```python
import numpy as np
import qutip as qt
import datetime as dt
import matplotlib.pyplot as plt
import time
from tqdm import tqdm
from qutip.measurement import measure, measurement_statistics

plt.rcParams.update({
    "text.usetex": True,
    "font.family": "serif"
})
plt.rcParams['figure.figsize'] = [5, 3]
```

Three gates

```python
#CNOT
one_projector = qt.basis(2,1)*(qt.basis(2,1).dag())
zero_projector = qt.basis(2,0)*(qt.basis(2,0).dag())
cnot = qt.tensor(zero_projector,qt.qeye(2)) +
qt.tensor(one_projector,qt.sigmax())


# Two qubit hadamard
had = (1/(np.sqrt(2))*qt.Qobj([[1,1],[1,-1]]))
had_2 = qt.tensor(had, had)

#Two qubit Pauli_x
paulix_2 = qt.tensor(qt.sigmax(), qt.sigmax())
```

Dimensions of all three of them is 2 X 2 X 2 X 2

State vector s

```python
#State vector simulator using tensor product notation
#input: number of qubits
#output: Simulated state vector
def statevector_simulator_2(no_of_qubits):
    #size of hilbert space
    size = (2**no_of_qubits)

    # simulated only for even number of qubits
    # since CNOT gate is used
    if(no_of_qubits%2==0):
        #set initial state to be |00> in tensor notation
```

```
        ini = qt.tensor(qt.basis(2,0) , qt.basis(2,0))

        # Applying three gates sequentially
        output_2 = cnot*had_2*paulix_2*ini
        out_temp = output_2
        output = output_2

        #fixing the number of iterations
        iterr = round((no_of_qubits-2)/2)

        #Starting the time counter
        start = time.time()

        #Loop to simulate state vector for higher qubits
        for i in range(iterr):
            output = qt.tensor(out_temp , output_2)
            out_temp = output
        end = time.time()

        #Time interval caluculated after end of loop
        intrval = end-start
        return output , intrval
    else:
        print("Invalid number of qubits")


#State vector simulator using only two dimensional matrices
def statevector_simulator_1(no_of_qubits):
    #size of hilbert space
    size = (2**no_of_qubits)
    if(no_of_qubits%2==0):
        #set initial state to be |00> in tensor notation
        ini = qt.basis(4 , 0)

        #Changing dimensions of gates to make them 2D matrices
        cnot.dims = [[4],[4]]
        had_2.dims = [[4],[4]]
        paulix_2.dims = [[4],[4]]

        # Applying three gates sequentially
        output_2 = cnot*had_2*paulix_2*ini
        out_temp = output_2
        output = output_2

        #fixing the number of iterations for computing state vector
        iterr = round((no_of_qubits-2)/2)

        #Loop to simulate state vector for higher qubits
        start = time.time()
        for i in range(iterr):
```

```
            output = qt.tensor(out_temp , output_2)
            #Changing dimenions of output to make it a 1D vector
            dimen = 16*(4**i)
            output.dims = [[dimen],[1]]
            out_temp = output
        #Time interval caluculated after end of loop
        end = time.time()
        intrval = end-start
        return output , intrval
    else:
        print("Invalid number of qubits")
```

State vector simulation using 2D matrix (Sub task -1)

```
# Time display
pbar = tqdm()
# Number of qubits
noq = 25

#List of time intervals
time_int_l = []
i = 0
noq_l = []
while(i<noq):
    i = i + 2
    # Simulating the final state using 2D matrices
    a , time_int = statevector_simulator_1(i)
    # recording the time take for each iteration
    time_int_l.append(time_int)
    noq_l.append(i)
    pbar.update(1)
pbar.close()

13it [00:00, 23.39it/s]
```

