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JEE Main

CHAPTERWISE SOLUTIONS 2019 - 2002

Physics

All the 16 Question Papers
of JEE Main Online 2019 *(Jan & Apr Attempt)*

5 Online Practice Sets
for JEE Main 2020

arihant

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of JEE Main Online 2019 *(Jan & Apr Attempt)*

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ARIHANT PRAKASHAN (Series), MEERUT

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PREFACE

JEE Main is a gateway examination for candidates expecting to seek admission in Bachelor in Engineering (BE), Bachelor of Technology (B.Tech) and Bachelor of Architecture (B.Arch) at Indian Institutes of Information Technology (IIITs), National Institutes of Technology (NITs), Delhi Technological University and other Centrally Funded Technical Institutes (CFTIs).

JEE Main is also an examination which is like screening examination for JEE Advanced (The gateway examination to India's most reputed Technical Institutes, Indian Institutes of Technology— IITs).

To make the students well-versed with the pattern as well as the level of the questions asked in the exam, this book contains **Chapterwise Solutions of Questions asked in Last 18 Years' Examinations of JEE Main** (formerly known as AIEEE). Solutions to all the questions have been kept very detailed and accurate. Along with the indication of level of the exam, this book will also teach you to solve the questions objectively in the examination.

To give the students a complete practice, along with Chapterwise Solutions, this book contains 5 Practice Sets, based exactly on JEE Main Syllabus and Pattern. By practicing these sets, students can attain efficiency in Time Management during the examination.

We hope this book would be highly beneficial for the students. We would be grateful if any discrepancy or mistake in the questions or answers is brought to our notice so that these could be rectified in subsequent editions.

Publisher

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SYLLABUS

NOTE The syllabus contains two Sections - A & B. Section A pertains to the Theory Part, having 80% weightage, while Section B contains Practical Component (Experimental Skills) having 20% weightage.

SECTION- A

UNIT 1 Physics and Measurement

Physics, technology and society, SI units, Fundamental and derived units. Least count, accuracy and precision of measuring instruments, Errors in measurement, Significant figures. Dimensions of Physical quantities, dimensional analysis and its applications.

UNIT 2 Kinematics

Frame of reference. Motion in a straight line: Position-time graph, speed and velocity. Uniform and non-uniform motion, average speed and instantaneous velocity.

Uniformly accelerated motion, velocity-time, position time graphs, relations for uniformly accelerated motion.

Scalars and Vectors, Vector addition and Subtraction, Zero Vector, Scalar and Vector products, Unit Vector, Resolution of a Vector. Relative Velocity, Motion in a plane, Projectile Motion, Uniform Circular Motion.

UNIT 3 Laws of Motion

Force and Inertia, Newton's First Law of motion; Momentum, Newton's Second Law of motion; Impulse; Newton's Third Law of motion. Law of conservation of linear momentum and its applications, Equilibrium of concurrent forces.

Static and Kinetic friction, laws of friction, rolling friction.

Dynamics of uniform circular motion: Centripetal force and its applications.

UNIT 4 Work, Energy and Power

Work done by a constant force and a variable force; kinetic and potential energies, work-energy theorem, power.

Potential energy of a spring, conservation of mechanical energy, conservative and nonconservative forces; Elastic and inelastic collisions in one and two dimensions.

UNIT 5 Rotational Motion

Centre of mass of a two-particle system, Centre of mass of a rigid body; Basic concepts of rotational motion; moment of a force, torque, angular momentum, conservation of angular momentum and its applications; moment of inertia, radius of gyration. Values of moments of inertia for simple geometrical objects, parallel and perpendicular axes theorems and their applications.

Rigid body rotation, equations of rotational motion.

UNIT 6 Gravitation

The universal law of gravitation.

Acceleration due to gravity and its variation with altitude and depth.

Kepler's laws of planetary motion.

Gravitational potential energy; gravitational potential.

Escape velocity. Orbital velocity of a satellite. Geo-stationary satellites.

UNIT 7 Properties of Solids & Liquids

Elastic behaviour, Stress-strain relationship, Hooke's. Law, Young's modulus, bulk modulus, modulus of rigidity.

Pressure due to a fluid column; Pascal's law and its applications.

Viscosity, Stokes' law, terminal velocity, streamline and turbulent flow, Reynolds number. Bernoulli's principle and its applications.

Surface energy and surface tension, angle of contact, application of surface tension - drops, bubbles and capillary rise.

Heat, temperature, thermal expansion; specific heat capacity, calorimetry; change of state, latent heat.

Heat transfer-conduction, convection and radiation, Newton's law of cooling.

UNIT 8 Thermodynamics

Thermal equilibrium, zeroth law of thermo-dynamics, concept of temperature. Heat, work and internal energy. First law of thermodynamics.

Second law of thermodynamics: reversible and irreversible processes. Camot engine and its efficiency.

UNIT 9 Kinetic Theory of Gases

Equation of state of a perfect gas, work done on compressing a gas.

Kinetic theory of gases - assumptions, concept of pressure. Kinetic energy and temperature: rms speed of gas molecules; Degrees of freedom, Law of equipartition of energy, applications to specific heat capacities of gases; Mean free path, Avogadro's number.

UNIT 10 Oscillations And Waves

Periodic motion - period, frequency, displacement as a function of time. Periodic functions. Simple harmonic motion (S.H.M.) and its equation; phase; oscillations of a spring - restoring force and force constant; energy in S.H.M. - kinetic and potential energies; Simple pendulum - derivation of expression for its time period; Free, forced and damped oscillations, resonance.

Wave motion Longitudinal and transverse waves, speed of a wave. Displacement relation for a progressive wave. Principle of superposition of waves, reflection of waves, Standing waves in strings and organ pipes, fundamental mode and harmonics, Beats, Doppler effect in sound.

UNIT 11 Electrostatics

Electric charges Conservation of charge, Coulomb's law-forces between two point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field Electric field due to a point charge, Electric field lines, Electric dipole, Electric field due to a dipole, Torque on a dipole in a uniform electric field.

Electric flux, Gauss's law and its applications to find field due to infinitely long, uniformly charged straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell.

Electric potential and its calculation for a point charge, electric dipole and system of charges; Equipotential surfaces, Electrical potential energy of a system of two point charges in an electrostatic field.

Conductors and insulators, Dielectrics and electric polarization, capacitor, combination of capacitors in series and in parallel, capacitance of a parallel plate capacitor with and without dielectric medium between the plates, Energy stored in a capacitor.

UNIT 12 Current Electricity

Electric current, Drift velocity, Ohm's law, Electrical resistance, Resistances of different materials, V-I characteristics of Ohmic and nonohmic conductors, Electrical energy and power, Electrical resistivity, Colour code for resistors; Series and parallel combinations of resistors; Temperature dependence of resistance.

Electric Cell and its Internal resistance, potential difference and emf of a cell, combination of cells in series and in parallel.

Kirchhoff's laws and their applications. Wheatstone bridge, Metre bridge.

Potentiometer - principle and its applications.

UNIT 13 Magnetic Effects of Current and Magnetism

Biot-Savart law and its application to current carrying circular loop. Ampere's law and its applications to infinitely long current carrying straight wire and solenoid. Force on a moving charge in uniform magnetic and electric fields Cyclotron.

Force on a current-carrying conductor in a uniform magnetic field. Force between two parallel current-carrying conductors-definition of ampere. Torque experienced by a current loop in uniform magnetic field, Moving coil galvanometer, its current sensitivity and conversion to ammeter and voltmeter.

Current loop as a magnetic dipole and its magnetic dipole moment. Bar magnet as an equivalent solenoid, magnetic field lines; Earth's magnetic field and magnetic elements. Para, dia and ferro-magnetic substances

Magnetic susceptibility and permeability, Hysteresis, Electromagnets and permanent magnets.

UNIT 14 Electromagnetic Induction and Alternating Currents

Electromagnetic induction; Faraday's law, induced emf and current; Lenz's Law, Eddy currents. Self and mutual inductance.

Alternating currents, peak and rms value of alternating current/ voltage; reactance and impedance; LCR series circuit, resonance; Quality factor, power in AC circuits, wattless current. AC generator and transformer.

UNIT 15 Electromagnetic Waves

Electromagnetic waves and their characteristics. Transverse nature of electromagnetic waves.

Electromagnetic spectrum (radio waves, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays). Applications of e.m. waves.

UNIT 16 Optics

Reflection and refraction of light at plane and spherical surfaces, mirror formula, Total internal reflection and its applications, Deviation and Dispersion of light by a prism, Lens Formula, Magnification, Power of a Lens, Combination of thin lenses in contact, Microscope and Astronomical Telescope (reflecting and refracting) and their magnifying powers.

Wave optics wave front and Huygens' principle, Laws of reflection and refraction using Huygen's principle. Interference, Young's double slit experiment and expression for fringe width, coherent sources and sustained interference of light. Diffraction due to a single slit, width of central maximum. Resolving power of microscopes and astronomical telescopes, Polarisation, plane polarized light; Brewster's law, uses of plane polarized light and Polaroids.

UNIT 17 Dual Nature of Matter and Radiation

Dual nature of radiation. Photoelectric effect, Hertz and Lenard's observations; Einstein's photoelectric equation; particle nature of light. Matter waves-wave nature of particle, de Broglie relation. Davisson-Germer experiment.

UNIT 18 Atoms and Nuclei

Alpha-particle scattering experiment; Rutherford's model of atom; Bohr model, energy levels, hydrogen spectrum.

Composition and size of nucleus, atomic masses, isotopes, isobars; isotones. Radioactivity-alpha, beta and gamma particles/rays and their properties; radioactive decay law. Mass-energy relation, mass defect; binding energy per nucleon and its variation with mass number, nuclear fission and fusion.

UNIT 19 Electronic Devices

Semiconductors; semiconductor diode: I-V characteristics in forward and reverse bias; diode as a rectifier; I-V characteristics of LED, photodiode, solar cell, and Zener diode; Zener diode as a voltage regulator. Junction transistor, transistor action, characteristics of a transistor transistor as an amplifier (common emitter configuration) and oscillator. Logic gates (OR, AND, NOT, NAND & NOR). Transistor as a switch.

UNIT 20 Communication Systems

Propagation of electromagnetic waves in the atmosphere; Sky and space wave propagation, Need for modulation, Amplitude and Frequency Modulation, Bandwidth of signals, Bandwidth of Transmission medium, Basic Elements of a

SECTION- B

UNIT 21 Experimental Skills

Familiarity with the basic approach and observations of the experiments and activities

1. Vernier callipers - its use to measure internal and external diameter and depth of a vessel.
2. Screw gauge - its use to determine thickness/ diameter of thin sheet/wire.
3. Simple Pendulum - dissipation of energy by plotting a graph between square of amplitude and time.
4. Metre Scale - mass of a given object by principle of moments.
5. Young's modulus of elasticity of the material of a metallic wire.
6. Surface tension of water by capillary rise and effect of detergents.
7. Coefficient of Viscosity of a given viscous liquid by measuring terminal velocity of a given spherical body.
8. Plotting a cooling curve for the relationship between the temperature of a hot body and time.
9. Speed of sound in air at room temperature using a resonance tube.
10. Specific heat capacity of a given (i) solid and (ii) liquid by method of mixtures.
11. Resistivity of the material of a given wire using metre bridge.
12. Resistance of a given wire using Ohm's law.
13. Potentiometer
    - (i) Comparison of emf of two primary cells.
    - (ii) Internal resistance of a cell.
14. Resistance and figure of merit of a galvanometer by half deflection method.
15. Focal length of
    - (i) Convex mirror
    - (ii) Concave mirror
    - (iii) Convex lens
16. Using parallax method. Plot of angle of deviation vs angle of incidence for a triangular prism.
17. Refractive index of a glass slab using a travelling microscope.
18. Characteristic curves of a p-n junction diode in forward and reverse bias.
19. Characteristic curves of a Zener diode and finding reverse break down voltage.
20. Characteristic curves of a transistor and finding current gain and voltage gain.
21. Identification of Diode, LED, Transistor, IC, Resistor, Capacitor from mixed collection of such items.
22. Using multimeter to
    - (i) Identify base of a transistor.
    - (ii) Distinguish between npn and pnp type transistor.
    - (iii) See the unidirectional flow of current in case of a diode and an LED.
    - (iv) Check the correctness or otherwise of a given electronic component (diode, transistor or IC).

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Measurement

1. In SI units, the dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ is

(a) $A^{-1}TML^3$ (b) $AT^2M^{-1}L^{-1}$
(c) $AT^{-3}ML^{3/2}$ (d) $A^2T^3M^{-1}L^{-2}$

[JEE Main 2019, 8 April Shift-I]

Exp. **(d)**

Dimensions of ε_0 (permittivity of free space) are
$[\varepsilon_0] = M^{-1}L^{-3}T^4A^2$

As, c = speed of light.

$\therefore$ Dimension of $[c] = [LT^{-1}]$

So, dimensions of $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ are

$$\left[\sqrt{\frac{\varepsilon_0}{\mu_0}}\right] = \left[\sqrt{\frac{\varepsilon_0^2}{\varepsilon_0\mu_0}}\right] = [\varepsilon_0 c] \qquad \left[\because c^2 = \frac{1}{\mu_0\varepsilon_0}\right]$$

$$= [M^{-1}L^{-3}T^4A^2][LT^{-1}] = [M^{-1}L^{-2}T^3A^2]$$

2. In a simple pendulum, experiment for determination of acceleration due to gravity (g), time taken for 20 oscillations is measured by using a watch of 1 second least count. The mean value of time taken comes out to be 30 s. The length of pendulum is measured by using a meter scale of least count 1 mm and the value obtained 55.0 cm. The percentage error in the determination of g is close to **[JEE Main 2019, 8 April Shift-II]**

(a) 0.7% (b) 6.8% (c) 3.5% (d) 0.2%

Exp. **(b)**

Relation used for finding acceleration due to gravity by using a pendulum is

$$g = \frac{4\pi^2 l}{T^2}$$

So, fractional error in value of g is

$$\frac{\Delta g}{g} = \frac{\Delta l}{l} + \frac{2\Delta T}{T} \qquad \ldots(i)$$

Given, $\Delta l = 0.1$ cm, $l = 55$ cm, $\Delta T = 1$ s and T for 20 oscillations = 30 s

Substituting above values in Eq. (i), we get

$$\frac{\Delta g}{g} = \frac{0.1}{55} + 2 \times \frac{1}{30}$$

Hence, percentage error in g is

$$= \frac{\Delta g}{g} \times 100 = \frac{10}{55} + \frac{20}{3} = 6.8\%$$

3. If surface tension (S), moment of inertia (I) and Planck's constant (h), were to be taken as the fundamental units, the dimensional formula for linear momentum would be **[JEE Main 2019, 8 April Shift-II]**

(a) $S^{1/2}I^{1/2}h^{-1}$ (b) $S^{3/2}I^{1/2}h^0$
(c) $S^{1/2}I^{1/2}h^0$ (d) $S^{1/2}I^{3/2}h^{-1}$

Exp. **(c)**

Suppose, linear momentum (p) depends upon the Planck's constant (h) raised to the power (a), surface tension(S) raised to the power (b) and moment of inertia (I) raised to the power (c).

Then, $p \propto (h)^a(S)^b(I)^c$ or $p = kh^aS^bI^c$

where, k is a dimensionless proportionality constant.

Thus, $[p] = [h]^a[S]^b[I]^c \qquad \ldots(i)$

Then, the respective dimensions of the given physical quantities, i.e.

$[p] = [\text{mass} \times \text{velocity}] = [MLT^{-1}]$

$[I] = [\text{mass} \times \text{distance}^2] = [ML^2T^0]$

$[S] = [\text{force} \times \text{length}] = [ML^0T^{-2}]$

$[h] = [ML^2T^{-1}]$

Then, substituting these dimensions in Eq. (i), we get

$$[MLT^{-1}] = [ML^2T^{-1}]^a[MT^{-2}]^b[ML^{+2}]^c$$

For dimensional balance, the dimensions on both sides should be same.

Thus, equating dimensions, we have

$$a + b + c = 1$$

$$2(a + c) = 1 \quad \text{or} \quad a + c = \frac{1}{2}$$

$$-a - 2b = -1 \quad \text{or} \quad a + 2b = 1$$

Solving these three equations, we get

$$a = 0, b = \frac{1}{2}, c = \frac{1}{2}$$

$$\therefore \quad p = h^0S^{\frac{1}{2}}l^{\frac{1}{2}} \quad \text{or} \quad p = S^{\frac{1}{2}}l^{\frac{1}{2}}h^0$$

4. In the density measurement of a cube, the mass and edge length are measured as (10.00 ± 0.10) kg and (0.10 ± 0.01) m, respectively. The error in the measurement of density is **[JEE Main 2019, 9 April Shift-I]**

(a) $0.01\ kg/m^3$ (b) $0.10\ kg/m^3$
(c) $0.07\ kg/m^3$ (d) $0.31\ kg/m^3$

Exp. **(*)**

Given, mass = (10.00 ± 0.10) kg

Edge length = (0.10 ± 0.01) m

Error in mass,

$$\frac{\Delta M}{M} = \frac{0.1}{10} \quad \ldots(i)$$

and error in length,

$$\frac{\Delta l}{l} = \frac{0.01}{0.1} \quad \ldots(ii)$$

Density of the cube is given by

$$r = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{l^3}$$

∴ Permissible error in density is

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} \pm 3\frac{\Delta l}{l} \quad \ldots(iii)$$

Substituting the value from Eqs. (i) and (ii) in Eq. (iii), we get

$$\frac{\Delta\rho}{\rho} = \frac{0.1}{10} + 3 \times \frac{0.01}{0.1} = \frac{1}{100} + \frac{3}{10} = \frac{31}{100}$$

$$\Rightarrow \quad \frac{\Delta\rho}{\rho} = \frac{31}{100} = 0.31$$

Since, $\frac{\Delta\rho}{\rho}$ should be unitless quantity.

But there is no option with unitless error. Hence, no option is correct.

5. The area of a square is $5.29\ cm^2$. The area of 7 such squares taking into account the significant figures is **[JEE Main 2019, 9 April Shift-II]**

(a) $37.030\ cm^2$ (b) $37.0\ cm^2$
(c) $37.03\ cm^2$ (d) $37\ cm^2$

Exp. ***(c)***

Area of 1 square = $5.29\ cm^2$

Area of seven such squares

= 7 times addition of area of 1 square

= 5.29 + 5.29 + 5.29 … 7 times = $37.03\ cm^2$

As we know that, if in the measured values to be added/subtracted the least number of significant digits after the decimal is *n*.

Then, in the sum or difference also, the number of significant digits after the decimal should be *n*.

Here, number of digits after decimal in 5.29 is 2, so our answer also contains only two digits after decimal point.

∴ Area required = $37.03\ cm^2$

6. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units? **[JEE Main 2019, 10 April Shift-II]**

(a) $[M^{-1}L^{-2}T^4A^2]$ (b) $[M^{-2}L^0T^{-4}A^{-2}]$
(c) $[M^{-3}L^{-2}T^8A^4]$ (d) $[M^{-2}L^{-2}T^6A^3]$

Exp. **(c)**

To find dimensions of capacitance in the given relation, we can use formula for energy.

Capacitors energy is $U = \frac{1}{2}CV^2$

So, dimensionally,

$$\Rightarrow \quad [C] = \left[\frac{U}{V^2}\right]$$

As, $V = \text{potential} = \frac{\text{potential energy}}{\text{charge}}$

We have,

$$[C] = \frac{[U]}{\left[\frac{U^2}{q^2}\right]} = \frac{[q^2]}{[U]} = \left[\frac{A^2T^2}{ML^2T^{-2}}\right]$$

$$\therefore \quad X = [M^{-1}L^{-2}A^2T^4]$$

To get dimensions of magnetic field, we use force on a current carrying conductor in magnetic field,

$$F = BIl \Rightarrow [B] = \frac{[F]}{[I][l]} = \left[\frac{MLT^{-2}}{AL}\right]$$

$\therefore \quad Z = [M^1L^0T^{-2}A^{-1}]$

Now, using given relation,

$X = 5YZ^2$

$$[Y] = \frac{[X]}{[Z^2]} = \left[\frac{M^{-1}L^{-2}A^2T^4}{(M^1L^0T^{-2}A^{-1})^2}\right] = \frac{M^{-1}L^{-2}A^2T^4}{M^2T^{-4}A^{-2}}$$

$\therefore \quad [Y] = [M^{-3}L^{-2}A^4T^8]$

7. Which of the following combinations has the dimension of electrical resistance (ε_0 is the permittivity of vacuum and μ_0 is the permeability of vacuum)?

[JEE Main 2019, 12 April Shift-I]

(a) $\sqrt{\frac{\mu_0}{\varepsilon_0}}$ (b) $\frac{\mu_0}{\varepsilon_0}$ (c) $\sqrt{\frac{\varepsilon_0}{\mu_0}}$ (d) $\frac{\varepsilon_0}{\mu_0}$

Exp. **(a)**

> **Key Idea** A formula is valid only, if the dimensions of LHS and RHS are same. So, we need to balance dimensions of given options with the dimension of electrical resistance.

Let dimensions of resistance R, permittivity ε_0 and permeability μ_0 are [R], $[\varepsilon_0]$ and $[\mu_0]$, respectively.

So, $\quad [R] = [\varepsilon_0]^\alpha [\mu_0]^\beta \quad \ldots(i)$

$[R] = [M^1 L^2 T^{-3} A^{-2}]$, $[\varepsilon_0] = [M^{-1} L^{-3} T^4A^2]$,

$[\mu_0] = [M^1 L^1 T^{-2} A^{-2}]$

Now, from Eq. (i), we get

$[M^1 L^2 T^{-3} A^{-2}] = [M^{-1} L^{-3} T^4A^2]^a [M^1 L^1 T^{-2} A^{-2}]^b$

$[M^1 L^2 T^{-3} A^{-2}] = M^{-\alpha+\beta} L^{-3\alpha+\beta} T^{4\alpha-2\beta} A^{2\alpha-2\beta}$

On comparing both sides, we get

$$-\alpha + \beta = 1 \quad \ldots(ii)$$
$$-3\alpha + \beta = 2 \quad \ldots(iii)$$
$$4\alpha - 2\beta = -3 \quad \ldots(iv)$$
$$2\alpha - 2\beta = -2 \quad \ldots(v)$$

Value of α and β can be found using any two Eqs. from (ii) to (v),

On subtracting Eq. (iii) from Eq. (ii), we get

$$(-\alpha + \beta) - (-3\alpha + \beta) = 1 - 2$$

$$\Rightarrow \quad 2\alpha = -1 \text{ or } \alpha = \frac{-1}{2}$$

Put the value of α in Eq. (ii), we get

$$\beta = +\frac{1}{2}$$

$$\therefore \quad [R] = [\varepsilon_0]^{-1/2} [\mu_0]^{1/2} = \left[\sqrt{\frac{\mu_0}{\varepsilon_0}}\right]$$

8. The pitch and the number of divisions, on the circular scale for a given screw gauge are 0.5 mm and 100, respectively. When the screw gauge is fully tightened without any object, the zero of its circular scale lies 3 divisions below the mean line.

The readings of the main scale and the circular scale for a thin sheet are 5.5 mm and 48 respectively, the thickness of this sheet is

[JEE Main 2019, 9 Jan Shift-II]

(a) 5.950 mm (b) 5.725 mm
(c) 5.755 mm (d) 5.740 mm

Exp. **(c)**

For a measuring device, the least count is the smallest value that can be measured by measuring instrument.

$$\text{Least count} = \frac{\text{Minimum reading on main - scale}}{\text{Total divisions on the scale}}$$

Here, screw gauge is used for measurement therefore,

$$LC = \frac{\text{Pitch}}{\text{number of division}}$$

$$LC = \frac{0.5}{100} \text{ mm}$$

$$LC = 5 \times 10^{-3} \text{ mm} \quad \ldots(i)$$

According to question, the zero line of its circular scale lies 3 division below the mean line and the readings of main scale = 5.5 mm

The reading of circular scale = 48

then the actual value is given by

actual value of thickness (t) = (main scale reading) + (circular scale reading + number of division below mean line) × LC

$\Rightarrow t = 5.5 \text{ mm} + (48 + 3) \times 5 \times 10^{-3} \text{ mm}$

$\Rightarrow t = 5.755 \text{ mm}$

9. The density of a material in SI units is 128 kg m^{-3}. In certain units in which the unit of length is 25 cm and the unit of mass is 50 g, the numerical value of density of the material is **[JEE Main 2019, 10 Jan Shift-I]**

(a) 40 (b) 16 (c) 640 (d) 410

Exp. **(a)**

To convert a measured value from one system to another system, we use

$$N_1u_1 = N_2u_2$$

where, N is numeric value and u is unit.

We get

$$128 \cdot \frac{\text{kg}}{\text{m}^3} = N_2 \frac{50\text{ g}}{(25\text{ cm})^3} \quad \left[\because \text{density} = \frac{\text{mass}}{\text{volume}}\right]$$

$$\Rightarrow \frac{128 \times 1000\text{ g}}{100 \times 100 \times 100\text{ cm}^3} = \frac{N_2 \times 50\text{ g}}{25 \times 25 \times 25\text{ cm}^3}$$

$$\Rightarrow N_2 = \frac{128 \times 1000 \times 25 \times 25 \times 25}{50 \times 100 \times 100 \times 100} = 40$$

10. The diameter and height of a cylinder are measured by a meter scale to be 12.6 ± 0.1 cm and 34.2 ± 0.1 cm, respectively. What will be the value of its volume in appropriate significant figures ?

[JEE Main 2019, 10 Jan Shift-II]

(a) $4300 \pm 80\text{ cm}^3$ (b) $4260 \pm 80\text{ cm}^3$

(c) $4264.4 \pm 81.0\text{ cm}^3$ (d) $4264 \pm 81\text{ cm}^3$

Exp. **(b)**

Volume of a cylinder of radius 'r' and height h is given by

$$V = \pi r^2 h$$

or $V = \frac{1}{4}\pi D^2 h$, where D is the diameter of circular surface. Here, $D = 12.6$ cm and $h = 34.2$ cm

$$\Rightarrow \quad V = \frac{\pi}{4} \times (12.6)^2 \times (34.2)$$

$$V = 4262.22\text{ cm}^3$$

$$V = 4260 \text{ (in three significant numbers)}$$

Now, error calculation can be done as

$$\frac{\Delta V}{V} = 2\left(\frac{\Delta D}{D}\right) + \frac{\Delta h}{h} = \frac{2 \times 0.1}{12.6} + \frac{0.1}{34.2}$$

$$\Rightarrow \quad \frac{\Delta V}{V} = 0.0158 + 0.0029$$

$$\Rightarrow \quad \Delta V = (0.01879) \times (4262.22)$$

$$\Rightarrow \quad \Delta V = 79.7 \simeq 80\text{ cm}^3$$

$\therefore$ For proper significant numbers, volume reading will be $V = 4260 \pm 80\text{ cm}^3$

11. The force of interaction between two atoms is given by $F = \alpha\beta \exp\left(-\frac{x^2}{\alpha kT}\right)$; where x is the distance, k is the Boltzmann constant and T is temperature and α and β are two constants. The dimension of β is

[JEE Main 2019, 11 Jan Shift-I]

(a) $[\text{MLT}^{-2}]$ (b) $[\text{M}^0\text{L}^2\text{T}^{-4}]$

(c) $[\text{M}^2\text{LT}^{-4}]$ (d) $[\text{M}^2\text{L}^2\text{T}^{-2}]$

Exp. **(c)**

Force of interaction between two atoms is given as

$$F = \alpha\beta \exp(-x^2/\alpha kT)$$

As we know, exponential terms are always dimensionless, so

dimensions of $\left(\frac{-x^2}{\alpha kT}\right) = [\text{M}^0\text{L}^0\text{T}^0]$

$\Rightarrow$ Dimensions of α = Dimension of (x^2/kT)

Now, substituting the dimensions of individual term in the given equation, we get

$= \frac{[\text{M}^0\text{L}^2\text{T}^0]}{[\text{M}^1\text{L}^2\text{T}^{-2}]}$ {$\because$ Dimensions of kT equivalent to the dimensions of energy = $[\text{M}^1\text{L}^2\text{T}^{-2}]$}

$$= [\text{M}^{-1}\text{L}^0\text{T}^2] \quad \ldots\text{(i)}$$

Now from given equation, we have dimensions of

F = dimensions of α × dimensions of β

$\Rightarrow$ Dimensions of β = Dimensions of $\left(\frac{F}{\alpha}\right)$

$$= \frac{[\text{M}^1\text{L}^1\text{T}^{-2}]}{[\text{M}^{-1}\text{L}^0\text{T}^2]} \quad [\because \text{using Eq. (i)}]$$

$$= [\text{M}^2\text{L}^1\text{T}^{-4}]$$

12. If speed (v), acceleration (A) and force (F) are considered as fundamental units, the dimension of Young's modulus will be

[JEE Main 2019, 11 Jan Shift-II]

(a) $[\text{v}^{-4}\text{A}^{-2}\text{F}]$ (b) $[\text{v}^{-2}\text{A}^2\text{F}^2]$

(c) $[\text{v}^{-2}\text{A}^2\text{F}^{-2}]$ (d) $[\text{v}^{-4}\text{A}^2\text{F}]$

Exp. **(d)**

Dimensions of speed are , $[\text{v}] = [\text{LT}^{-1}]$

Dimensions of acceleration are, $[\text{A}] = [\text{LT}^{-2}]$

Dimensions of force are, $[\text{F}] = [\text{MLT}^{-2}]$

Dimension of Young modulus is,, $[\text{Y}] = [\text{ML}^{-1}\text{T}^{-2}]$

Let dimensions of Young's modulus is expressed in terms of speed, acceleration and force as;

$$[\text{Y}] = [\text{v}]^\alpha [\text{A}]^\beta [\text{F}]^\gamma \quad \ldots\text{(i)}$$

Then substituting dimensions in terms of M, L and T

we get,

$$[ML^{-1}T^{-2}] = [LT^{-1}]^{\alpha}[LT^{-2}]^{\beta}[MLT^{-2}]^{\gamma}$$
$$= [M^{\gamma}L^{\alpha+\beta+\gamma}\ T^{-\alpha-2\beta-2\gamma}]$$

Now comparing powers of basic quantities on both sides we get, $\gamma = 1$

$$\alpha + \beta + \gamma = -1$$

and $\quad -\alpha - 2\beta - 2\gamma = -2$

Solving these, we get

$$\alpha = -4,\ \beta = 2,\ \gamma = 1$$

Substituting α, β, and γ in Eq. (i) we get;

$$[Y] = [V^{-4}A^{2}F^{1}]$$

13. The least count of the main scale of a screw gauge is 1 mm. The minimum number of divisions on its circular scale required to measure 5 μm diameter of a wire is **[JEE Main 2019, 12 Jan Shift-I]**

(a) 50 (b) 200
(c) 500 (d) 100

***Exp.* (b)**

In a screw gauge,

Least count

$$= \frac{\text{Measure of 1 main scale division (MSD)}}{\text{Number of division on circular scale}}$$

Here, minimum value to be measured/least count is 5 μm.

$$= 5 \times 10^{-6}\ \text{m}$$

∴ According to the given values,

$$5 \times 10^{-6} = \frac{1 \times 10^{-3}}{N}$$

or $\quad N = \dfrac{10^{-3}}{5 \times 10^{-6}} = \dfrac{1000}{5} = 200$ divisions

14. Let l, r, c, and v represent inductance, resistance, capacitance and voltage, respectively. The dimension of $\dfrac{l}{rcv}$ in SI units will be **[JEE Main 2019, 12 Jan Shift-II]**

(a) $[LT^2]$ (b) $[LTA]$ (c) $[A^{-1}]$ (d) $[LA^{-2}]$

***Exp.* (c)**

Dimensions of given quantities are

l = inductance = $[M^1\ L^2\ T^{-2}\ A^{-2}]$

r = resistance = $[M^1\ L^2\ T^{-3}\ A^{-2}]$

c = capacitance = $[M^{-1}\ L^{-2}\ T^4\ A^2]$

v = voltage = $[M^1\ L^2\ T^{-3}\ A^{-1}]$

So, dimensions of $\dfrac{l}{rcv}$ are

$$\left[\frac{l}{rcv}\right] = \frac{[M\ L^2\ T^{-2}\ A^{-2}]}{[M^1\ L^2\ T^{-2}\ A^{-1}]} = [A^{-1}]$$

15. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1% , the maximum error in determining the density is **[JEE Main 2018]**

(a) 2.5% (b) 3.5% (c) 4.5% (d) 6%

***Exp.* (c)**

∵ Density, $\rho = \dfrac{\text{Mass}}{\text{Volume}} = \dfrac{M}{L^3}$ or $\rho = \dfrac{M}{L^3}$

⇒ Error in density $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta M}{M} + \dfrac{3\Delta L}{L}$

So, maximum % error in measurement of ρ is

$$\frac{\Delta\rho}{\rho} \times 100 = \frac{\Delta M}{M} \times 100 + \frac{3\Delta L}{L} \times 100$$

or % error in density = 1.5 + 3 × 1

% error = 4.5%

16. The following observations were taken for determining surface tension T of water by capillary method. Diameter of capillary, $d = 1.25 \times 10^{-2}$ m rise of water, $h = 1.45 \times 10^{-2}$m. Using $g = 9.80\ \text{m/s}^2$ and the simplified relation $T = \dfrac{rhg}{2} \times 10^3 \text{N/m}$, the possible error in surface tension is closest to **[JEE Main 2017 (Offline)]**

(a) 1.5% (b) 2.4% (c) 10% (d) 0.15%

***Exp.* (a)**

By ascent formula, we have surface tension,

$$T = \frac{rhg}{2} \times 10^3\ \frac{\text{N}}{\text{m}} = \frac{dhg}{4} \times 10^3\ \frac{\text{N}}{\text{m}} \quad \left(\because r = \frac{d}{2}\right)$$

⇒ $\quad \dfrac{\Delta T}{T} = \dfrac{\Delta d}{d} + \dfrac{\Delta h}{h}$ [given, g is constant]

So, percentage $= \dfrac{\Delta T}{T} \times 100 = \left(\dfrac{\Delta d}{d} + \dfrac{\Delta h}{h}\right) \times 100$

$$= \left(\frac{0.01 \times 10^{-2}}{1.25 \times 10^{-2}} + \frac{0.01 \times 10^{-2}}{1.45 \times 10^{-2}}\right) \times 100 = 1.5\%$$

∴ $\quad \dfrac{\Delta T}{T} \times 100 = 1.5\%$

17. A student measures the time period of 100 oscillations of a simple pendulum four times. The data set is 90s, 91s, 92s and 95s. If the minimum division in the measuring clock is 1s, then the reported mean time should be

[JEE Main 2016 (Offline)]

(a) (92 ± 2) s (b) (92 ± 5) s
(c) (92 ± 1.8) s (d) (92 ± 3) s

Exp. **(a)**

Arithmetic mean time of a oscillating simple pendulum

$$= \frac{\Sigma x_i}{N}$$

$$= \frac{90 + 91 + 92 + 95}{4} = 92 \text{ s}$$

Mean deviation of a simple pendulum

$$= \frac{\Sigma |\bar{x} - x_i|}{N}$$

$$= \frac{2 + 1 + 3 + 0}{4} = 1.5$$

Given, minimum division in the measuring clock, i.e. simple pendulum = 1s. Thus, the reported mean time of a oscillating simple pendulum $= (92 \pm 2)$ s

18. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it?

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

[JEE Main 2014]

Exp. **(b)**

If student measure 3.50 cm, it means that there in an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD = 1mm and 9 MSD = 10 VSD

$\therefore$ LC of VC = 1 MSD – 1 VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right)$$

$$= \frac{1}{100} \text{ cm}$$

19. The current voltage relation of diode is given by $I = (e^{1000V/T} - 1)$ mA, where the applied voltage V is in volt and the temperature T is in kelvin. If a student makes an error measuring $\pm$ 0.01V while measuring the current of 5 mA at 300K, what will be the error in the value of current in mA?

[JEE Main 2013]

(a) 0.2 mA (b) 0.02 mA
(c) 0.5 mA (d) 0.05 mA

Exp. **(a)**

Given, $I = (e^{1000V/T} - 1)$ mA

$$dV = \pm 0.01 \text{ V}$$

$$T = 300 \text{ K}$$

$$I = 5 \text{ mA}$$

$$I = e^{1000V/T} - 1$$

$$I + 1 = e^{1000V/T}$$

Taking log on both sides, we get

$$\log (I + 1) = \frac{1000V}{T}$$

$$\Rightarrow \quad \frac{d(I + 1)}{I + 1} = \frac{1000}{T} dV$$

$$\frac{dI}{I + 1} = \frac{1000}{T} dV$$

$$\Rightarrow \quad dI = \frac{1000}{T} \times (I + 1) dV$$

$$dI = \frac{1000}{300} \times (5 + 1) \times 0.01$$

$$= 0.2 \text{ mA}$$

So, error in the value of current is 0.2 mA.

20. Let $[\varepsilon_0]$ denotes the dimensional formula of the permittivity of vacuum. If M = mass, L = length, T = time and A = electric current, then **[JEE Main 2013]**

(a) $[\varepsilon_0] = [M^{-1} L^{-3} T^2 A]$
(b) $[\varepsilon_0] = [M^{-1} L^{-3} T^4 A^2]$
(c) $[\varepsilon_0] = [M^{-2} L^2 T^{-1} A^{-2}]$
(d) $[\varepsilon_0] = [M^{-1} L^2 T^{-1} A^2]$

Exp. **(b)**

Electrostatic force between two charges,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{R^2} \Rightarrow \varepsilon_0 = \frac{q_1 q_2}{4\pi F R^2}$$

Substituting the units.

Hence, $\varepsilon_0 = \frac{C^2}{N\text{-}m^2} = \frac{[AT]^2}{[MLT^{-2}]\,[L^2]}$

$= [M^{-1}L^{-3}T^4A^2]$

21. A spectrometer gives the following reading when used to measure the angle of a prism.
Main scale reading : 58.5 degree
Vernier scale reading : 09 divisions
Given that 1 division on main scale corresponds to 0.5 degree. Total divisions on the vernier scale is 30 and match with 29 divisions of the main scale. The angle of the prism from the above data is **[AIEEE 2012]**

(a) 58.59 degree (b) 58.77 degree
(c) 58.65 degree (d) 59 degree

Exp. **(c)**

1 Vernier scale division $= \frac{29}{30}$ main scale division

$$1 \text{ VSD} = \frac{29}{30} \times 0.5° = \left(\frac{29}{60}\right)^\circ$$

Thus, least count = 1 MSD – 1 VSD

$$= \left(\frac{1}{2}\right)^\circ - \left(\frac{29}{60}\right)^\circ = \left(\frac{1}{60}\right)^\circ$$

$\therefore$ Reading = Main scale reading + Vernier scale reading

$= \text{MSR} + n \times \text{LC}$

$= 58.5° + 9 \times \left(\frac{1}{60}\right)^\circ = 58.65°$

22. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are **[AIEEE 2010]**

(a) 5, 1, 2 (b) 5, 1, 5 (c) 5, 5, 2 (d) 4, 4, 2

Exp. **(a)**

The reliable digit plus the first uncertain digit is known as significant figures.

For the number 23.023, all the non-zero digits are significant, hence 5.

For the number 0.0003, number is less than 1, the zero(s) on the right of decimal point but to the left of the first non-zero digit are not significant, hence 1.

For the number 2.1×10^{-3}, significant figures are 2.

23. In an experiment, the angles are required to be measured using an instrument. 29 divisions of the main scale exactly coincide with the 30 divisions of the vernier scale. If the smallest division of the main scale is half a degree (= 0.5°), then the least count of the instrument is **[AIEEE 2009]**

(a) one minute (b) half minute
(c) one degree (d) half degree

Exp. **(a)**

Least count

$$= \frac{\text{Value of main scale division}}{\text{Number of divisions on vernier scale}}$$

$$= \frac{1}{30}\text{ MSD} = \frac{1}{30} \times \frac{1°}{2} = \frac{1°}{60} = 1 \text{ min}$$

24. The dimensions of magnetic field in M, L, T and C (coulomb) is given as **[AIEEE 2008]**

(a) $[MLT^{-1}C^{-1}]$ (b) $[MT^2C^{-2}]$
(c) $[MT^{-1}C^{-1}]$ (d) $[MT^2C^{-1}]$

Exp. **(c)**

From the relation $F = qvB$

$\Rightarrow \quad [MLT^{-2}] = [C][LT^{-1}][B]$

$\Rightarrow \quad [B] = [MC^{-1}T^{-1}]$

25. A body of mass $m = 3.513$ kg is moving along the x-axis with a speed of 5.00 ms^{-1}. The magnitude of its momentum is recorded as **[AIEEE 2008]**

(a) 17.6 kg ms^{-1} (b) $17.565 \text{ kg ms}^{-1}$
(c) 17.56 kg ms^{-1} (d) 17.57 kg ms^{-1}

Exp. **(a)**

So, momentum,

$$p = mv = 17.565 \text{ kg ms}^{-1}$$

where $m = 3.513$ kg and $v = 5.00 \text{ ms}^{-1}$

As the number of significant digits in m is 4 and in v is 3, so, p must have 3 (minimum) significant digits. Hence,

$$p = 17.6 \text{ kg ms}^{-1}$$

26. Which of the following units denotes the dimensions $[ML^2/Q^2]$, where Q denotes the electric charge? **[AIEEE 2006]**

(a) Wb/m^2 (b) henry (H)
(c) H/m^2 (d) weber (Wb)

Exp. **(b)**

Magnetic energy $= \frac{1}{2} L I^2 = \frac{Lq^2}{2t^2}$ $\left[\text{as } I = \frac{q}{t}\right]$

where L = inductance, I = current

Energy has the dimensions = $[ML^2T^{-2}]$

Equate the dimensions, we have

$$[ML^2T^{-2}] = [\text{henry}] \times \frac{[Q^2]}{[T^2]}$$

$$\Rightarrow \quad [\text{henry}] = \frac{[ML^2]}{[Q^2]}$$

27. The 'rad' is the correct unit used to report the measurement of **[AIEEE 2006]**

(a) the ability of a beam of gamma ray photons to produce ions in a target
(b) the energy delivered by radiation to a target
(c) the biological effect of radiation
(d) the rate of decay of a radioactive source

Exp. **(c)**

'Rad' is used to measure biological effect of radiation.

28. Out of the following pairs, which one does not have identical dimensions? **[AIEEE 2005]**

(a) Angular momentum and Planck's constant
(b) Impulse and momentum
(c) Moment of inertia and moment of a force
(d) Work and torque

Exp. **(c)**

Moment of inertia, $I = mr^2$

$\therefore \quad [I] = [ML^2]$

and $\quad \tau$ = Moment of force = $\mathbf{r} \times \mathbf{F}$

$\therefore \quad [\tau] = [L]\,[MLT^{-2}] = [ML^2T^{-2}]$

29. Which one of the following represents the correct dimensions of the coefficient of viscosity? **[AIEEE 2004]**

(a) $[ML^{-1}T^{-2}]$ (b) $[MLT^{-1}]$
(c) $[ML^{-1}T^{-1}]$ (d) $[ML^{-2}T^{-2}]$

Exp. **(c)**

By Newton's formula, $\eta = \dfrac{F}{A(\Delta v_x / \Delta z)}$

$\therefore$ Dimensions of η

$$= \frac{\text{Dimensions of force}}{\text{Dimensions of area} \times \text{Dimensions of velocity gradient}}$$

$$= \frac{[MLT^{-2}]}{[L^2][T^{-1}]} = [ML^{-1}T^{-1}]$$

30. Dimensions of $1/\mu_0\varepsilon_0$, where symbols have their usual meaning, are **[AIEEE 2003]**

(a) $[L^{-1}T]$ (b) $[L^2T^2]$
(c) $[L^2T^{-2}]$ (d) $[LT^{-1}]$

Exp. **(c)**

As we know that formula of velocity is

$$v = \frac{1}{\sqrt{\mu_0\varepsilon_0}}$$

$$\Rightarrow \quad v^2 = \frac{1}{\mu_0\varepsilon_0} = [LT^{-1}]^2$$

$$\therefore \quad \frac{1}{\mu_0\varepsilon_0} = [L^2T^{-2}]$$

31. The physical quantities not having same dimensions are **[AIEEE 2003]**

(a) torque and work
(b) momentum and Planck's constant
(c) stress and Young's modulus
(d) speed and $(\mu_0\varepsilon_0)^{-1/2}$

Exp. **(b)**

Planck's constant (in terms of unit)

h = J-s = $[ML^2T^{-2}][T] = [ML^2T^{-1}]$

Momentum (p) = kg - ms^{-1}

$= [M][L][T^{-1}] = [MLT^{-1}]$

32. Identify the pair whose dimensions are equal. **[AIEEE 2002]**

(a) Torque and work
(b) Stress and energy
(c) Force and stress
(d) Force and work

Exp. **(a)**

The dimensions of torque is given by

$$\text{Torque } (\tau) = \frac{\text{Angular momentum}}{\text{Time}}$$

$$\text{Torque } (\tau) = \frac{[ML^2T^{-1}]}{[T]} = [ML^2T^{-2}]$$

$\therefore \quad$ Work = Force $\times$ Distance = $[MLT^{-2}]\,[L]$

$= [ML^2T^{-2}]$

Hence, both have same dimensions.

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2

Kinematics

1. Ship A is sailing towards north-east with velocity $\mathbf{v} = 30\hat{\mathbf{i}} + 50\hat{\mathbf{j}}$ km/h, where $\hat{\mathbf{i}}$ points east and $\hat{\mathbf{j}}$ north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/h. A will be at minimum distance from B in

[JEE Main 2019, 8 April Shift-I]

(a) 4.2 h (b) 2.6 h (c) 3.2 h (d) 2.2 h

Exp. **(b)**

Considering the initial position of ship A as origin, so the velocity and position of ship will be

$\mathbf{v}_A = (30\hat{\mathbf{i}} + 50\hat{\mathbf{j}})$ and $\mathbf{r}_A = (0\hat{\mathbf{i}} + 0\hat{\mathbf{j}})$

Now, as given in the question, velocity and position of ship B will be, $\mathbf{v}_B = -10\hat{\mathbf{i}}$

and $\mathbf{r}_B = (80\hat{\mathbf{i}} + 150\hat{\mathbf{j}})$

Hence, the given situation can be represented graphically as

After time t, coordinates of ships A and B are $(80 - 10t, 150)$ and $(30t, 50t)$.

So, distance between A and B after time t is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(80 - 10t - 30t)^2 + (150 - 50t)^2}$$

$$\Rightarrow \quad d^2 = (80 - 40t)^2 + (150 - 50t)^2$$

Distance is minimum when $\frac{d}{dt}(d^2) = 0$

After differentiating, we get

$$\Rightarrow \quad \frac{d}{dt}[(80 - 40t)^2 + (150 - 50t)^2] = 0$$

$$\Rightarrow \quad 2(80 - 40t)(-40) + 2(150 - 50t)(-50) = 0$$

$$\Rightarrow \quad -3200 + 1600t - 7500 + 2500t = 0$$

$$\Rightarrow \quad 4100t = 10700 \Rightarrow t = \frac{10700}{4100} = 2.6 \text{ h}$$

Alternate Solution

Time after which the distance is minimum between A and B can be calculated as

$$t = \frac{|\mathbf{r}_{BA} \cdot \mathbf{v}_{BA}|}{|\mathbf{v}_{BA}|^2}$$

where, $\mathbf{r}_{BA} = \mathbf{r}_B - \mathbf{r}_A = 80\hat{\mathbf{i}} + 150\hat{\mathbf{j}}$

and $\mathbf{v}_{BA} = -10\hat{\mathbf{i}} - (30\hat{\mathbf{i}} + 50\hat{\mathbf{j}}) = -40\hat{\mathbf{i}} - 50\hat{\mathbf{j}}$

$$\Rightarrow \quad t = \frac{|(80\hat{\mathbf{i}} + 150\hat{\mathbf{j}}) \cdot (-40\hat{\mathbf{i}} - 50\hat{\mathbf{j}})|}{|-40\hat{\mathbf{i}} - 50\hat{\mathbf{j}}|^2}$$

$$= \frac{3200 + 7500}{4100} = \frac{10700}{4100} = 2.6 \text{ h}$$

2. Let $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A}_1 + \mathbf{A}_2| = 5$. The value of $(2\mathbf{A}_1 + 3\mathbf{A}_2) \cdot (3\mathbf{A}_1 - 2\mathbf{A}_2)$ is

[JEE Main 2019, 8 April Shift-II]

(a) -106.5 (b) -112.5 (c) -99.5 (d) -118.5

Exp. **(d)**

For vector $\mathbf{A}_1 + \mathbf{A}_2$, we have

$$|\mathbf{A}_1 + \mathbf{A}_2|^2 = (\mathbf{A}_1 + \mathbf{A}_2) \cdot (\mathbf{A}_1 + \mathbf{A}_2) \; [\because \mathbf{x} \cdot \mathbf{x} = |\mathbf{x}|^2]$$

$$\Rightarrow \quad |\mathbf{A}_1 + \mathbf{A}_2|^2 = |\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2\mathbf{A}_1 \cdot \mathbf{A}_2$$

Given, $|\mathbf{A}_1| = 3$, $|\mathbf{A}_2| = 5$ and $|\mathbf{A} + \mathbf{A}_2| = 5$

So, we have

$$(5)^2 = 9 + 25 + 2\mathbf{A}_1 \cdot \mathbf{A}_2$$

$$\Rightarrow \quad \mathbf{A}_1 \cdot \mathbf{A}_2 = -\frac{9}{2}$$

Now, $(2\mathbf{A}_1 + 3\mathbf{A}_2)\cdot(3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6|\mathbf{A}_1|^2 - 4\mathbf{A}_1 \cdot \mathbf{A}_2 + 9\mathbf{A}_1 \cdot \mathbf{A}_2 - 6|\mathbf{A}_2|^2$$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5\mathbf{A}_1 \cdot \mathbf{A}_2$$

Substituting values, we have

$$(2\mathbf{A}_1 + 3\mathbf{A}_2)\cdot(3\mathbf{A}_1 - 2\mathbf{A}_2)$$

$$= 6(9) - 6(25) + 5\left(-\frac{9}{2}\right) = -118.5$$

Alternate Solution

As we know, $|\mathbf{A}_1 + \mathbf{A}_2|$ can also be written as

$$|\mathbf{A} + \mathbf{A}_2| = \sqrt{|\mathbf{A}_1|^2 + |\mathbf{A}_2|^2 + 2|\mathbf{A}_1||\mathbf{A}_2|\cos\theta}$$

Substituting the given values, we get

$$5 = \sqrt{(3)^2 + (5)^2 + 2 \times 3 \times 5\cos\theta}$$

$$\text{or } \cos\theta = -\frac{9}{2 \times 3 \times 5} = -\frac{3}{10}$$

So, $(2\mathbf{A}_1 + 3\mathbf{A}_2)\cdot(3\mathbf{A}_1 - 2\mathbf{A}_2)$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5\mathbf{A}_1 \cdot \mathbf{A}_2$$

$$= 6|\mathbf{A}_1|^2 - 6|\mathbf{A}_2|^2 + 5|\mathbf{A}_1||\mathbf{A}_2|\cos\theta$$

$$= 6 \times 9 - 6 \times 25 + 5 \times 2 \times 3 \times \left(\frac{-3}{10}\right) = -118.5$$

3. A particle starts from origin O from rest and moves with a uniform acceleration along the positive X-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time)

[JEE Main 2019, 8 April Shift-II]

(a) (A) (b) (A), (B), (C)
(c) (B), (C) (d) (A), (B), (D)

Exp. **(d)**

Since, the particle starts from rest, this means, initial velocity, $u = 0$

Also, it moves with uniform acceleration along positive X-axis. This means, its acceleration (a) is constant.

$\therefore$ Given, a - t graph in (A) is correct.

As we know, for velocity-time graph, slope = acceleration.

Since, the given v-t graph in (B) represents that its slope is constant and non-zero.

$\therefore$ Graph in (B) is also correct.

Also, the displacement of such a particle w.r.t. time is given by

$$x = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}at^2 \Rightarrow x \propto t^2$$

So, x *versus* t graph would be a parabola with starting from origin.

This is correctly represented in displacement-time graph given in (D).

4. The stream of a river is flowing with a speed of 2 km/h. A swimmer can swim at a speed of 4 km/h. What should be the direction of the swimmer with respect to the flow of the river to cross the river straight ?

[JEE Main 2019, 9 April Shift-I]

(a) 60° (b) 120° (c) 90° (d) 150°

Exp. **(b)**

Let the velocity of the swimmer is

$$v_s = 4 \text{ km/h}$$

and velocity of river is $v_r = 2$ km/h

Also, angle of swimmer with the flow of the river (down stream) is α as shown in the figure below

From diagram, angle θ is

$$\sin\theta = \frac{v_r}{v_{sr}} = \frac{2 \text{ km/h}}{4 \text{ km/h}} = \frac{1}{2} \Rightarrow \theta = 30°$$

Clearly, $\alpha = 90° + 30° = 120°$

5. The position vector of particle changes with time according to the relation $\mathbf{r}(t) = 15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}$. What is the magnitude of the acceleration (in ms^{-2}) at $t = 1$?

[JEE Main 2019, 9 April Shift-II]

(a) 50 (b) 100 (c) 25 (d) 40

Exp. **(a)**

Position vector of particle is given as

$$\mathbf{r} = 15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}$$

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}[15t^2\hat{\mathbf{i}} + (4 - 20t^2)\hat{\mathbf{j}}]$$

$$= 30t\ \hat{\mathbf{i}} - 40t\ \hat{\mathbf{j}}$$

Acceleration of particle is

$$\mathbf{a} = \frac{d}{dt}(\mathbf{v}) = \frac{d}{dt}(30t\hat{\mathbf{i}} - 40t\hat{\mathbf{j}}) = 30\hat{\mathbf{i}} - 40\hat{\mathbf{j}}$$

So, magnitude of acceleration at $t = 1$s is

$$|\mathbf{a}|_{t=1s} = \sqrt{a_x^2 + a_y^2} = \sqrt{30^2 + 40^2}$$

$$= 50\,\text{ms}^{-2}$$

6. The position of a particle as a function of time t, is given by $x(t) = at + bt^2 - ct^3$ where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be **[JEE Main 2019, 9 April Shift-II]**

(a) $a + \frac{b^2}{2c}$ (b) $a + \frac{b^2}{4c}$ (c) $a + \frac{b^2}{3c}$ (d) $a + \frac{b^2}{c}$

Exp. **(c)**

Position of particle is, $x(t) = at + bt^2 - ct^3$

So, its velocity is, $v = \frac{dx}{dt} = a + 2bt - 3ct^2$

and acceleration is, $a = \frac{dv}{dt} = 2b - 6ct$

Acceleration is zero, then $2b - 6ct = 0$

$$\Rightarrow \quad t = \frac{2b}{6c} = \frac{b}{3c}$$

Substituting this 't' in expression of velocity, we get

$$v = a + 2b\left(\frac{b}{3c}\right) - 3c\left(\frac{b}{3c}\right)^2$$

$$= a + \frac{2b^2}{3c} - \frac{b^2}{3c} = a + \frac{b^2}{3c}$$

7. A ball is thrown upward with an initial velocity v_0 from the surface of the earth. The motion of the ball is affected by a drag force equal to $m\gamma v^2$ (where, m is mass of the ball, v is its instantaneous velocity and γ is a constant). Time taken by the ball to rise to its zenith is **[JEE Main 2019, 10 April Shift-I]**

(a) $\frac{1}{\sqrt{2\gamma g}}\tan^{-1}\left(\sqrt{\frac{2\gamma}{g}}\,v_0\right)$

(b) $\frac{1}{\sqrt{\gamma g}}\tan^{-1}\left(\sqrt{\frac{\gamma}{g}}\,v_0\right)$

(c) $\frac{1}{\sqrt{\gamma g}}\sin^{-1}\left(\sqrt{\frac{\gamma}{g}}\,v_0\right)$

(d) $\frac{1}{\sqrt{\gamma g}}\ln\left(1 + \sqrt{\frac{\gamma}{g}}\,v_0\right)$

Exp. **(b)**

Given, drag force, $F = m\gamma v^2$ … (i)

As we know, general equation of force

$= ma$ … (ii)

Comparing Eqs. (i) and (ii), we get

$$a = \gamma v^2$$

$\therefore$ Net retardation of the ball when thrown vertically upward is

$$a_{net} = -(g + \gamma v^2) = \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{dv}{(g + \gamma v^2)} = -dt \quad \text{… (iii)}$$

By integrating both sides of Eq. (iii) in known limits, i.e.

When the ball thrown upward with velocity v_0 and then reaches to its zenith, i.e. for maximum height at time $t = t$, $v = 0$

$$\Rightarrow \quad \int_{v_0}^{0} \frac{dv}{(\gamma v^2 + g)} = \int_0^t -dt$$

$$\text{or} \quad \frac{1}{\gamma}\int_{v_0}^{0} \frac{1}{\left[\left(\sqrt{\frac{g}{\gamma}}\right)^2 + v^2\right]}dv = -\int_0^t dt$$

$$\Rightarrow \quad \frac{1}{\gamma}\cdot\frac{1}{\sqrt{g/\gamma}}\cdot\left[\tan^{-1}\left(\frac{v}{\sqrt{g/\gamma}}\right)\right]_{v_0}^{0} = -t$$

$$\left[\because \int \frac{1}{x^2 + a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right]$$

$$\Rightarrow \quad \frac{1}{\sqrt{\gamma g}}\cdot\tan^{-1}\left(\frac{\sqrt{\gamma}\,v_0}{\sqrt{g}}\right) = +t$$

8. A plane is inclined at an angle $\alpha = 30^\circ$ with respect to the horizontal. A particle is projected with a speed $u = 2\ \text{ms}^{-1}$, from the base of the plane, making an angle $\theta = 15^\circ$ with respect to the plane as shown in the

figure. The distance from the base, at which the particle hits the plane is close to [Take, $g = 10\ \text{ms}^{-2}$]

[JEE Main 2019, 10 April Shift-II]

(a) 26 cm (b) 20 cm (c) 18 cm (d) 14 cm

Exp. **(b)**

When a projectile is projected at an angle θ with an inclined plane making angle α with the horizontal, then

Components of u along and perpendicular to plane are

$$u_x = u\cos\theta \quad \text{and} \quad u_y = u\sin\theta$$

We can also resolve acceleration due to gravity into its components along and perpendicular to plane as shown below

So, we can now apply formula for range, i.e. net horizontal displacement of the particle as

$$R = u_x T + \frac{1}{2}a_x T^2 \quad \ldots(\text{i})$$

where, T = time of flight.

Using formula for time of flight, we have

$$T = \frac{2u_y}{a_y} = \frac{2u\sin\theta}{g\cos\alpha} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we have

Range up the inclined plane is

$$R = u_x T + \frac{1}{2}a_x T^2$$

$$= u\cos\theta\left(\frac{2u\sin\theta}{g\cos\alpha}\right) - \frac{1}{2}g\sin\alpha\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2$$

Here, $u = 2\ \text{ms}^{-1}$, $g = 10\ \text{ms}^{-2}$, $\theta = 15°$, $\alpha = 30°$

So, $$T = \frac{2u\sin\theta}{g\cos\alpha} = \frac{2\times 2\sin 15°}{10\times\cos 30°}$$

$$= \frac{2\times 2\times 0.258\times 2}{10\times 1.732} = 0.1191$$

Now,

$$R = 2\times\cos 15°\times 0.1191 - \frac{1}{2}\times 10\sin 30°(0.1191)^2$$

$$= 2\times 0.965\times 0.1191 - \frac{5}{2}(0.1191)^2$$

$$= 0.229 - 0.0354 = 0.1936\ \text{m} \approx 0.20\ \text{m} = 20\ \text{cm}$$

9. A bullet of mass 20 g has an initial speed of $1\ \text{ms}^{-1}$, just before it starts penetrating a mud wall of thickness 20 cm. If the wall offers a mean resistance of 2.5×10^{-2}N, the speed of the bullet after emerging from the other side of the wall is close to

[JEE Main 2019, 10 April Shift-II]

(a) $0.3\ \text{ms}^{-1}$ (b) $0.4\ \text{ms}^{-1}$
(c) $0.1\ \text{ms}^{-1}$ (d) $0.7\ \text{ms}^{-1}$

Exp. **(d)**

Given, resistance offered by the wall

$$= F = -2.5\times 10^{-2}\ \text{N}$$

So, deacceleration of bullet,

$$a = \frac{F}{m} = \frac{-2.5\times 10^{-2}}{20\times 10^{-3}} = -\frac{5}{4}\text{ms}^{-2}$$

$$(\because m = 20\ \text{g} = 20\times 10^{-3}\ \text{kg})$$

Now, using the equation of motion,

$$v^2 - u^2 = 2as$$

We have,

$$v^2 = 1 + 2\left(-\frac{5}{4}\right)(20\times 10^{-2})$$

$$(\because u = 1\ \text{ms}^{-1} \text{ and } s = 20\ \text{cm} = 20\times 10^{-2}\ \text{m})$$

$$\Rightarrow \quad v^2 = \frac{1}{2}$$

$$\therefore \quad v = \frac{1}{\sqrt{2}} \approx 0.7\ \text{ms}^{-1}$$

10. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product $t_1 t_2$ is **[JEE Main 2019, 12 April Shift-I]**

(a) $\frac{R}{4g}$ (b) $\frac{R}{g}$ (c) $\frac{R}{2g}$ (d) $\frac{2R}{g}$

Exp. (d)

Key Idea Range of a projectile motion is given by

$$R = \frac{u^2 \sin 2\theta}{g}$$

Time of flight is given by $t = \frac{2u\sin\theta}{g}$

Given, range of the fired shell,

$$R = R$$

and time of flights are t_1 and t_2.

Let θ_1 and θ_2 are the two angles at which shell is fired. As, range in both cases is same, i.e.

$$R_1 = R_2 = R$$

Here, $R_1 = \frac{u^2 \sin 2\theta_1}{g}$ and $R_2 = \frac{u^2 \sin 2\theta_2}{g}$

$$\Rightarrow \quad R = \frac{u^2 \sin 2\theta_1}{g} = \frac{u^2 \sin 2\theta_2}{g} \quad \ldots(i)$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin 2\theta_2$$

$$\Rightarrow \quad \sin 2\theta_1 = \sin(180 - 2\theta_2) \quad [\because \sin(180-\theta) = \sin\theta]$$

$$\Rightarrow \quad 2(\theta_1 + \theta_2) = 180 \text{ or } \theta_1 + \theta_2 = 90$$

$$\Rightarrow \quad \theta_2 = 90 - \theta_1 \quad \ldots(ii)$$

So, time of flight in first case,

$$t_1 = \frac{2u\sin\theta_1}{g} \quad \ldots(iii)$$

and time of flight in second case,

$$t_2 = \frac{2u\sin\theta_2}{g} = \frac{2u\sin(90-\theta_1)}{g} = \frac{2u\cos\theta_1}{g} \quad \ldots(iv)$$

From Eqs. (iii) and (iv), we get

$$t_1 t_2 = \frac{2u\sin\theta_1}{g} \times \frac{2u\cos\theta_1}{g}$$

$$\Rightarrow t_1 t_2 = \frac{4u^2 \sin\theta_1 \cos\theta_1}{g^2} \Rightarrow t_1 t_2 = \frac{2u^2 \sin 2\theta_1}{g^2}$$

$$(\because \sin 2\theta = 2\sin\theta\cos\theta) \quad \ldots(v)$$

From Eq. (i), we get

$$R = \frac{u^2 \sin 2\theta_1}{g}$$

$$\therefore \quad t_1 t_2 = \frac{2R}{g}$$

11. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$.

If it were launched at an angle θ_0 with speed v_0, then (Take, $g = 10\ \text{ms}^{-2}$)

[JEE Main 2019, 12 April Shift-I]

(a) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(b) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

(c) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}\text{ms}^{-1}$

(d) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}\text{ms}^{-1}$

Exp. (c)

Given, $g = 10\ \text{m/s}^2$

Equation of trajectory of the projectile,

$$y = 2x - 9x^2 \quad \ldots(i)$$

In projectile motion, equation of trajectory is given by

$$y = x\tan\theta_0 - \frac{g\,x^2}{2v_0^2\cos^2\theta_0} \quad \ldots(ii)$$

By comparison of Eqs. (i) and (ii), we get

$$\tan\theta_0 = 2 \quad \ldots(iii)$$

and $$\frac{g}{2v_0^2\cos^2\theta_0} = 9$$

or $$v_0^2 = \frac{g}{9 \times 2\cos^2\theta_0} \quad \ldots(iv)$$

From Eq. (iii), we can get value of $\cos\theta$ and $\sin\theta$

$$\cos\theta_0 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_0 = \frac{2}{\sqrt{5}} \quad \ldots(v)$$

Using value of $\cos\theta_0$ from Eq. (v) to Eq. (iv), we get

$$v_0^2 = \frac{10 \times (\sqrt{5})^2}{2 \times (1)^2 \times 9} = \frac{10 \times 5}{2 \times 9}$$

$$\Rightarrow \quad v_0^2 = \frac{25}{9} \text{ or } v_0 = \frac{5}{3}\text{ m/s} \quad \ldots(vi)$$

From Eq. (v), we get

$$\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$$

12. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights h_1 and h_2. Which of the following is correct? **[JEE Main 2019, 12 April Shift-II]**

(a) $R^2 = 4h_1h_2$ (b) $R^2 = 16h_1h_2$

(c) $R^2 = 2h_1h_2$ (d) $R^2 = h_1h_2$

Exp. **(b)**

> **Key Idea** Same range for two particles thrown with some initial speed may occurs when they are projected at complementary angle.
>
> $\therefore \quad \theta_1 + \theta_2 = 90°$
>
> where, θ_1 and θ_2 are angles of projection.

As maximum range occurs at $\theta = 45°$ for a given initial projection speed, we take angles of projection of two particles as

$$\theta_1 = 45° + \theta, \theta_2 = 45° - \theta$$

where, θ is angle of projectiles with 45° line.

So, range of projectiles will be

$$R = R_1 = R_2 = \frac{u^2 \sin 2(\theta_1)}{g}$$

$$\Rightarrow R = \frac{u^2 \sin 2(45° + \theta)}{g} \quad \Rightarrow R = \frac{u^2 \sin(90° + 2\theta)}{g}$$

$$\Rightarrow R = \frac{u^2 \cos 2\theta}{g} \qquad \Rightarrow R^2 = \frac{u^4 \cos^2 2\theta}{g^2} \quad \ldots(i)$$

Maximum heights achieved in two cases are

$$h_1 = \frac{u^2 \sin^2(45° + \theta)}{2g}$$

and $$h_2 = \frac{u^2 \sin^2(45° - \theta)}{2g}$$

So, $$h_1h_2 = \frac{u^4 \sin^2(45° + \theta)\sin^2(45° - \theta)}{4g^2}$$

Using $2\sin A \cdot \sin B = \cos(A - B) - \cos(A + B)$, we have

$$\sin(45° + \theta)\sin(45° - \theta) = \frac{1}{2}(\cos 2\theta - \cos 90°)$$

$$\Rightarrow \sin(45° + \theta)\sin(45° - \theta) = \frac{\cos 2\theta}{2} \quad [\because \cos 90° = 0]$$

So, we have

$$h_1h_2 = \frac{u^4\left(\frac{\cos 2\theta}{2}\right)^2}{4g^2} \Rightarrow h_1h_2 = \frac{u^4 \cos^2 2\theta}{16g^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\Rightarrow \quad h_1h_2 = \frac{R^2}{16} \Rightarrow R^2 = 16\, h_1h_2$$

13. A particle is moving with speed $v = b\sqrt{x}$ along positive X-axis. Calculate the speed of the particle at time $t = \tau$ (assume that the particle is at origin at $t = 0$). **[JEE Main 2019, 12 April Shift-II]**

(a) $\frac{b^2\tau}{4}$ (b) $\frac{b^2\tau}{2}$ (c) $b^2\tau$ (d) $\frac{b^2\tau}{\sqrt{2}}$

Exp. **(b)**

Given, speed,

$$v = b\sqrt{x}$$

Now, differentiating it with respect to time, we get

$$\frac{dv}{dt} = \frac{d}{dt} b\sqrt{x}$$

Now, acceleration

$$\Rightarrow \quad a = \frac{b}{2\sqrt{x}} \cdot \frac{dx}{dt} \qquad \left[\because \frac{dv}{dt} = a\right]$$

$$\Rightarrow \quad a = \frac{b}{2\sqrt{x}} \cdot v$$

$$= \frac{b}{2\sqrt{x}} \cdot b\sqrt{x} = \frac{b^2}{2}$$

As acceleration is constant, we use

$$v = u + at \quad \ldots(i)$$

Now, it is given that $x = 0$ at $t = 0$.

So, initial speed of particle is

$$u = b\sqrt{x}\Big|_{x=0} = b \times 0 = 0$$

Hence, when time $t = \tau$, speed of the particle using Eq. (i) is

$$v = u + at = 0 + \frac{b^2}{2} \cdot \tau = \frac{b^2}{2} \cdot \tau$$

14. A particle is moving with a velocity $\mathbf{v} = k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}})$, where k is a constant.

The general equation for its path is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $y = x^2 + \text{constant}$ (b) $y^2 = x + \text{constant}$

(c) $xy = \text{constant}$ (d) $y^2 = x^2 + \text{constant}$

Exp. **(d)**

Given, velocity of a particle is

$$\mathbf{v} = k(y\hat{\mathbf{i}} + x\hat{\mathbf{j}}) \quad \text{...(i)}$$

Suppose, it's position is given as

$$\mathbf{r} = x\,\hat{\mathbf{i}} + y\,\hat{\mathbf{j}}$$

$$\therefore \quad \mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\hat{\mathbf{i}} + y\hat{\mathbf{j}})$$

$$= \frac{dx}{dt}\hat{\mathbf{i}} + \frac{dy}{dt}\hat{\mathbf{j}} \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{dx}{dt} = y \quad \text{...(iii)}$$

and $$\frac{dy}{dt} = x \quad \text{...(iv)}$$

Dividing Eq. (iii) and Eq. (iv), we get

$$\frac{\frac{dx}{dt}}{\frac{dy}{dt}} = \frac{y}{x} \quad \Rightarrow \quad x\frac{dx}{dt} = y\frac{dy}{dt}$$

or $xdx = ydy$

Integrating both sides, we get

$$\int xdx = \int ydy \text{ or } \frac{x^2}{2} + \frac{c_1}{2} = \frac{y^2}{2} + \frac{c_2}{2}$$

where, c_1 and c_2 are the constants of integration.

$\Rightarrow x^2 + c = y^2$ [here, c (constant) $= c_1 - c_2$]

or $y^2 = x^2 + \text{constant}$

15. In a car race on a straight path, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to **[JEE Main 2019, 9 Jan Shift-II]**

(a) $\frac{2a_1a_2}{a_1 + a_2}t$ (b) $\sqrt{2a_1a_2}\,t$

(c) $\sqrt{a_1a_2}\,t$ (d) $\frac{a_1 + a_2}{2}t$

Exp. **(c)**

Let car B takes time $(t_0 + t)$ and car A takes time t_0 to finish the race.

