Major Exam: Real Analysis II

School of Mathematics and Statistics
University of Hyderabad

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Duration: 3 Hours
Maximum Score: 60 points

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Course Code: MM 451

Instructions: You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. State and prove Chain rule. [1 + 6]

2. State and prove Inverse Function Theorem. [1 + 6]

3. State and prove Implicit Function Theorem. [1 + 5]

4. Evaluate the following triple integral and make a sketch of the region of integration:

$$\iiint_S \frac{1}{(1+x+y+z)^3}\, dx\, dy\, dz$$

where S is the solid bounded by the coordinate planes $x = 0,\ y = 0,\ z = 0$ and the plane $x + y + z = 1$. [4]

5. Let $R = \left\{(x,\ y) \in \mathbb{R}^2 :\ 9x^2 + 4y^2 \leq 1\right\}$. Find the $\iint_R \sin\left(9x^2 + 4y^2\right) dxdy$. [4]

6. Evaluate the following integral:

$$\iiint_S (x^2 + y^2)\, dx\, dy\, dz$$

where S is the solid bounded by the surface $2z = x^2 + y^2$ and the plane $z = 2$. [4]

7. Evaluate the following integral:

$$\iiint_S dx\, dy\, dz$$

where S is sphere of radius a and center at the origin. [4]

8. Use Green's theorem to evaluate the line integral $\oint_C y^2\, dx + x\, dy$, when C has the vector equation $\alpha(t) = (2\cos^3 t,\ 2\sin^3 t),\ \ 0 \le t \le 2\pi$.

[4]

9. Let S denote the hemisphere $x^2 + y^2 + z^2 = 1,\ z \ge 0$ and $F(x,\ y,\ z) = (x,\ y,\ 0)$. Let η be the unit outward normal to S. Compute the value of the surface integral $\iint_S F \cdot \eta\, dS$ using:

   (a) Considering S as a graph: $z = f(x,\ y)$. [2]

   (b) Considering S as a parametric surface (but not as a graph). [2]

10. Compute the area of the surface

$$x = uv,\ \ y = u + v,\ \ z = u - v$$

where

$$(u,\ v) \in R := \left\{(s,\ t) \in \mathbb{R}^2\ :\ s^2 + t^2 \le 1\right\}.$$

[4]

11. Let S be the sphere $x^2 + y^2 + z^2 = 9$, with $z \ge 0$. Find

$$\iint_S \left((z + \cos z)x + y^2 + xz\right) d\sigma.$$

[4]

12. Let S be the sphere $x^2 + y^2 + z^2 = 9$. Assume that for some $\beta \in \mathbb{R}$,

$$\iint_S \left(zx + \beta y^2 + xz\right) d\sigma = \frac{4\pi}{3},\ \ \text{Find } \beta.$$

[4]

13. Let $F(x,\ y,\ z) = (y^2,\ xy,\ xz)$, where S is the hemisphere $x^2 + y^2 + z^2 = 1,\ z \ge 0$ and η is the unit normal having a nonnegative z- component. Using the Stokes' theorem, transform the surface integral

$$\iint_S \operatorname{curl} F \cdot \eta\, dS$$

to a line integral and then evaluate the line integral

[2 + 2]

All the best !