MM 212 (Probability and Statistics)

Practice Problem Set - 6

(Jointly Distributed Random Variables)

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1. Suppose that 15 percent of the families in a certain community have no children, 20 percent have 1 child, 35 percent have 2 children and 30 percent have 3. Suppose further that in each family each child is equally likely (independently) to be a boy or a girl. If a family is chosen at random from this community then $B :=$ the number of boys and $G :=$ the number of girls in this family. Find the joint probability mass function of B and G. Also, find the prob. mass function of B and G.

2. Two fair dice are rolled. Find the joint probability mass function of X and Y when:
   (a) X is the largest value obtained on any die and Y is the sum of the values.
   (b) X is the value on the first die and Y is the larger of the two values.
   (c) X is the smallest and Y is the largest value obtained on the dice.

3. Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls.
   (a) Let X_i equal to 1 if the i^{th} ball selected is white and let it equal to 0 otherwise. Find the joint pmf of $X_1,\ X_2$, and $X_1,\ X_2,\ X_3$.
   (b) Suppose that the white balls are numbered and Y_i equal to 1 if the i^{th} white ball is selected and 0 otherwise. Find the joint pmf of Y_1, Y_2 and Y_1, Y_2, Y_3.

4. Repeat Problem 3, when the ball selected is replaced in the urn before the next selection.

5. The joint probability density function of random variables X and Y is given by
$$f(x,\ y)\ =\ c\Big(y^2 - x^2\Big)\ e^{-y}, \quad -y \le x \le y,\ \ 0 < y < \infty.$$
   (a) Find c.
   (b) Find the marginal densities of X and Y.
   (c) Find $E[X]$.

6. The joint probability density function of random variables X and Y is given by
$$f(x,\ y)\ =\ \frac{6}{7}\left(x^2 + \frac{xy}{2}\right), \quad 0 < x < 1,\ \ 0 < y < 2.$$
   (a) Verify that this is indeed a joint density function.
   (b) Find the marginal densities of X and Y.
   (c) Find $P\{X > Y\}$.
   (d) Find $P\Big\{Y > \frac{1}{2} \mid X < \frac{1}{2}\Big\}$.
   (e) Find $E[X]$.
   (f) Find $E[Y]$.

7. The joint probability density function of random variables X and Y is given by

$$f(x,\ y) \ = \ e^{-(x+y)}, \quad 0 \leq x < \infty, \ \ 0 \leq y < \infty.$$

Find:
(a) $P\{X < Y\}$,
(b) $P\{X < a\}$.

8. A television store owner figures that 45 percent of the customers entering his store will purchase an ordinary television set, 15 percent will purchase a plasma television set, and 40 percent will just be browsing. If 5 customers enter his store on a given day, what is the probability that he will sell exactly 2 ordinary sets and 1 plasma set on that day ?

9. The number of people who enter a drugstore in a given hour is a Poisson random variable with parameter $\lambda = 10$. Compute the conditional probability that at most 3 men entered the drugstore, given that 10 women entered in that hour. What assumptions have you made ?

10. A man and a woman agree to meet at a certain location about 12 : 30 P.M. If the man arrives at a time uniformly distributed between 12 : 15 and 12 : 45 and if the woman independently arrives at a time uniformly distributed between 12 : 00 and 1 P.M. Find the probability that the first to arrive waits no longer than 5 minutes. What is the probability that the man arrives first ?

11. The joint density of X and Y is given by

$$f(x,y) \ = \ \begin{cases} x\ e^{-(x+y)} & x > 0, y > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent ?

12. The joint density of X and Y is given by

$$f(x,y) \ = \ \begin{cases} 2 & 0 < x < y,\ 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Are X and Y independent ?

13. The joint probability mass function of the random variables X, Y, Z is

$$p(1,2,3) = p(2,1,1) = p(2,2,1) = p(2,3,2) = \frac{1}{4}.$$

Find $E[XYZ]$ and $E[XY + XZ + YZ]$.