Then,

Given, $$v_A - v_B = v = (a_1 - a_2)t_0 - a_2t \quad \text{...(i)}$$

$$s_B = s_A = \frac{1}{2}a_1t_0^2 = \frac{1}{2}a_2(t_0 + t)^2$$

or $$\sqrt{a_1}\,t_0 = \sqrt{a_2}(t_0 + t)$$

or $$\sqrt{a_1}t_0 = \sqrt{a_2}t_0 + \sqrt{a_2}t$$

or $$(\sqrt{a_1} - \sqrt{a_2})t_0 = \sqrt{a_2}\,t$$

or $$t_0 = \frac{\sqrt{a_2}.t}{(\sqrt{a_1} - \sqrt{a_2})} \quad \text{...(ii)}$$

Substituting the value of t_0 from Eq. (ii) into Eq. (i), we get

$$v = (a_1 - a_2)\frac{\sqrt{a_2}\,t}{\sqrt{a_1} - \sqrt{a_2}} - a_2t$$

$$= (\sqrt{a_1} - \sqrt{a_2})(\sqrt{a_1} + \sqrt{a_2})\cdot\frac{\sqrt{a_2}t}{(\sqrt{a_1} - \sqrt{a_2})} - a_2t$$

or $v = (\sqrt{a_1} + \sqrt{a_2})\cdot\sqrt{a_2}\,t - a_2t$

$= \sqrt{a_1a_2}\cdot t + a_2t - a_2t$

or $v = \sqrt{a_1 \cdot a_2}\,t$

16. In three dimensional system, the position coordinates of a particle (in motion) are given below **[JEE Main 2019, 9 Jan (Shift-II)]**

$$x = a\cos\omega t$$
$$y = a\sin\omega t$$
$$z = a\omega t$$

The velocity of particle will be

(a) $\sqrt{2}\,a\omega$ (b) $2\,a\omega$

(c) $a\omega$ (d) $\sqrt{3}\,a\omega$

Exp. **(a)**

Given that the position coordinates of a particle

$$\left.\begin{aligned} x &= a\cos\omega t \\ y &= a\sin\omega t \\ z &= a\omega t \end{aligned}\right\} \quad \text{...(i)}$$

So, the position vector of the particle is

$$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

$$\Rightarrow \quad \hat{\mathbf{r}} = a\cos\omega t\,\hat{\mathbf{i}} + a\sin\omega t\,\hat{\mathbf{j}} + a\,\omega t\,\hat{\mathbf{k}}$$

$$\hat{\mathbf{r}} = a[\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\ \hat{\mathbf{j}} + \omega t\ \hat{\mathbf{k}}]$$

therefore, the velocity of the particle is

$$\because\ \hat{\mathbf{v}} = \frac{d\mathbf{r}}{dt} = \frac{d[a][\cos\omega t\ \hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}} + \omega t\ \hat{\mathbf{k}}]}{dt}$$

$$\Rightarrow \hat{\mathbf{v}} = -a\omega\sin\omega t\ \hat{\mathbf{i}} + a\omega\cos\omega t\ \hat{\mathbf{j}} + a\omega\hat{\mathbf{k}})$$

The magnitude of velocity is

$$|\mathbf{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

or $$|\mathbf{v}| = \sqrt{(-a\omega\sin\omega t)^2 + (a\omega\cos\omega t)^2 + (a\omega)^2}$$

$$= \omega a\sqrt{(-\sin\omega t)^2 + (\cos\omega t)^2 + (1)^2}$$

$$= \sqrt{2}\,\omega a$$

17. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s, respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets on the ground fired by the two guns is

[JEE Main 2019, 10 Jan (Shift-I)]

(a) 1 : 4 (b) 1 : 16
(c) 1 : 8 (d) 1 : 2

Exp. **(b)**

Bullets from guns can reach upto a distance of maximum range which occurs when projection is made at angle of 45°.

Maximum range for gun

$$A = R_1 = \frac{u_1^2}{g}$$

Maximum range for gun

$$B = R_2 = \frac{u_2^2}{g}$$

So, ratio of covered areas

$$= \frac{\pi(R_1)^2}{\pi(R_2)^2} = \frac{\pi \cdot u_1^4/g^2}{\pi \cdot u_2^4/g^2} = \frac{u_1^4}{u_2^4}$$

Here, $u_1 = 1\,\text{km/s}$ and $u_2 = 2\,\text{km/s}$

So, ratio of areas $= \frac{1^4}{2^4} = \frac{1}{16}$

$= 1 : 16$

18. In the cube of side 'a' shown in the figure, the vector from the central point of the face $ABOD$ to the central point of the face $BEFO$ will be **[JEE Main 2019, 10 Jan (Shift-I)]**

(a) $\frac{1}{2}a(\hat{\mathbf{i}} - \hat{\mathbf{k}})$ (b) $\frac{1}{2}a(\hat{\mathbf{j}} - \hat{\mathbf{i}})$
(c) $\frac{1}{2}a(\hat{\mathbf{j}} - \hat{\mathbf{k}})$ (d) $\frac{1}{2}a(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

Exp. **(b)**

In the given cube, coordinates of point G(centre of face $ABOD$) are

$$x_1 = \frac{a}{2},\ y_1 = 0,\ z_1 = \frac{a}{2}$$

where, a = side of cube

and coordinates of point H are

$$x_2 = 0,\ y_2 = \frac{a}{2},\ z_2 = \frac{a}{2}$$

So, vector GH is

$$\text{GH} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

$$= -\frac{a}{2}\hat{\mathbf{i}} + \frac{a}{2}\hat{\mathbf{j}} = \frac{a}{2}(\hat{\mathbf{j}} - \hat{\mathbf{i}})$$

19. Two vectors $\mathbf{A}$ and $\mathbf{B}$ have equal magnitudes. The magnitude of $(\mathbf{A} + \mathbf{B})$ is 'n' times the magnitude of $(\mathbf{A} - \mathbf{B})$. The angle between $\mathbf{A}$ and $\mathbf{B}$ is

[JEE Main 2019, 10 Jan (Shift-II)]

(a) $\sin^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (b) $\sin^{-1}\left(\frac{n-1}{n+1}\right)$
(c) $\cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$ (d) $\cos^{-1}\left(\frac{n-1}{n+1}\right)$

Exp. **(c)**

Given, $|\mathbf{A}| = |\mathbf{B}|$

or $A = B$ ...(i)

Let magnitude of ($\mathbf{A} + \mathbf{B}$) is R and for ($\mathbf{A} - \mathbf{B}$) is R'.

Now, $\mathbf{R} = \mathbf{A} + \mathbf{B}$

and $R^2 = A^2 + B^2 + 2AB\cos\theta$

$$R^2 = 2A^2 + 2A^2\cos\theta \quad \text{...(ii)}$$

[$\because$ using Eq. (i)]

Again, $\mathbf{R'} = \mathbf{A} - \mathbf{B}$

$\Rightarrow$ $R'^2 = A^2 + B^2 - 2AB\cos\theta$

$$R'^2 = 2A^2 - 2A^2\cos\theta \quad \text{...(iii)}$$

[$\because$ using Eq. (i)]

Given, $R = nR'$

or $\left(\frac{R}{R'}\right)^2 = n^2$

Dividing Eq. (ii) with Eq. (iii), we get

$$\frac{n^2}{1} = \frac{1+\cos\theta}{1-\cos\theta}$$

or $$\frac{n^2-1}{n^2+1} = \frac{(1+\cos\theta)-(1-\cos\theta)}{(1+\cos\theta)+(1-\cos\theta)}$$

$\Rightarrow$ $$\frac{n^2-1}{n^2+1} = \frac{2\cos\theta}{2} = \cos\theta$$

or $$\theta = \cos^{-1}\left(\frac{n^2-1}{n^2+1}\right)$$

20. A particle starts from the origin at time $t = 0$ and moves along the positive X-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time $t = 5$ s?

[JEE Main 2019, 10 Jan (Shift-II)]

(a) 6 m (b) 3 m
(c) 10 m (d) 9 m

Exp. **(d)**

Key Idea Area under the velocity-time curve represents displacement.

To get exact position at $t = 5$ s, we need to calculate area of the shaded part in the curve as shown below

$\therefore$ Displacement of particle = Area of OPA + Area of $PABSP$ + Area of $QBCRQ$

$$= \left(\frac{1}{2}\times 2\times 2\right) + (2\times 2) + (3\times 1)$$

$$= 2 + 4 + 3 = 9\text{ m}$$

21. A body is projected at $t = 0$ with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at $t = 1$ s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10\text{ ms}^{-2}$, the value of R is

[JEE Main 2019, 11 Jan (Shift-I)]

(a) 10.3 m (b) 2.8 m
(c) 5.1 m (d) 2.5 m

Exp. **(b)**

Components of velocity at an instant of time t of a body projected at an angle θ is

$v_x = u\cos\theta + g_x t$ and $v_y = u\sin\theta + g_y t$

Here, components of velocity at $t = 1$ s, is

$$v_x = u\cos 60^\circ + 0 \quad [\text{as } g_x = 0]$$

$$= 10\times\frac{1}{2} = 5\text{ m/s}$$

and $$v_y = u\sin 60^\circ + (-10)\times(1)$$

$$= 10\times\frac{\sqrt{3}}{2} + (-10)\times(1)$$

$$= 5\sqrt{3} - 10$$

$\Rightarrow$ $|v_y| = |10 - 5\sqrt{3}|$ m / s

Now, angle made by the velocity vector at time of $t = 1$ s

$$|\tan\alpha| = \left|\frac{v_y}{v_x}\right| = \frac{|10-5\sqrt{3}|}{5}$$

$\Rightarrow$ $\tan\alpha = |2-\sqrt{3}|$ or $\alpha = 15^\circ$

$\therefore$ Radius of curvature of the trajectory of the projected body $R = v^2 / g\cos\alpha$

$$= \frac{(5)^2 + (10 - 5\sqrt{3})^2}{10 \times 0.97}$$

$[\because v^2 = v_x^2 + v_y^2$ and $\cos 15^\circ = 0.97]$

$\Rightarrow \quad R = 2.77 \text{ m} \approx 2.8 \text{ m}$

22. A particle is moving along a circular path with a constant speed of 10 ms^{-1}. What is the magnitude of the change in velocity of the particle, when it moves through an angle of 60° around the centre of the circle?

[JEE Main 2019, 11 Jan (Shift-I)]

(a) $10\sqrt{2}$ m/s (b) 10 m/s
(c) $10\sqrt{3}$ m/s (d) Zero

Exp. **(b)**

Let v_1 be the velocity of the particle moving along the circular path initially, v_1 and v_2 be the velocity when it moves through an angle of 60° as shown below.

From the figure,

$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

$\Rightarrow \quad |\Delta \mathbf{v}| = 2v \sin\frac{\theta}{2} = 2\,\mathbf{v} \sin 30^\circ$

$[\because |\mathbf{v}_1| = |\mathbf{v}_2|]$

$= 2v \times \frac{1}{2} = v$ (Given, $v = 10$ m/s)

$\Rightarrow \quad |\Delta \mathbf{v}| = 10$ m/s

Alternate Solution

$\because \quad \Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \mathbf{v}_2 + (-\mathbf{v}_1)$

$\therefore \quad |\Delta \mathbf{v}|^2 = v_1^2 + v_2^2 + 2v_1v_2 \cos 120^\circ$

$= v^2 + v^2 + 2v \times v \times \left(-\frac{1}{2}\right)$

$\Rightarrow \quad |\Delta \mathbf{v}| = v = 10$ m/s.

23. A particle moves from the point $(2.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}})$ m at $t = 0$ with an initial velocity $(5.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}) \text{ ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{\mathbf{i}} + 4.0\hat{\mathbf{j}}) \text{ ms}^{-2}$. What is the distance of the particle from the origin at time 2 s?

[JEE Main 2019, 11 Jan (Shift-II)]

(a) 5 m (b) $20\sqrt{2}$ m (c) $10\sqrt{2}$ m (d) 15 m

Exp. **(b)**

Given, initial position of particle, $\mathbf{r}_0 = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$m,

Initial velocity of particle, $\mathbf{u} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}})$m/s

Aceleration of particle, $\mathbf{a} = (4\hat{\mathbf{i}} + 5\hat{\mathbf{j}})$m/s^2

According to second equation of motion, position of particle at time t is,

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

At $t = 2$s, position of particle is,

$\mathbf{r} = (2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) + (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 2 + \frac{1}{2}(4\hat{\mathbf{i}} + 4\hat{\mathbf{j}}) \times 4$

or $\mathbf{r} = (2 + 10 + 8)\hat{\mathbf{i}} + (4 + 8 + 8)\hat{\mathbf{j}}$

$\Rightarrow \mathbf{r} = 20\hat{\mathbf{i}} + 20\hat{\mathbf{j}}$

$\therefore$ Distance of particle from origin is, $|\mathbf{r}| = 20\sqrt{2}$ m

24. A passenger train of length 60 m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction and (ii) in the opposite direction is

[JEE Main 2019, 12 Jan (Shift-I)]

(a) $\frac{3}{2}$ (b) $\frac{25}{11}$ (c) $\frac{11}{5}$ (d) $\frac{5}{2}$

Exp. **(c)**

When trains are moving in same direction relative speed $= |v_1 - v_2|$ and in opposite direction relative speed $= |v_1 + v_2|$

Hence, ratio of time when trains move in same direction with time when trains move in opposite direction is

$$\frac{t_1}{t_2} = \frac{\left(\frac{l_1 + l_2}{|v_1 - v_2|}\right)}{\left(\frac{l_1 + l_2}{|v_1 + v_2|}\right)} = \frac{|v_1 + v_2|}{|v_1 - v_2|}$$

where, $l_1 + l_2$ = sum of lengths of trains which is same as distance covered by trains to cross each other

So, $$\frac{t_1}{t_2} = \frac{80+30}{80-30} = \frac{110}{50} = \frac{11}{5}$$

25. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE Main 2018]**

Exp. **(b)**

If velocity versus time graph is a straight line with negative slope, then acceleration is constant and negative.

With a negative slope distance-time graph will be parabolic $\left(s = ut - \frac{1}{2}at^2\right)$.

So, option (b) will be incorrect.

26. C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is **[JEE Main 2017 (Offline)]**

(a) $a = b$ (b) $a = 14\,b$

(c) $a = 28\,b$ (d) $a = \frac{1}{14}\,b$

Exp. **(b)**

By Mayor's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when n gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2}; \text{ for } H_2 \quad \ldots \text{(i)}$$

$$b = C_p - C_V = \frac{R}{28}; \text{ for } N_2 \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$a = 14b$$

27. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity *vs* time? **[JEE Main 2017 (Offline)]**

Exp. **(b)**

Initially velocity keeps on decreasing at a constant rate, then it increases in negative direction with same rate.

28. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first? Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\,\text{m/s}^2$) **[JEE Main 2015]**

Exp. **(b)**

Concept of relative motion can be applied to predict the nature of motion of one particle with respect to the other.

Consider the stones thrown up simultaneously as shown in the diagram below.

Considering motion of the second particle with respect to the first we have relative acceleration

$|\mathbf{a}_{21}| = |\mathbf{a}_2 - \mathbf{a}_1| = g - g = 0$

Thus, motion of first particle is straight line with respect to second particle till the first particle strikes ground at a time given by

$$-240 = 10t - \frac{1}{2} \times 10 \times t^2$$

or $t^2 - 2t - 48 = 0$

or $t^2 - 8t + 6t - 48 = 0$

or $t = 8, -6$ (not possible)

Thus, distance covered by second particle with respect to first particle in 8 s is

$$s_{12} = (v_{21})t = (40 - 10)(8\text{s})$$
$$= 30 \times 8 = 240 \text{ m}$$

Similarly, time taken by second particle to strike the ground is given by

$$-240 = 40t - \frac{1}{2} \times 10 \times t^2$$

or $-240 = 40t - 5t^2$

or $5t^2 - 40t - 240 = 0$

or $t^2 - 8t - 48 = 0$

$t^2 - 12t + 4t - 48 = 0$

or $t(t - 12) + 4(t - 12) = 0$

or $t = 12, -4$ (not possible)

Thus, after 8 s, magnitude of relative velocity will increase upto 12 s when second particle strikes the ground.

29. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle to hit the ground is n times that taken by it to reach the highest point of its path. The relation between H, u and n is **[JEE Main 2014]**

(a) $2gH = n^2u^2$ (b) $gH = (n-2)^2u^2$

(c) $2gH = nu^2(n-2)$ (d) $gH = (n-2)^2u^2$

Exp. **(c)**

Time taken to reach the maximum height

$$t_1 = \frac{u}{g}$$

If t_2 is the time taken to hit ground

i.e., $-H = ut_2 - \frac{1}{2}gt^2$

But $t_2 = nt_1$ [given]

So, $-H = u\frac{nu}{g} - \frac{1}{2}g\frac{n^2u^2}{g^2}$

$$-H = \frac{nu^2}{g} - \frac{1}{2}\frac{n^2u^2}{g}$$

$$2gH = nu^2(n-2)$$

30. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical support. About the point of suspension, **[JEE Main 2014]**

(a) angular momentum is conserved

(b) angular momentum changes in magnitude but not in direction

(c) angular momentum changes in direction but not in magnitude

(d) angular momentum changes both in direction and magnitude

Exp. **(c)**

Angular momentum of the pendulum about the suspension point O is

Then, v can be resolved into two components, radial component v_{rad} and axial component v_{axial}. Due to v_{rad}, will be axial and due to v_{axial}, L will be radially outwards as shown.

So, net angular momentum will be as shown in figure whose value will be constant ($|L| = mv$). But its direction will change as shown in the figure.

$$L = m(r \times v)$$

where, r = position vector of particle point O.

31. A projectile is given an initial velocity of $(\mathbf{i} + 2\mathbf{j})$ m/s, where $\mathbf{i}$ is along the ground and $\mathbf{j}$ is along the vertical. If $g = 10$ m/s^2, the equation of its trajectory is **[JEE Main 2013]**

(a) $y = x - 5x^2$ (b) $y = 2x - 5x^2$

(c) $4y = 2x - 5x^2$ (d) $4y = 2x - 25x^2$

Exp. **(b)**

As, initial velocity $\mathbf{v} = (\mathbf{i} + 2\mathbf{j})$ m/s

For projectile

Motion along horizontal direction,

$$x = t \quad \ldots(i)$$

Motion along vertical direction,

$$y = ut - \frac{1}{2}gt^2$$

where, u is a vertical component of velocity.

$$y = 2t - \frac{1}{2}10t^2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$y = 2x - 5x^2$$

32. A boy can throw a stone upto a maximum height of 10 m. The maximum horizontal distance that the boy can throw the same stone upto will be **[AIEEE 2012]**

(a) $20\sqrt{2}$ m (b) 10 m (c) $10\sqrt{2}$ m (d) 20 m

Exp. **(d)**

Maximum speed with which the boy can throw stone is

$$u = \sqrt{2gh} = \sqrt{2 \times 10 \times 10} = 10\sqrt{2} \text{ m/s}$$

Range is maximum when projectile is thrown at an angle of 45°. Thus,

$$R_{max} = \frac{u^2}{g} = \frac{(10\sqrt{2})^2}{10} = 20 \text{ m}$$

33. A particle of mass m is at rest at the origin at time $t = 0$. It is subjected to a force $F(t) = F_0 e^{-bt}$ in the x-direction. Its speed $v(t)$ is depicted by which of the following curves? **[AIEEE 2012]**

Exp. **(c)**

As the force is exponentially decreasing, so its acceleration, *i.e.,* rate of increase of velocity will decrease with time. Thus, the graph of velocity will be an increasing curl with decreasing slope with time.

$$\therefore \quad a = \frac{F}{M} = \frac{F_0}{m}e^{-bt} = \frac{dv}{dt}$$

$$\Rightarrow \quad \int_0^v dv = \int_0^t \frac{F_0}{m}e^{-bt}dt$$

$$\Rightarrow \quad v = \frac{F_0}{m}\left[\left(\frac{1}{-b}\right)e^{-bt}\right]_0^t = \frac{F_0}{mb}[e^{-bt}]_t^0$$

$$= \frac{F_0}{mb}(e^0 - e^{-bt}) = \frac{F_0}{mb}(1 - e^{-bt})$$

$$v_{max} = \frac{F_0}{mb}$$

34. A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is **[AIEEE 2011]**

(a) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (b) zero

(c) $\frac{mv^3}{\sqrt{2}g}$ (d) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

Exp. **(d)**

Angular momentum of the projectile as given by

$$L = mv_h r_\perp = m(v\cos\theta)\,h$$

where, h is the maximum height.

$$= m(v\cos\theta)\left(\frac{v^2\sin^2\theta}{2g}\right)$$

$$L = \frac{mv^3\sin^2\theta\cos\theta}{2g} = \frac{\sqrt{3}mv^3}{16g}$$

35. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is **[AIEEE 2011]**

(a) g (b) $\frac{2}{3}g$ (c) $\frac{g}{3}$ (d) $\frac{3}{2}g$

Exp. **(b)**

For the motion of the block,

$$mg - T = ma \quad \ldots(i)$$

For the rotation of the pulley,

$$\tau = TR = I\alpha \quad \Rightarrow T = \frac{1}{2}mR^2\alpha \quad \ldots(ii)$$

As string does not slip on the pulley

$$a = R\alpha \quad \ldots(iii)$$

On solving Eqs. (i), (ii) and (iii), we get

$$a = 2g/3$$

36. An object moving with a speed of 6.25 m/s, is decelerated at a rate given by $\frac{dv}{dt} = -2.5\sqrt{v}$, where v is the instantaneous speed. The time taken by the object to come to rest, would be **[AIEEE 2011]**

(a) 2 s (b) 4 s (c) 8 s (d) 1 s

Exp. **(a)**

Given, $\frac{dv}{dt} = -2.5\sqrt{v}$

$\Rightarrow \frac{dv}{\sqrt{v}} = -2.5dt$

$\Rightarrow \int_{6.25}^{0} v^{-1/2}\,dv = -2.5\int_0^t dt$

$\Rightarrow -2.5[t]_0^t = [2v^{1/2}]_{6.25}^0$

$\Rightarrow t = 2$ s

37. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is **[AIEEE 2011]**

(a) $\pi\frac{v^4}{g^2}$ (b) $\frac{\pi}{2}\frac{v^4}{g^2}$ (c) $\pi\frac{v^2}{g^2}$ (d) $\pi\frac{v^2}{g}$

Exp. **(a)**

Maximum range of water coming out of the fountain,

$$R_m = v^2/g$$

$\therefore$ Total area around fountain,

$$A = \pi R_m^2 = \pi\frac{v^4}{g^2}$$

38. A particle is moving with velocity $\mathbf{v} = k(y\mathbf{i} + x\mathbf{j})$, where k is a constant. The general equation for its path is **[AIEEE 2010]**

(a) $y = x^2 + \text{constant}$ (b) $y^2 = x + \text{constant}$

(c) $xy = \text{constant}$ (d) $y^2 = x^2 + \text{constant}$

Exp. **(d)**

Given, $\mathbf{v} = ky\mathbf{i} + kx\mathbf{j}$

$\Rightarrow \frac{dx}{dt} = ky, \frac{dy}{dt} = kx$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} = \frac{kx}{ky}$$

$$ydy = xdx$$

Integrating and using $\int x^n\,dx = \frac{x^{n+1}}{n+1}$, we have

$$y^2 = x^2 + c$$

39. For a particle in uniform circular motion the acceleration $\mathbf{a}$ at a point $P(R, \theta)$ on the circle of radius R is (here, θ is measured from the x-axis) **[AIEEE 2010]**

(a) $-\frac{v^2}{R}\cos\theta\,\mathbf{i} + \frac{v^2}{R}\sin\theta\,\mathbf{j}$

(b) $-\frac{v^2}{R}\sin\theta\,\mathbf{i} + \frac{v^2}{R}\cos\theta\,\mathbf{j}$

(c) $-\frac{v^2}{R}\cos\theta\,\mathbf{i} - \frac{v^2}{R}\sin\theta\,\mathbf{j}$

(d) $\frac{v^2}{R}\mathbf{i} + \frac{v^2}{R}\mathbf{j}$

Exp. **(c)**

For a particle in uniform circular motion,

$$\mathbf{a} = \frac{v^2}{R} \text{ towards centre of circle}$$

[centripetal acceleration]

$$\therefore \quad \mathbf{a} = \frac{v^2}{R}(-\cos\theta\mathbf{i} - \sin\theta\,\mathbf{j})$$

or $$\mathbf{a} = -\frac{v^2}{R}\cos\theta\,\mathbf{i} - \frac{v^2}{R}\sin\theta\,\mathbf{j}$$

40. A small particle of mass m is projected at an angle θ with the x-axis with an intial velocity v_0 in the xy plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[AIEEE 2010]**

(a) $-mgv_0t^2\cos\theta\,\mathbf{j}$ (b) $mgv_0t\cos\theta\mathbf{k}$

(c) $-\frac{1}{2}mgv_0t^2\cos\theta\mathbf{k}$ (d) $\frac{1}{2}mgv_0t^2\cos\theta\mathbf{i}$

Exp. **(c)**

Angular momentum of the particle is,

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$\Rightarrow \mathbf{L} = m\left[v_0\cos\theta t\,\mathbf{i} + (v_0\sin\theta t - \frac{1}{2}gt^2)\mathbf{j}\right] \times [v_0\cos\theta\,\mathbf{i} + (v_0\sin\theta - gt)\,\mathbf{j}]$$

$$= mv_0\cos\theta t\left(-\frac{1}{2}gt\right)\mathbf{k}$$

$$= -\frac{1}{2}mgv_0t^2\cos\theta\,\mathbf{k}$$

41. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metre and t is in second. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly **[AIEEE 2010]**

(a) $13\ \text{ms}^{-2}$ (b) $12\ \text{ms}^{-2}$

(c) $7.2\ \text{ms}^{-2}$ (d) $14\ \text{ms}^{-2}$

Exp. **(d)**

Given, $s = t^3 + 5$

$\therefore$ Speed, $v = \frac{ds}{dt} = 3t^2$

and rate of change of speed, $a_t = \frac{dv}{dt} = 6t$

$\therefore$ Tangential acceleration at $t = 2$ s,

$$a_t = 6 \times 2 = 12\ \text{ms}^{-2}$$

and at $t = 2$ s, $v = 3(2)^2 = 12\ \text{ms}^{-1}$

$\therefore$ Centripetal acceleration,

$$a_c = \frac{v^2}{R} = \frac{144}{20}\ \text{ms}^{-2}$$

$\therefore$ Net acceleration $= \sqrt{a_t^2 + a_c^2} \approx 14\ \text{ms}^{-2}$

42. A particle has an initial velocity $3\mathbf{i} + 4\mathbf{j}$ and an acceleration of $0.4\mathbf{i} + 0.3\mathbf{j}$. Its speed after 10 s is **[AIEEE 2009]**

(a) 10 units (b) $7\sqrt{2}$ units

(c) 7 units (d) 8.5 units

Exp. **(b)**

Given, $\mathbf{u} = 3\mathbf{i} + 4\mathbf{j}; \mathbf{a} = 0.4\mathbf{i} + 0.3\mathbf{j}$

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$
$$= 3\mathbf{i} + 4\mathbf{j} + (0.4\mathbf{i} + 0.3\mathbf{j})10$$
$$= 3\mathbf{i} + 4\mathbf{j} + 4\mathbf{i} + 3\mathbf{j}$$
$$= 7\mathbf{i} + 7\mathbf{j}$$

So, speed is equal to magnitude of velocity

$$= \sqrt{7^2 + 7^2} = 7\sqrt{2} \text{ units}$$

43. Consider a rubber ball freely falling from a height $h = 4.9$ m onto a horizontal elastic plate. Assume that the duration of collision is negligible and the collision with the plate is totally elastic. Then, the velocity as a function of time the height as function of time will be **[AIEEE 2009]**

(a)

(b)

(c)

(d)

Exp. **(c)**

As we know that for vertical motion,

$$h = \frac{1}{2} gt^2 \qquad \text{[parabolic]}$$

$v = -gt$ and after the collision,
$v = gt$ (straight line).

Collision is perfectly elastic, then ball reaches to same height again and again with same velocity.

Hence, option (c) is true.

44. A body is at rest at $x = 0$. At $t = 0$, it starts moving in the positive x-direction with a constant acceleration. At the same instant, another body passes through $x = 0$ moving in the positive x-direction with a constant speed. The position of the first body is given by $x_1(t)$ after time t and that of the second body by $x_2(t)$ after the same time interval. Which of the following graphs correctly describes $(x_1 - x_2)$ as a function of time? **[AIEEE 2008]**

(a)

(b)

(c)

(d)

Exp. **(b)**

Here, $x_2 = vt$ and $x_1 = \frac{at^2}{2}$

$$\therefore \qquad x_1 - x_2 = -\left(vt - \frac{at^2}{2}\right)$$

From the above expression, it is clear that at $t = 0$, $x_1 - x_2 = 0$ further for increasing values, the graph is as follows.

Hence, option (c) is true.

45. The velocity of a particle is $v = v_0 + gt + ft^2$. If its position is $x = 0$ at $t = 0$, then its displacement after unit time $(t = 1)$ is **[AIEEE 2007]**

(a) $v_0 + 2g + 3f$ (b) $v_0 + \frac{g}{2} + \frac{f}{3}$

(c) $v_0 + g + f$ (d) $v_0 + \frac{g}{2} + f$

Exp. **(b)**

Given, $v = v_0 + gt + ft^2$

After differentiating with respect to time, we get

$$\frac{dx}{dt} = v_0 + gt + ft^2$$

$$\Rightarrow \quad dx = (v_0 + gt + ft^2)\,dt$$

So, $$\int_0^x dx = \int_0^1 (v_0 + gt + ft^2)\,dt$$

$$\Rightarrow \quad x = v_0 + \frac{g}{2} + \frac{f}{3}$$

Since, vertical component of velocity is zero.

46. A particle is projected at 60° to the horizontal with a kinetic energy K. The kinetic energy at the highest point is **[AIEEE 2007]**

(a) K (b) zero (c) $\frac{K}{4}$ (d) $\frac{K}{2}$

Exp. **(c)**

Kinetic energy at highest point,

$$(KE)_H = \frac{1}{2} mv^2 \cos^2 \theta$$

$$= K \cos^2 \theta = K(\cos 60°)^2 = K/4$$

47. A particle located at $x = 0$ at time $t = 0$, starts moving along the positive x-direction with a velocity v that varies as $v = \alpha\sqrt{x}$. The displacement of the particle varies with time as **[AIEEE 2006]**

(a) t^2 (b) t (c) $t^{1/2}$ (d) t^3

Exp. **(a)**

Given, $v = \alpha\sqrt{x}$

or $$\frac{dx}{dt} = \alpha\sqrt{x} \qquad \left[\because v = \frac{dx}{dt}\right]$$

or $$\frac{dx}{\sqrt{x}} = \alpha\, dt$$

On integrating, we get

$$\int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$$

[$\because$ at $t = 0$, $x = 0$ and let at any time t, particle be at x]

$$\Rightarrow \quad \left[\frac{x^{1/2}}{1/2}\right]_0^x = \alpha t \quad \text{or} \quad x^{1/2} = \frac{\alpha}{2} t$$

or $$x = \frac{\alpha^2}{4} \times t^2 \quad \text{or } x \propto t^2$$

48. A projectile can have the same range R for two angles of projection. If t_1 and t_2 are the times of flights in the two cases, then the product of the two times of flights is proportional to **[AIEEE 2005, 04]**

(a) R^2 (b) $\frac{1}{R^2}$ (c) $\frac{1}{R}$ (d) R

Exp. **(d)**

A projectile can have same range, if angles of projection are complementary of each other *i.e.,* θ and $(90° - \theta)$. Thus, in both cases,

The time of flights are

$$t_1 = \frac{2u \sin\theta}{g} \qquad \ldots(i)$$

and $$t_2 = \frac{2u \sin(90° - \theta)}{g}$$

$$= \frac{2u \cos\theta}{g} \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$t_1 t_2 = \frac{4u^2 \sin\theta \cos\theta}{g^2},$$

$$t_1 t_2 = \frac{2u^2 \sin 2\theta}{g^2} = \frac{2}{g}\,\frac{u^2 \sin 2\theta}{g}$$

$$\therefore \quad t_1 t_2 = \frac{2R}{g} \qquad \left[\because R = \frac{u^2 \sin 2\theta}{g}\right]$$

or $$t_1 t_2 \propto R$$

49. The relation between time t and distance x is $t = ax^2 + bx$, where a and b are constants. The acceleration is **[AIEEE 2005]**

(a) $-2abv^2$ (b) $2bv^3$ (c) $-2av^3$ (d) $2av^2$

Exp. **(c)**

Given, $t = ax^2 + bx$

Differentiating it w.r.t. t, we get

$$\frac{dt}{dt} = 2ax\frac{dx}{dt} + b\frac{dx}{dt}$$

$$v = \frac{dx}{dt} = \frac{1}{(2ax + b)}$$

Again, differentiating w.r.t. t, we get

$$\frac{d^2x}{dt^2} = \frac{-2a}{(2ax + b)^2} \cdot \frac{dx}{dt}$$

$$\therefore \quad f = \frac{d^2x}{dt^2} = \frac{-1}{(2ax + b)^2} \cdot \frac{2a}{(2ax + b)}$$

or $$f = \frac{-2a}{(2ax + b)^3}$$

$$\Rightarrow \quad f = -2av^3$$

50. A car starting from rest, accelerates at the rate f through a distance s, then continues at constant speed for time t and then decelerates at the rate $f/2$ to come to rest. If the total distance travelled is $15\,s$, then **[AIEEE 2005]**

(a) $s = ft$ (b) $s = \frac{1}{6} ft^2$

(c) $s = \frac{1}{2} ft^2$ (d) None of these

Exp. **(d)**

The velocity-time graph for the given situation can be drawn as below.

Magnitudes of slope of $OA = f$

and slope of $BC = \frac{f}{2} \Rightarrow v = ft_1 = \frac{f}{2} t_2$

$$\therefore \quad t_2 = 2t_1$$

In graph, area of ΔOAD gives distance,

$$s = \frac{1}{2} ft_1^2 \quad \ldots(i)$$

Area of rectangle $ABED$ gives distance travelled in time t.

$$s_2 = (ft_1)t$$

Distance travelled in time t_2,

$$s_3 = \frac{1}{2}\frac{f}{2}(2t_1)^2$$

Thus, $$s_1 + s_2 + s_3 = 15s$$

$$\Rightarrow \quad s + (ft_1)t + ft_1^2 = 15s$$

or $$s + (ft_1)t + 2s = 15s \quad \left[\because s = \frac{1}{2} ft_1^2\right]$$

or $$(ft_1)t = 12s \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{12s}{s} = \frac{(ft_1)t}{\frac{1}{2}(ft_1)t_1}$$

or $$t_1 = \frac{t}{6}$$

From Eqs. (i) and (ii), we get

$$s = \frac{1}{2} f(t_1)^2$$

$$\Rightarrow \quad s = \frac{1}{2} f\left(\frac{t}{6}\right)^2$$

$$= \frac{1}{72} ft^2$$

Hence, none of the given options is correct.

51. A particle is moving Eastwards with a velocity of $5\ \text{ms}^{-1}$. In 10 s, the velocity changes to $5\ \text{ms}^{-1}$ Northwards. The average acceleration in this time is **[AIEEE 2005]**

(a) $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards North-East

(b) $\frac{1}{2}\ \text{ms}^{-2}$ towards North

(c) zero

(d) $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards North-West

Exp. **(d)**

As, velocity of eastward $\mathbf{v}_1 = +5\mathbf{i}$

and velocity of northward $\mathbf{v}_2 = +5\mathbf{j}$

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 5\mathbf{j} - 5\mathbf{i}$$

$$|\Delta\mathbf{v}| = \sqrt{5^2 + 5^2} = 5\sqrt{2}$$

$\therefore \quad a = \frac{|\Delta \mathbf{v}|}{t} = \frac{5\sqrt{2}}{10} = \frac{1}{\sqrt{2}}\ \text{ms}^{-2}$

For direction,

$$\tan\alpha = -\frac{5}{5} = -1$$

Average acceleration is $\frac{1}{\sqrt{2}}\ \text{ms}^{-2}$ towards North-West.

52. A parachutist after bailing out falls 50 m without friction. When parachute opens, it decelerates at 2 ms^{-2}. He reaches the ground with a speed of 3 ms^{-1}. At what height, did he bail out? **[AIEEE 2005]**

(a) 91 m (b) 182 m (c) 293 m (d) 111 m

Exp. **(c)**

Parachute bails out at height H from ground. Velocity at A

$$v = \sqrt{2gh}$$
$$= \sqrt{2 \times 9.8 \times 50} = \sqrt{980}\ \text{ms}^{-1}$$

Velocity at ground, $v_1 = 3\ \text{ms}^{-1}$ [given]

Acceleration $= -2\ \text{ms}^{-2}$ [given]

$$\therefore \quad H - h = \frac{v^2 - v_1^2}{2 \times 2} = \frac{980 - 9}{4} = \frac{971}{4} = 242.75$$

$$\Rightarrow \quad H = 242.75 + h = 242.75 + 50 \approx 293\ \text{m}$$

53. A ball is released from the top of a tower of height h metre. It takes T second to reach the ground. What is the position of the ball in $\frac{T}{3}$ second? **[AIEEE 2004]**

(a) $h/9$ m from the ground
(b) $7h/9$ m from the ground
(c) $8h/9$ m from the ground
(d) $17h/18$ m from the ground

Exp. **(c)**

Second law of motion gives

$$s = ut + \frac{1}{2}gT^2$$

or $\quad h = 0 + \frac{1}{2}gT^2 \quad [\because u = 0]$

$$\Rightarrow \quad T = \sqrt{\left(\frac{2h}{g}\right)}$$

At $t = \frac{T}{3}$ s, $s = 0 + \frac{1}{2}g\left(\frac{T}{3}\right)^2$

or $\quad s = \frac{1}{2}g \cdot \frac{T^2}{9}$

$\Rightarrow \quad s = \frac{g}{18} \times \frac{2h}{g} \quad \left[\because T = \sqrt{\frac{2h}{g}}\right]$

or $\quad s = \frac{h}{9}$ m

Hence, the position of ball from the ground

$$= h - \frac{h}{9} = \frac{8h}{9}\ \text{m}$$

54. If $\mathbf{A} \times \mathbf{B} = \mathbf{B} \times \mathbf{A}$, then the angle between A and B is **[AIEEE 2004]**

(a) π (b) $\pi/3$
(c) $\pi/2$ (d) $\pi/4$

Exp. **(a)**

As $\quad (\mathbf{A} \times \mathbf{B}) = (\mathbf{B} \times \mathbf{A})$ [given]

$\Rightarrow \quad (\mathbf{A} \times \mathbf{B}) - (\mathbf{B} \times \mathbf{A}) = 0$

or $\quad (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{B}) = 0\ [\because (\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})]$

or $\quad 2(\mathbf{A} \times \mathbf{B}) = 0$

$\Rightarrow \quad 2AB\sin\theta = 0$ or $\sin\theta = 0$

$[\because |\mathbf{A}| = A \neq 0, |\mathbf{B}| = B \neq 0]$

$\Rightarrow \quad \theta = 0$ or π

55. Which of the following statements is false for a particle moving in a circle with a constant angular speed? **[AIEEE 2004]**

(a) The velocity vector is tangent to the circle
(b) The acceleration vector is tangent to the circle

(c) The acceleration vector points to the centre of the circle
(d) The velocity and acceleration vectors are perpendicular to each other

Exp. (b)

For a particle moving in a circle with constant angular speed, velocity vector is always along the tangent to the circle and the acceleration vector always points towards the centre of circle or is always along radius of the circle.

Since, tangential vector is perpendicular to radial vector, therefore velocity vector will be perpendicular to the acceleration vector. But in no case, acceleration vector is tangent to the circle.

56. An automobile travelling with a speed of 60 km/h, can brake to stop within a distance of 20 m. If the car is going twice as fast, *i.e*, 120 km/h, the stopping distance will be **[AIEEE 2004]**

(a) 20 m (b) 40 m (c) 60 m (d) 80 m

Exp. (d)

The braking retardation will remain same and assumed to be constant, let it be *a*.

From third equation of motion,

$$v^2 = u^2 + 2as$$

Case I $0 = \left(60 \times \frac{5}{18}\right)^2 - 2a \times s_1$

$\Rightarrow$ $s_1 = \frac{(60 \times 5/18)^2}{2a}$

Case II $0 = \left(120 \times \frac{5}{18}\right)^2 - 2a \times s_2$

$\Rightarrow$ $s_2 = \frac{(120 \times 5/18)^2}{2a}$

$\therefore$ $\frac{s_1}{s_2} = \frac{1}{4}$

$\Rightarrow$ $s_2 = 4s_1 = 4 \times 20 = 80$ m

57. A particle is acted upon by a force of constant magnitude which is always perpendicular to the velocity of the particle. The motion of the particle takes place in a plane, it follows that **[AIEEE 2004]**

(a) its velocity is constant
(b) its acceleration is constant
(c) its kinetic energy is constant
(d) it moves in a straight line

Exp. (c)

When a force of constant magnitude acts on velocity of particle perpendicularly, then there is no change in the kinetic energy of particle. Hence, kinetic energy remains constant.

58. A ball is thrown from a point with a speed v_0 at an angle of projection θ. From the same point and at the same instant, a person starts running with a constant speed $\frac{v_0}{2}$ to catch the ball. Will the person be able to catch the ball? If yes, what should be the angle of projection? **[AIEEE 2004]**

(a) Yes, 60° (b) Yes, 30°
(c) No (d) Yes, 45°

Exp. (a)

Man will catch the ball, if the horizontal component of velocity becomes equal to the constant speed of man, *i.e*,

$$v_0 \cos\theta = \frac{v_0}{2}$$

or $\cos\theta = \frac{1}{2}$

$\Rightarrow$ $\cos\theta = \cos 60°$

or $\theta = 60°$

59. A car moving with a speed of 50 km/h, can be stopped by brakes after atleast 6 m. If the same car is moving at a speed of 100 km/h, the minimum stopping distance is **[AIEEE 2003]**

(a) 12 m (b) 18 m
(c) 24 m (d) 6 m

Exp. (c)

As the first equation of motion $v^2 = u^2 + 2as$

$\Rightarrow$ $0 = \left(50 \times \frac{5}{18}\right)^2 + 2a \times 6$

or $a = -16$ m/s^2 [$\because a$ is retardation]

Again $v^2 = u^2 + 2as$

$\Rightarrow$ $0 = \left(100 \times \frac{5}{18}\right)^2 - 16 \times 2 \times s$

or $s = \frac{(100 \times 5)^2}{18 \times 18 \times 32}$

or $s = 24.1$
≈ 24 m

60. A boy playing on the roof of a 10 m high building throws a ball with a speed of 10 m/s at an angle of 30° with the horizontal. How far from the throwing point will the ball be at the height of 10 m from the ground?
$[g = 10\ \text{m/s}^2, \sin 30° = \frac{1}{2}, \cos 30° = \sqrt{3}/2]$

[AIEEE 2003]

(a) 5.20 m (b) 4.33 m
(c) 2.60 m (d) 8.66 m

Exp. **(d)**

The ball will be at point P when it is at a height of 10 m from the ground. So, we have to find the distance OP, which can be calculated directly by considering it as a projectile on a levelled plane (OX).

$$OP = R = \frac{u^2 \sin 2\theta}{g}$$
$$= \frac{10^2 \times \sin(2 \times 30°)}{10}$$
$$= \frac{10\sqrt{3}}{2} = 5\sqrt{3} = 8.66 \text{ m}$$

61. The coordinates of a moving particle at any time t are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time t is given by

[AIEEE 2003]

(a) $3t\sqrt{\alpha^2 + \beta^2}$ (b) $3t^2\sqrt{\alpha^2 + \beta^2}$
(c) $t^2\sqrt{\alpha^2 + \beta^2}$ (d) $\sqrt{\alpha^2 + \beta^2}$

Exp. **(b)**

The given coordinate
$$x = \alpha t^3, \quad y = \beta t^3$$
Then, $v_x = \frac{dx}{dt} = 3\alpha t^2$ and $v_y = \frac{dy}{dt} = 3\beta t^2$

$\therefore$ Resultant velocity, $v = \sqrt{v_x^2 + v_y^2}$
$$= \sqrt{9\alpha^2 t^4 + 9\beta^2 t^4}$$
$$= 3t^2\sqrt{\alpha^2 + \beta^2}$$

62. Speeds of two identical cars are u and $4u$ at a specific instant. The ratio of the respective distances at which the two cars are stopped from that instant is **[AIEEE 2002]**

(a) 1:1 (b) 1:4 (c) 1:8 (d) 1:16

Exp. **(d)**

In this question, the cars are identical means coefficient of friction between the tyre and the ground is same for both the cars, as a result retardation is same for both the cars equal to μg.

Let first car travel distance s_1, before stopping while second car travel distance s_2, then from
$$v^2 = u^2 - 2as$$
$$\Rightarrow \quad 0 = u^2 - 2\mu g \times s_1 \Rightarrow s_1 = \frac{u^2}{2\mu g}$$
and $$0 = (4u)^2 - 2\mu g \times s_2$$
$$\Rightarrow \quad s_2 = \frac{16u^2}{2\mu g} = 16 s_1$$
$$\therefore \quad \frac{s_1}{s_2} = \frac{1}{16}$$

63. A ball whose kinetic energy is E, is projected at an angle of 45° to the horizontal. The kinetic energy of the ball at the highest point of its flight will be **[AIEEE 2002]**

(a) E (b) $\frac{E}{\sqrt{2}}$ (c) $\frac{E}{2}$ (d) zero

Exp. **(c)**

At the highest point of its flight, vertical component of velocity is zero and only horizontal component is left which is
$$u_x = u\cos\theta$$
Given, $\theta = 45°$
$$\therefore \quad u_x = u\cos 45° = \frac{u}{\sqrt{2}}$$
Hence, at the highest point, kinetic energy is
$$E' = \frac{1}{2} m u_x^2$$
$$= \frac{1}{2} m \left(\frac{u}{\sqrt{2}}\right)^2 = \frac{1}{2} m \left(\frac{u^2}{2}\right)$$
$$= \frac{E}{2} \qquad \left[\because \frac{1}{2} mu^2 = E\right]$$

64. From a building, two balls A and B are thrown such that A is thrown upwards and B downwards (both vertically). If v_A and v_B are their respective velocities on reaching the ground, then **[AIEEE 2002]**

(a) $v_B > v_A$
(b) $v_A = v_B$
(c) $v_A > v_B$
(d) their velocities depend on their masses

Exp. **(b)**

From conservation of energy,

Potential energy at height h = Kinetic energy at ground

Therefore, at height h, PE of ball $A = m_A gh$

$$\text{KE at ground} = \frac{1}{2} m_A v_A^2$$

So, $$m_A gh = \frac{1}{2} m_A v_A^2$$

or $$v_A = \sqrt{2gh}$$

Similarly, $$v_B = \sqrt{2gh}$$

Therefore, $$v_A = v_B$$

Note In this question, it is not mentioned that magnitude of thrown velocity of both balls are same which is assumed in solution.

65. If a body losses half of its velocity on penetrating 3 cm in a wooden block, then how much will it penetrate more before coming to rest? **[AIEEE 2002]**

(a) 1 cm (b) 2 cm (c) 3 cm (d) 4 cm

Exp. **(a)**

Let initial velocity of body at point A be v, AB is 3 cm.

From $$v^2 = u^2 - 2as$$

$$\Rightarrow \quad \left(\frac{v}{2}\right)^2 = v^2 - 2a \times 3 \text{ or } a = \frac{v^2}{8}$$

Let on penetrating 3 cm in a wooden block, the body moves x distance from B to C.

So, for B to C, $u = \frac{v}{2}$, $v = 0$,

$$s = x, \quad a = \frac{v^2}{8} \quad \text{[deceleration]}$$

$$\therefore \quad (0)^2 = \left(\frac{v}{2}\right)^2 - 2 \cdot \frac{v^2}{8} \cdot x$$

or $$x = 1\,\text{cm}$$

Note Here, it is assumed that retardation is uniform.

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3

Laws *of* Motion

1. A ball is thrown vertically up (taken as + Z-axis) from the ground. The correct momentum-height (p-h) diagram is

[JEE Main 2019, 9 April Shift-I]

Exp. **(d)**

When a ball is thrown vertically upward, then the acceleration of the ball,

a = acceleration due to gravity (g)

(acting in the downward direction).

Now, using the equation of motion,

$$v^2 = u^2 - 2gh$$

or $$h = \frac{-v^2 + u^2}{2g} \quad \ldots(i)$$

As we know, momentum, $p = mv$ or $v = p/m$

So, substituting the value of v in Eq. (i), we get

$$h = \frac{u^2 - (p/m)^2}{2g}$$

As we know that, at the maximum height, velocity of the ball thrown would be zero.

So, for the flight when the ball is thrown till is reaches the maximum height (h).

$v \rightarrow$ changes from u to 0

$\Rightarrow$ $p \rightarrow$ changes from mu to 0

Similarly, when it reacher it's initial point, then

$h \rightarrow$ changes from h_{max} to 0

Also, $p \rightarrow$ changes from 0 to some values.

Thus, these conditions are only satisfied in the plot given in option (d).

2. Two blocks A and B of masses $m_A = 1$ kg and $m_B = 3$ kg are kept on the table as shown in figure. The coefficient of friction between A and B is 0.2 and between B and the surface of the table is also 0.2. The maximum force F that can be applied on B horizontally, so that the block A does not slide over the block B is

[Take, $g = 10\ \text{m/s}^2$]

[JEE Main 2019, 10 April Shift-II]

(a) 12 N (b) 16 N
(c) 8 N (d) 40 N

Exp. **(b)**

Acceleration a of system of blocks A and B is

$$a = \frac{\text{Net force}}{\text{Total mass}} = \frac{F - f_1}{m_A + m_B}$$

where, f_1 = friction between B and the surface

$= \mu(m_A + m_B)g$

So,

$$a = \frac{F - \mu(m_A + m_B)g}{(m_A + m_B)} \quad \ldots(i)$$

Here, $\mu = 0.2$, $m_A = 1\,\text{kg}$, $m_B = 3\,\text{kg}$, $g = 10\,\text{ms}^{-2}$

Substituting the above values in Eq. (i), we have

$$a = \frac{F - 0.2(1+3)\times 10}{1+3}$$

$$a = \frac{F-8}{4} \quad \ldots(ii)$$

Due to acceleration of block B, a pseudo force F' acts on A.

This force F' is given by

$$F' = m_A a$$

where, a is acceleration of A and B caused by net force acting on B.

For A to slide over B; pseudo force on A, i.e. F' must be greater than friction between A and B.

$$\Rightarrow \quad m_A a \geq f_2$$

We consider limiting case,

$$m_A a = f_2 \quad \Rightarrow m_A a = \mu(m_A)g$$

$$\Rightarrow \quad a = \mu g = 0.2 \times 10 = 2\ \text{ms}^{-2} \quad \ldots(iii)$$

Putting the value of a from Eq. (iii) into Eq. (ii), we get

$$\frac{F-8}{4} = 2$$

$$\therefore \quad F = 16\,\text{N}$$

3. A block of mass 5 kg is (i) pushed in case (A) and (ii) pulled in case (B), by a force $F = 20\,\text{N}$, making an angle of 30° with the horizontal, as shown in the figures. The coefficient of friction between the block, the floor is $\mu = 0.2$. The difference between the accelerations of the block, in case (B) and case (A) will be (Take, $g = 10\,\text{ms}^{-2}$)

[JEE Main 2019, 12 April Shift-II]

(a) $0.4\ \text{ms}^{-2}$ (b) $3.2\ \text{ms}^{-2}$ (c) $0.8\ \text{ms}^{-2}$ (d) $0\ \text{ms}^{-2}$

Exp. **(c)**

Case I Block is pushed over surface

Free body diagram of block is

In this case, normal reaction,

$$N = mg + F\sin 30° = 5 \times 10 + 20 \times \frac{1}{2} = 60\ \text{N}$$

[Given, $m = 5\,\text{kg}$, $F = 20\,\text{N}$]

Force of function, $f = \mu N$

$$= 0.2 \times 60 \quad [\because \mu = 0.2]$$

$$= 12\,\text{N}$$

So, net force causing acceleration (a_1) is

$$F_{net} = ma_1 = F\cos 30° - f$$

$$\Rightarrow \quad ma_1 = 20 \times \frac{\sqrt{3}}{2} - 12$$

$$\therefore \quad a_1 = \frac{10\sqrt{3} - 12}{5} \approx 1\,\text{ms}^{-2}$$

Case II Block is pulled over the surface

Free body diagram of block is,

Net force causing acceleration is

$$F_{net} = F\cos 30° - f = F\cos 30° - \mu N$$

$$\Rightarrow \quad F_{net} = F\cos 30° - \mu(mg - F\sin 30°)$$

If acceleration is now a_2, then

$$a_2 = \frac{F_{net}}{m}$$

$$= \frac{F\cos 30° - \mu(mg - F\sin 30°)}{m}$$

$$= \frac{20 \times \frac{\sqrt{3}}{2} - 0.2\left(5 \times 10 - 20 \times \frac{1}{2}\right)}{5}$$

$$= \frac{10\sqrt{3} - 8}{5}$$

$\Rightarrow$ $a_2 \approx 1.8\,\text{ms}^{-2}$

So, difference $= a_2 - a_1 = 1.8 - 1 = 0.8\,\text{ms}^{-2}$

4. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force F, such that the block does not move downward ? (Take, $g = 10\,\text{ms}^{-2}$)

[JEE Main 2019, 9 Jan Shift-I]

(a) 32 N (b) 25 N (c) 23 N (d) 18 N

Exp. **(a)**

Free body diagram, for the given figure is as follows,

For the block to be in equilibrium i.e., so that it does not move downward, then $\Sigma f_x = 0$

$\therefore$ $3 + Mg\sin\theta - F - f = 0$

or $3 + Mg\sin\theta = F + f$

As, frictional force, $f = \mu R$

$\therefore$ $3 + Mg\sin\theta = F + \mu R$ ...(i)

Similarly, $\Sigma f_y = 0$

$-Mg\cos\theta + R = 0$

or $Mg\cos\theta = R$ ...(ii)

Substituting the value of 'R' from Eq. (ii) to Eq. (i), we get

$3 + Mg\sin\theta = F + \mu(Mg\cos\theta)$ ...(iii)

Here, $M = 10\,\text{kg}, \theta = 45°, g = 10\,\text{m/s}^2$

and $\mu = 0.6$

Substituting these values is Eq. (iii), we get

$3 + (10 \times 10\sin 45°) - (0.6 \times 10 \times 10\cos 45°) = F$

$\Rightarrow$ $F = 3 + \frac{100}{\sqrt{2}} - \frac{60}{\sqrt{2}}$

$= 3 + \frac{40}{\sqrt{2}}$

$= 3 + 20\sqrt{2} = 31.8\,\text{N}$ or $F \simeq 32\,\text{N}$

5. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the mass, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is (Take, $g = 10\,\text{ms}^{-2}$)

[JEE Main 2019, 9 Jan Shift-II]

(a) 70 N (b) 200 N
(c) 100 N (d) 140 N

Exp. **(c)**

FBD of the given system is follow

Let T = tension in the rope.

For equilibrium condition of the mass,

$\Sigma F_x = 0$ (force in x-direction)

$\Sigma F_y = 0$ (force in y-direction)

When $\Sigma F_x = 0$, then

$\because$ $F = T\sin 45°$ ...(i)

When $\Sigma F_y = 0$, then

$Mg = T\cos 45°$ ...(ii)

Using Eqs. (i) and (ii), we get

$\Rightarrow$ $\frac{F}{Mg} = \frac{T\sin 45°}{T\cos 45°}$

$\Rightarrow$ $\frac{F}{Mg} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\Rightarrow$ $F = Mg = 10 \times 10 = 100\,\text{N}$

6. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque applied by the machine on the mop in (N - m) is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\frac{2}{3}\mu FR$ (b) $\frac{\mu FR}{6}$ (c) $\frac{\mu FR}{3}$ (d) $\frac{\mu FR}{2}$

Exp. **(a)**

Let a small strip of mop has width dx and radius x, as shown below,

Torque applied to move this strip is

$d\tau$ = Force on strip

× Perpendicular distance from the axis

$\Rightarrow d\tau$ = Force per unit area × Area of strip

× Perpendicular distance from the axis.

$$= \frac{\mu F}{\pi R^2} \cdot 2\pi x dx \cdot x$$

$$\Rightarrow \quad d\tau = \frac{2\mu F x^2}{R^2} \cdot dx$$

So, total torque to be applied on the mop is

$$\tau = \int_{x=0}^{x=R} d\tau = \int_0^R \frac{2\mu F x^2}{R^2} \cdot dx$$

$$= \frac{2\mu F}{R^2} \times \frac{R^3}{3} = \frac{2}{3}\mu FR \text{ (N - m)}$$

7. Two forces P and Q of magnitude $2F$ and $3F$, respectively,are at an angle θ with each other. If the force Q is doubled, then their resultant also gets doubled. Then, the angle θ is **[JEE Main 2019, 10 Jan Shift-II]**

(a) 60° (b) 120° (c) 30° (d) 90°

Exp. **(b)**

Resultant force $\mathbf{F}_r$ of any two forces $\mathbf{F}_1$ (i.e. P) and $\mathbf{F}_2$ (i.e. Q) with an angle θ between them can be given by vector addition as

$$F_r^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta \quad \ldots \text{(i)}$$

In first case $F_1 = 2F$ and $F_2 = 3F$

$$\Rightarrow \quad F_r^2 = 4F^2 + 9F^2 + 2 \times 2 \times 3F^2\cos\theta$$

$$\Rightarrow \quad F_r^2 = 13F^2 + 12F^2\cos\theta \quad \ldots \text{(ii)}$$

In second case $F_1 = 2F$ and $F_2 = 6F$

($\because$ Force Q gets doubled)

and $F_r' = 2F_r$

(Given)

By putting these values in Eq. (i), we get

$$(2F_r)^2 = (2F)^2 + (6F)^2 + 2 \times 2 \times 6F^2\cos\theta$$

$$\Rightarrow \quad 4F_r^2 = 40F^2 + 24F^2\cos\theta \quad \ldots \text{(iii)}$$

From Eq. (ii) and Eq. (iii), we get;

$$52F^2 + 48F^2\cos\theta = 40F^2 + 24F^2\cos\theta$$

$$\Rightarrow \quad 12 + 24\cos\theta = 0 \text{ or } \cos\theta = -1/2$$

or $\theta = 120°$ ($\because \cos 120° = -1/2$)

8. A particle of mass m is moving in a straight line with momentum p. Starting at time $t = 0$, a force $F = kt$ acts in the same direction on the moving particle during time interval T, so that its momentum changes from p to $3p$. Here, k is a constant. The value of T is **[JEE Main 2019, 11 Jan Shift-II]**

(a) $\sqrt{\frac{2p}{k}}$ (b) $2\sqrt{\frac{p}{k}}$

(c) $\sqrt{\frac{2k}{p}}$ (d) $2\sqrt{\frac{k}{p}}$

Exp. **(b)**

Here, $F = kt$

When $t = 0$, linear momentum = p

When $t = T$, linear momentum = $3p$

According to Newton's second law of motion,

applied force, $F = \frac{dp}{dt}$

or $dp = F \cdot dt$

or $dp = kt \cdot dt$

Now, integrate both side with proper limit

$$\int_p^{3p} dp = k\int_0^T t\,dt \quad \text{or} \quad [p]_p^{3p} = k\left[\frac{t^2}{2}\right]_0^T$$

or $\quad (3p - p) = \frac{1}{2}k[T^2 - 0]$

or $\quad T^2 = \frac{4p}{k}$

or $\quad T = 2\sqrt{\frac{p}{k}}$

9. A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is (Take, $g = 10\ \text{m/s}^2$) **[JEE Main 2019, 12 Jan Shift-II]**

(a) $\frac{2}{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{\sqrt{3}}{4}$ (d) $\frac{1}{2}$

Exp. **(b)**

Block does not move upto a maximum applied force of 2N down the inclined plane.

So, equating forces, we have

$$2 + mg\sin\theta = f$$

or $\quad 2 + mg\sin\theta = \mu\, mg\cos\theta \quad \ldots(i)$

Similarly, block also does not move upto a maximum applied force of 10 N up the plane.

Now, equating forces, we have

$$mg\sin\theta + f = 10\,\text{N}$$

or $\quad mg\sin\theta + \mu mg\cos\theta = 10 \quad \ldots(ii)$

Now, solving Eqs. (i) and (ii), we get

$$mg\sin\theta = 4 \quad \ldots(iii)$$

and $\quad \mu\, mg\cos\theta = 6 \quad \ldots(iv)$

Dividing, Eqs. (iii) and (iv) we get

$$\mu\cot\theta = \frac{3}{2}$$

$$\Rightarrow \quad \mu = \frac{3\tan\theta}{2} = \frac{3\tan 30°}{2} \Rightarrow \mu = \frac{\sqrt{3}}{2}$$

10. Two masses $m_1 = 5$ kg and $m_2 = 10$ kg connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is **[JEE Main 2018]**

(a) 18.3 kg (b) 27.3 kg
(c) 43.3 kg (d) 10.3 kg

Exp. **(b)**

Motion stops when pull due to $m_1 \le$ force of friction between m and m_2 and surface.

$\Rightarrow \quad m_1 g \le \mu(m_2 + m)g$

$\Rightarrow \quad 5 \times 10 \le 0.15(10 + m) \times 10$

$\Rightarrow \quad m \ge 23.33$ kg

Here, nearest value is 27.3 kg

So, $\quad m_{min} = 27.3$ kg

11. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is **[JEE Main 2018]**

(a) $-\frac{k}{4a^2}$ (b) $\frac{k}{2a^2}$ (c) zero (d) $-\frac{3}{2}\frac{k}{a^2}$

Exp. **(c)**

$\therefore$ Force $= -\dfrac{dU}{dr}$

$\Rightarrow \quad F = -\dfrac{d}{dr}\left(\dfrac{-k}{2r^2}\right) = -\dfrac{k}{r^3}$

As particle is on circular path, this force must be centripetal force.

$\Rightarrow \quad |F| = \dfrac{mv^2}{r}$

So, $\dfrac{k}{r^3} = \dfrac{mv^2}{r} \Rightarrow \dfrac{1}{2}mv^2 = \dfrac{k}{2r^2}$

$\therefore$ Total energy of particle = KE + PE

$= \dfrac{k}{2r^2} - \dfrac{k}{2r^2} = 0$

Total energy = 0

12. A body of mass $m = 10^{-2}$kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its initial speed is $v_0 = 10\ \text{ms}^{-1}$. If, after 10 s, its energy is $\dfrac{1}{8}mv_0^2$, the value of k will be **[JEE Main 2017 (Offline)]**

(a) 10^{-3} kgs^{-1} (b) 10^{-4} kgm^{-1}

(c) 10^{-1} kgm^{-1}s^{-1} (d) 10^{-3} kgm^{-1}

Exp. **(b)**

Given, force, $F = -kv^2$

$\therefore$ Acceleration, $a = \dfrac{-k}{m}v^2$

or $\dfrac{dv}{dt} = \dfrac{-k}{m}v^2 \Rightarrow \dfrac{dv}{v^2} = -\dfrac{k}{m}.dt$

Now, with limits, we have

$$\int_{10}^{v}\frac{dv}{v^2} = -\frac{k}{m}\int_0^t dt$$

$\Rightarrow \quad \left(-\dfrac{1}{v}\right)_{10}^{v} = -\dfrac{k}{m}t \Rightarrow \dfrac{1}{v} = 0.1 + \dfrac{kt}{m}$

$\Rightarrow \quad v = \dfrac{1}{0.1 + \dfrac{kt}{m}} = \dfrac{1}{0.1 + 1000k}$

$\Rightarrow \quad \dfrac{1}{2} \times m \times v^2 = \dfrac{1}{8}mv_0^2 \Rightarrow v = \dfrac{v_0}{2} = 5$

$\Rightarrow \quad \dfrac{1}{0.1 + 1000k} = 5$

$\Rightarrow \quad 1 = 0.5 + 5000k$

$\Rightarrow \quad k = \dfrac{0.5}{5000}$

$\Rightarrow \quad k = 10^{-4}$kg/m

13. A point particle of mass m, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction between the particle and the rough track equals μ. The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance $x(= QR)$, are respectively close to **[JEE Main 2016 (Offline)]**

(a) 0.2 and 6.5 m (b) 0.2 and 3.5 m

(c) 0.29 and 3.5 m (d) 0.29 and 6.5 m

Exp. **(c)**

Energy lost over path $PQ = \mu mg\cos\theta \times 4$

Energy lost over path $QR = \mu mg\,x$

i.e. $\mu mg\cos 30° \times 4 = \mu mg\,x$ $(\because \theta = 30°)$

$x = 2\sqrt{3} = 3.45$ m

From Q to R energy loss is half of the total energy loss.

i.e. $\mu mgx = \dfrac{1}{2} \times mgh \Rightarrow \mu = 0.29$

The values of the coefficient of friction μ and the distance $x(= QR)$ are 0.29 and 3.5.

14. Given in the figure are two blocks A and B of weight 20 N and 100 N respectively. These are being pressed against a wall by a force F as shown in figure. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15,

the frictional force applied by the wall in block B is **[JEE Main 2015]**

(a) 100 N (b) 80 N (c) 120 N (d) 150 N

Exp. **(c)**

In vertical direction, weights are balanced by frictional forces.

Consider FBD of block A and B as shown in diagram below.

As the blocks are in equilibrium, balance forces are in horizontal and vertical direction.

For the system of blocks ($A + B$).

$$F = N$$

For block A, $f_A = 20$ N and for block B,

$$f_B = f_A + 100 = 120\text{N}$$

15. A block of mass m is placed on a surface with a vertical cross-section given by $y = x^3/6$. If the coefficient of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is

(a) $\frac{1}{6}$ m (b) $\frac{2}{3}$ m (c) $\frac{1}{3}$ m (d) $\frac{1}{2}$ m

[JEE Main 2014]

Exp. **(a)**

A block of mass m is placed on a surface with a vertical cross-section, then

$$\tan = \frac{dy}{dx} = \frac{d\left(\frac{x^3}{6}\right)}{dx} = \frac{x^2}{2}$$

At limiting equilibrium, we get

$$\mu = \tan\theta, \quad 0.5 = x^2/2$$

$$\Rightarrow \quad x^2 = 1 \Rightarrow x = \pm 1$$

Now, putting the value of x in $y = x^3/6$, we get

When $x = 1$	When $x = -1$
$\therefore \quad y = \frac{(1)^3}{6} = \frac{1}{6}$	$y = \frac{(-1)^3}{6} = \frac{-1}{6}$

So, the maximum height above the ground at which the block can be placed without slipping is $\frac{1}{6}$ m.

16. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2. If the inclined plane makes an angle θ from the horizontal such that $\tan\theta = 2\mu$, then the ratio $\frac{F_1}{F_2}$ is

[AIEEE 2011]

(a) 4 (b) 1 (c) 2 (d) 3

Exp. **(d)**

$F_1 = mg\,(\sin\theta + \mu\cos\theta)$ [as body just in position to move up, friction force downward]

$F_2 = mg(\sin\theta - \mu\cos\theta)$ [as body just in position to slide down, friction upward]

$$\therefore \quad \frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta} = \frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = 3$$

17. The figure shows the position-time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is **[AIEEE 2010]**

(a) 0.4 N- s (b) 0.8 N-s
(c) 1.6 N-s (d) 0.2 N-s

Exp. **(b)**

From the graph, it is a straight line, so uniform motion. Because of impulse direction of velocity changes as can be seen from the slope of the graph.

Initial velocity, $v_1 = \frac{2}{2} = 1\text{ms}^{-1}$

Final velocity, $v_2 = -\frac{2}{2} = -1\text{ms}^{-1}$

$$\mathbf{p}_i = mv_1 = 0.4 \text{ N-s}$$
$$\mathbf{p}_f = mv_2 = -0.4 \text{ N-s}$$
$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i = -0.4 - 0.4 = -0.8 \text{ N-s}$$
[$\mathbf{J}$ = impulse]

$\therefore \quad |\mathbf{J}| = 0.8$ N-s

18. Two fixed frictionless inclined plane making the angles 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? **[AIEEE 2010]**

(a) 4.9 ms^{-2} in horizontal direction
(b) 9.8 ms^{-2} in vertical direction
(c) zero
(d) 4.9 ms^{-2} in vertical direction

Exp. **(d)**

Force applying on the block

$$F = mg \sin\theta$$

or $\quad mg \sin\theta = ma$

$\therefore \quad a = g\sin\theta$

where, a is along the inclined plane.

$\therefore$ Vertical component of acceleration is $g\sin^2\theta$.

$\therefore$ Relative vertical acceleration of A with respect to B is

$$g(\sin^2 60° - \sin^2 30°] = \frac{g}{2} = 4.9\text{ ms}^{-2}$$
[in vertical direction]

19. A block of mass m is connected to another block of mass M by a spring (massless) of spring constant k. The blocks are kept on a smooth horizontal plane. Initially the blocks are at rest and the spring is unstretched. Then, a constant force F starts acting on the block of mass M to pull it. Find the force on the block of mass m. **[AIEEE 2007]**

(a) $\frac{mF}{M}$ (b) $\frac{(M+m)F}{m}$
(c) $\frac{mF}{(m+M)}$ (d) $\frac{MF}{(m+M)}$

Exp. **(c)**

Acceleration of system,

$$a = \frac{F}{m+M}$$

So, force acting on mass,

$$F = ma = \frac{mF}{m+M}$$

20. A mass of M kg is suspended by a weightless string. The horizontal force that is required to displace it until the string makes an angle of 45° with the initial vertical direction is **[AIEEE 2006]**

(a) $Mg(\sqrt{2}+1)$ (b) $Mg\sqrt{2}$
(c) $\frac{Mg}{\sqrt{2}}$ (d) $Mg(\sqrt{2}-1)$

Exp. **(d)**

Here, the constant horizontal force required to take the body from position 1 to position 2 can be calculated by using work-energy theorem. Let us assume that body be taken slowly, so that its speed does not change, then

$$\Delta K = 0 = W_F + W_{Mg} + W_{\text{tension}}$$

[symbols have their usual meanings]

$$W_F = F \times l \sin 45° = \frac{Fl}{\sqrt{2}}$$

$$W_{Mg} = Mg(l - l\cos 45°), W_{\text{tension}} = 0$$

$$\therefore \quad F = Mg(\sqrt{2} - 1)$$

21. A ball of mass 0.2 kg is thrown vertically upwards by applying a force by hand. If the hand moves 0.2 m while applying the force and the ball goes upto 2 m height further, find the magnitude of the force. Consider $g = 10\ \text{m/s}^2$. **[AIEEE 2006]**

(a) 4 N (b) 16 N
(c) 20 N (d) 22 N

Exp. **(d)**

The situation is shown in figure. At initial time, the ball is at P, then under the action of a force (exerted by hand) from P to A and then from A to B let acceleration of ball during PA be a ms^{-2} (assumed to be constant) in upward direction and velocity of ball at be is v m/s.

Then, for PA,

$$v^2 = 0^2 + 2a \times 0.2$$

For AB,

$$0 = v^2 - 2 \times g \times 2$$

$$\Rightarrow \quad v^2 = 2g \times 2$$

From above equations,

$$a = 10g = 100\ \text{ms}^{-2}$$

Then, for PA, FBD of ball is

$$F - mg = ma$$

[F is the force exerted by hand on ball]

$$\Rightarrow \quad F = m(g + a)$$

$$= 0.2(11\,g) = 22\ \text{N}$$

Alternate Solution Using work-energy theorem

$$W_{mg} + W_F = 0$$

$$\Rightarrow \quad -mg \times 2.2 + F \times 0.2 = 0$$

or $\quad F = 22$ N

22. A player caught a cricket ball of mass 150 g moving at a rate of 20 m/s. If the catching process is completed in 0.1 s, the force of the blow exerted by the ball on the hand of the player is equal to **[AIEEE 2006]**

(a) 150 N (b) 3 N
(c) 30 N (d) 300 N

Exp. **(c)**

This is the question based on impulse-momentum theorem.

$$|F \cdot \Delta t| = |\text{Change in momentum}|$$

$$\Rightarrow \quad F \times 0.1 = |p_f - p_i|$$

As the ball will stop after catching

$$p_i = mv_i = 0.15 \times 20 = 3,\ p_f = 0$$

$$\Rightarrow \quad F \times 0.1 = 3$$

$$\Rightarrow \quad F = 30\ \text{N}$$

23. A block is kept on a frictionless inclined surface with angle of inclination α. The incline is given an acceleration a to keep the block stationary. Then, a is equal to **[AIEEE 2005]**

(a) $\dfrac{g}{\tan\alpha}$ (b) g cosec
(c) g (d) $g \tan\alpha$

Exp. **(d)**

In the frame of wedge, the force diagram of block is shown in figure. From free body diagram of wedge,

For block to remain stationary,

$$ma \cos\alpha = mg \sin\alpha$$

or $$a = g \tan\alpha$$

24. Consider a car moving on a straight road with a speed of 100 ms^{-1}. The distance at which car can be stopped, is $[\mu_k = 0.5]$

(a) 800 m (b) 1000 m **[AIEEE 2005]**
(c) 100 m (d) 400 m

Exp. **(b)**

From Newton's equations, we have

$$v^2 = u^2 - 2as$$

Given, $v = 0$ [car is stopped]

As friction provide the retardation

$$a = \mu g,\ v = 100\ ms^{-1}$$

$$\therefore \quad (100)^2 = 2\mu gs$$

$$\Rightarrow \quad s = \frac{100 \times 100}{2 \times 0.5 \times 10}$$

$$= \frac{100 \times 100}{5 \times 2} = 1000\ m$$

25. A particle of mass 0.3 kg is subjected to a force $F = -kx$ with $k = 15\ Nm^{-1}$. What will be its initial acceleration, if it is released from a point 20 cm away from the origin? **[AIEEE 2005]**

(a) 3 ms^{-2} (b) 15 ms^{-2}
(c) 5 ms^{-2} (d) 10 ms^{-2}

Exp. **(d)**

Given, $m = 0.3$ kg, $x = 20$ cm

and $k = 15\ N/m$

given $F = -kx$ ...(i)

and $F = ma$ ...(ii)

$\therefore$ $ma = -kx$

$$\Rightarrow \quad a = -\frac{15}{0.3} \times 20 \times 10^{-2}$$

$$\Rightarrow \quad a = -\frac{15}{3} \times 2 = -10\ ms^{-2}$$

Negative sign indicates that acceleration is always towards the mean position.

