End Semester Exam

School of Mathematics and Statistics
University of Hyderabad

December 08, 2025

Duration: 3 Hours
Maximum Score: 60 points

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Course: Eng Math - I

Instructions: You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. (a) Prove that a homogeneous system of linear equations with fewer equations than unknowns always admits nontrivial solutions. [5]

   (b) Let A and B be nilpotent matrices of the same order. Show by an example that $A + B$ need not be nilpotent. [5]

   (c) Examine whether the set of vectors

$$\left\{v = (v_1,\ v_2,\ v_3)^T \in \mathbb{R}^3 \ :\ 3v_1 - 2v_2 + v_3 = 0,\ 4v_1 + 5v_2 = 0\right\}$$

   form a vector subspace of $\mathbb{R}^3$. If so, find its dimension. [5]

2. (a) Let A be a square matrix. Let B be a matrix of same size such that $AB = I$. Then show that A is invertible and $B = A^{-1}$. [5]

   (b) Find a basis for the row space of the following matrix:

$$\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$$

   consisting entirely of the row vectors of A. [5]

   (c) Show that

$$A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 4 & 2 \\ -2 & 1 & 5 \end{bmatrix}$$

   is diagonalizable. Find A^{2026}. [5]

3. (a) If y_1 and y_2 are solutions (defined on interval I) of the following linear homogeneous ODE:

$$y'' + a(x)y' + b(x)y = 0.$$

Then show that

$$W(x_0) = 0 \text{ at some point } x_0 \Leftrightarrow W(x) = 0 \text{ at every point } x \in I.$$

[5]

(b) Using the method of variation of parameters, solve the following ODE:

$$y'' + 4y = x \sin 2x.$$

[5]

4. (a) Let

$$F(x,\ y,\ z) = (2xy^2 + 3x^2,\ 2yx^2,\ 1).$$

Evaluate

$$\int_C F \cdot dR$$

where C is the curve $\left(t^{2026},\ \sin^{2026}(\frac{\pi t}{2}),\ t\right)$, $t \in [0,\ 1]$.

[5]

(b) Let $f : \mathbb{R}^2 \longmapsto \mathbb{R}$ by

$$f(x,y) = \begin{cases} \frac{y}{|y|}\sqrt{x^2+y^2} & \text{if } y \neq 0, \\ 0\} & \text{if } y = 0. \end{cases}$$

Examine whether f is differentiable at $(0,\ 0)$. [5]

(c) Write the statement of Lagrange Multiplier Method. Does this contradict the following minimization problem of the function

$$f(x,y) = x^2 + y^2$$

subject to the condition

$$g(x,y) = (x-1)^3 - y^2 = 0.$$

[5]

(d) State the Greens Theorem. Find the area bounded by ellipse:

$$C : \frac{x^2}{2^2} + \frac{y^2}{3^2} = 1.$$

[5]

All the best !