Minor- II: Real Analysis - II

School of Mathematics and Statistics
University of Hyderabad

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Duration: 60 minutes
Maximum Score: 20 points

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Course Code: MM 451

Instructions: You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x,y) = \begin{cases} \frac{x^2y(x-y)}{x^2+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$
   Examine whether $f_{xy}(0,0) = f_{yx}(0,0)$. [3]

2. Find the 3rd order Taylor polynomial of $f(x,y,z) = x^2y + z$ about the point $(1,2,1)$.

   [4]

3. Determine all the points of local maximum, local minimum and all the saddle points of $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(x,y) = y^2 - x^3, \quad \forall \ (x,\ y) \in \mathbb{R}^2$.

   [2]

4. State TRUE or FALSE with justification for each of the following statements:

   (a) Let $f : \mathbb{R}^n \to \mathbb{R}$ be differentiable function and attain a local maximum at $b \in \mathbb{R}^n$. Then $\nabla f(b) = 0$. [3]

   (b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be differentiable function such that $\nabla f(b) = 0$ for some $b \in \mathbb{R}^2$. Then f attain either a local maximum or a local minimum at b.

   [3]

5. Let $f : \mathbb{R}^m \to \mathbb{R}^m$ be a C^1 - map such that $Df(b)$ is invertible. Then $\exists$ a neighborhood U of b such that f is $1-1$ in U.

   [5]

All the best !