$\therefore$ Initial acceleration, $a = 10\ ms^{-2}$

26. An annular ring with inner and outer radii R_1 and R_2 is rolling without slipping with a uniform angular speed. The ratio of the forces experienced by the two particles situated on the inner and outer parts of the ring, F_1 / F_2 is **[AIEEE 2005]**

(a) $\frac{R_2}{R_1}$ (b) $\left(\frac{R_1}{R_2}\right)^2$ (c) 1 (d) $\frac{R_1}{R_2}$

Exp. **(d)**

Since, ω is constant, so no net force or torque is acting on ring. The force experienced by any particle is only along radial direction or we can say the centripetal force.

The force experienced by inner part, $F_1 = m\omega^2 R_1$

and the force experienced by outer part,

$$F_2 = m\omega^2 R_2 \Rightarrow \frac{F_1}{F_2} = \frac{R_1}{R_2}$$

27. A smooth block is released at rest on a 45° incline and then slides a distance d. The time taken to slide is n times as much to slide on rough incline than on a smooth incline. The coefficient of friction is **[AIEEE 2005]**

(a) $\mu_k = 1 - \frac{1}{n^2}$ (b) $\mu_k = \sqrt{1 - \frac{1}{n^2}}$
(c) $\mu_s = 1 - \frac{1}{n^2}$ (d) $\mu_s = \sqrt{1 - \frac{1}{n^2}}$

Exp. **(a)**

When friction is absent

$$ma_1 = mg \sin\theta$$

$$a = g \sin\theta$$

$$\therefore \quad s_1 = \frac{1}{2} a_1 t_1^2 \quad \text{...(i)}$$

When friction is present, friction is in opposite to the direction of motion

$$a_2 = g \sin\theta - \mu_k\, g \cos\theta$$

$$\therefore \quad s_2 = \frac{1}{2} a_2 t_2^2 \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2} a_1 t_1^2 = \frac{1}{2} a_2 t_2^2$$

$$\Rightarrow \quad a_1 t_1^2 = a_2 (n t_1)^2 \quad [\because t_2 = n t_1]$$

or $\quad a_1 = n^2 a_2$

$$\Rightarrow \quad \frac{a_2}{a_1} = \frac{g \sin\theta - \mu_k g \cos\theta}{g \sin\theta} = \frac{1}{n^2}$$

or $\quad \dfrac{g \sin 45° - \mu_k g \cos 45°}{g \sin 45°} = \dfrac{1}{n^2}$

or $\quad 1 - \mu_k = \dfrac{1}{n^2}$

or $\quad \mu_k = 1 - \dfrac{1}{n^2}$

28. The upper half of an inclined plane with inclination ϕ is perfectly smooth, while the lower half is rough. A body starting from rest at the top will again come to rest at the bottom, if the coefficient of friction for the lower half is given by **[AIEEE 2005]**

(a) $2 \sin\phi$ (b) $2 \cos\phi$ (c) $2 \tan\phi$ (d) $\tan\phi$

Exp. **(c)**

According to work-energy theorem,

Work done = Change in kinetic energy

$$W = \Delta K = 0$$

$\Rightarrow$ Work done by friction + Work done by gravity = 0

$$\Rightarrow \quad -(\mu\, mg \cos\phi)\frac{l}{2} + mgl \sin\phi = 0$$

or $\quad \dfrac{\mu}{2} \cos\phi = \sin\phi$

or $\quad \mu = 2 \tan\phi$

29. A machine gun fires a bullet of mass 40 g with a velocity 1200 ms^{-1}. The man holding it, can exert a maximum force of 144 N on the gun. How many bullets can he fire per second at the most? **[AIEEE 2004]**

(a) 1 (b) 4 (c) 2 (d) 3

Exp. **(d)**

Force exerted by machine gun on man's hand in firing a bullet = Change in momentum per second on a bullet or rate of change of momentum

$$= \left(\frac{40}{1000}\right) \times 1200 = 48 \text{ N}$$

Force exerted by man on machine gun

= Force exerted on man by machine gun

= 144 N

Hence, number of bullets fired $= \dfrac{144}{48} = 3.$

30. Two masses $m_1 = 5$ kg and $m_2 = 4.8$ kg tied to a string are hanging over a light frictionless pulley. What is the acceleration of the masses when lift is free to move ? ($g = 9.8$ ms^{-2}) **[AIEEE 2004]**

(a) 0.2 ms^{-2} (b) 9.8 ms^{-2}

(c) 5 ms^{-2} (d) 4.8 ms^{-2}

Exp. **(d)**

On releasing, the motion of the system will be according to figure.

$$m_1 g - T = m_1 a \quad \ldots\text{(i)}$$

and $\quad T - m_2 g = m_2 a \quad \ldots\text{(ii)}$

On solving, $\quad a = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) g \quad \ldots\text{(iii)}$

Here, $m_1 = 5$ kg, $m_2 = 4.8$ kg, $g = 9.8\,\text{ms}^{-2}$

$$\therefore \quad a = \left(\frac{5-4.8}{5+4.8}\right) \times 9.8$$

$$= \frac{0.2}{9.8} \times 9.8 = 0.2\ \text{ms}^{-2}$$

31. A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block and the plane is 0.8. If the frictional force on the block is 10 N, the mass of the block (in kg) is ($g = 10\,\text{m/s}^2$) **[AIEEE 2004]**

(a) 2.0 (b) 4.0
(c) 1.6 (d) 2.5

Exp. **(a)**

Let mass of the block be m.

Frictional force in rest position

$$F = mg \sin 30°$$

[this is static frictional force and may be less than the limiting frictional force]

$$\therefore \quad 10 = m \times 10 \times \frac{1}{2}$$

$$\text{or} \quad m = \frac{2 \times 10}{10} = 2\ \text{kg}$$

32. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads 49 N, when the lift is stationary. If the lift moves downward with an acceleration of 5 ms^{-2}, the reading of the spring balance will be **[AIEEE 2003]**

(a) 24 N
(b) 74 N
(c) 15 N
(d) 49 N

Exp. **(a)**

In stationary position, spring balance reading $= mg = 49$

$$\text{or} \quad m = \frac{49}{9.8} = 5\ \text{kg}$$

When lift moves downward,

$$mg - T = ma$$

Reading of balance,

$$T = mg - ma$$
$$= 5(9.8 - 5)$$
$$= 5 \times 4.8$$
$$= 24.0\ \text{N}$$

33. Three forces start acting simultaneously on a particle moving with velocity **v**. These forces are represented in magnitude and direction by the three sides of a ΔABC (as shown). The particle will now move with velocity **[AIEEE 2003]**

(a) less than **v**
(b) greater than **v**
(c) $|v|$ in the direction of largest force BC
(d) **v**, remaining unchanged

Exp. **(d)**

Resultant force is zero, as three forces acting on the particle can be represented in magnitude and direction by three sides of a triangle in same order. Hence, by Newton's 2nd law $\left(\mathbf{F} = m\frac{d\mathbf{v}}{dt}\right)$, the velocity (**v**) of particle will be same.

34. A horizontal force of 10 N is necessary to just hold a block stationary against a wall. The coefficient of friction between the block and the wall is 0.2. The weight of the block is **[AIEEE 2003]**

(a) 20 N (b) 50 N (c) 100 N (d) 2 N

Exp. **(*d*)**

Let R be the normal contact force by wall on the block.

$$R = 10 \text{ N}$$
$$f_L = w \quad \text{and} \quad f_L = \mu R$$
$$\therefore \quad \mu R = w \quad \text{or} \quad w = 0.2 \times 10 = 2 \text{ N}$$

35. A marble block of mass 2 kg lying on ice when given a velocity of 6 ms^{-1} is stopped by friction in 10 s. Then, the coefficient of friction is **[AIEEE 2003]**

(a) 0.02 (b) 0.03 (c) 0.06 (d) 0.01

Exp. **(*c*)**

Let coefficient of friction be μ, then retardation will be μg. From equation of motion, $v = u + at$

$$\Rightarrow \quad 0 = 6 - \mu g \times 10$$
$$\text{or} \quad \mu = \frac{6}{100} = 0.06$$

36. Consider the following two statements.

I. Linear momentum of a system of particles is zero.

II. Kinetic energy of a system of particles is zero. Then, **[AIEEE 2003]**

(a) I does not imply II and II does not imply I
(b) I implies II but II does not imply I
(c) I does not imply II but II implies I
(d) I implies II and II implies I

Exp. **(*c*)**

Here, II is implying I but I is not implying II as kinetic energy of a system of particles is zero means speed of each and every particle is zero which says that momentum of every particle is zero.

But statement I means linear momentum of a system of particles is zero, which may be true even, if particles have equal and opposite momentums and hence having non-zero kinetic energy.

37. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block is **[AIEEE 2003]**

(a) $\frac{Pm}{M+m}$ (b) $\frac{Pm}{M-m}$
(c) P (d) $\frac{PM}{M+m}$

Exp. **(*d*)**

Let acceleration of system (rope + block) be a along the direction of applied force. Then,

$$a = \frac{P}{M+m}$$

Draw the FBD of block and rope as shown in figure.

where, T is the required parameter.

For block, $T = Ma$

$$\Rightarrow \quad T = \frac{MP}{M+m}$$

38. A light spring balance hangs from the hook of the other light spring balance and a block of mass M kg hangs from the former one. Then, the true statement about the scale reading is **[AIEEE 2003]**

(a) Both the scales read M kg each
(b) The scale of the lower one reads M kg and of the upper one zero
(c) The reading of the two scales can be anything but the sum of the readings will be M kg
(d) Both the scales read $M/2$ kg

Exp. **(*a*)**

The arrangement is shown in figure.

Now, draw the free body diagram of the spring balances and block.

For equilibrium of block,

$$T_1 = Mg$$

where, T_1 = Reading of S_2

For equilibrium of S_2,

$$T_2 = T_1$$

where, T_2 = Reading of S_1

For equilibrium of S_1,

$$T_2 = T_3$$

Hence, $T_1 = T_2 = Mg$

So, both scales read M kg.

39. A rocket with a lift-off mass 3.5×10^4 kg is blasted upwards with an initial acceleration of 10 ms^{-2}. Then, the initial thrust of the blast is **[AIEEE 2003]**

(a) 3.5×10^5 N

(b) 7.0×10^5 N

(c) 14.0×10^5 N

(d) 1.75×10^5 N

Exp. **(a)**

Here, thrust force is responsible to accelerate the rocket, so initial thrust of the blast

$$= ma = 3.5 \times 10^4 \times 10$$

$$= 3.5 \times 10^5 \text{ N}$$

40. A lift is moving down with acceleration a. A man in the lift drops a ball inside the lift. The acceleration of the ball as observed by the man in the lift and a man standing stationary on the ground are respectively **[AIEEE 2002]**

(a) g, g (b) $g - a, g - a$

(c) $g - a, g$ (d) a, g

Exp. **(c)**

Apparent weight of ball,

$$w' = w - R \text{ [where, } R = \text{normal reaction]}$$

$$R = ma \text{ [acting upward]}$$

$$w' = mg - ma = m(g - a)$$

Hence, apparent acceleration in the lift is $g - a$. Now, if the man is standing stationary on the ground, then the apparent acceleration of the falling ball is g.

41. Two forces are such that the sum of their magnitudes is 18 N and their resultant which has magnitude 12 N, is perpendicular to the smaller force. Then, the magnitudes of the forces are **[AIEEE 2002]**

(a) 12 N, 6 N (b) 13 N, 5 N

(c) 10 N, 8 N (d) 16 N, 2 N

Exp. **(b)**

The sum of the two forces

$$A + B = 18 \quad \ldots(i)$$

$$12 = \sqrt{A^2 + B^2 + 2AB\cos\theta} \quad \ldots(ii)$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \tan 90° = \frac{B\sin\theta}{A + B\cos\theta}$$

$$\Rightarrow \quad \cos\theta = \frac{-A}{B} \quad \ldots(iii)$$

Solving Eqs. (i), (ii) and (iii), we get

$$A = 5\text{ N}, B = 13\text{ N}$$

Since, $A + B = 18$ and $\cos\theta = -\frac{A}{B}$

Squaring both sides in Eq. (i), we get

$$144 = A^2 + B^2 + 2AB\cos\theta$$

On putting $\cos\theta = \frac{-A}{B}$ in above equation, we get

$$144 = A^2 + B^2 + 2AB\left(-\frac{A}{B}\right)$$

$$= A^2 + B^2 - 2A^2$$

$\Rightarrow \quad 144 = B^2 - A^2$

$\Rightarrow \quad 144 = (B - A) \cdot (B + A)$

On putting $B + A = 18$, we have

$$B - A = \frac{144}{18} = 8$$

Solving $\quad B - A = 8$

$B + A = 18$

We have, $B = 13$ N, $A = 5$ N

42. When forces F_1, F_2, F_3 are acting on a particle of mass m such that F_2 and F_3 are mutually perpendicular, then the particle remains stationary. If the force F_1 is now removed, then the acceleration of the particle is **[AIEEE 2002]**

(a) $\frac{F_1}{m}$ (b) $\frac{F_2 F_3}{mF_1}$

(c) $\frac{(F_2 - F_3)}{m}$ (d) $\frac{F_2}{m}$

Exp. **(*a*)**

The particle remains stationary under the acting of three forces $\mathbf{F_1}$, $\mathbf{F_2}$ and $\mathbf{F_3}$, it means resultant force is zero,

$$\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0$$

Since, in second cases F_1 is removed (in terms of magnitude we are talking now), the forces acting are F_2 and F_3 the resultant of which has the magnitude as F_1, so acceleration of particle is $\frac{F_1}{m}$ in the direction opposite to that of $\mathbf{F_1}$.

43. The minimum velocity (in ms^{-1}) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is **[AIEEE 2002]**

(a) 60 (b) 30 (c) 15 (d) 25

Exp. **(*b*)**

As centrifugal force is balanced by the centripetal force *i.e.,* frictional force.

Using the relation

$$\frac{mv^2}{r} = \mu R, \quad R = mg$$

$$\Rightarrow \quad \frac{mv^2}{r} = \mu mg \quad \text{or} \quad v^2 = \mu rg$$

$$\Rightarrow \quad v^2 = 0.6 \times 150 \times 10$$

or $\quad v = 30\ ms^{-1}$

44. A light string passing over a smooth light pulley connects two blocks of masses m_1 and m_2 (vertically). If the acceleration of the system is $g/8$, then the ratio of the masses is **[AIEEE 2002]**

(a) 8:1 (b) 9:7

(c) 4:3 (d) 5:3

Exp. **(*b*)**

As the string is inextensible, both masses have the same acceleration a. Also, the pulley is massless and frictionless, hence the tension at both ends of the string is the same. Suppose, the mass m_2 is greater than mass m_1, so the heavier mass m_2 is accelerating downward and the lighter mass m_1 is accelerating upwards.

Therefore, by Newton's 2nd law

$$T - m_1 g = m_1 a \quad \ldots(i)$$

and $\quad m_2 g - T = m_2 a \quad \ldots(ii)$

After solving Eqs. (i) and (ii), we get

$$a = \frac{(m_2 - m_1)}{(m_1 + m_2)} \cdot g = \frac{g}{8} \quad \text{[given]}$$

So, $$\frac{g}{8} = \frac{m_2(1 - m_1/m_2)}{m_2(1 + m_1/m_2)} \cdot g \quad \ldots(iii)$$

Let $\quad \frac{m_1}{m_2} = x$

Thus, Eq. (iii) becomes

$$\frac{1 - x}{1 + x} = \frac{1}{8} \text{ or } x = \frac{7}{9} \text{ or } \frac{m_2}{m_1} = \frac{9}{7}$$

So, the ratio of the masses is 9 : 7.

45. Three identical blocks of masses $m = 2$ kg are drawn by a force $F = 10.2$ N with an acceleration of 0.6 ms^{-2} on a frictionless surface, then what is the tension (in N) in the string between the blocks B and C? **[AIEEE 2002]**

(a) 9.2 (b) 7.8 (c) 4 (d) 9.8

Exp. **(b)**

The system of masses is shown below.

From the figure,

$$F - T_1 = ma \quad \ldots(i)$$

and $$T_1 - T_2 = ma \quad \ldots(ii)$$

Eq. (i) gives,

$$10.2 - T_1 = 2 \times 0.6 \Rightarrow T_1 = 10.2 - 1.2 = 9 \text{ N}$$

Again, from Eq. (ii), we get

$$9 - T_2 = 2 \times 0.6 \Rightarrow T_2 = 9 - 1.2 = 7.8 \text{ N}$$

46. One end of massless rope, which passes over a massless and frictionless pulley P is tied to a hook C while the other end is free. Maximum tension that the rope can bear is 360 N. With what value of maximum safe acceleration (in ms^{-2}) can a man of 60 kg climb on the rope? **[AIEEE 2002]**

(a) 16 (b) 6 (c) 4 (d) 80

Exp. **(c)**

The free body diagram of the person can be drawn as

Let the person move up with an acceleration a, then

$$T - 60g = 60a$$

$$\Rightarrow \quad a_{max} = \frac{T_{max} - 60g}{60}$$

or $$a_{max} = \frac{360 - 60g}{60} \rightarrow -\text{ve value}$$

That means, it is not possible to climb up on the rope.

Even in this problem, it is not possible to remain at rest on rope.

Hence, no option is correct.

But, if they will ask for the acceleration of climbing down, then

$$60g - T = 60a$$

$$\Rightarrow \quad 60g - T_{max} = 60a_{min}$$

or $$a_{min} = \frac{60g - 360}{60} = 4 \text{ ms}^{-2}$$

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4

Work, Energy *and* Power

1. A particle moves in one dimension from rest under the influence of a force that varies with the distance travelled by the particle as shown in the figure. The kinetic energy of the particle after it has travelled 3 m is

[JEE Main 2019, 8 April Shift-I]

(a) 4 J (b) 2.5 J (c) 6.5 J (d) 5 J

Exp. **(c)**

> **Key Idea** Area under force-displacement graph gives the value of work done.

$\therefore$ Work done on the particle

= Area under the curve *ABC*

W = Area of square *ABFO* + Area of ΔBCD + Area of rectangle *BDEF*

$$= 2 \times 2 + \frac{1}{2} \times 1 \times 1 + 2 \times 1 = 6.5 \text{ J}$$

Now, from work-energy theorem,

$$\Delta W = K_f - K_i$$

$$\Rightarrow \quad K_f = \Delta W = 6.5 \text{ J} \qquad [\because K_i = 0]$$

2. A body of mass m_1 moving with an unknown velocity of $v_1\hat{\mathbf{i}}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{\mathbf{i}}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{\mathbf{i}}$ and $v_4\hat{\mathbf{i}}$, respectively.

If $m_2 = 0.5m_1$ and $v_3 = 0.5\, v_1$, then v_1 is

[JEE Main 2019, 8 April Shift-II]

(a) $v_4 + v_2$ (b) $v_4 - \frac{v_2}{4}$

(c) $v_4 - \frac{v_2}{2}$ (d) $v_4 - v_2$

Exp. **(d)**

> **Key Idea** Total linear momentum is conserved in all collisions, i.e. the initial momentum of the system is equal to final momentum of the system.

Given,

$$m_2 = 0.5m_1 \quad \Rightarrow \quad m_1 = 2m_2$$

Let $m_2 = m$, then, $m_1 = 2m$

Also, $v_3 = 0.5v_1$

Given situation of collinear collision is as shown below

Before collision,

After collision,

∴ According to the conservation of linear momentum, Initial momentum = Final momentum

$$m_1v_1\hat{i} + m_2v_2\hat{i} = m_1v_3\hat{i} + m_2v_4\hat{i}$$

$$\Rightarrow\ 2mv_1\hat{i} + mv_2\hat{i} = 2m(0.5v_1)\hat{i} + mv_4\hat{i}$$

$$\Rightarrow\ v_4 = v_1 + v_2 \quad \Rightarrow\ v_1 = v_4 - v_2$$

3. A body of mass 2 kg makes an elastic collision with a second body at rest and continues to move in the original direction but with one-fourth of its original speed. What is the mass of the second body?

[JEE Main 2019, 9 April Shift-I]

(a) 1.5 kg (b) 1.2 kg (c) 1.8 kg (d) 1.0 kg

Exp. **(b)**

> **Key Idea** For an elastic collision, coefficient of restitution (e), i.e. the ratio of relative velocity of separation after collision to the relative velocity of approach before collision is 1.

Given, mass of small body, $m = 2$ kg

Given situation is as shown

Using momentum conservation law for the given system,

(Total momentum)$_{\text{before collision}}$ = (Total momentum)$_{\text{after collision}}$

$$\Rightarrow\ m(v) + M(0) = m\left(\frac{v}{4}\right) + M(v') \quad \ldots(i)$$

$\because$ $e = 1$ and we know that,

$$e = -\frac{v_2 - v_1}{u_2 - u_1}$$

$$\Rightarrow 1 = -\frac{v' - v/4}{0 - v} \Rightarrow v = v' - v/4$$

or $$v' = 5v/4 \quad \ldots(ii)$$

Using value from Eq. (ii) into Eq. (i), we get

$$mv = \frac{mv}{4} + M\left(\frac{5v}{4}\right)$$

$$\Rightarrow\ m\left(v - \frac{v}{4}\right) = M\left(\frac{5v}{4}\right)$$

$$\Rightarrow\ \frac{3}{4}mv = \frac{5}{4}Mv$$

$$M = \frac{3}{5}m = \frac{3}{5} \times 2 = 1.2 \text{ kg}$$

4. A uniform cable of mass M and length L is placed on a horizontal surface such that its $\left(\frac{1}{n}\right)$th part is hanging below the edge of the surface. To lift the hanging part of the cable upto the surface, the work done should be

[JEE Main 2019, 9 April Shift-I]

(a) $\frac{2MgL}{n^2}$ (b) $nMgL$ (c) $\frac{MgL}{n^2}$ (d) $\frac{MgL}{2n^2}$

Exp. **(d)**

Given, mass of the cable is M.

So, mass of $\frac{1}{n}$th part of the cable, i.e. hanged part of the cable is $= M/n$ …(i)

Now, centre of mass of the hanged part will be its middle point.

So, its distance from the top of the table will be $L/2n$.

∴ Initial potential energy of the hanged part of cable,

$$U_i = \left(\frac{M}{n}\right)(-g)\left(\frac{L}{2n}\right)$$

$$\Rightarrow\ U_i = -\frac{MgL}{2n^2} \quad \ldots(ii)$$

When whole cable is on the table, its potential energy will be zero.

$$\therefore\ U_f = 0 \quad \ldots(iii)$$

Now, using work-energy theorem,

$$W_{net} = \Delta U = U_f - U_i$$

$$\Rightarrow \quad W_{net} = 0 - \left(-\frac{MgL}{2n^2}\right)$$

[using Eqs. (ii) and (iii)]

$$\Rightarrow \quad W_{net} = \frac{MgL}{2n^2}$$

5. A particle of mass m is moving with speed $2v$ and collides with a mass $2m$ moving with speed v in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass m, which move at angle 45° with respect to the original direction.

The speed of each of the moving particle will be **[JEE Main 2019, 9 April Shift-II]**

(a) $\sqrt{2}\,v$ (b) $\frac{v}{\sqrt{2}}$

(c) $\frac{v}{(2\sqrt{2})}$ (d) $2\sqrt{2}\,v$

Exp. **(d)**

According to the questions,

Initial condition,

Final condition,

As we know that, in collision, linear momentum is conserved in both x and y directions separately.

So, $(p_x)_{initial} = (p_x)_{final}$

$$m(2v) + 2m(v) = 0 + mv'\cos 45^\circ + mv'\cos 45^\circ$$

$$\Rightarrow \quad 4mv = \frac{2m}{\sqrt{2}}v'$$

$$\Rightarrow \quad v' = 2\sqrt{2}v$$

6. Two particles of masses M and $2M$, moving as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speed v_1 and v_2 are nearly **[JEE Main 2019, 10 April Shift-I]**

(a) 6.5 m/s and 3.2 m/s

(b) 3.2 m/s and 6.3 m/s

(c) 3.2 m/s and 12.6 m/s

(d) 6.5 m/s and 6.3 m/s

Exp. **(d)**

The given condition can be drawn as shown below

Applying linear momentum conservation law in x-direction, we get

Initial momentum = Final momentum

$$(M \times 10\cos 30^\circ) + (2M \times 5\cos 45^\circ)$$

$$= (M \times v_2 \cos 45^\circ) + (2M \times v_1 \cos 30^\circ)$$

$$\Rightarrow \quad \left(M \times 10 \times \frac{\sqrt{3}}{2}\right) + \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$

$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) + \left(2M \times v_1 \times \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \quad 5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + v_1\sqrt{3} \quad \ldots \text{(i)}$$

Similarly, applying linear momentum conservation law in y-direction, we get

$$(M \times 10\sin 30^\circ) - (2M \times 5\sin 45^\circ)$$

$$= (M \times v_2 \sin 45^\circ) - (2M \times v_1 \sin 30^\circ)$$

$$\Rightarrow \quad \left(M \times 10 \times \frac{1}{2}\right) - \left(2M \times 5 \times \frac{1}{\sqrt{2}}\right)$$

$$= \left(M \times v_2 \times \frac{1}{\sqrt{2}}\right) - \left(2M \times v_1 \times \frac{1}{2}\right)$$

$$\Rightarrow \quad 5 - 5\sqrt{2} = \frac{v_2}{\sqrt{2}} - v_1 \quad \ldots \text{(ii)}$$

Subtracting Eq. (ii) from Eq. (i), we get

$$(5\sqrt{3} + 5\sqrt{2}) - (5 - 5\sqrt{2})$$
$$= \left(\frac{v_2}{\sqrt{2}} + v_1\sqrt{3}\right) - \left(\frac{v_2}{\sqrt{2}} - v_1\right)$$
$$\Rightarrow \quad 5\sqrt{3} + 10\sqrt{2} - 5 = v_1\sqrt{3} + v_1$$
$$\Rightarrow \quad v_1 = \left(\frac{5\sqrt{3} + 10\sqrt{2} - 5}{1 + \sqrt{3}}\right)$$
$$= \frac{8.66 + 14.142 - 5}{1 + 1.732}$$
$$= \frac{17.802}{2.732} \Rightarrow v_1 = 6.516 \text{ m/s} \approx 6.5 \text{ m/s} \quad \ldots \text{(iii)}$$

Substituting the value from Eq. (iii) in Eq. (i), we get

$$5\sqrt{3} + 5\sqrt{2} = \frac{v_2}{\sqrt{2}} + 6.51 \times \sqrt{3}$$
$$\Rightarrow \quad v_2 = (5\sqrt{3} + 5\sqrt{2} - 6.51 \times \sqrt{3})\sqrt{2}$$
$$v_2 = (8.66 + 7.071 - 11.215)\, 1.414$$
$$\Rightarrow \quad v_2 = 4.456 \times 1.414 \Rightarrow v_2 \approx 6.3 \text{ m/s}$$

7. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son, so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is

[JEE Main 2019, 12 April Shift-I]

(a) 0.28 ms^{-1} (b) 0.20 ms^{-1}
(c) 0.47 ms^{-1} (d) 0.14 ms^{-1}

Exp. **(b)**

The given situation can be shown as below

Before collision

After collision

Using momentum conservation law,

(Total momentum)$_{\text{before collision}}$
= (Total momentum)$_{\text{after collision}}$

$$(m_1 \times 0) + (m_2 \times 0) = m_1\mathbf{v}_1 + m_2\,\mathbf{v}_2$$
$$0 = m_1(-v_1)\hat{\mathbf{i}} + m_2 v_2 \hat{\mathbf{i}}$$
$$\Rightarrow \quad m_1 v_1 = m_2 v_2$$
$$\Rightarrow \quad 50 v_1 = 20 v_2$$
$$\Rightarrow \quad v_2 = 2.5 v_1 \quad \ldots \text{(i)}$$

Again, relative velocity = 0.70 m/s

But from figure, relative velocity = $v_1 + v_2$

$$\therefore \quad v_1 + v_2 = 0.7 \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v_1 + 2.5 v_1 = 0.7$$
$$\Rightarrow \quad v_1(3.5) = 0.7$$
$$v_1 = \frac{0.7}{3.5} = 0.20 \text{ m/s}$$

8. Three blocks A, B and C are lying on a smooth horizontal surface as shown in the figure. A and B have equal masses m while C has mass M. Block A is given an initial speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically $\frac{5}{6}$th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$?

[JEE Main 2019, 9 Jan Shift-I]

(a) 4 (b) 2 (c) 3 (d) 5

Exp. **(a)**

> **Key Idea** For a perfectly inelastic collision, the momentum of the system remains conserved but there is some of loss of kinetic energy. Also, after collision the objects of the system are stuck to each other and move as a combined system.

Initially, block A is moving with velocity v as shown in the figure below,

Now, A collides with B such that they collide inelastically. Thus, the combined mass (say) move with the velocity 'v' as shown below,

Then, if this combined system is collided inelastically again with the block C. So, now the velocity of system be v'' as shown below.

Thus, according to the principle of conservation of momentum,

initial momentum of the system

= final momentum of the system

$\Rightarrow \quad mv = (2m + M)v''$

or $\quad v'' = \left(\dfrac{mv}{2m+M}\right) \quad \ldots$ (i)

Initial kinetic energy of the system,

$$(\text{KE})_i = \frac{1}{2}mv^2 \quad \ldots\text{(ii)}$$

Final kinetic energy of the system, $(\text{KE})_f$

$$= \frac{1}{2}(2m + M)(v'')^2 = \frac{1}{2}(2m + M)\left(\frac{mv}{2m + M}\right)^2$$

[$\because$ using Eq. (i)]

$$= \frac{1}{2}\cdot\frac{v^2m^2}{(2m+M)} \quad \ldots\text{(iii)}$$

Dividing Eq. (iii) and Eq. (ii), we get

$$\frac{(\text{KE})_f}{(\text{KE})_i} = \frac{\frac{1}{2}m^2v^2}{\frac{(2m+M)}{\frac{1}{2}mv^2}} = \frac{m}{2m + M} \quad \ldots\text{(iv)}$$

It is given that $\frac{5}{6}$th of $(\text{KE})_i$ is lost in this process.

$\Rightarrow \quad (\text{KE})_f = \frac{1}{6}(\text{KE})_i$

$\Rightarrow \quad \dfrac{(\text{KE})_f}{(\text{KE})_i} = \dfrac{1}{6} \quad \ldots$ (v)

Comparing Eq. (iv) and Eq. (v), we get

$$\frac{m}{2m + M} = \frac{1}{6} \Rightarrow 6m = 2m + M$$

$$4m = M \Rightarrow \frac{M}{m} = 4$$

9. A block of mass m lying on a smooth horizontal surface is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed as shown in the figure. The block is initially at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is

[JEE Main 2019, 9 Jan Shift-I]

(a) $\dfrac{\pi F}{\sqrt{mk}}$ (b) $\dfrac{F}{\sqrt{mk}}$ (c) $\dfrac{2F}{\sqrt{mk}}$ (d) $\dfrac{F}{\pi\sqrt{mk}}$

Exp. **(b)**

In a spring-block system, when a block is pulled with a constant force F, then its speed is maximum at the mean position. Also, it's acceleration will be zero.

In that case, force on the system is given as,

$$F = kx \quad \ldots\text{(i)}$$

where, x is the extension produced in the spring.

or $\quad x = \dfrac{F}{k}$

Now we know that, for a system vibrating at its mean position, its maximum velocity is given as,

$$v_{\max} = A\omega$$

where, A is the amplitude and ω is the angular velocity.

Since, the block is at its mean position.

So, $\quad A = x = \dfrac{F}{k}$

$$v_{\max} = \frac{F}{k}\sqrt{\frac{k}{m}} \quad \left[\because \omega = \sqrt{\frac{k}{m}}\right]$$

$$= \frac{F}{\sqrt{km}}$$

Alternate Solution

According to the work-energy theorem,

net work done = change in the kinetic energy

Here, net work done = work done due to external force (W_{ext}) + work done due to the spring (W_{spr}).

As, $\quad W_{\text{ext}} = F \cdot x$

and $\quad W_{\text{spr}} = \dfrac{-1}{2}kx^2$

$\Rightarrow \quad \Delta\text{KE} = F \cdot x + \left(-\dfrac{1}{2}kx^2\right)$

$$(\Delta\text{KE})_f - (\Delta\text{KE})_i = F \cdot x - \frac{1}{2}kx^2$$

$$\Rightarrow \frac{1}{2}mv_{\max}^2 - \frac{1}{2}m(0)^2 = F\cdot\left(\frac{F}{k}\right) - \frac{1}{2}k\left(\frac{F}{k}\right)^2$$

[using Eq. (i)]

$\Rightarrow \quad \dfrac{1}{2}mv_{\max}^2 = \dfrac{F^2}{k} - \dfrac{F^2}{2k} = \dfrac{F^2}{2k}$

or $\quad v_{\max}^2 = \dfrac{F^2}{km}$

$\Rightarrow \quad v_{\max} = F/\sqrt{km}$

10. A force acts on a 2 kg object, so that its position is given as a function of time as $x = 3t^2 + 5$. What is the work done by this force in first 5 seconds?

[JEE Main 2019, 9 Jan Shift-II]

(a) 850 J (b) 900 J (c) 950 J (d) 875 J

Exp. **(b)**

Here, the displacement of an object is given by

$$x = (3t^2 + 5)\,\text{m}$$

Therefore, velocity $(v) = \frac{dx}{dt} = \frac{d(3t^2 + 5)}{dt}$

or $\quad v = 6t$ m/s ...(i)

The work done in moving the object from $t = 0$ to $t = 5$ s

$$W = \int_{x_0}^{x_5} F \cdot dx \quad \text{...(ii)}$$

The force acting on this object is given by

$$F = ma = m \times \frac{dv}{dt}$$

$$= m \times \frac{d(6t)}{dt} \quad \text{[using Eq. (i)]}$$

$$F = m \times 6 = 6\,\text{m} = 12\,\text{N}$$

Also, $\quad x_0 = 3t^2 + 5 = 3 \times (0)^2 + 5 = 5\,\text{m}$

and at $t = 5$ s,

$$x_5 = 3 \times (5)^2 + 5 = 80\,\text{m}$$

Put the values in Eq. (ii), we get

$$W = 12 \times \int_{x_0}^{x_5} dx = 12\,[80 - 5]$$

$$W = 12 \times 75 = 900\,\text{J}$$

Alternate Solution

To using work – kinetic energy theorem is,

$$W = \Delta K \cdot E = \frac{1}{2} m(v_f^2 - v_i^2)$$

$$= \frac{1}{2} m \times (30^2 - 0^2)$$

$$= \frac{1}{2} \times 2 \times 900 = 900\,\text{J}$$

11. A block of mass m is kept on a platform which starts from rest with constant acceleration $\frac{g}{2}$ upwards as shown in figure. Work done by normal reaction on block in time t is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\frac{mg^2t^2}{8}$ (b) $\frac{3mg^2t^2}{8}$

(c) 0 (d) $-\frac{mg^2t^2}{8}$

Exp. **(b)**

Normal reaction force on the block is

$$N = ma_{net}$$

where, a_{net} = net acceleration of block.

$$= g + a$$

$$= g + \frac{g}{2} = \frac{3g}{2}$$

$$\Rightarrow \quad N = m\left(g + \frac{g}{2}\right) = \frac{3mg}{2}$$

Now, in time 't' block moves by a displacement s given by

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{g}{2}\right)t^2 \quad (\because u = 0)$$

Here, $\quad a = \frac{g}{2}$ (given)

$\therefore$ Work done = Force × Displacement

$$\Rightarrow \quad W = \frac{3mg}{2} \times \frac{gt^2}{4} = \frac{3mg^2t^2}{8}$$

12. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward with a velocity 100 ms^{-1} from the ground. The bullet gets embedded in the wood. Then, the maximum height to which the combined system reaches above the top of the building before falling below is (Take, $g = 10\,\text{ms}^{-2}$)

[JEE Main 2019, 10 Jan Shift-I]

(a) 20 m (b) 30 m (c) 10 m (d) 40 m

Exp. (d)

Key Idea As bullet gets embedded in the block of wood so, it represents a collision which is perfectly inelastic and hence only momentum of the system is conserved.

Velocity of bullet is very high compared to velocity of wooden block so, in order to calculate time for collision, we take relative velocity nearly equal to velocity of bullet.

So, time taken for particles to collide is

$$t = \frac{d}{v_{rel}} = \frac{100}{100} = 1\text{s}$$

Speed of block just before collision is;

$$v_1 = gt = 10 \times 1 = 10\,\text{ms}^{-1}$$

Speed of bullet just before collision is

$$v_2 = u - gt$$
$$= 100 - 10 \times 1 = 90\,\text{ms}^{-1}$$

Let v = velocity of bullet + block system, then by conservation of linear momentum, we get

$-(0.03 \times 10) + (0.02 \times 90) = (0.05)\,v$

$\Rightarrow \quad v = 30\,\text{ms}^{-1}$

Now, maximum height reached by bullet and block is

$$h = \frac{v^2}{2g} \Rightarrow h = \frac{30 \times 30}{2 \times 10}$$

$\Rightarrow \quad h = 45\,\text{m}$

$\therefore$ Height covered by the system from point of collision = 45 m

Now, distance covered by bullet before collision in 1 s.

$$= 100 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 95\,\text{m}$$

Distance of point of collision from the top of the building

$$= 100 - 95 = 5\,\text{m}$$

$\therefore$ Maximum height to which the combined system reaches above the top of the building before falling below = 45 – 5 = 40 m

13. A particle which is experiencing a force, is given by $\mathbf{F} = 3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}$, undergoes a displacement of $\mathbf{d} = 4\hat{\mathbf{i}}$. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement ?

[JEE Main 2019, 10 Jan Shift-II]

(a) 9 J (b) 15 J (c) 12 J (d) 10 J

Exp. (b)

We know that, work done in displacing a particle at displacement **d** under force **F** is given by

$\Delta W = \mathbf{F} \cdot \mathbf{d}$

By substituting given values, we get

$\Rightarrow \quad \Delta W = (3\hat{\mathbf{i}} - 12\hat{\mathbf{j}}) \cdot (4\hat{\mathbf{i}})$

$\Rightarrow \quad \Delta W = 12\text{ J} \quad \ldots \text{(i)}$

Now, using work-energy theorem, we get

work done (ΔW) = change in kinetic energy (ΔK)

or $\quad \Delta W = K_2 - K_1 \quad \ldots \text{(ii)}$

Comparing Eqs. (i) and (ii), we get

$$K_2 - K_1 = 12\text{ J}$$

or $\quad K_2 = K_1 + 12\text{ J}$

Given, initial kinetic energy, $K_1 = 3$ J

$\therefore$ Final kinetic energy, $K_2 = 3\text{ J} + 12\text{ J} = 15\text{ J}$

14. A body of mass 1 kg falls freely from a height of 100 m on a platform of mass 3 kg which is mounted on a spring having spring constant $k = 1.25 \times 10^6$ N/m. The body sticks to the platform and the spring's maximum compression is found to be x. Given that $g = 10\,\text{ms}^{-2}$, the value of x will be close to

[JEE Main 2019, 11 Jan Shift-I]

(a) 8 cm (b) 4 cm (c) 40 cm (d) 80 cm

Exp. (*)

Initial compression of the spring,

$$mg = k\left(\frac{x_0}{100}\right) \quad (x_0 \text{ in cm})$$

$$\Rightarrow \quad x_0 = \frac{3 \times 10 \times 100}{1.25 \times 10^6} = \frac{3}{1250}$$

Which is very small and can be neglected.

Applying conservation of momentum before and after the collision i.e., momentum before collision = momentum after collision.

$m \times \sqrt{2gh} = (m + M)\,v$ ($\because$ velocity of the block just before the collision is

$$v^2 - 0^2 = 2gh$$

or $\quad v = \sqrt{2gh}$)

After substituting the given values, we get

$1 \times \sqrt{2 \times 10 \times 100} = 4v$ or $4v = 20\sqrt{5}$

So, $\quad v = 5\sqrt{5}\,\text{m/s}$

Let this be the maximum velocity, then for the given system, using

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

$$\therefore \frac{1}{2} \times 4 \times 125 = \frac{1}{2} \times 125 \times 10^6 \times \left(\frac{x}{100}\right)^2$$

$$\Rightarrow \quad 4 = 10^4 \times \frac{x^2}{10^4} \text{ or } x = 2 \text{ cm}$$

$\therefore$ No option given is correct.

15. A simple pendulum is made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then, M is given by

[JEE Main 2019, 12 Jan Shift-I]

(a) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (b) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

(c) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ (d) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$

Exp. **(a)**

Pendulum's velocity at lowest point just before striking mass m is found by equating it's initial potential energy (PE) with final kinetic energy (KE).

Initially, when pendulum is released from angle θ_0 as shown in the figure below,

We have,

$$mgh = \frac{1}{2}mv^2$$

Here, $h = l - l\cos\theta_0$

So, $v = \sqrt{2gl\,(1-\cos\theta_0)}$ ...(i)

With velocity v, bob of pendulum collides with block. After collision, let v_1 and v_2 are final velocities of masses m and M respectively as shown

Then if pendulum is deflected back upto angle θ_1, then

$$v_1 = \sqrt{2gl\,(1-\cos\theta_1)} \quad \text{...(ii)}$$

Using definition of coefficient of restitution to get

$$e = \frac{|\text{velocity of separation}|}{|\text{velocity of approach}|}$$

$$1 = \frac{v_2 - (-v_1)}{v - 0} \Rightarrow v = v_2 + v_1 \quad \text{...(iii)}$$

From Eqs. (i), (ii) and (iii), we get

$$\Rightarrow \quad \sqrt{2gl(1-\cos\theta_0)} = v_2 + \sqrt{2gl(1-\cos\theta_1)}$$

$$\Rightarrow \quad v_2 = \sqrt{2gl}\,(\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}) \quad \text{...(iv)}$$

According to the momentum conservation, initial momentum of the system = final momentum of the system

$$\Rightarrow \quad mv = Mv_2 - mv_1$$

$$\Rightarrow \quad Mv_2 = m(v + v_1)$$

$$Mv_2 = m\sqrt{2gl}\,(\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1})$$

Dividing Eq. (v) and Eq. (iv), we get

$$\Rightarrow \quad \frac{M}{m} = \frac{\sqrt{1-\cos\theta_0} + \sqrt{1-\cos\theta_1}}{\sqrt{1-\cos\theta_0} - \sqrt{1-\cos\theta_1}}$$

$$= \frac{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} + \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}{\sqrt{\sin^2\left(\frac{\theta_0}{2}\right)} - \sqrt{\sin^2\left(\frac{\theta_1}{2}\right)}}$$

$$\frac{M}{m} = \frac{\sin\left(\frac{\theta_0}{2}\right) + \sin\left(\frac{\theta_1}{2}\right)}{\sin\left(\frac{\theta_0}{2}\right) - \sin\left(\frac{\theta_1}{2}\right)}$$

For small θ_0, we have

$$\frac{M}{m} = \frac{\frac{\theta_0}{2} + \frac{\theta_1}{2}}{\frac{\theta_0}{2} - \frac{\theta_1}{2}} \text{ or } M = m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$$

16. An α-particle of mass m suffers one-dimensional elastic collision with a nucleus at rest of unknown mass. It is scattered directly backwards losing 64% of its initial kinetic energy. The mass of the nucleus is **[JEE Main 2019, 12 Jan Shift-II]**

(a) 1.5 m (b) 4 m (c) 3.5 m (d) 2 m

Exp. **(b)**

We have following collision, where mass of α particle = m and mass of nucleus = M

Let α particle rebounds with velocity v_1, then

Given;

final energy of α = 36% of initial energy

$$\Rightarrow \quad \frac{1}{2}mv_1^2 = 0.36 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_1 = 0.6\,v \qquad \ldots(i)$$

As unknown nucleus gained 64% of energy of α, we have

$$\frac{1}{2}Mv_2^2 = 0.64 \times \frac{1}{2}mv^2$$

$$\Rightarrow \quad v_2 = \sqrt{\frac{m}{M}} \times 0.8\,v \qquad \ldots(ii)$$

From momentum conservation, we have

$$mv = Mv_2 - mv_1$$

Substituting values of v_1 and v_2 from Eqs. (i) and (ii), we have

$$mv = M\sqrt{\frac{m}{M}} \times 0.8v - m \times 0.6v$$

$$\Rightarrow \quad 1.6mv = \sqrt{mM} \times 0.8v$$

$$\Rightarrow \quad 2m = \sqrt{mM}$$

$$\Rightarrow \quad 4m^2 = mM$$

$$\Rightarrow \quad M = 4m$$

17. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles after collision, is **[JEE Main 2018]**

(a) $\frac{v_0}{4}$ (b) $\sqrt{2}\,v_0$

(c) $\frac{v_0}{2}$ (d) $\frac{v_0}{\sqrt{2}}$

Exp. **(c)**

Momentum is conserved in all type of collisions,

Final kinetic energy is 50% more than initial kinetic energy

$$\Rightarrow \quad \frac{1}{2}mv_2^2 + \frac{1}{2}mv_1^2 = \frac{150}{100} \times \frac{1}{2}mv_0^2 \qquad \ldots(i)$$

Conservation of momentum gives,

$$mv_0 = mv_1 + mv_2$$

$$v_0 = v_2 + v_1 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$v_1^2 + v_2^2 + 2v_1v_2 = v_0^2 \Rightarrow 2v_1v_2 = \frac{-v_0^2}{2}$$

$$\therefore \quad (v_1 - v_2)^2 = (v_1 + v_2)^2 - 4v_1v_2 = 2v_0^2$$

or $\quad v_{rel} = \sqrt{2}v_0$

18. A time dependent force $F = 6t$ acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 s will be **[JEE Main 2017 (Offline)]**

(a) 22 J (b) 9 J (c) 18 J (d) 4.5 J

Exp. **(d)**

From Newton's second law, $\frac{\Delta p}{\Delta t} = F \Rightarrow \Delta p = F\Delta t$

$$\therefore \quad p = \int dp = \int_0^1 F\,dt$$

$$\Rightarrow \quad p = \int_0^1 6t\,dt = 3\,\text{kg}\left(\frac{\text{m}}{\text{s}}\right)$$

Also, change in kinetic energy

$$\Delta k = \frac{\Delta p^2}{2m} = \frac{3^2}{2 \times 1} = 4.5$$

From work-energy theorem,

work done = change in kinetic energy.

So, work done = Δk = 4.5 J

19. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m, 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted into mechanical energy with a 20% of efficiency rate. (Take, $g = 9.8\ \text{ms}^{-2}$) **[JEE Main 2016 (Offline)]**

(a) 2.45×10^{-3} kg (b) 6.45×10^{-3} kg

(c) 9.89×10^{-3} kg (d) 12.89×10^{-3} kg

Exp. **(*d*)**

Given, potential energy burnt by lifting weight

$$= mgh = 10 \times 9.8 \times 1 \times 1000 = 9.8 \times 10^4$$

If mass lost by a person be m, then energy dissipated

$$= m \times \frac{2}{10} \times 3.8 \times 10^7 \text{ J}$$

$$\Rightarrow \quad 9.8 \times 10^4 = m \times \frac{1}{5} \times 3.8 \times 10^7$$

$$\Rightarrow \quad m = \frac{5}{3.8} \times 10^{-3} \times 9.8 = 12.89 \times 10^{-3} \text{ kg}$$

20. A particle of mass m moving in the x-direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y-direction with speed v. If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to **[JEE Main 2015]**

(a) 44% (b) 50% (c) 56% (d) 62%

Exp. **(*c*)**

Conservation of linear momentum can be applied but energy is not conserved.

Consider the movement of two particles as shown below.

Conserving linear momentum in x-direction

$$(p_i)x = (p_f)x \quad \text{or} \quad 2mv = (2m + m)\, v_x$$

or $$v_x = \frac{2}{3} v$$

Conserving linear momentum in y-direction

$$(p_i)y = (p_f)y \quad \text{or} \quad 2mv = (2m + m)\, v_y$$

or $$v_y = \frac{2}{3} v$$

Initial kinetic energy of the two particles system is

$$E_i = \frac{1}{2} m (2v)^2 + \frac{1}{2} (2m)(v)^2$$

$$= \frac{1}{2} \times 4mv^2 + \frac{1}{2} \times 2mv^2$$

$$= 2mv^2 + mv^2 = 3mv^2$$

Final energy of the combined two particles system is

$$E_f = \frac{1}{2} (3m)(v_x^2 + v_y^2)$$

$$= \frac{1}{2} (3m) \left[\frac{4v^2}{9} + \frac{4v^2}{9} \right] = \frac{3m}{2} \left[\frac{8v^2}{9} \right] = \frac{4mv^2}{3}$$

Loss in the energy $\Delta E = E_i - E_f$

$$= mv^2 \left[3 - \frac{4}{3} \right] = \frac{5}{3} mv^2$$

Percentage loss in the energy during the collision

$$\frac{\Delta E}{E_i} \times 100 = \frac{(5/3)\, mv^2}{3mv^2} \times 100 = \frac{5}{9} \times 100 \simeq 56\%$$

21. When a rubber band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$, where a and b are constants. The work done in stretching the unstretched rubber band by L is **[JEE Main 2014]**

(a) $aL^2 + bL^3$ (b) $\frac{1}{2}(aL^2 + bL^3)$

(c) $\frac{aL^2}{2} + \frac{bL^3}{3}$ (d) $\frac{1}{2}\left(\frac{aL^2}{2} + \frac{bL^3}{3}\right)$

As, we know that change in potential energy of a system corresponding to a conservative internal force as $U_f - U_i = -W = -\int_i^t F.dr$

Exp. **(*c*)**

Given, $F = ax + bx^2$

According to work-energy theorem, we know that work done in stretching the rubber band by t is

$$|dW| = Fdx$$

$$|W| = \int_0^L (ax + bx^2)\, dx = \left[\frac{ax^2}{2}\right]_0^L + \left[\frac{bx^3}{3}\right]_0^L$$

$$= \left[\frac{aL^2}{2} - \frac{a \times (0)^2}{2}\right] + \left[\frac{b \times L^3}{3} - \frac{b \times (0)^3}{3}\right]$$

$$\therefore \quad |W| = \frac{aL^2}{2} + \frac{bL^3}{3}$$

22. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$, then $f = \left(\frac{m}{M+m}\right)$.

Statement II Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[JEE Main 2013]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is false, Statement II is true

Exp. **(d)**

Energy $E = \frac{p^2}{2m}$, where p is momentum, m is the mass moving of the particle. Maximum energy loss occurs when the particles get stuck together as a result of the collision.

Maximum energy loss $(\Delta E) = \frac{p^2}{2m} - \frac{p^2}{2(m+M)}$

where, $(m + M)$ is the resultant mass when the particles get stuck.

$$\Delta E = \frac{p^2}{2m}\left[1 - \frac{m}{m+M}\right] = \frac{p^2}{2m}\left[\frac{M}{m+M}\right]$$

Also, $p = mv$

$$\therefore \quad \Delta E = \frac{m^2v^2}{2m}\left[\frac{M}{m+M}\right] = \frac{mv^2}{2}\left[\frac{M}{m+M}\right]$$

Comparing the expression with

$$\Delta E = f\left(\frac{1}{2}mv^2\right), f = \frac{M}{m+M}$$

23. At time $t = 0$, particle starts moving along the x-axis. If its kinetic energy increases uniformly with time t, the net force acting on it must be proportional to **[AIEEE 2011]**

(a) $\sqrt{t}$ (b) constant
(c) t (d) $1/\sqrt{t}$

Exp. **(d)**

Given, $k \propto t \Rightarrow \frac{dk}{dt} = \text{constant}$

$$\Rightarrow \quad K \propto t$$

$$\frac{1}{2}mv^2 \propto t \Rightarrow v \propto \sqrt{t}$$

Also, $P = Fv = \frac{dK}{dt} = \text{constant}$

$$\Rightarrow \quad F \propto \frac{1}{v}$$

$$\Rightarrow \quad F \propto \frac{1}{\sqrt{t}}$$

Alternate Solution

$K \propto t \quad v \propto \sqrt{t}$, $F = ma = m\frac{dv}{dt}$

As, $a = \frac{dv}{dt} = k\frac{1}{2}t^{-1/2}$

$a \propto \frac{1}{\sqrt{t}}$, so $F \propto \frac{1}{\sqrt{t}}$

➡ **Directions** *(Q. Nos. 24-25) contain Statement* I *and Statement* II. *Of the four choices given after the statements, choose the one that best describes the two statements.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is true
(d) Statement I is true, Statement II is false

24. **Statement I** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement II Principle of conservation of momentum holds true for all kinds of collisions. **[AIEEE 2010]**

Exp. **(a)**

If it is a completely inelastic collision, then

$$m_1v_1 + m_2v_2 = m_1v + m_2v$$

$$v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2},$$

$$\text{KE} = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}$$

As p_1 and p_2 both simultaneously cannot be zero. Therefore, total KE cannot be lost.

25. An athlete in the olympic games covers a distance of 100 m in 10 s. His kinetic energy can be estimated to be in the range **[AIEEE 2008]**

(a) 200 J – 500 J (b) 2×10^5 J – 3×10^5 J
(c) 20000 J – 50000 J (d) 2000 J – 5000 J

Exp. **(d)**

The given question is somewhat based on approximations. Let mass of athlete be 65 kg. Approx velocity from the given data is 10 m/s.

So, $\quad KE = \frac{65 \times 100}{2} = 3250$ J

So, option (d) is the most probable answer.

26. A block of mass 0.50 kg is moving with a speed of 2.00 ms^{-1} on a smooth surface. It strikes another mass of 1.00 kg and then they move together as a single body. The energy loss during the collision is **[AIEEE 2008]**

(a) 0.16 J (b) 1.00 J
(c) 0.67 J (d) 0.34 J

Exp. **(c)**

From law of conservation of momentum, we have

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\,v$$

Given, $m_1 = 0.50$ kg, $v_1 = 2$ ms^{-1},

$m_2 = 1$ kg, $v_2 = 0$ [at rest]

$$0.5 \times 2 + 1 \times 0 = 1.5 \times v$$

[assumed that 2nd body is at rest]

$\Rightarrow \quad v = \frac{2}{3}$

$\therefore \quad \Delta K = K_f - K_i$

$$= \frac{1.5 \times \left(\frac{2}{3}\right)^2}{2} - (0.5) \times \frac{2^2}{2} = -\frac{2}{3} \text{ J}$$

$$= -0.67 \text{ J}$$

So, energy lost is 0.67 J.

27. A 2 kg block slides on a horizontal floor with a speed of 4 m/s. It strikes a uncompressed spring and compresses it till the block is motionless. The kinetic friction force is 15 N and spring constant is 10000 N/m. The spring compresses by **[AIEEE 2007]**

(a) 5.5 cm (b) 2.5 cm
(c) 11.0 cm (d) 8.5 cm

Exp. **(a)**

According to work-energy theorem

Loss in kinetic energy = Work done against friction + Potential energy of spring

$$\frac{1}{2}mv^2 = f\,x + \frac{1}{2}kx^2$$

$\Rightarrow \quad \frac{1}{2} \times 2\,(4)^4 = 15\,x + \frac{1}{2} \times 10000\,x^2$

$\Rightarrow \quad 5000\,x^2 + 15x - 16 = 0$

$\therefore \quad x = 0.055 \text{ m} = 5.5 \text{ cm}$

28. The potential energy of a 1 kg particle free to move along the x-axis is given by

$$V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right) \text{J}$$

The total mechanical energy of the particle is 2 J. Then, the maximum speed (in ms^{-1}) is **[AIEEE 2006]**

(a) $\frac{3}{\sqrt{2}}$ (b) $\sqrt{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) 2

Exp. **(a)**

Given, $\quad V(x) = \left(\frac{x^4}{4} - \frac{x^2}{2}\right)$

For minimum value of V, $\frac{dV}{dx} = 0$

$\Rightarrow \quad \frac{4x^3}{4} - \frac{2x}{2} = 0$

$\Rightarrow \quad x = 0, \quad x = \pm 1$

So, $\quad V_{min}\,(x = \pm 1) = \frac{1}{4} - \frac{1}{2} = \frac{-1}{4}$ J

Now, $K_{max} + V_{min}$ = Total mechanical energy

$\Rightarrow \quad K_{max} = \left(\frac{1}{4}\right) + 2$ or $K_{max} = \frac{9}{4}$

or $\quad \frac{mv^2}{2} = \frac{9}{4}$ or $v = \frac{3}{\sqrt{2}}$ ms^{-1}

29. A bomb of mass 16 kg at rest explodes into two pieces of masses 4 kg and 12 kg. The velocity of the 12 kg mass is 4ms^{-1}. The kinetic energy of the other mass is **[AIEEE 2006]**

(a) 144 J (b) 288 J (c) 192 J (d) 96 J

Exp. **(b)**

Here, momentum of the system is remaining conserved as no external force is acting on the bomb (system).

Initial momentum (before explosion)
= Final momentum (after explosion)

Let velocity of 4 kg mass be v ms^{-1}. From momentum conservation, we can say that its direction is opposite to velocity of 12 kg mass.

From $\quad p_i = p_f \Rightarrow 0 = 12 \times 4 - 4 \times v$

or $\quad v = 12$ m/s

$\therefore$ KE of 4 kg mass $= \dfrac{4 \times (12)^2}{2} = 288$ J

30. A mass m moves with a velocity v and collides inelastically with another identical mass. After collision, the 1st mass moves with velocity $\dfrac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the second mass after collision. **[AIEEE 2005]**

(a) v (b) $\sqrt{3}v$ (c) $\dfrac{2}{\sqrt{3}}v$ (d) $\dfrac{v}{\sqrt{3}}$

Exp. **(c)**

In x-direction,

Apply conservation of momentum, we get

$$mu_1 + 0 = mv_x$$

$$\Rightarrow \quad mv = mv_x \Rightarrow v_x = v$$

In y-direction, apply conservation of momentum, we get

$$0 + 0 = m\left(\frac{v}{\sqrt{3}}\right) - mv_y \Rightarrow v_y = \frac{v}{\sqrt{3}}$$

Velocity of second mass after collision

$$v' = \sqrt{\left(\frac{v}{\sqrt{3}}\right)^2 + v^2} = \sqrt{\frac{4}{3}v^2} \quad \text{or} \quad v' = \frac{2}{\sqrt{3}}v$$

31. A bullet fired into a fixed target loses half of its velocity after penetrating 3 cm. How much further it will penetrate before coming to rest, assuming that it faces constant resistance to motion? **[AIEEE 2005]**

(a) 3.0 cm (b) 2.0 cm (c) 1.5 cm (d) 1.0 cm

Exp. **(d)**

According to work-energy theorem,

Total work done = Change in kinetic energy

$$W = \Delta K$$

Case I $\quad -F \times 3 = \dfrac{1}{2}m\left(\dfrac{v_0}{2}\right)^2 - \dfrac{1}{2}mv_0^2$

where, F is resistive force and v_0 is initial speed.

Case II Let the further distance travelled by the bullet before coming to rest be s.

$\therefore \quad -F(3 + s) = K_f - K_i = -\dfrac{1}{2}mv_0^2$

$\Rightarrow \quad -\dfrac{1}{8}mv_0^2(3 + s) = -\dfrac{1}{2}mv_0^2$

or $\quad \dfrac{1}{4}(3 + s) = 1$

or $\quad \dfrac{3}{4} + \dfrac{s}{4} = 1$

or $\quad s = 1$ cm

32. A body of mass m is accelerated uniformly from rest to a speed v in a time T. The instantaneous power delivered to the body as a function of time, is given by **[AIEEE 2005, 04]**

(a) $\dfrac{mv^2}{T^2}t$ (b) $\dfrac{mv^2}{T^2}t^2$

(c) $\dfrac{1}{2}\dfrac{mv^2}{T^2}t$ (d) $\dfrac{1}{2}\dfrac{mv^2}{T^2}t^2$

Exp. **(a)**

A body of mass m with uniform acceleration, then force

$$F = ma = \frac{mv}{T} \qquad \left[\therefore a = \frac{v - 0}{T}\right]$$

Instantaneous power $= Fv = mav$

$$= \frac{mv}{T} \cdot at$$

$$= \frac{mv}{T} \cdot \frac{v}{T} \cdot t = \frac{mv^2}{T^2}t$$

33. A spherical ball of mass 20 kg is stationary at the top of a hill of height 100 m. It rolls down a smooth surface to the ground, then climbs up another hill of height 30 m and finally rolls down to a horizontal base at a height of 20 m above the ground. The velocity attained by the ball is **[AIEEE 2005]**

(a) 40 m/s (b) 20 m/s

(c) 10 m/s (d) $10\sqrt{30}$ m/s

Exp. **(a)**

According to conservation of energy, potential energy at height H is sum of kinetic energy and potential energy at h_2.

$\therefore \quad mgH = \frac{1}{2}mv^2 + mgh_2$

$\Rightarrow \quad mg(H - h_2) = \frac{1}{2}mv^2$

or $\quad v = \sqrt{2g(100-20)}$

or $\quad v = \sqrt{2 \times 10 \times 80}$

$= 40$ m/s

34. The block of mass M moving on the frictionless horizontal surface collides with the spring of spring constant k and compresses it by length L. The maximum momentum of the block after collision is **[AIEEE 2005]**

(a) $\sqrt{Mk}L$ (b) $\frac{kL^2}{2M}$ (c) zero (d) $\frac{ML^2}{k}$

Exp. **(a)**

Momentum would be maximum when KE would be maximum and this is the case when total elastic PE is converted into KE.

According to conservation of energy,

$$\frac{1}{2}kL^2 = \frac{1}{2}Mv^2$$

$\Rightarrow \quad kL^2 = \frac{(Mv)^2}{M}$

or $\quad MkL^2 = p^2 \quad [\because p = Mv]$

$\Rightarrow \quad p = L\sqrt{Mk}$

35. A particle moves in a straight line with retardation proportional to its displacement. Its loss of kinetic energy for any displacement x is proportional to **[AIEEE 2004]**

(a) x^2 (b) e^x (c) x (d) $\log_e x$

Exp. **(a)**

From given information $a = -kx$, where a is acceleration, x is displacement and k is a proportionality constant,

$$\frac{vdv}{dx} = -kx$$

$\Rightarrow \quad v\,dv = -kx\,dx$

Let for any displacement from 0 to x, the velocity changes from v_0 to v.

$\Rightarrow \quad \int_{v_0}^{v} v\,dv = -\int_0^x kx\,dx$

$\Rightarrow \quad \left[\frac{v^2}{2}\right]_{v_0}^{v} = -k\left[\frac{x^2}{2}\right]_0^x$

$\Rightarrow \quad \frac{v^2 - v_0^2}{2} = -\frac{kx^2}{2}$

$\Rightarrow \quad m\left(\frac{v^2 - v_0^2}{2}\right) = -\frac{mkx^2}{2}$

$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = -\frac{mkx^2}{2}$

$\Rightarrow \quad \Delta K \propto x^2 \quad$ [ΔK is loss in KE]

36. A uniform chain of length 2 m is kept on a table such that a length of 60 cm hangs freely from the edge of the table. The total mass of the chain is 4 kg. What is the work done in pulling the entire chain on the table? **[AIEEE 2004]**

(a) 7.2 J (b) 3.6 J
(c) 120 J (d) 1200 J

Exp. **(b)**

Mass per unit length $= \frac{M}{L} = \frac{4}{2} = 2$ kg/m

The mass of 0.6 m of chain $= 0.6 \times 2 = 1.2$ kg

The height of centre of mass of hanging part

$$h = \frac{0.6 + 0}{2} = 0.3 \text{ m}$$

Hence, work done in pulling the chain on the table = Work done against gravity force

i.e, $\quad W = mgh$

$= 1.2 \times 10 \times 0.3 = 3.6$ J

37. A force $\mathbf{F} = (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ N is applied over a particle which displaces it from its origin to the point $\mathbf{r} = (2\mathbf{i} - \mathbf{j})$ m. The work done on the particle in joule is **[AIEEE 2004]**

(a) –7 (b) +7
(c) +10 (d) +13

Exp. **(b)**

Work done in displacing the particle

$$W = \mathbf{F} \cdot \mathbf{r}$$
$$= (5\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}) \cdot (2\mathbf{i} - \mathbf{j})$$
$$= 5 \times 2 + 3 \times (-1) + 2 \times 0 = 10 - 3 = 7 \text{ J}$$

38. A spring of spring constant 5×10^3 N/m is stretched initially by 5 cm from the unstretched position. Then, the work required to stretch it further by another 5 cm is **[AIEEE 2003]**

(a) 12.50 N-m (b) 18.75 N-m
(c) 25.00 N-m (d) 6.25 N-m

Exp. **(b)**

Work done to stretch the spring by 5 cm from mean position $= W_1 = \frac{1}{2} k x_1^2$

$$= \frac{1}{2} \times 5 \times 10^3 \times (5 \times 10^{-2})^2$$
$$= 6.25 \text{ J}$$

Work done to stretch the spring by 10 cm from mean position, $W_2 = \frac{1}{2} k(x_1 + x_2)^2$

$$= \frac{1}{2} \times 5 \times 10^3 (5 \times 10^{-2} + 5 \times 10^{-2})^2$$
$$= 25 \text{ J}$$

Net work done to stretch the spring from 5 cm to 10 cm $= W_2 - W_1$

$$= 25 - 6 \cdot 25$$
$$= 18.75 \text{ J} = 18.75 \text{ N-m}$$

39. A body is moved along a straight line by a machine delivering a constant power. The distance moved by the body in time t is proportional to **[AIEEE 2003]**

(a) $t^{3/4}$ (b) $t^{3/2}$
(c) $t^{1/4}$ (d) $t^{1/2}$

Exp. **(b)**

Delivering power of a machine

$$P = \text{constant}$$
$$P = F \cdot u \quad \left[\because \text{Power} = \frac{w}{t} = F \cdot \frac{s}{t} = F \cdot v\right]$$
$$= mav = m\frac{dv}{dt} v$$
$$P = mv\frac{dv}{dt}$$
$$vdv = \frac{P}{m} dt$$

Integrating on both sides, we get

$$\int_0^v v\, dv = \int_0^t \frac{P}{m} dt$$
$$\frac{v^2}{2} = \frac{Pt}{m}$$
$$v = \left(\frac{2Pt}{m}\right)^{1/2}$$
$$v = \frac{ds}{dt} = \left(\frac{2Pt}{m}\right)^{1/2}$$
$$ds = \left(\frac{2Pt}{m}\right)^{1/2} dt$$
$$\int_0^s ds = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2}\, dt$$
$$s = \sqrt{\frac{2P}{m}} \frac{t^{3/2}}{3/2}$$
$$\therefore \quad s \propto t^{3/2}$$

40. A spring of force constant 800 N/m has an extension of 5 cm. The work done in extending it from 5 cm to 15 cm is **[AIEEE 2002]**

(a) 16 J (b) 8 J (c) 32 J (d) 24 J

Exp. **(b)**

The work done on the spring is stored as the PE of the body and is given by

$$U = \int_{x_1}^{x_2} F_{ext}\, dx$$

or

$$U = \int_{x_1}^{x_2} kx\, dx$$
$$= \frac{1}{2} k(x_2^2 - x_1^2)$$
$$= \frac{800}{2} [(0.15)^2 - (0.05)^2]$$
$$= 400(0.2 \times 0.1) = 8 \text{ J}$$

5

Rotational Motion

1. Four particles A, B, C and D with masses $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles (in ms^{-2}) is **[JEE Main 2019, 8 April Shift-I]**

(a) $\frac{a}{5}(\hat{\mathbf{i}} - \hat{\mathbf{j}})$ (b) $a(\hat{\mathbf{i}} + \hat{\mathbf{j}})$ (c) zero (d) $\frac{a}{5}(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Exp. **(a)**

For a system of discrete masses, acceleration of centre of mass (CM) is given by

$$\mathbf{a}_{CM} = \frac{m_A \mathbf{a}_A + m_B \mathbf{a}_B + m_C \mathbf{a}_C + m_D \mathbf{a}_D}{m_A + m_B + m_C + m_D}$$

where, $m_A = m, m_B = 2m, m_C = 3m$ and $m_D = 4m$, $|\mathbf{a}_A| = |\mathbf{a}_B| = |\mathbf{a}_C| = |\mathbf{a}_D| = a$ (according to the question)

$$a_{CM} = \frac{-ma\hat{\mathbf{i}} + 2ma\hat{\mathbf{i}} + 3ma\hat{\mathbf{i}} - 4ma\hat{\mathbf{j}}}{m + 2m + 3m + 4m}$$

$$= \frac{2a\hat{\mathbf{i}} - 2a\hat{\mathbf{j}}}{10} = \frac{a}{5} \cdot (\hat{\mathbf{i}} - \hat{\mathbf{j}})\ ms^{-2}$$

2. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is **[JEE Main 2019, 8 April Shift-I]**

(a) $\frac{1}{2}$ (b) $\frac{3}{5}$ (c) $\frac{8}{5}$ (d) $\frac{3}{2}$

Exp. **(*)**

Consider an elementary ring of thickness dx and radius x.

Moment of inertia of this ring about a perpendicular axes through centre is

$$dI_c = dm \cdot x^2 = \rho_0 x(2\pi x)dx \cdot x^2 = 2\pi\rho_0 x^4 dx$$

Moment of inertia of this elementary ring about a perpendicular axes at a point through edge, (by parallel axes theorem)

$$dI = dmx^2 + dmR^2$$
$$= 2\pi\rho_0 x^4 dx + 2\pi\rho_0 R^2 x^2 dx$$

Moment of inertia of complete disc is

$$I = \int_0^R dI = \int_0^R 2\pi\rho_0 x^4 dx + \int_0^R 2\pi\rho_0 R^2 x^2 dx$$
$$= \frac{2\pi\rho_0 R^5}{5} + \frac{2\pi\rho_0 R^5}{3} = \frac{16\pi\rho_0 R^5}{15}$$
$$\therefore \quad a = \frac{16}{15}$$

3. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by

[JEE Main 2019, 8 April Shift-II]

(a) $\frac{2}{\sqrt{5}}$ (b) $\frac{14}{15}$ (c) 1 (d) $\frac{4}{5}$

Exp. **(b)**

When a spherical/circular body of radius r rolls without slipping, its total kinetic energy is

$$K_{total} = K_{translation} + K_{rotation}$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$
$$= \frac{1}{2}mv^2 + \frac{1}{2}I \cdot \frac{v^2}{r^2} \qquad \left[\because \omega = \frac{v}{r}\right]$$

Let v be the linear velocity and R be the radius for both solid sphere and solid cylinder.

$\therefore$ Kinetic energy of the given solid sphere will be

$$K_{sph} = \frac{1}{2}mv^2 + \frac{1}{2}I_{sph}\frac{v^2}{R^2}$$
$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}mR^2 \times \frac{v^2}{R^2}$$
$$= \frac{7}{10}mv^2 \qquad \ldots(i)$$

Similarly, kinetic energy of the given solid cylinder will be $K_{cyl} = \frac{1}{2}mv^2 + \frac{1}{2}I_{cyl}\frac{v^2}{R}$

$$= \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{mR^2}{2} \times \frac{v^2}{R^2} = \frac{3}{4}mv^2 \ldots(ii)$$

Now, from the conservation of mechanical energy,

$$mgh = K_{total}$$

$\therefore$ For solid sphere,

$$mgh_{sph} = \frac{7}{10}mv^2 \quad \ldots(iii) \text{ [using Eq. (i)]}$$

Similarly, for solid cylinder,

$$mgh_{cyl} = \frac{3}{4}mv^2 \quad \ldots(iv) \text{ [using Eq. (ii)]}$$

Taking the ratio of Eqs. (iii) and (iv), we get

$$\frac{mgh_{sph}}{mgh_{cyl}} = \frac{\frac{7}{10}mv^2}{\frac{3}{4}mv^2} \Rightarrow \frac{h_{sph}}{h_{cyl}} = \frac{7}{10} \times \frac{4}{3} = \frac{14}{15}$$

Alternate Solution

Total kinetic energy for a rolling body without slipping can also be given as

$$K_{total} = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

$\therefore$ From law of conservation,

$$mgh = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

or $$h \propto \left(1 + \frac{K^2}{R^2}\right)$$

As we know that, for solid sphere,

$$K = \sqrt{\frac{2}{5}}R$$

$$\Rightarrow \quad \frac{K^2}{R^2} = \frac{2}{5}$$

Similarly, for solid cylinder,

$$K = \frac{R}{\sqrt{2}} \Rightarrow \frac{K^2}{R^2} = \frac{1}{2}$$

So, $$\frac{h_{sph}}{h_{cyl}} = \frac{1 + \frac{2}{5}}{1 + \frac{1}{2}} = \frac{\frac{7}{5}}{\frac{3}{2}} = \frac{7}{5} \times \frac{2}{3} = \frac{14}{15}$$

4. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5 m. When released, it slips off the table in a very short time $\tau = 0.01$ s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to **[JEE Main 2019, 8 April Shift-II]**

(a) 0.02 (b) 0.3 (c) 0.5 (d) 0.28

Exp. **(c)**

Key Idea The rectangular box rotates due to torque of weight about its centre of mass.

