Minor- III: Real Analysis - II

School of Mathematics and Statistics
University of Hyderabad

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Duration: 60 minutes
Maximum Score: 20 points

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Course Code: MM 451

Instructions: You may use results proven in the lectures; however, answers without justification will receive a score of zero.

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1. (a) Write the statement of Implicit Function Theorem. [2]

   (b) Write the statement of Lagrange multipliers method. [2]

2. (a) Prove / Disprove: The equation $\sin(xyz) = z$ defines x implicitly as a differentiable function of y and z locally around the point $(x, y, z) = (\frac{\pi}{2}, 1, 1)$. [3]

   (b) Prove / Disprove: The equation $x^2 + y + \sin(xy) = 0$ defines x implicitly as a differentiable function of y locally around the point $(x, y) = (0, 0)$. [3]

3. Use Lagrange multipliers to find the maximum of

$$f(x) = \big(x_1 x_2 \cdots x_n\big)^2$$

   subject to the constraint $x_1^2 + x_2^2 + \cdots + x_n^2 = 1$. Using this, Prove that the AM-GM inequality. [3 + 3]

4. Let $f(x,\ y,\ z) = (2xy^2 + 3x^2,\ 2yx^2,\ 1)$. Calculate $\int_{C_i} f\ d\alpha$, where $\mathrm{C_i}$ $i = 1, 2$ are given by as follows:

   (a) C_1 is the curve $\big(t^{2025},\ \sin^{2025}(\frac{\pi t}{2}),\ t\big),\ \ 0 \leq t \leq 1.$ [2]

   (b) C_2 is the curve obtained by intersecting the surfaces $x^2 + y^2 = 1$ and $z = x^2 + y^2$. [2]

All the best !