Chapter 36

Process models

In the following sections, mathematical models of various physical processes are presented. Some of these models are used in examples and in problems in the book. They may be used in aditional problems, and as a basis of dynamic simulators.

36.1 Wood chip tank

36.1.1 System description

Figure 36.1 shows a wood chip tank with a feed screw with continuous feed of wood chip, conveyor belt, which runs with a fixed speed. There is a continuous outflow of wood chip.[1] The conveyor belt makes up a time delay or transport delay from the screw to the tank.

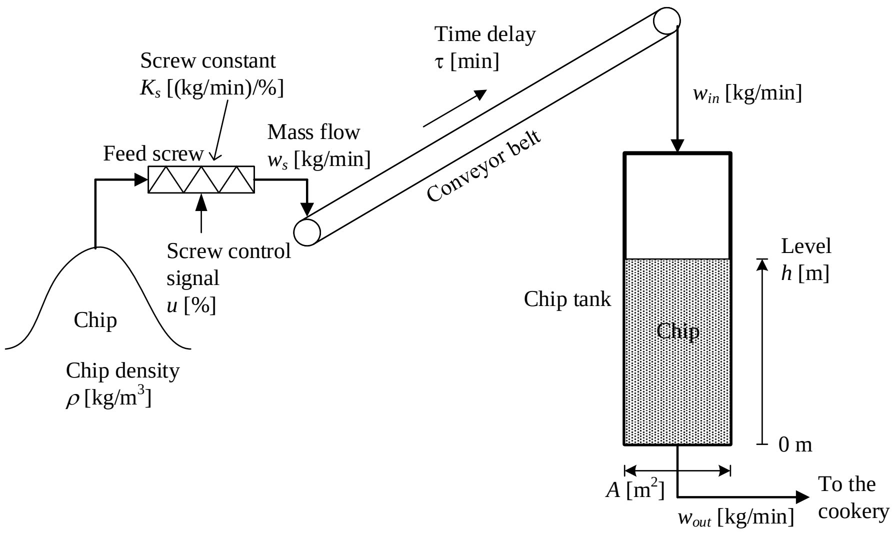

Figure 36.1: Wood chip tank.

[1] Typically, there is such a chip tank in the beginning of the production line of a paper and pulp factory.

36.1.2 Variables and parameters

Variables and parameters of the wood chip tank are defined in Table 36.1.[2] We assume that there is an overflow if the level exceeds the maximum level.

Table 36.1: Wood chip tank: Variables and parameters

Symbol	Value (default)	Unit	Description
h	10	m	Wood chip level
-	[0, 15]	m	Range of level
u	50	%	Control signal to feed screw
w_s	1500	kg/min	Feed screw flow (flow into conveyor belt)
w_{in}	1500	kg/min	Wood chip flow into tank (from belt)
w_{out}	1500	kg/min	Wood chip outflow from tank
ρ	145	kg/m^3	Wood chip density
A	13.4	m^2	Tank cross sectional area
K_s	30	(kg/min)/%	Feed screw gain (capacity)
τ	250 s	s	Transport time (time delay) on conveyor belt

36.1.3 Overall block diagram

Figure 36.2 shows a block diagram of the wood chip tank.

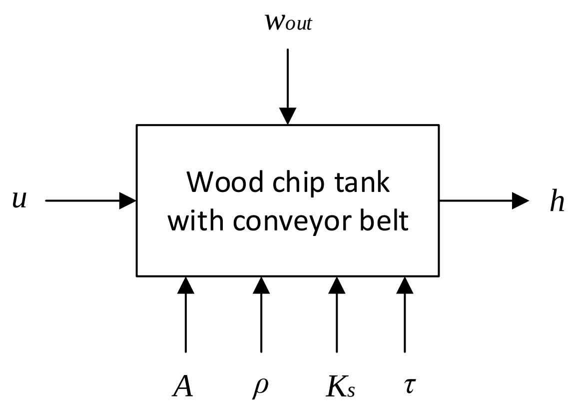

Figure 36.2: Block diagram of the wood chip tank.

[2]Courtesy of earlier Sødra Cell, and even earlier Norske Skog, Tofte, Norway.