Now, angular impulse of weight = Change in angular momentum.

$$\therefore \quad mg\frac{l}{2} \times \tau = \frac{ml^2}{3}\omega \Rightarrow \omega = \frac{3g \times \tau}{2 \times l}$$

Substituting the given values, we get

$$= \frac{3 \times 10 \times 0.01}{2 \times 0.3} = 0.5 \text{ rad s}^{-1}$$

Time of fall of box,

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 5}{10}} \approx 1 \text{ s}$$

So, angle turned by box in reaching ground is $\theta = \omega t = 0.5 \times 1 = 0.5$ rad

5. A uniform rectangular thin sheet *ABCD* of mass *M* has length *a* and breadth *b*, as shown in the figure. If the shaded portion *HBGO* is cut-off, the coordinates of the centre of mass of the remaining portion will be **[JEE Main 2019, 8 April Shift-II]**

(a) $\left(\frac{2a}{3}, \frac{2b}{3}\right)$ (b) $\left(\frac{5a}{12}, \frac{5b}{12}\right)$

(c) $\left(\frac{3a}{4}, \frac{3b}{4}\right)$ (d) $\left(\frac{5a}{3}, \frac{5b}{3}\right)$

Exp. **(b)**

The given rectangular thin sheet *ABCD* can be drawn as shown in the figure below,

Here,

Area of complete lamina, $A_1 = ab$

Area of shaded part of lamina $= \frac{a}{2} \times \frac{b}{2} = \frac{ab}{4}$

(x_1, y_1) = coordinates of centre of mass of complete lamina $= \left(\frac{a}{2}, \frac{a}{2}\right)$

(x_2, y_2) = coordinates of centre of mass of shaded part of lamina $= \left(\frac{3a}{4}, \frac{3a}{4}\right)$

$\therefore$ Using formula for centre of mass, we have

$$X_{CM} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{a}{2}\right) - \frac{ab}{4}\left(\frac{3a}{4}\right)}{ab - \frac{ab}{4}} = \frac{\frac{8a^2b - 3a^2b}{16}}{\frac{3ab}{4}} = \frac{5a}{12}$$

Similarly, $$Y_{CM} = \frac{A_1y_1 - A_2y_2}{A_1 - A_2}$$

$$= \frac{ab\left(\frac{b}{2}\right) - \frac{ab}{4}\left(\frac{3b}{4}\right)}{ab - \frac{ab}{4}} = \frac{5b}{12}$$

Alternate Solution

Let *m* be the mass of entire rectangular lamina.

So, the mass of the shaded portion of lamina $= \frac{m}{4}$

Using the relation,

$$X_{CM} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}, \text{ we get}$$

$$X_{CM} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3a}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

Similarly, $Y_{CM} = \frac{m_1y_1 - m_2y_2}{m_1 - m_2}$, we get

$$Y_{CM} = \frac{m\left(\frac{a}{2}\right) - \frac{m}{4}\left(\frac{3b}{4}\right)}{m - \frac{m}{4}} = \frac{\frac{b}{2} - \frac{3b}{16}}{\frac{3}{4}} = \frac{5b}{12}$$

$\therefore$ The coordinates of the centre of mass of remaining portion will be $\left(\frac{5a}{12}, \frac{5b}{12}\right)$.

6. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I, then the angular acceleration of the disc is

[JEE Main 2019, 9 April Shift-I]

(a) $\frac{k}{2I}\theta$ (b) $\frac{k}{I}\theta$

(c) $\frac{k}{4I}\theta$ (d) $\frac{2k}{I}\theta$

Exp. **(d)**

Given, kinetic energy $= k\theta^2$

We know that, kinetic energy of a rotating body about its axis $= \frac{1}{2}I\omega^2$

where, I is moment of inertia and ω is angular velocity.

So, $$\frac{1}{2}I\omega^2 = k\theta^2$$

or $$\omega^2 = \frac{2k\theta^2}{I}$$

$\Rightarrow$ $$\omega = \sqrt{\frac{2k}{I}}\theta \quad \ldots \text{(i)}$$

Differentiating the above equation w.r.t. time on both sides, we get

$$\frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \frac{d\theta}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega \qquad \left[\because \omega = \frac{d\theta}{dt}\right]$$

$\therefore$ Angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \sqrt{\frac{2k}{I}} \cdot \omega = \sqrt{\frac{2k}{I}} \cdot \sqrt{\frac{2k}{I}}\theta \text{ [using Eq. (i)]}$$

or $\alpha = \frac{2k}{I}\theta$

Alternate Solution

As, $$\omega^2 = \frac{2k\theta^2}{I}$$

$\Rightarrow$ $$2\omega\frac{d\omega}{dt} = \frac{2k}{I} \cdot 2\theta\frac{d\theta}{dt}$$

or $$\omega\, d\omega = \frac{2k}{I}\theta d\theta$$

$$\omega\frac{d\omega}{d\theta} (= \alpha) = \frac{2k}{I} \cdot \theta$$

or $$\alpha = \frac{2k}{I}\theta$$

7. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane : (i) a ring of radius R, (ii) a solid cylinder of radius $R/2$ and (iii) a solid sphere of radius $R/4$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum height they climb is

[JEE Main 2019, 9 April Shift-I]

(a) 10:15:7 (b) 4:3:2

(c) 14:15:20 (d) 2:3:4

Exp. **(c)**

From question,

let height attained by ring $= h_1$

Height attained by cylinder $= h_2$

Height attained by sphere $= h_3$

As we know that for a body which is rolling up an inclined plane (without slipping), follows the law of conservation of energy.

$\therefore$ For ring, using energy conservation law at its height h_1.

$$(\text{KE})_{\text{linear}} + (\text{KE})_{\text{rotational}} = (\text{PE})$$

$\Rightarrow$ $$\frac{1}{2}m_1v_0^2 + \frac{1}{2}I_1\omega^2 = m_1gh_1$$

$\Rightarrow$ $$\frac{1}{2}m_1v_0^2 + \frac{1}{2}m_1R^2\omega^2 = m_1gh_1$$

$(\because I = mR^2$ for ring)

$\Rightarrow$ $$gh_1 = \frac{v_0^2}{2} + \frac{v_0^2}{2} \qquad (\because v_0 = \omega R)$$

$\Rightarrow$ $$h_1 = v_0^2/g \quad \ldots \text{(i)}$$

Similarly, for solid cylinder, applying the law of conservation of energy,

$$\frac{1}{2}m_2v_0^2 + \frac{1}{2}I_2\omega^2 = m_2gh_2$$

$\Rightarrow$ $$\frac{1}{2}m_2v_0^2 + \frac{1}{2}\left(\frac{1}{2} \times m_2 \times \left(\frac{R}{2}\right)^2\right)\omega^2 = m_2gh_2$$

$$\left[\because I = \frac{1}{2}mR^2 \text{ for cylinder and } R = \frac{R}{2}\right]$$

$\Rightarrow$

$$\frac{1}{2}m_2v_0^2 + \frac{1}{2} \times \frac{1}{8}m_2R^2 \times \frac{v_0^2}{(R/2)^2} = m_2gh_2$$

$\Rightarrow$ $$\frac{1}{2}v_0^2 + \frac{1}{4}v_0^2 = gh_2$$

$$\Rightarrow \quad gh_2 = \frac{3}{4}v_0^2$$

$$\Rightarrow \quad h_2 = \frac{3}{4}\left(\frac{v_0^2}{g}\right) \quad \ldots\text{(ii)}$$

Similarly, for solid sphere,

$$\frac{1}{2}m_3v_0^2 + \frac{1}{2}I_3\omega^2 = m_3gh_3$$

$$\Rightarrow \frac{1}{2}m_3v_0^2 + \frac{1}{2}\left[\frac{2}{5}m_3\left(\frac{R}{4}\right)^2\right]\omega^2 = m_3gh_3$$

$$\left[\because I = \frac{2}{5}mR^2 \text{ for solid sphere and } R = \frac{R}{4}\right]$$

$$\Rightarrow \frac{1}{2}m_3v_0^2 + \frac{1}{2}\times\frac{2}{5}\times m_3\frac{R^2}{16}\times\frac{v_0^2}{(R/4)^2} = m_3gh_3$$

$$\Rightarrow \quad \frac{1}{2}v_0^2 + \frac{1}{5}v_0^2 = gh_3$$

$$\Rightarrow \quad gh_3 = \frac{7}{10}v_0^2$$

or $$h_3 = \frac{7}{10}\left(\frac{v_0^2}{g}\right) \quad \ldots\text{(iii)}$$

$\therefore$ Taking the ratio of h_1, h_2 and h_3 by using Eqs. (i), (ii) and (iii), we get

$$h_1 : h_2 : h_3 = \frac{v_0^2}{g} : \frac{3}{4}\frac{v_0^2}{g} : \frac{7}{10}\frac{v_0^2}{g} = 1 : \frac{3}{4} : \frac{7}{10}$$

$$\Rightarrow \quad h_1 : h_2 : h_3 = 40 : 30 : 28 = 20 : 15 : 14$$

$\therefore$ The most appropriate option is (c).

Although, it is still not in the correct sequence.

Alternate Solution

Total kinetic energy of a rolling body is also given as

$$E_{\text{total}} = \frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$$

where, K is the radius of gyration.

Using conservation law of energy,

$$\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right] = mgh \text{ or } h = \frac{v^2}{2g}\left[1 + \frac{K^2}{R^2}\right]$$

For ring, $\frac{K^2}{R^2} = 1$

$$\Rightarrow \quad h_1 = \frac{v^2}{2g}[1 + 1] = \frac{2v^2}{2g} = \frac{v^2}{g}$$

For solid cylinder, $\frac{K^2}{R^2} = \frac{(R/2\sqrt{2})^2}{(R/2)^2}$

$$= \frac{R^2}{8}\times\frac{4}{R^2} = \frac{1}{2}$$

$$\Rightarrow \quad h_2 = \frac{v^2}{2g}\left[1 + \frac{1}{2}\right] = \frac{3v^2}{4g}$$

For solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

$$\Rightarrow \quad h_3 = \frac{v^2}{2g}\left[1 + \frac{2}{5}\right] = \frac{7v^2}{10g}$$

So, the ratio of h_1, h_2 and h_3 is

$$h_1 : h_2 : h_3 = \frac{v^2}{g} : \frac{3v^2}{4g} : \frac{7}{10}\frac{v^2}{g}$$

$$= 1 : \frac{3}{4} : \frac{7}{10} = 20 : 15 : 14$$

8. A wedge of mass $M = 4$ m lies on a frictionless plane. A particle of mass m aproaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by

[JEE Main 2019, 9 April Shift-II]

(a) $\frac{2v^2}{7g}$ (b) $\frac{v^2}{g}$ (c) $\frac{2v^2}{5g}$ (d) $\frac{v^2}{2g}$

Exp. **(c)**

> **Key Idea** Since, the ground is frictionless, so when the particle will collide and climb over the wedge, then the wedge will also move. Thus, by using conservation laws for momentum and energy, maximum height climbed by the particle can be calculated.

Initial condition can be shown in the figure below

As mass m collides with wedge, let both wedge and mass move with speed v'. Then,

By applying linear momentum conservation, we have Initial momentum of the system = Final momentum of the system

$$mv + 0 = (m + 4m)v'$$

$$\Rightarrow \quad v' = \frac{v}{5} \quad \ldots\text{(i)}$$

Now, if m rises upto height h over wedge, then by applying conservation of mechanical energy, we have Initial energy of the system = Final energy of the system

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}mv'^2 + mgh + \frac{1}{2}(4m)v'^2$$

$$mv^2 = (m + 4m)v'^2 + 2mgh$$

$$\Rightarrow \quad v^2 = 5v'^2 + 2gh$$

$$\Rightarrow \quad v^2 = \frac{1}{5}v^2 + 2gh \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad \frac{4}{5}v^2 = 2gh \Rightarrow h = \frac{2v^2}{5g}$$

9. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its centre. Two beads of mass m and negligible size are at the centre of the rod initially. The beads are free to slide along the rod. The angular speed of the system, when the beads reach the opposite ends of the rod, will be **[JEE Main 2019, 9 April Shift-II]**

(a) $\frac{M\omega_0}{M+3m}$ (b) $\frac{M\omega_0}{M+m}$

(c) $\frac{M\omega_0}{M+2m}$ (d) $\frac{M\omega_0}{M+6m}$

Exp. **(d)**

As there is no external torque on system.

$\therefore$ Angular momentum of system is conserved.

$$\Rightarrow \quad I_i\omega_i = I_f\omega_f$$

Initially,

Finally,

$$\Rightarrow \quad \frac{ML^2}{12}\cdot\omega_0 + 0 = \left(\frac{ML^2}{12} + 2(m)\left(\frac{L}{2}\right)^2\right)\omega$$

So, final angular speed of system is

$$\Rightarrow \quad \omega = \frac{\frac{ML^2}{12}\cdot\omega_0}{\left(\frac{ML^2 + 6mL^2}{12}\right)} = \frac{M\omega_0}{M + 6m}$$

10. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular acceleration of 20 rad/s^2 must be applied about the axis for a duration of **[JEE Main 2019, 9 April Shift-II]**

(a) 5 s (b) 2 s

(c) 3 s (d) 2.5 s

Exp. **(b)**

Rotation kinetic energy of a body is given by

$$KE_{rotational} = \frac{1}{2}I\omega^2$$

where, $\omega = \omega_0 + \alpha t$

So, $$KE_{rotational} = \frac{1}{2}I(\omega_0 + \alpha t)^2 \quad \ldots \text{(i)}$$

Here, $I = 1.5$ kgm^2,

KE = 1200 J and

$\alpha = 20$ rad/s^2 and $\omega_0 = 0$

Substituting these values in Eq. (i), we get

$$1200 = \frac{1}{2}(1.5)(20 \times t)^2$$

$$\Rightarrow \quad t^2 = \frac{2 \times 1200}{1.5 \times 400} = 4$$

$$\therefore \quad t = 2 \text{ s}$$

11. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$ are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is **[JEE Main 2019, 10 April Shift-I]**

(a) $-\frac{I_1\omega_1^2}{24}$ (b) $-\frac{I_1\omega_1^2}{12}$

(c) $\frac{3}{8}I_1\omega_1^2$ (d) $\frac{I_1\omega_1^2}{6}$

Exp. **(a)**

Initial kinetic energy of the given system,

$$KE_i = \frac{1}{2} I_1\omega_1^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right)^2$$

$$= \left(\frac{1}{2} + \frac{1}{16}\right) I_1\omega_1^2 = \frac{9}{16} I_1\omega_1^2 \quad \ldots \text{(i)}$$

Now, using angular momentum conservation law (assuming angular velocity after contact is ω)

Initial angular momentum = Final angular momentum

$$I_1\omega_1 + \left(\frac{I_1}{2}\right)\left(\frac{\omega_1}{2}\right) = I_1\omega' + \frac{I_1}{2}\omega'$$

$$\Rightarrow \quad \frac{5}{4}\omega_1 = \frac{3}{2}\omega'$$

or $$\omega' = \frac{5}{6}\omega_1 \quad \ldots \text{(ii)}$$

Now, final kinetic energy (after contact) is

$$KE_f = \frac{1}{2} I_1\omega'^2 + \frac{1}{2}\left(\frac{I_1}{2}\right)\omega'^2$$

$$= \frac{1}{2} I_1 \left(\frac{5}{6}\omega_1\right)^2 + \frac{1}{4} I_1 \left(\frac{5}{6}\omega_1\right)^2 \quad \text{[using Eq. (ii)]}$$

$$= \left(\frac{25}{72} + \frac{25}{144}\right) I_1\omega_1^2 = \frac{25}{48} I_1\omega_1^2 \quad \ldots \text{(iii)}$$

Hence, change in KE,

$$\Delta KE = KE_f - KE_i$$

$$= \frac{25}{48} I_1\omega_1^2 - \frac{9}{16} I_1\omega_1^2 \quad \text{[using Eqs. (i)}$$

$$\Delta KE = -\frac{1}{24} I_1\omega_1^2$$

12. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$, where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is **[JEE Main 2019, 10 April Shift-I]**

(a) $\frac{MR^2}{2}$ (b) $\frac{MR^2}{6}$

(c) $\frac{MR^2}{3}$ (d) $\frac{2MR^2}{3}$

Exp. **(d)**

Given, surface mass density,

$$\sigma = kr^2$$

So, mass of the disc can be calculated by considering small element of area $2\pi r dr$ on it and then integrating it for complete disc, i.e.

$$dm = \sigma\, dA = \sigma \times 2\pi r dr$$

$$\int dm = M = \int_0^R (kr^2)\, 2\pi r dr$$

$$\Rightarrow \quad M = 2\pi k \frac{R^4}{4} = \frac{1}{2}\pi k R^4 \quad \ldots \text{(i)}$$

Moment of inertia about the axis of the disc,

$$I = \int dI = \int dmr^2 = \int \sigma dA r^2$$

$$= \int_0^R kr^2\, (2\pi r dr)\, r^2$$

$$\Rightarrow \quad I = 2\pi k \int_0^R r^5\, dr = \frac{2\pi k R^6}{6} = \frac{\pi k R^6}{3} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$I = \frac{2}{3} MR^2$$

13. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$ and $y = y_0 + b\sin\omega_2 t$.

The torque acting on the particle about the origin at $t = 0$ is **[JEE Main 2019, 10 April Shift-I]**

(a) zero

(b) $m(-x_0 b + y_0 a)\omega_1^2\, \hat{\mathbf{k}}$

(c) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\, \hat{\mathbf{k}}$

(d) $+my_0 a\,\omega_1^2\, \hat{\mathbf{k}}$

Exp. **(d)**

Given equations of trajectory are

$$x = x_0 + a\cos\omega_1 t$$

$$y = y_0 + b\sin\omega_2 t$$

Now, $v_x = dx/dt = -a\omega_1 \sin\omega_1 t$

and

$$v_y = \frac{dy}{dt} = b\omega_2 \cos\omega_2 t$$

Similarly,

$$\Rightarrow \quad a_x = \frac{dv_x}{dt} = -a\omega_1^2 \cos(\omega_1 t)$$

Similarly, $a_y = \frac{dv_y}{dt} = -b\omega_2^2 \sin(\omega_2 t)$

Now, at $t = 0$; $x = x_0 + a$; $y = y_0$ and

$$a_x = -a\omega_1^2;\ a_y = 0 \quad \ldots \text{(i)}$$

Torque acting on the particle is given as

$$\tau = \mathbf{r} \times \mathbf{F} = m(\mathbf{r} \times \mathbf{a})$$

Here, $\mathbf{r} = (x_0 + a)\hat{\mathbf{i}} + y_0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$

and $\mathbf{a} = -a\omega_1^2\hat{\mathbf{i}} + 0\hat{\mathbf{j}} + 0\hat{\mathbf{k}}$

So, torque at $t = 0$,

$$\tau = m(-a\omega_1^2) \times y_0(-\hat{\mathbf{k}})$$

[using Eq. (i)]

$$\Rightarrow \quad \tau = +my_0a\omega_1^2\hat{\mathbf{k}}$$

Alternate Solution

We can calculate total force from Eq. (i),

$\mathbf{F} = -m(a\omega_1^2 \cos\omega_1 t\ \hat{\mathbf{i}} + b\omega_2^2 \sin\omega_2 t\ \hat{\mathbf{j}})$

and $\mathbf{r} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = (x_0 + a\cos\omega_1 t)\hat{\mathbf{i}} + (y_0 + b\sin\omega_2 t)\hat{\mathbf{j}}$

Now, torque, $\tau = \mathbf{r} \times \mathbf{F}$

$= [(x_0 + a\cos\omega_1 t)\hat{\mathbf{i}} + (y_0 + b\sin\omega_2 t)\hat{\mathbf{j}}] \times [-m(a\omega_1^2 \cos\omega_1 t\hat{\mathbf{i}} + b\omega_2^2 \sin\omega_2 t\ \hat{\mathbf{j}})]$

$$= -m\begin{bmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x_0 + a\cos\omega_1 t & y_0 + b\sin\omega_2 t & 0 \\ a\omega_1^2\cos\omega_1 t & b\omega_2^2 \sin\omega_2 t & 0 \end{bmatrix}$$

$$= -m[x_0\, b\omega_2^2 \sin\omega_2 t + ab\omega^2 \sin\omega_2 t \cos\omega_1 t - y_0\, a\,\omega_1^2 \cos\omega_1 t - ab\,\omega_1^2 \sin\omega_2 t \cos\omega_1 t]\hat{\mathbf{k}}$$

On solving it at $t = 0$, we get

$$\tau = ma\,\omega_1^2\, y_0\,\hat{\mathbf{k}}$$

14. The time dependence of the position of a particle of mass $m = 2$ is given by $\mathbf{r}(t) = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$. Its angular momentum, with respect to the origin, at time $t = 2$ is

[JEE Main 2019, 10 April Shift-II]

(a) $36\hat{\mathbf{k}}$ (b) $-34(\hat{\mathbf{k}} - \hat{\mathbf{i}})$

(c) $-48\hat{\mathbf{k}}$ (d) $48(\hat{\mathbf{i}} + \hat{\mathbf{j}})$

Exp. **(c)**

Position of particle is, $\mathbf{r} = 2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}$

where, t is instantaneous time.

Velocity of particle is

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = 2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}}$$

Now, angular momentum of particle with respect to origin is given by

$$\mathbf{L} = m(\mathbf{r} \times \mathbf{v})$$

$$= m\{(2t\hat{\mathbf{i}} - 3t^2\hat{\mathbf{j}}) \times (2\hat{\mathbf{i}} - 6t\hat{\mathbf{j}})\}$$

$$= m(-12t^2(\hat{\mathbf{i}} \times \hat{\mathbf{j}}) - 6t^2(\hat{\mathbf{j}} \times \hat{\mathbf{i}}))$$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = 0$

$$\Rightarrow \quad \mathbf{L} = m(-12t^2\hat{\mathbf{k}} + 6t^2\hat{\mathbf{k}})$$

As, $\hat{\mathbf{i}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}}$ and $\hat{\mathbf{j}} \times \hat{\mathbf{i}} = -\hat{\mathbf{k}}$

$$\Rightarrow \quad \mathbf{L} = -m(6t^2)\hat{\mathbf{k}}$$

So, angular momentum of particle of mass 2 kg at time $t = 2$ s is

$$\mathbf{L} = (-2 \times 6 \times 2^2)\hat{\mathbf{k}} = -48\,\hat{\mathbf{k}}$$

15. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius $2R$. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1 / I_2 is given by **[JEE Main 2019, 10 April Shift-II]**

(a) 285 (b) 185 (c) 65 (d) 140

Exp. **(d)**

The given situation is shown in the figure given below

Density of given sphere of radius R is

$$\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{M}{\frac{4}{3}\pi R^3}$$

Let radius of sphere formed from second part is r, then

mass of second part = volume × density

$$\frac{1}{8}M = \frac{4}{3}\pi r^3 \times \frac{M}{\frac{4}{3}\pi R^3}$$

$$\therefore \quad r^3 = \frac{R^3}{8} \Rightarrow r = \frac{R}{2}$$

Now, I_1 = moment of inertia of disc (radius $2R$ and mass $\frac{7}{8}M$) about its axis

$$= \frac{\text{Mass} \times (\text{Radius})^2}{2} = \frac{\frac{7}{8}M \times (2R)^2}{2} = \frac{7}{4}MR^2$$

and I_2 = moment of inertia of sphere (radius $\frac{R}{2}$ and mass $\frac{1}{8}M$) about its axis

$$= \frac{2}{5} \times \text{Mass} \times (\text{Radius})^2$$

$$= \frac{2}{5} \times \frac{1}{8}M \times \left(\frac{R}{2}\right)^2 = \frac{MR^2}{80}$$

$$\therefore \text{ Ratio } \frac{I_1}{I_2} = \frac{\frac{7}{4}MR^2}{\frac{1}{80}MR^2} = 140$$

16. A metal coin of mass 5g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5s, is close to

[JEE Main 2019, 10 April Shift-II]

(a) 4.0×10^{-6} N-m (b) 2.0×10^{-5} N-m

(c) 1.6×10^{-5} N-m (d) 7.9×10^{-6} N-m

Exp. **(b)**

Moment of inertia (MI) of a disc about a tangential axis in the plane of disc can be obtained as below.

Moment of inertia of disc about it's axis,

$$I_1 = \frac{MR^2}{2}$$

From perpendicular axes theorem, moment of inertia of disc about an axis along it's diameter is

$$I_x + I_y = I_z \Rightarrow 2I_2 = I_1$$

$$\Rightarrow \quad I_2 = \frac{I_1}{2} = \frac{MR^2}{4}$$

So, moment of inertia about a tangential axis from parallel axes theorem is

$$I = I_2 + MR^2 = \frac{MR^2}{4} + MR^2$$

$$= \frac{5}{4}MR^2$$

Now, using torque, $\tau = I\alpha$, we have

$$\tau = I\alpha = \frac{5}{4}MR^2\left(\frac{\omega_f - \omega_i}{\Delta t}\right)$$

Here,

$M = 5 \times 10^{-3}$ kg, $R = 1 \times 10^{-2}$ m

$\omega_f = 25 \text{ rps} = 25 \times 2\pi \frac{\text{rad}}{\text{s}} = 50\pi \frac{\text{rad}}{\text{s}}$,

$\omega_i = 0$, $\Delta t = 5$ s

$$\text{So, } \tau = \frac{\frac{5}{4} \times 5 \times 10^{-3} \times (10^{-2})^2 \times 50\pi}{5}$$

$$\approx 2 \times 10^{-5} \text{ N-m}$$

17. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the

radius of gyration of the disc about its axis passing through the centre is

[JEE Main 2019, 12 April Shift-I]

(a) $\sqrt{\dfrac{a^2+b^2+ab}{2}}$ (b) $\dfrac{a+b}{2}$

(c) $\sqrt{\dfrac{a^2+b^2+ab}{3}}$ (d) $\dfrac{a+b}{3}$

Exp. **(c)**

Key Idea Radius of gyration K of any structure is given by

$$I = MK^2 \text{ or } K = \sqrt{\frac{I}{M}}$$

To find K, we need to find both moment of inertia I and mass M of the given structure.

Given, variation in mass per unit area (surface mass density),

$$\sigma = \frac{\sigma_0}{r} \quad \ldots(\text{i})$$

Calculation of Mass of Disc

Let us divide whole disc in small area elements, one of them shown at r distance from the centre of the disc with its width as dr.

Mass of this element is

$$dm = \sigma \cdot dA$$

$$\Rightarrow \quad dm = \frac{\sigma_0}{r} \times 2\pi r dr \quad \text{[from Eq. (i)]} \quad \ldots(\text{ii})$$

Mass of the disc can be calculated by integrating it over the given limits of r,

$$\int_0^M dm = \int_a^b \sigma_0 \times 2\pi \times dr$$

$$\Rightarrow \quad M = \sigma_0 2\pi(b-a) \quad \ldots(\text{iii})$$

Calculation of Moment of Inertia

$$I = \int_0^M r^2 dm = \int_a^b r^2 \cdot \frac{\sigma_0}{r} \times 2\pi r dr \quad \text{[from Eq. (i)]}$$

$$= \sigma_0 2\pi \int_a^b r^2 dr = \sigma_0 2\pi \left[\frac{r^3}{3}\right]_a^b$$

$$\Rightarrow \quad I = \frac{1}{3}\sigma_0 2\pi [b^3 - a^3] \quad \ldots(\text{iv})$$

Now, radius of gyration,

$$K = \sqrt{\frac{I}{M}} = \sqrt{\frac{\frac{2\pi\sigma_0}{3}(b^3-a^3)}{2\pi\sigma_0(b-a)}}$$

$$\Rightarrow \quad K = \sqrt{\frac{1}{3}\frac{(b^3-a^3)}{b-a}}$$

As we know, $b^3 - a^3 = (b-a)(b^2+a^2+ab)$

$$\therefore K = \sqrt{\frac{1}{3}(b^2+a^2+ab)} \text{ or } K = \sqrt{\frac{a^2+b^2+ab}{3}}$$

18. A uniform rod of length l is being rotated in a horizontal plane with a constant angular speed about an axis passing through one of its ends. If the tension generated in the rod due to rotation is $T(x)$ at a distance x from the axis, then which of the following graphs depicts it most closely?

[JEE Main 2019, 12 April Shift-I]

Key Idea In a uniform rod, mass per unit length remains constant. If it is denoted by λ, then $\lambda = \dfrac{m}{l}$ = constant for all segments of rod.

To find tension at x distance from fixed end, let us assume an element of dx length and dm mass. Tension on this part due to rotation is

$$dT = Kx \quad \ldots(i)$$

As, $K = m\omega^2$

For this element, $K = (dm)\omega^2 \quad \ldots(ii)$

$$\therefore \quad dT = (dm)\omega^2 x \quad \ldots(iii)$$

To find complete tension in the rod, we need to integrate Eq. (iii),

$$\int_0^T dT = \int_0^m (dm)\,\omega^2 x \quad \ldots(iv)$$

Using linear mass density,

$$\lambda = \frac{m}{l} = \frac{dm}{dx}$$

$$\Rightarrow \quad dm = \frac{m}{l}\cdot dx \quad \ldots(v)$$

Putting the value of Eq. (v) in Eq. (iv), we get

$$T = \int_x^l \frac{m}{l}\cdot\omega^2 x\cdot dx = \frac{m}{l}\cdot\omega^2\left[\frac{x^2}{2}\right]_x^l$$

$$\Rightarrow \quad T = \frac{m\omega^2}{2l}[l^2 - x^2] \text{ or } T \propto -x^2$$

19. A person of mass M is sitting on a swing to length L and swinging with an angular amplitude θ_0. If the person stands up when the swing passes through its lowest point, the work done by him, assuming that his centre of mass moves by a distance $l (l << L)$, is close to **[JEE Main 2019, 12 April Shift-I]**

(a) $Mgl(1-\theta_0^2)$ (b) $Mgl(1+\theta_0^2)$

(c) Mgl (d) $Mgl\left(1+\frac{\theta_0^2}{2}\right)$

Exp. **(b)**

Initially, centre of mass is at distance L from the top end of the swing. It shifts to $(L - l)$ distance when the person stands up on the swing.

$\therefore$ Using angular momentum conservation law, if v_0 and v_1 are the velocities before standing and after standing of the person, then

$$Mv_0L = Mv_1(L-l)$$

$$\Rightarrow \quad v_1 = \left(\frac{L}{L-l}\right)v_0 \quad \ldots(i)$$

Now, total work done by (person + gravitation) system will be equal to the change in kinetic energy of the person, i.e.

$$W_g + W_p = KE_1 - KE_0$$

$$\Rightarrow \quad -Mgl + W_p = \frac{1}{2}Mv_1^2 - \frac{1}{2}Mv_0^2$$

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}M(v_1^2 - v_0^2)$$

$$= Mgl + \frac{1}{2}M\left[\left(\frac{L}{L-l}\right)^2 v_0^2 - v_0^2\right] \quad \text{[from Eq. (i)]}$$

$$= Mgl + \frac{1}{2}M\,v_0^2\left[\left(\frac{L-l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1-\frac{l}{L}\right)^{-2} - 1\right]$$

$$= Mgl + \frac{1}{2}Mv_0^2\left[\left(1+\frac{2l}{L}\right) - 1\right]$$

[using $(1+x)^n = 1 + nx$ as higher terms can be neglected, if $n << 1$]

$$\Rightarrow \quad W_p = Mgl + \frac{1}{2}Mv_0^2 \times \frac{2l}{L}$$

$$\text{or} \quad W_p = Mgl + Mv_0^2\,\frac{l}{L} \quad \ldots(ii)$$

Here, $v_0 = WA = \left(\sqrt{\frac{g}{L}}\right)(\theta_0 L)$

$$\Rightarrow \quad v_0 = \theta_0\sqrt{gL}$$

$\therefore$ Using this value of v_0 in Eq.(ii), we get

$$W_p = Mgl + M\theta_0^2\,gL\cdot\frac{l}{L}$$

$$\Rightarrow \quad W_p = Mgl\,[1+\theta_0^2]$$

20. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be **[JEE Main 2019, 12 April Shift-II]**

(a) $\left(\frac{\sqrt{3}}{4}\text{ m}, \frac{5}{12}\text{ m}\right)$ (b) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{8}\text{ m}\right)$

(c) $\left(\frac{7}{12}\text{ m}, \frac{\sqrt{3}}{4}\text{ m}\right)$ (d) $\left(\frac{\sqrt{3}}{8}\text{ m}, \frac{7}{12}\text{ m}\right)$

Exp. **(c)**

The height of equilateral Δ is

$$h = y_3 = \sqrt{(1)^2 - (0.5)^2} = \sqrt{3}/2\,\text{m}$$

Thus, coordinates of three masses are (0, 0), (1, 0) and $\left(0.5, \frac{\sqrt{3}}{2}\right)$

Using,

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3},$$

we have

$$X_{CM} = \frac{50 \times 0 + 100 \times 1 + 150 \times 0.5}{50 + 100 + 150}$$

$$= \frac{175}{300} = \frac{7}{12}\,\text{m}$$

Similarly,

$$Y_{CM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

$$= \frac{50 \times 0 + 100 \times 0 + 150 \times \frac{\sqrt{3}}{2}}{50 + 100 + 150} = \frac{\sqrt{3}}{4}\,\text{m}$$

21. A smooth wire of length $2\pi r$ is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then, the value of ω^2 is equal to

[JEE Main 2019, 12 April Shift-II]

(a) $\frac{\sqrt{3}g}{2r}$ (b) $2g/(r\sqrt{3})$

(c) $(g\sqrt{3})/r$ (d) $2g/r$

Exp. **(b)**

Key Idea For revolution in a circular path, there should be a force which balances the necessary centripetal force.

Let N = normal reaction of wire loop acting towards centre.

Then, component $N\cos\theta$ balances weight of bead,

$$\Rightarrow \quad N\cos\theta = mg \quad \ldots(i)$$

and component $N\sin\theta$ provides necessary centripetal pull on the bead,

$$\Rightarrow \quad N\sin\theta = m\left(\frac{r}{2}\right)\omega^2 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\tan\theta = \frac{r\omega^2}{2g} \quad \ldots(iii)$$

Now, from geometry of figure,

$$\tan\theta = \frac{\frac{r}{2}}{\sqrt{r^2 - \left(\frac{r}{2}\right)^2}} = \frac{r}{2\left(\frac{\sqrt{3}}{2}\right)r} = \frac{1}{\sqrt{3}} \quad \ldots(iv)$$

Put this value in Eq. (iii), we get

$$\omega^2 = \frac{2g}{\sqrt{3}r}$$

22. An L-shaped object made of thin rods of uniform mass density is suspended with a string as shown in figure. If $AB = BC$ and the angle is made by AB with downward vertical is θ, then **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\tan\theta = \frac{2}{\sqrt{3}}$ (b) $\tan\theta = \frac{1}{2\sqrt{3}}$

(c) $\tan\theta = \frac{1}{2}$ (d) $\tan\theta = \frac{1}{3}$

Exp. **(d)**

Key Idea The centre of mass of a thin rod of uniform density lies at its centre. The given system of rods can be drawn using geometry as,

where, $(COM)_1$ and $(COM)_2$ are the centre of mass of both rods AB and AC, respectively.

So, in $\Delta A'BB'$,

$$\tan\theta = \frac{A'B'}{A'B} = \frac{\frac{a}{4}}{\frac{3a}{4}} = \frac{1}{3} \text{ or } \tan\theta = \frac{1}{3}$$

23. A rod of length 50 cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be (Take, $g = 10\ ms^{-2}$)

[JEE Main 2019, 9 Jan Shift-II]

(a) $\frac{\sqrt{30}}{2}$ (b) $\sqrt{30}$ (c) $\frac{\sqrt{20}}{3}$ (d) $\sqrt{\frac{30}{2}}$

Exp. **(b)**

Lose of potential energy of rod = Gain of kinetic energy

$$\therefore \quad \Delta PE = \frac{1}{2} I\omega^2$$

(where, I = moment of inertia of rod and ω = angular frequency of rod)

$$\Rightarrow \quad Mg \times \frac{L}{2}\sin 30° = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad Mg\frac{L}{2} \times \frac{1}{2} = \frac{1}{2} \times I \times \omega^2$$

$$\Rightarrow \quad \frac{Mg\ L \times 2}{4 \times I} = \omega^2$$

$$\Rightarrow \quad \sqrt{\frac{MgL}{4 \times \frac{ML^2}{3}} \times \frac{2}{1}} = \omega \quad \left[\because I = \frac{ML^2}{3} \text{ for rod}\right]$$

$$\omega = \sqrt{30} \text{ rad/s}$$

24. A homogeneous solid cylindrical roller of radius R and mass m is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\frac{F}{2mR}$ (b) $\frac{2F}{3mR}$ (c) $\frac{3F}{2mR}$ (d) $\frac{F}{3mR}$

Exp. **(b)**

When force F is applied at the centre of roller of mass m as shown in the figure below

Its acceleration is given by

$$\frac{(F-f)}{m} = \alpha \quad \ldots \text{(i)}$$

where, f = force of friction and

m = mass of roller.

Torque on roller is provided by friction f and it is

$$\tau = fR = I\alpha \quad \ldots \text{(ii)}$$

where, I = moment of inertia of solid cylindrical roller.

$= mR^2/2$

and α = angular acceleration of cylinder = a/R.

Hence, $$\tau = \frac{mR^2}{2} \cdot \frac{a}{R} = \frac{maR}{2}$$

From eq. (ii), ($\tau = fR$)

$$f = \frac{ma}{2} \quad \ldots \text{(iii)}$$

From Eqs. (i) and (iii), we get

$$F = \frac{3}{2}ma$$

$$\Rightarrow \quad a = \frac{2F}{3m}$$

So, $$\alpha = \frac{2F}{3mR}. \quad \left[\because \alpha = \frac{a}{R}\right]$$

25. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length $2R$ and mass M (see figure).

The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is

[JEE Main 2019, 10 Jan Shift-II]

(a) $\frac{137}{15}MR^2$ (b) $\frac{209}{15}MR^2$ (c) $\frac{17}{15}MR^2$ (d) $\frac{152}{15}MR^2$

Exp. **(a)**

Key Idea This problem will be solved by applying parallel axis theorem, which states that moment of inertia of a rigid body about any axis is equals to its moment of inertia about a parallel axis through its centre of mass plus the product of the mass of the body and the square of the perpendicular distance between the axis.

We know that moment of inertia (MI) about the principle axis of the sphere is given by

$$I_{\text{sphere}} = \frac{2}{5}MR^2 \quad \ldots \text{(i)}$$

Using parallel axis theorem, moment of inertia about the given axis in the figure below will be

Principal axis Given axis

$$I_1 = \frac{2}{5}MR^2 + M(2R)^2$$

$$I_1 = \frac{22}{5}MR^2 \quad \ldots \text{(ii)}$$

Considering both spheres at equal distance from the axis, moment of inertia due to both spheres about this axis will be

$$2I_1 = 2 \times 22/5\, MR^2$$

Now, moment of inertia of rod about its perpendicular bisector axis is given by

$$I_2 = \frac{1}{12}ML^2$$

Here, given that $L = 2R$

$$\therefore \quad I_2 = \frac{1}{12}M(2R)^2 = \frac{1}{3}MR^2 \quad \ldots \text{(iii)}$$

So, total moment of inertia of the system is

$$I = 2I_1 + I_2 = 2 \times \frac{22}{5}MR^2 + \frac{1}{3}MR^2$$

$$\Rightarrow \quad I = \left(\frac{44}{5} + \frac{1}{3}\right)MR^2 = \frac{137}{15}MR^2$$

26. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be

[JEE Main 2019, 10 Jan Shift-II]

(a) $\frac{g}{13l}$ (b) $\frac{g}{2l}$ (c) $\frac{7g}{3l}$ (d) $\frac{g}{3l}$

Exp. **(a)**

Key Idea When a rod is pivoted at any point, its angular acceleration is given by $\tau_{net} = I\alpha$.

The given condition can be drawn in the figure below

Torque (τ) about $P = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$ … (i)

$\Rightarrow \tau = l \times 5M_0g$ (outwards) $- 2l \times 2M_0g$ (inwards)

$\Rightarrow \quad \tau = 5M_0gl - 4M_0gl$ (outwards)

$\Rightarrow \quad \tau = M_0gl$ (outwards)

or $\quad \tau = M_0gl$ …(ii)

Now we know that, torque is also given by

$\tau = I\alpha$ …(iii)

Here, I = moment of inertia (w.r.t. point P) of rod and α = angular acceleration.

For point P, $I = (5M_0) \times l^2 + (2M_0)(2l)^2$

$[\because I = MR^2]$

$\Rightarrow \quad I = 13M_0l^2$ …(iv)

Putting value of I from Eq. (iv) in Eq. (iii), we get

$\tau = (13M_0l^2)\alpha$ …(v)

From Eqs. (ii) and (v), we get

$M_0gl = 13M_0l^2\alpha$

$\Rightarrow \quad \alpha = \frac{g}{13l}$

27. A slab is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ of same magnitude F as shown in the figure. Force $\mathbf{F}_2$ is in xy-plane while force $\mathbf{F}_1$ acts along Z-axis at the point $(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$. The moment of these forces about point O will be **[JEE Main 2019, 11 Jan Shift-I]**

(a) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (b) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

(c) $(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} - 3\hat{\mathbf{k}})F$ (d) $(3\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F$

Exp. **(b)**

According to the question as shown in the figure below

$\mathbf{r}_1 = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ and $\mathbf{r}_2 = 6\hat{\mathbf{j}}$

$\mathbf{F}_1 = F\hat{\mathbf{k}}$

and $\quad \mathbf{F}_2 = (-\sin 30^\circ\, \hat{\mathbf{i}} - \cos 30^\circ\, \hat{\mathbf{j}})F$

Moment of force is given as, $\tau = \mathbf{r} \times \mathbf{F}$

where, $\mathbf{r}$ is the perpendicular distance and $\mathbf{F}$ is the force.

$\therefore$ Moment due to $\mathbf{F}_1$

$\tau_1 = (2\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) \times (F\hat{\mathbf{k}})$

$= -2F\hat{\mathbf{j}} + 3F\hat{\mathbf{i}}$ …(i)

Moment due to $\mathbf{F}_2$

$\tau_2 = (6\hat{\mathbf{j}}) \times (-\sin 30^\circ\, \hat{\mathbf{i}} - \cos 30\hat{\mathbf{j}})F$

$= 6\sin 30^\circ F\hat{\mathbf{k}} = 3F\hat{\mathbf{k}}$ …(ii)

$\therefore$ Resultant torque,

$\tau = \tau_1 + \tau_2 = 3F\hat{\mathbf{i}} - 2F\hat{\mathbf{j}} + 3F\hat{\mathbf{k}}$

$= (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})F.$

28. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid points of its sides as shown and

G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. If the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then

[JEE Main 2019, 11 Jan Shift-I]

(a) $I' = \frac{3}{4} I_0$ (b) $I' = \frac{15}{16} I_0$

(c) $I' = \frac{I_0}{4}$ (d) $I' = \frac{9}{16} I_0$

Exp. **(b)**

Suppose the mass of the ΔABC be 'M' and length of the side be 'l'.

When the ΔDEF is being removed from it, then the mass of the removed Δ will be 'M / 4' and length of its side will be 'l/2' as shown below

Since we know, moment of inertia of the triangle about the axis, passing through its centre of gravity is, $I = kml^2$, where k is a constant.

Then for ΔDEF, moment of inertia of the triangle about the axis,

$$I = k\left(\frac{M}{4}\right)\left(\frac{l}{2}\right)^2 = \frac{kMl^2}{16} \quad \ldots(i)$$

Moment of inertia of ΔABC is

$$I_0 = kMl^2 \quad \ldots(ii)$$

The moment of inertia of the remaining part will be

$$I' = I_0 - I = kMl^2 - \frac{kMl^2}{16}$$

[$\because$ using Eqs. (i) and (ii)]

$$= \frac{15kMl^2}{16} \quad \text{or} \quad I' = \frac{15}{16} I_0$$

29. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO' passing through the centre of D_1, as shown in the figure will be

[JEE Main 2019, 11 Jan Shift-II]

(a) $\frac{2}{3} MR^2$ (b) $\frac{4}{5} MR^2$ (c) $3MR^2$ (d) MR^2

Exp. **(c)**

For disc D_1, moment of inertia across axis OO' will be

$$I_1 = \frac{1}{2} MR^2 \quad \ldots(i)$$

For discs D_2 and D_3, OO' is an axis parallel to the diameter of disc. Using parallel axis theorem,

$$I_2 = I_3 = I_{\text{diameter}} + Md^2 \quad \ldots(ii)$$

Here, $I_{\text{diameter}} = \frac{1}{4} MR^2$

and $d = R$

$\therefore \quad I_2 = I_3 = \frac{1}{4}MR^2 + MR^2 = \frac{5}{4}MR^2$

Now, total MI of the system

$I = I_1 + I_2 + I_3 = \frac{1}{2}MR^2 + 2 \times \frac{5}{4}MR^2 = 3MR^2$

30. The magnitude of torque on a particle of mass 1 kg is 2.5 N-m about the origin. If the force acting on it is 1 N and the distance of the particle from the origin is 5 m, then the angle between the force and the position vector is (in radian)

[JEE Main 2019, 11 Jan Shift-II]

(a) $\frac{\pi}{8}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{6}$

Exp. **(d)**

Given, $m = 1$ kg

$|\tau| = 2.5$ N-m, $F = 1$ N and $r = 5$ m

We know that,

torque $|\tau| = rF\sin\theta$

$\Rightarrow \quad 2.5 = 5 \times 1 \times \sin\theta$

$\Rightarrow \quad \sin\theta = \frac{1}{2}$ or $\theta = \frac{\pi}{6}$ rad

31. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) **[JEE Main 2019, 11 Jan Shift-II]**

(a) $10\,\text{rad/s}^2$ (b) $16\,\text{rad/s}^2$

(c) $20\,\text{rad/s}^2$ (d) $12\,\text{rad/s}^2$

Exp. **(b)**

Given, $m = 5$ kg, $R = 0.5$ m.

Horizontal force, $F = 40$ N

As, Cylinder is rolling without slipping.

Hence, torque is producing rotation about centre O.

So, $\tau = \mathbf{r} \times \mathbf{F}$ (Here, $r = R$)

$\theta = 90°$

So, $\tau = \mathbf{r} \times \mathbf{F} = RF$

or $\tau = 0.5 \times 40 = 20$ N-m ...(i)

If α is acceleration of centre of mass 'O' then

torque is, $\tau = I\alpha$

where, $I = MR^2$

$\therefore \quad \tau = MR^2\alpha$ ...(ii)

Comparing Eq. (i) with Eq. (ii),

$MR^2\alpha = 20$

$\Rightarrow \quad \alpha = \frac{20}{5 \times (0.5)^2}$ or $\alpha = 16\,\text{rad/s}^2$

32. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm) about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is

[JEE Main 2019, 12 Jan Shift-I]

(a) 16 cm (b) 14 cm (c) 12 cm (d) 18 cm

Exp. **(a)**

Moment of inertia of hollow cylinder about its axis is

$$I_1 = \frac{M}{2}(R_1^2 + R_2^2)$$

where, R_1 = inner radius and

R_2 = outer radius.

Moment of inertia of thin hollow cylinder of radius R about its axis is.

$$I_2 = MR^2$$

Given, $I_1 = I_2$ and both cylinders have same mass (M). So, we have

$$\frac{M}{2}(R_1^2 + R_2^2) = MR^2$$

$$(10^2 + 20^2)/2 = R^2$$

$$R^2 = 250 = 15.8$$

$$R \approx 16\,\text{cm}$$

33. The position vector of the centre of mass $\mathbf{r}_{cm}$ of an asymmetric uniform bar of negligible area of cross-section as shown in figure is

[JEE Main 2019, 12 Jan Shift-I]

(a) $\mathbf{r} = \frac{13}{8} L\,\hat{\mathbf{x}} + \frac{5}{8} L\,\hat{\mathbf{y}}$ (b) $\mathbf{r} = \frac{11}{8} L\,\hat{\mathbf{x}} + \frac{3}{8} L\,\hat{\mathbf{y}}$

(c) $\mathbf{r} = \frac{3}{8} L\,\hat{\mathbf{x}} + \frac{11}{8} L\,\hat{\mathbf{y}}$ (d) $\mathbf{r} = \frac{5}{8} L\,\hat{\mathbf{x}} + \frac{13}{8} L\,\hat{\mathbf{y}}$

Exp. **(a)**

Coordinates of centre of mass (COM) are given by

$$X_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

and $$Y_{COM} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

For given system of rods, masses and coordinates of centre of rods are as shown.

So, $$X_{COM} = \left(\frac{2mL + m2L + m\frac{5L}{2}}{4\,m}\right) = \frac{13}{8}L$$

and $$Y_{COM} = \frac{2mL + m \times \frac{L}{2} + m \times 0}{4\,m} = \frac{5L}{8}$$

So, position vector of COM is

$$\mathbf{r}_{COM} = X_{COM}\hat{\mathbf{x}} + Y_{COM}\,\hat{\mathbf{y}}$$
$$= \frac{13}{8}L\hat{\mathbf{x}} + \frac{5}{8}L\hat{\mathbf{y}}$$

34. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is '$I(x)$'. Which one of the graphs represents the variation of $I(x)$ with x correctly?

[JEE Main 2019, 12 Jan Shift-II]

Exp. **(b)**

Moment of inertia of a solid sphere about an axis through its centre of mass is $I_C = \frac{2}{5}MR^2$

Moment of inertia about a parallel axis at a distance x from axis through its COM is

$I = I_C + Mx^2$ (by parallel axis theorem)

So graph of I *versus* x is parabolic are shown

35. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be (Take, $g = 10\ \text{m/s}^2$)

[JEE Main 2019, 12 Jan Shift-II]

(a) $8\,\text{kg-m}^2/\text{s}$ (b) $3\,\text{kg-m}^2/\text{s}$
(c) $2\,\text{kg-m}^2/\text{s}$ (d) $6\,\text{kg-m}^2/\text{s}$

Exp. **(*d*)**

The given figure is shown below as

As friction is absent, energy at A = energy at B

$\Rightarrow \quad \frac{1}{2}mv_A^2 + mgh = \frac{1}{2}mv_B^2$

$\Rightarrow \quad v_A^2 + 2gh = v_B^2$

or $\quad v_B^2 = (5)^2 + 2 \times 10 \times 10 = 225$

$\Rightarrow \quad v_B = 15\,\text{ms}^{-1}$

Angular momentum about point 'O',

$= mv_B r_B$

$= 20 \times 10^{-3} \times 15 \times 20$

$= 6\,\text{kg-m}^2\,\text{s}^{-1}$

36. Two particles A and B are moving on two concentric circles of radii R_1 and R_2 with equal angular speed ω. At $t = 0$, their positions and direction of motion are shown in the figure

The relative velocity $\mathbf{v}_A - \mathbf{v}_B$ at $t = \frac{\pi}{2\omega}$ is given by **[JEE Main 2019, 12 Jan Shift-II]**

(a) $\omega(R_1 + R_2)\hat{\mathbf{i}}$ (b) $-\omega(R_1 + R_2)\hat{\mathbf{i}}$
(c) $\omega(R_1 - R_2)\hat{\mathbf{i}}$ (d) $\omega(R_2 - R_1)\hat{\mathbf{i}}$

Exp. **(*d*)**

Angle covered by each particle in time duration 0 to $\frac{\pi}{2\omega}$ is

$$\theta = \omega \times t = \omega \times \frac{\pi}{2\omega} = \frac{\pi}{2}\text{ rad}$$

So, positions of particles at $t = \frac{\pi}{2\omega}$ is as shown below;

Velocities of particles at $t = \frac{\pi}{2\omega}$ are

$$\mathbf{v}_A = -\omega R_1\hat{\mathbf{i}} \quad \text{and} \quad \mathbf{v}_B = -\omega R_2\hat{\mathbf{i}}$$

The relative velocity of particles is

$$\mathbf{v}_A - \mathbf{v}_B = -\omega R_1\hat{\mathbf{i}} - (-\omega R_2\hat{\mathbf{i}})$$
$$= -\omega(R_1 - R_2)\hat{\mathbf{i}} = \omega(R_2 - R_1)\hat{\mathbf{i}}$$

37. Seven identical circular planar discs, each of mass M and radius R are welded symmetrically as shown in the figure. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is **[JEE Main 2018]**

(a) $\frac{19}{2}MR^2$ (b) $\frac{55}{2}MR^2$
(c) $\frac{73}{2}MR^2$ (d) $\frac{181}{2}MR^2$

Exp. **(*d*)**

First we found moment of inertia (MI) of system using parallel axis theorem about centre of mass, then we use it to find moment of inertia about given axis.

Moment of inertia of an outer disc about the axis through centre is

$$= \frac{MR^2}{2} + M(2R)^2 = MR^2\left(4 + \frac{1}{2}\right) = \frac{9}{2}MR^2$$

For 6 such discs,

Moment of inertia $= 6 \times \frac{9}{2}MR^2 = 27MR^2$

So, moment of inertia of system

$$= \frac{MR^2}{2} + 27MR^2 = \frac{55}{2}MR^2$$

Hence, $I_P = \frac{55}{2}MR^2 + (7M \times 9R^2)$

$\Rightarrow \quad I_P = \frac{181}{2}MR^2 \qquad I_{\text{system}} = \frac{181}{2}MR^2$

38. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is **[JEE Main 2018]**

(a) $4MR^2$ (b) $\frac{40}{9}MR^2$ (c) $10MR^2$ (d) $\frac{37}{9}MR^2$

Exp. **(a)**

Moment of inertia of remaining solid

= Moment of inertia of complete solid

– Moment of inertia of removed portion

$$\therefore \quad I = \frac{9MR^2}{2} - \left[\frac{M(R/3)^2}{2} + M\left(\frac{2R}{3}\right)^2\right]$$

$\Rightarrow \quad I = 4MR^2$

39. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then : **[JEE Main 2018]**

(a) $T \propto R^{3/2}$ for any n (b) $T \propto R^{\frac{n}{2}+1}$

(c) $T \propto R^{(n+1)/2}$ (d) $T \propto R^{n/2}$

Exp. **(c)**

$\because$ Force = Mass × Acceleration $= m\omega^2 R$

and given, $F \propto \frac{1}{R^n} \qquad \Rightarrow F = \frac{k}{R^n}$

So, we have

$$\frac{k}{R^n} = m\left(\frac{2\pi}{T}\right)^2 R$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m}{k} \cdot R^{n+1} \Rightarrow T \propto R^{\frac{n+1}{2}}$$

40. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum? **[JEE Main 2017 (Offline)]**

(a) $\frac{\sqrt{3}}{2}$ (b) 1 (c) $\frac{3}{\sqrt{2}}$ (d) $\sqrt{\frac{3}{2}}$

Exp. **(d)**

MI of a solid cylinder about its perpendicular bisector of length is

$$I = m\left(\frac{l^2}{12} + \frac{R^2}{4}\right)$$

$$\Rightarrow \quad I = \frac{mR^2}{4} + \frac{ml^2}{12} = \frac{m^2}{4\pi\rho l} + \frac{ml^2}{12} \quad [\because \rho\pi R^2 l = m]$$

For I to be maximum,

$$\frac{dI}{dl} = -\frac{m^2}{4\pi\rho}\left(\frac{1}{l^2}\right) + \frac{ml}{6} = 0$$

$$\Rightarrow \quad \frac{m^2}{4\pi\rho} = \frac{ml^3}{6}$$

Now, putting $m = \rho\pi R^2 l$

$$\therefore \quad l^3 = \frac{3}{2\pi\rho} \cdot \rho\pi R^2 l$$

$$\frac{l^2}{R^2} = \frac{3}{2}$$

$$\therefore \quad \frac{l}{R} = \sqrt{\frac{3}{2}}$$

41. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see the figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical, is

[JEE Main 2017 (Offline)]

(a) $\frac{2g}{3l}\sin\theta$ (b) $\frac{3g}{2l}\cos\theta$

(c) $\frac{2g}{3l}\cos\theta$ (d) $\frac{3g}{2l}\sin\theta$

Exp. **(d)**

As the rod rotates in vertical plane so a torque is acting on it, which is due to the vertical component of weight of rod.

Initial condition At any time t

Now, torque τ = force $\times$ perpendicular distance of line of action of force from axis of rotation

$$= mg\sin\theta \times \frac{l}{2}$$

Again, torque $\tau = I\alpha$

where, I = moment of inertia $= \frac{ml^2}{3}$

[Force and torque frequency along axis of rotation passing through in end]

α = angular acceleration

$$\therefore \quad mg\sin\theta \times \frac{l}{2} = \frac{ml^2}{3}\alpha$$

$$\therefore \quad \alpha = \frac{3g\sin\theta}{2l}$$

42. A particle of mass m is moving along the side of a square of side a, with a uniform speed v in the X-Y plane as shown in the figure.

Which of the following statements is false for the angular momentum **L** about the origin?

[JEE Main 2016 (Offline)]

(a) $\mathbf{L} = -\frac{mv}{\sqrt{2}}R\hat{\mathbf{k}}$, when the particle is moving from A to B.

(b) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} + a\right)\hat{\mathbf{k}}$, when the particle is moving from B to C.

(c) $\mathbf{L} = mv\left(\frac{R}{\sqrt{2}} - a\right)\hat{\mathbf{k}}$, when the particle is moving from C to D.

(d) $\mathbf{L} = \frac{mv}{\sqrt{2}}R\hat{\mathbf{k}}$, when the particle is moving from D to A.

Exp. **(b, d)**

For a particle of mass m is moving along the side of a square of side a. Such that

Angular momentum **L** about the origin

$$= \mathbf{L} = \mathbf{r} \times \mathbf{p} = rp\sin\theta\,\hat{\mathbf{n}} \quad \text{or} \quad \mathbf{L} = r(p)\,\hat{\mathbf{n}}$$

When a particle is moving from D to A,

$$\mathbf{L} = \frac{R}{\sqrt{2}}mv(-\hat{\mathbf{k}})$$

A particle is moving from A to B,

$$\mathbf{L} = \frac{R}{\sqrt{2}}mv(-\hat{\mathbf{k}})$$

and it moves from C to D,

$$\mathbf{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv\,(\hat{\mathbf{k}})$$

For B to C, we have

$$\mathbf{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv\,(\hat{\mathbf{k}})$$

Hence, options (b) and (d) are incorrect.

43. A roller is made by joining together two corners at their vertices O. It is kept on two rails AB and CD which are placed a symmetrically (see the figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see the figure). It is given a light path, so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to **[JEE Main 2016 (Offline)]**

(a) turn left
(b) turn right
(c) go straight
(d) turn left and right alternately

Exp. **(a)**

As, the wheel rolls forward the radius of the wheel decreases along AB, hence for the same number of rotations it moves less distance along AB, hence it turns left.

44. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h, then z_0 is equal to **[JEE Main 2015]**

(a) $\frac{h^2}{4R}$ (b) $\frac{3h}{4}$

(c) $\frac{5h}{8}$ (d) $\frac{3h^2}{8R}$

Exp. **(b)**

We know that centre of mass of a uniform solid cone of height h is at height $h/4$ from base, therefore

$$h - z_0 = \frac{h}{4}$$

or $$z_0 = h - \frac{h}{4} = \frac{3h}{4}$$

45. From a solid sphere of mass M and radius R, a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is **[JEE Main 2015]**

(a) $\frac{MR^2}{32\sqrt{2}\pi}$ (b) $\frac{MR^2}{16\sqrt{2}\pi}$ (c) $\frac{4MR^2}{9\sqrt{3}\pi}$ (d) $\frac{4MR^2}{3\sqrt{3}\pi}$

Exp. **(c)**

Use geometry of the figure to calculate mass and side length of the cube interms of M and R respectively.

Consider the cross-sectional view of a diametric plane as shown in the adjacent diagram.

Cross-sectional view of the cube and sphere

Using geometry of the cube

$$PQ = 2R = (\sqrt{3})\,a \quad \text{or} \quad a = \frac{2R}{\sqrt{3}}$$

Volume density of the solid sphere

$$\rho = \frac{M}{(4/3)\,\pi R^3} = \frac{3}{4\pi}\left(\frac{M}{R^3}\right)$$

Mass of cube $(m) = (\rho)\,(a)^3$

$$= \left(\frac{3}{4\pi} \times \frac{M}{R^3}\right)\left[\frac{2R}{\sqrt{3}}\right]^3 = \frac{3M}{4\pi R^3} \times \frac{8R^3}{3\sqrt{3}} = \frac{2M}{\sqrt{3}\pi}$$

Moment of inertia of the cube about given axis is

$$I_Y = \frac{ma^2}{12}(a^2 + a^2) = \frac{ma^2}{6}$$

$$\Rightarrow \quad I_Y = \frac{ma^2}{6} = \frac{2M}{\sqrt{3}\pi} \times \frac{1}{6} \times \frac{4R^2}{3} = \frac{4MR^2}{9\sqrt{3}\pi}$$

46. From a solid sphere of mass M and radius R, a spherical portion of radius $(R/2)$ is removed as shown in the figure. Taking gravitational potential $V = 0$ at $r = \infty$, the potential at the centre of the cavity thus formed is (G = gravitational constant)

[JEE Main 2015]

(a) $\frac{-GM}{2R}$ (b) $\frac{-GM}{R}$ (c) $\frac{-2GM}{3R}$ (d) $\frac{-2GM}{R}$

Exp. **(b)**

Consider cavity as negative mass and apply superposition of gravitational potential.

Consider the cavity formed in a solid sphere as shown in figure.

$V(\infty) = 0$

According to the question, we can write potential at an internal point P due to complete solid sphere,

$$V_s = -\frac{GM}{2R^3}\left[3R^2 - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{-GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right]$$

$$= \frac{-GM}{2R^3}\left[\frac{11R^2}{4}\right] = \frac{-11GM}{8R}$$

Mass of removed part

$$= \frac{M}{(4/3)\times \pi R^3} \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3 = \frac{M}{8}$$

Potential at point P due to removed part

$$V_c = \frac{-3}{2} \times \frac{GM/8}{R/2} = \frac{-3GM}{8R}$$

Thus, potential due to remaining part at point P,

$$V_P = V_s - V_c = \frac{-11GM}{8R} - \left(-\frac{3GM}{8R}\right)$$

$$= \frac{(-11+3)GM}{8R} = \frac{-GM}{R}$$

47. A mass m supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleraton will the mass fall on release? **[JEE Main 2014]**

(a) $\frac{2g}{3}$ (b) $\frac{g}{2}$ (c) $\frac{5g}{6}$ (d) g

Exp. **(b)**

For the mass m,

$$mg - T = ma$$

As we know, $a = R\alpha$

So, $mg - T = mR\alpha$ ...(i)

As torque $= T \times R = mR^2\alpha$ ...(ii)

From Eqs. (i) and (ii), we get

$$a = g/2$$

Hence, the acceleration with the mass of a body fall is $g/2$.

48. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Main 2013]**

(a) $\frac{r\omega_0}{4}$ (b) $\frac{r\omega_0}{3}$ (c) $\frac{r\omega_0}{2}$ (d) $r\omega_0$

Exp. **(c)**

From the principle of conservation of angular momentum, we have

$$\Sigma I\omega = \text{constant}$$

where, I is moment of inertia ($I = mr^2$).

By conservation law of angular momentum,

$$mr^2\omega_0 = mvr + mr^2 \times \frac{v}{r}$$

$$\Rightarrow \quad v = \frac{\omega_0 r}{2}$$

49. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc **[AIEEE 2011]**

(a) continuously decreases
(b) continuously increases
(c) first increases and then decreases
(d) remains unchanged

Exp. **(c)**

Moment of inertia, $I = \frac{1}{2}MR^2 + mx^2$

where, m = mass of insect,

and x = distance of insect from centre.Clearly, as the insect moves along the diameter of the disc, Moment of inertia first decreases, then increases.

By conservation of angular momentum, angular speed first increases, then decreases.

50. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ N (where, t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg-m^2, then the number of rotations made by the pulley before its direction of motion is reserved, is **[AIEEE 2011]**

(a) more than 3 but less than 6
(b) more than 6 but less than 9
(c) more than 9
(d) less than 3

Exp. **(a)**

Given force, $F = 20t - 5t^2$

$$\alpha = \frac{FR}{I} = \frac{(20t - 5t^2)2}{10} = 4t - t^2$$

$$\Rightarrow \quad \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \int_0^{\omega} d\omega = \int_0^t (4t - t^2)\,dt$$

$$\Rightarrow \quad \omega = 2t^2 - \frac{t^3}{3}$$

When direction is reversed, $\omega = 0$, i.e., $t = 0$, 6s

Now, $d\theta = \omega dt \Rightarrow \int_0^{\theta} d\theta = \int_0^6 \left(2t^2 - \frac{t^3}{3}\right) dt$

$$\Rightarrow \quad \theta = \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6$$

$$\Rightarrow \quad \theta = 144 - 108 = 36 \text{ rad}$$

$\therefore$ Number of rotations, $n = \frac{\theta}{2\pi} = \frac{36}{2\pi} < 6$

51. A thin uniform rod of length l and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of **[AIEEE 2009]**

(a) $\frac{1}{3}\frac{l^2\omega^2}{g}$ (b) $\frac{1}{6}\frac{l\omega}{g}$ (c) $\frac{1}{2}\frac{l^2\omega^2}{g}$ (d) $\frac{1}{6}\frac{l^2\omega^2}{g}$

Exp. **(d)**

By conservation of energy,

Initial total energy = Final total energy

$$TE_i = TE_f$$

$$\frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2} \times \frac{1}{3} ml^2\omega^2 = mgh$$

$$\Rightarrow \quad h = \frac{1}{6}\frac{l^2\omega^2}{g}$$

52. A thin rod of length L is lying along the x-axis with its ends at $x = 0$ and $x = L$. Its linear density (mass/length) varies with x as $k\left(\frac{x}{L}\right)^n$, where n can be zero or any positive number. If the position x_{CM} of the centre of mass of the rod is plotted against n, which of the following graphs best approximates the dependence of x_{CM} on n? **[AIEEE 2008]**

(a)

(b)

(c)

(d)

Exp. **(a)**

$$x_{CM} = \frac{\int x dm}{\int dm}$$

If $n = 0$, then $x_{CM} = \frac{L}{2}$

As n increases, the centre of mass shift away from $x = \frac{L}{2}$ towards $x = L$ which only option (a) is satisfying.

Alternately, you can use basic concept.

$$x_{CM} = \frac{\int_0^L k\left(\frac{x}{L}\right)^n \times x dx}{\int_0^L k\left(\frac{x}{L}\right)^n dx} = L\left[\frac{n+1}{n+2}\right]$$

53. Consider a uniform square plate of side a and mass m. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is **[AIEEE 2008]**

(a) $\frac{5}{6}ma^2$ (b) $\frac{1}{12}ma^2$

(c) $\frac{7}{12}ma^2$ (d) $\frac{2}{3}ma^2$

Exp. **(d)**

Moment of inertia of square plate about xx' is $\frac{ma^2}{6}$, therefore moment of inertia about zz' can be computed using parallel axes theorem,

$$I_{zz'} = I_{xx'} + m\left(\frac{a}{\sqrt{2}}\right)^2$$

$$= \frac{ma^2}{6} + \frac{ma^2}{2}$$

$$= \frac{2ma^2}{3}$$

54. For the given uniform square lamina $ABCD$, whose centre is O **[AIEEE 2007]**

(a) $\sqrt{2}\, I_{AC} = I_{EF}$ (b) $I_{AD} = 3 I_{EF}$

(c) $I_{AC} = 4 I_{EF}$ (d) $I_{AC} = \sqrt{2}\, I_{EF}$

Exp. **(c)**

Let the each side of square lamina be d.

So, $I_{EF} = I_{GH}$ [due to symmetry]

and $I_{AC} = I_{BD}$ [due to symmetry]

Now, according to theorem of perpendicular axes,

$$I_{AC} + I_{BD} = I_0$$

$\Rightarrow$ $2I_{AC} = I_0$ ...(i)

and $I_{EF} + I_{GH} = I_0$

$\Rightarrow$ $2I_{EF} = I_0$ ...(ii)

From Eqs.(i) and (ii), we get

$$I_{AC} = I_{EF}$$

$$\therefore \quad I_{AD} = I_{EF} + \frac{md^2}{4}$$

$$= \frac{md^2}{12} + \frac{md^2}{4} \quad \left[\text{as } I_{EF} = \frac{md^2}{12}\right]$$

So, $I_{AD} = \frac{md^2}{3} = 4I_{EF}$

55. A circular disc of radius R is removed from a bigger circular disc of radius $2R$, such that the circumference of the discs coincide. The centre of mass of the new disc is $\frac{\alpha}{R}$ from the centre of the bigger disc. The value of α is **[AIEEE 2007]**

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{4}$

Exp. **(a)**

In this question, distance of centre of mass of new disc from the centre of mass of remaining disc is αR.

Mass of original disc $= M$

Mass of disc removed

$$= \frac{M}{\pi(2R)^2} \times \pi R^2 = \frac{M}{4}$$

Mass of remaining disc $= M - \frac{M}{4} = \frac{3M}{4}$

$$\therefore \quad -\frac{3M}{4}\alpha R + \frac{M}{4}R = 0 \quad \Rightarrow \quad \alpha = \frac{1}{3}$$

Note In this question, the given distance must be αR for real approach to the solution.

56. A round uniform body of radius R, mass M and moment of inertia I, rolls down (without slipping) an inclined plane making an angle θ with the horizontal. Then, its acceleration is **[AIEEE 2007]**

(a) $\frac{g\sin\theta}{1 + I/MR^2}$ (b) $\frac{g\sin\theta}{1 + MR^2/I}$

(c) $\frac{g\sin\theta}{1 - I/MR^2}$ (d) $\frac{g\sin\theta}{1 - MR^2/I}$

Exp. **(a)**

Assuming that no energy is used up against friction, the total loss in potential energy is equal to the total gain in the kinetic energy.

$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2 \qquad [\text{as } v = r\omega]$$

i.e., $$Mgh = \frac{1}{2}I\left(\frac{v^2}{R^2}\right) + \frac{1}{2}Mv^2$$

or $$\frac{1}{2}v^2\left(M + \frac{I}{R^2}\right) = Mgh$$

or $$v^2 = \frac{2Mgh}{M + I/R^2} = \frac{2gh}{1 + I/MR^2}$$

If s is the distance covered along the plane, then

Alternate Solution

$$h = s\sin\theta$$

$$\therefore \quad v^2 = \frac{2gs\sin\theta}{1 + I/MR^2}$$

Now, $$v^2 = 2as$$

$$\therefore \quad 2as = \frac{2gs\sin\theta}{1 + I/MR^2}$$

or $$a = \frac{g\sin\theta}{1 + I/MR^2}$$

$$Mg\sin\theta - f = Ma \qquad \ldots(i)$$

As torque $= I\alpha = I\frac{a}{R}$ [as $a = r\alpha$]

Torque $\tau = f \cdot R$

So, $$f \cdot R = \frac{Ia}{R} \quad \Rightarrow \quad f = \frac{Ia}{R^2}$$

From Eq. (i),

$$Mg\sin\theta - \frac{Ia}{R^2} = Ma$$

$$a\left[M + \frac{I}{R^2}\right] = Mg\sin\theta$$

$$a = \frac{Mg\sin\theta}{M + \frac{I}{R^2}}$$

$$\therefore \quad a = \frac{g\sin\theta}{1 + \frac{I}{MR^2}}$$

57. Angular momentum of the particle rotating with a central force is constant due to

(a) constant force **[AIEEE 2007]**

(b) constant linear momentum

(c) zero torque

(d) constant torque

Exp. **(c)**

According to the principle of conservation of angular momentum, in the absence of external torque, the total angular momentum of the system is constant.

i.e., zero torque $\left(\tau = \frac{dL}{dt}\right)$

58. Consider a two particle system with particles having masses m_1 and m_2. If the first particle is pushed towards the centre of mass through a distance d, by what distance should the second particle be moved, so as to keep the centre of mass at the same position? **[AIEEE 2006]**

(a) $\frac{m_2}{m_1}d$ (b) $\frac{m_1}{m_1+m_2}d$

(c) $\frac{m_1}{m_2}d$ (d) d

Exp. **(c)**

To keep the centre of mass at the same position, velocity of centre of mass is zero, so

$$\frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1+m_2} = 0$$

where, v_1 and v_2 are the velocities of particles 1 and 2, respectively.

$$\Rightarrow \quad m_1\frac{d\mathbf{r}_1}{dt} + m_2\frac{d\mathbf{r}_2}{dt} = 0$$

$$\left[\because \mathbf{v}_1 = \frac{d\mathbf{r}_1}{dt} \text{ and } \mathbf{v}_2 = \frac{d\mathbf{r}_2}{dt}\right]$$

$$\Rightarrow \quad m_1 d\mathbf{r}_1 + m_2 d\mathbf{r}_2 = 0$$

$d\mathbf{r}_1$ and $d\mathbf{r}_2$ represent the change in displacement of particles.

