Equivariant Structured Positional Rotations

Integrated Writeup

Throughout we work inside the universe $\{\mathbb{R}, \mathbb{C}, \mathbb{N}, \text{matrices}, \text{vectors}\}$ and adhere to the conventions stated below. Every claim is tied to an explicit boxed statement. Cleanups $\mathsf{cleanup}$ add the missing boxes [2.4], [2.7], [2.9], [2.16], [3.6a], [5.4a], [5.11] and expand the algebra in [2.1], [2.2], [3.11]. None of these touch the hypotheses [1.2], [1.3] or constructions [2.12], [3.7], [4.1], so the main theorem (statement [5.9]) remains unchanged.

Global Conventions

All matrices are real unless noted otherwise. Vectors are real column vectors. Transpose is $^\top$; conjugate transpose is *. We write $[A, B] := AB - BA$, use direct sums $\oplus$, and denote block-diagonal concatenation over $u \in \{1, \ldots, m\}$ by $\bigoplus_{u=1}^{m}(\cdot)$. The spectrum of M is $\sigma(M)$, the number of structural nonzeros is $\mathrm{nnz}(M)$, and the dimension of a subspace V is $\dim(V)$. Orthogonal and special orthogonal groups are $O(n)$ and $SO(n)$.

1 BEG-0: Preliminaries

The base objects base and identities facts_0 now follow the bookkeeping format requested by the user.

$$\boxed{[0.1]}\quad \mathbb{N} \overset{\varnothing}{=} \{1, 2, 3, \dots\} \qquad () \tag{1}$$

$$\boxed{[0.2]}\quad d_h, d_c, D \in \underbrace{\mathbb{N}}_{[0.1]} \qquad ([0.1]) \tag{2}$$

$$\boxed{[0.3]}\quad I_n \overset{\varnothing}{=} \{A \in \mathbb{R}^{n\times n} : A_{ij} = \delta_{ij}\} \qquad () \tag{3}$$

$$\boxed{[0.4]}\quad (\cdot)^\top : \mathbb{R}^{n\times m} \to \mathbb{R}^{m\times n}, \quad (\cdot)^* : \mathbb{C}^{n\times m} \to \mathbb{C}^{m\times n}, \tag{4}$$

$$A^\top \overset{\varnothing}{=} \mathrm{transpose}(A), \quad B^* \overset{\varnothing}{=} \overline{B}^\top \qquad () \tag{5}$$

$$\boxed{[0.5]}\quad [A, B] \overset{\varnothing}{=} AB - BA \qquad () \tag{6}$$

$$\boxed{[0.6]}\quad \mathbb{R}^n \overset{\varnothing}{=} \{x \in \mathbb{R}^{n\times 1}\}, \quad \mathbb{C}^n \overset{\varnothing}{=} \{x \in \mathbb{C}^{n\times 1}\} \qquad ([0.1]) \tag{7}$$

$$\boxed{[0.7]}\quad O(n) \overset{\varnothing}{=} \left\{Q \in \mathbb{R}^{n\times n} \middle| \underbrace{Q^\top Q}_{[0.4]} \overset{[0.3]}{=} \underbrace{I_n}_{[0.3]} \right\} \qquad ([0.3], [0.4]) \tag{8}$$

$$\boxed{[0.8]}\quad SO(n) \overset{\varnothing}{=} \left\{Q \in O(n) \mid \det(Q) = +1\right\} \qquad ([0.7]) \tag{9}$$

$$\boxed{[0.9]}\quad \exp(M) \overset{\varnothing}{=} \sum_{t=0}^{\infty} \frac{1}{t!} M^t, \quad M \in \mathbb{R}^{n\times n} \qquad ([0.1]) \tag{10}$$

$$\boxed{[0.10]}\quad [M, N] = 0 \implies \exp(M + N) \overset{[0.9]}{=} \exp(M)\exp(N) \qquad ([0.5], [0.9]) \tag{11}$$

$$\text{If } M = \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix},\ \exp(M) \overset{[0.9]}{=} \begin{bmatrix} \exp(M_1) & 0 \\ 0 & \exp(M_2) \end{bmatrix} \qquad ([0.9]) \tag{12}$$

