MM 212 (Probability and Statistics)

Practice Problem Set - 4

(Discrete Random Variables)

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1. Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win 2 rupees for each black ball selected and we lose 1 rupees for each white ball selected. Let X denote our winnings. What are the possible values of X, and what are the probabilities associated with each value ?

2. Two fair dice are rolled. Let X equal the product of the 2 dice. Compute $P(X = i)$ for $i = 1, \cdots, 36$.

3. Three dice are rolled. Find the probabilities attached to the possible values that X can take on, where X is the sum of the 3 dice.

4. 5 men and 5 women are ranked according to their scores on an examination. Assume that no two scores are alike and all 10! possible rankings are equally likely. Let X denote the highest ranking achieved by a woman (For instance, $X = 1$ if the top-ranked person is female).
$$\text{Find} \quad P(X = i), \quad i = 1, 2, 3, \cdots, 8, 9, 10.$$

5. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X ?. For $n = 3$, if the coin is assumed fair, what are the probabilities associated with the values that X can take on ?

6. Suppose that a die is rolled twice. What are the possible values that the following random variables can take on:
   (a) the maximum value to appear in the two rolls,
   (b) the minimum value to appear in the two rolls,
   (c) the sum of the two rolls,
   (d) the value of the first roll minus the value of the second roll ?.

   Find the probabilities associated with the random variables in parts (a) through (d).

7. Four balls are to be randomly selected with replacement from an urn that contains 20 balls numbered 1 through 20. If X is the largest numbered ball selected. What are the possible values that X can take on. Also find the value of $P(X > 10)$.

8. Plot the graph of the pmf of the random variable representing the sum when two dice are rolled.

9. Let X be the winnings of a gambler and $p(i) = P(X = i)$ be pmf of X. Assume that
$$p(0) = \frac{1}{3}, \;\; p(1) = p(-1) = \frac{13}{55}, \;\; p(2) = p(-2) = \frac{1}{11}, \;\; p(3) = p(-3) = \frac{1}{165}.$$

Find the conditional probability that the gambler wins i, $i = 1, 2, 3$ given that he/she wins a positive amount.

10. Four independent flips of a fair coin are made. Let X denote the number of heads obtained. Plot the pmf of the random variable $X - 2$.

11. If the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{1}{2} & 0 \le b < 1 \\ \frac{3}{5} & 1 \le b < 2 \\ \frac{4}{5} & 2 \le b < 3 \\ \frac{9}{10} & 3 \le b < 3.5 \\ 1 & b \ge 3.5. \end{cases}$$

Find the probability mass function of X.

12. Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \le b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \le b < 2 \\ \frac{11}{12} & 2 \le b < 3 \\ 1 & b \ge 3. \end{cases}$$

(a) Find $P(X = i)$ for $i = 1, 2, 3$ and $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$.

13. Suppose that two teams play a series of games that ends when one of them has won i games. Suppose that each game played is independently won by team A with probability p. Find the expected number of games that are played when (a) $i = 2$ and (b) $i = 3$. Also, show in both cases that this number is maximised when $p = \frac{1}{2}$.

14. If $\mathrm{E}[X] = 1$ and $\mathrm{Var}(X) = 5$ then find:

(a) $\mathrm{E}\big[(2 + X)^2\big]$,

(b) $\mathrm{Var}(4 + 3X)$.

15. When coin 1 is flipped, it lands on heads with probability .4. When coin 2 is flipped, it lands on heads with probability .7. One of these coins is randomly chosen and flipped 10 times.

    (a) What is the probability that the coin lands on heads on exactly 7 of the 10 flips ?

    (b) Given that the first of these 10 flips lands heads, what is the conditional probability that exactly 7 of the 10 flips land on heads ?

16. Suppose that a biased coin that lands on heads with probability p is flipped 10 times. Given that a total of 6 heads results, find the conditional probability that the first 3 outcomes are:

    (a) h, t, t (means the first flip results in heads, the second in tails, and the third in tails)

    (b) t, h, t.

17. Compare the Poisson approximation with the correct binomial probability for the following cases:

    (a) $P(X = 2)$ when $n = 8, \quad p = .1,$

    (b) $P(X = 9)$ when $n = 10, \quad p = .95,$

    (c) $P(X = 0)$ when $n = 10, \quad p = .1,$

    (d) $P(X = 4)$ when $n = 9, \quad p = .2.$

18. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $\frac{1}{100}$, what is the (approximate) probability that you will win a prize:

    (a) at least once ?

    (b) exactly once ?

    (c) at least twice ?

19. A fair coin is continually flipped until heads appear for the 10th time. Let X denote the number of tails that occur. Compute the probability mass function of X.

20. Let X be random variable such that

    $$P(X = 1) = p = 1 - P(X = -1).$$

    Find $c \neq 1$ such that $E[c^X] = 1$.

21. Find Var(X) if

    $$P(X = a) = p = 1 - P(X = b).$$

22. If X is a binomial random variable such that $E[X] = 6$ and $\text{Var}(X) = 2.4$. Find $P(X = 5)$.