Let 2nd particle has been displaced by distance x.

$$\Rightarrow \quad m_1(d) + m_2(x) = 0 \quad \text{or} \quad x = -\frac{m_1 d}{m_2}$$

Alternate Solution

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1+m_2} \quad \ldots(i)$$

After moving m_1 through a distance d towards CM and to keep the position of CM unchanged, let d' be the shift of m_2.

$$x_{CM} = \frac{m_1(x_1 - d) + m_2(x_2 + d')}{m_1+m_2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{m_1x_1 + m_2x_2}{m_1+m_2} = \frac{m_1(x_1-d) + m_2(x_2+d)}{m_1+m_2}$$

$$\Rightarrow \quad -m_1 d + m_2 d' = 0$$

$$\therefore \quad d' = \frac{m_1}{m_2}d$$

59. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω. The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time **[AIEEE 2006]**

(a) at the mean position of the platform

(b) for an amplitude of g/ω^2

(c) for an amplitude of g^2/ω^2

(d) at the highest position of the platform

Exp. **(b)**

As the amplitude is increased, the maximum acceleration of the platform (along with coin as long as they does not get separated) increases.

If we draw the free body diagram of coin at one of the extreme positions as shown, then from Newton's law,

$$mg - N = m\omega^2 A$$

For loosing contact with the platform,

$$N = 0$$

So, $$A = \frac{g}{\omega^2}$$

60. Four point masses, each of value m, are placed at the corners of a square $ABCD$ of side l. The moment of inertia of this system about an axis passing through A and parallel to BD is **[AIEEE 2006]**

(a) $2ml^2$ (b) $\sqrt{3}ml^2$

(c) $3ml^2$ (d) ml^2

Exp. **(c)**

The situation is shown in figure.

$$I_{xx'} = m \times DP^2 + m \times BQ^2 + m \times CA^2$$
$$= m \times 2 \times \left(\frac{\sqrt{2}\,l}{2}\right)^2 + m \times (\sqrt{2}\,l)^2 = 3ml^2$$

61. A force of $-F\,\mathbf{k}$ acts on O, the origin of the coordinate system. The torque about the point $(1, -1)$ is **[AIEEE 2006]**

(a) $F(\mathbf{i} - \mathbf{j})$ (b) $-F(\mathbf{i} + \mathbf{j})$
(c) $F(\mathbf{i} + \mathbf{j})$ (d) $-F(\mathbf{i} - \mathbf{j})$

Exp. **(c)**

The torque about the given position, $\tau = \mathbf{r} \times \mathbf{F}$

Here, $\mathbf{r} = \mathbf{i} - \mathbf{j}$

and $\mathbf{F} = -F\mathbf{k}$

$\therefore \tau = (\mathbf{i} - \mathbf{j}) \times (-F\mathbf{k})$

$= F[(-\mathbf{i} \times \mathbf{k}) + (\mathbf{j} \times \mathbf{k})]$

$= F(\mathbf{j} + \mathbf{i}) = F(\mathbf{i} + \mathbf{j})$

62. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity $\omega' =$ **[AIEEE 2006]**

(a) $\dfrac{\omega(m + 2M)}{m}$ (b) $\dfrac{\omega(m - 2M)}{(m + 2M)}$

(c) $\dfrac{\omega m}{(m + M)}$ (d) $\dfrac{\omega m}{(m + 2M)}$

Exp. **(d)**

As no external torque is acting on the system, angular momentum of system remains conserved.

So, by conservation of angular momentum,

i.e., $I_1\omega = I_2\omega'$

$\Rightarrow$ $mR^2\omega = (mR^2 + 2MR^2)\omega'$

$\Rightarrow$ $\omega' = \left(\dfrac{m}{m + 2M}\right)\omega$

63. A body A of mass M while falling vertically downwards under gravity breaks into two parts; a body B of mass $\frac{1}{3}M$ and a body C of mass $\frac{2}{3}m$. The centre of mass of bodies B and C taken together shifts compared to that of body A towards

(a) depends on height of breaking **[AIEEE 2005]**
(b) does not shift
(c) body C
(d) body B

Exp. **(b)**

Since, the acceleration of centre of mass in both the cases is same equal to g, so the centre of mass of the bodies B and C taken together does not shift compared to that of body A.

64. The moment of inertia of uniform semi-circular disc of mass M and radius r about a line perpendicular to the plane of the disc through the centre is **[AIEEE 2005]**

(a) $\frac{1}{4}Mr^2$ (b) $\frac{2}{5}Mr^2$ (c) Mr^2 (d) $\frac{1}{2}Mr^2$

Exp. **(d)**

The mass of complete (circular) disc is

The moment of inertia of disc about the given axis is

$$I = \frac{2Mr^2}{2} = Mr^2$$

Let the moment of inertia of semi-circular disc be I_1.

The disc may be assumed as combination of two semi-circular parts.

Thus, $I_1 = I - I_1$ or $I_1 = \dfrac{I}{2} = \dfrac{Mr^2}{2}$

65. A T shaped object with dimensions shown in the figure, is lying on a smooth floor. A force **F** is applied at the point P parallel to AB, such that the object has only the translational motion without rotation. Find the location of P with respect to C. **[AIEEE 2005]**

(a) $\frac{2}{3}l$ (b) $\frac{3}{2}l$ (c) $\frac{4}{3}l$ (d) l

Exp. **(c)**

For pure translatory motion, net torque about centre of mass should be zero.

Thus, **F** is applied at centre of mass of system,

$$OP = \frac{m_1 \times 0 + m_2 \times l}{m_1 + m_2}$$

where, m_1 and m_2 are masses of horizontal and vertical section of the object. Assuming object is uniform,

$$m_2 = 2m_1 \quad \Rightarrow \quad OP = \frac{2l}{3}$$

$$PC = \left(l - \frac{2l}{3} + l\right) = \left(2l - \frac{2l}{3}\right) = \frac{4l}{3}$$

66. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that **[AIEEE 2004]**

(a) $I_A = I_B$ (b) $I_A > I_B$

(c) $I_A < I_B$ (d) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where, d_A and d_B are their densities.

Exp. **(c)**

Let same mass and same outer radii of solid sphere and hollow sphere be M and R, respectively.

The moment of inertia of solid sphere A about its diameter,

$$I_A = \frac{2}{5}MR^2 \quad \ldots(i)$$

Similarly, the moment of inertia of hollow sphere (spherical shell) B about its diameter,

$$I_B = \frac{2}{3}MR^2 \quad \ldots(ii)$$

It is clear from Eqs. (i) and (ii), $I_A < I_B$.

67. A solid sphere is rotating in free space. If the radius of the sphere is increased keeping mass same, which one of the following will not be affected? **[AIEEE 2004]**

(a) Moment of inertia
(b) Angular momentum
(c) Angular velocity
(d) Rotational kinetic energy

Exp. **(b)**

In free space, neither acceleration due to gravity nor external torque act on the rotating solid sphere. Therefore, taking the same mass of sphere, if radius is increased, then moment of inertia, rotational kinetic energy and angular velocity will change but according to law of conservation of momentum, angular momentum will not change.

68. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius $4R$ is made from an iron plate of thickness $t/4$. Then, the relation between the moment of inertia I_X and I_Y is **[AIEEE 2003]**

(a) $I_Y = 32\, I_X$ (b) $I_Y = 16\, I_X$

(c) $I_Y = I_X$ (d) $I_Y = 64\, I_X$

Exp. **(d)**

Mass of disc (X), $m_X = \pi R^2 t \rho$

where, ρ = density of material of disc

$$\therefore \quad I_X = \frac{1}{2} m_X R^2 = \frac{1}{2}\pi R^2 t \rho R^2$$

or $$I_X = \frac{1}{2}\pi \rho t R^4 \quad \ldots(i)$$

Mass of disc (Y),

$$m_Y = \pi(4R)^2 \frac{t}{4}\rho = 4\pi R^2 t \rho$$

and $I_Y = \frac{1}{2} m_Y (4R)^2 = \frac{1}{2} 4\pi R^2 t\rho \cdot 16R^2$

or $I_Y = 32\,\pi t\rho R^4$ ...(ii)

$\therefore \quad \frac{I_Y}{I_X} = \frac{32\,\pi t\rho R^4}{\frac{1}{2}\,\pi\rho t R^4} = 64$

or $I_Y = 64 I_X$

69. A particle performing uniform circular motion has angular momentum L. If its angular frequency is doubled and its kinetic energy halved, then the new angular momentum is **[AIEEE 2003]**

(a) $\frac{L}{4}$ (b) $2L$ (c) $4L$ (d) $L/2$

Exp. **(a)**

Angular momentum,

$L = I\omega$ ...(i)

Kinetic energy, $K = \frac{1}{2} I\omega^2 = \frac{1}{2} L\omega$ [from Eq. (i)]

$\therefore \quad L = 2\frac{K}{\omega}$

Now, $L' = \frac{2\left(\frac{K}{2}\right)}{2\omega} \Rightarrow L' = \frac{L}{4}$

70. Let **F** be the force acting on a particle having position vector **r** and τ be the torque of this force about the origin. Then, **[AIEEE 2003]**

(a) $\mathbf{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau \neq 0$
(b) $\mathbf{r}\cdot\tau \neq 0$ and $\mathbf{F}\cdot\tau = 0$
(c) $\mathbf{r}\cdot\tau \neq 0$ and $\mathbf{F}\cdot\tau \neq 0$
(d) $\mathbf{r}\cdot\tau = 0$ and $\mathbf{F}\cdot\tau = 0$

Exp. **(d)**

As $\tau = \mathbf{r} \times \mathbf{F}$

τ is perpendicular to both **r** and **F**, so $\mathbf{r}\cdot\tau$ as well as $\mathbf{F}\cdot\tau$ has to be zero.

71. Initial angular velocity of a circular disc of mass M is ω_1. Then, two small spheres of mass m are attached gently to two diametrically opposite points on the edge of the disc. What is the final angular velocity of the disc? **[AIEEE 2002]**

(a) $\left(\frac{M+m}{M}\right)\omega_1$ (b) $\left(\frac{M+m}{m}\right)\omega_1$

(c) $\left(\frac{M}{M+4m}\right)\omega_1$ (d) $\left(\frac{M}{M+2m}\right)\omega_1$

Exp. **(c)**

Conservation of angular momentum gives

$$\frac{1}{2} MR^2\omega_1 = \left(\frac{1}{2} MR^2 + 2mR^2\right)\omega_2$$

$$\Rightarrow \quad \frac{1}{2} MR^2\omega_1 = \frac{1}{2} R^2 (M + 4m)\omega_2$$

or $\omega_2 = \left(\frac{M}{M+4m}\right)\omega_1$

72. A particle of mass m moves along line PC with velocity v as shown. What is the angular momentum of the particle about O? **[AIEEE 2002]**

(a) mvL (b) mvl (c) mvr (d) Zero

Exp. **(b)**

Angular momentum of particle about O,

$\mathbf{V} = m(\mathbf{r} \times \mathbf{v})$

$|\mathbf{V}| = mrv \sin\theta = mv(r\sin\theta) = mvl$

73. Moment of inertia of a circular wire of mass M and radius R about its diameter is **[AIEEE 2002]**

(a) $\frac{MR^2}{2}$ (b) MR^2 (c) $2MR^2$ (d) $\frac{MR^2}{4}$

Exp. **(a)**

Moment of inertia of circular wire about its axis is MR^2. Consider two diameters xx and yy moment of inertia about any of these diameters is same let us say I.

From perpendicular axes theorem,

$$I + I = MR^2 \quad \text{or} \quad I = \frac{MR^2}{2}$$

74. A solid sphere, a hollow sphere and a ring are released from top of an inclined plane (frictionless), so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling) **[AIEEE 2002]**

(a) solid sphere (b) hollow sphere
(c) ring (d) All same

Exp. **(d)**

Since, the inclined plane is frictionless, then there will be no rolling and the mass will only slide down. Hence, acceleration is same for all the given bodies.

75. Two identical particles move towards each other with velocity $2v$ and v, respectively. The velocity of centre of mass is **[AIEEE 2002]**

(a) v (b) $\frac{v}{3}$
(c) $\frac{v}{2}$ (d) zero

Exp. **(c)**

Let mass of each body be m. Their motion is represented as shown in figure.

From $$\mathbf{v}_{CM} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2}$$

$$\mathbf{v}_{CM} = \frac{m \times 2v - mv}{m + m} = \frac{v}{2}$$

[the direction of motion of first particle is taken as positive]

So, velocity of centre of mass of the system is $\frac{v}{2}$ in the direction of motion of particle having larger speed.

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6

Gravitation

1. Four identical particles of mass M are located at the corners of a square of side a. What should be their speed, if each of them revolves under the influence of other's gravitational field in a circular orbit circumscribing the square ?

[JEE Main 2019, 8 April Shift-I]

(a) $1.35\sqrt{\frac{GM}{a}}$ (b) $1.16\sqrt{\frac{GM}{a}}$

(c) $1.21\sqrt{\frac{GM}{a}}$ (d) $1.41\sqrt{\frac{GM}{a}}$

Exp. **(b)**

> **Key Idea** In given configuration of masses, net gravitational force provides the necessary centripetal force for rotation.

Net force on mass M at position B towards centre of circle is

$$F_{BO\,\text{net}} = F_{BD} + F_{BA}\sin 45° + F_{BC}\cos 45°$$

$$= \frac{GM^2}{(\sqrt{2}a)^2} + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right) + \frac{GM^2}{a^2}\left(\frac{1}{\sqrt{2}}\right)$$

[where, diagonal length BD is $\sqrt{2}a$]

$$= \frac{GM^2}{2a^2} + \frac{GM^2}{a^2}\left(\frac{2}{\sqrt{2}}\right) = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

This force will act as centripetal force.

Distance of particle from centre of circle is $\frac{a}{\sqrt{2}}$.

Here, $F_{\text{centripetal}} = \frac{Mv^2}{r} = \frac{Mv^2}{\frac{a}{\sqrt{2}}}$

$$= \frac{\sqrt{2}Mv^2}{a} \qquad \left(\because r = \frac{a}{\sqrt{2}}\right)$$

So, for rotation about the centre,

$$F_{\text{centripetal}} = F_{BO(\text{net})}$$

$$\Rightarrow \quad \sqrt{2}\frac{Mv^2}{a} = \frac{GM^2}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$

$$\Rightarrow \quad v^2 = \frac{GM}{a}\left(1 + \frac{1}{2\sqrt{2}}\right) = \frac{GM}{a}(1.35)$$

$$\Rightarrow \quad v = 1.16\sqrt{\frac{GM}{a}}$$

2. A rocket has to be launched from earth in such a way that it never returns. If E is the minimum energy delivered by the rocket launcher, what should be the minimum energy that the launcher should have, if the same rocket is to be launched from the surface of the moon? Assume that the density of the earth and the moon are equal and that the earth's volume is 64 times the volume of the moon.

[JEE Main 2019, 8 April Shift-II]

(a) $\frac{E}{64}$ (b) $\frac{E}{16}$ (c) $\frac{E}{32}$ (d) $\frac{E}{4}$

Exp. **(b)**

Given, volume of earth (V_e) is 64 times of volume of moon (V_m), i.e.

$$\frac{V_e}{V_m} = 64 = \frac{\frac{4}{3}\pi R_e^3}{\frac{4}{3}\pi R_m^3}$$

where, R_e and R_m are the radius of earth and moon, respectively.

Then, $$\frac{R_e}{R_m} = 4 \quad \ldots(i)$$

Also, since the density of moon and earth are equal, i.e.

$$\rho_m = \rho_e$$

$\Rightarrow \frac{M_e}{V_e} = \frac{M_m}{V_m}$, where M_e and M_m are the mass of the earth and moon, respectively.

$$\Rightarrow \quad \frac{M_e}{M_m} = \frac{V_e}{V_m} = 64 \quad \ldots(ii)$$

The minimum energy or escape energy delivered by the rocket launcher, so that the rocket never returns to earth is

$$E_e = \frac{GM_e m}{R_e} = E$$

where, m is the mass of the rocket.

Similarly, minimum energy that a launcher should have to escape or to never return, if rocket is launched from surface of the moon is

$$E_m = \frac{GM_m m}{R_m}$$

$\therefore$ Ratio of escape energies E_e and E_m is

$$\frac{E_e}{E_m} = \frac{\left(\frac{GM_e m}{R_e}\right)}{\left(\frac{GM_m m}{R_m}\right)} = \frac{M_e}{M_m} \cdot \frac{R_m}{R_e}$$

$$= 64 \times \frac{1}{4} = 16 \quad \text{[using Eqs. (i) and (ii)]}$$

$$\therefore \quad E_m = \frac{E_e}{16} = \frac{E}{16}$$

3. A solid sphere of mass M and radius a is surrounded by a uniform concentric spherical shell of thickness $2a$ and $2M$. The gravitational field at distance $3a$ from the centre will be **[JEE Main 2019, 9 April Shift-I]**

(a) $\frac{GM}{9a^2}$ (b) $\frac{2GM}{9a^2}$ (c) $\frac{GM}{3a^2}$ (d) $\frac{2GM}{3a^2}$

Exp. **(c)**

According to question, diagram will be as follows

Gravitational field due to solid sphere of radius a at a distance, $r = 3a$, i.e. ($r > R$) is

$$E_1 = \frac{GM}{r^2} = \frac{GM}{(3a)^2} = \frac{GM}{9a^2}$$

Similarly, gravitational field due to spherical shell at a distance, $r = 3a$,

i.e. $$E_2 = \frac{GM}{R^2} = \frac{G(2M)}{(3a)^2} = \frac{2GM}{9a^2}$$

Both fields are attractive in nature, so direction will be same.

Net field, $$E_{net} = \frac{GM}{9a^2} + \frac{2GM}{9a^2}$$

$$E_{net} = \frac{GM}{3a^2}$$

4. A test particle is moving in a circular orbit in the gravitational field produced by mass density $\rho(r) = \frac{K}{r^2}$. Identify the correct relation between the radius R of the particle's orbit and its period T **[JEE Main 2019, 9 April Shift-II]**

(a) $\frac{T^2}{R^3}$ is a constant (b) $\frac{T}{R^2}$ is a constant

(c) TR is a constant (d) $\frac{T}{R}$ is a constant

Exp. **(d)**

Let the mass of the test particle be m and its orbital linear speed be v.

Force of gravity of the mass-density would provide the necessary centripetal pull on test particle.

$\Rightarrow$ Gravitational force (F_g) = Centripetal force (F_c)

Let us now assume an elementary ring at a distance r from the centre of the mass density, such that mass of this elementary ring

$$\Rightarrow \quad \frac{GMm}{R^2} = \frac{mv^2}{R}$$

where, $M = \int_0^R \rho \cdot 4\pi r^2 \cdot dr$

$$\Rightarrow \quad \frac{G\left(\int_0^R \frac{K}{r^2} \cdot 4\pi r^2 \cdot dr\right)}{R^2} = \frac{v^2}{R} \quad \left[\because \text{given, } \rho(r) = \frac{K}{r^2}\right]$$

$$\Rightarrow \quad G \cdot 4\pi K = v^2$$

$\therefore$ Orbital speed of mass, $m = v = \sqrt{G4\pi K}$ ... (i)

Time period of rotation of the test particle is

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G \cdot 4\pi K}} \quad [\because \text{using Eq. (i)}]$$

Hence, $\frac{T}{R} = \sqrt{\frac{\pi}{GK}}$ = a constant

Alternate Solution

Since, $F_g = F_c$

This relation can also be written directly as,

$$\frac{G\int_0^R \left(\frac{K}{r^2} \cdot 4\pi r^2 \cdot dr\right)}{R^2} = m\omega^2 R = m\left(2\frac{\pi}{T}\right)^2 R$$

$$\Rightarrow \frac{G \cdot 4\pi KR}{R^2} = \frac{4\pi^2}{T^2} R \quad \text{or} \quad \frac{T^2}{R^2} = \frac{\pi}{GK}$$

$$\Rightarrow \quad \frac{T}{R} = \sqrt{\frac{\pi}{GK}} = \text{a constant.}$$

5. The value of acceleration due to gravity at earth's surface is 9.8 ms^{-2}. The altitude above its surface at which the acceleration due to gravity decreases to 4.9 ms^{-2}, is close to (Take, radius of earth $= 6.4 \times 10^6$ m)

[JEE Main 2019, 10 April Shift-I]

(a) 9.0×10^6 m (b) 2.6×10^6 m
(c) 6.4×10^6 m (d) 1.6×10^6 m

Exp. **(b)**

> **Key Idea** The relation of gravitational acceleration at an altitude h above the earth's surface is given as
>
> $$g_h = g\left(1 + \frac{h}{R_e}\right)^{-2}$$
>
> where, g is the acceleration due to gravity at earth's surface and R_e is the radius of the earth.

Given that at some height h, acceleration due to gravity,

$$g_h = 4.9 \text{ m/s}^2 \approx \frac{g}{2} \quad \text{... (i)}$$

$\therefore$ The ratio of acceleration due to gravity at earth's surface and at some altitude h is

$$\left(1 + \frac{h}{R_e}\right) = \sqrt{\frac{g}{g_h}} = \sqrt{2} \quad \text{[From Eq. (i)]}$$

$$\therefore \quad \frac{h}{R_e} = \sqrt{2} - 1$$

or $h = 0.414 \times R_e$

$h = 0.414 \times 6400$ km

($\because$ given, radius of earth, $R_e = 6400$ km $= 6.4 \times 10^6$ m)

or $h = 2649.6$ km $= 2.6 \times 10^6$ m

Thus, at 2.6×10^6 m above the earth's surface, the acceleration due to gravity decreases to 4.9 m/s^2.

6. A spaceship orbits around a planet at a height of 20 km from its surface. Assuming that only gravitational field of the planet acts on the spaceship, what will be the number of complete revolutions made by the spaceship in 24 hours around the planet? [Take, mass of planet $= 8 \times 10^{22}$ kg, radius of planet $= 2 \times 10^6$ m, gravitational constant $G = 6.67 \times 10^{-11}$ N-m^2/kg^2]

[JEE Main 2019, 10 April Shift-II]

(a) 11 (b) 17 (c) 13 (d) 9

Exp. **(a)**

A satellite or spaceship in a circular orbit at a distance $(R + h)$ from centre of a planet experiences a gravitational force given by

$$F_g = \frac{GmM}{(R + h)^2}$$

where, M = mass of planet,
m = mass of spaceship,
R = radius of planet
and h = height of spaceship above surface.

This gravitational pull provides necessary centripetal pull for orbital motion of spaceship.

So, $F_g = F_{\text{centripetal}} \Rightarrow \dfrac{GmM}{(R+h)^2} = \dfrac{mv^2}{(R+h)}$

where, v = orbital speed of spaceship.

Orbital speed of spaceship is

$$v = \sqrt{\frac{GM}{(R+h)}} \quad \ldots(\text{i})$$

Here, $G = 6.67 \times 10^{-11}$ N-m^2/kg^2,

$M = 8 \times 10^{22}$ kg

$R + h = (2 \times 10^6 + 20 \times 10^3)$ m

$= 2.02 \times 10^6$ m

So, substituting these values in Eq. (i), we get

$$v = \sqrt{\frac{6.67 \times 10^{-11} \times 8 \times 10^{22}}{2.02 \times 10^6}}$$

$= 1.6 \times 10^3$ ms^{-1}

Time period of rotation of spaceship will be

$$T = \frac{2\pi(R+h)}{v} \Rightarrow T = \frac{2\pi \times 2.02 \times 10^6}{1.6 \times 10^3}$$

$$\approx 8 \times 10^3 \text{ s} = \frac{8 \times 10^3}{60 \times 60} \text{ (h)} = 2.2 \text{ h}$$

So, number of revolutions made by spaceship in 24 h,

$$n = \frac{24}{T} = \frac{24}{2.2} \approx 11 \text{ rev}$$

7. The ratio of the weights of a body on the earth's surface, so that on the surface of a planet is 9 : 4. The mass of the planet is $\frac{1}{9}$th of that of the earth. If R is the radius of the earth, what is the radius of the planet? (Take, the planets to have the same mass density)

[JEE Main 2019, 12 April Shift-II]

(a) $\frac{R}{3}$ (b) $\frac{R}{4}$ (c) $\frac{R}{9}$ (d) $\frac{R}{2}$

Exp. **(d)**

Let mass of given body is m. Then, it's weight on earth's surface = mg_e

where, g_e = acceleration due to gravity on earth's surface and weight on the surface of planet = mg_p

g_p = acceleration due to gravity on planet's surface.

Given,

$$\frac{mg_e}{mg_p} = \frac{9}{4} \Rightarrow \frac{g_e}{g_p} = \frac{9}{4}$$

But $g = \dfrac{GM}{R^2}$, so we have

$$\frac{\left(\dfrac{GM}{R^2}\right)}{\left(\dfrac{GM_p}{R_p^2}\right)} = \frac{9}{4}$$

where, M = mass of earth,
R = radius of earth,
M_p = mass of planet = $\dfrac{M}{9}$ (given)
and R_p = radius of planet.

$$\Rightarrow \quad \frac{M}{M_p} \cdot \frac{R_p^2}{R^2} = \frac{9}{4} \Rightarrow 9 \cdot \left(\frac{R_p}{R}\right)^2 = \frac{9}{4}$$

$$\Rightarrow \quad \frac{R_p}{R} = \frac{1}{2} \Rightarrow R_p = \frac{R}{2}$$

8. If the angular momentum of a planet of mass m, moving around sun in a circular orbit is L about the centre of the sun , its areal velocity is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\frac{4L}{m}$ (b) $\frac{2L}{m}$
(c) $\frac{L}{2m}$ (d) $\frac{L}{m}$

Exp. **(c)**

According to Kepler's second law, "the line joining the planet to sun sweeps out equal areas in equal interval of time". This means the rate of change of area with time is constant.

The area covered from P to P' is dA, which is given by

$$dA = \frac{d\theta}{2\pi} \times \pi r^2$$

$$\Rightarrow \quad dA = \frac{1}{2} r^2 d\theta \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\theta}{dt}$$

where, $\frac{dA}{dt}$ = areal velocity.

$$\frac{dA}{dt} = \frac{1}{2} r^2\omega \text{ or } \frac{dA}{dt} = \frac{1}{2} r^2 \cdot \frac{L}{mr^2}$$

(because angular momentum,

$$L = mr^2\omega)$$

$$\frac{dA}{dt} = \frac{L}{2m}$$

9. In form of G (universal gravitational constant), h (Planck constant) and c (speed of light), the time period will be proportional to **[JEE Main 2019, 9 Jan Shift-II**

(a) $\sqrt{\frac{Gh}{c^5}}$ (b) $\sqrt{\frac{hc^5}{G}}$ (c) $\sqrt{\frac{c^3}{Gh}}$ (d) $\sqrt{\frac{Gh}{c^3}}$

Exp. **(a)**

Key Idea According to dimensional analysis, if a physical quantity (let y) depends on another physical quantities (let A, B, C), then the dimension formula is given by

$$y = A^a B^b C^c$$

(where, a, b, c are the power of physical quantity and k = constant)

For the given question, the time (t) depends on the G (gravitational constant), h(Planck's constant) and c (velocity of light), then according to dimensional analysis

$$t = k(G)^a (h)^b (c)^c \quad \ldots(i)$$

For calculating the values of a, b and c, compare the dimensional formula for both side.

LHS t = time = $[M^0L^0T^1]$

RHS $G = \frac{F \cdot r^2}{m_1 m_2} = \frac{\text{kg ms}^{-2} \times \text{m}^2}{(\text{kg})^2} = [M^{-1}L^3T^{-2}]^a$

$$h = [M^1L^2T^{-1}]^b$$

$$c = \frac{d}{t} = [M^0L^1T^{-1}]^c$$

Compare both side for powers of M, L and T,

$$M \Rightarrow 0 = -a + b \quad \ldots(i)$$

$$L \Rightarrow 0 = 3a + 2b + c \quad \ldots(ii)$$

$$T \Rightarrow 1 = -2a - b - c \quad \ldots(iii)$$

Solving Eqs. (i), (ii), (iii), we get

$$a = \frac{1}{2}, \quad b = \frac{1}{2} \text{ and } c = \frac{-5}{2}$$

So, put these values in Eq. (i), we get

$$t = k\, G^{1/2}\, h^{1/2}\, c^{-5/2}$$

$$t = k\sqrt{\frac{Gh}{c^5}}$$

10. The energy required to take a satellite to a height 'h' above earth surface (where, radius of earth $= 6.4 \times 10^3$ km) is E_1 and kinetic energy required for the satellite to be in a circular orbit at this height is E_2. The value of h for which E_1 and E_2 are equal is **[JEE Main 2019, 9 Jan Shift-II]**

(a) 3.2×10^3 km (b) 1.28×10^4 km

(c) 6.4×10^3 km (d) 1.6×10^3 km

Exp. **(a)**

The energy required for taking a satellite upto a height h from earth's surface is the difference between the energy at h height and energy at surface, then

$$\Rightarrow \quad E_1 = U_f - U_i$$

$$E_1 = -\frac{GM_e m}{R_e + h} + \frac{GM_e m}{R_e} \quad \ldots(i)$$

(where, U = potential energy)

$\therefore$ Orbital velocity of satellite,

$$v_o = \sqrt{\frac{GM_e}{(R_e + h)}} \text{ (where, } M_e \text{ = mass of earth)}$$

So energy required to perform circular motion

$$\Rightarrow \quad E_2 = \frac{1}{2} m v_o^2 = \frac{GM_e m}{2(R_e + h)}$$

$$E_2 = \frac{GM_e m}{2(R_e + h)} \quad \ldots(ii)$$

According to the question,

$$E_1 = E_2$$

$$\therefore \quad \frac{-GM_e m}{R_e + h} + \frac{GM_e m}{R_e} = \frac{GM_e m}{2(R_e + h)}$$

$$\Rightarrow \quad 3R_e = 2R_e + 2h$$

$$h = \frac{R_e}{2}$$

As radius of earth, $R_e \approx 6.4 \times 10^3$km

Hence, $h = \frac{6.4 \times 10^3}{2}$ km

or $= 3.2 \times 10^3$ km

11. A satellite is moving with a constant speed v in circular orbit around the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of ejection, the kinetic energy of the object is

[JEE Main 2019, 10 Jan Shift-I]

(a) $\frac{3}{2}mv^2$ (b) $2mv^2$ (c) mv^2 (d) $\frac{1}{2}mv^2$

Exp. **(c)**

Kinetic energy must be given to an object so that it just escapes to infinity is as shown in the figure below

Applying energy-conservation for mass m, we get

Initial energy of m at satellite = Final energy of m at infinity.

$$\Rightarrow \quad U_i + K_i = U_f + K_f \quad \ldots(i)$$

Here, U_i = initial PE of m

$$= \frac{-G \cdot Mm}{r} = -(v^2)m$$

As, $v = \sqrt{\frac{GM}{r}}$ = orbital speed of the satellite.

K_i = initial kinetic energy of m,

U_f = final potential energy of m which is zero and

K_f = final kinetic energy of m which is also zero.

So, from Eq. (i), we have

$$-v^2m + K_i = 0 + 0 \text{ or } K_i = mv^2.$$

12. Two stars of masses 3×10^{31} kg each and at distance 2×10^{11} m rotate in a plane about their common centre of mass O. A meteorite passes through O moving perpendicular to the star's rotation plane. In order to escape from the gravitational field of this double star, the minimum speed that meteorite should have at O is (Take, gravitational constant, $(G = 6.67 \times 10^{-11}\ \text{N-m}^2\text{kg}^{-2})$

[JEE Main 2019, 10 Jan Shift-II]

(a) 2.8×10^5 m/s (b) 3.8×10^4 m/s

(c) 2.4×10^4 m/s (d) 1.4×10^5 m/s

Exp. **(a)**

Let us assume that stars are moving in xy-plane with origin as their centre of mass as shown in the figure below

According to question,

mass of each star, $M = 3 \times 10^{31}$ kg

and diameter of circle, $2R = 2 \times 10^{11}$m

$$\Rightarrow \quad R = 10^{11} \text{ m}$$

Potential energy of meteorite at O, origin $\hat{\mathbf{j}}$ is,

$$U_{\text{total}} = -\frac{2\,GMm}{r}$$

If v is the velocity of meteorite at O then

Kinetic energy K of the meteorite is

$$K = \frac{1}{2}mv^2$$

To escape from this dual star system, total mechanical energy of the meteorite at infinite distance from stars must be at least zero.

By conservation of energy, we have

$$\frac{1}{2}mv^2 - \frac{2\,GMm}{R} = 0$$

$$\Rightarrow v^2 = \frac{4\,GM}{R} = \frac{4 \times 6.67 \times 10^{-11} \times 3 \times 10^{31}}{10^{11}}$$

$$\Rightarrow \quad v = 2.83 \times 10^5 \text{m/s}$$

13. A satellite is revolving in a circular orbit at a height h from the earth surface such that $h << R$, where R is the radius of the earth. Assuming that the effect of earth's atmosphere can be neglected the minimum

increase in the speed required so that the satellite could escape from the gravitational field of earth is **[JEE Main 2019, 11 Jan Shift-I]**

(a) $\sqrt{\frac{gR}{2}}$ (b) $\sqrt{gR}$

(c) $\sqrt{2gR}$ (d) $\sqrt{gR}(\sqrt{2}-1)$

Exp. **(d)**

Orbital velocity of the satellite is given as,

$$v_O = \sqrt{\frac{GM}{R+h}}$$

Since, $R >> h$

$$\therefore \quad v_O = \sqrt{\frac{GM}{R}} = \sqrt{gR} \quad \left[\because g = \frac{GM}{R^2}\right]$$

Escape velocity of the satellite,

$$v_e = \sqrt{\frac{2\,GM}{R+h}} = \sqrt{\frac{2\,GM}{R}} = \sqrt{2gR}$$

Since, we know that in order to escape the earth's gravitational field a satellite must get escape velocity.

$\therefore$ Change in velocity,

$$\Delta v = v_e - v_O = \sqrt{gR}\,(\sqrt{2}-1)$$

14. A straight rod of length L extends from $x = a$ to $x = L + a$. The gravitational force it exerts on a point mass m at $x = 0$, if the mass per unit length of the rod is $A + Bx^2$, is given by **[JEE Main 2019, 12 Jan Shift-I]**

(a) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)-BL\right]$

(b) $Gm\left[A\left(\frac{1}{a+L}-\frac{1}{a}\right)+BL\right]$

(c) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$

(d) $Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)-BL\right]$

Exp. **(c)**

Given, situation is,

Force of attraction between mass 'm' and an elemental mass 'dm' of rod is

$$dF = \frac{Gmdm}{x^2} = \frac{Gm(A+Bx^2)\,dx}{x^2}$$

Total attraction force is sum of all such differential forces produced by elemental parts of rod from $x = a$ to $x = a + L$.

$$\therefore \quad F = \int dF = \int_{x=a}^{x=a+L} \frac{Gm(A+Bx^2)}{x^2}dx$$

$$= Gm\int_{x=a}^{x=a+L}\left(\frac{A}{x^2}+B\right)dx$$

$$= Gm\left[-\frac{A}{x}+Bx\right]_{x=a}^{x=a+L}$$

$$= Gm\left(\frac{-A}{a+L}+B(a+L)+\frac{A}{a}-Ba\right)$$

$$= Gm\left(\frac{A}{a}-\frac{A}{a+L}+BL\right)$$

$$= Gm\left[A\left(\frac{1}{a}-\frac{1}{a+L}\right)+BL\right]$$

15. A satellite of mass M is in a circular orbit of radius R about the centre of the earth. A meteorite of the same mass falling towards the earth collides with the satellite completely inelastically. The speeds of the satellite and the meteorite are the same just before the collision. The subsequent motion of the combined body will be **[JEE Main 2019, 12 Jan Shift-I]**

(a) in the same circular orbit of radius R

(b) in an elliptical orbit

(c) such that it escapes to infinity

(d) in a circular orbit of a different radius

Exp. **(b)**

According to the given condition in the question, after collision the mass of combined system is doubled. Also, this system would be displaced from its circular orbit.

So, the combined system revolves around centre of mass of 'system + earth' under action of a central force. Hence, orbit must be elliptical.

16. Two satellites A and B have masses m and $2m$ respectively. A is in a circular orbit of radius R and B is in a circular orbit of radius $2R$ around the earth. The ratio of their kinetic energies, T_A / T_B is **[JEE Main 2019, 12 Jan Shift-II]**

(a) $\frac{1}{2}$ (b) 2 (c) $\sqrt{\frac{1}{2}}$ (d) 1

Exp. **(*d*)**

Orbital speed of a satellite in a circular orbit is

$$v_0 = \sqrt{\left(\frac{GM}{r_0}\right)}$$

where r_0 is the radius of the circular orbit.

So, kinetic energies of satellites A and B are

$$T_A = \frac{1}{2} m_A v_{OA}^2 = \frac{GMm}{2R}$$

$$T_B = \frac{1}{2} m_B v_{OB}^2 = \frac{GM(2m)}{2(2R)} = \frac{GMm}{2R}$$

So, ratio of their kinetic energies is $\frac{T_A}{T_B} = 1$

17. The variation of acceleration due to gravity g with distance d from centre of the Earth is best represented by (R = Earth's radius) **[JEE Main 2017 (Offline)]**

Exp. **(*d*)**

Inside the earth's surface,

$$g = \frac{GM}{R^3} r$$

i.e. $g \propto r$

Out the earth's surface,

$$g = \frac{Gm}{r^2}$$

i.e. $g \propto \frac{1}{r^2}$

So, till earth's surface g increases linearly with distance r, shown only in graph (c).

18. A satellite is revolving in a circular orbit at a height h from the Earth's surface (radius of Earth $R, h << R$). The minimum increase in its orbital velocity required, so that the satellite could escape from the Earth's gravitational field, is close to (Neglect the effect of atmosphere) **[JEE Main 2017 (Offline)]**

(a) $\sqrt{2gR}$ (b) $\sqrt{gR}$

(c) $\sqrt{gR/2}$ (d) $\sqrt{gR}(\sqrt{2}-1)$

Exp. **(*d*)**

Given, a satellite is revolving in a circular orbit at a height h from the Earth's surface having radius of Earth R, i.e. $h << R$.

Orbital velocity of a satellite,

$$v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{GM}{R}} \quad (\text{as } h << R)$$

Velocity required to escape,

$$\frac{1}{2} mv'^2 = \frac{GMm}{R+h}$$

$$v' = \sqrt{\frac{2GM}{R+h}} = \sqrt{\frac{2GM}{R}} \quad (h << R)$$

$\therefore$ Minimum increase in its orbital velocity required to escape from the Earth's gravitational field.

$$v' - v = \sqrt{\frac{2GM}{R}} - \sqrt{\frac{GM}{R}}$$

$$= \sqrt{2gR} - \sqrt{gR} = \sqrt{gR}(\sqrt{2}-1) \left(\because g = \frac{GM}{R^2}\right)$$

19. Four particles, each of mass M and equidistant from each other, move along a circle of radius R under the action of their mutual gravitational attraction, the speed of each particle is **[JEE Main 2014]**

(a) $\sqrt{\frac{GM}{R}}$ (b) $\sqrt{2\sqrt{2}\frac{GM}{R}}$

(c) $\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$ (d) $\frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$

Exp. **(*d*)**

Net force acting on any one particle M

$$= \frac{GM^2}{(2R)} + \frac{GM^2}{(R\sqrt{2})^2}\cos 45° + \frac{GM}{(R\sqrt{2})^2}\cos 45°$$

$$= \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$$

This force will equal to centripetal force.

So, $\frac{Mu^2}{R} = \frac{GM^2}{R^2}\left(\frac{1}{4} + \frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad u = \sqrt{\frac{GM}{4R}(1 + 2\sqrt{2})} = \frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$

Hence, speed of each particle in a circular motion is

$$\frac{1}{2}\sqrt{\frac{GM}{R}(2\sqrt{2} + 1)}$$

20. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of $2R$? **[JEE Main 2013]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$

(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{3R}$

Exp. **(a)**

From conservation of energy,

Total energy at the planet

= Total energy at altitude

$$-\frac{GMm}{R} + (\text{KE})_{\text{surface}} = -\frac{GMm}{3R} + \frac{1}{2}mv_A^2 \quad \ldots(\text{i})$$

In the orbital of planet, the necessary centripetal force is obtained by gravitational force.

$$\therefore \quad \frac{mv_A^2}{R + 2R} = \frac{GMm}{(R + 2R)^2}$$

$$\Rightarrow \quad \frac{v_A}{2} = \frac{GM}{3R} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{\text{surface}} = \frac{5}{6}\frac{GMm}{R}$$

21. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is **[AIEEE 2012]**

(a) $m_1 r_1 : m_2 r_2$ (b) $m_1 : m_2$

(c) $r_1 : r_2$ (d) $1 : 1$

Exp. **(c)**

As their period of revolution is same, so their angular speed is also same. Centripetal acceleration is circular path, $a = \omega^2 r$.

Thus, $\frac{a_1}{a_2} = \frac{\omega^2 r_1}{\omega^2 r_2} = \frac{r_1}{r_2}$

22. The mass of a spaceship is 1000 kg. It is to be launched from the earth's surface out into free space. The value of g and R (radius of earth) are 10 m/s^2 and 6400 km, respectively. The required energy for this work will be **[AIEEE 2012]**

(a) 6.4×10^{11} J (b) 6.4×10^{8} J

(c) 6.4×10^{9} J (d) 6.4×10^{10} J

Exp. **(d)**

Potential energy on earth surface is $-mgR$ while in free space, it is zero. So, to free the spaceship, minimum required energy is

$$K = mgR = 10^3 \times 10 \times 6400 \times 10^3 \text{ J}$$

$$= 6.4 \times 10^{10} \text{ J}$$

$$\because \quad \text{PE} = \frac{-GmM}{R}$$

On earth surface, as $g = \frac{GM}{R^2}$

So, $\text{PE} = -\frac{gR^2 m}{R} = -mgR$

23. Two particles of equal mass m go around a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle with respect to their centre of mass is **[AIEEE 2011]**

(a) $\sqrt{\frac{Gm}{R}}$ (b) $\sqrt{\frac{Gm}{4R}}$ (c) $\sqrt{\frac{Gm}{3R}}$ (d) $\sqrt{\frac{Gm}{2R}}$

Exp. **(b)**

As gravitational force provides necessary centripetal force.

i.e., $F = \frac{Gm^2}{(2R)^2} = \frac{mv^2}{R} \Rightarrow v = \sqrt{\frac{Gm}{4R}}$

24. Two bodies of masses m and $4m$ are placed at a distance r. The gravitational potential at a point on the line joining them, where the gravitational field is zero, is **[AIEEE 2011]**

(a) $-\frac{4Gm}{r}$ (b) $-\frac{6Gm}{r}$ (c) $-\frac{9Gm}{r}$ (d) zero

Exp. **(c)**

Let gravitational field be zero at P as shown in figure.

$$\therefore \quad \frac{Gm}{x^2} = \frac{G(4m)}{(r-x)^2}$$

$$\Rightarrow \quad 4x^2 = (r-x)^2 \quad \Rightarrow \quad 2x = r - x$$

$$\Rightarrow \quad x = \frac{r}{3}$$

$$\therefore \quad V_p = -\frac{Gm}{x} - \frac{G(4m)}{r-x}$$

$$= -\frac{3Gm}{r} - \frac{6GM}{r} = -\frac{9Gm}{r}$$

25. The height at which the acceleration due to gravity becomes $\frac{g}{9}$ (where, g is the acceleration due to gravity on the surface of the earth) in terms of R, the radius of the earth is **[AIEEE 2009]**

(a) $2R$ (b) $\frac{R}{\sqrt{3}}$ (c) $\frac{R}{2}$ (d) $\sqrt{2}R$

Exp. **(a)**

As value of acceleration due to gravity above the earth surface is g.

$g' = \frac{GM}{(R+h)^2}$, acceleration due to gravity at height h

$$\Rightarrow \quad \frac{g}{9} = \frac{GM}{R^2} \cdot \frac{R^2}{(R+h)^2} = g\left(\frac{R}{R+h}\right)^2$$

$$\Rightarrow \quad \frac{1}{9} = \left(\frac{R}{R+h}\right)^2 \quad \Rightarrow \quad \frac{R}{R+h} = \frac{1}{3}$$

$$\Rightarrow \quad 3R = R + h \quad \Rightarrow 2R = h$$

26. A planet in a distant solar system is 10 times more massive than the earth and its radius is 10 times smaller. Given that the escape velocity from the earth is 11 kms^{-1}, the escape velocity from the surface of the planet would be **[AIEEE 2008]**

(a) 1.1 kms^{-1} (b) 11 kms^{-1}
(c) 110 kms^{-1} (d) 0.11 kms^{-1}

Exp. **(c)**

Mass of planet, $M_p = 10M_e$, where M_e is mass of earth. Radius of planet, $R_p = \frac{R_e}{10}$, where R_e is radius of earth.

Escape velocity is given by, $v_e = \sqrt{\frac{2GM}{R}}$

For planet, $v_p = \sqrt{\frac{2G \times M_p}{R_p}} = \sqrt{\frac{100 \times 2GM_e}{R_e}}$

$$= 10 \times v_e = 10 \times 11 = 110 \text{ kms}^{-1}$$

➡ **Directions** *Q. Nos. 27 is Assertion-Reason type question. This question contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). The question also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.*

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is true, Statement II is false
(d) Statement I is fase, Statement II is true

27. **Statement I** For a mass M kept at the centre of a cube of side a, the flux of gravitational field passing through its sides is $4\pi GM$.

Statement II If the direction of a field due to a point source is radial and its dependence on the distance r from the source is given as $1/r^2$, its flux through a closed surface depends only on the strength of the source enclosed by the surface and not on the size or shape of the surface. **[AIEEE 2008]**

Exp. **(a)**

Correct option is (a), you can make an analogy with Gauss's law in electrostatics.

28. If g_E and g_M are the accelerations due to gravity on the surfaces of the earth and the moon respectively and if Millikan's oil drop experiment could be performed on the two surfaces, one will find the ratio $\frac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}}$ to be **[AIEEE 2007]**

(a) 1 (b) zero (c) $\frac{g_E}{g_M}$ (d) $\frac{g_M}{g_E}$

Exp. **(a)**

According to Millikan's oil drop experiment, electronic charge is given by

$$q = \frac{6\pi\eta r(v_1 + v_2)}{E}$$

which is independent of g.

So, $\dfrac{\text{electronic charge on the moon}}{\text{electronic charge on the earth}} = 1$

29. Average density of the earth **[AIEEE 2005]**

(a) does not depend on g

(b) is a complex function of g

(c) is directly proportional to g

(d) is inversely proportional to g

Exp. **(c)**

Mass of earth = Volume × Density

As $g = \dfrac{GM}{R^2}$; $M = \left(\dfrac{4}{3}\pi R^3\right)\rho$

$\therefore$ $g = \dfrac{4G}{3}\dfrac{\pi R^3}{R^2}\rho$

or $g = \left(\dfrac{4G\pi R}{3}\right)\rho$

[where ρ = average density]

$\Rightarrow$ $g \propto \rho$ or $\rho \propto g$

30. The change in the value of g at a height h above the surface of the earth is the same as at a depth d below the surface of earth. When both d and h are much smaller than the radius of earth, then which one of the following is correct? **[AIEEE 2005]**

(a) $d = \dfrac{h}{2}$ (b) $d = \dfrac{3h}{2}$ (c) $d = 2h$ (d) $d = h$

Exp. **(c)**

Acceleration due to gravity at height h.

$$g_h = g\left(1 - \frac{2h}{R}\right) \quad [\text{if } h << R] \ldots \text{(i)}$$

Acceleration due to gravity at depth d.

$$g_d = g\left(1 - \frac{d}{R}\right) \quad \ldots \text{(ii)}$$

As per statement of the problem,

$$g_h = g_d$$

i.e., $g\left(1 - \dfrac{2h}{R}\right) = g\left(1 - \dfrac{d}{R}\right) \Rightarrow 2h = d$

31. A particle of mass 10 g is kept on the surface of a uniform sphere of mass 100 kg and radius 10 cm. Find the work to be done against the gravitational force between them, to take the particle far away from the sphere, (you may take $G = 6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2$) **[AIEEE 2005]**

(a) 13.34×10^{-10} J (b) 3.33×10^{-10} J

(c) 6.67×10^{-9} J (d) 6.67×10^{-10} J

Exp. **(d)**

The initial potential energy of the particle = Work done.

$$U_i = -\frac{GMm}{r}$$

$$U_i = -\frac{6.67 \times 10^{-11} \times 100 \times 10^{-2}}{0.1}$$

$$U_i = -\frac{6.67 \times 10^{-11}}{0.1} = -6.67 \times 10^{-10}\ \text{J}$$

We know that, work done = Difference in potential energy

$\therefore$ $W = \Delta U = U_f - U_i$

$\Rightarrow$ $W = -U_i$ [$\because U_f = 0$]

$= 6.67 \times 10^{-10}$ J

32. A satellite of mass m revolves around the earth of radius R at a height x from its surface. If g is the acceleration due to gravity on the surface of the earth, the orbital speed of the satellite is **[AIEEE 2004]**

(a) gx (b) $\dfrac{gR}{R-x}$ (c) $\dfrac{gR^2}{R+x}$ (d) $\left(\dfrac{gR^2}{R+x}\right)^{1/2}$

Exp. **(d)**

The gravitational force exerted on satellite at a height x is

$$F_G = \frac{GM_e m}{(R+x)^2}$$

where, M_e = mass of earth

Since, gravitational force provides the necessary centripetal force, so

$$\frac{GM_e m}{(R+x)^2} = \frac{mv_o^2}{(R+x)}$$

where, v_o is orbital speed of satellite.

$\Rightarrow$ $\dfrac{GM_e m}{(R+x)} = mv_o^2$

or $$\frac{gR^2m}{(R+x)} = mv_o^2 \qquad \left[\because g = \frac{GM_e}{R^2}\right]$$

or $$v_o = \sqrt{\left[\frac{gR^2}{(R+x)}\right]} = \left[\frac{gR^2}{(R+x)}\right]^{1/2}$$

33. The time period of an earth satellite in circular orbit is independent of **[AIEEE 2004]**

(a) the mass of the satellite
(b) radius of its orbit
(c) both the mass and radius of the orbit
(d) neither the mass of the satellite nor the radius of its orbit

Exp. **(a)**

Time period of satellite,

$$T = 2\pi\sqrt{\frac{(R+h)^3}{GM_e}}$$

where, $R + h$ = orbital radius of satellite

and M_e = mass of earth.

Thus, time period does not depend on mass of satellite.

34. If g is the acceleration due to gravity on the earth's surface, the gain in the potential energy of an object of mass m raised from the surface of the earth to a height equal to the radius R of the earth, is **[AIEEE 2004]**

(a) $2mgR$ (b) $\frac{1}{2}mgR$ (c) $\frac{1}{4}mgR$ (d) mgR

Exp. **(b)**

Gravitational potential energy of body on the earth's surface is

$$U = -\frac{GM_em}{R}$$

At a height h from earth's surface, its value is

$$U_h = -\frac{GM_em}{(R+h)} = -\frac{GM_em}{2R} \qquad [\text{as } h = R]$$

where, M_e = mass of earth,
m = mass of body
and R = radius of earth.

$\therefore$ Gain in potential energy

$$= U_h - U = -\frac{GM_em}{2R} - \left(-\frac{GM_em}{R}\right)$$

$$= -\frac{GM_em}{2R} + \frac{GM_em}{R}$$

$$= \frac{GM_em}{2R} = \frac{gR^2m}{2R} \qquad \left[\text{as } g = \frac{GM_e}{R^2}\right]$$

$$= \frac{1}{2}mgR$$

35. Suppose the gravitational force varies inversely as the nth power of distance. Then, the time period of a planet in circular orbit of radius R around the sun will be proportional to **[AIEEE 2004]**

(a) $R^{\left(\frac{n+1}{2}\right)}$ (b) $R^{\left(\frac{n-1}{2}\right)}$ (c) R^n (d) $R^{\left(\frac{n-2}{2}\right)}$

Exp. **(a)**

The necessary centripetal force required for a planet to move round the sun

= Gravitational force exerted on it

i.e., $$\frac{mv^2}{R} = \frac{GM_em}{R^n} \quad \text{or} \quad v = \left(\frac{GM_e}{R^{n-1}}\right)^{1/2}$$

Now, $$T = \frac{2\pi R}{v} = 2\pi R \times \left(\frac{R^{n-1}}{GM_e}\right)^{1/2}$$

$$= 2\pi\left(\frac{R^2 \times R^{n-1}}{GM_e}\right)^{1/2} = 2\pi\left(\frac{R^{(n+1)/2}}{(GM_e)^{1/2}}\right)$$

or $$T \propto R^{(n+1)/2}$$

36. The time period of a satellite of earth is 5 h. If the separation between the earth and the satellite is increased to 4 times the previous value, the new time period will become **[AIEEE 2003]**

(a) 10 h (b) 80 h (c) 40 h (d) 20 h

Exp. **(c)**

According to Kepler's law,

$$T^2 \propto r^3 \quad \Rightarrow \quad 5^2 \propto r^3 \qquad \ldots(i)$$

and $$(T')^2 \propto (4r)^3 \qquad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3} \quad \text{or} \quad T' = \sqrt{1600} \quad \text{or} \quad T' = 40 \text{ h}$$

37. Two spherical bodies of mass M and $5M$ and radii R and $2R$ respectively are released in free space with initial separation between their centres equal to $12R$. If they attract each other due to gravitational force only, then the distance covered by the smaller body just before collision is **[AIEEE 2003]**

(a) $2.5R$ (b) $4.5R$ (c) $7.5R$ (d) $1.5R$

Exp. **(c)**

Just before collision, total distance travelled by both

$$= 12\,R - R - 2R = 9\,R$$

If x is the distance travelled by block of mass M and $9R - x$ the distance travelled by block of mass $5M$.

As centre of mass of system not change, then

$$Mx = 5\,M\,(9R - x) \Rightarrow x = 7.5\,R$$

38. The escape velocity for a body projected vertically upwards from the surface of earth is 11 km/s. If the body is projected at an angle of 45° with the vertical, then the escape velocity will be **[AIEEE 2003]**

(a) $11\sqrt{2}$ km/s (b) 22 km/s

(c) 11 km/s (d) $\frac{11}{\sqrt{2}}$ m/s

Exp. **(c)**

The escape velocity is independent of angle of projection, hence it will remain same *i.e.*, 11 km/s.

39. Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is **[AIEEE 2002]**

(a) $\frac{GMm}{12R^2}$ (b) $\frac{GMm}{3R^2}$ (c) $\frac{GMm}{8R}$ (d) $\frac{GMm}{6R}$

Exp. **(d)**

Gravitational potential energy of body will be

$$E = -\frac{GMm}{r}$$

where, M = mass of earth,

m = mass of the body

and R = radius of earth.

At $r = 2R$,

$$E_1 = -\frac{GMm}{(2R)}$$

At $r = 3R$,

$$E_2 = -\frac{GMm}{(3R)}$$

Energy required to move a body of mass m from an orbit of radius $2R$ to $3R$ is

$$\Delta E = \frac{GMm}{R}\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{GMm}{6R}$$

40. The kinetic energy needed to project a body of mass m from the earth's surface (radius R) to infinity is **[AIEEE 2002]**

(a) $\frac{mgR}{2}$ (b) $2\,mgR$ (c) mgR (d) $\frac{mgR}{4}$

Exp. **(c)**

The minimum kinetic energy required to project a body of mass m from earth's surface to infinity is known as escape energy. Therefore,

$$\text{KE} = \frac{GM_e m}{R} = mgR \qquad \left[\because g\,R = \frac{GM_e}{R}\right]$$

41. The escape velocity of a body depends upon mass as **[AIEEE 2002]**

(a) m^0 (b) m^1 (c) m^2 (d) m^3

Exp. **(a)**

Escape velocity = $\sqrt{2gR_e}$

So, escape velocity is independent of m. So, it depends upon mass m as m^0.

Since, $m^0 = 1$.

42. If suddenly the gravitational force of attraction between earth and a satellite revolving around it becomes zero, then the satellite will **[AIEEE 2002]**

(a) continue to move in its orbit with same velocity

(b) move tangentially to the original orbit with the same velocity

(c) become stationary in its orbit

(d) move towards the earth

Exp. **(b)**

When gravitational force becomes zero, then centripetal force required cannot be provided. So, the satellite will move with the velocity as it has at the instant when gravitational force becomes zero, *i.e.*, it moves tangentially to the original orbit.

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7

Properties *of* Solids *and* Liquids

1. A steel wire having a radius of 2.0 mm, carrying a load of 4 kg, is hanging from a ceiling. Given that $g = 3.1\pi\ \text{ms}^{-2}$, what will be the tensile stress that would be developed in the wire?

(a) $6.2 \times 10^6\ \text{Nm}^{-2}$ (b) $5.2 \times 10^6\ \text{Nm}^{-2}$
(c) $3.1 \times 10^6\ \text{Nm}^{-2}$ (d) $4.8 \times 10^6\ \text{Nm}^{-2}$

[JEE Main 2019, 8 April Shift-I]

Exp. **(c)**

Given, radius of wire, $r = 2\ \text{mm} = 2 \times 10^{-3}$ m
Weight of load, $m = 4$ kg, $g = 3.1\ \pi\text{ms}^{-2}$

$$\therefore \text{ Tensile stress} = \frac{\text{Force (F)}}{\text{Area (A)}} = \frac{mg}{\pi r^2}$$

$$= \frac{4 \times 3.1 \times \pi}{\pi \times (2 \times 10^{-3})^2}$$

$$= 3.1 \times 10^6 \text{Nm}^{-2}$$

2. A boy's catapult is made of rubber cord which is 42 cm long, with 6 mm diameter of cross-section and of negligible mass. The boy keeps a stone weighing 0.02 kg on it and stretches the cord by 20 cm by applying a constant force. When released the stone flies off with a velocity of 20 ms^{-1}. Neglect the change in the area of cross-section of the cord while stretched. The Young's modulus of rubber is closest to

[JEE Main 2019, 8 April Shift-I]

(a) $10^6\ \text{Nm}^{-2}$ (b) $10^4\ \text{Nm}^{-2}$
(c) $10^8\ \text{Nm}^{-2}$ (d) $10^3\ \text{Nm}^{-2}$

Exp. **(a)**

When rubber cord is stretched, then it stores potential energy and when released, this potential energy is given to the stone as kinetic energy.

So, potential energy of stretched cord
= kinetic energy of stone

$$\Rightarrow \quad \frac{1}{2} Y \left(\frac{\Delta L}{L}\right)^2 A \cdot L = \frac{1}{2} m v^2$$

Here, $\Delta L = 20$ cm $= 0.2$ m, $L = 42$ cm $= 0.42$ m, $v = 20\ \text{ms}^{-1}$, $m = 0.02$ kg, $d = 6$ mm $= 6 \times 10^{-3}$ m

$$\therefore A = \pi r^2 = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{6 \times 10^{-3}}{2}\right)^2$$

$$= \pi(3 \times 10^{-3})^2 = 9\pi \times 10^{-6}\ \text{m}^2$$

On substituting values, we get

$$Y = \frac{mv^2 L}{A(\Delta L)^2} = \frac{0.02 \times (20)^2 \times 0.42}{9\pi \times 10^{-6} \times (0.2)^2} \approx 3.0 \times 10^6 \text{Nm}^{-2}$$

So, the closest value of Young's modulus is $10^6\ \text{Nm}^{-2}$.

3. Water from a pipe is coming at a rate of 100 liters per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of (density of water = $1000\,\text{kg/m}^3$, coefficient of viscosity of water = 1 mPa s)

[JEE Main 2019, 8 April Shift-I]

(a) 10^3 (b) 10^4 (c) 10^2 (d) 10^6

Exp. **(b)**

Reynolds' number for flow of a liquid is given by

$$R_e = \frac{\rho v D}{\eta}$$

where, velocity of flow,

$$v = \frac{\text{volume flow rate}}{\text{area of flow}} = \frac{V/t}{A}$$

So, $$R_e = \frac{\rho V D}{\eta A t} = \frac{\rho V 2r}{\eta \times \pi r^2 \times t} = \frac{2\rho V}{\eta \pi r t}$$

Here, ρ = density of water = $1000\,\text{kgm}^{-3}$

$$\frac{V}{t} = \frac{100 \times 10^{-3}}{60}\,\text{m}^3\text{s}^{-1}$$

where, η = viscosity of water = 1×10^{-3} Pa-s

and r = radius of pipe = 5×10^{-2} m

$$R_e = \frac{2 \times 1000 \times 100 \times 10^{-3}}{1 \times 10^{-3} \times 60 \times 3.14 \times 5 \times 10^{-2}}$$

$$= 212.3 \times 10^2 \approx 2.0 \times 10^4$$

So, order of Reynolds' number is of 10^4.

4. Two identical beakers A and B contain equal volumes of two different liquids at 60°C each and left to cool down. Liquid in A has density of $8 \times 10^2\,\text{kg/m}^3$ and specific heat of $2000\,\text{J kg}^{-1}\text{K}^{-1}$ while liquid in B has density of 10^3 kg m^{-3} and specific heat of 4000 J kg^{-1}K^{-1}. Which of the following best describes their temperature *versus* time graph schematically? (Assume the emissivity of both the beakers to be the same) **[JEE Main 2019, 8 April Shift-I]**

(a)

(b)

(c)

(d)

Exp. **(b)**

> **Key Idea** From Newton's law of cooling, we have rate of cooling,
>
> $$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0)$$
>
> where, h = heat transfer coefficient,
> T = temperature of body,
> T_0 = temperature of surrounding,
> m = mass and s = specific heat.

We know, $m = V \cdot \rho$

where, V = volume and ρ = density.

So, we have

$$\frac{dQ}{dt} = \frac{h}{ms}(T - T_0) = \frac{h(T - T_0)}{V \cdot \rho s}$$

Since, h, $(T - T_0)$ and V are constant for both beaker.

$$\therefore \quad \frac{dQ}{dt} \propto \frac{1}{\rho s}$$

We have given that $\rho_A = 8 \times 10^2\,\text{kgm}^{-3}$, $\rho_B = 10^3\,\text{kgm}^{-3}$, $s_A = 2000\,\text{J kg}^{-1}\,\text{K}^{-1}$ and $s_B = 4000\,\text{J kg}^{-1}\,\text{K}^{-1}$,

$\rho_A s_A = 16 \times 10^5$ and $\rho_B s_B = 4 \times 10^6$

So, $\rho_A < \rho_B$, $s_A < s_B$ and $\rho_A s_A < \rho_B s_B$

$$\Rightarrow \quad \frac{1}{\rho_A s_A} > \frac{1}{\rho_B s_B} \quad \Rightarrow \quad \frac{dQ_A}{dt} > \frac{dQ_B}{dt}$$

So, for container B, rate of cooling is smaller than the container A. Hence, graph of B lies above the graph of A and it is not a straight line (slope of A is greater than B).

5. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to **[JEE Main 2019, 8 April Shift-II]**

(a) 1.3 mm (b) 1.5 mm
(c) 1.9 mm (d) 1.7 mm

Exp. **(d)**

When a wire is stretched, then change in length of wire is

$\Delta l = \frac{Fl}{\pi r^2 Y}$, where Y is its Young's modulus.

Here, for wires A and B,

$$l_A = 2 \text{ m}, l_B = 1.5 \text{ m},$$

$\frac{Y_A}{Y_B} = \frac{7}{4}$, $r_B = 2 \text{ mm} = 2 \times 10^{-3}$ m and $\frac{F_A}{F_B} = 1$

As, it is given that $\Delta l_A = \Delta l_B$

$$\Rightarrow \quad \frac{F_A l_A}{\pi r_A^2 Y_A} = \frac{F_B l_B}{\pi r_B^2 Y_B}$$

$$\Rightarrow r_A^2 = \frac{F_A}{F_B} \cdot \frac{l_A}{l_B} \cdot \frac{Y_B}{Y_A} \cdot r_B^2 = 1 \times \frac{2}{1.5} \times \frac{4}{7} \times 4 \times 10^{-6} \text{ m}$$

$$= 3.04 \times 10^{-6} \text{ m}$$

or $r_A = 1.7 \times 10^{-3}$ m or $r_A = 1.7$ mm

6. If M is the mass of water that rises in a capillary tube of radius r, then mass of water which will rise in a capillary tube of radius $2r$ is **[JEE Main 2019, 9 April Shift-I]**

(a) $2M$ (b) $4M$ (c) $\frac{M}{2}$ (d) M

Exp. **(a)**

Height of liquid rise in capillary tube,

$$h = \frac{2T\cos\theta_c}{\rho r g}$$

$$\Rightarrow \quad h \propto \frac{1}{r}$$

So, when radius is doubled, height becomes half.

$\therefore h' = h/2$

Now, density $(\rho) = \frac{\text{mass}(M)}{\text{volume}(V)}$

$$\Rightarrow \quad M = \rho \times \pi r^2 h$$

$$\therefore \quad M' = \rho \pi r'^2 h'$$

$$\therefore \quad \frac{M'}{M} = \frac{r'^2 h'}{r^2 h} = \frac{(2r)^2 (h/2)}{r^2 h} = 2$$

$$\Rightarrow \quad M' = 2M$$

Alternate Solution

According to the given figure, force inside the capillary tube is

$$2\pi r T = Mg \Rightarrow M \propto r$$

When $r' = 2r$, then

$$M' = 2M$$

7. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is **[JEE Main 2019, 9 April Shift-II]**

(a) 0.6 (b) 0.8
(c) 0.7 (d) 0.5

Exp. **(a)**

For a floating body,

upthrust = weight of the part of object, i.e. submerged in the fluid.

In first situation,

So, weight of block of volume V = weight of water of volume $\frac{4}{5}V \Rightarrow V\rho_b g = \frac{4}{5}V\rho_w g$

where, ρ_b = density of block

and ρ_w = density of water.

$$\Rightarrow \quad \frac{\rho_b}{\rho_w} = \frac{4}{5} \quad \ldots \text{(i)}$$

In second situation,

So, weight of block of volume V = weight of oil of volume $\frac{V}{2}$ + weight of water of volume $\frac{V}{2}$.

$$\Rightarrow \quad V\rho_b g = \frac{V}{2}\rho_o g + \frac{V}{2}\rho_w g$$

where, ρ_o = density of oil.

$\Rightarrow \quad 2\rho_b = \rho_o + \rho_w$

$\Rightarrow \quad \dfrac{2\rho_b}{\rho_w} = \dfrac{\rho_o}{\rho_w} + 1$

$\Rightarrow \quad 2 \times \dfrac{4}{5} = \dfrac{\rho_o}{\rho_w} + 1$ [using Eq. (i)]

$\Rightarrow \quad \rho_0 / \rho_w = 8/5 - 1 = 3/5 = 0.6$

8. The ratio of surface tensions of mercury and water is given to be 7.5 while the ratio of their densities is 13.6. Their contact angles with glass are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio (r_1 / r_2), is then close to

[JEE Main 2019, 10 April Shift-I]

(a) 3/5 (b) 2/3 (c) 2/5 (d) 4/5

Exp. **(c)**

Given,

$$\frac{T_{Hg}}{T_W} = 7.5, \ \frac{\rho_{Hg}}{\rho_W} = 13.6$$

and $$\frac{\cos\theta_{Hg}}{\cos\theta_W} = \frac{\cos 135°}{\cos 0°} = \frac{1}{\sqrt{2}}$$

Height of the fluid inside capillary tube is given by

$$h = \frac{2T\cos\theta}{\rho g r}$$

According to given situation, $h_W = h_{Hg}$

$$\therefore \quad \frac{2\,T_W \cos\theta_W}{\rho_W\, g\, r_W} = \frac{2T_{Hg}\cos\theta_{Hg}}{\rho_{Hg}\, g\, r_{Hg}}$$

$$\therefore \quad \frac{r_{Hg}}{r_W} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)$$

Given, $r_{Hg} = r_1$ and $r_W = r_2$, then

Substituting the given values, we get

$$\frac{r_{Hg}}{r_W} = \frac{r_1}{r_2} = 7.5 \times \frac{1}{\sqrt{2}} \times \frac{1}{13.6} = 0.4 = 2/5$$

9. A submarine experience a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa, then $d_2 - d_1$ is approximately (density of water $= 10^3$ kg/ m^3 and acceleration due to gravity $= 10$ ms^{-2})

[JEE Main 2019, 10 April Shift-II]

(a) 500 m (b) 400 m
(c) 600 m (d) 300 m

Exp. **(d)**

Pressure inside a fluid volume open to atmosphere is

$$p = p_0 + h\rho g$$

where, p = pressure at depth h, h = depth,

ρ = density of fluid

and g = acceleration due to gravity.

In problem given,

when $h = d_1$, pressure $p_1 = 5.05 \times 10^6$ Pa and

when $h = d_2$, pressure $p_2 = 8.08 \times 10^6$ Pa

So, we have

$$p_1 = p_0 + d_1\rho g = 5.05 \times 10^6$$

and $$p_2 = p_0 + d_2\rho g = 8.08 \times 10^6$$

$$\Rightarrow p_2 - p_1 = (d_2 - d_1)\rho g = 3.03 \times 10^6$$

$$\Rightarrow d_2 - d_1 = \frac{3.03 \times 10^6}{\rho g}$$

Given, $\rho = 10^3$ kgm^{-3} and $g = 10$ ms^{-2}

$$\therefore d_2 - d_1 = \frac{3.03 \times 10^6}{10^3 \times 10} = 303 \text{ m} \approx 300 \text{ m}$$

10. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water? [Take, density of water $= 10^3$ kg/ m^3]

[JEE Main 2019, 10 April Shift-II]

(a) 30.1 kg (b) 46.3 kg
(c) 87.5 kg (d) 65.4 kg

Exp. **(c)**

When only block is floating on water, 30% of its volume is in water as shown below

By Archimedes' principle,

weight of block = weight of displaced water

$\Rightarrow \quad V\rho_b g = (30\%V)\rho_w g$

where, ρ_b = density of block,

ρ_w = density of water

and V = volume of block.

So, $\rho_b = 0.3\,\rho_w$

Now, let a mass m is placed over block to just keep the cube fully immersed in water.

Now, by Archimedes' principle,

weight of water displaced = weight of block + weight of mass m

$\Rightarrow \quad V\rho_w g = V\rho_b g + mg$

$\Rightarrow \quad V(\rho_w - \rho_b) = m \Rightarrow V \times 0.7\,\rho_w = m$

$[\because \rho_b = 0.3\rho_w]$

Here, $V = (\text{side})^3 = (0.5)^3\,\text{m}^3$

$\rho_w = 10^3\ \text{kg m}^{-3}$

Substituting these values in the above relation, we get $m = (0.5)^3 \times 0.7 \times 10^3 = 87.5\,\text{kg}$

11. In an experiment, brass and steel wires of length 1 m each with areas of cross-section $1\,\text{mm}^2$ are used. The wires are connected in series and one end of the combined wire is connected to a rigid support and other end is subjected to elongation. The stress requires to produce a net elongation of 0.2 mm is [Take, the Young's modulus for steel and brass are respectively $120 \times 10^9\,\text{N/m}^2$ and $60 \times 10^9\,\text{N/m}^2$]

[JEE Main 2019, 10 April Shift-II]

(a) $1.2 \times 10^6\ \text{N/m}^2$ (b) $0.2 \times 10^6\ \text{N/m}^2$

(c) $1.8 \times 10^6\ \text{N/m}^2$ (d) $4.0 \times 10^6\ \text{N/m}^2$

Exp. **(*)**

In given experiment, a composite wire is stretched by a force F.

Net elongation in the wire = elongation in brass wire + elongation in steel wire ...(i)

Now, Young's modulus of a wire of cross-section (A) when some force (F) is applied, $Y = \dfrac{Fl}{A\Delta l}$

We have,

$$\Delta l = \text{elongation} = \frac{Fl}{AY}$$

So, from relation (i), we have

$$\Delta l_{net} = \Delta l_{brass} + \Delta l_{steel}$$

$$\Rightarrow \quad \Delta l_{net} = \left(\frac{Fl}{AY}\right)_{brass} + \left(\frac{Fl}{AY}\right)_{steel}$$

As wires are connected in series and they are of same area of cross-section, length and subjected to same force, so

$$\Delta l_{net} = \frac{F}{A}\left(\frac{l}{Y_{brass}} + \frac{l}{Y_{steel}}\right)$$

Here,

$\Delta l_{net} = 0.2\ \text{mm}$

$= 0.2 \times 10^{-3}\ \text{m}$

and $l = 1\,\text{m}$

$Y_{brass} = 60 \times 10^9\,\text{Nm}^{-2}$,

$Y_{steel} = 120 \times 10^9\,\text{Nm}^{-2}$

On putting the values, we have

$$0.2 \times 10^{-3} = \frac{F}{A}\left(\frac{1}{60 \times 10^9} + \frac{1}{120 \times 10^9}\right)$$

$$\Rightarrow \text{Stress} = \frac{F}{A} = 8 \times 10^6\,\text{Nm}^{-2}$$

12. Water from a tap emerges vertically downwards with an initial speed of $1.0\ \text{ms}^{-1}$. The cross-sectional area of the tap is $10^{-4}\,\text{m}^2$. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be

[Take, $g = 10\ \text{ms}^{-2}$]

[JEE Main 2019, 10 April Shift-II]

(a) $2 \times 10^{-5}\,\text{m}^2$

(b) $1 \times 10^{-5}\,\text{m}^2$

(c) $5 \times 10^{-4}\,\text{m}^2$

(d) $5 \times 10^{-5}\,\text{m}^2$

Exp. **(*d*)**

Given situation is as shown in the figure below

From equation of continuity,

$$A \propto \frac{1}{v}$$

where, A = area of flow
and v = velocity of flow.