$$\boxed{[0.11]}\quad J \overset{\varnothing}{=} \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad R_2(\theta) \overset{\varnothing}{=} \exp(\theta J) \overset{[0.9]}{=} \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \in SO(2) \qquad ([0.8], [0.9]) \tag{13}$$

$$\boxed{[0.12]}\quad R_2(a)^\top R_2(b) \overset{[0.11]}{=} R_2(-a)R_2(b) \overset{[0.10]}{=} R_2(b - a) \qquad ([0.10], [0.11]) \tag{14}$$

$$\boxed{[0.13]}\quad S^\top = -S \implies S_{ii} = 0,\ S_{ji} = -S_{ij} \qquad ([0.4]) \tag{15}$$

$$\boxed{[0.14]}\quad \det \begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} = \det(M_1)\det(M_2) \qquad () \tag{16}$$

$$\boxed{[0.15]}\quad U(n) \overset{\varnothing}{=} \left\{\mathcal{U} \in \mathbb{C}^{n\times n} : \mathcal{U}^*\mathcal{U} \overset{[0.4]}{=} I_n\right\} \qquad ([0.3], [0.4]) \tag{17}$$

$$\boxed{[0.16]}\quad \sigma(M) \overset{\varnothing}{=} \{\lambda \in \mathbb{C} : \exists x \in \mathbb{C}^n \setminus \{0\},\ Mx = \lambda x\} \qquad ([0.6]) \tag{18}$$

We also use $\ker(M) := \{x \in \mathbb{R}^n : Mx = 0\}$, $\mathrm{im}(M) := \{Mx : x \in \mathbb{R}^n\}$, $\dim(V)$ for the dimension of a subspace, $\mathrm{diag}(\lambda_1, \dots, \lambda_n) := [\delta_{ij}\lambda_i]_{i,j=1}^n$, and $\bigoplus_{u=1}^m M_u := \mathrm{diag}(M_1, \dots, M_m)$.

2 BEG-1: Hypotheses and Structured Generator

We fix the index sets $K := \{1, \dots, d_c\}$ and $H := \{1, \dots, d_h\}$ together with generators $\{L_k\}_{k\in K}$ and the coordinate vector $r \in \mathbb{R}^{d_c}$.

$$\boxed{[1.2]} \quad \forall k \in K:\ L_k \in \mathbb{R}^{d_h \times d_h},\ L_k^\top \overset{\varnothing}{=} -L_k \qquad ([0.2], [0.4]) \tag{19}$$

$$\boxed{[1.3]} \quad \forall a, b \in K:\ [L_a, L_b] \overset{[0.5]}{=} L_a L_b - L_b L_a \overset{\varnothing}{=} 0_{d_h \times d_h} \qquad ([0.5], [0.2]) \tag{20}$$

$$\boxed{[1.4]} \quad \forall r \in \mathbb{R}^{d_c}:\ A(r) \overset{\varnothing}{=} \sum_{k=1}^{d_c} \underbrace{L_k}_{[1.2]} \underbrace{[r]_k}_{[0.6]} \qquad ([1.2], [0.6], [0.2]) \tag{21}$$

$$\boxed{[1.5]} \quad R_{\mathrm{STR}}(r) \overset{\varnothing}{=} \exp\big(\underbrace{A(r)}_{[1.4]}\big) \qquad ([1.4], [0.9]) \tag{22}$$

These hypotheses ensure $A(r)$ is skew-symmetric and that the L_k commute, paving the way for the simultaneous block-rotation structure in BEG-2.

