Minor- I

School of Mathematics and Statistics
University of Hyderabad

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Duration: 60 minutes
Maximum Score: 20 points

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Course: Eng Math - I

Instructions: You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. Recall that trace of a square matrix $A = [a_{ij}]$ denoted by $tr(A)$ is defined as the sum of its diagonal elements: $tr(A) = \sum_i a_{ii}$. For square matrices A, B of the same size, show that

$$tr(AB) = \mathrm{tr}(BA).$$

[5]

2. Find the values of real numbers $\alpha,\ \beta,\ \gamma$ such that the following system of algebraic equations

$$x + 2y + \alpha z = 0, \ \ 2x + \beta y + 5z = \gamma$$

admits no solution.

[5]

3. Let A, B be symmetric square matrices of the same size. Show that AB is a symmetric matrix if and only if $AB = BA$. [5]

4. If $v = (v_1,\ v_2,\ \cdots, v_n)$ and $w = (w_1,\ w_2,\ \cdots, w_n)$ are vectors in $\mathbb{R}^n$, we define the inner product of v and w by

$$v \cdot w = v_1 w_1 + v_2 w_2 + \cdots + v_n w_n.$$

Let x be a fixed vector in $\mathbb{R}^n$.

Examine whether the set of vectors $\left\{ v \ : \ v \cdot x = 1 \right\}$ form a vector subspace of $\mathbb{R}^n$. Justify your answer.

[5]

All the best !