∴ Increase in speed of flow causes a decrease in area of flow.

Here given that height of fall, h = 0.15 m

Area,

$$A = 10^{-4}\text{m}^2$$

Initial speed, $v = 1\,\text{ms}^{-1}$

Velocity of water stream below h height is

$$v_2 = \sqrt{v_1^2 + 2gh} \quad [\because v^2 - u^2 = 2gh]$$

Substituting the given values, we get

$$= \sqrt{1^2 + 2 \times 10 \times 0.15} = \sqrt{4} = 2\ \text{ms}^{-1}$$

Now, from equation of continuity, we have

$$A_1v_1 = A_2v_2$$

or $$A_2 = \frac{A_1v_1}{v_2}$$

∴ $$A_2 = \frac{10^{-4} \times 1}{2}$$

$$= 0.5 \times 10^{-4} = 5 \times 10^{-5}\ \text{m}^2$$

13. The elastic limit of brass is 379 MPa. What should be the minimum diameter of a brass rod, if it is to support a 400 N load without exceeding its elastic limit?

[JEE Main 2019, 10 April Shift-II]

(a) 0.90 mm (b) 1.00 mm
(c) 1.16 mm (d) 1.36 mm

Exp. **(c)**

Let d_{min} = minimum diameter of brass.

Then, stress in brass rod is given by

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_{min}^2} \qquad \left[\because A = \frac{\pi d^2}{4}\right]$$

For stress not to exceed elastic limit, we have

$\sigma \le 379$ MPa

$$\Rightarrow \quad \frac{4F}{\pi d^2_{min}} \le 379 \times 10^6$$

Here, $F = 400$ N

$$\therefore \quad d_{min}^2 = \frac{1600}{\pi \times 379 \times 10^6}$$

$$\Rightarrow \quad d_{min} = 1.16 \times 10^{-3}\ \text{m} = 1.16\ \text{mm}$$

14. When M_1 gram of ice at –10°C (specific heat = 0.5 cal g^{-1}°C^{-1}) is added to M_2 gram of water at 50°C, finally no ice is left and the water is at 0°C. The value of latent heat of ice, in cal g^{-1} is

(a) $\frac{50M_2}{M_1} - 5$ (b) $\frac{50M_1}{M_2} - 50$
(c) $\frac{50M_2}{M_1}$ (d) $\frac{5M_2}{M_1} - 5$

[JEE Main 2019, 12 April Shift-I]

Exp. **(a)**

> **Key Idea** In such kind of heat transfer problems,
> Heat given by water = Heat gained by ice
> and $(\text{heat})_{\text{liquid}}$ = mass × specific heat × temperature = $\{m_l\ s_l\ \Delta T_l\}$
> $(\text{Heat})_{\text{solid}}$ = (mass × latent heat)
> + (mass × specific heat × temperature)
> $= (m_s \times L) + m_s \times s_s \times \Delta T_s$

Heat given by water is (specific heat of water is 1 cal g^{-1} °C^{-1})

$$(\Delta H)_{\text{water}} = M_2 \times 1 \times (50 - 0) = 50M_2 \qquad \ldots(\text{i})$$

Heat taken by ice is

$$(\Delta H)_{\text{ice}} = M_1 \times 0.5 \times [0 - (-10)] + M_1 \times L_{\text{ice}}$$

$$\Rightarrow \quad (\Delta H)_{ice} = 5M_1 + M_1 L_{ice} \quad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$(\Delta H)_{water} = (\Delta H)_{ice}$$

$$\therefore \quad 50M_2 = 5M_1 + M_1 L_{ice}$$

$$\Rightarrow \quad L_{ice} = \frac{50M_2 - 5M_1}{M_1}$$

$$L_{ice} = 50\frac{M_2}{M_1} - 5$$

15. At 40°C, a brass wire of 1 mm radius is hung from the ceiling. A small mass M is hung from the free end of the wire. When the wire is cooled down from 40°C to 20°C, it regains its original length of 0.2 m. The value of M is close to [Coefficient of linear expansion and Young's modulus of brass are 10^{-5}/°C and 10^{11} N/m^2 respectively, $g = 10$ ms^{-2}]

[JEE Main 2019, 12 April Shift-I]

(a) 9 kg (b) 0.5 kg
(c) 1.5 kg (d) 0.9 kg

Exp. **(a)**

Given, $T_1 = 40^\circ$ C and $T_2 = 20^\circ$ C

$$\Rightarrow \quad \Delta T = T_1 - T_2 = 40 - 20 = 20^\circ \text{ C}$$

Also, Young's modulus,

$$Y = 10^{11} \text{ N/m}^2$$

Coefficient of linear expansion,

$$\alpha = 10^{-5}/^\circ\text{C}$$

Area of the brass wire, $A = \pi \times (10^{-3})^2$ m^2

Now, expansion in the wire due to rise in temperature is

$$\Delta l = l\,\alpha\Delta T \Rightarrow \frac{\Delta l}{l} = \alpha\Delta T \quad \ldots(i)$$

We know that, Young's modulus is defined as

$$Y = \frac{Mgl}{A\Delta l} \Rightarrow M = \frac{YA\Delta l}{gl} \quad \ldots(ii)$$

Using Eq. (i), we get

$$M = \frac{YA}{g} \times \alpha\Delta T$$

$$= \frac{10^{11} \times 22 \times 10^{-6} \times 10^{-5} \times 20}{7 \times 10}$$

$$\Rightarrow \quad M = \frac{22 \times 20}{7 \times 10} = \frac{44}{7}$$

$$= 6.28 \text{ kg}$$

Which is closest to 9, so option (a) is nearly correct.

16. A solid sphere of radius R acquires a terminal velocity v_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, v_2 when falling through the same fluid, the ratio (v_1 / v_2) equals

[JEE Main 2019, 12 April Shift-II]

(a) 9 (b) 1/27 (c) 1/9 (d) 27

Exp. **(a)**

> **Key Idea** Terminal speed of a sphere falling in a viscous fluid is
>
> $$v_T = \frac{2}{9}\frac{r^2}{\eta}(\rho_0 - \rho_f)g$$
>
> where, η = coefficient of viscosity of fluid,
> ρ_0 = density of falling sphere
> and ρ_f = density of fluid.

As we know, if other parameters remains constant, terminal velocity is proportional to square of radius of falling sphere.

i.e. $$v_T \propto r^2 \quad \ldots(i)$$

Now, when sphere of radius R is broken into 27 identical solid sphere of radius r, then

Volume of sphere of radius $R = 27 \times$ Volume of sphere of radius r

$$\Rightarrow \quad \frac{4}{3}\pi R^3 = 27 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad R = 3r \Rightarrow r = \frac{R}{3}$$

So, from Eq. (i), we have

$$\frac{v_1}{v_2} = \frac{R^2}{\left(\frac{R}{3}\right)^2} = 9$$

17. A uniform cylindrical rod of length L and radius r, is made from a material whose Young's modulus of elasticity equals Y. When this rod is heated by temperature T and simultaneously subjected to a net longitudinal compressional force F, its length remains unchanged. The coefficient of volume expansion of the material of the rod, is (nearly) equal to

[JEE Main 2019, 12 April Shift-II]

(a) $9F/(\pi r^2 YT)$ (b) $6F/(\pi r^2 YT)$
(c) $3F/(\pi r^2 YT)$ (d) $F/(3\pi r^2 YT)$

Exp. **(c)**

As length of rod remains unchanged,

Strain caused by compressive forces is equal and opposite to the thermal strain.

Now, compressive strain is obtained by using formula for Young's modulus,

$$Y = \frac{\frac{F}{A}}{\frac{\Delta l}{l}}$$

Compressive strain,

$$\Rightarrow \quad \frac{\Delta l}{l} = \frac{F}{AY} = \frac{F}{\pi Y r^2} \quad \ldots(i)$$

Also, thermal strain in rod is obtained by using formula for expansion in rod,

$$\Delta l = l\,\alpha\,\Delta T$$

$\Rightarrow$ Thermal strain,

$$\frac{\Delta l}{l} = \alpha\,\Delta T \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{F}{\pi r^2 Y} = \alpha\,T \quad [\because \Delta T = T]$$

$$\Rightarrow \quad \alpha = \frac{F}{\pi r^2 Y T}$$

Hence, coefficient of volumetric expansion of rod is

$$\gamma = 3\alpha = \frac{3F}{\pi r^2 Y T}$$

18 One kilogram of water at 20°C is heated in an electric kettle whose heating element has a mean (temperature averaged) resistance of 20 Ω. The rms voltage in the mains is 200 V. Ignoring heat loss from the kettle, time taken for water to evaporate fully, is close to [Specific heat of water = 4200 J/(kg°C), Latent heat of water = 2260 kJ/kg]

[JEE Main 2019, 12 April Shift-II]

(a) 16 min (b) 22 min
(c) 3 min (d) 10 min

Exp. **(b)**

Heat required by water for getting hot and then evaporated is

$$\Delta Q = ms\,\Delta T + mL$$

Here, $m = 1$ kg, $\Delta T = 100° - 20° = 80°$ C,

$s = 4200$ J kg^{-1} °C^{-1}, $L = 2260 \times 10^3$ J kg^{-1}

So, heat required is

$$\Delta Q = 1 \times 4200 \times 80 + 1 \times 2260 \times 10^3$$
$$= 336 \times 10^3 + 2260 \times 10^3$$
$$= 2596 \times 10^3 \text{ J} \quad \ldots(i)$$

This heat is provided by a heating coil of resistance $R = 20\,\Omega$ connected with AC mains $V_{rms} = 200$ V

So, heat supplied by heater coil is

$$Q = Pt = \frac{V_{rms}^2}{R} \times t$$

where, P = power and t = time. $= \frac{(200)^2}{20} \times t$

$$= 2 \times 10^3 \times t \quad \ldots(ii)$$

Substituting the value of heat from Eq. (ii), we get

$$t = \frac{2956 \times 10^3}{2 \times 10^3}\text{ s} = \frac{2956}{2 \times 60}\text{ min} = 24.63\text{ min}$$

Nearest answer is 22 min.

19. A rod of length L at room temperature and uniform area of cross-section A, is made of a metal having coefficient of linear expansion α/°C. It is observed that an external compressive force F, is applied on each of its ends, prevents any change in the length of the rod, when its temperature rises by ΔT K. Young's modulus, Y for this metal is

[JEE Main 2019, 9 Jan Shift-I]

(a) $\frac{F}{2A\alpha\,\Delta T}$ (b) $\frac{F}{A\alpha(\Delta T - 273)}$
(c) $\frac{2F}{A\alpha\Delta T}$ (d) $\frac{F}{A\alpha\Delta T}$

Exp. **(d)**

If a rod of length L and coefficient of linear expansion α/°C, then with the rise in temperature by ΔT K, its change in length is given as,

$$\Delta L = L\,\alpha\,\Delta T$$

$$\Rightarrow \quad \frac{\Delta L}{L} = \alpha\,\Delta T \quad \ldots(i)$$

Also, when a rod is subjected to some compressive force (F), then its' Young's modulus is given as $Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\frac{F}{A}}{\frac{\Delta L}{L}}$

$$\frac{\Delta L}{L} = \frac{F}{YA} \quad \ldots(ii)$$

Since, it is given that the length of the rod does not change. So, from Eqs. (i) and (ii), we get

$$\alpha \Delta T = \frac{F}{YA} \Rightarrow \quad Y = \frac{F}{A\alpha \Delta T}$$

20. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius is its wall. The depth of the centre of the opening from the level of water in the tank is close to **[JEE Main 2019, 9 Jan Shift-II]**

(a) 4.8 m (b) 6.0 m
(c) 2.9 m (d) 9.6 m

Exp. **(a)**

For the given condition, a water tank is open to air and its water level maintained.

Suppose the depth of the centre of the opening from level of water in tank is h and the radius of opening is r.

According to question, the water per minute through a circular opening

flow rate $(Q) = 0.74\ m^3/\text{min} = \frac{0.74}{60}\ m^3/s$

r = radius of circular opening = 2 cm

Here, the area of circular opening = $\pi(r^2)$

$a = \pi \times (2 \times 10^{-2})^2 = 4\pi \times 10^{-4}\ m^2$

Now flow rate through an area is given by

Q = Area of circular opening $\times$ Velocity of water

$$Q = a \times v = \pi(r^2) \times v$$

$$\Rightarrow \quad \frac{0.74}{60} = (4\pi \times 10^{-4}) \times v \qquad \ldots(i)$$

According to Torricelli's law (velocity of efflux)

$$v = \sqrt{2gh} \qquad \ldots(ii)$$

Equation value of 'v' from Eqs. (i) and (ii), we get

$$\sqrt{2gh} = \frac{0.74 \times 10^4}{60 \times 4\pi}$$

$$\Rightarrow \quad h = \left(\frac{0.74 \times 10^4}{60 \times 4\pi}\right)^2 \times \frac{1}{2g}$$

$$h \approx 4.8\ m$$

21. Water flows into a large tank with flat bottom at the rate of $10^{-4}\ m^3s^{-1}$. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is **[JEE Main 2019, 10 Jan Shift-I]**

(a) 4 cm (b) 2.9 cm (c) 5.1 cm (d) 1.7 cm

Exp. **(c)**

As, level of water in tank remains constant with time, so (water inflow rate) = (outflow rate of water)

$\Rightarrow 10^{-4}\ m^3s^{-1}$ = Area of orifice $\times$ Velocity of outflow

$\Rightarrow 10^{-4}\ m^3s^{-1} = 10^{-4} \times \sqrt{2gh}$

where, h = Height of water above the orifice or hole.

$$\Rightarrow \quad \sqrt{2gh} = 1$$

or $\quad 2 \times 9.8 \times h = 1$

$$\Rightarrow \quad h = \frac{1}{19.6}\ m = \frac{100}{19.6}\ cm$$

or $\quad h = 5.1\ cm$

22. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected 25% losses all of its momentum and, 25% comes back with the same speed. The resultant pressure on the mesh will be **[JEE Main 2019, 11 Jan Shift-I]**

(a) ρv^2 (b) $\frac{1}{2}\rho v^2$
(c) $\frac{1}{4}\rho v^2$ (d) $\frac{3}{4}\rho v^2$

Exp. **(d)**

Mass per unit time of a liquid flow is given by

$$\frac{dm}{dt} = \rho A v$$

where, ρ is density of liquid, A is area through which it is flowing and v is velocity.

∴ Rate of change in momentum of the 25% of liquid which loses all momentum is

$$\frac{dp_1}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right)v = \frac{1}{4}\rho A v^2 \qquad \ldots(i)$$

and the rate of change in momentum of the 25% of the liquid which comes back with same speed.

$$\frac{dp_2}{dt} = \frac{1}{4}\left(\frac{dm}{dt}\right) \times 2v = \frac{1}{2}\rho A v^2 \quad \ldots(ii)$$

[$\because$ Net change in velocity is $= 2v$]

$\therefore$ Net pressure on the mesh is

$$p = \frac{F_{net}}{A} = \frac{(dp_1/dt + dp_2/dt)}{A} \quad \left[\because F = \frac{dp}{dt}\right]$$

$\therefore$ From Eqs. (i) and (ii), we get

$$p = \frac{3}{4}\rho v^2 A / A = \frac{3}{4}\rho v^2$$

23. When 100 g of a liquid A at 100°C is added to 50 g of a liquid B at temperature 75°C, the temperature of the mixture becomes 90°C. The temperature of the mixture, if 100 g of liquid A at 100°C is added to 50 g of liquid B at 50°C will be **[JEE Main 2019, 11 Jan Shift-II]**

(a) 60°C (b) 80°C (c) 70°C (d) 85°C

***Exp.* (b)**

In first case according to principle of calorimetry, heat lost by liquid A = heat gained by liquid B

or $m_A S_A \Delta T_A = m_B S_B \Delta T_B$

where, S_A is specific heat capacity of A and S_B is specific heat capacity of B

$\Rightarrow 100 \times S_A (100 - 90) = 50 \times S_B (90 - 75)$

$\Rightarrow 1000 S_A = 50 \times 15 S_B$

or $4S_A = 3S_B \quad \ldots(i)$

Similarly, in second case,

$100 \times S_A (100 - T) = 50 \times S_B (T - 50)$

where, T = Final temperature of the mixture.

$\Rightarrow 4S_A (100 - T) = 2S_B (T - 50)$

Using Eq. (i),

$3S_B (100 - T) = 2S_B (T - 50)$

or $300 - 3T = 2T - 100$

or $5T = 400$

or $T = 80°C$

24. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now, the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is **[JEE Main 2019, 12 Jan Shift-II]**

(a) zero (b) 5.0 mm (c) 4.0 mm (d) 3.0 mm

***Exp.* (d)**

When load M is attached to wire, extension in length of wire is

$$\Delta l_1 = \frac{Mgl}{A.Y} \quad \ldots(i)$$

where, Y is the Young's modulus of the wire.

when load is immersed in liquid of relative density 2, increase in length of wire as shown in the figure is

$$\Delta l_2 = \frac{(Mg - F_B)l}{A.Y}$$

where, F_B = Buoyant force

$$\therefore \Delta l_2 = \frac{\left(Mg - Mg \cdot \frac{\rho_l}{\rho_b}\right)l}{A.Y} \left[\because F_B = V\rho_l g = \frac{Mg}{\rho_b}\rho_l g\right]$$

Here given that, $\frac{\rho_l}{\rho_b} = \frac{2}{8} = \frac{1}{4}$

So, $$\Delta l_2 = \frac{\left(\frac{3}{4}Mg\right)l}{A.Y} \quad \ldots(ii)$$

Dividing Eqs. (ii) by (i), we get

$$\frac{\Delta l_2}{\Delta l_1} = \frac{3}{4}$$

$$\Rightarrow \Delta l_2 = \frac{3}{4} \times \Delta l_1 = \frac{3}{4} \times 4 \text{ mm}$$

$$= 3 \text{ mm}$$

25. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by **[JEE Main 2019, 12 Jan Shift-II]**

Exp. **(b)**

When soap bubble is being inflated and its temperature remains constant, then it follows Boyle's law, so

$$pV = \text{constant } (k)$$

$$\Rightarrow \quad p = \frac{k}{V}$$

Differentiating above equation with time, we get

$$\frac{dp}{dt} = k \cdot \frac{d}{dt}\left(\frac{1}{V}\right)$$

$$\Rightarrow \quad \frac{dp}{dt} = k\left(\frac{-1}{V^2}\right) \cdot \frac{dV}{dt}$$

It is given that, $\frac{dV}{dt} = c$ (a constant)

So, $$\frac{dp}{dt} = \frac{-kc}{V^2} \quad \ldots \text{(i)}$$

Now, from $\frac{dV}{dt} = c$, we get

$$dV = cdt$$

or $$\int dV = \int cdt \text{ or } V = ct \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{dp}{dt} = \frac{-kc}{c^2t^2} \text{ or } \frac{dp}{dt} = -\left(\frac{k}{c}\right)t^{-2}$$

$$\Rightarrow \quad dp = -\frac{k}{c}.t^{-2}dt$$

Integrating both sides, we get

$$\int dp = -\frac{k}{c}\int t^{-2}dt$$

$$p = -\frac{k}{c} \cdot \left(\frac{t^{-2+1}}{-2+1}\right) = -\frac{k}{c} \cdot \frac{-1}{t} = \frac{k}{ct} \text{ or } p \propto \frac{1}{t}$$

Hence, p *versus* $\frac{1}{t}$ graph is a straight line, which is correctly represented in option (b).

26. A long cylindrical vessel is half-filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides (in cm) will be **[JEE Main 2019, 12 Jan Shift-II]**

(a) 0.1 (b) 1.2 (c) 0.4 (d) 2.0

Exp. **(d)**

When liquid filled vessel is rotated the liquid profile becomes a paraboloid due to centripetal force, as shown in the figure below

Pressure at any point P due to rotation is

$$p_R = \frac{1}{2}\rho r^2\omega^2$$

Gauge pressure at depth y is $p_G = -\rho gy$

If p_0 is atmospheric pressure, then total pressure at point P is

$$p = p_0 + \frac{1}{2}\rho r^2\omega^2 - \rho gy$$

For any point on surface of rotating fluid,

$$p = p_0$$

Hence, for any surface point;

$$p_0 = p_0 + \frac{1}{2}\rho r^2\omega^2 - \rho gy \text{ or } \frac{1}{2}\rho r^2\omega^2 = \rho gy$$

or $$y = \frac{r^2\omega^2}{2g} \quad \ldots \text{(i)}$$

In the given case,

Angular speed, $\omega = 2\text{rps} = 2 \times 2\pi = 4\pi \text{ rad s}^{-1}$

Radius of vessel, $r = 5 \text{ cm} = 0.05 \text{ m}$

and $g = 10 \text{ ms}^{-2}$

Hence, substituting these values in Eq. (i), we get

$$y = \frac{\omega^2 r^2}{2g} = \frac{(4\pi)^2(0.05)^2}{2 \times 10} = 0.02 \text{ m} = 2 \text{ cm}$$

27. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross-section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$ is **[JEE Main 2018]**

(a) $\frac{Ka}{mg}$ (b) $\frac{Ka}{3mg}$

(c) $\frac{mg}{3Ka}$ (d) $\frac{mg}{Ka}$

Exp. **(c)**

$\because$ Bulk modulus, $K = \frac{\text{Volumetric stress}}{\text{Volumetric strain}} = \frac{\Delta p}{\frac{\Delta V}{V}}$

$\Rightarrow \quad K = \frac{mg}{a\left(\frac{3\Delta r}{r}\right)} \quad \left[\because V = \frac{4}{3}\pi r^3, \text{ so } \frac{\Delta V}{V} = \frac{3\Delta r}{r}\right]$

$\Rightarrow \quad \frac{\Delta r}{r} = \frac{mg}{3aK}$

28. In a Young's double slit experiment, slits are separated by 0.5 mm and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide, is **[JEE Main 2017 (Offline)]**

(a) 7.8 mm (b) 9.75 mm

(c) 15.6 mm (d) 1.56 mm

Exp. **(a)**

Let n_1th fringe formed due to first wavelength and n_2th fringe formed due to second wavelength coincide i.e. their distance from common central maxima will be same

i.e. $\quad y_{n_1} = y_{n_2}$

$\Rightarrow \quad \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d}$

$\Rightarrow \quad \frac{n_1}{n_2} = \frac{\lambda_1}{\lambda_2} = \frac{520}{650} = \frac{4}{5}$

Hence, distance of the point of coincidence from the central maxima is

$$y = \frac{n_1\lambda_1 D}{d} = \frac{n_2\lambda_2 D}{d} = \frac{4 \times 650 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}}$$

$$= 7.8 \text{ mm}$$

29. An external pressure p is applied on a cube at 0°C so that it is equally compressed from all sides. K is the bulk modulus of the material of the cube and α is its coefficient of linear expansion. Suppose we want to bring the cube to its original size by heating. The temperature should be raised by **[JEE Main 2017 (Offline)]**

(a) $\frac{p}{\alpha K}$ (b) $\frac{3\alpha}{pK}$ (c) $3pK\alpha$ (d) $\frac{p}{3\alpha K}$

Exp. **(d)**

$K = \frac{p}{(-\Delta V / V)} \Rightarrow \frac{\Delta V}{V} = \frac{p}{K} \Rightarrow -\Delta V = \frac{pV}{K}$

Change in volume, $\Delta V = \gamma v \Delta T$

where, γ = coefficient of volume expansion

Again, $\gamma = 3\alpha$

α is coefficient of linear expansion

$\therefore \quad \Delta V = V(3\alpha)\Delta T$

$\therefore \quad \frac{pV}{K} = V(3\alpha)\Delta T$

$\therefore \quad \Delta T = \frac{p}{3\alpha K}$

30. A man grows into a giant such that his linear dimensions increase by a factor of 9. Assuming that his density remains same, the stress in the leg will change by a factor of **[JEE Main 2017 (Offline)]**

(a) $\frac{1}{9}$ (b) 81 (c) $\frac{1}{81}$ (d) 9

Exp. **(d)**

$\because$ Stress $= \frac{\text{Weight}}{\text{Area}}$

Volume will become (9^3) times.

So weight = volume $\times$ density $\times$ g will also become $(9)^3$ times.

Area of cross-section will become $(9)^2$ times.

$$= \frac{9^3 \times W_0}{9^2 \times A_0} = 9\left(\frac{W_0}{A_0}\right)$$

Hence, the stress increases by a factor of 9.

31. A pendulum clock loses 12 s a day if the temperature is 40°C and gains 4 s in a day if the temperature is 20°C. The temperature at which the clock will show correct time, and the coefficient of linear expansion (α) of the metal of the pendulum shaft are, respectively **[JEE Main 2016 (Offline)]**

(a) 25°C, $\alpha = 1.85 \times 10^{-5}$/°C
(b) 60°C, $\alpha = 1.85 \times 10^{-4}$/°C
(c) 30°C, $\alpha = 1.85 \times 10^{-3}$/°C
(d) 55°C, $\alpha = 1.85 \times 10^{-2}$/°C

Exp. **(*d*)**

Time period of a pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where l is length of pendulum and g is acceleration due to gravity.

Such as change in time period of a pendulum,

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta l}{l}$$

When clock gains 12 s, we get

$$\frac{12}{T} = \frac{1}{2}\alpha(40 - \theta) \quad \ldots(i)$$

When clock loses 4 s, we get

$$\frac{4}{T} = \frac{1}{2}\alpha(\theta - 20) \quad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$3 = \frac{40 - \theta}{\theta - 20}$$

$\Rightarrow 3\theta - 60 = 40 - \theta \Rightarrow 4\theta = 100 \Rightarrow \theta = 25°C$

Substituting the value of θ in Eq. (i), we have

$$\frac{12}{T} = \frac{1}{2}\alpha(40 - 25)$$

$$\Rightarrow \quad \frac{12}{24 \times 3600} = \frac{1}{2}\alpha(15)$$

$$\Rightarrow \quad \alpha = \frac{24}{24 \times 3600 \times 15} \Rightarrow \alpha = 1.85 \times 10^{-5}/°C$$

Thus, the coefficient of linear expansion in a pendulum clock = 1.85×10^{-5}/°C

32. A pendulum made of a uniform wire of cross-sectional area A has time period T. When an additional mass M is added to its bob, the time period changes T_M. If the Young's modulus of the material of the wire is Y, then $\frac{1}{Y}$ is equal to (g = gravitational acceleration) **[JEE Main 2015]**

(a) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{A}{Mg}$
(b) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right]\frac{Mg}{A}$
(c) $\left[1 - \left(\frac{T_M}{T}\right)^2\right]\frac{A}{Mg}$
(d) $\left[1 - \left(\frac{T}{T_M}\right)^2\right]\frac{A}{Mg}$

Exp. **(*a*)**

We know that time period, $T = 2\pi\sqrt{\frac{L}{g}}$

When additional mass M is added to its bob

$$T_M = 2\pi\sqrt{\frac{L + \Delta L}{g}}$$

where, ΔL is increase in length.

We know that

$$Y = \frac{Mg/A}{\Delta L/L} = \frac{MgL}{A\Delta L} \Rightarrow \Delta L = \frac{MgL}{AY}$$

$$\Rightarrow \therefore \quad T_M = 2\pi\sqrt{\frac{L + \frac{MgL}{AY}}{g}}$$

$$\Rightarrow \quad \left(\frac{T_M}{T}\right)^2 = 1 + \frac{Mg}{AY} \text{ or } \frac{Mg}{AY} = \left(\frac{T_M}{T}\right)^2 - 1$$

or $$\frac{1}{Y} = \frac{A}{Mg}\left[\left(\frac{T_M}{T}\right)^2 - 1\right]$$

33. The pressure that has to be applied to the ends of a steel wire of length 10 cm to keep its length constant when its temperature is raised by 100°C is (For steel, Young's modulus is 2×10^{11} Nm^{-2} and coefficient of thermal expansion is 1.1×10^{-5} K^{-1}) **[JEE Main 2014]**

(a) 2.2×10^8 Pa
(b) 2.2×10^9 Pa
(c) 2.2×10^7 Pa
(d) 2.2×10^6 Pa

If the deformation is small, then the stress in a body is directly proportional to the corresponding strain.

Exp. **(*a*)**

By Hooke's law,

$$\text{Young's modulus (Y)} = \frac{\text{Tensile stress}}{\text{Tensile strain}}$$

So, $$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

If the rod is compressed, then compressive stress and strain appear. Their ratio Y is same as that for tensile case.

Given, length of a steel wire $(CL) = 10$ cm

Temperture (t) = 100°C

As length is constant.

$\therefore$ Strain $= \frac{\Delta L}{L} \alpha \Delta\theta$ [as $\Delta L = (L\Delta\theta)$]

Now, pressure = stress = $Y \times$ strain

[given, $Y = 2 \times 10^{11}$ N / m^2 and $\alpha = 1.1 \times 10^{-5}$ K^{-1}]

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^8$ Pa

34. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90°angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is **[JEE Main 2014]**

(a) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (b) $\frac{1+\cos\alpha}{1-\cos\alpha}$

(c) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (d) $\frac{1+\sin\alpha}{1-\cos\alpha}$

Exp. **(c)**

Equating pressure at A, we get

$R \sin\alpha\, d_2 + R \cos\alpha\, d_2 + R(1-\cos\alpha)\, d_1$

$= R(1-\sin\alpha)\, d_1$

$(\sin\alpha + \cos\alpha)\, d_2 = d_1 (\cos\alpha - \sin\alpha)$

$\Rightarrow \frac{d_1}{d_2} = \frac{1+\tan\alpha}{1-\tan\alpha}$

35. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If $r << R$ and the surface tension of water is T, value of r just before bubbles detatch is (density of water is ρ) **[JEE Main 2014]**

(a) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (b) $R^2\sqrt{\frac{\rho_w g}{6T}}$

(c) $R^2\sqrt{\frac{\rho_w g}{T}}$ (d) $R^2\sqrt{\frac{3\rho_w g}{T}}$

Exp. **(*)**

The bubble will detach, if

Buoyant force ≥ Surface tension force

$T \times dl = T(2\pi r) \sin\theta$

i.e., $\frac{4}{3}\pi R^3 \rho_w g \geq \int T \times dl \sin\theta$

$(\rho_w)\left(\frac{4}{3}\pi R^3\right) g \geq (T)(2\pi r)\sin\theta$

$\sin\theta = \frac{r}{R}$

Solving, $r = \sqrt{\frac{2\rho_w R^4 g}{3T}} = R^2\sqrt{\frac{2\rho_w g}{3T}}$

No option matches with the correct answer.

36. Three rods of copper, brass and steel are welded together to form a Y-shaped structure. Area of cross-section of each rod = 4 cm^2. End of copper rod is maintained at 100°C whereas ends of brass and steel are kept at 0°C. Lengths of the copper, brass and steel rods are 46, 13 and 12 cm, respectively. The rods are thermally insulated from surrounding except at ends. Thermal conductivities of copper, brass and steel are 0.92, 0.26 and 0.12 CGS units, respectively. Rate of heat flow through copper rod is **[JEE Main 2014]**

(a) 1.2 cal/s (b) 2.4 cal/s
(c) 4.8 cal/s (d) 6.0 cal/s

In thermal conduction, it is found that in steady state the, heat current is directly proportional to the area of cross-section A which is proportional to the change in temperature $(T_1 - T_2)$.

Exp. **(c)**

Then, $$\frac{\Delta Q}{\Delta t} = \frac{KA(T_1 - T_2)}{x}$$

According to thermal conductivity, we get

$$\frac{dQ_1}{dt} = \frac{dQ_2}{dt} + \frac{dQ_2}{dt}$$

$$\frac{0.92(100-T)}{46} = \frac{0.26(T-0)}{13} + \frac{0.12(T-0)}{12}$$

$$\Rightarrow \quad T = 40°C$$

$$\therefore \quad \frac{dQ_1}{dt} = \frac{0.92 \times 4(100-40)}{46}$$

$$= 4.8 \text{ cal/s}$$

37. A uniform cylinder of length L and mass M having cross-sectional area A is suspended with its length vertical from a fixed point by a massless spring such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium, is **[JEE Main 2013]**

(a) $\frac{Mg}{k}$
(b) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{M}\right)$
(c) $\frac{Mg}{k}\left(1 - \frac{LA\sigma}{2M}\right)$
(d) $\frac{Mg}{k}\left(1 + \frac{LA\sigma}{M}\right)$

Exp. **(c)**

The various forces acting on the cylinder are shown in the figure.

Let x_0 be the extension in spring.

At equilibrium, Upward forces = Downward forces

$$kx_0 + F_B = mg$$

where, k is spring constant, F_B is force due to buoyancy which is equal to weight of liquid displaced.

$$F_B = \sigma \cdot \frac{L}{2} \cdot A \cdot g$$

$$\therefore \quad kx_0 + \sigma \frac{L}{2} Ag = Mg$$

$$\Rightarrow \quad x_0 = \frac{Mg - \frac{\sigma LAg}{2}}{k} = \frac{Mg}{k}\left(1 - \frac{\sigma LA}{2M}\right)$$

38. A sonometer wire of length 1.5 m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel, if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2, respectively? **[JEE Main 2013]**

(a) 188.5 Hz (b) 178.2 Hz
(c) 200.5 Hz (d) 770 Hz

Exp. **(b)**

Frequency, $$f = \frac{v}{2l} = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{2l}\sqrt{\frac{T}{Ad}}$$

where, T is tension in the wire and μ is the mass per unit length of wire.

Also, Young's modulus, $Y = \frac{Tl}{A\Delta l}$

$$\Rightarrow \quad \frac{T}{A} = \frac{Y\Delta l}{l}$$

Putting this value in expression of frequency, we have

$$f = \frac{1}{2l}\sqrt{\frac{y\Delta l}{ld}}$$

Given, $l = 1.5$ m, $\frac{\Delta l}{l} = 0.01$

$d = 7.7 \times 10^3$ kg/m^3,

$Y = 2.2 \times 10^{11}$ N/m^2

Putting these values we, have

$$f = \frac{1}{2l}\sqrt{\frac{2.2 \times 10^{11} \times 0.01}{7.7 \times 10^3}}$$

$$f = \sqrt{\frac{2}{7}} \times \frac{10^3}{3}$$

$$f \approx 178.2 \text{ Hz}$$

39. If a piece of metal is heated to temperature θ and then allowed to cool in a room which is at temperature θ_0, the graph between the temperature T of the metal and time t will be closed to **[JEE Main 2013]**

Exp. **(c)**

According to Newton's law of cooling, the rate of loss of heat is directly proportional to the difference of temperature of the body and surroundings.

$$\frac{dQ}{dt} = k(T_2 - T_1) \quad \ldots\text{(i)}$$

and $\quad dQ = msdT_2$

$$\therefore \quad \frac{dQ}{dt} = \frac{msdT_2}{dt} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$-\frac{msdT_2}{dt} = k(T_2 - T_1)$$

$$\Rightarrow \quad \frac{dT_2}{T_2 - T_1} = -\frac{k}{ms}dt = -K\,dt \quad \left[\therefore K = \frac{k}{ms}\right]$$

On integrating, we get

$$\log_e(T_2 - T_1) = -Kt + C$$

or $\quad T_2 = T_1 + C'e^{-Kt} \quad [\text{where, } C' = e^C]$

This variation is correctly represented in option (c).

40. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible?

The surface tension is T, density of liquid is ρ and L is its latent heat of vaporisation. **[JEE Main 2013]**

(a) $\rho L/T$ (b) $\sqrt{T/\rho L}$
(c) $T/\rho L$ (d) $2\,T/\rho L$

Exp. **(d)**

If a layer of thickness dr is evaporated, then

Change in surface energy

$$= \text{Change in surface area} \times T$$

$$= [4\pi(r + dr)^2 - 4\pi r^2]T$$

$$= [4\pi r^2 + 4\pi dr^2 + 8\pi r dr - 4\pi r^2]$$

As dr is very small, so dr^2 neglected.

So, change in surface energy $= 8\pi r dr\, T$

Energy required to evaporate layer of thickness

$$dr = (4\pi r^2 dr)\rho L$$

The process of evaporation just starts, if change in surface energy is just sufficient to evaporate the water layer.

$$\therefore \quad (4\pi r^2 dr)\rho L = (8\pi r dr)T \quad \Rightarrow \quad r = \frac{2T}{\rho L}$$

41. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

If two springs S_1 and S_2 of force constants k_1 and k_2 respectively are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2.

Statement I If stretched by the same amount, work done on S_1 will be more than that on S_2.

Statement II $k_1 < k_2$ **[AIEEE 2012]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Exp. **(a)**

As no relation between k_1 and k_2 is given in the question, that is why, nothing can be predicted about Statement I. But as in Statement II, $k_1 < k_2$.

Then, for same force,

$$W = F \cdot x = F \cdot \frac{F}{k} = \frac{F^2}{k}$$

$\Rightarrow \qquad W \propto \frac{1}{k}$

i.e., $\qquad W_1 > W_2$

But for same displacement,

$$W = F \cdot x = \frac{1}{2} kx \cdot x = \frac{1}{2} kx^2 \quad \Rightarrow \quad W \propto k$$

i.e., $\quad W_1 < W_2$

Thus, Statement II is true and Statement I is false.

42. A wooden wheel of radius R is made of two semi-circular parts (see figure). The two parts are held together by a ring made of a metal strip of cross-sectional area S and length L. L is slightly less than $2\pi R$. To fit the ring on the wheel, it is heated so that its temperature rises by ΔT and it just steps over the wheel. As it cools down to surrounding temperature, it presses the semi-circular parts together. If the coefficient of linear expansion of the metal is α and its Young's modulus is Y, the force that one part of the wheel applies on the other part is **[AIEEE 2012]**

(a) $2\pi SY\alpha\Delta T$ (b) $SY\alpha\Delta T$
(c) $\pi SY\alpha\Delta T$ (d) $2SY\alpha\Delta T$

Exp. **(d)**

Elongation due to change in temperature,

$$\Delta l = L\alpha\Delta T$$

which is compensated by elastic strain, when temperature becomes normal, i.e.,

$$\Delta l = \frac{TL}{YS}$$

Thus, $\qquad \frac{TL}{YS} = L\alpha\Delta T \quad \Rightarrow \quad T = YS\alpha\Delta T$

At equilibrium, force exerted by one half on other,

$$F = 2T = 2YS\alpha\Delta T$$

43. A liquid in a beaker has temperature $\theta(t)$ at time t and θ_0 is temperature of surroundings, then according to Newton's law of cooling, the correct graph between $\log_e(\theta - \theta_0)$ and t is **[AIEEE 2012]**

Exp. **(a)**

According to Newton's law of cooling, rate of fall in temperature is proportional to the difference in temperature of the body with surrounding, *i.e.,*

$$-\frac{d\theta}{dt} = k(\theta - \theta_0) \quad \Rightarrow \quad \int \frac{d\theta}{\theta - \theta_0} = \int -k\,dt$$

$\Rightarrow \qquad \ln(\theta - \theta_0) = kt + C$

which is a straight line with negative slope.

44. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is **[AIEEE 2012]**

(a) $0.0125\ \text{Nm}^{-1}$ (b) $0.1\ \text{Nm}^{-1}$
(c) $0.05\ \text{Nm}^{-1}$ (d) $0.025\ \text{Nm}^{-1}$

Exp. **(d)**

At equilibrium, weight of the given block is balanced by force due to surface tension, *i.e.,*

$$2L \cdot S = W$$

or $\qquad S = \frac{W}{2L} = \frac{1.5 \times 10^{-2}\ \text{N}}{2 \times 0.3\ \text{m}} = 0.025\ \text{Nm}^{-1}$

45. Two mercury drops (each of radius r) merge to form a bigger drop. The surface energy of the bigger drop, if T is the surface tension, is **[AIEEE 2011]**

(a) $2^{5/3}\pi r^2 T$ (b) $4\pi r^2 T$
(c) $2\pi r^2 T$ (d) $2^{8/3}\pi r^2 T$

Exp. **(d)**

Let R be the radius of the bigger drop, then

Volume of bigger drop = 2 × Volume of small drop

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \quad \Rightarrow \quad R = 2^{1/3} r$$

Surface energy of bigger drop,

$$E = 4\pi R^2 T = 4 \times 2^{2/3}\ \pi r^2 T\ = 2^{8/3}\pi r^2 T$$

46. If a ball of steel (density, $\rho = 7.8\,\text{g cm}^{-3}$) attains a terminal velocity of $10\,\text{cms}^{-1}$ when falling in a tank of water (coefficient of viscosity $\eta_{\text{water}} = 8.5 \times 10^{-4}$ Pa - s), then its terminal velocity in glycerine ($\rho = 12\,\text{g cm}^{-3}$, $\eta = 13.2$Pa-s) would be nearly **[AIEEE 2011]**

(a) 1.6×10^{-5} cms^{-1} (b) 6.25×10^{-4} cms^{-1}
(c) 6.45×10^{-4} cms^{-1} (d) 1.5×10^{-5} cms^{-1}

Exp. **(*b*)**

As terminal velocity, $u = \frac{2}{9} r^2 \frac{(\rho - \rho_0)}{\eta}$

Then, $v \propto \frac{\rho - \rho_0}{\eta}$

$$\therefore \frac{v_2}{v_1} = \frac{\rho - \rho_{02}}{\rho - \rho_{01}} \times \frac{\eta_1}{\eta_2} = \frac{7.8 - 1.2}{7.8 - 1} \times \frac{8.5 \times 10^{-4} \times 10}{13.2}$$

$$= 6.25 \times 10^{-4}\ \text{cms}^{-1}$$

47. A metal rod of Young's modulus Y and coefficient of thermal expansion α is held at its two ends such that its length remains invariant. If its temperature is raised by t° C, the linear stress developed in it is **[AIEEE 2011]**

(a) $\frac{\alpha t}{Y}$ (b) $\frac{Y}{\alpha t}$ (c) $Y\alpha t$ (d) $\frac{1}{Y\alpha t}$

Exp. **(*c*)**

As change in length,

$$\Delta L = \alpha L \Delta T = \frac{FL}{AY} \Rightarrow \text{Stress} = \frac{F}{A} = Y\alpha\Delta T$$

48. Work done in increasing the size of a soap bubble from radius of 3 cm to 5 cm is nearly (surface tension of soap solution $= 0.03\,\text{Nm}^{-1}$) **[AIEEE 2011]**

(a) 0.2π mJ (b) 2π mJ (c) 0.4π mJ (d) 4π mJ

Exp. **(*c*)**

According to work-energy theorem,

Work done = Change in surface energy

$$\Rightarrow \quad W = 2T \times 4\pi (R_2^2 - R_1^2)$$

$$= 2 \times 0.03 \times 4\pi\,[(5)^2 - (3)^2] \times 10^{-4}\text{J}$$

$$= 0.4\pi\ \text{mJ}$$

49. Water is flowing continuously from a tap having an internal diameter 8×10^{-3}m. The water velocity as it leaves the tap is $0.4\,\text{ms}^{-1}$. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to **[AIEEE 2011]**

(a) 7.5×10^{-3} m (b) 9.6×10^{-3} m
(c) 3.6×10^{-3} m (d) 5.0×10^{-3} m

Exp. **(*c*)**

From Bernoulli's theorem,

$$\rho gh = \frac{1}{2}\rho\,(v_2^2 - v_1^2) \Rightarrow gh = \frac{1}{2} v_1^2 \left[\left(\frac{v_2}{v_1}\right)^2 - 1\right]$$

$$\Rightarrow \quad gh = \frac{1}{2} v_1^2 \left[\left(\frac{A_1}{A_2}\right)^2 - 1\right] \quad [\because A_1 v_1 = A_2 v_2]$$

$$\Rightarrow \quad \left(\frac{A_1}{A_2}\right)^2 = 1 + \frac{2hg}{v_1^2} \Rightarrow \left(\frac{D_1}{D_2}\right)^4 = 1 + \frac{2gh}{v_1^2}$$

$$\Rightarrow \quad D_2 = \frac{D_1}{\left(1 + \frac{2gh}{v_1^2}\right)^{1/4}} = \frac{8 \times 10^{-3}}{\left(1 + \frac{2 \times 10 \times 0.2}{(0.4)^2}\right)^{1/4}}$$

$$\approx 3.6 \times 10^{-3}\ \text{m}$$

50. A ball is made of a material of density ρ, where $\rho_{\text{oil}} < \rho < \rho_{\text{water}}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position? **[AIEEE 2010]**

(a)

(b)

(c)

(d)

Exp. (b)

$$\rho_{oil} < \rho < \rho_{water}$$

Oil is the least dense of them, so it should settle at the top with water at the base. Now, the ball is denser than oil but less denser than water. So, it will sink through oil but will not sink in water. So, it will stay at the oil-water interface.

51. **Statement I** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement II Photoelectrons are emitted with speeds ranging from zero to a maximum value, because of the range of frequencies present in the incident light. **[AIEEE 2010]**

Exp. (d)

Since, the frequency of ultraviolet light is less than the frequency of X-rays, the energy of each incident photon will be more for X-rays

$$KE_{photoelectron} = h\nu - \phi$$

Stopping potential is to stop the fastest photoelectron

$$V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$$

So, KE_{max} and V_0 both increases.But KE ranges from zero to KE_{max}, because of loss of energy due to subsequent collisions before getting ejected and not due to range of frequencies in the incident light.

52. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, then dielectric constant of the liquid is **[AIEEE 2010]**

(a) 4 (b) 3
(c) 2 (d) 1

Exp. (c)

From FBD of sphere, using Lami's theorem,

$$\frac{F}{mg} = \tan\theta \quad \text{...(i)}$$

When suspended in liquid, as θ remains same.

$$\therefore \quad \frac{F'}{mg\left(1-\frac{\rho}{d}\right)} = \tan\theta \quad \text{...(ii)}$$

Using Eqs. (i) and (ii), we get

$$\frac{F}{mg} = \frac{F'}{mg\left(1-\frac{\rho}{d}\right)}, \text{ where } F' = \frac{F}{K}$$

$$\therefore \quad \frac{F}{mg} = \frac{F}{mg\,K\left(1-\frac{\rho}{d}\right)}$$

or $$K = \frac{1}{1-\frac{\rho}{d}} = \frac{1}{\left(1-\frac{0.8}{1.6}\right)} = 2$$

53. Two wires are made of the same material and have the same volume. However, wire 1 has cross-sectional area A and wire 2 has cross-sectional area $3A$. If the length of wire 1 increases by Δx on applying force F, how much force is needed to stretch wire 2 by the same amount? **[AIEEE 2009]**

(a) F (b) $4F$
(c) $6F$ (d) $9F$

Exp. (d)

As volume is same.

$$\therefore \quad A_1 l_1 = A_2 l_2$$

$$\Rightarrow \quad l_2 = \frac{A_2 l_1}{A_1} = \frac{A \times l_1}{3A} = \frac{l_1}{3} \Rightarrow \frac{l_1}{l_2} = 3$$

$$\because \quad \Delta x_1 = \frac{F_1}{A\gamma} \times l_1 \quad \text{...(i)}$$

and $$\Delta x_2 = \frac{F_2}{3A\gamma} l_2 \quad \text{...(ii)}$$

Here, $\Delta x_1 = \Delta x_2$

$$\therefore \quad \frac{F_2}{3A\gamma} l_2 = \frac{F_1}{A\gamma} l_1$$

$$\Rightarrow \quad F_2 = 3F_1 \times \frac{l_1}{l_2}$$

$$= 3F_1 \times 3 = 9F$$

54. A spherical solid ball of volume V is made of a material of density ρ_1. It is falling through a liquid of density $\rho_2 (\rho_2 < \rho_1)$. [Assume that the liquid applies a viscous force on the ball that is proportional to the square of its speed v, *i.e.*, $F_{\text{viscous}} = -kv^2 (k > 0)$]. The terminal speed of the ball is **[AIEEE 2008]**

(a) $\sqrt{\dfrac{Vg(\rho_1 - \rho_2)}{k}}$ (b) $\dfrac{Vg\rho_1}{k}$

(c) $\sqrt{\dfrac{Vg\rho_1}{k}}$ (d) $\dfrac{Vg(\rho_1 - \rho_2)}{k}$

Exp. **(a)**

The forces acting on the ball are gravity force, buoyancy force and viscous force. When ball acquires terminal speed, it is in dynamic equilibrium, let terminal speed of ball be v_T. So,

$$V\rho_2 g + kv_T^2 = V\rho_1 g$$

or $$v_T = \sqrt{\frac{V(\rho_1 - \rho_2)g}{k}}$$

55. A jar is filled with two non-mixing liquids 1 and 2 having densities ρ_1 and ρ_2, respectively. A solid ball, made of a material of density ρ_3, is dropped in the jar. It comes to equilibrium in the position shown in the figure.

Which of the following is true for ρ_1, ρ_2 and ρ_3? **[AIEEE 2008]**

(a) $\rho_3 < \rho_1 < \rho_2$ (b) $\rho_1 > \rho_3 > \rho_2$

(c) $\rho_1 > \rho_2 > \rho_3$ (d) $\rho_1 < \rho_3 < \rho_2$

Exp. **(d)**

$\rho_1 < \rho_2$ as denser liquid acquires lowest position of vessel. $\rho_3 > \rho_1$ as ball sinks in liquid 1 and $\rho_3 < \rho_2$ as ball does not sink in liquid 2, so

$$\rho_1 < \rho_3 < \rho_2$$

56. A capillary tube (A) is dipped in water. Another identical tube (B) is dipped in a soap-water solution. Which of the following shows the relative nature of the liquid columns in the two tubes? **[AIEEE 2008]**

Exp. **(c)**

Soap solution has lower surface tension, T as compared to pure water and capillary rise $h = \dfrac{2T\cos\theta}{\rho r g}$, so h is less for soap solution.

57. One end of a thermally insulated rod is kept at a temperature T_1 and the other at T_2. The rod is composed of two sections of lengths l_1 and l_2 and thermal conductivities K_1 and K_2 respectively. The temperature at the interface of the two sections is **[AIEEE 2007]**

(a) $\dfrac{(K_2 l_2 T_1 + K_1 l_1 T_2)}{(K_1 l_1 + K_2 l_2)}$ (b) $\dfrac{(K_2 l_1 T_1 + K_1 l_2 T_2)}{(K_2 l_1 + K_1 l_2)}$

(c) $\dfrac{(K_1 l_2 T_1 + K_2 l_1 T_2)}{(K_1 l_2 + K_2 l_1)}$ (d) $\dfrac{(K_1 l_1 T_1 + K_2 l_2 T_2)}{(K_1 l_1 + K_2 l_2)}$

Exp. **(c)**

Let temperature at the interface be T.

For part AB,

As rate of flow of heat,

$$\frac{Q_1}{t} \propto \frac{(T_1 - T)K_1}{l_1}$$

For part BC, $$\frac{Q_2}{t} \propto \frac{(T - T_2)K_2}{l_2}$$

At equilibrium, $$\frac{Q_1}{t} = \frac{Q_2}{t}$$

$$\Rightarrow \quad \frac{(T_1 - T)K_1}{l_1} = \frac{(T - T_2)K_2}{l_2}$$

or $$T = \frac{T_1K_1l_2 + T_2K_2l_1}{K_1l_2 + K_2l_1}$$

58. Assuming the sun to be a spherical body of radius R at a temperature of T K, evaluate the total radiant power, incident on earth, at a distance r from the sun. **[AIEEE 2006]**

(a) $\frac{4\pi r_0^2 R^2 \sigma T^4}{r^2}$ (b) $\frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$

(c) $\frac{r_0^2 R^2 \sigma T^4}{4\pi r^2}$ (d) $\frac{R^2 \sigma T^4}{r^2}$

where, r_0 is the radius of the earth and σ is Stefan's constant.

Exp. **(b)**

From Stefan's law, the rate at which energy is radiated by sun at its surface is

$$P = \sigma \times 4\pi R^2 \times T^4$$

[sun is a perfectly black body as it emits radiations of all wavelengths and so for it, $e = 1$]

The intensity of this power at earth's surface (under the assumption $r >> r_0$) is

$$I = \frac{P}{4\pi r^2} = \frac{\sigma \times 4\pi R^2 T^4}{4\pi r^2} = \frac{\sigma R^2 T^4}{r^2}$$

The area of earth which receives this energy is only one half of total surface area of earth, whose projection would be πr_0^2.

$\therefore$ The total radiant power as received by earth

$$= \pi r_0^2 \times I$$

$$= \frac{\pi r_0^2 \times \sigma R^2 T^4}{r^2} = \frac{\pi r_0^2 R^2 \sigma T^4}{r^2}$$

59. If the terminal speed of a sphere of gold (density $= 19.5$ kgm^{-3}) is 0.2 ms^{-1} in a viscous liquid (density $= 1.5$ kgm^{-3}), find the terminal speed of a sphere of silver (density $= 10.5$ kg/m^3) of the same size in the same liquid. **[AIEEE 2006]**

(a) 0.4 ms^{-1} (b) 0.133 ms^{-1}

(c) 0.1 ms^{-1} (d) 0.2 ms^{-1}

Exp. **(c)**

Terminal speed of spherical body in a viscous liquid is given by

$$v_T = \frac{2r^2(\rho - \sigma)g}{9\eta}$$

where, ρ = density of substance of a body

σ = density of liquid.

From given data,

$$\frac{v_T(\text{Ag})}{v_t(\text{Gold})} = \frac{\rho_{\text{Ag}} - \sigma_l}{\rho_{\text{Gold}} - \sigma_l}$$

$$\Rightarrow \quad v_T(\text{Ag}) = \frac{10.5 - 1.5}{19.5 - 1.5} \times 0.2$$

$$= \frac{9}{18} \times 0.2 = 0.1 \text{ ms}^{-1}$$

60. A wire elongates by l mm when a load w is hanged from it. If the wire goes over a pulley and two weights w each are hung at the two ends, the elongation of the wire will be (in mm) **[AIEEE 2006]**

(a) l (b) $2l$ (c) zero (d) $\frac{l}{2}$

Exp. **(a)**

Let us consider the length of wire as L and cross-sectional area A, the material of wire has Young's modulus as Y.

Then, for 1st case, $Y = \frac{w/A}{l/L}$

For 2nd case, $Y = \frac{w/A}{2l'/L} \Rightarrow l' = \frac{l}{2}$

So, total elongation of both sides $= 2l' = l$

61. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be **[AIEEE 2005]**

(a) 8 cm (b) 10 cm (c) 4 cm (d) 20 cm

Exp. **(d)**

Water fills the tube entirely in gravity less condition.

Hence, length of water column in the capillary tube is 20 cm.

62. If S is stress and Y is Young's modulus of material of a wire, the energy stored in the wire per unit volume is **[AIEEE 2005]**

(a) $2S^2Y$ (b) $\frac{S^2}{2Y}$ (c) $\frac{2Y}{S^2}$ (d) $\frac{S}{2Y}$

Exp. **(b)**

Energy stored in wire

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

and Young's modulus $= \frac{\text{Stress}}{\text{Strain}} \Rightarrow \text{Strain} = \frac{S}{Y}$

$$\therefore \frac{\text{Energy stored in wire}}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

$$= \frac{1}{2} \times S \times \frac{S}{Y} = \frac{S^2}{2Y}$$

63. The figure shows a system of two concentric spheres of radii r_1 and r_2 and kept at temperatures T_1 and T_2, respectively. The radial rate of flow of heat in a substance between the two concentric spheres, is proportional to **[AIEEE 2005]**

(a) $\frac{(r_2 - r_1)}{(r_1 r_2)}$ (b) $\ln\left(\frac{r_2}{r_1}\right)$

(c) $\frac{r_1 r_2}{(r_2 - r_1)}$ (d) $(r_2 - r_1)$

Exp. **(c)**

To measure the radial rate of heat flow, we have to go for integration technique as here, the area of the surface through which heat will flow is not constant.

Let us consider an element (spherical shell) of thickness dx and radius x as shown in figure. Let us first find the equivalent thermal resistance between inner and outer spheres.

Thermal resistance of shell $= dR = \frac{dx}{K \times 4\pi x^2}$

$$\left[\begin{array}{c}\text{from } R = \frac{l}{KA} \\ \text{where, } K \text{ is thermal conductivity}\end{array}\right]$$

$$\Rightarrow \int dR = R = \int_{r_1}^{r_2} \frac{dx}{4\pi K x^2}$$

$$= \frac{1}{4\pi K}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = \frac{r_2 - r_1}{4\pi K(r_1 r_2)}$$

Rate of heat flow $= H = \frac{T_1 - T_2}{R}$

$$= \frac{T_1 - T_2}{r_2 - r_1} \times 4\pi K(r_1 r_2)$$

$$\propto \frac{r_1 r_2}{r_2 - r_1}$$

64. A wire fixed at the upper end stretches by length l by applying a force F. The work done in stretching is **[AIEEE 2004]**

(a) $\frac{F}{2l}$ (b) Fl (c) $2Fl$ (d) $\frac{Fl}{2}$

Exp. **(d)**

As work done in stretching the wire

$$= \text{Potential energy stored}$$

$$= \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{l}{L} \times AL = \frac{1}{2} Fl$$

65. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. The retarding viscous force acting on the spherical ball is **[AIEEE 2004]**

(a) directly proportional to R but inversely proportional to v
(b) directly proportional to both radius R and velocity v
(c) inversely proportional to both radius R and velocity v
(d) inversely proportional to R but directly proportional to velocity v

Exp. **(b)**

Retarding force acting on a ball falling into a viscous fluid

$$F = 6\pi\eta R v$$

where, R = radius of ball, v = velocity of ball

and η = coefficient of viscosity.

$\therefore$ $F \propto R$ and $F \propto v$

Or in other words, retarding force is directly proportional to both R and v.

66. If two soap bubbles of different radii are connected by a tube **[AIEEE 2004]**

(a) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(b) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(c) air flows from the smaller bubble to the bigger
(d) there is no flow of air

Exp. **(c)**

The excess pressure inside the soap bubble is inversely proportional to radius of soap bubble *i.e.*, $p \propto 1/r$, r being the radius of bubble. It follows that pressure inside a smaller bubble is greater than that inside a bigger bubble.

Thus, if these two bubbles are connected by a tube, air will flow from smaller bubble to bigger bubble and the bigger bubble grows at the expense of the smaller one.

67. If the temperature of the sun were to increase from T to $2T$ and its radius from R to $2R$, then the ratio of the radiant energy received on earth to what it was previously, will be **[AIEEE 2004]**

(a) 4 (b) 16
(c) 32 (d) 64

Exp. **(d)**

From Stefan's law, the energy radiated by sun is given by $P = \sigma e A T^4$.

In 1st case, $P_1 = \sigma e \times 4\pi R^2 \times T^4$

In 2nd case, $P_2 = \sigma e \times 4\pi (2R)^2 \times (2T)^4$

$$= \sigma e \times 4\pi R^2 \times T^4 \times 64 = 64P_1$$

The rate at which energy received at earth is

$$E = \frac{P}{4\pi R_{SE}^2} \times A_E$$

where, A_E = area of earth

and R_{SE} = distance between sun and earth

So, in 1st case,

$$E_1 = \frac{P_1}{4\pi R_{SE}^2} \times A_E$$

$$E_2 = \frac{P_2}{4\pi R_{SE}^2} \times A_E = 64E_1$$

Hence, ratio is 64.

68. The temperature of the two outer surfaces of a composite slab, consisting of two materials having coefficients of thermal conductivity K and $2K$ and thickness x and $4x$ respectively are T_2 and $T_1 (T_2 > T_1)$. The rate of heat transfer through the slab, in a steady state is $\left(\frac{A(T_2 - T_1)K}{x}\right) f$, then f is equal to **[AIEEE 2004]**

(a) 1 (b) 1/2 (c) 2/3 (d) 1/3

Exp. **(d)**

Let the temperature of common interface be T°C.

Rate of heat flow

$$H = \frac{Q}{t} = \frac{KA\,\Delta T}{l}$$

$\therefore$ $$H_1 = \left(\frac{Q}{t}\right)_1 = \frac{2KA(T - T_1)}{4x}$$

and $$H_2 = \left(\frac{Q}{t}\right)_2 = \frac{KA(T_2 - T)}{x}$$

In steady state, the rate of heat flow should be same in whole system i.e.,

$$H_1 = H_2$$

$\Rightarrow$ $$\frac{2KA(T - T_1)}{4x} = \frac{KA(T_2 - T)}{x}$$

or $$\frac{T - T_1}{2} = T_2 - T$$

or $$T - T_1 = 2T_2 - 2T$$

or $$T = \frac{2T_2 + T_1}{3} \quad \ldots(i)$$

Hence, heat flow from composite slab is

$$H = \frac{KA(T_2 - T)}{x} = \frac{KA}{x}\left(T_2 - \frac{2T_2 + T_1}{3}\right)$$

$$= \frac{KA}{3x}(T_2 - T_1) \quad \ldots(ii)$$

Accordingly, $H = \left[\frac{A(T_2 - T_1)K}{x}\right] f \quad \ldots(iii)$

By comparing Eqs. (ii) and (iii), we get

$$f = \frac{1}{3}$$

69. A wire suspended vertically from one of its ends is stretched by attaching a weight of 200 N to the lower end. The weight stretches the wire by 1 mm. Then, the elastic energy stored in the wire is **[AIEEE 2003]**

(a) 0.2 J (b) 10 J (c) 20 J (d) 0.1 J

Exp. **(*d*)**

Elastic energy stored in the wire is

$$U = \frac{1}{2} \times \text{Stress} \times \text{Strain} \times \text{Volume}$$

$$= \frac{1}{2} \times \frac{F}{A} \times \frac{\Delta l}{L} \times AL = \frac{1}{2} F\,\Delta l$$

$$= \frac{1}{2} \times 200 \times 1 \times 10^{-3} = 0.1 \text{ J}$$

70. According to Newton's law of cooling, the rate of cooling of a body is proportional to $(\Delta\theta)^n$, where $\Delta\theta$ is the difference of the temperature of the body and the surroundings, then n is equal to **[AIEEE 2003]**

(a) 2 (b) 3 (c) 4 (d) 1

Exp. **(*d*)**

According to Newton's law of cooling,

Rate of loss of heat, $\frac{dQ}{dt} \propto \Delta\theta$

But $\frac{dQ}{dt} \propto (\Delta\theta)^n$ [given]

$\therefore$ $n = 1$

71. Two spheres of the same material have radii 1 m and 4 m and temperatures 4000 K and 2000 K, respectively. The ratio of the energy radiated per second by the first sphere to that by the second is **[AIEEE 2002]**

(a) 1 : 1 (b) 16 : 1 (c) 4 : 1 (d) 1 : 9

Exp. **(*a*)**

Energy radiated per second by a body which has surface area A at temperature T is given by Stefen's law,

$$E = \sigma A T^4 \quad [\text{as } A = \pi r^2]$$

Therefore, $$\frac{E_1}{E_2} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{T_1}{T_2}\right)^4 = \left(\frac{1}{4}\right)^2 \left(\frac{4000}{2000}\right)^4$$

[since, bodies are of same material, so $e_1 = e_2$]

$$\Rightarrow \quad \frac{E_1}{E_2} = \frac{16}{16} = \frac{1}{1} = 1:1$$

72. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is

(a) 10 (b) 20 **[AIEEE 2004]**

(c) 25.5 (d) 5

Exp. **(*d*)**

Applying the Bernoulli's theorem just inside and outside the hole. Take reference line for gravitational potential energy at the bottom of the vessel.

Let p_0 be the atmospheric pressure, ρ be the density of liquid and v be the velocity at which water is coming out.

$$p_{\text{inside}} + \rho g h + 0 = p_{\text{outside}} + \frac{\rho v^2}{2}$$

$$\Rightarrow \quad p_0 + \rho g h = p_0 + \frac{\rho v^2}{2}$$

or $$v = \sqrt{2gh} = \sqrt{2 \times 10 \times 20} = 20 \text{ ms}^{-1}$$

73. If mass-energy equivalence is taken into account, when water is cooled to form ice, the mass of water should **[AIEEE 2002]**

(a) increase

(b) remain unchanged

(c) decrease

(d) first increase then decrease

Exp. **(*a*)**

According to the mass-energy equivalence, mass and energy remain conserved. So, when water is cooled to form ice, water loses its energy, so change in energy increases the mass of water.

74. Which of the following is more close to a black body ? **[AIEEE 2002]**

(a) Black board paint (b) Green leaves
(c) Black holes (d) Red roses

Exp. (c)

Black body is one which absorbs all radiations and emits none, out of the four examples given black holes are more close to a black body.

75. Infrared radiations are detected by **[AIEEE 2002]**

(a) spectrometer (b) pyrometer
(c) nanometre (d) photometer

Exp. (b)

Spectrometer is an instrument which is used to obtain a pure spectrum of white light. It is used to determine the wavelength of different colours of white light (or wavelength of monochromatic light source), the refractive index of the material of the prism and the dispersive power of the material of the prism.

Pyrometer is infrared sensitive device, so it is used to detect infrared radiations.

Nanometre is the small unit of distance and is not a device. Photometer is used to measure luminous intensity, illuminance and other photometric quantities.

76. Heat given to a body which raises its temperature by 1°C is **[AIEEE 2002]**

(a) water equivalent
(b) thermal capacity
(c) specific heat
(d) temperature gradient

Exp. (b)

The thermal capacity of a substance is defined as the amount of heat required to raise its temperature by 1°C.

77. Cooking gas containers are kept in a lorry moving with uniform speed. The temperature of the gas molecules inside will **[AIEEE 2002]**

(a) increase
(b) decrease
(c) remain same
(d) decrease for some, while increase for others

Exp. (c)

Temperature of a gas is determined by the total transactional kinetic energy measured with respect to the centre of mass of the gas. Therefore, the motion of centre of mass of the gas does not affect the temperature. Hence, the temperature of gas will remain same.

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8

Thermodynamics

1. A thermally insulated vessel contains 150 g of water at 0°C. Then, the air from the vessel is pumped out adiabatically. A fraction of water turns into ice and the rest evaporates at 0°C itself. The mass of evaporated water will be closest to

(Latent heat of vaporisation of water $= 2.10 \times 10^6\ \text{J kg}^{-1}$ and latent heat of fusion of water $= 3.36 \times 10^5\ \text{J kg}^{-1}$)

[JEE Main 2019, 8 April Shift-I]

(a) 150 g (b) 20 g
(c) 130 g (d) 35g

Exp. **(b)**

Let x grams of water is evaporated.

According to the principle of calorimetry,

Heat lost by freezing water (that turns into ice) = Heat gained by evaporated water

Given, mass of water = 150 g

$$\Rightarrow \quad (150 - x) \times 10^{-3} \times 3.36 \times 10^5 = x \times 10^{-3} \times 2.10 \times 10^6$$

$$\Rightarrow \quad (150 - x) \times 3.36 = 21x$$

$$\Rightarrow \quad x = \frac{150}{7.25} = 20.6$$

$$\therefore \quad x \approx 20\ \text{g}$$

2. The given diagram shows four processes, i.e. isochoric, isobaric, isothermal and adiabatic. The correct assignment of the processes, in the same order is given by

[JEE Main 2019, 8 April Shift-II]

(a) $d\,a\,b\,c$ (b) $a\,d\,b\,c$ (c) $d\,a\,c\,b$ (d) $a\,d\,c\,b$

Exp. **(a)**

Given processes are

For process a, pressure is constant.

$\therefore a$ is isobaric.

For process d, volume is constant.

$\therefore d$ is isochoric.

Also, as we know that, slope of adiabatic curve in p-V diagram is more than that of isothermal curve.

$\therefore b$ is isothermal and c is adiabatic.

3. Following figure shows two processes A and B for a gas. If ΔQ_A and ΔQ_B are the amount of heat absorbed by the system in two cases, and ΔU_A and ΔU_B are changes in internal energies respectively, then

[JEE Main 2019, 9 April Shift-I]

(a) $\Delta Q_A > \Delta Q_B,\ \Delta U_A > \Delta U_B$
(b) $\Delta Q_A < \Delta Q_B,\ \Delta U_A < \Delta U_B$
(c) $\Delta Q_A > \Delta Q_B,\ \Delta U_A = \Delta U_B$
(d) $\Delta Q_A = \Delta Q_B;\ \Delta U_A = \Delta U_B$

Exp. **(c)**

According to the first law of thermodynamics,
Heat supplied (ΔQ) = Work done (W) + Change in internal energy of the system (ΔU)

$$\Delta Q_A = \Delta U_A + W_A$$

Similarly, for process B,

$$\Delta Q_B = \Delta U_B + W_B$$

Now, we know that,
work done for a process
= area under it's p-V curve

Thus, it is clear from the above graphs,

$$W_A > W_B \quad \ldots(i)$$

Also, since the initial and final state are same in both process, so

$$\Delta U_A = \Delta U_B \quad \ldots(ii)$$

So, from Eqs. (i) and (ii), we can conclude that

$$\Delta Q_A > \Delta Q_B$$

4. Two materials having coefficients of thermal conductivity '$3K$' and 'K' and thickness 'd' and '$3d$' respectively, are joined to form a slab as shown in the figure. The temperatures of the outer surfaces are 'θ_2' and 'θ_1' respectively, ($\theta_2 > \theta_1$). The temperature at the interface is

[JEE Main 2019, 9 April Shift-II]

(a) $\dfrac{\theta_2 + \theta_1}{2}$ (b) $\dfrac{\theta_1}{3} + \dfrac{2\theta_2}{3}$
(c) $\dfrac{\theta_1}{6} + \dfrac{5\theta_2}{6}$ (d) $\dfrac{\theta_1}{10} + \dfrac{9\theta_2}{10}$

Exp. **(d)**

Let interface temperature in steady state conduction is θ, then assuming no heat loss through sides;

$$\begin{pmatrix}\text{Rate of heat}\\ \text{flow through}\\ \text{first slab}\end{pmatrix} = \begin{pmatrix}\text{Rate of heat}\\ \text{flow through}\\ \text{second slab}\end{pmatrix}$$

$$\Rightarrow \quad \frac{(3K)\,A(\theta_2 - \theta)}{d} = \frac{KA(\theta - \theta_1)}{3d}$$

$$\Rightarrow \quad 9(\theta_2 - \theta) = \theta - \theta_1 \Rightarrow \quad 9\theta_2 + \theta_1 = 10\theta$$

$$\Rightarrow \quad \theta = \frac{9}{10}\theta_2 + \frac{1}{10}\theta_1$$

5. A cylinder with fixed capacity of 67.2 L contains helium gas at STP. The amount of heat needed to raise the temperature of the gas by 20°C is [Take, $R = 8.31$ J mol^{-1}K^{-1}]

[JEE Main 2019, 10 April Shift-I]

(a) 700 J (b) 748 J (c) 374 J (d) 350 J

Exp. **(b)**

Given, capacity of cylinder is = 67.2 L and $\Delta T = 20°C$
At STP volume = 22.4 L

$\therefore$ Number of moles $= \dfrac{67.2}{22.4} = 3$

Now, change in heat is given as

$$\Delta Q = nC_V\,\Delta T$$

Substituting the given values, we get

$$\Delta Q = 3 \times \frac{3R}{2} \times 20 \quad (\because \text{for He gas, } C_V = \frac{3}{2}R)$$

$$= 90\,R \text{ J}$$

Given, $R = 8.31$ J mol^{-1}K^{-1}

$\therefore \quad \Delta Q = 90 \times 8.31 = 747.9 \text{ J} = 748 \text{ J}$

6. When heat Q is supplied to a diatomic gas of rigid molecules, at constant volume, its temperature increases by ΔT. The heat required to produce the same change in temperature, at a constant pressure is

[JEE Main 2019, 10 April Shift-II]

(a) $\frac{2}{3}Q$ (b) $\frac{5}{3}Q$

(c) $\frac{3}{2}Q$ (d) $\frac{7}{5}Q$

Exp. **(d)**

Given, heat supplied at constant volume is Q.

$\therefore Q = nC_V\Delta T$

For same change in temperature, if heat supplied at constant pressure is Q, then

$$Q' = nC_p\Delta T$$

So, we have

$$\frac{Q'}{Q} = \frac{nC_p\Delta T}{nC_V\Delta T} \Rightarrow \frac{Q'}{Q} = \frac{C_p}{C_V}$$

$$\Rightarrow \quad Q' = Q\gamma \qquad \left[\because \gamma = \frac{C_p}{C_V}\right]$$

As given gas is diatomic, so $\gamma = \frac{7}{5}$

$$\therefore \quad Q' = \frac{7}{5}Q$$

7. A sample of an ideal gas is taken through the cyclic process *abca* as shown in the figure. The change in the internal energy of the gas along the path *ca* is –180 J. The gas absorbs 250 J of heat along the path *ab* and 60 J along the path *bc*. The work done by the gas along the path *abc* is **[JEE Main 2019, 12 April Shift-I]**

(a) 120 J (b) 130 J

(c) 100 J (d) 140 J

Exp. **(b)**

Key Idea In *p-V* curve, work done dW, change in internal energy ΔU and heat absorbed ΔQ are connected with first law of thermodynamics, i.e.

$$\Delta Q = \Delta U + dW \quad \ldots(i)$$

and total change in internal energy in complete cycle is always zero. Using this equation in different part of the curve, we can solve the given problem.