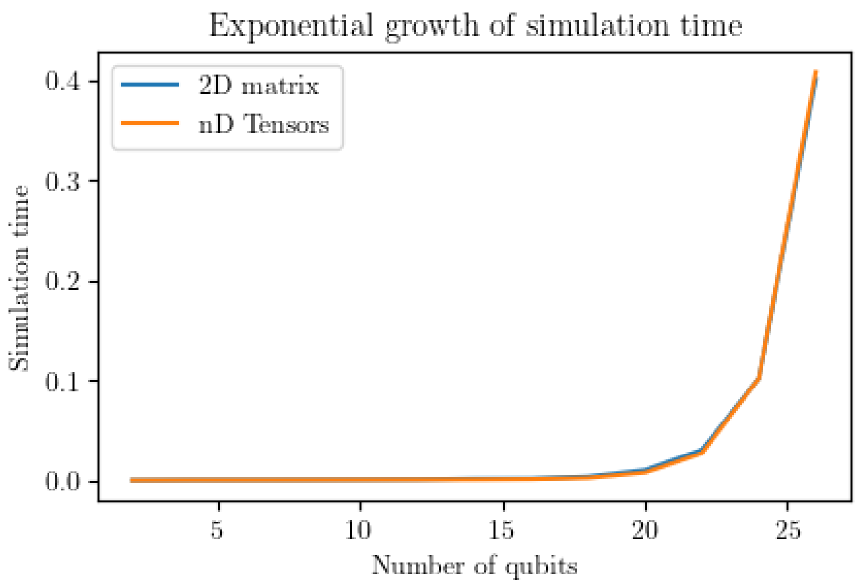
State vector simulation using nD tensors (Subtask 2)

```
#CNOT
one_projector = qt.basis(2,1)*(qt.basis(2,1).dag())
zero_projector = qt.basis(2,0)*(qt.basis(2,0).dag())
cnot = qt.tensor(zero_projector,qt.qeye(2)) +
qt.tensor(one_projector,qt.sigmax())
```

```
# Two qubit hadamard
had = (1/(np.sqrt(2))*qt.Qobj([[1,1],[1,-1]]))
had_2 = qt.tensor(had, had)

#Two qubit Pauli_x
paulix_2 = qt.tensor(qt.sigmax(), qt.sigmax())

# Time display
pbar = tqdm()
# Number of qubits
noq = 25

#List of time intervals
time_int_l2 = []
i = 0
noq_l = []
while(i<noq):
    i = i + 2
    # Simulating the final state using nD tensor
    a , time_int = statevector_simulator_2(i)
    time_int_l2.append(time_int)
    noq_l.append(i)
    pbar.update(1)
pbar.close()
```

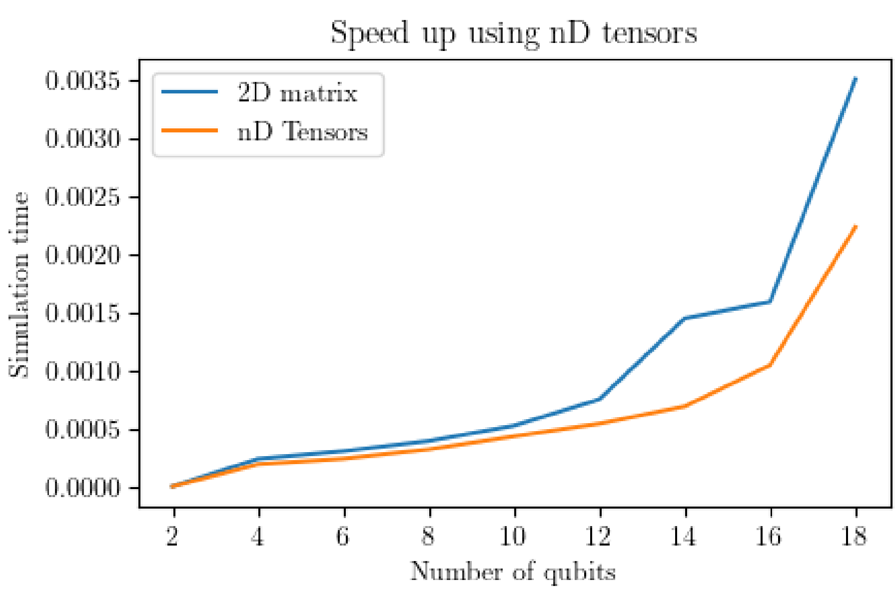
```
13it [00:00, 23.22it/s]
```

```
#rang = 8
rang = len(noq_l)
plt.plot(noq_l[0:rang], time_int_l[0:rang] , label = '2D matrix')
plt.plot(noq_l[0:rang], time_int_l2[0:rang] , label = 'nD Tensors')
plt.xlabel('Number of qubits')
plt.ylabel('Simulation time')
plt.title('Exponential growth of simulation time')
plt.legend()
plt.show()
#exponential scaling
```



Zoom in of previous plot

```
rang = 9
#rang = len(noq_l)
plt.plot(noq_l[0:rang], time_int_l[0:rang] , label = '2D matrix')
plt.plot(noq_l[0:rang], time_int_l2[0:rang] , label = 'nD Tensors')
plt.xlabel('Number of qubits')
plt.ylabel('Simulation time')
plt.title('Speed up using nD tensors')
plt.legend()
plt.show()
```



