**MATH MODELLING**

**First Round Questions**

1. Prove that in a group of N people, if some of them shake hands, there are at least two who have the same number of handshakes.

2. Pauline was walking through a train tunnel on her way to town. Suddenly, she heard the whistle of a train approaching from behind her! Pauline knew that the train always traveled at an even 60 miles per hour. She also knew that she was exactly three-eighths of the way through the tunnel. She could tell from the train whistle how far the train was from the tunnel. Pauline wasn't sure if she should run forward as fast as she could or run back to the near end of the tunnel. She did some lightning fast calculations, based on how fast she could run and the length of the tunnel. She figured out that whichever way she ran, she would just barely make it out of the tunnel before the train reached her. Whew! How fast could Pauline run?

3. Sam had never fully understood the points system in soccer (football), and felt that the scoring of goals should be encouraged. His idea is that 10 points should be awarded for a win, 5 points for a draw and 1 point for each goal scored, whatever the result of the match. This was tried with three teams, Hubs, Dins and Rungs. Each team scored at least one goal in every mach and no team played another more than once. Hubs scored 8 points, Dins 14 points and Rungs scored 9 points. Find the score in each match.

4. If you have two 1 litre bottles of wine, one red, the other white, and you pour 100 ml of the red into the white, and then return100 ml of the mixture, what are proportions of red and white in the two bottles ?

5. In a single elimination tournament, you need ceil (log2(players)) rounds.(2 players, 1 round; 3-4 players, 2 rounds; 5-8 players 3 rounds, etc.)In double elimination, players get a second chance. After losing in the winners' bracket, they enter a second bracket, and the winner of that emerges as a tournament finalist. How many rounds are there in the losers' bracket ?

6.Consider the sequence of numbers u(1), u(2), u(3), ... where u(n+1) = [u(n)² + 4] / [u(n) +2]As n --> infinity, u(n) --> a. Find a, and show that it's the limit.

7. MEMO+ FROM = HOMER. How?

8. A fruit vendor sells fruit in three combinations:

-- Small bag: 1 apple and 2 bananas.

-- Medium bag: 4 apples and 3 bananas.

-- Large bag: 8 apples and 7 bananas.

Noticing that the large bags are selling better than the others ,he decides to scrap the smaller sizes and just sell the big ones. In what proportions can be combine small and medium bags to make large ones, without any fruit left over ?

9. Suppose you have a grassy field, and cows eat grass at a constant rate.Keep in mind, the grass keeps growing continuously. 48 cows can clear all the grass off the field in 90 days. 120 cows can clear all the grass off the field in 30 days. How many cows would be needed to clear all of the grass in 16 days? Round up to the nearest whole cow.

10. A lorry, a van and a car set off at the same time travelling at a constant speed of 60km/h, 80km/h and 120km/h respectively.The lorry and the van were travelling from Town G to Town H,while the car was travelling from Town H to Town G. The car passed the lorry 2 minutes after passing the van. Find the distances between the towns and where the passes occurred.

11. A man dies and leave his estate to his sons. The estate is divided as follows:1st son gets 100 crowns + 1/10 of remainder of estate.2nd son gets 200 crowns + 1/10 of remainder of estate ...(n)th son gets 100 × (n) crowns + 1/10 of the remainder. Each son receives same amount. How many sons were there, what did each receive, and what was the estate?

12.If a triangle of area 9 square units is formed by the x-axis and the lines x= 1 and y = mx - 4 (m < 0), what are the coordinates of the vertices ?

13. A power house, P, is on one bank of a straight river 200 m wide, and a factory, F, is on the opposite bank 400m downstream from P. The cable has to be taken across the river,under water at a cost of $6/m. On land the cost is $3/m. What path should be chosen so that the cost is minimized?

14. Verna decided to sell her collection of books. To Fred, she sold 2 books, and one fifth of what was left. Later to Joan she sold 6 books, and one fifth of what then remained. If she sold more books to Fred than to Joan,what was the least possible number of books in her original collection ?

