MM 212 (Probability and Statistics)

Practice Problem Set - 3

(Conditional Probability and Independence)

Instructor: Dharmendra Kumar

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1. Urmi is 80 percent certain that his missing key is in one of the two pockets of his hanging jacket, being 40 percent certain it is in the left-hand pocket and 40 percent certain it is in the right-hand pocket. If a search of the left-hand pocket does not find the key, what is the conditional probability that it is in the other pocket ?

2. A coin is flipped twice. Assuming that all four points in the sample space
$$\mathbb{S} \ = \ \big\{(H,H),(H,T),(T,H),(T,T)\big\}$$
are equally likely.
What is the conditional probability that both flips land on heads, given that:
(a) the first flip lands on heads,
(b) at least one flip lands on heads ?

3. In the card game bridge, the 52 cards are dealt out equally to 4 players–called East, West, North and South. If North and South have a total of 8 spades among them. What is the probability that East has 3 of the remaining 5 spades ?

4. Suppose that an urn contains 8 red balls and 4 white balls. We draw 2 balls from the urn without replacement. If we assume that at each draw, each ball in the urn is equally likely to be chosen, what is the probability that both balls drawn are red ?

5. An ordinary deck of 52 playing cards is randomly divided into 4 piles of 13 cards each. Compute the probability that each pile has exactly 1 ace.

6. Two fair dice are rolled. What is the conditional probability that at least one lands on 6 given that the dice land on different numbers ?

7. If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i ? Compute for all values of i between 2 and 12.

8. Compute in the hand of bridge the conditional probability that East has 3 spades given that North and South have a combined total of 8 spades.

9. What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i, $\ i = 2, 3, \cdots, 11, 12$.

10. An urn contains 6 white and 9 black balls. If 4 balls are to be randomly selected without replacement, what is the probability that the first 2 selected are white and the last 2 black ?

11. The king comes from a family of 2 children. What is the probability that the other child is his sister ?

12. A couple has 2 children. What is the probability that both are girls if the older of the two is a girl ?

13. An urn initially contains 5 white and 7 black balls. Each time a ball is selected, its colour is noted and it is replaced in the urn along with 2 other balls of the same colour. Compute the probability that:
(a) the first 2 balls selected are black and the next 2 are white,
(b) of the first 4 balls selected, exactly 2 are black ?

14. In a certain community, 36 percent of the families own a dog and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is:

    (a) the probability that a randomly selected family owns both a dog and a cat ?
    (b) the conditional probability that a randomly selected family owns a dog given that it owns a cat ?

15. There are 3 coins in a box. One is a two-headed coin, another is a fair coin, and the third is a biased coin that comes up heads 75 percent of the time. When one of the 3 coins is selected at random and flipped, it shows heads. What is the probability that it was the two-headed coin ?

16. Show that if $P(A/B) = 1$, then $P(B^c/A^c) = 1$.

17. There is a 60 percent chance that event A will occur. If A does not occur, then there is a 10 percent chance that B will occur. What is the probability that at least one of the events A or B will occur ?

18. Show that if $P(A) > 0$, then
$$P\big(A \cap B/A\big) \;\;\geq\;\; P\big(A \cap B/A \cup B\big).$$

19. Let $A \subset B$. Express the following probabilities as simply as possible:
$$P\big(A/B\big),\;\; P\big(A/B^c\big),\;\; P\big(B/A\big),\;\; P\big(B/A^c\big).$$

20. A family has n children with probability p_n, where $p_1 = .1,\ p_2 = .25,\ p_3 = .35,\ p_4 = .3$. A child from this family is randomly chosen. Given that this child is the eldest child in the family, find the conditional probability that the family has:

    (a) only 1 child,
    (b) 4 children.

    Redo (a) and (b) when the randomly selected child is the youngest child of the family.

21. Suppose that we toss 2 fair dice. Let E denote the event that the sum of the dice is 6 and F denote the event that the first die equals 4. Examine whether E and F are independent.

22. Multiple-choice test consists of 20 questions. Each question has four choices, exactly one of which is correct. For each question, a student is able to correctly identify one of the choices as wrong and chooses one answer at random from the remaining three choices. The student will get a scholarship if she answers at least 18 questions correctly. What is the probability that she gets the scholarship ?