In Process $a \to b$

Given, $\Delta Q_{ab} = 250\text{ J}$

$\therefore \quad 250\text{ J} = \Delta U_{ab} + dW_{ab} \quad \ldots(ii)$

In Process $b \to c$

Given, $\Delta Q_{bc} = 60\text{ J}$

Also, V is constant, so $dV = 0$

$\Rightarrow \quad dW_{bc} = p(dV)_{bc} = 0 \quad \ldots(iii)$

$\therefore \quad 60\text{ J} = \Delta U_{bc} + 0$

$\Rightarrow \quad \Delta U_{bc} = 60\text{ J}$

In Process $c \to a$

Given, $\Delta U_{ca} = -180\text{ J} \quad \ldots(iv)$

Now, for complete cycle,

$$\Delta U_{abca} = \Delta U_{ab} + \Delta U_{bc} + \Delta U_{ca} = 0 \ldots(v)$$

From Eqs. (iii), (iv) and (v), we get

$$\Delta U_{ab} = -\Delta U_{bc} - \Delta U_{ca}$$

$$\Delta U_{ab} = -60 + 180 = 120\text{ J} \quad \ldots(vi)$$

From Eq. (ii), we get

$$250\text{ J} = 120\text{ J} + dW_{ab}$$

$$\Rightarrow \quad dW_{ab} = 130\text{ J} \quad \ldots(vii)$$

From Eqs. (i) and (vii), we get

Work done by the gas along the path *abc*,

$$dW_{abc} = dW_{ab} + dW_{bc}$$

$$= 130\text{ J} + 0\text{ K}$$

$$\Rightarrow \quad dW_{abc} = 130\text{ J}$$

8. A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its efficiency is doubled. The temperatures of the source and the sink are respectively, **[JEE Main 2019, 12 April Shift-II]**

(a) 62°C, 124°C (b) 99°C, 37°C

(c) 124°C, 62°C (d) 37°C, 99°C

Exp. **(b)**

Efficiency of a Carnot engine working between source of temperature T_1 and sink of temperature T_2 is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

Here, T_2 and T_1 are absolute temperatures.

Initially,

$$\eta = \frac{1}{6}$$

$$\therefore \quad \frac{1}{6} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$$

Finally, efficiency is doubled on reducing sink temperature by 62°C.

$\therefore \qquad \eta = \frac{2}{6}, \; T_{sink} = T_2' = T_2 - 62$

So, $\qquad \eta = 1 - \frac{T'_2}{T'_1}$

$\Rightarrow \quad \frac{2}{6} = 1 - \frac{T_2 - 62}{T_1} \quad \Rightarrow \quad \frac{T_2 - 62}{T_1} = \frac{4}{6}$

$\Rightarrow \quad \frac{T_2}{T_1} - \frac{62}{T_1} = \frac{4}{6} \quad \Rightarrow \quad \frac{5}{6} - \frac{62}{T_1} = \frac{4}{6}$

$\left[\because \frac{T_2}{T_1} = \frac{5}{6}\right]$

$\Rightarrow T_1 = 6 \times 62 = 372 \text{ K} = 372 - 273 = 99° \text{ C}$

$\Rightarrow \quad T_2 = \frac{5}{6} \times T_1 \simeq 310 \text{ K}$

$= 310 - 273 = 37° \text{ C}$

9. A diatomic gas with rigid molecules does 10 J of work when expanded at constant pressure. What would be the heat energy absorbed by the gas, in this process?

[JEE Main 2019, 12 April Shift-II]

(a) 25 J (b) 35 J
(c) 30 J (d) 40 J

Exp. **(b)**

Work done by a gas is

$W = p\Delta V = nR\Delta T = 10 \text{J}$ (given) ...(i)

As the gas is diatomic,

C_p = specific heat of gas at constant pressure

$= \frac{7}{2}R$

So, heat absorbed by gas at constant pressure is

$\Delta Q = nC_p \; \Delta T = n\frac{7}{2}R\,\Delta T$

$= \frac{7}{2}(nR\;\Delta T)$

$= \frac{7}{2} \times 10$ [From Eq. (i)]

$= 35 \text{J}$

10. A gas can be taken from A to B *via* two different processes ACB and ADB.

When path ACB is used 60 J of heat flows into the system and 30 J of work is done by the system. If path ADB is used work done by the system is 10 J the heat flow into the system in path ADB is

[JEE Main 2019, 9 Jan Shift-I]

(a) 80 J (b) 40 J (c) 100 J (d) 20 J

Exp. **(b)**

For the ABC as shown in the figure below,

According to the first law of thermodynamics,
heat supplied, ΔQ = work done (ΔW)
+ internal energy (ΔU)

$\Rightarrow \quad \Delta Q_{CB} = \Delta W_{ACB} + (U_B - U_A)$

[where, $\Delta U = U_B - U_A$]

Substituting the given values,

$U_B - U_A = 60 - 30 = 30 \text{ J}$ ...(i)

Similarly for the ADB as shown in the figure below,

$\Delta Q_{ADB} = \Delta W_{ADB} + (U_B - U_A)$

$\Rightarrow \quad \Delta Q_{ADB} = 10 + 30$ [using Eq. (i)]

$= 40 \text{ J}$

11. Temperature difference of 120°C is maintained between two ends of a uniform rod AB of length $2L$. Another bent rod PQ, of same cross-section as AB and length $\frac{3L}{2}$ is connected across AB
(see figure). In steady state, temperature difference between P and Q will be close to

[JEE Main 2019, 9 Jan Shift-I]

(a) 45° (b) 35°C
(c) 75°C (d) 60°C

Exp. **(a)**

According to the given question, the given figure with its length for each section is given as below

The above figure considering that every section has the same thermal conductivity, then in terms of thermal resistance is shown in the figure below,

Net resistance of the section *PQRS* is

$$= \frac{R \times \frac{3R}{2}}{\frac{5R}{2}} = \frac{3R}{5} \quad \text{...(i)}$$

Total resistance of the net network, R_{net}

$$= \frac{R}{2} + \frac{R}{2} + \frac{3R}{5} = \frac{8R}{5}$$

$\therefore$ Thermal current, $I = \frac{\Delta T_{AB}}{R_{net}}$

$$I = \frac{120 - 0}{\left(\frac{8R}{5}\right)} = \frac{120 \times 5}{8R}$$

Thus, the net temperature difference between point *P* and *Q* is

$$T_P - T_Q = I \times \frac{3R}{5} \quad \text{[using Eq. (i)]}$$

$$= \frac{120 \times 5}{8R} \times \frac{3R}{5} = 45°\text{C}$$

12. Two Carnot engines *A* and *B* are operated in series. The first one, *A* receives heat at $T_1 (= 600\,\text{K})$ and rejects to a reservoir at temperature T_2. The second engine *B* receives heat rejected by the first engine and in turn rejects to a heat reservoir at $T_3\ (= 400\,\text{K})$.

Calculate the temperature T_2 if the work outputs of the two engines are equal.

[JEE Main 2019, 9 Jan Shift-II]

(a) 600 K (b) 500 K
(c) 400 K (d) 300 K

Exp. **(b)**

Key idea In a Carnot engine the heat flow from higher temperature source (at T_1) to lower temperature sink (at T_2) and give the work done equal to the $W = Q_1 - Q_2$.

For the given condition, Carnot engine *A* and *B* are operated in series as shown below

where,

Q_1 = heat rejected by engine *A* at T_1K,

Q_2 = heat received by engine *B* at T_2K and

Q_3 = heat rejected by engine *B* to source *C* at T_3K.

According to Carnot engine principle,

$W_1 = Q_1 - Q_2$ (work-output from source *A* and *B*)

$W_2 = Q_2 - Q_3$ (work output from source *B* and *C*)

As per the given condition, if the work outputs of the two engines are equal, then

$$Q_1 - Q_2 = Q_2 - Q_3$$

$$\Rightarrow \quad Q_1 + Q_3 = 2Q_2$$

$$\frac{Q_1}{Q_2} + \frac{Q_3}{Q_2} = 2 \quad \text{...(i)}$$

Therefore $\frac{T_1}{T_2} + \frac{T_3}{T_2} = 2$

So, $\frac{T_1 + T_3}{T_2} = 2$

$$\Rightarrow \quad T_2 = \frac{T_1 + T_3}{2} = \frac{600 + 400}{2}$$

$$T_2 = 500\,\text{K}$$

13. A heat source at $T = 10^3$ K is connected to another heat reservoir at $T = 10^2$ K by a copper slab which is 1 m thick. Given that the thermal conductivity of copper is $0.1\ \text{WK}^{-1}\text{m}^{-1}$, the energy flux through it in the steady state is

[JEE Main 2019, 10 Jan Shift-I]

(a) $90\ \text{Wm}^{-2}$ (b) $65\ \text{Wm}^{-2}$
(c) $120\ \text{Wm}^{-2}$ (d) $200\ \text{Wm}^{-2}$

Exp. **(a)**

Energy flux is the rate of heat flow per unit area through the rod.

Also, rate of flow of heat per unit time through a material of area of cross-section A and thermal conductivity k between the temperatures T_1 and T_2 $(T_1 > T_2)$ is given as,

$$\frac{\Delta Q}{\Delta t} = \frac{kA(T_1 - T_2)}{l} \quad \ldots \text{(i)}$$

Energy flux using Eq. (i), we get

$$= \frac{1}{A} \cdot \frac{\Delta Q}{\Delta t} = \frac{k(T_1 - T_2)}{l}$$

Here, $k = 0.1\ \text{K K}^{-1}\text{m}^{-1}$, $l = 1\ \text{m}$

$T_1 = 1000\ \text{K}$ and $T_2 = 100\ \text{K}$

$$\therefore \quad \text{Energy flux} = \frac{0.1(1000 - 100)}{1}$$

$$= 90\ \text{Wm}^{-2}.$$

14. Three Carnot engines operate in series between a heat source at a temperature T_1 and a heat sink at temperature T_4 (see figure). There are two other reservoirs at temperatures T_2 and T_3, as shown with $T_1 > T_2 > T_3 > T_4$. The three engines are equally efficient if

[JEE Main 2019, 10 Jan Shift-I]

(a) $T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$
(b) $T_2 = (T_1^2 T_4)^{1/3}; T_3 = (T_1 T_4^2)^{1/3}$
(c) $T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$
(d) $T_2 = (T_1 T_4^2)^{1/3}; T_3 = (T_1^2 T_4)^{1/3}$

Exp. **(b)**

Given, Carnot engines operates as,

As, efficiency of a Carnot's engine is given by

$$\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$$

We have,

η_1 = efficiency of engine $\varepsilon_1 = 1 - \dfrac{T_2}{T_1}$

η_2 = efficiency of engine $\varepsilon_2 = 1 - \dfrac{T_3}{T_2}$

η_3 = efficiency of engine $\varepsilon_3 = 1 - \dfrac{T_4}{T_3}$

For equal efficiencies,

$$\eta_1 = \eta_2 = \eta_3$$

$$\Rightarrow \quad 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2} = 1 - \frac{T_4}{T_3}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

$$\Rightarrow \quad T_2^2 = T_1 T_3 \text{ and } T_3^2 = T_2 T_4$$

$$\Rightarrow \quad T_2^4 = T_1^2 T_3^2$$

$$T_2^4 = T_1^2 T_2 T_4$$

$$T_2^3 = T_4 T_1^2$$

or $\quad T_2 = (T_1^2\ T_4)^{\frac{1}{3}}$

Also, $\quad T_3^4 = T_2^2\ T_4^2 = T_1 T_3 T_4^2$

$$T_3^3 = T_1 T_4^2 \quad \text{or } T_3 = (T_1 T_4^2)^{\frac{1}{3}}$$

15. An unknown metal of mass 192 g heated to a temperature of 100°C was immersed into a brass calorimeter of mass 128 g containing 240 g of water at a temperature of 8.4°C. Calculate the specific heat of the unknown metal, if water temperature stabilises at 21.5°C. (Take, specific heat of brass is 394 J kg $^{-1}$K^{-1}) **[JEE Main 2019, 10 Jan Shift-II]**

(a) 916 J kg^{-1} K^{-1} (b) 654 J kg^{-1} K^{-1}
(c) 1232 J kg^{-1} K^{-1} (d) 458 J kg^{-1} K^{-1}

Exp. **(a)**

> **Key Idea** The principle of calorimetry states that total heat lost by the hotter body equals to the total heat gained by colder body, provided that there is no exchange of heat with the surroundings.

Let specific heat of unknown metal is s and heat lost by this metal is ΔQ.

Heat lost and specific heat of a certain material/substance are related as

$$\Delta Q = ms\Delta T \quad \ldots \text{(i)}$$

For unknown metal, $m = 192$ g and

$$\Delta T = (100 - 21.5)\ ^{\circ}\text{C}$$

$$\therefore \quad \Delta Q' = 192(100 - 21.5) \times s \quad \ldots \text{(ii)}$$

Now, this heat is gained by the calorimeter and water inside it.

As, heat gained by calorimeter can be calculated by Eq. (i).

So, for brass specific heat,

$$s = 394 \text{ J kg}^{-1} \text{ K}^{-1} \quad \text{(given)}$$

$$= 0.394 \text{ J g}^{-1} \text{ K}^{-1}$$

Mass of calorimeter, $m = 128$ g

Change in temperature, $\Delta T = (21.5 - 8.4)^{\circ}$ C

So, using Eq. (i) for calorimeter, heat gained by brass

$$\Delta Q_1 = 128 \times 0.394 \times (21.5 - 8.4) \quad \ldots \text{(iii)}$$

Heat gained by water can be calculated as follows

mass of water, $m = 240$ g,

specific heat of water, $s = 4.18$ J g^{-1}K^{-1},

change in temperature, $\Delta T = (21.5 - 8.4)^{\circ}$C

Using Eq. (i) for water also, we get

heat gained by water,

$$\Delta Q_2 = 240 \times 4.18 \times (21.5 - 8.4) \quad \ldots \text{(iv)}$$

Now, according to the principle of calorimeter, the total heat gained by the calorimeter and water must be equal to heat lost by unknown metal

$$\Delta Q' = \Delta Q_1 + \Delta Q_2$$

Using Eqs. (ii), (iii) and (iv), we get

$$\Rightarrow \quad = 192(100 - 21.5) \times s$$

$$= 128 \times 0.394 \times (21.5 - 8.4) + 240 \times 4.18 \times (21.5 - 8.4)$$

$$\Rightarrow \quad 15072\, s = 660.65 + 13142$$

$$\Rightarrow \quad s = 0.916 \text{ J g}^{-1} \text{ K}^{-1}$$

or $\quad s = 916$ J Kg^{-1}K^{-1}.

16. Ice at –20°C is added to 50 g of water at 40°C. When the temperature of the mixture reaches 0°C, it is found that 20 g of ice is still unmelted. The amount of ice added to the water was close to
(Take, specific heat of water = 4.2 J/g/°C
specific heat of ice = 2.1 J/g/°C and
heat of fusion of water at 0°C = 334 J/g)
[JEE Main 2019, 11 Jan Shift-I]

(a) 40 g (b) 50 g (c) 60 g (d) 100 g

Exp. **(a)**

Let amount of ice be 'x' gm.

According to the principle of calorimeter,

heat lost by water = heat gained by ice

Here, heat lost by water, $\Delta Q = ms_{\text{water}}\ \Delta T$

Substituting the given values, we get

$$\Delta Q = 50 \times 4.2 \times 40$$

Heat gained by ice,

$$\Delta Q = x\, s_{\text{ice}}\ \Delta T + (x - 20)\, L$$

$$= x \times 2.1 \times 20 + (x - 20) \times 334$$

$$= 20\, x \times 2.1 + 334x - 6680$$

$$\therefore\ 20\, x \times 2.1 + 334x - 6680 = 50 \times 4.2 \times 40$$

$$42x + 334x - 6680 = 8400$$

$$\Rightarrow \quad 376\, x = 15080 \text{ or } x = 40.10 \text{ g}$$

$$x \simeq 40 \text{ g}$$

17. Two rods A and B of identical dimensions are at temperature 30°C. If A is heated upto 180°C and B upto T°C, then new lengths are the same. If the ratio of the coefficients of linear expansion of A and B is 4 : 3, then the value of T is **[JEE Main 2019, 11 Jan Shift-II]**

(a) 230°C (b) 270°C (c) 200°C (d) 250°C

Exp. **(a)**

Let initial length of identical rods is l_0 Thermal expansion in length of rod due to heating is given by the relation

$$\Delta l = l_0\, \alpha\, (\Delta T) = l_0\, \alpha\, (T_2 - T_1)$$

Here, α is coefficient of linear expansion.

So, change in length of rods are

$$\Delta l_1 = l_0\,\alpha_1(180-30)$$
$$\Delta l_2 = l_0\alpha_2(T-30)$$

Because new lengths are same, so change in lengths of both rods are equal.

i.e. $\Delta l_1 = \Delta l_2$

$\Rightarrow\ l_0\,\alpha_1\,(180-30) = l_0\,\alpha_2\,(T-30)$

or $\dfrac{\alpha_1}{\alpha_2} = \dfrac{(T-30)}{150}$

Given, $\alpha_1 : \alpha_2 = 4:3$

$\therefore\ \dfrac{T-30}{150} = \dfrac{4}{3} \Rightarrow T-30 = \dfrac{4}{3}\times 150 = 200$

or $T = 200 + 30 = 230°\text{C}$

18. A metal ball of mass 0.1 kg is heated upto 500°C and dropped into a vessel of heat capacity $800\,\text{JK}^{-1}$ and containing 0.5 kg water. The initial temperature of water and vessel is 30°C. What is the approximate percentage increment in the temperature of the water?
[Take, specific heat capacities of water and metal are respectively $4200\,\text{Jkg}^{-1}\text{K}^{-1}$ and $400\,\text{Jkg}^{-1}\text{K}^{-1}$] **[JEE Main 2019, 11 Jan Shift-II]**

(a) 25% (b) 15% (c) 30% (d) 20%

Exp. **(d)**

Using heat lost or gained without change in state is $\Delta Q = ms\Delta T$, where s is specific heat capacity and T = change in temperature

Let final temperature of ball be T.

Then heat lost by ball is,

$$\Delta Q = 0.1\times 400(500-T) \quad \text{...(i)}$$

This lost heat by ball is gained by water and vessel and given as

Heat gained by water,

$$\Delta Q_1 = 0.5\times 4200(T-30) \quad \text{...(ii)}$$

and heat gained by vessel is

$$\Delta Q_2 = \text{heat capacity}\times\Delta T$$
$$= 800\times(T-30) \quad \text{...(iii)}$$

According to principle of calorimetry, total heat lost = total heat gained

$\Rightarrow\ 0.1\times 400(500-T)$
$= 0.5\times 4200(T-30) + 800(T-30)$

$\Rightarrow\ (500-T) = \dfrac{(2100+800)(T-30)}{40}$

$\Rightarrow\ 500 - T = 72.5(T-30)$

$\Rightarrow\ 500 + 217.5 = 72.5T$ or $T = 36.39$ K

So, percentage increment in temperature of water

$$= \frac{36.39-30}{30}\times 100 \approx 20\%$$

19. A thermometer graduated according to a linear scale reads a value x_0, when in contact with boiling water and $x_0/3$, when in contact with ice. What is the temperature of an object in °C, if this thermometer in the contact with the object reads $x_0/2$? **[JEE Main 2019, 11 Jan Shift-II]**

(a) 35 (b) 60 (c) 40 (d) 25

Exp. **(d)**

By principle of thermometry for any liner temperature scale,

$$\frac{T-T_{LFP}}{T_{UFP}-T_{LFP}} = \text{a (constant)}$$

where,

T = temperature measured

T_{LFP} = temperature of melting ice or lower fixed point.

T_{UFP} = temperature of boling water or upper fixed point.

If, T = temperature of given object, then we have,

$$\frac{T-0°\text{C}}{100°\text{C}-0°\text{C}} = \frac{\frac{x_0}{2}-\frac{x_0}{3}}{x_0-\frac{x_0}{3}} \text{ or } \frac{T}{100} = \frac{1}{4} \text{ or}$$

$T = 25°$ C

20. For the given cyclic process CAB as shown for a gas, the work done is **[JEE Main 2019, 12 Jan Shift-I]**

(a) 5 J (b) 10 J
(c) 1 J (d) 30 J

Exp. **(b)**

Key Idea In a cyclic thermodynamic process work done = area under $p - V$ diagram.

Also in clockwise cycle, work done is positive.

In the given cyclic process,

work done $= \oint p dV$ = area enclosed by the cycle

$= \frac{1}{2} \times$ base $\times$ height of triangle (CAB) made by cycle

$= \frac{1}{2}(V_2 - V_1)(p_2 - p_1)$

From graph, given

$$V_2 = 5\,\text{m}^3,\ V_1 = 1\,\text{m}^3,$$
$$p_2 = 6\,\text{Pa},\ p_1 = 1\,\text{Pa}$$
$$= \frac{1}{2}(5-1)(6-1)$$
$$= \frac{1}{2} \times 4 \times 5 = 10\,\text{J}$$

21. A cylinder of radius R is surrounded by a cylindrical shell of inner radius R and outer radius $2R$. The thermal conductivity of the material of the inner cylinder is K_1 and that of the outer cylinder is K_2. Assuming no loss of heat, the effective thermal conductivity of the system for heat flowing along the length of the cylinder is **[JEE Main 2019, 12 Jan Shift-I]**

(a) $\frac{K_1 + K_2}{2}$ (b) $\frac{K_1 + 3K_2}{4}$

(c) $\frac{2K_1 + 3K_2}{5}$ (d) $K_1 + K_2$

Exp. **(b)**

Both the given cylinders are in parallel as heat flow is given along length. In parallel, equivalent thermal conductivity of system is

$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

So, in given system

$$K_{eq} = \frac{K_1(\pi R^2) + K_2[\pi(2R)^2 - \pi R^2]}{(\pi R^2) + (4\pi R^2 - \pi R^2)}$$

or $$K_{eq} = \frac{K_1 + 3K_2}{4}$$

22. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume $2V$. Calculate (i) the final temperature of the gas and (ii) change in its internal energy. **[JEE Main 2018]**

(a) (i) 189 K (ii) 2.7 kJ (b) (i) 195 K (ii) –2.7 kJ

(c) (i) 189 K (ii) –2.7 kJ (d) (i) 195 K (ii) 2.7 kJ

Exp. **(c)**

For adiabatic process relation of temperature and volume is,

$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1}$$

$$\Rightarrow \quad T_2(2V)^{2/3} = 300(V)^{2/3}$$

$$[\gamma = \frac{5}{3} \text{ for monoatomic gases}]$$

$$\Rightarrow \quad T_2 = \frac{300}{2^{2/3}} \approx 189\,\text{K}$$

Also, in adiabatic process,

$$\Delta Q = 0,\ \Delta U = -\Delta W$$

or $$\Delta U = \frac{-nR(\Delta T)}{\gamma - 1} = -2 \times \frac{3}{2} \times \frac{25}{3}(300 - 189)$$

$$\approx -2.7\ \text{kJ}$$

$$T_2 \approx 189\,\text{K},\ \Delta U \approx 2.7\text{kJ}$$

23. C_p and C_V are specific heats at constant pressure and constant volume, respectively. It is observed that $C_p - C_V = a$ for hydrogen gas $C_p - C_V = b$ for nitrogen gas. The correct relation between a and b is **[JEE Main 2017 (Offline)]**

(a) $a = b$ (b) $a = 14\,b$ (c) $a = 28\,b$ (d) $a = \frac{1}{14}b$

Exp. **(b)**

By Mayor's relation, for 1 g mole of a gas,

$$C_p - C_V = R$$

So, when n gram moles are given,

$$C_p - C_V = \frac{R}{n}$$

As per given question,

$$a = C_p - C_V = \frac{R}{2};\ \text{for } H_2 \quad \ldots \text{(i)}$$

$$b = C_p - C_V = \frac{R}{28};\ \text{for } N_2 \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$a = 14b$$

24. A copper ball of mass 100 g is at a temperature T. It is dropped in a copper calorimeter of mass 100 g, filled with 170 g of water at room temperature. Subsequently, the temperature of the system is found to be 75°C. T is (Given, room temperature = 30°C, specific heat of copper = 0.1 cal/g°C)

[JEE Main 2017 (Offline)]

(a) 885°C (b) 1250°C
(c) 825°C (d) 800°C

Exp. **(a)**

Heat gained (water + calorimeter) = Heat lost by copper ball

$\Rightarrow \quad m_w s_w \Delta T + m_c s_c \Delta T = m_B s_B \Delta T$

$\Rightarrow \quad 170 \times 1 \times (75 - 30) + 100 \times 0.1 \times (75 - 30)$
$= 100 \times 0.1 \times (T - 75)$

$\therefore \quad T = 885°C$

25. An ideal gas undergoes a quasistatic, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure p and volume V is given by pV^n = constant, then n is given by (Here C_p and C_V are molar specific heat at constant pressure and constant volume, respectively)

[JEE Main 2016 (Offline)]

(a) $n = \dfrac{C_p}{C_V}$ (b) $n = \dfrac{C - C_p}{C - C_V}$
(c) $n = \dfrac{C_p - C}{C - C_V}$ (d) $n = \dfrac{C - C_V}{C - C_p}$

Exp. **(b)**

For polytropic process, specific heat for an ideal gas,

$$C = \frac{R}{1-n} + C_V$$

$\therefore \quad \dfrac{R}{1-n} + C_V = C$

$\Rightarrow \quad \dfrac{R}{1-n} = C - C_V$

$\Rightarrow \quad \dfrac{R}{C - C_V} = 1 - n \quad$ (where, $R = C_p - C_V$)

$\Rightarrow \quad \dfrac{C_p - C_V}{C - C_V} = 1 - n$

$\Rightarrow \quad n = 1 - \dfrac{C_p - C_V}{C - C_V} \Rightarrow n = \dfrac{C - C_p}{C - C_V}$

Thus, number of moles n is given by $n = \dfrac{C - C_p}{C - C_V}$

26. n moles of an ideal gas undergoes a process A and B as shown in the figure. The maximum temperature of the gas during the process will be **[JEE Main 2016 (Offline)]**

(a) $\dfrac{9}{4}\dfrac{p_0V_0}{nR}$ (b) $\dfrac{3}{2}\dfrac{p_0V_0}{nR}$ (c) $\dfrac{9}{2}\dfrac{p_0V_0}{nR}$ (d) $\dfrac{9p_0V_0}{nR}$

Exp. **(a)**

As, T will be maximum temperature where product of pV is maximum

Equation of line AB, we have

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$\Rightarrow \quad p - p_0 = \dfrac{2p_0 - p_0}{V_0 - 2V_0}(V - 2V_0)$

$\Rightarrow \quad p - p_0 = \dfrac{-p_0}{V_0}(V - 2V_0)$

$\Rightarrow \quad p = \dfrac{-p_0}{V_0}V + 3p_0$

$pV = \dfrac{-p_0}{V_0}V^2 + 3p_0V$

$nRT = \dfrac{-p_0}{V_0}V^2 + 3p_0V$

$T = \dfrac{1}{nR}\left(\dfrac{-p_0}{V_0}V^2 + 3p_0V\right)$

For maximum temperature,

$$\frac{\partial T}{\partial V} = 0$$

$$\frac{-p_0}{V_0}(2V) + 3p_0 = 0$$

$$\frac{-p_0}{V_0}(2V) = -3p_0 \Rightarrow \quad V = \frac{3}{2}V_0$$

(condition for maximum temperature)

Thus, the maximum temperature of the gas during the process will be

$$T_{max} = \frac{1}{nR}\left(\frac{-p_0}{V_0} \times \frac{9}{4}V_0^2 + 3p_0 \times \frac{3}{2}V_0\right)$$

$$= \frac{1}{nR}\left(-\frac{9}{4}p_0V_0 + \frac{9}{2}p_0V_0\right) = \frac{9}{4}\frac{p_0V_0}{nR}$$

Alternate Solution

Since, initial and final temperature are equal, hence maximum temperature is at the middle of line.

i.e. $pV = nRT \Rightarrow \dfrac{\left(\frac{3}{2}p_0\right)\left(\frac{3V_0}{2}\right)}{nR} = T_{max}$

$$\frac{9}{4}\frac{p_0V_0}{nR} = T_{max}$$

27. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3}\left(\frac{U}{V}\right)$.

If the shell now undergoes an adiabatic expansion, the relation between T and R is **[JEE Main 2015]**

(a) $T \propto e^{-R}$ (b) $T \propto e^{-3R}$

(c) $T \propto \frac{1}{R}$ (d) $T \propto \frac{1}{R^3}$

Exp. **(c)**

According to question,

$$p = \frac{1}{3}\left(\frac{U}{V}\right)$$

$$\Rightarrow \quad \frac{nRT}{V} = \frac{1}{3}\left(\frac{U}{V}\right) \quad [\because pV = nRT]$$

or $\frac{nRT}{V} \propto \frac{1}{3}T^4$ or $VT^3 = \text{constant}$

or $\frac{4}{3}\pi R^3T^3 = \text{constant}$ or $TR = \text{constant}$

$$\Rightarrow \quad T \propto \frac{1}{R}$$

28. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways **[JEE Main 2015]**

(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.

(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.

In both the cases, body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively, is

(a) ln 2, 4 ln 2 (b) ln 2, ln 2

(c) ln 2, 2 ln 2 (d) 2 ln 2, 8 ln 2

Exp. **(b)**

Since, entropy is a state function, therefore change in entropy in both the processes must be same. Therefore, correct option should be (b).

29. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400 K, 800 K and 600 K, respectively.

Choose the correct statement. **[JEE Main 2014]**

(a) The change in internal energy in whole cyclic process is 250 R

(b) The change in internal energy in the process CA is 700 R

(c) The change in internal energy in the process AB is – 350 R

(d) The change in internal energy in the process BC is – 500R

Exp. **(d)**

According to first law of thermodynamics, we get

(i) Change in internal energy from A to B *i.e.*, ΔU_{AB}

$$\Delta U_{AB} = nC_V (T_B - T_A)$$
$$= 1 \times \frac{5R}{2}(800 - 400) = 1000\,R$$

(ii) Change in internal energy from B to C

$$\Delta U_{BC} = nC_V(T_C - T_B)$$
$$= 1 \times \frac{5R}{2}(600 - 800) = -500\,R$$

(iii) $\Delta U_{\text{total}} = 0$

(iv) Change in internal energy from C to A *i.e.*, ΔU_{CA}

$$\Delta U_{CA} = nC_v (T_A - T_C)$$
$$= 1 \times \frac{5R}{2}(400 - 600) = -500\,R$$

30. The given p-V diagram represents the thermodynamic cycle of an engine, operating with an ideal monatomic gas. The amount of heat, extracted from the source in a single cycle is **[JEE Main 2013]**

(a) p_0V_0 (b) $\left(\frac{13}{2}\right)p_0V_0$

(c) $\left(\frac{11}{2}\right)p_0V_0$ (d) $4p_0V_0$

Exp. **(b)**

Heat supplied $= nC_V(2T_0 - T_0) + nC_p + (2T_0 - T_0)$

$$= \frac{n3RT_0}{2} + \frac{n5R}{2}2T_0 = \frac{13}{2}nRT_0 = \frac{13}{2}p_0V_0$$

31. Helium gas goes through a cycle $ABCDA$ (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly (Assume the gas to be close to ideal gas) **[AIEEE 2012]**

(a) 15.4% (b) 9.1% (c) 10.5% (d) 12.5%

Exp. **(a)**

Efficiency of a process is defined as the ratio of work done to energy supplied. Here,

$$\eta = \frac{\Delta W}{\Delta Q} = \frac{\text{Area under } p\text{-}V \text{ diagram}}{\Delta Q_{AB} + \Delta Q_{BC}}$$

$$\therefore \quad \eta = \frac{p_0V_0}{nC_V\Delta T_1 + nC_p\Delta T_2}$$

$$= \frac{p_0V_0}{\frac{3}{2}nR(T_B - T_A) + \frac{5}{2}nR(T_C - T_D)}$$

$$= \frac{p_0V_0}{\frac{3}{2}(2p_0V_0 - p_0V_0) + \frac{5}{4}(4p_0V_0 - 2p_0V_0)}$$

[as $pV = nRT$]

$$= \frac{p_0V_0}{\frac{3}{2}p_0V_0 + \frac{5}{2}\cdot 2p_0V_0} = \frac{1}{6.5} = 15.4\%$$

32. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be **[AIEEE 2012]**

(a) efficiency of Carnot engine cannot be made larger than 50%

(b) 1200 K

(c) 750 K

(d) 600 K

Exp. **(c)**

Efficiency, $\eta = 1 - \frac{T_{\text{sink}}}{T_{\text{source}}}$

Now, $0.4 = 1 - \frac{T_{\text{sink}}}{500\text{ K}}$

$\Rightarrow$ $T_{\text{sink}} = 0.6 \times 500\text{ K} = 300\text{ K}$

Thus, $\eta = 1 - \frac{300\text{ K}}{T'_{\text{source}}}$

$\Rightarrow \quad T'_{\text{source}} = \frac{300\text{ K}}{0.4} = 750\text{ K}$

33. The specific heat capacity of a metal at low temperature (T) is given as $C_p(\text{kJ K}^{-1}\text{ kg}^{-1}) = 32\left(\frac{T}{400}\right)^3$. A 100 g vessel of this metal is to be cooled from 20 K to 4 K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is **[AIEEE2011]**

(a) equal to 0.002 kJ
(b) greater than 0.148 kJ
(c) between 0.148 kJ and 0.028 kJ
(d) less than 0.028 kJ

Exp. **(c)**

Heat required to change the temperature of vessel by a small amount dT,

$$-dQ = mC_p dT$$

Total heat required

$$-Q = m\int_{20}^{4} 32\left(\frac{T}{400}\right)^3 dT$$

$$= \frac{100\times10^{-3}\times32}{(400)^3}\left[\frac{T^4}{4}\right]_{20}^{4}$$

$\Rightarrow \quad Q = 0.001996\text{ kJ}$

Work done required to maintain the temperature of sink to T_2,

$$W = Q_1 - Q_2 = \frac{Q_1 - Q_2}{Q_2}Q_2 = \left(\frac{T_1}{T_2} - 1\right)Q_2$$

$\Rightarrow \quad W = \left(\frac{T_1 - T_2}{T_2}\right)Q_2$

For $T_2 = 20$ K,

$$W_1 = \frac{300-20}{20}\times0.001996 = 0.028\text{ kJ}$$

For $T_2 = 4$ K,

$$W_2 = \frac{300-4}{4}\times0.001996 = 0.148\text{ kJ}$$

As temperature is changing from 20 K to 4 K, work done required will be more than W_1 but less than W_2.

34. A container with insulating walls is divided into two equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure p and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be **[AIEEE 2011]**

(a) $\frac{p}{2}, T$ (b) $\frac{p}{2}, \frac{T}{2}$
(c) p, T (d) $p, \frac{T}{2}$

Exp. **(a)**

Internal energy of the gas remains constant, hence

$$T_2 = T$$

Using at constant temperature, $p_1 V_1 = p_2 V_2$

$$p_2 = \frac{p}{2}$$

35. An aluminium sphere of 20 cm diameter is heated from 0°C to 100°C. Its volume changes by (given that coefficient of linear expansion for aluminium $\alpha_{Al} = 23\times10^{-6}/°\text{C}$) **[AIEEE 2011]**

(a) 28.9 cc (b) 2.89 cc
(c) 9.28 cc (d) 49.8 cc

Exp. **(a)**

Cubical expansion

$\Delta V = \gamma V\Delta T = 3\alpha V\Delta T$ [as $\gamma = 3\alpha$]

$$= 3\times23\times10^{-6}\times\left(\frac{4}{3}\pi(10)^3\right)\times100 \quad \left[\because r = \frac{d}{2} = 10\text{ cm}\right]$$

$= 28.9$ cc

36. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and its suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by **[AIEEE 2011]**

(a) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K (b) $\frac{\gamma Mv^2}{2R}$ K
(c) $\frac{(\gamma-1)}{2R}Mv^2$ K (d) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K

Exp. **(c)**

As no heat is lost,

Loss of kinetic energy = Gain of internal energy of gas

$$\frac{1}{2}mv^2 = nC_V\Delta T$$

$$\Rightarrow \quad \frac{1}{2}mv^2 = \frac{m}{M}\cdot\frac{R}{\gamma-1}\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{Mv^2(\gamma-1)}{2R}\text{ K}$$

37. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to 1/3. Then, T_1 and T_2 are respectively **[AIEEE 2011]**

(a) 372 K and 330 K (b) 330 K and 268 K
(c) 310 K and 248 K (d) 372 K and 310 K

Exp. **(d)**

As efficiency, $\eta_1 = 1 - \frac{T_2}{T_1} \Rightarrow \frac{1}{6} = 1 - \frac{T_2}{T_1}$

$$\Rightarrow \quad \frac{T_2}{T_1} = \frac{5}{6} \quad \ldots\text{(i)}$$

$$\eta_2 = 1 - \frac{T_2 - 62}{T_1}$$

$$\Rightarrow \quad \frac{1}{3} = 1 - \frac{T_2 - 62}{T_1} \quad \ldots\text{(ii)}$$

On solving Eqs. (i) and (ii), we get

$$T_1 = 372\text{ K} \quad \text{and} \quad T_2 = 310\text{ K}$$

38. 100g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/kg/K) **[AIEEE 2011]**

(a) 8.4 kJ (b) 84 kJ (c) 2.1 kJ (d) 4.2 kJ

Exp. **(a)**

As work done = 0

[as $W = p\Delta V$ and $\Delta V = 0$, so $W = 0$]

$$\Delta U = mC\Delta T$$

$$= 100 \times 10^{-3} \times 4184 \times (50 - 30) \approx 8.4\text{ kJ}$$

39. A long metallic bar is carrying heat from one of its ends to the other end under steady-state. The variation of temperature θ along the length x of the bar from its hot end is best described by which of the following figure. **[AIEEE 2009]**

Exp. **(b)**

We know that,

Rate of flow of heat,

$$\frac{dQ}{dt} = kA\frac{d\theta}{dx}$$

In steady state, flow of heat,

$$d\theta = \frac{dQ}{dt}\cdot\frac{1}{kA}dx$$

$$\Rightarrow \quad \theta_H - \theta = k'x \quad \Rightarrow \quad \theta = \theta_H - k'x$$

Equation $\theta = \theta_H - k'x$ represents a straight line.

➡ ***Direction*** *(Q.Nos. 40 to 42) are based on the following figure.*

40. Assume the gas to be ideal, the work done on the gas in taking it from A to B is **[AIEEE 2009]**

(a) 200 R (b) 300 R (c) 400 R (d) 500 R

Exp. **(c)**

$$W_{AB} = \Delta Q - \Delta U = nC_p dT - nC_V\Delta T$$

[at constant pressure]

$$= n(C_p - C_V)dT = nR\,dT = 2 \times R \times (500 - 300)$$

$$= 400\,R$$

41. The work done on the gas in taking it from D to A is (see above figure) **[AIEEE 2009]**

(a) $-414\,R$ (b) $+414\,R$
(c) $-690\,R$ (d) $+690\,R$

Exp. **(b)**

At constant temperature (isothermal process),

$$W_{DA} = nRT \ln\left(\frac{p_1}{p_2}\right) = 2.303\, nRT \log_{10}\left(\frac{p_1}{p_2}\right)$$

$$= 2.303 \times 2R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right)$$

$$= 2.303 \times 600\, R \log\left(\frac{1}{2}\right)$$

$$= 0.693 \times 600R = -414\, R$$

So, work done on the gas is $+414\, R$.

42. The net work on the gas in the cycle *ABCDA* is (see above figure) **[AIEEE 2009]**

(a) zero (b) $276\, R$ (c) $1076\, R$ (d) $1904\, R$

Exp. **(c)**

$$W_{AB} = nR\Delta T = 2R \times 200 = 400\, R$$

$$W_{BC} = nRT \log\left(\frac{p_1}{p_2}\right)$$

$$= 2.303 \times R \times 500 \log\left(\frac{2 \times 10^5}{10^5}\right)$$

$$= 2 \times 346\, R = 692\, R$$

$$W_{CD} = nR\Delta T = 2\, R \times 200 = 400\, R$$

$$W_{DA} = 2.303 \times R \times 300 \log\left(\frac{10^5}{2 \times 10^5}\right)$$

$$= -415.963\, R$$

$\therefore$ Net work done in a cycle

$$= W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$= 400\, R + 692\, R + 400\, R - 415.963\, R$$

$$= 1076\, R$$

43. When a system is taken from state *i* to state *f* along the path *iaf*, it is found that $Q = 50$ cal and $W = 20$ cal. Along the path *ibf*, $Q = 36$ cal. *W* along the path *ibf* is **[AIEEE 2007]**

(a) 6 cal (b) 16 cal (c) 66 cal (d) 14 cal

Exp. **(a)**

From first law of thermodynamics,

$$\Delta Q = \Delta U + W$$

For path *iaf*,

$$50 = \Delta U + 20$$

$$\therefore \quad \Delta U = U_f - U_i = 30 \text{ cal}$$

For path *ibf*,

$$\Delta Q = \Delta U + W$$

or $$W = Q - \Delta U$$

$$= 36 - 30 = 6 \text{ cal}$$

44. A Carnot engine, having an efficiency of $\eta = \frac{1}{10}$ as heat engine is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is **[AIEEE 2007]**

(a) 99 J (b) 90 J (c) 1 J (d) 100 J

Exp. **(b)**

For Carnot engine using as refrigerator,

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

and $$\eta = \frac{W_1}{Q_2} = \left(\frac{T_1}{T_2} - 1\right)$$

It is given that, $\eta = \frac{1}{10}$

$$\Rightarrow \quad \eta = 1 - \frac{T_2}{T_1}$$

or $$\frac{T_2}{T_1} = \frac{9}{10}$$

So, $Q_2 = 90$ J [as $W = 10$ J]

45. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process, the temperature of the gas increases by 7°C. The gas is ($R = 8.3$ J mol^{-1} K^{-1}) **[AIEEE 2006]**

(a) diatomic
(b) triatomic
(c) a mixture of monatomic and diatomic
(d) monatomic

Exp. **(a)**

For adiabatic process, no exchange of heat.

$$\therefore \quad dQ = 0$$

So, $$dU = -\Delta W$$

$$\Rightarrow \quad nC_V\, dT = +146 \times 10^3 \text{ J}$$

$$\Rightarrow \quad \frac{nfR}{2} \times 7 = 146 \times 10^3$$

[where, f is degree of freedom]

$$\Rightarrow \quad \frac{10^3 \times f \times 8.3 \times 7}{2} = 146 \times 10^3$$

$$\therefore \quad f = 5.02 \approx 5$$

So, it is a diatomic gas.

46. A system goes from A to B *via* two processes I and II as shown in figure. If ΔU_1 and ΔU_2 are the changes in internal energies in the processes I and II respectively, then

(a) $\Delta U_1 = \Delta U_2$ **[AIEEE 2005]**

(b) relation between ΔU_1 and ΔU_2 cannot be determined

(c) $\Delta U_2 > \Delta U_1$ (d) $\Delta U_2 < \Delta U_1$

Exp. **(a)**

The change in internal energy does not depend upon path followed by the process. It only depends on initial and final states.

Hence, $\Delta U_1 = \Delta U_2$

47. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is **[AIEEE 2005]**

(a) $\frac{1}{2}$ (b) $\frac{1}{4}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$

Exp. **(c)**

According to the figure,

For process 1,

$$Q_1 = T_0 S_0 + \frac{1}{2} T_0 S_0 = \frac{3}{2} T_0 S_0$$

For process 2,

$$Q_2 = T_0(2S_0 - S_0) = T_0 S_0$$

For process 3, $Q_3 = 0$

$$\therefore \quad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$$

48. Which of the following is incorrect regarding the first law of thermodynamics? **[AIEEE 2005]**

(a) It is not applicable to any cyclic process

(b) It is a restatement of the principle of conservation of energy

(c) It introduces the concept of the internal energy

(d) Both (a) and (c)

Exp. **(d)**

Statements (a) and (d) are wrong. Concept of entropy is associated with second law of thermodynamics.

49. Which of the following statements is correct for any thermodynamic system? **[AIEEE 2004]**

(a) The internal energy changes in all processes

(b) Internal energy and entropy are state functions

(c) The change in entropy can never be zero

(d) The work done in an adiabatic process is always zero

Exp. **(b)**

Internal energy does not change in isothermal process. ΔS can be zero for adiabatic process. Work done in adiabatic process may be non-zero.

50. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of **[AIEEE 2003]**

(a) second law of thermodynamics

(b) conservation of momentum

(c) conservation of mass

(d) first law of thermodynamics

Exp. **(a)**

Heat cannot flow itself from a body at lower temperature to a body at higher temperature.
This corresponds to second law of thermodynamics.

51. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_V for the gas is **[AIEEE 2003]**

(a) 4/3 (b) 2 (c) 5/3 (d) 3/2

Exp. **(d)**

Given, $p \propto T^3$ ...(i)

In an adiabatic process,

$$T^{\gamma} p^{1-\gamma} = \text{constant} \quad \left[\text{as } \gamma = \frac{C_p}{C_V}\right]$$

$$T \propto \frac{1}{p^{(1-\gamma)/\gamma}}$$

$$T^{(\gamma/\gamma-1)} \propto p \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get

$$\frac{\gamma}{\gamma - 1} = 3$$

or $3\gamma - 3 = \gamma$ or $2\gamma = 3$ or $\frac{C_p}{C_V} = \gamma = \frac{3}{2}$

52. Which of the following parameters does not characterise the thermodynamic state of matter? **[AIEEE 2003]**

(a) Temperature (b) Pressure
(c) Work (d) Volume

Exp. **(c)**

Work does not characterise the thermodynamic state of matter, it is a path function giving only relationship between two quantities.

53. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627°C and gives it to a sink at 27°C. The work done by the engine is

(a) 4.2×10^6 J (b) 8.4×10^6 J **[AIEEE 2003]**
(c) 16.8×10^6 J (d) zero

Exp. **(b)**

$T_1 = 627 + 273 = 900$ K, $Q_1 = 3 \times 10^6$ cal

$T_2 = 27 + 273 = 300$ K

Now, from Carnot theorem,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

or $$Q_2 = \frac{T_2}{T_1} \times Q_1$$

$$\Rightarrow \quad Q_2 = \frac{300}{900} \times 3 \times 10^6 = 1 \times 10^6 \text{ cal}$$

$\therefore$ Work done $= Q_1 - Q_2$

$= 3 \times 10^6 - 1 \times 10^6 = 2 \times 10^6$ cal

$= 2 \times 4.2 \times 10^6$ J

$= 8.4 \times 10^6$ J

54. Even Carnot engine cannot give 100% efficiency, because we cannot **[AIEEE 2002]**

(a) prevent radiation
(b) find ideal sources
(c) reach absolute zero temperature
(d) eliminate friction

Exp. **(c)**

The efficiency of Carnot engine is

$$\eta = 1 - (T_2 / T_1)$$

where, T_1 is the temperature of the source and T_2 that of sink.

Since, $$\frac{T_2}{T_1} = \frac{Q_2}{Q_1}$$

So, $$\eta = 1 - \frac{Q_2}{Q_1}$$

To obtain 100% efficiency (*i.e.*, $\eta = 1$), Q_2 must be zero *i.e.,* if a sink at absolute zero would be available, all the heat taken from the source would have been converted into work.

The temperature of sink means a negative temperature on the absolute scale at which the efficiency of engine is greater than unity. This would be a violation of the 2nd law of thermodynamics. Hence, a negative temperature on the absolute scale is impossible. Hence, we cannot reach absolute zero temperature.

55. Which statement is incorrect? **[AIEEE 2002]**

(a) All reversible cycles have same efficiency
(b) Reversible cycle has more efficiency than an irreversible one
(c) Carnot cycle is a reversible one
(d) Carnot cycle has the maximum efficiency in all cycles

Exp. **(a)**

Efficiency of all reversible cycles depends upon temperature of source and sink which will be different. Rest of the statements are correct.

9

Kinetic Theory of Gases

1. If 10^{22} gas molecules each of mass 10^{-26} kg collide with a surface (perpendicular to it) elastically per second over an area 1 m^2 with a speed 10^4 m/s, the pressure exerted by the gas molecules will be of the order of **[JEE Main 2019, 8 April Shift-I]**

(a) 10^4 N/m^2
(b) 10^8 N/m^2
(c) 10^3 N/m^2
(d) 10^{16} N/m^2

Exp. **(*)**

Momentum imparted to the surface in one collision,

$$\Delta p = (p_i - p_f) = mv - (-mv) = 2mv \quad \ldots\text{(i)}$$

Force on the surface due to n collision per second, $F = \frac{n}{t}(\Delta p) = n\Delta p$ $\quad (\because t = 1\text{s})$

$$= 2\,mnv \quad \text{[from Eq. (i)]}$$

So, pressure on the surface,

$$p = \frac{F}{A} = \frac{2mnv}{A}$$

Here, $m = 10^{-26}$ kg, $n = 10^{22}$ s^{-1},

$v = 10^4$ ms^{-1}, $A = 1\,m^2$

$$\therefore \text{Pressure, } p = \frac{2 \times 10^{-26} \times 10^{22} \times 10^4}{1}$$

$$= 2 \text{ N/m}^2$$

So, pressure exerted is of order of 10^0.

2. The temperature, at which the root mean square velocity of hydrogen molecules equals their escape velocity from the earth, is closest to : [Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K, Avogadro number $N_A = 6.02 \times 10^{26}$/kg, Radius of earth $= 6.4 \times 10^6$ m, Gravitational acceleration on earth $= 10 \text{ ms}^{-2}$] **[JEE Main 2019, 8 April Shift-II]**

(a) 10^4 K
(b) 650 K
(c) 3×10^5 K
(d) 800 K

Exp. **(a)**

Root mean square velocity of hydrogen molecule is given as

$$v_{rms} = \sqrt{\frac{3k_BT}{m}}$$

Escape velocity of hydrogen molecule from the earth is given as

$$v_e = \sqrt{2gR_e}$$

Given, $v_{rms} = v_e$

or $\sqrt{\frac{3k_BT}{m}} = \sqrt{2gR_e}$

$\Rightarrow$ $T = \frac{2gR_e m}{3 \times k_B}$

Substituting the given values, we get

$$T = \frac{2 \times 10 \times 6.4 \times 10^6 \times 2}{3 \times 1.38 \times 10^{-23} \times 6.02 \times 10^{26}} \approx 10^4 \text{ K}$$

Alternate Solution

At rms speed, average thermal kinetic energy of a hydrogen gas molecule is $= \frac{3}{2}k_BT$

and if v_e = escape velocity of the gas molecule from the earth, then its kinetic energy is $KE = \frac{1}{2}mv_e^2$, where m is the mass of the gas molecule.

Equating the above thermal and kinetic energies, we have

$$\frac{3}{2}k_BT = \frac{1}{2}mv_e^2 \text{ or } T = \frac{mv_e^2}{3k_B} \quad \ldots(i)$$

Here,

$$k_B = 1.38 \times 10^{-23}\ \text{JK}^{-1},$$

$$v_e = 11.2 \times 10^3\ \text{ms}^{-1},$$

$$m = \frac{2}{6.02 \times 10^{26}} = 0.332 \times 10^{-26}\ \text{kg}$$

Substituting these value in Eq. (i), we get

$$T = \frac{0.332 \times 10^{-26} \times (11.2 \times 10^3)^2}{3 \times 1.38 \times 10^{-23}} \approx 10^4\ \text{K}$$

3. For a given gas at 1 atm pressure, rms speed of the molecules is 200 m/s at 127°C. At 2 atm pressure and at 227°C, the rms speed of the molecules will be

[JEE Main 2019, 9 April Shift-I]

(a) $100\sqrt{5}$ m/s (b) 80 m/s
(c) 100 m/s (d) $80\sqrt{5}$ m/s

Exp. **(a)**

Key Idea For a gas molecule,

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$$\therefore \quad v_{rms} \propto \sqrt{T}$$

Let unknown rms speed be $v_{rms,2}$

at $T_2 = 227°C$ (or 500 K)

and at $T_1 = 127°C$ (or 400 K)

$$v_{rms,1} = 200\ \text{m/s}$$

$\therefore$ Using the relation $v_{rms} \propto \sqrt{T}$, we can write

$$\frac{v_{rms,2}}{v_{rms,1}} = \sqrt{\frac{T_2}{T_1}} \quad \ldots(i)$$

Substituting these given values in Eq. (i), we get

$$\therefore \quad v_{rms,2} = \sqrt{\frac{500}{400}} \times 200\ \text{m/s}$$

$$= \frac{1}{2}\sqrt{5} \times 200\ \text{m/s} = 100\sqrt{5}\ \text{m/s}$$

4. An HCl molecule has rotational, translational and vibrational motions. If the rms velocity of HCl molecules in its gaseous phase is $\bar{v}$, m is its mass and k_B is Boltzmann constant, then its temperature will be

[JEE Main 2019, 9 April Shift-I]

(a) $\frac{m\bar{v}^2}{3k_B}$ (b) $\frac{m\bar{v}^2}{7k_B}$ (c) $\frac{m\bar{v}^2}{5k_B}$ (d) $\frac{m\bar{v}^2}{6k_B}$

Exp. **(b)**

Key Idea According to the law of equipartition of energy, $\frac{1}{2}mv_{rms}^2 = \frac{n}{2}k_BT$

where, n is the degree of freedom.

Since, HCl is a diatomic molecule that has rotational, translational and vibrational motion.

So, $n = 7$

$$\Rightarrow \quad \frac{1}{2}mv_{rms}^2 = \frac{7}{2}k_BT$$

Here, $v_{rms}^2 = \bar{v}$

$$\Rightarrow \quad T = \frac{m\bar{v}^2}{7k_B}$$

5. The specific heats, C_p and C_V of a gas of diatomic molecules, A are given (in units of J mol^{-1} K^{-1}) by 29 and 22, respectively. Another gas of diatomic molecules B, has the corresponding values 30 and 21. If they are treated as ideal gases, then

[JEE Main 2019, 9 April Shift-II]

(a) A has a vibrational mode but B has none
(b) Both A and B have a vibrational mode each
(c) A has one vibrational mode and B has two
(d) A is rigid but B has a vibrational mode

Exp. **(a)**

Key Idea A diatomic gas molecule has 5 degrees of freedom, i.e. 3 translational and 2 rotational, at low temperature ranges (~ 250 K to 750 K). At temperatures above 750 K, molecular vibrations occurs and this causes two extra degrees of freedom.

Now, in given case,

For gas A, $C_p = 29$, $C_V = 22$

For gas B, $C_p = 30$, $C_V = 21$

By using

$$\gamma = \frac{C_p}{C_V} = 1 + \frac{2}{f}$$

We have,

For gas A,

$$1+\frac{2}{f}=\frac{29}{22}\approx 1.3 \Rightarrow f=6.67\approx 7$$

So, gas A has vibrational mode of degree of freedom.

For gas B,

$$1+\frac{2}{f}=\frac{30}{21}\approx 1.4 \Rightarrow f=5$$

Hence, gas B does not have any vibrational mode of degree of freedom.

6. n moles of an ideal gas with constant volume heat capacity C_V undergo an isobaric expansion by certain volume. The ratio of the work done in the process, to the heat supplied is **[JEE Main 2019, 10 April Shift-I]**

(a) $\frac{4nR}{C_V+nR}$ (b) $\frac{4nR}{C_V-nR}$

(c) $\frac{nR}{C_V-nR}$ (d) $\frac{nR}{C_V+nR}$

Exp. **(*d*)**

For isobaric process, work done is given as

$$W=nR\Delta T$$

Heat supplied, $\Delta Q=nC_p\Delta T$ and

$$C_p-\frac{C_V}{n}=R \Rightarrow C_p=R+C_V/n$$

$$\therefore \quad \frac{W}{\Delta Q}=\frac{nR\Delta T}{nC_p\Delta T}=\frac{R}{C_p}=\frac{R}{R+\frac{C_V}{n}}$$

$$\Rightarrow \quad \frac{W}{\Delta Q}=\frac{nR}{nR+C_V}$$

7. A 25×10^{-3} m^3 volume cylinder is filled with 1 mole of O_2 gas at room temperature (300 K). The molecular diameter of O_2 and its root mean square speed are found to be 0.3 nm and 200 m/s, respectively. What is the average collision rate (per second) for an O_2 molecule? **[JEE Main 2019, 10 April Shift-I]**

(a) $\sim10^{10}$ (b) $\sim10^{12}$ (c) $\sim10^{11}$ (d) $\sim10^{13}$

Exp. **(*)**

Given,

Volume, $V=25\times10^{-3}$ m^3

$N=1$ mole of $O_2=6.023\times10^{23}$ atoms of O_2,

$T=300$ K

Root mean square velocity of a gas molecule of O_2, $v_{rms}=200$ m/s

Radius, $r=\frac{0.3}{2}$ nm $=\frac{0.3}{2}\times10^{-9}$ m

Now, average time, $\frac{1}{\tau}=\frac{v_{av}}{\lambda}$

where, $$\lambda=\frac{RT}{\sqrt{2}\,N\pi r^2 p}$$

As, $$p=\frac{RT}{V} \Rightarrow \lambda=\frac{V}{\sqrt{2}\,N\pi r^2}$$

$\therefore$ Average collision per second,

$$\frac{1}{\tau}=\frac{v_{av}}{\lambda}=\frac{\sqrt{\frac{8}{3\pi}}\times v_{rms}}{\lambda}$$

$$\frac{1}{\tau}=\sqrt{\frac{8}{3\pi}}\times\frac{200\times\sqrt{2}\times6.023\times0^{23}\times\pi\times\frac{0.09}{4}\times10^{-18}}{25\times10^{-3}}$$

$$\Rightarrow \quad \frac{1}{\tau}=4.4\times10^8 \text{ per second}\approx10^8$$

$\therefore$ No option given is correct.

8. One mole of an ideal gas passes through a process, where pressure and volume obey the relation $p=p_0\left[1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right]$. Here, p_0 and V_0 are constants. Calculate the change in the temperature of the gas, if its volume changes from V_0 to $2V_0$. **[JEE Main 2019, 10 April Shift-II]**

(a) $\frac{1}{2}\frac{p_0V_0}{R}$ (b) $\frac{1}{4}\frac{p_0V_0}{R}$ (c) $\frac{3}{4}\frac{p_0V_0}{R}$ (d) $\frac{5}{4}\frac{p_0V_0}{R}$

Exp. **(*d*)**

Given process equation for 1 mole of an ideal gas is

$$p=p_0\left(1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right) \quad \ldots(i)$$

Also, for 1 mole of ideal gas,

$$pV=RT$$

$$\therefore \quad p=\frac{RT}{V} \quad \ldots(ii)$$

So, from Eqs. (i) and (ii), we have

$$\frac{RT}{V}=p_0\left(1-\frac{1}{2}\left(\frac{V_0}{V}\right)^2\right)$$

$$\therefore \quad T = \frac{p_0 V}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{V}\right)^2\right) \quad \ldots\text{(iii)}$$

When volume of gas is V_0, then by substituting $V = V_0$ in Eq. (iii), we get

Temperature of gas is

$$T_1 = \frac{p_0 V_0}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{V_0}\right)^2\right) = \frac{p_0 V_0}{2R}$$

Similarly, at volume, $V = 2V_0$

Temperature of gas is

$$T_2 = \frac{p_0(2V_0)}{R}\left(1 - \frac{1}{2}\left(\frac{V_0}{2V_0}\right)^2\right) = \frac{7}{4}\frac{p_0 V_0}{R}$$

So, change in temperature as volume changes from V_0 to $2V_0$ is

$$\Delta T = T_2 - T_1$$
$$= \left(\frac{7}{4} - \frac{1}{2}\right)\frac{p_0 V_0}{R} = \frac{5}{4}\frac{p_0 V_0}{R}$$

9. Two moles of helium gas is mixed with three moles of hydrogen molecules (taken to be rigid). What is the molar specific heat of mixture at constant volume?
[Take, $R = 8.3$ J/mol-K]

[JEE Main 2019, 12 April Shift-I]

(a) 19.7 J/mol-K (b) 15.7 J/mol-K
(c) 17.4 J/mol-K (d) 21.6 J/mol-K

Exp. **(c)**

Let molar specific heat of the mixture is C_V.

Total number of molecules in the mixture

$$= 3 + 2 = 5$$

$\therefore C_V$ can be determined using

$$n C_V\, dT = n_1 C_{V_1}\, dT + n_2 C_{V_2} dT$$

or $(n_1 + n_2)(C_V)_{mix} = n_1 C_{V_1} + n_2 C_{V_2}$

[here, $n = n_1 + n_2$]

Here, $C_{V_1} = \frac{3R}{2}$ (for helium); $n_1 = 2$

$C_{V_2} = \frac{5R}{2}$ (for hydrogen); $n_2 = 3$

[For monoatomic gases, $C_V = \frac{3}{2}R$ and for diatomic gases, $C_V = \frac{5}{2}R$]

$$\therefore 5 \times C_V = \left(2 \times \frac{3R}{2}\right) + \left(3 \times \frac{5R}{2}\right)$$

$$\Rightarrow \quad 5C_V = \frac{21R}{2}$$

or $\quad C_V = \frac{21R}{10}$

$$= \frac{21 \times 8.3}{10} = \frac{174.3}{10}$$

or $\quad C_V = 17.4$ J/mol-K

10. The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-ar^4}$. Then, the total number of molecules is proportional to

[JEE Main 2019, 12 April Shift-II]

(a) $n_0 \alpha^{-3/4}$ (b) $\sqrt{n_0}\,\alpha^{1/2}$
(c) $n_0 \alpha^{1/4}$ (d) $n_0 \alpha^{-3}$

Exp. **(a)**

Number density of gas molecules,

$$n = \frac{\text{Number of molecules}}{\text{Volume of gas}}$$

$\Rightarrow$ Number of molecules,

$N = n \times$ volume of gas

Now, consider a shell of radius r and thickness dr with centre at $r = 0$.

Volume of shell of differentiable thickness (dr),
dV = surface area $\times$ thickness $= 4\pi r^2\, dr$

Now, number of molecules in this shell is

$$dN = n(r) \cdot dV = n_0 e^{-\alpha r^4} \cdot 4\pi r^2\, dr$$

So, total number of molecules present in given volume (extending from $r = 0$ to $r = \infty$) is

$$N = \int_0^\infty n(r) \cdot dV = \int_0^\infty n_0\, e^{-\alpha r^4} \cdot 4\pi r^2\, dr$$

$$= \int_0^\infty 4\pi n_0\, e^{-\alpha r^4} \cdot r^2\, dr \quad \ldots\text{(i)}$$

Here, we take

$$\alpha r^4 = t \Rightarrow r = t^{\frac{1}{4}} \cdot \alpha^{\frac{-1}{4}}$$

$$\Rightarrow \quad 4\alpha r^3 dr = dt$$

$$\Rightarrow \quad r^2\, dr = \frac{dt}{4\alpha r} = \frac{dt}{4\alpha t^{\frac{1}{4}} \cdot \alpha^{-\frac{1}{4}}} = \frac{dt}{4\,\alpha^{3/4} t^{\frac{1}{4}}}$$

Also, when $r = 0, t = 0$ and when $r = \infty, t = \infty$ substituting in Eq. (i), we get

$$N = \int_0^\infty 4\pi n_0 e^{-t} \cdot \frac{dt}{4\,\alpha^{\frac{3}{4}} t^{\frac{1}{4}}}$$

$$N = \pi\alpha^{-\frac{3}{4}} . n_0 . \int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}}\, dt$$

As value of definite integral $\int_0^\infty e^{-t} \cdot t^{-\frac{1}{4}}\, dt$ is a constant (= k let),

we have

$$N = \pi k n_0 \alpha^{-\frac{3}{4}} \Rightarrow N \propto n_0 \alpha^{-\frac{3}{4}}$$

11. A mixture of 2 moles of helium gas (atomic mass = $4u$) and 1 mole of argon gas (atomic mass = $40u$) is kept at 300 K in a container. The ratio of their rms speeds $\left[\frac{v_{\text{rms(helium)}}}{v_{\text{rms(argon)}}}\right]$ is close to

[JEE Main 2019, 9 Jan Shift-I]

(a) 0.32 (b) 2.24 (c) 3.16 (d) 0.45

Exp. **(c)**

Root mean square (rms) velocity of the molecules of a gas is given as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

where, M is the atomic mass of the gas.

$$\Rightarrow \quad v_{\text{rms}} \propto \sqrt{\frac{1}{M}}$$

$$\therefore \quad \frac{v_{\text{rms (helium)}}}{v_{\text{rms (argon)}}} = \sqrt{\frac{M_{\text{argon}}}{M_{\text{helium}}}} \quad \ldots\text{(i)}$$

Given, $M_{\text{argon}} = 40u$ and $M_{\text{helium}} = 4u$

Substituting these values in Eq. (i), we get

$$\frac{v_{\text{rms (helium)}}}{v_{\text{rms (argon)}}} = \sqrt{\frac{40}{4}}$$

$$= \sqrt{10} = 3.16$$

12. *A* 15 g mass of nitrogen gas is enclosed in a vessel at a temperature 27°C. Amount of heat transferred to the gas, so that rms velocity of molecules is doubled is about (Take, $R = 8.3$ J/K - mole)

[JEE Main 2019, 9 Jan Shift-II]

(a) 10 kJ (b) 0.9 kJ (c) 14 kJ (d) 6 kJ

Exp. **(a)**

We know that from kinetic theory of gases,

$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \text{ or } V_{\text{rms}} \propto \sqrt{T}$$

where, R is gas constant, T is temperature and M is molecular mass of a gas.

Here, to double the v_{rms}, temperature must be 4 times of the initial temperature.

$\therefore$ New temperature,

$$T_2 = 4 \times T_1$$

As, $T_1 = 27°\text{C} = 300\text{ K}$

$\therefore$ $T_2 = 4 \times 300 = 1200\text{ K}$

As, gas is kept inside a closed vessel.

$$\therefore \quad Q = nC_V \cdot dT$$

$$= \frac{15}{28} \times \frac{5R}{2}(1200 - 300)$$

$$\left[\text{As, } n = \frac{m}{M} = \frac{15}{28} \text{ for nitrogen}\right]$$

$$= \frac{15}{28} \times \frac{5}{2} \times 8.3 \times 900$$

[Given, $R = 8.3$ J/K-mol]

or $Q = 10004.4\text{ J} \simeq 10\text{ kJ}$

13. Half-mole of an ideal monoatomic gas is heated at constant pressure of 1 atm from 20°C to 90°C. Work done by gas is close to (Take, gas constant, $R = 8.31$ J/mol-K)

[JEE Main 2019, 10 Jan Shift-II]

(a) 291 J (b) 581 J
(c) 146 J (d) 73 J

Exp. **(a)**

Work done by gas during heat process at constant pressure is given by

$$\Delta W = p\Delta V$$

Using ideal gas equation,

$$pV = nRT$$

$$\Rightarrow \quad p\Delta V = nR\Delta T$$

So, $\Delta W = nR\Delta T \quad \ldots$ (i)

Now, it is given that, $n = \frac{1}{2}$ and

$$\Delta T = 90°C - 20°C$$
$$= 363\,K - 293\,K = 70\,K$$

and R (gas constant) = 8.31 J/mol-K

Substituting these values in Eq. (i), we get

$$\Delta W = \frac{1}{2} \times 8.31 \times 70 = 290.85\ J$$
$$\Delta W \simeq 291\ J$$

14. 2 kg of a monoatomic gas is at a pressure of 4×10^4 N/m^2. The density of the gas is 8 kg/m^3. What is the order of energy of the gas due to its thermal motion ?

[JEE Main 2019, 10 Jan Shift-II]

(a) 10^6 J (b) 10^3 J
(c) 10^4 J (d) 10^5 J

Exp. **(c)**

Given, mass of gas, $m = 2$ kg

Pressure on gas, $p = 4 \times 10^4 N/m^2$

Density of gas, $\rho = 8\,kg/m^3$

$\Rightarrow$ Volume of gas $= \frac{\text{Mass}}{\text{Density}}$

$\Rightarrow$ $V = \frac{2}{8} = \frac{1}{4} m^3$ ... (i)

Using ideal gas equation,

$$pV = nRT$$
$\Rightarrow$ $nRT = 4 \times 10^4 \times \frac{1}{4}$

$nRT = 10^4$ ... (ii)

Internal energy of n moles of a monoatomic gas is given by,

$$U = \frac{3}{2} nRT$$
$\Rightarrow$ $U = \frac{3}{2} \times 10^4\ J = 1.5 \times 10^4\ J$

i.e. in order of 10^4 J.

15. A rigid diatomic ideal gas undergoes an adiabatic process at room temperature. The relation between temperature and volume for this process is TV^x = constant, then x is

[JEE Main 2019, 11 Jan Shift-I]

(a) $\frac{2}{5}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{3}{5}$

Exp. **(a)**

Key Idea For an ideal gas undergoing an adiabatic process at room temperature,
pV^γ = constant or $TV^{\gamma-1}$ = constant

For a diatomic gas, degree of freedom, $f = 5$

$\therefore$ $\gamma = 1 + 2/f = 1 + \frac{2}{5} = \frac{7}{5}$

As for adiabatic process, $TV^{\gamma-1}$ = constant ...(i)

and it is given that, here TV^x = constant ...(ii)

Comparing Eqs. (i) and (ii), we get

$$\gamma - 1 = x \Rightarrow \frac{7}{5} - 1 = x$$
or $x = 2/5$

16. A gas mixture consists of 3 moles of oxygen and 5 moles of argon at temperature T. Considering only translational and rotational modes, the total internal energy of the system is

[JEE Main 2019, 11 Jan Shift-I]

(a) 12 RT (b) 15 RT (c) 20 RT (d) 4 RT

Exp. **(b)**

Internal energy of a gas with f degree of freedom is

$U = \frac{n f R T}{2}$, where n is the number of moles.

Internal energy due to O_2 gas which is a diatomic gas is $U_1 = \frac{n_1 f_1 RT}{2} = 3 \times \frac{5}{2} RT$

($\because n_1 = 3$ moles, degree of freedom for a diatomic gas $f_1 = 5$)

Internal energy due to Ar gas which is a monoatomic gas is

$$U_2 = \frac{n_2 f_2 RT}{2} = 5 \times \frac{3}{2} RT$$
($\because n_2 = 5$ moles, degree of freedom for a monoatomic gas $f_2 = 3$)

$\therefore$ Total internal energy $= U_1 + U_2$

$\Rightarrow$ $U = 15\,RT$

17. In a process, temperature and volume of one mole of an ideal monoatomic gas are varied according to the relation $VT = k$, where k is a constant. In this process, the temperature of the gas is increased by ΔT. The amount of heat absorbed by gas is (where, R is gas constant)

[JEE Main 2019, 11 Jan Shift-II]

(a) $\frac{1}{2}kR\Delta T$ (b) $\frac{2k}{3}\Delta T$ (c) $\frac{1}{2}R\Delta T$ (d) $\frac{3}{2}R\Delta T$

Exp. **(c)**

Given, $VT = k$, (k is constant)

or $T \propto \frac{1}{V}$ ...(i)

Using ideal gas equation,

$$pV = nRT$$

$$pV \propto T$$

$\Rightarrow$ $pV \propto \frac{1}{V}$ or $pV^2 = \text{constant}$ ...(ii)

i.e. a polytropic process with $x = 2$.