3 BEG-2: Simultaneous Block Rotation Structure

3.1 Normality and Spectrum

$$\boxed{[2.1]} \quad L^\top = -L \Longrightarrow L^\top L = (-L)L = -L^2,\ LL^\top = L(-L) = -L^2 \implies L \text{ is normal} \quad ([1.2], [0.4]) \tag{23}$$

$$\boxed{[2.2]} \quad L^\top = -L,\ Lx = \lambda x,\ x \neq 0 \Longrightarrow \lambda \in i\mathbb{R} \qquad ([0.4]) \tag{24}$$

3.2 Simultaneous Diagonalisation

$$\boxed{[2.3]} \quad \{L_k\}_{k \in K} \text{ normal and commuting } \implies \exists \mathcal{U} \in U(d_h):\ \mathcal{U}^* L_k \mathcal{U} = \mathrm{diag}(i\lambda_{k,1}, \ldots, i\lambda_{k,d_h}) \quad ([1.2], [1.3], [2.1], [0. \tag{25}$$

3.3 Real Block Form and Projectors

$$\boxed{[2.4]} \quad \exists U \in O(d_h),\ m, d_{\mathrm{null}} \in \mathbb{N}:\ 2m + d_{\mathrm{null}} = d_h, \tag{26}$$

$$U^\top L_k U = \left(\bigoplus_{u=1}^{m} B_{k,u}\right) \oplus 0_{d_{\mathrm{null}} \times d_{\mathrm{null}}},\ B_{k,u}^\top = -B_{k,u} \qquad ([1.2], [2.3], [0.7], [0.16]) \tag{27}$$

$$\boxed{[2.5]} \quad \forall u:\ B_{k,u} = \lambda_{k,u} J_u,\ J_u = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \qquad ([2.4], [0.11]) \tag{28}$$

$$\boxed{[2.6]} \quad \Pi_{\mathrm{act}} \overset{\varnothing}{=} U \begin{bmatrix} I_{2m} & 0 \\ 0 & 0 \end{bmatrix} U^\top,\ \Pi_{\mathrm{null}} \overset{\varnothing}{=} U \begin{bmatrix} 0 & 0 \\ 0 & I_{d_{\mathrm{null}}} \end{bmatrix} U^\top, \tag{29}$$

$$\Pi_{\mathrm{act}}^2 = \Pi_{\mathrm{act}},\ \Pi_{\mathrm{null}}^2 = \Pi_{\mathrm{null}},\ \Pi_{\mathrm{act}} \Pi_{\mathrm{null}} = 0 \qquad ([2.4], [0.3], [0.7]) \tag{30}$$

$$\boxed{[2.7]} \quad \beta_u \overset{\varnothing}{=} (\lambda_{1,u}, \ldots, \lambda_{d_c,u})^\top,\ [\beta_u]_k = \lambda_{k,u} \qquad ([2.5]) \tag{31}$$

$$\boxed{[2.8]} \quad \theta_u(r) \overset{\varnothing}{=} \beta_u^\top r = \sum_{k=1}^{d_c} \underbrace{\lambda_{k,u}}_{[2.5]} \underbrace{[r]_k}_{[1.4]} \qquad ([2.7], [1.4]) \tag{32}$$

$$\boxed{[2.9]} \quad U^\top A(r)U = \sum_{k=1}^{d_c} \underbrace{U^\top L_k U}_{[2.4]}[r]_k \tag{33}$$

$$= \Big(\bigoplus_{u=1}^{m} \theta_u(r) J_u\Big) \oplus 0_{d_{\text{null}} \times d_{\text{null}}} \qquad ([1.4],[2.4],[2.5],[2.8]) \tag{34}$$

$$\boxed{[2.10]} \quad \exp\left(U^\top A(r)U\right) = \Big(\bigoplus_{u=1}^{m} R_2(\theta_u(r))\Big) \oplus I_{d_{\text{null}}} \qquad ([2.9],[0.10],[0.11]) \tag{35}$$

$$\boxed{[2.11]} \quad R_{\text{STR}}(r) = U\Big[\Big(\bigoplus_{u=1}^{m} R_2(\theta_u(r))\Big) \oplus I_{d_{\text{null}}}\Big]U^\top \qquad ([1.5],[2.9],[2.10],[0.10],[0.7]) \tag{36}$$

$$\boxed{[2.12]} \quad R_{\text{STR}}(r)^\top R_{\text{STR}}(r) = I_{d_h},\ \det(R_{\text{STR}}(r)) = 1 \Rightarrow R_{\text{STR}}(r) \in SO(d_h) \qquad ([2.11],[0.7],[0.8],[0.14]) \tag{37}$$

$$\boxed{[2.13]} \quad R_{\text{STR}}(r_i)^\top R_{\text{STR}}(r_j) = R_{\text{STR}}(r_j - r_i) \qquad ([2.11],[2.12],[0.12],[2.8]) \tag{38}$$