36.1.4 Mathematical model

A mathematical model of the tank based on material balance is

$$\begin{aligned}\rho A h'(t) &= w_{\text{in}}(t) - w_{\text{out}}(t) &(36.1)\\ &= w_s(t-\tau) - w_{\text{out}}(t) &(36.2)\\ &= K_s u(t-\tau) - w_{\text{out}}(t) &(36.3)\end{aligned}$$

where w_{in} is the chip flow through the feed screw assumed being proportional to the control signal, u. w_s is the inflow to the conveyor belt, and it arrives time delayed to the tank.

36.2 Ship

36.2.1 System description

Figure 36.3 shows a ship. In this example we will only take the longitudinal motion relative to the ship, also denoted the surge motion, into account. [3]

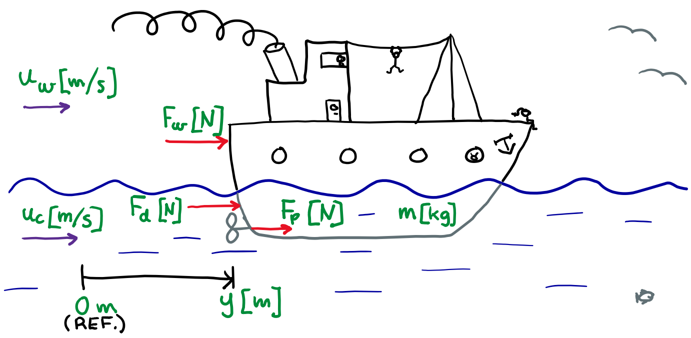

Figure 36.3: Ship.

36.2.2 Variables and parameters

Variables and parameters are defined in Table 36.2. The parameter values are realistic.[4]

Figure 36.4 shows the wind scale which characterizes various ranges of wind speed V_w.

36.2.3 Overall block diagram

Figure 36.5 shows a block diagram of the ship.

[3]Courtesy of Kongsberg Maritime AS, Norway for providing realistic parameter values.
[4]Courtesy of Kongsberg Maritime AS, Norway.

Table 36.2: Ship: Variables and parameters

Symbol	Value (default)	Unit	Description
$y = x_1$	-	m	Longitudinal or surge position of the ship
$\dot{y} = x_2$	-	m/s	Ship speed
F_p	$[-467 \cdot 10^3, +552 \cdot 10^3]$	N	Propeller force applied to move the ship
F_h	-	N	Hydrodynamic force on the ship
F_w	-	N	Wind force on the ship
u_c	$[-3\ ,\ +3]$	m/s	Water current speed
V_w	Cf. windscale in Fig. 36.4	m/s	Wind speed
m	$71164 \cdot 10^3$	kg	Mass of ship
D_h	$8.4 \cdot 10^3$	N/(m/s)^2	Hydrodynamic force constant
D_w	$0.177 \cdot 10^3$	N/(m/s)^2	Wind force constant

36.2.4 Mathematical model

We use the Newton's Second Law to model the motion of the ship:

$$\text{mass times accelaration} = \text{sum of forces}$$

In mathematical terms:

$$my'' = F_p + F_h + F_w \tag{36.4}$$

where:

- F_h is proportional to the difference between the water speed and the ship speed:
$$F_h = D_h \left(u_c - y'\right) \left|u_c - y'\right| \tag{36.5}$$
The absolute value in the last term ensures that the sign of F_h correct.
- F_w is proportional to the square of the difference between the wind speed and the ship speed:
$$F_w = D_w \left(V_w - y'\right) \left|V_w - y'\right| \tag{36.6}$$
The absolute value in the last term ensures that the sign of F_w correct.

When the ship moves, there is also a motion of an amount of water. The mass of the water is denoted the added mass. However, we disregard the added mass here.

Often it is convenient to represent the model as a state-space model. To this end, we define two state variables as follows:

- Position:
$$x_1 = y \tag{36.7}$$

m/s	Description
0.5 - 1.8 m/sec	light air
1.9 - 3.3 m/sec	light breeze
3.4 - 5.4 m/sec	gentle breeze
5.5 - 7.9 m/sec	breeze
8.0 - 11.0 m/sec	fresh breeze
11.1 - 14.1 m/sec	strong breeze
14.2 - 17.2 m/sec	near gale
17.3 - 20.8 m/sec	gale
20.9 - 24.4 m/sec	strong gale
24.5 - 28.5 m/sec	storm
28.6 - 32.6 m/sec	violent storm
> 32.6 m/sec	hurricane

