classmate

Date ______
Page ______

3A (i) To prove

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$$

i.e.,

$$\underbrace{\sum_{i=1}^{n} (x_i - \bar{x})^2}_{①} = \underbrace{\sum_{i=1}^{n} x_i^2 - n\bar{x}^2}_{②}$$

Expanding part ①

$$S_{xx} = \sum_{i=1}^{n} (x_i^2 - 2x_i\bar{x} + \bar{x}^2)$$

Expanding the summation

$$\boxed{S_{xx} = \sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2} \quad \text{— eq ①}$$

We know that

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} \Rightarrow \boxed{n\bar{x} = \sum_{i=1}^{n} x_i}$$

$$\Rightarrow -2\bar{x}\sum_{i=1}^{n} x_i = -2\bar{x}(n\bar{x})$$
$$= -2n\bar{x}^2$$

$$\sum_{i=1}^{n} \bar{x}^2 = n\bar{x}^2$$ [We are adding constant $\bar{x}^2$ for n times].

Now eq ① will be,

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2$$

$S_{xx} = \sum_{i=1}^{n} x_i^2 - n\bar{x}^2$ [Hence proved]

(ii) To prove,

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2$$

$$\Rightarrow S_{yy} = \underbrace{\sum_{i=1}^{n} (y_i - \bar{y})^2}_{1} = \underbrace{\sum_{i=1}^{n} y_i^2 - n\bar{y}^2}_{2}$$

Lets expand Part 1

$$\sum_{i=1}^{n} (y_i^2 - 2\bar{y}y_i + \bar{y}^2)$$

Expanding Summation:

$$\left(\sum_{i=1}^{n} y_i^2 - 2\bar{y}\sum y_i + \sum \bar{y}^2\right) \text{ — eq } 2$$

We know that,

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \Rightarrow \sum y_i = n\bar{y}$$

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - 2\bar{y}(n\bar{y}) + n\bar{y}^2$$

$$\boxed{S_{yy} = \sum_{i=1}^{n} y_i^2 - n\bar{y}^2}$$ Hence proved

iii, To prove,

$$S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}.$$

$$\Rightarrow S_{xy} = \underbrace{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}_{1} = \underbrace{\sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}_{2}$$

Expanding 1

$$\sum_{i=1}^{n} (x_i y_i - x_i\bar{y} - \bar{x}y_i + \bar{x}\bar{y}) = S_{xy}$$

$$\Rightarrow S_{xy} = \sum_{i=1}^{n} x_i y_i - \bar{y}\underbrace{\sum_{i=1}^{n} x_i}_{n\bar{x}} - \bar{x}\underbrace{\sum_{i=1}^{n} y_i}_{n\bar{y}} + \underbrace{\sum_{i=1}^{n} \bar{x}\bar{y}}_{n\bar{x}\bar{y}}$$

$$= \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y} - \cancel{n\bar{x}\bar{y}} + \cancel{n\bar{x}\bar{y}}$$

$$\boxed{S_{xy} = \sum_{i=1}^{n} x_i y_i - n\bar{x}\bar{y}}$$ Hence proved.