

13.5

Name: ~~XXXXXXXXXX~~

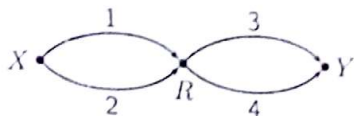
Student number: ~~XXXXXXXXXX~~

1. [9] Consider the following table of events and their probabilities

event	A	B	C	D
probability	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{8}$	$\frac{1}{4}$

- (a) [3] Suppose A and B are mutually exclusive. Find $\Pr[A \cup B]$.
 (b) [3] Suppose A and B are independent. Find $\Pr[A + B]$.
 (c) [3] Is it possible that the events A, B, C and D are mutually exclusive? Justify your answer.

2. [6] A communication network consisting of three nodes and four links is shown below.



Symbol	Event
F	No path from X to Y exists
A_i	Link i fails

- (a) [3] Write an expression for F in terms of A_1, A_2, A_3 and A_4 .
 (b) [3] Let $\Pr[A_i] = p_i$ and write an expression for $\Pr[F]$ in terms of p_i 's. Assume that links fail independent of each other.

① (a) $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B) \xrightarrow{0} \text{(mutually ex.)}$
 $= \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$ ✓

(b) $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
 $= \Pr(A) + \Pr(B) - \Pr(A) \cdot \Pr(B) \quad (i: \text{ind.})$
 $= \frac{1}{4} + \frac{1}{2} - \frac{1}{8}$ ✓
 $= \frac{3}{4} - \frac{1}{8} = \frac{5}{8}$

(c) ~~Answer~~: No, they cannot because
 $\Pr[A] + \Pr[B] + \Pr[C] + \Pr[D] > 1$,
 and hence there should be some non-zero quantity among

Q2 (a) $F = A_1 A_2 + A_3 A_4$ ✓ ③

(b) $\Pr[F] = p_1 p_2 + p_3 p_4$ 1.5

~~$\Pr[A \cap B]$ or $\Pr[B \cap C]$ or $\Pr[A \cap C]$ or $\Pr[A \cap D]$ or $\Pr[B \cap D]$ or $\Pr[C \cap D]$~~
 $\Pr[A \cap B \cap C]$ or $\Pr[A \cap B \cap D]$ or $\Pr[A \cap C \cap D]$ or $\Pr[B \cap C \cap D]$

9