Figure 36.4: Wind scale

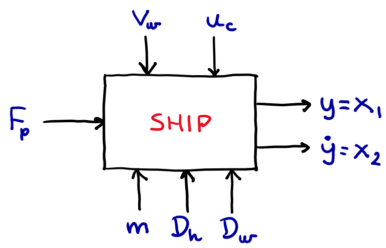

Figure 36.5: Block diagram of the ship

- Speed:

$$x_2 = y' \tag{36.8}$$

Now, the original second order differential equation, (36.4), can be represented with the following two first order differential equations:

$$x_1{}' = x_2 \tag{36.9}$$

$$x_2{}' = \frac{1}{m}\left[F_p + D_h\left(u_c - x_2\right)\left|u_c - x_2\right| + D_w\left(V_w - x_2\right)\left|V_w - x_2\right|\right] \tag{36.10}$$

36.3 Buffer tank

36.3.1 System description

Figure 36.6 shows a water tank. The tank may represent a buffer tank, or an equalization magazine, at the inlet of a plant, e.g. a water resource recovery facility (WWRF). The geometrical design of such a magazine may not have straight walls as in Figure 36.6, see Figure 1.16. However, if relatively small variations in the level are assumed, a tank with straight walls approximates the magazine with non-straight walls.

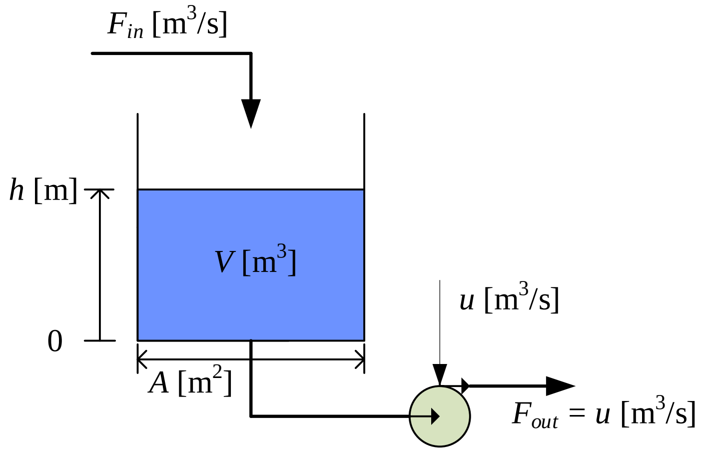

Figure 36.6: Water tank with pump outlet

36.3.2 Variables and parameters

Variables and parameters of the water tank are defined in Table 36.3. The numerical values resembles a typical operating point of the VEAS WWRF, Slemmestad, Norway.

Table 36.3: Water ank: Variables and parameters

Symbol	Value (default)	Unit	Description
h	2.0	m	Water level
F_{in}	3.0	m^3/s	Inflow
F_{out}	3.0	m^3/s	Outflow through pump
u	3.0	m^3/s	Control signal to pump
A	2000	m^2	Inner cross sectional area of tank
V	4000	m^3	Volume of water in tank

36.3.3 Overall block diagram

Figure 36.7 shows an overall block diagram of the water tank.

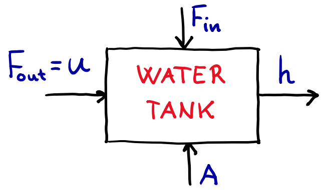

Figure 36.7: Block diagram of the water tank

36.3.4 Mathematical model

A mathematical model expressing the level variations can be derived from material balance of the water in the tank. The model is:

$$Ah' = F_{\text{in}} - F_{\text{out}} \tag{36.11}$$

where

$$F_{\text{out}} = u \tag{36.12}$$

36.4 Heated liquid tank

36.4.1 System description

Figure 36.8 shows a heated tank. Liquid (assumed water) flows into and out of the tank. The volume of liquid is constant (this can be realized with overflow or level of regulation). The inflow and outflow are thus equal. There is a heat transfer between the liquid and the air outside the tank. In the tank there are homogeneous conditions thanks to a mixer (there is thus no spatial variations in temperature). It is assumed that the mixer does not add power to the liquid. It is assumed that there is a time delay in the response in the temperature if there is a change in the supplied power. A time delay will be observed in any practical tank due to the unavoidable imperfect mixing.

36.4.2 Variables and parameters

Variables and parameters of the heated tank are defined in Table 36.4.

