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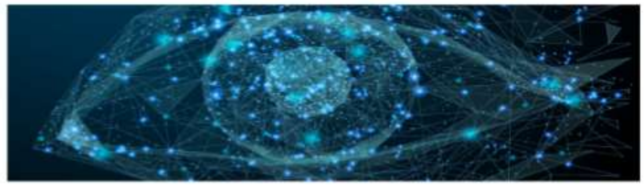
THE ROBOTICS INSTITUTE Computer Vision
16720-B Fall 2022



Hough Transform - Assignment 1

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n, Sheng-Yu
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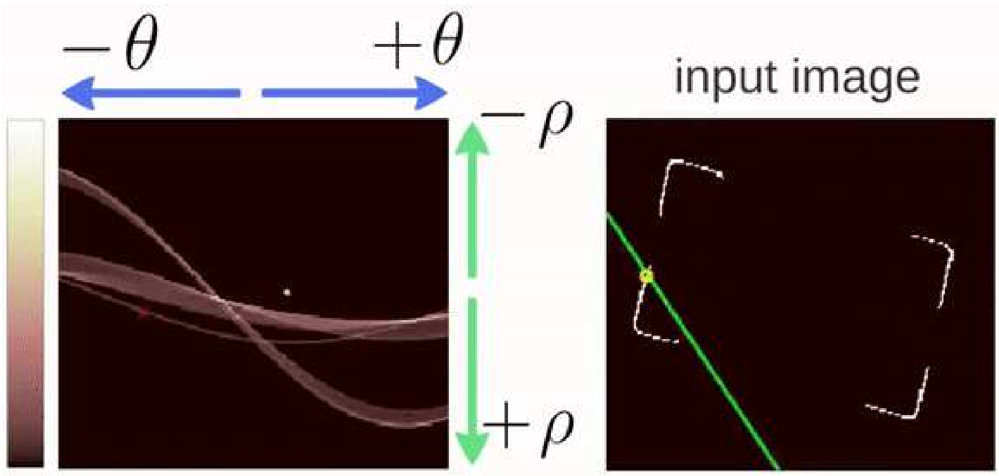
In this assignment you will be implementing a Hough Transform based line detector.



Theory Questions (25 points) ¶

Grading:

- Each question is 5 points.
- Please add your answers to the writeup. Insert images whenever necessary.
- Show all your work to obtain full credit.

Q1: Show that using $x \cos\theta + y \sin\theta - \rho = 0$, each image point (x, y) results in a sinsuoid in (ρ, θ) hough space.
Use the formulation, $a \sin\theta + b \cos\theta = A \sin(\theta + \phi)$ for the sinsuoid.
Write the amplitude A and phase ϕ of the sinusoid as a function of (x, y).

Since there are infinite lines that pass through a point, each with a unique (ρ, θ), we can say that the point can be respresented as the set of each of these pairs.

Given an (x, y) and θ, we have

$$\sin(\theta) = \frac{\rho}{(x + y\tan(\theta))}$$

implying that

$$\rho = x\cos(\theta) + y\sin(\theta)$$

Now, by dividing and multiplying with $\sqrt{x^2 + y^2}$, we get

$$\rho = \sqrt{x^2 + y^2}\,\sin(\theta + \phi)$$

where $\tan\phi = \frac{x}{y}$

Hence for a given point (x, y) the above equation represents a sinusoid in the parameter space

Q2: Why do we parameterize the line in terms of ρ, θ instead of slope and intercept (m, c)?
Also write the slope m and intercept c as a function of ρ and θ.

For the slope tangent formulation, the limits on both m and c are from $-\infty$ to $+\infty$, which makes discretizing this space challenging and computationally intensive. The (ρ, θ) formulation has the advantage of being easily boundable by the dimensions of the image and the fact that theta is in $[-\pi, \pi]$.

$$m = -\cot\theta$$
$$c = \frac{\rho}{\sin\theta}$$

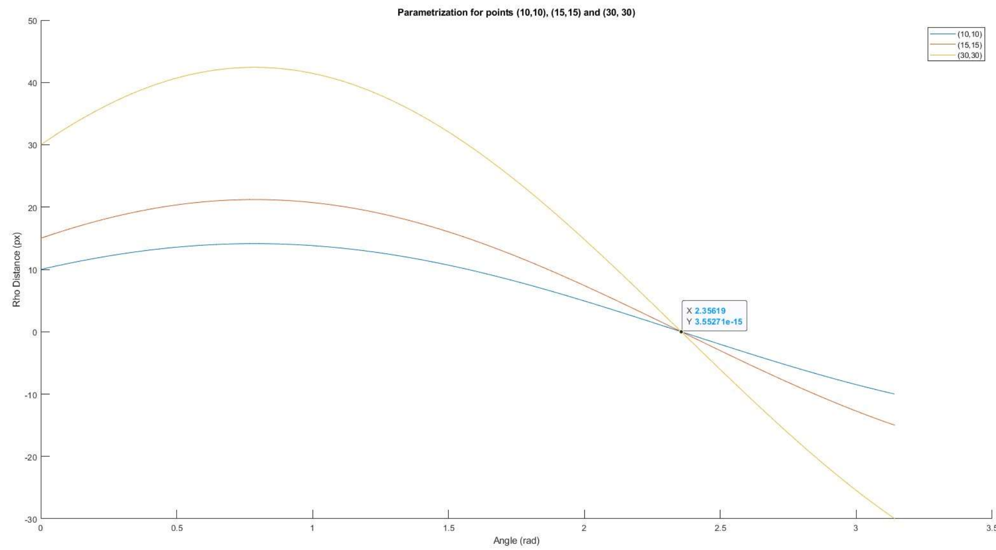
Q3: Assume the image points (x, y) are in an image of width W and height H, $x \in [1, W], y \in [1, H]$.
What is the maximum absolute value of ρ and what is the range of θ?

The max absolute value of ρ is for a point on the opposite corner of the image to the origin, with $\theta = \frac{\pi}{4}$

$$\rho_{\max} = \sqrt{W^2 + H^2}$$

Since the lines in an image must pass through the first quadrant, the possible range of values for θ is $[0, \pi]$

Q4: For points $(10, 10)$, $(15, 15)$ and $(30, 30)$ in the image, plot the corresponding sinusoid waves in Hough space (ρ, θ).
Also visualize how their intersection point defines the line (what is (m, c) for this line?).
Please add the plot as image.



The intersection point is at $\rho = 0$ and $\theta = \frac{3\pi}{4}$, signifying the line passes through the origin and is at $\frac{\pi}{4}$ since θ is perpendicular to actual line, which signifies the line $y = x$, with $m = 1$ and $c = 0$

Q5: How does the dimension of parameter space affects Hough Transform method? What would you do when the parameter space is high, i.e., 3D or 4D instead of 2D? Briefly explain your method.

Higher dimensions will allow us to capture more complex geometries in the image space as point representations in the Hough parameter space. For the 3D case, we can use the example of a circle. A circle needs three parameters to represent it, the location of the center and it's radius. Say we use polar coordinates r_c, θ for location and R as the radius - in the 3D space $r_c \times \theta \times R$ a circle would be a point, a family of concentric cirlces a plane and intersecting points of circles can be expressed as the line passing through them

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