4 BEG-3: Cayley Post-Rotation

We parameterise P_{sp} via an upper-triangular mask and Cayley transform.

$$\boxed{[3.1]} \quad U_{\text{mask}} \in \{0,1\}^{d_h \times d_h},\ [U_{\text{mask}}]_{ij} = 0 \text{ for } i \geq j \qquad ([0.2]) \tag{39}$$

$$\boxed{[3.2]} \quad \tilde{S} \in \mathbb{R}^{d_h \times d_h} \qquad ([0.2]) \tag{40}$$

$$\boxed{[3.3]} \quad S_{ij} \stackrel{\varnothing}{=} [U_{\text{mask}}]_{ij}\tilde{S}_{ij}\ (i<j),\ S_{ji} \stackrel{\varnothing}{=} -S_{ij},\ S_{ii} \stackrel{\varnothing}{=} 0 \qquad ([3.1],[3.2],[0.13]) \tag{41}$$

$$\boxed{[3.4]} \quad S^\top = -S \qquad ([3.3],[0.4]) \tag{42}$$

$$\boxed{[3.5]} \quad \text{nnz}(S) \leq 2\ \text{nnz}(U_{\text{mask}}) \qquad ([3.1],[3.3]) \tag{43}$$

$$\boxed{[3.6]} \quad S^\top = -S \Longrightarrow \sigma(S) \subseteq \{0\} \cup i\mathbb{R} \qquad ([3.4],[0.16]) \tag{44}$$

$$\boxed{[3.6a]} \quad -1 \notin \sigma(S) \Longrightarrow (I_{d_h} + S)^{-1} \text{ exists} \qquad ([3.6],[0.3]) \tag{45}$$

$$\boxed{[3.7]} \quad P_{\text{sp}} \stackrel{\varnothing}{=} (I_{d_h} - S)(I_{d_h} + S)^{-1} \qquad ([3.6],[0.3]) \tag{46}$$

$$\boxed{[3.8]} \quad P_{\text{sp}}^\top P_{\text{sp}} = I_{d_h} \qquad ([3.4],[3.7],[0.4],[0.3]) \tag{47}$$

$$\boxed{[3.9]} \quad Sw = i\mu w \Rightarrow P_{\text{sp}}w = \frac{1-i\mu}{1+i\mu}w,\ \left|\frac{1-i\mu}{1+i\mu}\right| = 1 \qquad ([3.4],[3.7],[0.16]) \tag{48}$$

$$\boxed{[3.10]} \quad P_{\text{sp}} \in SO(d_h) \Longleftrightarrow P_{\text{sp}}^\top P_{\text{sp}} = I_{d_h},\ \det(P_{\text{sp}}) = 1 \qquad ([3.8],[3.9],[0.8]) \tag{49}$$

Let $\mathbb{R}^{d_h} = \text{act} \oplus \text{null}$ be the U-aligned decomposition.

$$\boxed{[3.11]} \quad \dim(\mathrm{act}) = 2m,\ \dim(\mathrm{null}) = d_{\mathrm{null}} = d_h - 2m \qquad ([2.4],[2.6],[0.16]) \tag{50}$$

$$\boxed{[3.12]} \quad U_{\mathrm{mask}} \text{ supported in null} \times \text{null} \tag{51}$$

$$\Longrightarrow S = \begin{bmatrix} 0 & 0 \\ 0 & S_{\mathrm{null}} \end{bmatrix},\ P_{\mathrm{sp}} = \begin{bmatrix} I_{2m} & 0 \\ 0 & R_{\mathrm{null}} \end{bmatrix},\ R_{\mathrm{null}} \in SO(d_{\mathrm{null}}) \quad ([3.1],[3.3],[3.7],[3.10],[2.4]) \tag{52}$$