36.4.3 Overall block diagram

Figure 36.9 shows an overall block diagram of the heated tank.

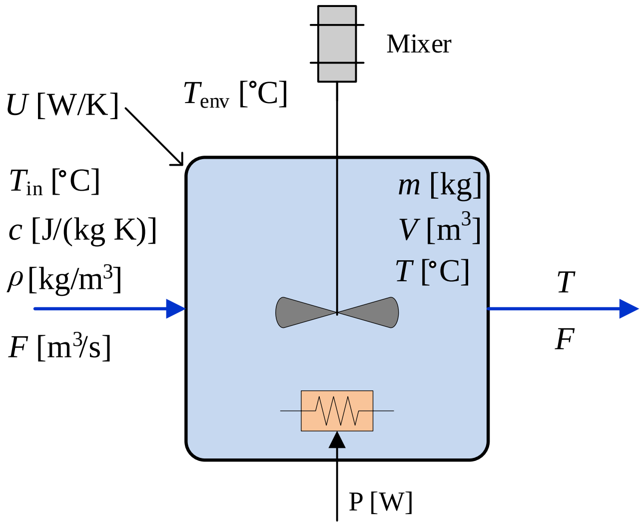

Figure 36.8: Heated tank

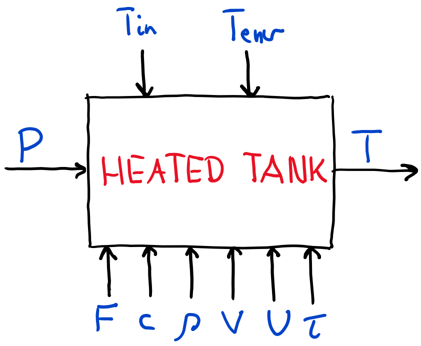

Figure 36.9: Block diagram of the heated tank

36.4.4 Mathematical model

A mathematical model of the temperature variation, based on energy balance of the liquid in the tank, is:

$$c\rho V T'(t) = P(t-\tau) + c\rho F\left[T_{\text{in}}(t) - T(t)\right] + U\left[T_{\text{env}}(t) - T(t)\right] \tag{36.13}$$

Table 36.4: Heated tank: Variables and parameters

Symbol	**Value (default)**	**Unit**	**Description**
P	0	W	Supplied power
T	20	°C	Temperature of liquid
T_{env}	20	°C	Environmental temperature
T_{in}	20	°C	Temperature of liquid inflow
F	$0.25{\cdot}10^{-3}$	m^3/s	Liquid flow
c	4200	J/(kg·K)	Specific heat capacity of liquid
ρ	1000	kg/m^3	Density of liquid
V	0.2	m^3	Liquid volume in tank
U	1000	W/K	Heat transfer coefficient of tank
τ	60 s	s	Time delay in the temperature response

36.5 Air heater

36.5.1 System description

The physical laboratory rig air heater is described on http://techteach.no/air_heater. Figure 36.10 shows the air heater.[5]

36.5.2 Variables and parameters

Variables and parameters and assumed parameter values are defined in Table 36.5.

36.5.3 Overall block diagram

Figure 36.11 shows an overall block diagram of the air heater.

36.5.4 Mathematical model

A mathematical model that has proven to describe quite well the dynamic behaviour of the outlet air temperature is given by the following differential equation representing "time constant with time delay" dynamics from control signal u to outlet temperature T:

$$\theta T'(t) = K_h \left[u(t-\tau)\right] + \left[T_{\text{env}}(t) - T(t)\right] \tag{36.14}$$

[5] University South-Eastern Norway, campus Porsgrunn, has 26 of identical units of this lab station, being used in several control courses in both bachelor and master programmes in technology.

