# MM 212 (Probability and statistics )

## Practice Problem Set - 2 (Axioms of Probability)

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1. If two dice are rolled, what is the probability that the sum of the upturned faces will equal 9 ?

2. If 4 balls are "randomly drawn" from a bowl containing 8 white and 6 black balls, what is the probability that one of the balls is white and the other three black ?

3. A committee of 7 is to be selected from a group of 8 men and 9 women. If the selection is made randomly, what is the probability that the committee consists of 5 men and 2 women ?

4. An urn contains 100 balls, one of which is special. If 10 of these balls are withdrawn one at a time, with each selection being equally likely to be any of the balls that remain at the time, what is the probability that the special ball is chosen ?

5. If 23 people are present in a room, what is the probability that no two of them celebrate their birthday on the same day of the year ?

6. A box contains 3 marbles: 1 red, 1 green, and 1 blue. Consider an experiment that consists of taking 1 marble from the box and then replacing it in the box and drawing a second marble from the box. Describe the sample space. Repeat when the second marble is drawn without replacing the first marble.

7. Two dice are thrown. Let $E$ be the event that the sum of the dice is odd, let $F$ be the event that at least one of the dice lands on 1, and let $G$ be the event that the sum is 5. Describe the events $E \cap F, E \cup F, F \cap G, E \cap F^c$, and $E \cap F \cap G$.

8. Suppose that $A$ and $B$ are mutually exclusive events for which $P(A) = .3$ and $P(B) = .5$. What is the probability that:
   (a) either $A$ or $B$ occurs ?
   (b) $A$ occurs but $B$ does not ?
   (c) both $A$ and $B$ occur ?

9. An elementary school is offering 3 language classes: one in Spanish, one in French, and one in German. The classes are open to any of the 100 students in the school. There are 28 students in the Spanish class, 26 in the French class. and 16 in the German class.There are 12 students who are in both Spanish and French, 4 who are in both Spanish and German, and 6 who are in both French and German. In addition, there are 2 students taking all 3 classes.

   (a) If a student is chosen randomly, what is the probability that he or she is not in any of the language classes ?
   (b) If a student is chosen randomly, what is the probability that he or she is taking exactly one language class ?
   (c) If 2 students are chosen randomly, what is the probability that at least 1 is taking a language class ?

10. Poker dice is played by simultaneously rolling 5 dice.   Show that:

   (a)  P(no two alike ) $=$ .0926
   (b)  P(one pair ) $=$ .4630
   (c)  P (two pair ) $=$ .2315
   (d)  P (three alike ) $=$ .1543
   (e)  P (full house ) $=$ .0386
   (f)  P (four alike ) $=$ .0193
   (g)  P (five alike ) $=$ .0008.

11. An urn contains 5 red, 6 blue and 8 green balls. If a set of 3 balls is randomly selected, what is the probability that each of the balls will be :
   (a)  of the same color ?
   (b)  of different colors ?


12. Repeat the previous problem 11, under the assumption that whenever a ball is selected, its color is noted and it is then replaced in the urn before the next selection. This is known as sampling *with replacement*.

13. An urn contains $n$ white and $m$ black balls, where $n$ and $m$ are positive numbers.
   (a) If two balls are randomly withdrawn, what is the probability that they are the same color ?
   (b)  If a ball is randomly withdrawn and then replaced before the second one is drawn, what is the probability that the withdrawn balls are the same color ?
   (c)  Show that the probability in part $(b)$ is always larger than the one in part $(a)$.

14. A pair of fair dice is rolled. What is the probability that the second die lands on a higher value than does the first ?

15. If two dice are rolled, what is the probability that the sum of the upturned faces equals $i$  ? Find it for $i = 2, 3, ..., 11, 12$.

16. Seven balls are randomly withdrawn from an urn that contains 12 red, 16 blue and 18 green balls. Find the probability that:
   (a)  3 red, 2 blue and 2 green balls are withdrawn
   (b)  at least 2 red balls are withdrawn
   (c)  all withdrawn balls are the same color
   (d)  either exactly 3 red balls or exactly 3 blue balls are withdrawn.

17. Five people, designated as $A$, $B$, $C$, $D$, $E$ are arranged in linear order. Assuming that each possible order is equally likely, what is the probability that:
   (a)  there is exactly one person between $A$ and $B$
   (b)  there are exactly two people between $A$ and $B$
   (c)  there are three people between $A$ and $B$ ?

18. From a group of 3 first-year students, 4 sophomores, 4 juniors and 3 seniors, a committee of size 4 is randomly selected. Find the probability that the committee will consist:
   (a)  1 from each class
   (b)  2 sophomores and 2 juniors
   (c)  only sophomores or juniors.

19. A six digit number is chosen at random. What is the probability that at least two consecutive digits in the chosen number are equal ?