End Semester Exam: Probability and Statistics

College For Integrated Studies
University of Hyderabad

November 29, 2025

Duration: 3 Hours
Maximum Score: 60 points

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Course Code: MM 212

Instructions: Answers without justification will receive a score of zero.

1. (a) How many different 7 place license plates are possible if the first 2 places are for letters and the other 5 for numbers under the assumption that no letter or number can be repeated in a single license plate.

   [6]

   (b) How many different 7 place license plates are possible when 3 of the entries are letters and 4 are digits under the assumption that repetition of letters and numbers is allowed and that there is no restriction on where the letters or numbers can be placed. [6]

2. (a) How many terms are there in the multinomial expansion of $\left(x_1 \;+\; x_2 \;+ \cdots + x_{2026}\right)^{2026}$. [6]

   (b) Suppose that we toss 2 fair dice. Let E denote the event that the sum of the dice is 6 and F denote the event that the first die equals 4. Examine whether E and F are independent. [6]

3. (a) There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin ? [6]

   (b) If the distribution function of X is given by

$$
F(b) = \begin{cases}
0 & b < 0 \\[2mm]
\frac{1}{2} & 0 \le b < 1 \\[2mm]
\frac{3}{5} & 1 \le b < 2 \\[2mm]
\frac{4}{5} & 2 \le b < 3 \\[2mm]
\frac{9}{10} & 3 \le b < 3.5 \\[2mm]
1 & b \ge 3.5.
\end{cases}
$$

Find the probability mass function of X. [6]

4. (a) If X is a binomial random variable such that $E[X] = 6$ and $\text{Var}(X) = 2.4$. Find $P(X = 5)$. [4]

(b) With $\Phi(x)$ being the probability that a normal random variable with mean 0 and variance 1 is less than x.

Prove / Disprove: $\Phi(-x) = \Phi(x)$, [4]

(c) If it is assumed that all $\binom{52}{5}$ poker hands are equally likely, what is the probability of being dealt a FLUSH $\Big($ A hand is said to be a flush if all 5 cards are of the same suit$\Big)$? [4]

5. (a) The joint density of X and Y is given by

$$
f(x, y) = \begin{cases}
x\, e^{-(x+y)} & x > 0, y > 0, \\
0 & \text{otherwise.}
\end{cases}
$$

Are X and Y independent ? [6]

(b) State DeMoivre - Laplace limit Theorem.

[6]

All the best !