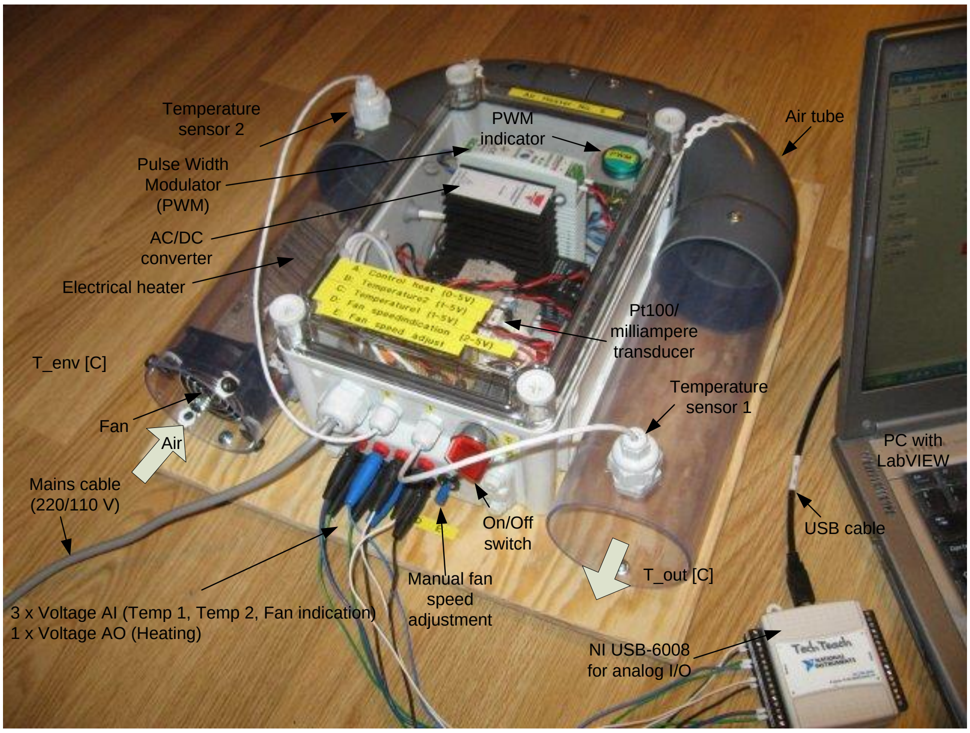

Figure 36.10: Air heater

This model may be derived from mechanistic (first-principles) modeling principles, i.e. a simple energy balance of the air, if we make the idealized assumption that the tube is a so-called CSTR (continuous stirred tank reactor) with air inflow and outflow and heat transfer with the environment through the "reactor" (tube) walls, and – in addition – we include the time delay as described above. In reality, the idealized CSTR conditions are not satisfied, but they lead to a useful model structure with parameter values that may be estimated from experimental data.

Many other stable physical processes show "time constant with time delay" dynamics, and such processes may be reasonably well represented with models similar to (36.14).

36.5.5 Data file

A datafile from an experiment with the air heater is available on:

http://techteach.no/control/python/airheater_logfile.txt

The time step (sampling time) is 0.1 s.

The datafile containt the following columns (from left in the file):

- Time t [s]

Table 36.5: Air heater: Variables and parameters

Symbol	Value (default)	Unit	Description
T	-	°C	Temperature of the air flowing out of tube. Measured by a sensor.
T_{env}	25	°C	The environmental, or ambient, temperature. It is the temperature in the outlet air of the air tube when the control signal to the heater has been set to zero for relatively long time (some minutes).
u	-	V	Control signal to heater.
K_h	3.5	°C/V	Heater gain.
θ	23.0	s	Time constant representing sluggishness of heater.
τ	3.0	s	Time delay representing air transportation and sluggishness of heater.

- Control signal u [V]
- Temperature measurement signal T [°C]

36.6 DC-motor

36.6.1 System description

Figure 36.12 shows a DC-motor with tachometer. The motor can be manipulated with an input voltage signal, u.

The rotational speed is measured with a tachometer which produces a output voltage signal which is proportional to the speed. The speed, S, is calculated continuously from the tachometer voltage, and hence the speed is assumed to be known at any instant of time. The motor and the tachometer are regarded as one unit, but we refer to it as “motor”.

36.6.2 Overall block diagram

Figure 36.13 shows an overall block diagram of the motor with tachometer.

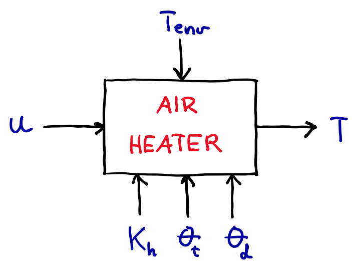

Figure 36.11: Block diagram of the air heater

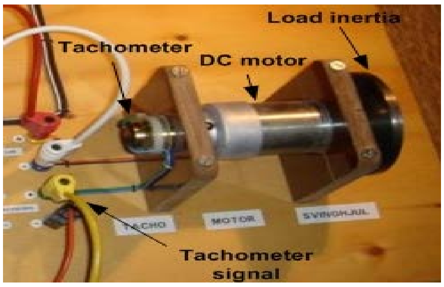

Figure 36.12: DC motor

36.6.3 Variables and parameters

Variables and parameters with assumed values are defined in Table 36.6.

