Minor- I: Functional Analysis

School of Mathematics and Statistics
University of Hyderabad

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Duration: 60 minutes | **Instructor: Dharmendra Kumar**
Maximum Score: 20 points | **Course Code: MM 501**

Instructions: No mark will be given for writing only TRUE or FALSE (without justification) in Question 1.

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1. State TRUE or FALSE giving proper justification for each of the following statements:

   (a) $\Big(C[0,\ 1],\ \ \|\cdot\|_\infty\Big)$ is separable. [2]

   (b) $\Big(C^1[0,\ 1],\ \ \|\cdot\|_\infty\Big)$ is Banach space. [1]

2. (a) State and prove Hölder's inequality for any complex numbers $x_1, x_2, \cdots, x_n$ and $y_1, y_2, \cdots, y_n$. [3]

   (b) If $1 \leq p < q < \infty$, then show that $\|x\|_p \leq n^{\frac{1}{p}-\frac{1}{q}}\|x\|_q$ for all $x \in \mathbb{K}^n$. [1]

   (c) Show that c_{00} is dense in $(c_0, \|\cdot\|_\infty)$ and also in $(\ell^p, \|\cdot\|_p)$ for $1 \leq p < \infty$.

   [2 + 2]

3. Examine whether $\|\cdot\|$ is a norm on $C[0,1]$, where $\|x\| = \min\{\|x\|_\infty, 2\|x\|_1\}$ for all $x \in C[0,1]$. [2]

4. Examine whether $\|\cdot\|$ is a norm on $C[0,1]$, where $\|x\| = \min\{\|x\|_\infty, \frac{1}{2}\|x\|_1\}$ for all $x \in C[0,1]$. [1]

5. Let E and F be normed linear spaces and $T : E \longrightarrow F$ be bounded linear map. Show that

$$\big\|T\big\| \; = \; \sup_{\|x\|=1} \; \|T(x)\|.$$

   [2]

6. Let E and F be normed linear spaces and $T : E \longrightarrow F$ be linear map. Show that TFAE:

   (a) T is continuous.

   (b) T sends Cauchy sequences in E to Cauchy sequences in F.

   [1 + 3]