tuned with the PI controller with the Ziegler-Nichols method as:

```
k_p = 6
ti = 750
e_k = h_sp - h_kp1 # control error
u_p_k = k_p*e_k # p-term
u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
u_k = u_man + u_p_k + u_i_k# total control signal
u_k = np.clip(u_k, u_min, u_max)
```

From the figure above it is clear that the stability of the control system is acceptable with the PI controller.

c. What is the steady-state control error if the outflow changes from 25 to 30 kg/s?

[12]:
```
for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 30  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 30  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 6
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
```

```
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)
```

```
The control error is:  -0.0005846509360605268
```

3. **Stability of the control system: Assume PI control.**

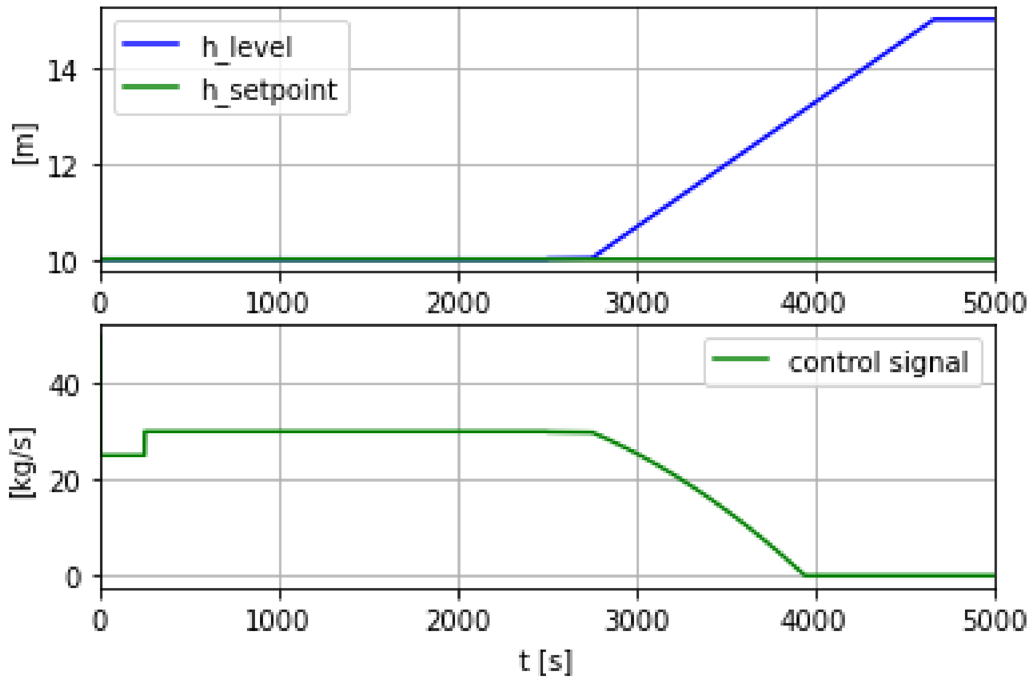
   a. Demonstrate that the control system becomes unstable if the time-delay in the control loop is too large (which may be due to a reduction of the conveyor belt speed). Specifically: Which time-delay makes the control system marginally stable (oscillatory with no damping)?

[14]:
```
# %% Time settings:

ts = 1  # Time-step [s]
t_start = 0.0  # [s]
t_stop = 5000.0  # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145  # [kg/m^3]
A = 13.4  # [m^2]
t_delay = 2500.0  # [s]
h_min = 0  # [m]
h_max = 15  # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:

u_delayed_init = 25  # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:
```

```
t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0  # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 25  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 25  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 6
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k

    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)
```

```python
    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