36.6.4 Mathematical model

A mathematical model of the motor is (omitting the time argument for simplicity):

$$TS' = K[u + L] - S \tag{36.15}$$

The speed measurement signal generated by the tachometer is

$$S_{\mathrm{m}} = K_t S \tag{36.16}$$

From (36.16) you can calculate the speed in krpm from the measurement in voltge:

$$S = S_m / K_t \tag{36.17}$$

36.6.5 Datafile

A datafile from an experiment is available on:

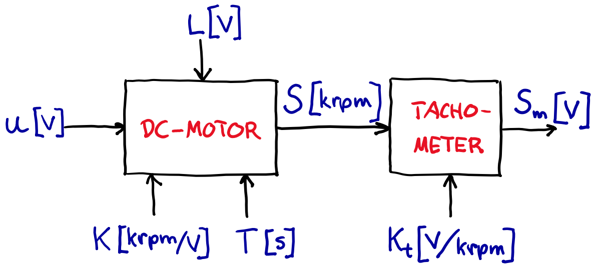

Figure 36.13: Block diagram of DC-motor with tachometer

Table 36.6: DC motor with tachometer: Variables and parameters

Symbol	**Value (default)**	**Unit**	**Description**
S	-	krpm	Rotational speed of the motor
S_m		V	Speed measurement signal generated by the tachometer
u	[−10 V, +10 V]	V	Control signal to the motor
L	0	V	Equivalent load torque, represented in the same unit as the control variable. L can be regarded as environmental variable or process disturbance.
K	0.17	krpm/V	Gain of motor with tachometer
T	0.30	s	Time constant of motor with tachometer
K_t	5.0	V/krpm	Tachometer gain

http://techteach.no/control/python/data_dc_motor.txt

The time step (sampling time) is 0.02 s.

The datafile containt the following columns (from left in the file):

- Time t [s]
- Control signal u [V]
- Tachometer measurement signal S_m [V]

36.7 Biogas reactor

36.7.1 System description

A biogas reactor is a vessel or tank in which organic matter, like food waste, slaughter waste, livestock manure, sewage sediments, etc., is converted by various cultures of microorganisms into energy-rich, combustable methane (CH_4) gas, which is the most important product, and carbondioxide (CO_2) gas. The two cultures assumed in the model presented here, are acidogens, which generate volatile fatty acids (VFAs), and methanogens which generate methane. The biological conversion process takes place without oxygen, and is often called an anaerobic digestion (AD) process. The AD process is assumed continuous, not “plug-flow”.

Biogas contains roughly 65% CH_4 and 35% CO_2. Combustion of the CH_4 gas results in CO_2 and H_2O. Biogas, raw or upgraded into approx. 98% CH_4, can be used in combustion motors to produce mechanical power to vehichles, or electrical power through a generator connected to the motor, or just heat in gas burners. The liquid phase can be used as fertilizer. The AD process is a part of a closed carbon cycle, opposite to processing and utilizing fossil fuel. The climate footprint is favourable as the combustion converts CH_4, which has (gives) a relatively high climate footprint, into CO_2 which has a considerably lower footprint, and H_2O, which has no footprint.

One of the results of my own research in model-based monitoring and control of biogas reactors Haugen (2014), is a mathematical model adapted to a pilot reactor[6] at Foss Biolab at Foss farm in Skien, Norway, using online measurements and laboratory analyses. The model may be referred to as the (modified) Hill model Hill (1983). Figure 36.14 shows the principal construction of the reactor.

36.7.2 Variables and parameters

Nomenclature of variables are shown in Table 36.7.

36.7.3 Overall block diagram

Figure 36.15 shows an overall block diagram of the mathetmatical model of the biogas reactor.

36.7.4 Mathematical model

The mathematical model is as presented below. Figure 36.15 shows an overall block diagram of the mathetmatical model. Nomenclature of variables is given in Table 36.7. Parameters with assumed values are defined in Table 36.8. Abbreviations are defined in Table 36.10.

[6]Reactor feed is filtered cow manure.