15. Given two unmarked jugs, one which holds 7 liters, and another which holds 11 liters, an unlimited supply of water, and no need to conserve, how do you measure exactly 6 liters?

16. What is the largest even integer that can NOT be written as a sum of 2 odd composite positive integers?

17. Find a problem in this format, where the answer in multiplication is the reverse of the answer in addition:

\_\_ \_\_ + \_\_ = \_\_ \_\_

\_\_ \_\_ × \_\_ = \_\_ \_\_

18. Suppose that two teams called the Bears and Wildcats are in a play off series where the first team to win 3 games wins the series. Suppose that for each game they play, the probability that the Bears win is 0.58 and the probability the Wildcats win is 0.42.

a) What is the probability that the Bears win the series in 4 games?

b) What is the probability that the Bears win the series in 5 games?

19. If mother looks at kid two hours a day and baby looks at mom a total of 3 hrs a day, what is the probability they will look at each other the same time?

20. Tim has a solid wooden cube with whole number dimensions. He paints the entire surface of the cube red. Then, with slices parallel to the faces of the cube, Tim cuts the cube into 1x1x1 cubes. A certain number of the small cubes are completely free of paint (x). A certain number of the small cubes are painted red on only one side (y). A certain number of the small cubes are painted red on two sides (z).

A) If y is twice as big as x, what was Tim's original cube size ?

B) If x is twice as big as y, what was Tim's original cube size ?

C) If y + z is 33% of x, what was Tim's original cube size ?

21.A gorilla harvests 3,000 bananas and needs to carry them 1,000 miles to the supermarket. He can only carry 1,000 at a time. Since he is a gorilla he eats 1 banana every mile he goes in any direction. He can(and will have to) leave bananas anywhere along the way. Once all his bananas have reached the end he DOES NOT need any to eat to get back .Remember he eats 1 banana every mile he goes even if he is going back to pick up more bananas. What is the maximum number of bananas he can get to the market?

**ROUND-2 QUESTIONS**

**(2Modelling+1Story writing)**

1. An increase in the level of tax on cigarettes will bring in extra tax revenue, however, it could lead to a reduction in the number of cigarettes sold, which could mean less revenue. If the tax is increased by x pence per packet, what is the limit on the value of x if the increase is to be worthwhile from the point of view of the Chancellor of the Exchequer?

List of factors is tabulated

**Description Type Symbol Unit**

Number of packets Currently sold daily Input parameter **N**  Tax on a single packet Input parameter t pence Tax increase on a single packet Variable x pence reduction in cigarette sales Variable r

Assume that

1. At present, N packets of cigarettes are sold everyday and there is a tax t on each packet.

2. The reduction in the number of cigarettes smoked is propotional to the increase in price.

2. consider the following problem:

An investor has $40,000 to invest.she is considering investments in savings at 7%,municipal bonds at 9% and stocks that have been consistently averaging 14%.Because there are varying degrees of risk involved in the various investments ,the investor has listed the following goals for the portfolio:

1.A yearly return of atleast $5000.

2.An investment of atleast $10000 in stocks.

3.The investment in stocks should not exceed the combined total in bonds and savings.

4.A liquid savings account between $5000 and $15000

5.The total investment must not exceed $40000.

we can see from the portfolio that the investor has more than one objective.Unfortunately,as is often the casewith real world problems,not all goals can be achieved simultaneously. If the investement returning the lowest yieldis set as low as possible(in this problem,$5000 into savings),the invest return possible without violating Goals2-5 is obtainbed by investing $15000 in bonds and $20000 in stocks. However,this portfolio falls short of thedesired yearly return of $5000. How are problems with more than one objective reconciled?

**STORY WRITING**

**(Using Mathematical words)**

1.Makeup a conversation you might have with an independent chimpanzee who is a stowaway on a submarine?

2.Write about a dreamworld with a sly sprite who is transformed into a willow tree?

3.Describe to your friend a humble musician who discovers shipwreck while scuba diving??