Key takeaway

Bonus Question

Part-1)

```
# works for both tensor representation or 2d notation
def exp_value_experimental(state , operator , no_of_shots = 100):

    # list of eigenvalues of the operator
    eig_list = operator.eigenenergies()

    # counter array to store occurence of each eigenvectore
    prob_list = np.zeros(len(eig_list))

    #loop for repeated measurement
    for i in tqdm(range(no_of_shots)):
        # post measurement state and its corresponding eigenvalue
using qutip measure
        eig_val ,eig_vect =  measure(state, operator)

        #counting occurence of each eigen value and updating the
counter
        for i in range(len(eig_list)):
            if(round(eig_val , 3) == round(eig_list[i] , 3)):
                prob_list[i] = prob_list[i] + 1

    if (sum(prob_list) != 0):
        #converting counts to probabilities
        prob_list = (1/sum(prob_list))*prob_list

        #finding expectation values
        exp_val = 0
        for i in range(len(eig_list)):
            exp_val = exp_val + (eig_list[i]*prob_list[i])
        #returns both expectation values and probablity distribution
        return exp_val , prob_list
    else:
        print("Incorrect probabilities calculated")
```

Part 2)

```
def exp_value_exact(state , operator):
    #using direct formula
```

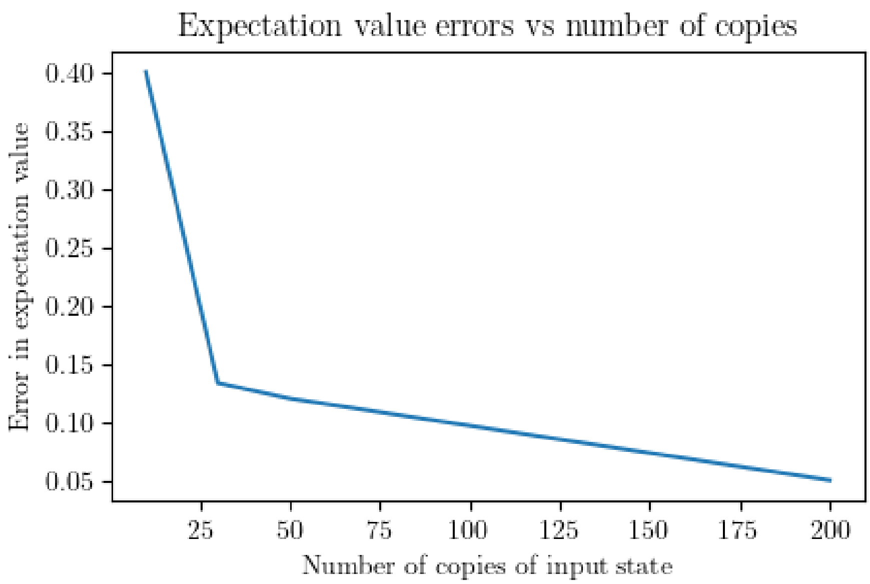
```
    exp_value = np.real((state.dag())*operator*state)
    return np.real(exp_value)
```

Example to test expectaion value

```
oper = qt.tensor(qt.sigmax() , qt.sigmay(),qt.sigmaz()\
                 ,qt.sigmax(),qt.sigmay(),qt.sigmaz())
state, interval = statevector_simulator_2(6)
val_10 , prob_list = exp_value_experimental(state , oper , 10)
val_30 , prob_list = exp_value_experimental(state , oper , 30)
val_50 , prob_list = exp_value_experimental(state , oper , 50)
val_200 , prob_list = exp_value_experimental(state , oper , 200)
val_act = exp_value_exact(state , oper)

error_list = []
val_list = [val_10 , val_30 , val_50 , val_200 ]
#val_list = [val_10 , val_100 ]
for val in val_list:
    error = abs(val_act - val)
    error_list.append(error)
```

```
100%|██████████| 10/10 [00:00<00:00,
83.42it/s]
100%|██████████| 30/30 [00:00<00:00,
82.70it/s]
100%|██████████| 50/50 [00:00<00:00,
83.05it/s]
100%|██████████| 200/200 [00:02<00:00,
86.02it/s]
```

```
no_of_shots = [10 , 30 , 50 , 200]
plt.plot(no_of_shots, error_list)
plt.xlabel("Number of copies of input state")
plt.ylabel("Error in expectation value")
plt.title("Expectation value errors vs number of copies")
plt.show()
```


Expectation value errors vs number of copies
Error in expectation value
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
25
50
75
100
125
150
175
200
Number of copies of input state