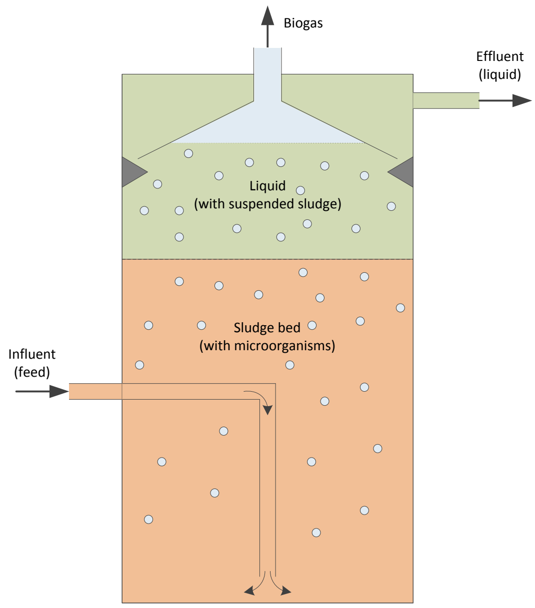

Figure 36.14: Principal construction of the biogas reactor

- Definition of the portion of the raw waste which can serve as substrate for the conversion done by microorganisms:

$$S_{\mathrm{bvs,in}} = B_0 S_{\mathrm{vs,in}} \tag{36.18}$$

- Definition of the portion of the biodegradable material which is initially in acid form:

$$S_{\mathrm{vfa,in}} = A_f S_{\mathrm{bvs,in}} \tag{36.19}$$

- Material balance of biodegradable volatile solids:

$$S_{\mathrm{bvs}}{}' = (S_{\mathrm{bvs,in}} - S_{\mathrm{bvs}})\, D - \mu k_1 X_{\mathrm{acid}} \tag{36.20}$$

- Material balance of total VFA:

$$S_{\mathrm{vfa}}{}' = (S_{\mathrm{vfa,in}} - S_{\mathrm{vfa}})\, D + \mu k_2 X_{\mathrm{acid}} - \mu_c k_3 X_{\mathrm{meth}} \tag{36.21}$$

- Material balance of acidogens:

$$X_{\mathrm{acid}}{}' = \left(\mu - K_d - \frac{D}{b}\right) X_{\mathrm{acid}} \tag{36.22}$$

- Material balance of methanogens:

$$X_{\mathrm{meth}}{}' = \left(\mu_c - K_{dc} - \frac{D}{b}\right) X_{\mathrm{meth}} \tag{36.23}$$

- Reactor volume-specific (normalized) methane gas flow rate – the gas production of the reactor:

$$q_{\mathrm{meth}} = V \mu_c k_5 X_{\mathrm{meth}} \tag{36.24}$$

Table 36.7: Biogas reactor: Variables

Symbol	**Unit**	**Description**
F	$\frac{\text{L}}{\text{d}}$	Flow, or load rate
F_{meth}	$\frac{\text{L } CH_4}{\text{d}}$	Methane gas flow
μ	d^{-1}	Reaction (growth) rate of acidogens
μ_c	d^{-1}	Reaction (growth) rate of methanogens
μ_m	d^{-1}	Maximum reaction rate for acidogens
μ_{mc}	d^{-1}	Maximum reaction rate for methanogens
$S_{\text{vs,in}}$	$\frac{\text{g VS}}{\text{L}}$	Concentration of VS in influent
$S_{\text{bvs,in}}$	$\frac{\text{g BVS}}{\text{L}}$	Concentration of BVS in influent
S_{bvs}	$\frac{\text{g BVS}}{\text{L}}$	Concentration of BVS in reactor
$S_{\text{vfa,in}}$	$\frac{\text{g VFA}}{\text{L}}$	Concentration of VFA in biodegradable part of influent
S_{vfa}	$\frac{\text{g VFA}}{\text{L}}$	Concentration of VFA acids in reactor
X_{acid}	$\frac{\text{g acidogens}}{\text{L}}$	Concentration of acidogens
X_{meth}	$\frac{\text{g methanogens}}{\text{L}}$	Concentration of methanogens
T_{reac}	°C	Reactor temperature

In the equations above:

- Actual (not normalized) methane gas flow rate:

$$F_{\text{meth}} = V q_{\text{meth}}$$

- The dilution rate is defined as reactor volume-specific (normalized) flow:

$$D = \frac{F}{V} \tag{36.25}$$

- Reaction rates, with Monod kinetics, are:

$$\mu = \mu_m \frac{S_{\text{bvs}}}{K_s + S_{\text{bvs}}} \tag{36.26}$$

$$\mu_c = \mu_{mc} \frac{S_{\text{vfa}}}{K_{sc} + S_{\text{vfa}}} \tag{36.27}$$

Table 36.8: Biogas reactor: Parameters

Symbol	Value (default)	Unit	Description
A_f	0.69	$\frac{\text{g VFA/L}}{\text{g BVS/L}}$	Acidity constant
b	2.90	d/d	Retention time ratio
B_0	0.25	$\frac{\text{g BVS/L}}{\text{g VS/L}}$	Biodegradability constant
k_1	3.89	$\frac{\text{g BVS}}{\text{g acidogens/L}}$	Yield constant
k_2	1.76	$\frac{\text{g VFA}}{\text{g acidogens/L}}$	Yield constant
k_3	31.7	$\frac{\text{g VFA}}{\text{g acidogens/L}}$	Yield constant
k_5	26.3	$\frac{\text{L}}{\text{g methanogens}}$	Yield constant
K_d	0.02	d^{-1}	Specific death rate of acidogens
K_{dc}	0.02	d^{-1}	Specific death rate of methanogens
K_s	15.5	$\frac{\text{g BVS}}{\text{L}}$	Monod half-velocity constant for acidogens
K_{sc}	3	$\frac{\text{g VFA}}{\text{L}}$	Monod half-velocity constant for methanogens
V	250	L	Reactor volume

where the maximum reaction rates are functions of the reactor temperature:

$$\mu_m\left(T_{\text{reac}}\right) = \mu_{mc}\left(T_{\text{reac}}\right) = 0.013T_{\text{reac}} - 0.129 \tag{36.28}$$
$$(20\,^\circ\text{C} < T_{\text{reac}} < 60\,^\circ\text{C})$$

The reactor temperature may be kept at a specified temperature setpoint, typically typically 35 °C, with an automatic temperature control system.

36.7.5 Operation point

In analysis of reactor dynamics and stability and in design of some types of state estimators and controllers it may be necessary to define a proper steady-state operating point. A steady-state operating point can be found from e.g. a simulation by reading off the value of the state variables at steady-state. One example of a steady-state operating point is given in Table 36.9.

Abbreviations are defined in Table 36.10.

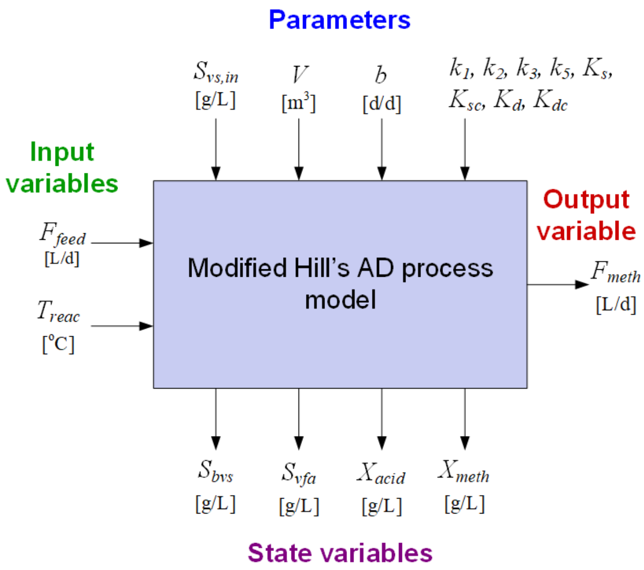

Figure 36.15: Overall block diagram of the mathematical model of the biogas reactor

Table 36.9: Values of inputs and states in one example of a steady-state operation point.

$F_{\text{feed}} = 45$ L/d
$T_{\text{reac}} = 35$ °C
$S_{\text{vs,in}} = 30.2$ g/L
$S_{\text{bvs}} = 5.2155$ g/L
$S_{\text{vfa}} = 1.0094$ g/L
$X_{\text{acid}} = 1.3128$ g/L
$X_{\text{meth}} = 0.3635$ g/L

Table 36.10: Abbreviations

VS	Volatile solids ("organic matter")
BVS	Biodegradable volatile solids
VFA	Volatile fatty acid