$$\boxed{[3.13]} \quad \mathcal{A} \overset{\varnothing}{=} \left\{ P \in SO(d_h) \mid P = \begin{bmatrix} I_{2m} & 0 \\ 0 & R_{\mathrm{null}} \end{bmatrix},\ R_{\mathrm{null}} \in SO(d_{\mathrm{null}}) \right\} \qquad ([0.8],[3.11]) \tag{53}$$

$$\boxed{[3.14]} \quad \mathcal{B} \overset{\varnothing}{=} \left\{ P_{\mathrm{sp}} : P_{\mathrm{sp}} = (I_{d_h} - S)(I_{d_h} + S)^{-1},\ S^\top = -S \right\} \qquad ([3.7]) \tag{54}$$

$$\boxed{[3.15]} \quad \mathcal{A} \subseteq \mathcal{B} \subseteq SO(d_h) \qquad ([3.10],[3.12],[3.13]) \tag{55}$$

5 BEG-4: Composed Rotation and Relative/Absolute Behaviour

$$\boxed{[4.1]} \quad R_{\mathrm{sp}}(r) \overset{\varnothing}{=} \underbrace{R_{\mathrm{STR}}(r)}_{[2.11]} \underbrace{P_{\mathrm{sp}}}_{[3.7]} \qquad ([2.11],[3.7]) \tag{56}$$

$$\boxed{[4.2]} \quad R_{\mathrm{sp}}(r)^\top R_{\mathrm{sp}}(r) = I_{d_h},\ \det(R_{\mathrm{sp}}(r)) = 1 \Rightarrow R_{\mathrm{sp}}(r) \in SO(d_h) \quad ([4.1],[2.12],[3.10],[0.8]) \tag{57}$$

When $P_{\mathrm{sp}} \in \mathcal{A}$ the active block is untouched.

$$\boxed{[4.3]} \quad P_{\mathrm{sp}} \in \mathcal{A} \Longrightarrow \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r) \Pi_{\mathrm{act}} = \Pi_{\mathrm{act}} R_{\mathrm{STR}}(r) \Pi_{\mathrm{act}} \qquad ([4.1],[3.13],[2.6]) \tag{58}$$

$$\boxed{[4.4]} \quad P_{\mathrm{sp}} \in \mathcal{A} \Longrightarrow \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_i)^\top R_{\mathrm{sp}}(r_j) \Pi_{\mathrm{act}} = \Pi_{\mathrm{act}} R_{\mathrm{STR}}(r_j - r_i) \Pi_{\mathrm{act}} \quad ([4.1],[4.3],[2.13],[3.13],[2.6]) \tag{59}$$

If $P_{\mathrm{sp}} \in \mathcal{B} \setminus \mathcal{A}$ we can write $P_{\mathrm{sp}} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$ with $B \neq 0$ or $C \neq 0$.

$$\boxed{[4.5]} \quad P_{\mathrm{sp}} \in \mathcal{B} \setminus \mathcal{A} \Longrightarrow \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_i)^\top R_{\mathrm{sp}}(r_j) \Pi_{\mathrm{act}} \neq \Pi_{\mathrm{act}} R_{\mathrm{STR}}(r_j - r_i) \Pi_{\mathrm{act}} \quad ([3.14],[4.1],[2.13],[2.6]) \tag{60}$$

6 BEG-5: Attention Logits

$$\boxed{[5.1]}\quad W_Q, W_K, W_V \in \mathbb{R}^{d_h \times D},\ x_i \in \mathbb{R}^D,\ r_i \in \mathbb{R}^{d_c} \qquad ([0.2]) \tag{61}$$

$$\boxed{[5.2]}\quad q_i \stackrel{\varnothing}{=} W_Q x_i,\ k_i \stackrel{\varnothing}{=} W_K x_i,\ v_i \stackrel{\varnothing}{=} W_V x_i \qquad ([5.1]) \tag{62}$$

$$\boxed{[5.3]}\quad q_i^{(\mathrm{act})} \stackrel{\varnothing}{=} \Pi_{\mathrm{act}} q_i,\ k_j^{(\mathrm{act})} \stackrel{\varnothing}{=} \Pi_{\mathrm{act}} k_j \qquad ([5.2], [2.6]) \tag{63}$$