```
The control error is:  -5.0
```



The figure above clerly represents that the control erros is 5 which means the system is unstable or there is no damping in the system.

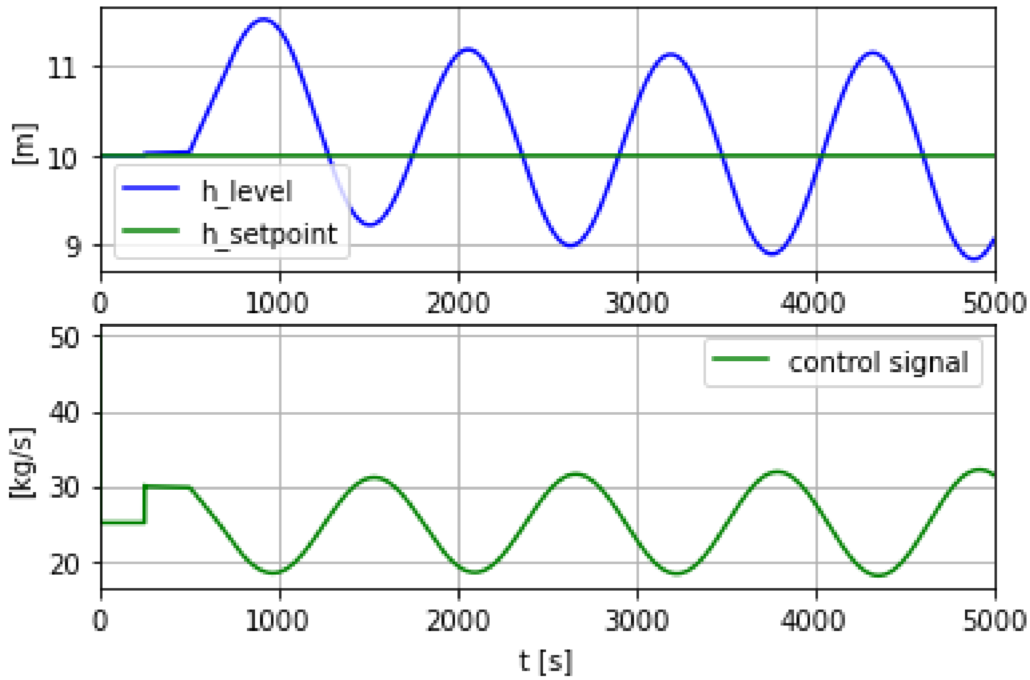
b. Demonstrate that the control system becomes unstable if the cross-sectional area of the tank is too small (in general, the area may decrease if the walls are not straight). Specifically: Which area value makes the control system marginally stable (oscillatory with no damping)?

[17]:
```
# %% Time settings:

ts = 1  # Time-step [s]
t_start = 0.0  # [s]
t_stop = 5000.0  # [s]
N_sim = int((t_stop-t_start)/ts) + 1

# %% Process params:

rho = 145  # [kg/m^3]
A = 7.5  # [m^2]
t_delay = 250.0  # [s]
h_min = 0  # [m]
h_max = 15  # [m]
u_min = 0 # [kg/s]
u_max = 50 # [kg/s]

# %% Initialization of time delay:
```

```python
u_delayed_init = 25  # [kg/s]
N_delay = int(round(t_delay/ts)) + 1
delay_array = np.zeros(N_delay) + u_delayed_init

# %% Arrays for plotting:

t_array = np.zeros(N_sim)
h_array = np.zeros(N_sim)
u_array = np.zeros(N_sim)
h_sp_array = np.zeros(N_sim)


# %% Initial state:

h_k = 10.0  # m setpoint
h_sp = h_k # just for graph
h_kp1 = 0.0 # m initializing only
u_i_k = 0

# %% Simulation for-loop:

for k in range(0, N_sim):

    t_k = k*ts

    if t_k <= 250:
        u_man = 25  # kg/s
        w_out_k = 25  # kg/s
    else:
        u_man = 30  # kg/s
        w_out_k = 25  # kg/s

    # PID Tuning with the Ziegler-Nichols

    k_p = 6
    ti = 1000
    e_k = h_sp - h_kp1 # control error
    u_p_k = k_p*e_k # p-term
    u_i_k = u_i_k + ((k_p*ts)/ti)*e_k
    u_k = u_man + u_p_k + u_i_k# total control signal
    u_k = np.clip(u_k, u_min, u_max)

    # Time delay:
    u_delayed_k = delay_array[-1]
    delay_array[1:] = delay_array[0:-1]
    delay_array[0] = u_k
```

```python
    # Euler-forward integration (Euler step):
    w_in_k = u_delayed_k  # kg/s
    dh_dt_k = (1/(rho*A))*(w_in_k - w_out_k)
    h_kp1 = h_k + ts*dh_dt_k
    h_kp1 = np.clip(h_kp1, h_min, h_max)

    # Storage for plotting:
    t_array[k] = t_k
    u_array[k] = u_k
    h_array[k] = h_k
    h_sp_array[k] = h_sp

    # Time shift:
    h_k = h_kp1

# %% Printing control error

print('The control error is: ', e_k)

# %% Plotting:

plt.close('all')
plt.figure(1)

plt.subplot(2, 1, 1)
plt.plot(t_array, h_array, 'b', label='h_level')
plt.plot(t_array, h_sp_array, 'g', label='h_setpoint')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[m]')

plt.subplot(2, 1, 2)
plt.plot(t_array, u_array, 'g', label='control signal')
plt.legend()
plt.grid()
plt.xlim(t_start, t_stop)
plt.xlabel('t [s]')
plt.ylabel('[kg/s]')

plt.show()
```

```
The control error is:  0.945915554908277
```



If the area is 7.5 meter square then it gives oscillation with no damping.