

# Minor- I: Real Analysis - II

School of Mathematics and Statistics  
University of Hyderabad

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**Duration:** 60 minutes

**Maximum Score:** 20 points

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**Course Code:** MM 451

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**Instructions:** You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. Examine whether the following limits exist (in  $\mathbb{R}$ ) and find their values if they exist (in  $\mathbb{R}$ ).

(a)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3-y^3}{x^2+y^2}$  [3]

(b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^3+y^2}{x^2+y}$  [3]

2. Examine the continuity of  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  at  $(0,0)$ , where for all  $(x, y) \in \mathbb{R}^2$ ,

$$f(x, y) = \begin{cases} \frac{x^2y^3}{x^6+y^4} & \text{if } (x, y) \neq (0, 0), \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

[3]

3. Let  $\Omega \subseteq \mathbb{R}^2$ ,  $(x_0, y_0) \in \mathbb{R}^2$  and  $r > 0$  be such that  $(B_r(x_0) \times B_r(y_0)) \setminus \{(x_0, y_0)\} \subseteq \Omega$ . Let  $\lim_{x \rightarrow x_0} f(x, y)$  exist (in  $\mathbb{R}$ ) for each  $y \in B_r(y_0) \setminus \{y_0\}$ ,  $\lim_{y \rightarrow y_0} f(x, y)$  exist (in  $\mathbb{R}$ ) for each  $x \in B_r(x_0) \setminus \{x_0\}$  and  $\lim_{(x,y) \rightarrow (x_0,y_0)} f(x, y) = \ell \in \mathbb{R}$ .

Show that  $\lim_{x \rightarrow x_0} \left( \lim_{y \rightarrow y_0} f(x, y) \right) = \lim_{y \rightarrow y_0} \left( \lim_{x \rightarrow x_0} f(x, y) \right) = \ell$ . Using this fact, show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2}{x^2+y^2}$  does not exist (in  $\mathbb{R}$ ). [4 + 1]

4. Let  $f(x, y) = (x + y^2, xy)$  for all  $(x, y) \in \mathbb{R}^2$ . Using directly the definition of differentiability, show that  $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  is differentiable and also find  $f'(x_0, y_0)$ , where  $(x_0, y_0) \in \mathbb{R}^2$ . [3]

5. Let  $f(x, y, z) = (x^3y + y^2z, xyz)$  and  $g(x, y) = (x^2y, xy, x - 2y, x^2 + 3y)$  for all  $x, y, z \in \mathbb{R}$ . Use chain rule to find  $(g \circ f)'(\mathbf{a})$ , where  $\mathbf{a} = (1, 2, -3)$ . [3]

All the best !