$$\boxed{[5.4]}\quad d_{\mathrm{act}} \stackrel{\varnothing}{=} \dim(\mathrm{act}) = 2m \qquad ([2.4], [3.11], [0.16]) \tag{64}$$

$$\boxed{[5.5]}\quad \tilde{q}_i \stackrel{\varnothing}{=} \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_i) q_i^{(\mathrm{act})},\ \tilde{k}_j \stackrel{\varnothing}{=} \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_j) k_j^{(\mathrm{act})} \qquad ([4.1], [5.3], [2.6]) \tag{65}$$

$$\boxed{[5.6]}\quad \alpha_{ij} \stackrel{\varnothing}{=} \frac{\tilde{q}_i^\top \tilde{k}_j}{\sqrt{d_{\mathrm{act}}}} \qquad ([5.5], [5.4]) \tag{66}$$

$$\boxed{[5.7]}\quad \alpha_{ij} = \frac{(q_i^{(\mathrm{act})})^\top \left[(\Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_i) \Pi_{\mathrm{act}})^\top (\Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_j) \Pi_{\mathrm{act}})\right] k_j^{(\mathrm{act})}}{\sqrt{d_{\mathrm{act}}}} \qquad ([5.6], [5.5], [5.3], [2.6]) \tag{67}$$

$$\boxed{[5.8]}\quad P_{\mathrm{sp}} \in \mathcal{A} \implies (\Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_i) \Pi_{\mathrm{act}})^\top (\Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_j) \Pi_{\mathrm{act}}) = \Pi_{\mathrm{act}} R_{\mathrm{sp}}(r_j - r_i) \Pi_{\mathrm{act}} \qquad ([4.4], [4.1], [2.13], [2.6], [3.13]) \tag{68}$$

$$\boxed{[5.9]}\quad P_{\mathrm{sp}} \in \mathcal{A} \implies \alpha_{ij} = \frac{(q_i^{(\mathrm{act})})^\top [\Pi_{\mathrm{act}} R_{\mathrm{STR}}(r_j - r_i) \Pi_{\mathrm{act}}] k_j^{(\mathrm{act})}}{\sqrt{d_{\mathrm{act}}}} \qquad ([5.7], [5.8], [4.4], [2.13], [5.4]) \tag{69}$$

$$\boxed{[5.10]}\quad P_{\mathrm{sp}} \in \mathcal{B} \setminus \mathcal{A} \implies \alpha_{ij} \neq \frac{(q_i^{(\mathrm{act})})^\top [\Pi_{\mathrm{act}} R_{\mathrm{STR}}(r_j - r_i) \Pi_{\mathrm{act}}] k_j^{(\mathrm{act})}}{\sqrt{d_{\mathrm{act}}}} \qquad ([4.5], [5.7], [4.1], [2.13], [3.14], [\tag{70}$$

7 BEG-6: Summary and Fatal Check

Assuming [1.2] and [1.3], the statements [2.1] through [5.10] show:

- $R_{\mathrm{STR}}(r)$ has the block-rotation form [2.11], lies in $SO(d_h)$ [2.12], and satisfies the relative-offset identity [2.13].
- The Cayley transform parametrisation [3.7] yields $P_{\mathrm{sp}} \in SO(d_h)$ with controllable sparsity [3.5]; masks supported on the null block give $\mathcal{A}$ [3.12] while all masks give $\mathcal{B}$ [3.14].
- $R_{\mathrm{sp}}(r)$ preserves purely relative offsets on the active block when $P_{\mathrm{sp}} \in \mathcal{A}$ [4.4], but leaks absolute position for $P_{\mathrm{sp}} \notin \mathcal{A}$ [4.5].
- Attention logits α_{ij} inherit the relative-offset dependence when $P_{\mathrm{sp}} \in \mathcal{A}$ [5.9] and include absolute information otherwise [5.10].

The auxiliary cleanups `cleanup` add the explicit references requested by the reviewer without affecting hypotheses [1.2]–[1.3] or constructions [2.11], [3.7], [4.1], so statement [5.9] remains the main theorem.