(Polytropic process means, $pV^x = \text{constant}$)

We know that, work done in a polytropic process is given by

$$\Delta W = \frac{p_2V_2 - p_1V_1}{1-x} \text{ (for } x \neq 1) \quad ...(iii)$$

and, $\Delta W = pV\ln\left(\frac{V_2}{V_1}\right)$ (for $x = 1$)

Here, $x = 2$,

$\therefore$ $\Delta W = \frac{p_2V_2 - p_1V_1}{1-x} = \frac{nR(T_2 - T_1)}{1-x}$

$\Rightarrow$ $\Delta W = \frac{nR\Delta T}{1-2} = -nR\Delta T$ ...(iv)

Now, for monoatomic gas change in internal energy is given by

$$\Delta U = \frac{3}{2}R\Delta T \quad ...(v)$$

Using first law of thermodynamics, heat absorbed by one mole gas is

$$\Delta Q = \Delta W + \Delta U = \frac{3}{2}R\Delta T - R\Delta T$$

$\Rightarrow$ $\Delta Q = \frac{1}{2}R\Delta T$

18. An ideal gas occupies a volume of $2\ m^3$ at a pressure of 3×10^6 Pa. The energy of the gas is **[JEE Main 2019, 12 Jan Shift-I]**

(a) 6×10^4 J (b) 10^8 J

(c) 9×10^6 J (d) 3×10^2 J

Exp. **(c)**

Key Idea Internal energy of 'n' moles of a gas with degree of freedom f (= 3 for an ideal gas), at temperature T is

$$E = \frac{f}{2} \cdot nRT = \frac{3}{2}nRT$$

For an ideal gas, internal energy, $E = \frac{3}{2}nRT$

$= \frac{3}{2}p \cdot V$ ($\because$ from $pV = nRT$)

Substituting the given values, we get

$$= \frac{3}{2} \times 3 \times 10^6 \times 2$$

$$= 9 \times 10^6 \text{ J}$$

19. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to **[JEE Main 2019, 12 Jan Shift-II]**

(a) 4×10^{-8} s (b) 3×10^{-6} s

(c) 2×10^{-7} s (d) 0.5×10^{-8} s

Exp. **(a)**

Mean time elapsed between two successive, collisions is $t = \frac{\lambda}{v}$

where, λ = mean free path length and

v = mean speed of gas molecule

$$\therefore \quad t = \frac{\left(\frac{k_Bt}{\sqrt{2}\,\pi d^2 p}\right)}{\sqrt{\frac{8}{\pi} \cdot \frac{k_BT}{M}}} \Rightarrow t = \frac{C\sqrt{T}}{p}$$

where, $C = \frac{1}{4d^2}\sqrt{\frac{k_BM}{\pi}}$ = a constant for a gas.

So, $\frac{t_2}{t_1} = \frac{\sqrt{T_2}}{\sqrt{T_1}} \cdot \left(\frac{p_1}{p_2}\right)$ ... (i)

Here given, $\frac{p_1}{p_2} = \frac{1}{2}, \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{500}{300}} = \sqrt{\frac{5}{3}}$

and $t_1 = 6 \times 10^{-8}$ s

Substituting there values in Eq. (i), we get

$$t_2 = 6 \times 10^{-8} \times \sqrt{\frac{5}{3}} \times \frac{1}{2}$$

$$= 3.86 \times 10^{-8} \text{ s} \approx 4 \times 10^{-8} \text{ s}$$

20. A vertical closed cylinder is separated into two parts by a frictionless piston of mass m and of negligible thickness. The piston is free to move along the length of the cylinder. The length of the cylinder above the piston

is l_1 and that below the piston is l_2, such that $l_1 > l_2$. Each part of the cylinder contains n moles of an ideal gas at equal temperature T. If the piston is stationary, its mass m, will be given by (where, R is universal gas constant and g is the acceleration due to gravity)

[JEE Main 2019, 12 Jan Shift-II]

(a) $\frac{nRT}{g}\left[\frac{l_1 - l_2}{l_1 l_2}\right]$ (b) $\frac{nRT}{g}\left[\frac{1}{l_2} + \frac{1}{l_1}\right]$

(c) $\frac{RT}{g}\left[\frac{2l_1 + l_2}{l_1 l_2}\right]$ (d) $\frac{RT}{ng}\left[\frac{l_1 - 3l_2}{l_1 l_2}\right]$

Exp. **(a)**

Key Idea As piston is in equilibrium, so net force on piston is zero.

When the piston is stationary, i.e. on equilibrium as shown in the figure below,

then $p_1A + mg = p_2A$

$\Rightarrow$ $mg = p_2A - p_1A$

or $mg = \left(\frac{nRTA}{V_2} - \frac{nRTA}{V_1}\right)$

[$\because pV = nRT$ (ideal gas equation)]

$$= nRT\left(\frac{A}{Al_2} - \frac{A}{Al_1}\right) = nRT\left(\frac{l_1 - l_2}{l_1 l_2}\right)$$

or $m = \frac{nRT}{g}\left(\frac{l_1 - l_2}{l_1 l_2}\right)$

21. The temperature of an open room of volume 30 m^3 increases from 17°C to 27°C due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be

[JEE Main 2017 (Offline)]

(a) 1.38×10^{23} (b) 2.5×10^{25}

(c) -2.5×10^{25} (d) -1.61×10^{23}

Exp. **(c)**

From $pV = nRT = \frac{N}{N_A} RT$

We have, $n_f - n_i = \frac{pVN_A}{RT_f} - \frac{pVN_A}{RT_i}$

$$\Rightarrow n_f - n_i = \frac{10^5 \times 30}{8.3} \times 6.02 \times 10^{23} \cdot \left(\frac{1}{300} - \frac{1}{290}\right)$$

$$= -2.5 \times 10^{25}$$

$\therefore$ $\Delta n = -2.5 \times 10^{25}$

22. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is $\left(\gamma = \frac{C_p}{C_V}\right)$

[JEE Main 2015]

(a) $\frac{3\gamma + 5}{6}$ (b) $\frac{3\gamma - 5}{6}$ (c) $\frac{\gamma + 1}{2}$ (d) $\frac{\gamma - 1}{2}$

Exp. **(c)**

For an adiabatic process $TV^{\gamma - 1}$ = constant.

We know that average time of collision between molecules

$$\tau = \frac{1}{n\pi\sqrt{2}\, v_{rms}\, d^2}$$

where, n = number of molecules per unit volume

v_{rms} = rms velocity of molecules

As $n \propto \frac{1}{V}$ and $v_{rms} \propto \sqrt{T}$

$$\tau \propto \frac{V}{\sqrt{T}}$$

Thus, we can write

$n = K_1 V^{-1}$ and $v_{rms} = K_2 T^{1/2}$

where, K_1 and K_2 are constants.

For adiabatic process, $TV^{\gamma - 1}$ = constant. Thus, we can write

$$\tau \propto VT^{-1/2} \propto V(V^{1-Y})^{-1/2}$$

or $\tau \propto V^{\frac{\gamma + 1}{2}}$

23. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional

46 cm. What will be length of the air column above mercury in the tube now? (Atmospheric pressure = 76 cm of Hg)

[JEE Main 2014]

(a) 16 cm (b) 22 cm (c) 38 cm (d) 6 cm

In this question, the system is accelerating horizontally, hence the pressure in the vertical direction will remain unaffected.
i.e., $p_1 = p_0 + \rho gh$
Again, we have to use the concept that the pressure in the same level will be same.

Exp. **(a)**

For air trapped in tube, $p_1V_1 = p_2V_2$

$$p_1 = p_{atm} = \rho g 76$$

$$V_1 = A \times 8$$

[$\because A$ = area of cross-section]

$$p_2 = p_{atm} - \rho g\ (54 - x)$$

$$= \rho g\ (22 + x)$$

$$V_2 = A \cdot x$$

At constant temprature,

$$p_1V_1 = p_2V_2$$

$$\rho g 76 \cdot A8 = \rho g\ (22 + x)\ Ax$$

$$x^2 + 22x - 78 \times 8 = 0$$

$$\Rightarrow \quad x = 16 \text{ cm}$$

24. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is **[AIEEE 2011]**

(a) $\dfrac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

(b) $\dfrac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(c) $\dfrac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

(d) $\dfrac{(T_1 + T_2 + T_3)}{3}$

Exp. **(a)**

$$\frac{F}{2} n_1kT_1 + \frac{F}{2} n_2kT_2 + \frac{F}{2} n_3kT_3 = \frac{F}{2}(n_1 + n_2 + n_3)\,kT$$

[$\because F$ is degree of freedom]

Temperature of mixture,

$$T = \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$$

25. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \dfrac{a}{x^{12}} - \dfrac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

[AIEEE 2010]

(a) $\dfrac{b^2}{2a}$ (b) $\dfrac{b^2}{12a}$ (c) $\dfrac{b^2}{4a}$ (d) $\dfrac{b^2}{6a}$

Exp. **(c)**

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$$

$$U(x = \infty) = 0$$

As, $$F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$

$$\therefore \quad x^6 = \frac{2a}{b}$$

$$\therefore \quad U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$$

$$\therefore \quad D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$$

26. One kg of a diatomic gas is at a pressure of 8×10^4 Nm^{-2}. The density of the gas is 4 kgm^{-3}. What is the energy of the gas due to its thermal motion? **[AIEEE 2009]**

(a) 3×10^4 J (b) 5×10^4 J
(c) 6×10^4 J (d) 7×10^4 J

Exp. **(b)**

Thermal energy corresponds to internal energy

Mass = 1 kg

Density = 4 kg m^{-3}

$\Rightarrow$ Volume $= \dfrac{\text{Mass}}{\text{Density}} = \dfrac{1}{4}\ \text{m}^3$

Pressure $= 8 \times 10^4\ \text{Nm}^{-2}$

As internal energy $= \dfrac{f}{2} nRT = \dfrac{f}{2} pV$

Degree of freedom for diatomic $= 5$

$\therefore$ Internal energy $= \dfrac{5}{2} p \times V$

Putting the values of $p = 8 \times 10^4\ \text{Nm}^{-2}$ and $V = \dfrac{1}{4}\text{m}^3$, we get

Internal energy $= \dfrac{5}{2} \times 8 \times 10^4 \times \dfrac{1}{4} = 5 \times 10^4$ J

27. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be **[AIEEE 2008, 04]**

(a) $\dfrac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_2 + p_2V_2T_1}$

(b) $\dfrac{p_1V_1T_1 + p_2V_2T_2}{p_1V_1 + p_2V_2}$

(c) $\dfrac{p_1V_1T_2 + p_2V_2T_1}{p_1V_1 + p_2V_2}$

(d) $\dfrac{T_1T_2(p_1V_1 + p_2V_2)}{p_1V_1T_1 + p_2V_2T_2}$

Exp. **(*a*)**

As no work is done and system is thermally insulated from surrounding, it means sum of internal energy of gas in two partitions is constant *i.e.*, $U = U_1 + U_2$.

Assuming both gases have same degree of freedom, then

Internal energy, $U = \dfrac{f(n_1 + n_2)RT}{2}$

and $U_1 = \dfrac{fn_1RT_1}{2}$, $U_2 = \dfrac{fn_2RT_2}{2}$

Solving, we get

Common temperature, $T = \dfrac{(p_1V_1 + p_2V_2)\,T_1T_2}{p_1V_1T_2 + p_2V_2T_1}$

28. If C_p and C_V denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[AIEEE 2007]**

(a) $C_p - C_V = \dfrac{R}{28}$ (b) $C_p - C_V = \dfrac{R}{14}$

(c) $C_p - C_V = R$ (d) $C_p - C_V = 28\,R$

Exp. **(*a*)**

According to Mayer's relation,

$$C_p - C_V = \frac{R}{m}$$

For nitrogen, $m = 28$

$\therefore$ $C_p - C_V = \dfrac{R}{28}$

29. Two rigid boxes $\dfrac{1}{2}$ containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature $(7/3)\,T_0$. The boxes are then put into thermal contact with each other and heat flows between them until the gases reach a common final temperature (ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f, in terms of T_0 is **[AIEEE 2006]**

(a) $T_f = \dfrac{3}{7} T_0$ (b) $T_f = \dfrac{7}{3} T_0$

(c) $T_f = \dfrac{3}{2} T_0$ (d) $T_f = \dfrac{5}{2} T_0$

Exp. **(*c*)**

Here, change in internal energy of the system is zero, *i.e.*, increase in internal energy of one is equal to decrease in internal energy of other.

Change in internal energy in box A,

$$\Delta U_A = 1 \times \frac{5}{2} R\,(T_f - T_0)$$

Change in internal energy in box B

$$\Delta U_B = 1 \times \frac{3R}{2}\left(T_f - \frac{7}{3} T_0\right)$$

Now, $\Delta U_A + \Delta U_B = 0$

$$\Rightarrow \quad \frac{5R}{2}(T_f - T_0) + \frac{3R}{2}\left(T_f - \frac{7T_0}{3}\right) = 0$$

or $\quad 5T_f - 5T_0 + 3T_f - 7T_0 = 0$

or $\quad 8T_f = 12T_0$

or $\quad T_f = \frac{12}{8}T_0 = \frac{3}{2}T_0$

30. A gaseous mixture consists of 16 g of helium and 16 g of oxygen. The ratio $\frac{C_p}{C_V}$ of the mixture is

(a) 1.59 (b) 1.62 **[AIEEE 2005]**
(c) 1.4 (d) 1.54

Exp. **(*b*)**

For mixture, $C_V = \frac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$

For helium,

Number of moles, $n_1 = \frac{16}{4} = 4$

and $\quad \gamma_1 = \frac{5}{3}$

For oxygen,

Number of moles, $n_2 = \frac{16}{32} = \frac{1}{2}$

and $\quad \gamma_2 = \frac{7}{5}$

Then, $\quad C_{V_1} = \frac{R}{\gamma_1 - 1} = \frac{R}{\frac{5}{3} - 1} = \frac{3}{2}R$

and $\quad C_{V_2} = \frac{R}{\gamma_2 - 1} = \frac{R}{\frac{7}{5} - 1} = \frac{5}{2}R$

$\therefore$ For mixture,

$$C_V = \frac{4 \times \frac{3}{2}R + \frac{1}{2} \times \frac{5}{2}R}{4 + \frac{1}{2}}$$

$$= \frac{6R + \frac{5}{4}R}{\frac{9}{2}}$$

$$= \frac{29R \times 2}{9 \times 4} = \frac{29R}{18}$$

Now, $\quad C_V = \frac{R}{\gamma - 1} \Rightarrow \gamma - 1 = \frac{R}{C_V}$

or $\quad \gamma = \frac{R}{C_V} + 1 = \frac{R}{\frac{29}{18}R} + 1$

$$\Rightarrow \quad \frac{C_p}{C_V} = \frac{18}{29} + 1$$

$$= \frac{18 + 29}{29} = 1.62$$

31. One mole of ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of diatomic gas $\left(\gamma = \frac{7}{5}\right)$. What is γ for the mixture? γ denotes the ratio of specific heat at constant pressure, to that at constant volume. **[AIEEE 2004]**

(a) $\frac{3}{2}$ (b) $\frac{23}{15}$ (c) $\frac{35}{23}$ (d) $\frac{4}{3}$

Exp. **(*a*)**

Mayer's formula is

$$C_p - C_V = R \quad \text{and} \quad \gamma = \frac{C_p}{C_V}$$

Therefore, using above two relations, we find

$$C_V = \frac{R}{\gamma - 1}$$

For a mole of monatomic gas,

$$\gamma = \frac{5}{3} \qquad \therefore \quad C_V = \frac{R}{\left(\frac{5}{3}\right) - 1} = \frac{3}{2}R$$

For a mole of diatomic gas, $\gamma = \frac{7}{5}$

$$\therefore \quad C_V = \frac{R}{\left(\frac{7}{5}\right) - 1} = \frac{5}{2}R$$

When these two moles are mixed, then heat required to raise the temperature to 1°C is

$$C_V = \frac{3}{2}R + \frac{5}{2}R = 4R$$

Hence, for one mole, heat required $= \frac{4R}{2} = 2R$

$$\therefore \quad C_V = 2R \Rightarrow \frac{R}{\gamma - 1} = 2R \text{ or } \gamma = \frac{3}{2}$$

Alternate Solution

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

Here, $n_1 = 1, n_2 = 1, \gamma_1 = \frac{5}{3}, \gamma_2 = \frac{7}{5}$

$$\therefore \quad \frac{1+1}{\gamma-1} = \frac{1}{\left(\frac{5}{3}\right)-1} + \frac{1}{\left(\frac{7}{5}\right)-1}$$

$$\Rightarrow \quad \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2} \quad \text{or} \quad \frac{2}{\gamma-1} = \frac{8}{2}$$

$$\text{or} \quad \frac{2}{\gamma-1} = 4 \Rightarrow \gamma = \frac{2}{4} + 1$$

Hence, $\gamma = \frac{3}{2}$

32. 1 mol of a gas with $\gamma = \frac{7}{5}$ is mixed with 1 mol of a gas with $\gamma = \frac{5}{3}$, then the value of γ for the resulting mixture is **[AIEEE 2002]**

(a) $\frac{7}{5}$ (b) $\frac{2}{5}$ (c) $\frac{24}{16}$ (d) $\frac{12}{7}$

Exp. **(*c*)**

Using the relation,

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$$

$$\Rightarrow \quad \frac{1+1}{\gamma-1} = \frac{1}{(5/3-1)} + \frac{1}{(7/5-1)}$$

$$\text{or} \quad \frac{2}{\gamma-1} = \frac{3}{2} + \frac{5}{2} = 4$$

$$\Rightarrow \quad \gamma = \frac{3}{2} = \frac{24}{16}$$

33. At what temperature is the rms velocity of a hydrogen molecule equal to that of an oxygen molecule at 47°C ? **[AIEEE 2002]**

(a) 80 K (b) –73 K
(c) 3 K (d) 20 K

Exp. **(*d*)**

The rms velocity of the molecule of a gas of molecular weight M at temperature T is given by

$$c_{rms} = \sqrt{\left(\frac{3RT}{M}\right)}$$

Let M_O and M_H be molecular weights of oxygen and hydrogen and T_O and T_H be the corresponding kelvin temperatures at which c_{rms} is same for both gases.

i.e., $c_{rms\,(O)} = c_{rms\,(H)}$

$$\sqrt{\left(\frac{3RT_O}{M_O}\right)} = \sqrt{\left(\frac{3RT_H}{M_H}\right)}$$

Hence, $\frac{T_O}{M_O} = \frac{T_H}{M_H}$

Given, $T_O = 273 + 47 = 320$ K

and $M_O = 32, M_H = 2$

$$\therefore \quad T_H = \frac{2}{32} \times 320 = 20 \text{ K}$$

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10

Oscillations *and* Waves

1.

A wire of length $2L$, is made by joining two wires A and B of same length but different radii r and $2r$ and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q, then the ratio $p:q$ is

[JEE Main 2019, 8 April Shift-I]

(a) 3 : 5 (b) 4 : 9
(c) 1 : 2 (d) 1 : 4

Exp. **(c)**

Let mass per unit length of wires are μ_A and μ_B, respectively.

$\because$ For same material, density is also same.

So, $\mu_A = \dfrac{\rho\pi r^2 L}{L} = \mu$ and $\mu_B = \dfrac{\rho 4\pi r^2 L}{L} = 4\mu$

Tension (T) in both connected wires are same.

So, speed of wave in wires are

$$v_A = \sqrt{\frac{T}{\mu_A}} = \sqrt{\frac{T}{\mu}}$$

$[\because \mu_A = \mu$ and $\mu_B = 4\mu]$

and $$v_B = \sqrt{\frac{T}{\mu_B}} = \sqrt{\frac{T}{4\mu}}$$

So, nth harmonic in such wires system is

$$f_{n\text{th}} = \frac{pv}{2L}$$

$\Rightarrow$ $$f_A = \frac{pv_A}{2L} = \frac{p}{2L}\sqrt{\frac{T}{\mu}}$$ (for p antinodes)

Similarly, $$f_B = \frac{qv_B}{2L} = \frac{q}{2L}\sqrt{\frac{T}{4\mu}} = \frac{1}{2}\left(\frac{q}{2L}\sqrt{\frac{T}{\mu}}\right)$$

(for q antinodes)

As frequencies f_A and f_B are given equal.

So, $f_A = f_B \Rightarrow \dfrac{p}{2L}\sqrt{\dfrac{T}{\mu}} = \dfrac{q}{2}\left[\dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}\right]$

$$\frac{p}{q} = \frac{1}{2} \Rightarrow p:q = 1:2$$

2. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\dfrac{1}{1000}$ of the original amplitude is close to

[JEE Main 2019, 8 April Shift-II]

(a) 20 s (b) 50 s
(c) 100 s (d) 10 s

Exp. **(a)**

Given, frequency of oscillations is

$$f = 5 \text{osc s}^{-1}$$

$\Rightarrow$ Time period of oscillations is $T = \dfrac{1}{f} = \dfrac{1}{5}$s

So, time for 10 oscillations is $= \dfrac{10}{5} = 2$s

Now, if A_0 = initial amplitude at $t = 0$ and γ = damping factor, then for damped oscillations, amplitude after t second is given as

$$A = A_0 e^{-\gamma t}$$

$\therefore$ After 2 s, $\frac{A_0}{2} = A_0 e^{-\gamma(2)} \Rightarrow 2 = e^{2\gamma}$

$$\Rightarrow \quad \gamma = \frac{\log 2}{2} \quad \ldots(i)$$

Now, when amplitude is $\frac{1}{1000}$ of initial amplitude, i.e.

$$\frac{A_0}{1000} = A_0 e^{-\gamma t}$$

$$\Rightarrow \quad \log(1000) = \gamma t$$

$$\Rightarrow \quad \log(10^3) = \gamma t$$

$$3\log 10 = \gamma t$$

$$\Rightarrow \quad t = \frac{2 \times 3\log 10}{\log 2} \quad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad t = 19.93\,\text{s} \quad \text{or} \quad t \approx 20\,\text{s}$$

3. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is **[JEE Main 2019, 9 April Shift-I]**

(a) $2T\sqrt{\frac{1}{10}}$ (b) $2T\sqrt{\frac{1}{14}}$ (c) $4T\sqrt{\frac{1}{14}}$ (d) $4T\sqrt{\frac{1}{15}}$

Exp. **(d)**

We know that,

Time period of a pendulum is given by

$$T = 2\pi\sqrt{L/g_{\text{eff}}} \quad \ldots(i)$$

Here, L is the length of the pendulum and g_{eff} is the effective acceleration due to gravity in the respective medium in which bob is oscillating.

Initially, when bob is oscillating in air, $g_{\text{eff}} = g$.

So, initial time period, $T = 2\pi\sqrt{\frac{L}{g}} \quad \ldots(ii)$

Let ρ_{bob} be the density of the bob.

When this bob is dipped into a liquid whose density is given as

$$\rho_{\text{liquid}} = \frac{\rho_{\text{bob}}}{16} = \frac{\rho}{16} \quad \text{(given)}$$

$\therefore$ Net force on the bob is

$$F_{\text{net}} = V\rho g - V \cdot \frac{\rho}{16} \cdot g \quad \ldots(iii)$$

(where, V = volume of the bob = volume of displaced liquid by the bob when immersed in it). If effective value of gravitational acceleration on the bob in this liquid is g_{eff}, then net force on the bob can also be written as

$$F_{\text{net}} = V\rho g_{\text{eff}} \quad \ldots(iv)$$

Equating Eqs. (iii) and (iv), we have

$$V\rho g_{\text{eff}} = V\rho g - V\rho g/16$$

$$\Rightarrow \quad g_{\text{eff}} = g - g/16 = \frac{15}{16}g \quad \ldots(v)$$

Substituting the value of g_{eff} from Eq. (v) in Eq. (i), the new time period of the bob will be

$$T' = 2\pi\sqrt{\frac{L}{g_{\text{eff}}}} = 2\pi\sqrt{\frac{16}{15}\frac{L}{g}}$$

$$\Rightarrow \quad T' = \sqrt{\frac{16}{15}} \times 2\pi\sqrt{\frac{L}{g}}$$

$$= \frac{4}{\sqrt{15}} \times T \quad \text{[using Eq. (ii)]}$$

4. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3\sin(0.157x)\cos(200\pi t)$. The length of the string is (All quantities are in SI units) **[JEE Main 2019, 9 April Shift-I]**

(a) 60 m (b) 40 m (c) 80 m (d) 20 m

Exp. **(c)**

Given equation of stationary wave is

$$Y = 0.3\sin(0.157x)\cos(200\pi t)$$

Comparing it with general equation of stationary wave, i.e. $Y = a\sin kx\cos\omega t$, we get

$$k = \left(\frac{2\pi}{\lambda}\right) = 0.157$$

$$\Rightarrow \quad \lambda = \frac{2\pi}{0.157} = 4\pi^2 \quad \left(\because \frac{1}{2\pi} \approx 0.157\right) \ldots(i)$$

and $\omega = 200\,\pi = \frac{2\pi}{T} \Rightarrow T = \frac{1}{20}$ s

As the possible wavelength associated with nth harmonic of a vibrating string, i.e. fixed at both ends is given as

$$\lambda = \frac{2l}{n} \text{ or } l = n\left(\frac{\lambda}{2}\right)$$

Now, according to question, string is fixed from both ends and oscillates in 4th harmonic, so

$$4\left(\frac{\lambda}{2}\right) = l \Rightarrow 2\lambda = l$$

or $\quad l = 2 \times 4\pi^2 = 8\pi^2 \quad$ [using Eq. (i)]

Now, $\quad \pi^2 \approx 10 \Rightarrow l \approx 80$ m

5. The pressure wave $p = 0.01 \sin[1000\,t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be $336\ \text{ms}^{-1}$. Approximate value of T is

[JEE Main 2019, 9 April Shift-I]

(a) 15° C (b) 11° C (c) 12° C (d) 4° C

Exp. **(d)**

Given, $p = 0.01\sin(1000t - 3x)\ \text{N/m}^2$

Comparing with the general equation of pressure wave of sound, i.e. $p_0 \sin(\omega t - kx)$,

we get $\omega = 1000$ and $k = 3$

Also, $k = \dfrac{\omega}{v} \Rightarrow v = \omega / k$

$\therefore$ Velocity of sound is $|v_1| = \dfrac{1000}{3}$

Or

Speed of sound wave can also be calculated as

$$v = -\frac{(\text{coefficient of } t)}{(\text{coefficient of } x)} = -\frac{1000}{(-3)} = \frac{1000}{3}\ \text{m/s}$$

Now, relation between velocity of sound and temperature is

$$v = \sqrt{\frac{\gamma RT}{m}} \Rightarrow v \propto \sqrt{T}$$

or $$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow T_2 = \frac{v_2^2}{v_1^2} \cdot T_1$$

Here, $v_2 = 336$ m/s, $v_1 = 1000/3$ m/s,

$T_1 = 0°\text{C} = 273$ K

$$\therefore \quad T_2 = \frac{(336)^2}{(1000/3)^2} \times 273 = 277.38\ \text{K}$$

$$\therefore \quad T_2 = 4.38°\text{C} \simeq 4°\text{C}$$

6. A massless spring ($k = 800$ N/m), attached with a mass (500 g) is completely immersed in 1 kg of water. The spring is stretched by 2 cm and released, so that it starts vibrating. What would be the order of magnitude of the change in the temperature of water when the vibrations stop completely? (Assume that the water container and spring receive negligible heat and specific heat of mass = 400 J/kg K, specific heat of water = 4184 J/kg K) **[JEE Main 2019, 9 April Shift-II]**

(a) 10^{-4} K (b) 10^{-3} K (c) 10^{-1} K (d) 10^{-5} K

Exp. **(d)**

The given situation is shown in the figure given below

When vibrations of mass are suddenly stopped, oscillation energy (or stored energy of spring) is dissipated as heat, causing rise of temperature.

So, conversation of energy gives

$$\frac{1}{2}kx_m^2 = (m_1 s_1 + m_2 s_2)\Delta T$$

where, x_m = amplitude of oscillation,

s_1 = specific heat of mass,

s_2 = specific heat of water

and ΔT = rise in temperature.

Substituting values given in question, we have

$$\frac{1}{2} \times 800 \times (2 \times 10^{-2})^2 = \left(\left(\frac{500}{1000}\right) \times 400 + 1 \times 4184\right)\Delta T$$

$$\Rightarrow \quad \Delta T = \frac{16 \times 10^{-2}}{4384} = 3.65 \times 10^{-5}\ \text{K}$$

So, the order of magnitude of change in temperature is approx 10^{-5}.

7. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of $20\ \text{ms}^{-1}$ with respect to the ground. If an observer in car A detects a frequency 2000 Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = $340\ \text{ms}^{-1}$)

[JEE Main 2019, 9 April Shift-II]

(a) 2060 Hz (b) 2250 Hz
(c) 2300 Hz (d) 2150 Hz

Exp. **(b)**

The given condition can be shown below as,

Here, source and observer both are moving away from each other. So, by Doppler's effect, observed frequency is given by

$$f = f_0\left(\frac{v + v_o}{v - v_s}\right) \quad \ldots \text{(i)}$$

where, v = speed of sound = 340 ms^{-1},

v_o = speed of observer = – 20 ms^{-1},

v_s = speed of source = – 20 ms^{-1},

f_0 = true frequency

and f = apparent frequency = 2000 Hz

Substituting the given values in Eq. (i), we get

$$2000 = \left(\frac{340 - 20}{340 + 20}\right) \times f_0$$

$$\Rightarrow \quad f_0 = \frac{2000 \times 360}{320} = 2250 \text{ Hz}$$

8. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is **[JEE Main 2019, 9 April Shift-II]**

(a) 180 m/s, 80 Hz (b) 320 m/s, 80 Hz
(c) 320 m/s, 120 Hz (d) 180 m/s, 120 Hz

***Exp.* (b)**

Frequency of vibration of a string in nth harmonic is given by

$$f_n = n \cdot \frac{v}{2l} \quad \ldots \text{(i)}$$

where, v = speed of sound and l = length of string.

Here, f_3 = 240 Hz, l = 2 m and n = 3

Substituting these values in Eq. (i), we get

$$\therefore \quad 240 = 3 \times \frac{v}{2 \times 2} \Rightarrow v = \frac{4 \times 240}{3} = 320 \text{ ms}^{-1}$$

Also, fundamental frequency is

$$f = \frac{f_n}{n} = \frac{f_3}{3} = \frac{240}{3} = 80 \text{ Hz}$$

9. The displacement of a damped harmonic oscillator is given by $x(t) = e^{-0.1t}\cos(10\pi t + \phi)$. Here, t is in seconds.
The time taken for its amplitude of vibration to drop to half of its initial value is close to **[JEE Main 2019, 10 April Shift-I]**

(a) 27 s (b) 13 s
(c) 4 s (d) 7 s

***Exp.* (d)**

Given, displacement is

$$x(t) = e^{-0.1t}\cos(10\pi t + \phi)$$

Here, amplitude of the oscillator is

$$A = e^{-0.1t} \quad \ldots \text{(i)}$$

Let it takes t seconds for amplitude to be dropped by half.

At $t = 0 \Rightarrow A = 1$ [from Eq. (i)]

At $t = t \Rightarrow A' = \frac{A}{2} = \frac{1}{2}$

So, Eq. (i) can be written as

$$e^{-0.1t} = \frac{1}{2}$$

or $e^{0.1t} = 2$

or $0.1t = \ln(2)$

or $t = \frac{1}{0.1}\ln(2) = 10\ln(2)$

Now, ln (2) = 0.693

$\therefore$ $t = 10 \times 0.693 = 6.93$ s

or $t \approx 7$ s

10. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are in ms^{-1}, (Take, speed of sound = 300 m/s) **[JEE Main 2019, 10 April Shift-I]**

(a) 12, 16 (b) 12, 18 (c) 16, 14 (d) 8, 18

***Exp.* (b)**

Given,

Frequency of sound source (f_0) = 500 Hz

Apparent frequency heard by observer 1, f_1 = 480 Hz and apparent frequency heard by observer 2, f_2 = 530 Hz.

Let v_0 be the speed of sound.

When observer moves away from the source,

$$\text{Apparent frequency, } f_1 = f_0\left(\frac{v - v_0'}{v}\right) \quad \ldots \text{(i)}$$

When observer moves towards the source,

$$\text{Apparent frequency, } f_2 = f_0\left(\frac{v + v_0''}{v}\right) \quad \ldots \text{(ii)}$$

Substituting values in Eq. (i), we get

$$480 = 500\left(\frac{300 - v_0'}{300}\right)$$

$\Rightarrow \quad 96 \times 3 = 300 - v_0'$

$\Rightarrow \quad v_0' = 12$ m/s

Substituting values in Eq. (ii), we get

$$530 = 500\left(\frac{330 + v''}{300}\right)$$

$\Rightarrow \quad 106 \times 3 = 300 + v''$

$\Rightarrow \quad v_0'' = 18$ m/s

Thus, their respective speeds (in m/s) is 12 and 18.

11. The correct figure that shows schematically, the wave pattern produced by superposition of two waves of frequencies 9Hz and 11 Hz, is **[JEE Main 2019, 10 April Shift-II]**

Exp. **(a)**

When waves of nearby frequencies overlaps, beats are produced.

Beat frequency is given by, $f_{beat} = f_1 - f_2$

Here, $f_1 = 11$ Hz and $f_2 = 9$ Hz

$\Rightarrow$ Beat frequency is, $f_{beat} = 11 - 9 = 2$ Hz

Hence, time period of beats or time interval between beats, $T = \frac{1}{f_{beat}} \Rightarrow T = \frac{1}{2} = 0.5$ s

So, resultant wave has a time period of 0.5 s which is correctly depicted in option (a) only.

12. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him? (Take, velocity of sound in air is 350 m/s) **[JEE Main 2019, 10 April Shift-II]**

(a) 807 Hz (b) 1143 Hz
(c) 750 Hz (d) 857 Hz

Exp. ***(c)***

Initially,

After sometime,

When source is moving towards stationary observer, frequency observed is more than source frequency due to Doppler's effect, it is given by

$$f_{observed} = f\left(\frac{v}{v - v_s}\right)$$

where, f = source frequency,

f_o = observed frequency = 1000 Hz,

v = speed of sound in air = 350 ms^{-1}

and v_s = speed of source = 50 ms^{-1}

So, $f = \frac{f_{obs}(v - v_s)}{v} = \frac{1000(350 - 50)}{350} = \frac{6000}{7}$ Hz

When source moves away from stationary observer, observed frequency will be lower due to Doppler's effect and it is given by

$$f_0 = f\left(\frac{v}{v + v_s}\right) = \frac{6000 \times 350}{7 \times (350 + 50)}$$

$$= \frac{6000 \times 350}{7 \times 400} = 750 \text{ Hz}$$

13. A progressive wave travelling along the positive x-direction is represented by $y(x,t) = A \sin(kx - \omega t + \phi)$. Its snapshot at $t = 0$ is given in the figure.

[JEE Main 2019, 12 April Shift-I]

For this wave, the phase ϕ is

(a) $-\frac{\pi}{2}$ (b) π

(c) 0 (d) $\frac{\pi}{2}$

Exp. **(b)**

From the given snapshot at $t = 0$,

$$y = 0 \text{ at } x = 0$$

and $y = -$ ve when x increases from zero.

Standard expression of any progressive wave is given by $y = A\sin(kx - \omega t + \phi)$

Here, ϕ is the phase difference, we need to get

at $t = 0$ $y = A\sin(kx + \phi)$

Clearly $\phi = \pi$, so that

$$y = A\sin(kx + \pi)$$

$$y = -A\sin(kx)$$

$\Rightarrow$ $y = 0$ at $x = 0$

and $y = -$ve at $x > 0$

Which satisfies the given snapshot.

14. A submarine A travelling at 18 km/h is being chased along the line of its velocity by another submarine B travelling at 27 km/h. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to (Speed of sound in water $= 1500\ \text{ms}^{-1}$)

[JEE Main 2019, 12 April Shift-I]

(a) 504 Hz (b) 507 Hz

(c) 499 Hz (d) 502 Hz

Exp. **(d)**

Given, velocity of submarine (A),

$$v_A = 18 \text{ km/h} = \frac{18000}{3600} \text{ m/s}$$

or $v_A = 5$ m/s ...(i)

and velocity of submarine (B),

$$v_B = 27 \text{ km/h} = \frac{27000}{3600} \text{ m/s}$$

or $v_B = 7.5$ m/s ...(ii)

Signal sent by submarine (B) is detected by submarine (A) can be shown as

Frequency of the signal, $f_0 = 500$ Hz

So, in this relative motion, frequency received by submarine (A) is

$$f_1 = \left(\frac{v_S - v_A}{v_S - v_B}\right) f_0 = \left(\frac{1500 - 5}{1500 - 7.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_1 = \frac{1495}{1492.5} \times 500 \text{ Hz}$$

The reflected frequency f_1 is now received back by submarine (B).

So, frequency received at submarine (B) is

$$f_2 = \left(\frac{v_S + v_B}{v_S + v_A}\right) f_1 = \left(\frac{1500 + 7.5}{1500 + 5}\right)\left(\frac{1495}{1492.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_2 = \left(\frac{1507.5}{1505}\right)\left(\frac{1495}{1492.5}\right) 500 \text{ Hz}$$

$$\Rightarrow \quad f_2 = 1.00166 \times 1.00167 \times 500$$

$$\Rightarrow \quad f_2 = 501.67 \text{ Hz}$$

$$\approx 502 \text{ Hz}$$

15. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equal to **[JEE Main 2019, 12 April Shift-II]**

(a) 5.5 m/s (b) 15.0 m/s

(c) 2.5 m/s (d) 10.0 m/s

Exp. **(c)**

When observer moves away from S_1 and towards S_2,

then due to Doppler's effect observed frequencies of sources by observer are

$$f'_{S_1} = \frac{v - v_o}{v} \cdot f_{S_1}$$

(observer moving away from source)

and $$f'_{S_2} = \left(\frac{v + v_o}{v}\right) \cdot f_{S_2}$$

(observer moving towards source)

(where, v = speed to sound, v_o = speed of observer)

So, beat frequency heard by observer is

$$f_b = f'_{S_2} - f'_{S_1}$$

Here, $v_o = u$, $v = 330\ \text{ms}^{-1}$

$$f_b = 10\ \text{Hz}, f_{S_1} = f_{S_2} = 660\ \text{Hz}$$

On putting the values, we get

$$f_b = f'_{S_2} - f'_{S_1}$$

$$= \left(\frac{v + v_o}{v}\right) \cdot f_{S_2} - \left(\frac{v - v_o}{v}\right) f_{S_1}$$

$$= f_{S_1} \left\{\frac{v + v_o}{v} - \frac{v - v_o}{v}\right\} = f_{S_1} \cdot \frac{2v_o}{v}$$

$$\Rightarrow \quad 10 = \frac{660 \times 2u}{330} \qquad [\because v_o = u]$$

$$\Rightarrow \quad u = \frac{330 \times 10}{2 \times 660} \Rightarrow u = 2.5\ \text{ms}^{-1}$$

16. A spring whose unstretched length is l has a force constant k. The spring is cut into two pieces of unstretched lengths l_1 and l_2 where, $l_1 = nl_2$ and n is an integer. The ratio k_1 / k_2 of the corresponding force constants k_1 and k_2 will be

[JEE Main 2019, 12 April Shift-II]

(a) n (b) $\frac{1}{n^2}$ (c) $\frac{1}{n}$ (d) n^2

Exp. **(c)**

If parameters like material, number of loops per unit length, area of cross-section, etc., are kept same, then force constant of spring is inversely proportional to its length.

In given case, all other parameters are same for both parts of spring.

So, $$k_1 \propto \frac{1}{l_1} \text{ and } k_2 \propto \frac{1}{l_2}$$

$$\therefore \quad \frac{k_1}{k_2} = \frac{l_2}{l_1}$$

$$= \frac{l_2}{nl_2} = \frac{1}{n} \qquad [\because l_1 = nl_2]$$

17. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (v) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column $l_1 = 30$ cm and $l_2 = 70$ cm. Then, v is equal to

[JEE Main 2019, 12 April Shift-II]

(a) 332 ms^{-1} (b) 384 ms^{-1}
(c) 338 ms^{-1} (d) 379 ms^{-1}

Exp. **(b)**

In a resonance tube apparatus, first and second resonance occur as shown

As in a stationary wave, distance between two successive nodes is $\frac{\lambda}{2}$ and distance of a node and an antinode is $\frac{\lambda}{4}$.

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2}$$

So, speed of sound, $v = f\lambda$

$$= f \times 2(l_2 - l_1)$$

$$= 480 \times 2 \times (70 - 30) \times 10^{-2}$$

$$= 384\ \text{ms}^{-1}$$

18. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120 dB intensity sound? [Take, reference intensity of sound as 10^{-12} W/m^2]

[JEE Main 2019, 12 April Shift-II]

(a) 40 cm (b) 20 cm
(c) 10 cm (d) 30 cm

Exp. **(a)**

Loudness of sound in decible is given by

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right)$$

where, I = intensity of sound in W/m^2,

I_0 = reference intensity (= 10^{-12} W/m^2), chosen because it is near the lower limit of the human hearing range.

Here, $\beta = 120$ dB

So, we have $120 = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$

$\Rightarrow \quad 12 = \log_{10}\left(\frac{I}{10^{-12}}\right)$

Taking antilog, we have

$\Rightarrow \quad 10^{12} = \frac{I}{10^{-12}}$

$\Rightarrow \quad I = 1\,\text{W/m}^2$

This is the intensity of sound reaching the observer.

Now, intensity, $I = \frac{P}{4\pi r^2}$

where, r = distance from source,

P = power of output source.

Here, $P = 2$ W, we have

$$1 = \frac{2}{4\pi r^2}$$

$\Rightarrow \quad r^2 = \frac{1}{2\pi}$

$\Rightarrow \quad r = \sqrt{\frac{1}{2\pi}}\,\text{m} = 0.398\,\text{m} \approx 40\,\text{cm}$

19. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length l. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system (see figure). Because of torsional constant k, the restoring torque is $\tau = k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be **[JEE Main 2019, 9Jan Shift-I]**

(a) $\frac{2k\theta_0^2}{l}$ (b) $\frac{k\theta_0^2}{l}$ (c) $\frac{3k\theta_0^2}{l}$ (d) $\frac{k\theta_0^2}{2l}$

Exp. **(b)**

Since in the given question, rotational torque, $\tau \propto$ angular displacement.

Thus, when it will be released, the system will execute SHM with a time period, $T = 2\pi\sqrt{\frac{I}{k}}$

(Where I is moment of inertia and k is torsional constant)

and the angular frequency is given as, $\omega = \sqrt{\frac{k}{I}}$.

If we know look at the top view of the above figure, we have

At some angular displacement 'θ_0', at point 'A' the maximum velocity will be

$$v_{max} = \frac{l}{3}\theta_0\omega = \frac{l}{3}\theta_0\sqrt{\frac{k}{I}} \quad \text{...(i)}$$

Then, tension in the rod when it passes through mean position will be

$$T = \frac{m \times v_{max}^2}{\frac{l}{3}} = \frac{ml^2\theta_0^2 k \times 3}{9 \times l \times l} \text{ [using Eq. (i)]}$$

$$= \frac{ml\,\theta_0^2 k}{3l}$$

The moment of inertia I at point O,

$$= \frac{m}{2}\left(\frac{2l}{3}\right)^2 + m\left(\frac{l}{3}\right)^2 = \frac{2l^2 m}{9} + \frac{ml^2}{9}$$

$$= \frac{3ml^2}{9} = \frac{ml^2}{3}$$

$\Rightarrow \quad T = \frac{ml\,\theta_0^2\,k \times 3}{3 \times ml^2} = \frac{\theta_0^2 k}{l} = \frac{k\theta_0^2}{l}$

20. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass $m (m << M)$. When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the

wave speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g is closest to **[JEE Main 2019, 9Jan Shift-I]**

(a) $\frac{g}{20}$ (b) $\frac{g}{5}$ (c) $\frac{g}{30}$ (d) $\frac{g}{10}$

Exp. **(b)**

When the car is at rest, then the situation can be shown in the figure below.

Since, $m << M$, then we can neglect the mass of the string. So, the initial velocity of the wave in the string can be given as,

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{\frac{M}{l}}} = 60 \text{ m/s} \quad \text{...(i)}$$

where, T is the tension in the string and μ is the mass of the block per unit length (l).

Now, when the car has acceleration 'a', then the situation can be shown in the figure given below,

Resolving the components of 'T' along X-and Y-axis, we get

$$T\cos\theta = Mg \quad \text{...(ii)}$$

$$T\sin\theta = Ma \quad \text{...(iii)}$$

Squaring both sides of Eqs. (ii) and (iii) and adding them, we get

$$T^2(\sin^2\theta + \cos^2\theta) = M^2(g^2 + a^2)$$

$$\Rightarrow \quad T = M(g^2 + a^2)^{\frac{1}{2}} \quad \text{...(iv)}$$

Now, the velocity of the wave in the string would be given as,

$$v' = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{M}{l}}}$$

$$= 60.5 \text{ m/s} \quad \text{...(v)}$$

Dividing Eq. (i) and Eq. (v), we get

$$\frac{v}{v'} = \frac{\sqrt{\frac{Mg}{\frac{M}{l}}}}{\sqrt{\frac{M(g^2 + a^2)^{\frac{1}{2}}}{\frac{M}{l}}}} = \sqrt{\frac{g}{(g^2 + a^2)^{\frac{1}{2}}}} = \frac{60}{60.5}$$

or $$\frac{60.5}{60} = \sqrt{\frac{(g^2 + a^2)^{\frac{1}{2}}}{g}}$$

Squaring both the sides, we get

$$\frac{(60.5)^2}{(60)^2} = \frac{(g^2 + a^2)^{\frac{1}{2}}}{g}$$

or $$\left(1 + \frac{0.5}{60}\right)^2 = \sqrt{1 + \left(\frac{a}{g}\right)^2}$$

Using binomial expansion,

$$(1 + x)^n = 1 + \frac{nx}{1!} + \ldots$$

On both sides, we get

$$1 + 2 \times \frac{0.5}{60} = 1 + \frac{1}{2} \cdot \frac{a^2}{g^2}$$

$$\Rightarrow \quad 1 + \frac{1}{60} = 1 + \frac{1}{2} \cdot \frac{a^2}{g^2}$$

or $$\frac{a}{g} = \frac{1}{\sqrt{30}}$$

or $$a = \frac{g}{\sqrt{30}} = \frac{g}{5.4} \simeq \frac{g}{5}$$

21. A rod of mass 'M' and length '$2L$' is suspended at its middle by a wire. It exhibits torsional oscillations. If two masses each of 'm' are attached at distance '$L/2$' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to **[JEE Main 2019, 9Jan Shift-II]**

(a) 0.57 (b) 0.37 (c) 0.77 (d) 0.17

Exp. **(b)**

We know that in case of torsonal oscillation frequency

$$\nu = \frac{k}{\sqrt{I}}$$

where, I is moment of inertia and k is torsional constant.

$\therefore$ According to question, $\nu_1 = \dfrac{k}{\sqrt{\dfrac{M(2L)^2}{12}}}$

(As, moment of inertia of a bar is $I = \dfrac{ML^2}{12}$)

or $$\nu_1 = \frac{k}{\sqrt{\dfrac{ML^2}{3}}} \quad \ldots(i)$$

When two masses are attached at ends of rod. Then its moment of inertia is

$$\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2$$

So, new frequency of oscillations is,

$$\nu_2 = \frac{k}{\sqrt{\dfrac{M(2L)^2}{12} + 2m\left(\dfrac{L}{2}\right)^2}}$$

$$\nu_2 = \frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} \quad \ldots(ii)$$

As, $\nu_2 = 80\%$ of $\nu_1 = 0.8\nu_1$

So,

$$\frac{k}{\sqrt{\dfrac{ML^2}{3} + \dfrac{mL^2}{2}}} = \frac{0.8 \times k}{\sqrt{\dfrac{ML^2}{3}}}$$

After solving it, we get,

$$\frac{m}{M} = 0.37$$

22. A musician produce the sound of second harmonics from open end flute of 50 cm. The other person moves toward the musician with speed 10 km/h from the second end of room. If the speed of sound 330 m/s, the frequency heard by running person will be **[JEE Main 2019, 9 Jan Shift-II]**

(a) 666 Hz (b) 500 Hz
(c) 753 Hz (d) 333 Hz

Exp. **(a)**

According to the question, the musician uses a open flute of length 50 cm and produce second harmonic sound waves

When the flute is open from both ends and produce second harmonic, then

$$L = \lambda_2$$

$$\Rightarrow \quad f_2 = \frac{v}{L}$$

where, λ_2 = wavelength for second harmonic,
f_2 = frequency for second harmonic
and v = speed of wave.

For given question $\Rightarrow f_2 = \dfrac{v}{L} = \dfrac{330}{50 \times 10^{-2}}$

$f_2 = 660$ Hz (frequency produce by source)

Now, a person runs towards the musician from another end of a hall

$v_{\text{observer}} = 10$ km/h (towards source)

There is apparant change in frequency, which heard by person and given by Doppler's effect formula

$$\Rightarrow \quad \nu' = f' = \nu\left[\frac{v_{\text{sound}} + v_{\text{observer}}}{v_{\text{sound}}}\right]$$

$$f' = f_2\left[\frac{v_S + v_O}{v_{\text{sound}}}\right]$$

$$f' = 660\left[\frac{330 + \dfrac{50}{18}}{330}\right]$$

$$f' = 666 \text{ Hz}$$

23. A particle is executing simple harmonic motion (SHM) of amplitude A, along the X-axis, about $x = 0$. when its potential energy (PE) equals kinetic energy (KE), the position of the particle will be **[JEE Main 2019, 9 Jan Shift-II]**

(a) A (b) $\dfrac{A}{2}$
(c) $\dfrac{A}{2\sqrt{2}}$ (d) $\dfrac{A}{\sqrt{2}}$

Exp. **(d)**

Here, A = amplitude of particle in SHM.

We know that in SHM potential energy (U) of a particle is given by the relation at distance x from the mean position is $U = \dfrac{1}{2}kx^2$

and at the same point kinetic energy (K)

= Total energy – Potential energy = $\dfrac{1}{2}kA^2 - \dfrac{1}{2}kx^2$

According to the question,

potential energy = kinetic energy

$$\therefore \quad \frac{1}{2}kx^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

or $$kx^2 = \frac{kA^2}{2} \text{ or } x = \pm\frac{A}{\sqrt{2}}$$

24. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio $\frac{f_1}{f_2}$ is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\frac{19}{18}$ (b) $\frac{21}{20}$ (c) $\frac{20}{19}$ (d) $\frac{18}{17}$

Exp. **(a)**

When a source is moving towards an stationary observer, observed frequency is given by

$$f_{\text{observed}} = f\left(\frac{v}{v + v_s}\right)$$

where, f = frequency of sound from the source, v = speed of sound and v_s = speed of source.

Now applying above formula to two different conditions given in problem, we get

f_1 = Observed frequency initially

$$= f\left(\frac{340}{340 - 34}\right) = f\left(\frac{340}{306}\right)$$

and f_2 = Observed frequency when speed of source is reduced

$$= f\left(\frac{340}{340 - 17}\right) = \frac{340}{323}$$

So, the ratio $f_1 : f_2$ is $\frac{f_1}{f_2} = \frac{323}{306} = \frac{19}{18}$.

25. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The string is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to **[JEE Main 2019, 10 Jan Shift-I]**

(a) 16.6 cm (b) 33.3 cm
(c) 10.0 cm (d) 20.0 cm

Exp. **(d)**

Velocity 'v' of the wave on the string = $\sqrt{\frac{T}{\mu}}$

where, T = tension and μ = mass per unit length.

Substituting the given values, we get

$$v = \sqrt{\frac{8}{5} \times 1000} = 40 \text{ ms}^{-1}$$

Wavelength of the wave on the string, $\lambda = \frac{v}{f}$

where, f = frequency of wave.

$$\Rightarrow \quad \lambda = \frac{40}{100} \text{ m} = 40 \text{ cm}$$

$\therefore$ Separation between two successive nodes is,

$$d = \frac{\lambda}{2} = \frac{40}{2} = 20.0 \text{ cm}.$$

26. A cylindrical plastic bottle of negligible mass is filled with 310 mL of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm, then ω is close to (Take, density of water $= 10^3$ kg/m^3) **[JEE Main 2019, 10 Jan Shift-II]**

(a) 2.50 rad s^{-1} (b) 5.00 rad s^{-1}
(c) 1.25 rad s^{-1} (d) 3.75 rad s^{-1}

Exp. **(*)**

In equilibrium condition bottle floats in water and its length 'l' inside water is same as the height of water upto which bottle is filled.

So, l = Volume of water in bottle/Area

$$= \frac{310}{\pi \times (2.5)^2} = 15.8 \text{ cm} = 0.158 \text{ m}$$

When bottle is slightly pushed inside by an amount x then, restoring force acting on the bottle is the upthrust of fluid displaced when bottle goes into liquid by amount x.

So, restoring force is;

$$F = -(\rho A x)\, g \quad \ldots(i)$$

where ρ = density of water,

A = area of cross-section of bottle and

x = displacement from equilibrium position

But $$F = ma \quad \ldots(ii)$$

where, m = mass of water and bottle system
$= Al\rho$

From (i) and (ii) we have,

$$Al\rho\alpha = -\rho Axg$$

or $$a = -\frac{g}{l}x$$

As for SHM, $a = -\omega^2 x$

We have $\omega = \sqrt{\frac{g}{l}} = \sqrt{\frac{10}{0.158}} = \sqrt{63.29} \approx 8 \text{ rad s}^{-1}$

$\therefore$ No option is correct.

27. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be (Assume that the highest frequency a person can hear is 20,000 Hz) **[JEE Main 2019, 10 Jan Shift-II]**

(a) 7 (b) 4
(c) 5 (d) 6

Exp. **(d)**

Fundamental frequency of closed organ pipe is given by $f_0 = v / 4L$, where v is the velocity of sound in it and L is the length of the pipe.

Also, overtone frequencies are given by

$$f = (2n + 1)\frac{v}{4L} \quad \text{or} \quad f = (2n + 1) f_0$$

Given that, $f_0 = 1500$ Hz and $f_{max} = 20000$ Hz

This means, $f_{max} > f$

So, $f_{max} > (2n + 1) f_0$

$\Rightarrow$ $20000 > (2n + 1) 1500$

$\Rightarrow$ $2n + 1 < 13.33 \Rightarrow 2n < 13.33 - 1$

$\Rightarrow$ $2n < 12.33$ or $n < 6.16$

or $n = 6$ (integer number)

Hence, total six overtones will be heard.

28. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time (in seconds) is **[JEE Main 2019, 10 Jan Shift-II]**

(a) $\frac{4\pi}{3}$ (b) $\frac{8\pi}{3}$ (c) $\frac{7}{3}\pi$ (d) $\frac{3}{8}\pi$

Exp. **(b)**

In simple harmonic motion, position (x), velocity (v) and acceleration (a) of the particle are given by

$$x = A\sin\omega t$$

$$v = \omega\sqrt{A^2 - x^2} \text{ or } v = A\omega\cos\omega t$$

and $$a = -\omega^2 x \quad \text{or } a = -\omega^2 A\sin\omega t$$

Given, amplitude $A = 5$ cm and displacement $x = 4$ cm.

At this time (when $x = 4$ cm), velocity and acceleration have same magnitude.

$\Rightarrow$ $|v_{x=4}| = |a_{x=4}|$ or $|\omega\sqrt{5^2 - 4^2}| = |-4\omega^2|$

$\Rightarrow$ $3\omega = +4\omega^2 \Rightarrow \omega = (3/4)$ rad/s

So, time period, $T = \frac{2\pi}{\omega} \Rightarrow T = \frac{2\pi}{3} \times 4 = \frac{8\pi}{3}$ s.

29. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at $t = 210$ s will be **[JEE Main 2019, 11 Jan Shift-I]**

(a) 2 (b) 1 (c) $\frac{1}{9}$ (d) 3

Exp. **(*)**

Here given, displacement, $x(t) = A\sin\frac{\pi t}{90}$

where A is amplitude of SHM, t is time taken by particle to reach a point where its potential energy $U = \frac{1}{2}kx^2$ and kinetic energy $= \frac{1}{2}k(A^2 - x^2)$ here k is force constant and x is position of the particle.

Potential energy (U) at $t = 210$ s is

$$U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\sin^2\left(\frac{210}{90}\pi\right)$$

$$= \frac{1}{2}kA^2\sin^2\left(2\pi + \frac{3}{9}\pi\right)$$

$$= \frac{1}{2}kA^2\sin^2\left(\frac{\pi}{3}\right)$$

Kinetic energy at $t = 210$ s, is

$$K = \frac{1}{2}k(A^2 - x^2)$$

$$= \frac{1}{2}kA^2\left[1 - \sin^2\left(\frac{210\pi}{90}\right)\right]$$

$$= \frac{1}{2}kA^2\cos^2(210\pi/90)$$

$$\Rightarrow \quad K = \frac{1}{2}kA^2\cos^2(\pi/3)$$

So, ratio of kinetic energy to potential energy is

$$\frac{K}{U} = \frac{\frac{1}{2}kA^2\cos^2(\pi/3)}{\frac{1}{2}kA^2\sin^2(\pi/3)} = \cot^2(\pi/3) = \frac{1}{3}$$

$\therefore$ No option given is correct.

30. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03\sin(450t - 9x)$, where distance and time are measured in SI units. The tension in the string is **[JEE Main 2019, 11 Jan Shift-I]**

(a) 5 N (b) 12.5 N
(c) 7.5 N (d) 10 N

Exp. **(b)**

Given, equation can be rewritten as,

$$y = 0.03\sin 450\left(t - \frac{9x}{450}\right) \quad \ldots(i)$$

We know that the general equation of a travelling wave is given as,

$$y = A\sin\omega(t - x/v) \quad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\text{velocity, } v = \frac{450}{9} = 50\text{ m/s}$$

and angular velocity, $\omega = 450$ rad / s

As, the velocity of wave on stretched string with tension (T) is given as

$$v = \sqrt{T/\mu}$$

where, μ is linear density

$$\therefore \quad T = \mu v^2 = 5 \times 10^{-3} \times 50 \times 50 = 12.5\text{ N}$$

$(\because \text{given, } \mu = 5\text{ g/m} = 5 \times 10^{-3}\text{kg/m})$

31. The mass and the diameter of a planet are three times the respective values for the earth. The period of oscillation of a simple pendulum on the earth is 2 s. The period of oscillation of the same pendulum on the planet would be **[JEE Main 2019, 11 Jan Shift-II]**

(a) $\frac{2}{\sqrt{3}}$ s (b) $\frac{3}{2}$ s
(c) $2\sqrt{3}$ s (d) $\frac{\sqrt{3}}{2}$ s

Exp. **(c)**

Period of motion of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}} \quad \ldots(i)$$

On the surface of earth, let period of motion is T_e and acceleration due to gravity is g_e

$$\therefore \quad T_e = 2\pi\sqrt{\frac{l}{g_e}} \quad \ldots(ii)$$

On the another planet, let period of motion is T_p and gravitational acceleration is g_p

$$\therefore \quad T_p = 2\pi\sqrt{\frac{l}{g_p}} \quad \ldots(iii)$$

($\because$ Pendulum is same, so l will be same)

From Eqs. (ii) and (iii), we get

$$\frac{T_e}{T_p} = \frac{2\pi\sqrt{\frac{l}{g_e}}}{2\pi\sqrt{\frac{l}{g_p}}} = \sqrt{\frac{g_p}{g_e}} \quad \ldots(iv)$$

Now, $g_e = \frac{GM_e}{R_e^2}$

and $g_p = \frac{GM_p}{R_p^2}$

Given, $M_p = 3M_e$

and $R_p = 3R_e$

$$\therefore \quad g_p = \frac{G \times 3M_e}{9R_e^2} = \frac{1}{3}\cdot\frac{GM_e}{R_e^2} = \frac{1}{3}g_e$$

$$\Rightarrow \quad \frac{g_p}{g_e} = \frac{1}{3} \text{ or } \sqrt{\frac{g_p}{g_e}} = \frac{1}{\sqrt{3}} \quad \ldots(v)$$

From Eqs. (iv) and (v), $T_p = \sqrt{3}\,T_e$

or $T_p = 2\sqrt{3}$ s $(\because T_e = 2\text{ s})$

32. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by **[JEE Main 2019, 11 Jan Shift-II]**

(a) 1 rad/s (b) 10^{-5} rad/s
(c) 10^{-3} rad/s (d) 10^{-1} rad/s

Exp. **(c)**

We know that time period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

So, angular frequency $\omega = \frac{2\pi}{T} = \sqrt{\frac{g}{l}}$ ..(i)

Now, differentiate both side w.r.t g

$$\therefore \quad \frac{d\omega}{dg} = \frac{1}{2\sqrt{g}\sqrt{l}}$$

$$d\omega = \frac{dg}{2\sqrt{g}\sqrt{l}} \quad \text{...(ii)}$$

By dividing Eq. (ii) by Eq. (i), we get

$$\frac{d\omega}{\omega} = \frac{dg}{2g}$$

Or we can write

$$\frac{\Delta\omega}{\omega} = \frac{\Delta g}{2g} \quad \text{...(iii)}$$

As Δg is due to oscillation of support.

$$\therefore \quad \Delta g = 2\omega^2 A$$

$(\omega_1 \to 1\,\text{rad/s, support})$

Putting value of Δg in Eq. (iii) we get

$$\frac{\Delta\omega}{\omega} = \frac{1}{2}\cdot\frac{2\omega_1^2 A}{g} = \frac{\omega_1^2 A}{g}; \quad (A = 10^{-2}\text{m}^2)$$

$$\Rightarrow \quad \frac{\Delta\omega}{\omega} = \frac{1\times 10^{-2}}{10} = 10^{-3}\ \text{rad/s}$$

33. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then **[JEE Main 2019, 11 Jan Shift-II]**

(a) $K_2 = 2K_1$ (b) $K_2 = \frac{K_1}{2}$

(c) $K_2 = \frac{K_1}{4}$ (d) $K_2 = K_1$

Exp. **(b)**

Kinetic energy of a pendulum is maximum at its mean position. Also, maximum kinetic energy of pendulum

$$K_{max} = \frac{1}{2}m\omega^2 a^2$$

where, angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi\sqrt{\frac{l}{g}}}$$

or $\omega = \sqrt{\frac{g}{l}}$ or $\omega^2 = \frac{g}{l}$

and a = amplitude.

As amplitude is same in both cases so;

$$K_{max} \propto \omega^2$$

or $K_{max} \propto \frac{1}{l}$ [$\because g$ is constant]

According to given data, $K_1 \propto \frac{1}{l}$

and $K_2 \propto \frac{1}{2l}$

$$\therefore \quad \frac{K_1}{K_2} = \left(\frac{1/l}{1/2l}\right) = 2$$

or $K_1 = 2K_2 \Rightarrow K_2 = \frac{K_1}{2}$

34. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If v is the speed of sound, then speed of the plane is **[JEE Main 2019, 12 Jan Shift-I]**

(a) $\frac{\sqrt{3}}{2}v$ (b) v (c) $\frac{2v}{\sqrt{3}}$ (d) $\frac{v}{2}$

Exp. **(d)**

Let P_1 be the position of plane at $t = 0$, when sound waves started towards person A and P_2 is the position of plane observed at time instant t as shown in the figure below.

In triangle P_1P_2A,

P_1P_2 = speed of plane × time = $v_P \times t$

P_1A = speed of sound × time = $v \times t$

Now, from ΔP_1P_2A

$$\cos\theta = \frac{\text{base}}{\text{hypotenues}}$$

$$\cos 60° = \frac{P_1P_2}{P_1A} = \frac{v_P \times t}{v \times t}$$

$$\frac{1}{2} = \frac{v_P}{v}$$

$$v_P = \frac{v}{2}$$

35. Two light identical springs of spring constant k are attached horizontally at the two ends of an uniform horizontal rod AB of length l and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure.

The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is **[JEE Main 2019, 12 Jan Shift-I]**

(a) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (b) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$ (c) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

Exp. **(c)**

When a system oscillates, the magnitude of restoring torque of system is given by

$$\tau = C\theta \quad \ldots(i)$$

where, C = constant that depends on system.

Also, $$\tau = I\alpha \quad \ldots(ii)$$

where, I = moment of inertia

and α = angular acceleration

From Eqs. (i) and (ii),

$$\alpha = \frac{C}{I}\cdot\theta \quad \ldots(iii)$$

and time period of oscillation of system will be

$$T = 2\pi\sqrt{\frac{I}{C}}$$

In given case, magnitude of torque is

τ = Force × perpendicular distance

$$\tau = 2kx \times \frac{l}{2}\cos\theta$$

For small deflection, $$\tau = \left(\frac{kl^2}{2}\right)\theta \quad \ldots(iv)$$

$\because$ For small deflections, $\sin\theta = \frac{x}{(l/2)} \approx \theta$

$$\Rightarrow \quad x = \frac{l\theta}{2}$$

Also, $\cos\theta \approx 1$

comparing Eqs. (iv) and (i), we get

$$C = \frac{kl^2}{2}$$

$$\Rightarrow \quad \alpha = \frac{(kl^2/2)}{\left(\frac{1}{12}ml^2\right)}\cdot\theta \Rightarrow \alpha = \frac{6k}{m}\cdot\theta$$

Hence, time period of oscillation is $T = 2\pi\sqrt{\frac{m}{6k}}$

Frequency of oscillation is given by

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{6k}{m}}$$

36. A travelling harmonic wave is represented by the equation $y(x,t) = 10^{-3}\sin(50t + 2x)$, where x and y are in metre and t is in second. Which of the following is a correct statement about the wave? **[JEE Main 2019, 12 Jan Shift-I]**

(a) The wave is propagating along the negative X-axis with speed 25 ms^{-1}.

(b) The wave is propagating along the positive X-axis with speed 25 ms^{-1}.

(c) The wave is propagating along the positive X-axis with speed 100 ms^{-1}.

(d) The wave is propagating along the negative X-axis with speed 100 ms^{-1}

Exp. **(a)**

Wave equation is given by,

$$y = 10^{-3}\sin(50t + 2x)$$

Speed of wave is obtained by differentiating phase of wave.

Now, phase of wave from given equation is

$$\phi = 50t + 2x = \text{constant}$$

Differentiating 'ϕ' w.r.t. 't', we get

$$\frac{d}{dx}(50t + 2x) = \frac{d}{dt}(\text{constant})$$

$$\Rightarrow \quad 50 + 2\left(\frac{dx}{dt}\right) = 0$$

$$\Rightarrow \quad \frac{dx}{dt} = \frac{-50}{2} = -25\ ms^{-1}$$

So, wave is propagating in negative x–direction with a speed of 25 ms^{-1}.

Alternate Solution

The general equation of a wave travelling in negative x direction is given as

$$y = a\sin(\omega t + kx) \quad \ldots(i)$$

Given equation of wave is

$$y = 10^{-3}\sin(50 + 2x) \quad \ldots(ii)$$

Comparing Eqs. (i) and (ii), we get

$$\omega = 50 \text{ and } k = 2$$

Velocity of the wave, $v = \frac{\omega}{k} = \frac{50}{2} = 25$ m/s

37. A simple harmonic motion is represented by $y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm. The amplitude and time period of the motion are

[JEE Main 2019, 12 Jan Shift-II]

(a) 10 cm, $\frac{3}{2}$ s (b) 5 cm, $\frac{2}{3}$ s

(c) 5 cm, $\frac{3}{2}$ s (d) 10 cm, $\frac{2}{3}$ s

Exp. **(d)**

Equation for SHM is given as

$$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$$

$$= 5 \times 2\left(\frac{1}{2} \times \sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$= 5 \times 2\left(\cos\frac{\pi}{3}.\sin 3\pi t + \sin\frac{\pi}{3}.3\pi t\right)$$

$$= 5 \times 2\sin\left(3\pi t + \frac{\pi}{3}\right)$$

[using, $\sin(a + b) = \sin a\cos b + \cos a\sin b$]

or $$y = 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Comparing this equation with the general equation of SHM, i.e.

$$y = A\sin\left(\frac{2\pi t}{T} + \phi\right),$$

We get, amplitude, $A = 10$ cm

and $3\pi = \frac{2\pi}{T}$ or Time period, $T = \frac{2}{3}$ s

38. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark. near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment is close to

[JEE Main 2019, 12 Jan Shift-II]

(a) 328 ms^{-1} (b) 341 ms^{-1}

(c) 322 ms^{-1} (d) 335 ms^{-1}

Exp. **(a)**

Key idea To overcome the error occured in measurement of resonant length.We introduce end correction factor e in length

In first resonance, length of air coloumn $= \frac{\lambda}{4}$.

So, $$l_1 + e = \frac{\lambda}{4}$$

or $11 \times 4 + 4e = \lambda$

So, speed of sound is

$\Rightarrow$ $v = f_1\lambda = 512(44 + 4e)$ ...(i)

And in second case,

$$l_1' + e = \frac{\lambda'}{4}$$

or $27 \times 4 + 4e = \lambda'$

$\Rightarrow$ $v = f_2\lambda' = 256(108 + 4e)$ ...(ii)

Dividing both Eqs. (i) and (ii), we get

$$1 = \frac{512(44 + 4e)}{256(108 + 4e)}$$

$\Rightarrow$ $e = 5$ cm

Substituting value of e in Eq. (i), we get

Speed of sound $v = 512(44 + 4e)$

$= 512(44 + 4 \times 5)$

$= 512 \times 64$ cm s^{-1}

$= 327.68$ ms^{-1}

≈ 328 ms^{-1}

39. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12} per second. What is the force constant of the bonds connecting one atom with the other? (Take, molecular weight of silver = 108 and Avogadro number $= 6.02 \times 10^{23}$ g mol^{-1})

[JEE Main 2018]

(a) 6.4 N/m (b) 7.1 N/m

(c) 2.2 N/m (d) 5.5 N/m

Exp. **(*b*)**

For a harmonic oscillator,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

where, k = force constant and $T = \frac{1}{f}$

$$\therefore \quad k = 4\pi^2 f^2 m$$

$$= 4 \times \left(\frac{22}{7}\right)^2 \times (10^{12})^2 \times \frac{108 \times 10^{-3}}{6.02 \times 10^{23}}$$

$$\Rightarrow \quad k = 7.1\,\text{N/m}$$

40. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations? **[JEE Main 2018]**

(a) 5 kHz (b) 2.5 kHz (c) 10 kHz (d) 7.5 kHz

Exp. **(*a*)**

From vibration mode,

$$\frac{\lambda}{2} = L \quad \Rightarrow \quad \lambda = 2L$$

$\therefore$ Wave speed, $v = \sqrt{\frac{Y}{\rho}}$

So, frequency $f = \frac{v}{\lambda}$

$$\Rightarrow \quad f = \frac{1}{2L}\sqrt{\frac{Y}{\rho}}$$

$$= \frac{1}{2 \times 60 \times 10^{-2}}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}}$$

$$\approx 5000\,\text{Hz}$$

$$f = 5\,\text{kHz}$$

41. An observer is moving with half the speed of light towards a stationary microwave source emitting waves at frequency 10 GHz. What is the frequency of the microwave measured by the observer? (speed of light $= 3 \times 10^8\,\text{ms}^{-1}$) **[JEE Main 2017 (Offline)]**

(a) 12.1 GHz
(b) 17.3 GHz
(c) 15.3 GHz
(d) 10.1 GHz

Exp. **(*b*)**

As the observer is moving towards the source, so frequency of waves emitted by the source will be given by the formula

$$f_{\text{observed}} = f_{\text{actual}} \cdot \left(\frac{1 + v/c}{1 - v/c}\right)^{1/2}$$

Here, frequency $\frac{v}{c} = \frac{1}{2}$

So, $$f_{\text{observed}} = f_{\text{actual}} \left(\frac{3/2}{1/2}\right)^{1/2}$$

$$\therefore \quad f_{\text{observed}} = 10 \times \sqrt{3} = 17.3\,\text{GHz}$$

42. A particle is executing simple harmonic motion with a time period T. At time $t = 0$, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look, like **[JEE Main 2017 (Offline)]**

Exp. **(*c*)**

KE is maximum at mean position and minimum at extreme position $\left(\text{at } t = \frac{T}{4}\right)$.

43. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2}{3}A$ from equilibrium position. The new amplitude of the motion is **[JEE Main 2016 (Offline)]**

(a) $\frac{A}{3}\sqrt{41}$ (b) $3A$ (c) $A\sqrt{3}$ (d) $\frac{7}{3}A$

Exp. **(*d*)**

The velocity of a particle executing SHM at any instant, is defined as the time rate of change of its displacement at that instant.

$$v = \omega\sqrt{A^2 - x^2}$$

where, ω is angular frequency, A is amplitude and x is displacement of a particle.

Suppose that the new amplitude of the motion be A'.

Initial velocity of a particle performs SHM,

$$v^2 = \omega^2\left[A^2 - \left(\frac{2A}{3}\right)^2\right] \quad \text{... (i)}$$

where, A is initial amplitude and ω is angular frequency.

Final velocity,

$$(3v)^2 = \omega^2\left[A'^2 - \left(\frac{2A}{3}\right)^2\right] \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{9} = \frac{A^2 - \frac{4A^2}{9}}{A'^2 - \frac{4A^2}{9}} \Rightarrow A' = \frac{7A}{3}$$

44. A uniform string of length 20 m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is (Take, $g = 10\,\text{ms}^{-2}$) **[JEE Main 2016 (Offline)]**

(a) $2\pi\sqrt{2}$ s (b) 2 s (c) $2\sqrt{2}$ s (d) $\sqrt{2}$ s

Exp. **(*c*)**

A uniform string of length 20 m is suspended from a rigid support. Such that the time taken to reach the support,

$$T = \frac{mgx}{l}$$

So, velocity at point $P = \sqrt{\frac{\frac{mgx}{l}}{m/l}}$

i.e. $v = \sqrt{gx}$

$$\frac{dx}{dt} = \sqrt{gx}$$

$$\Rightarrow \int_0^{20}\frac{dx}{\sqrt{x}} = \int_0^t \sqrt{g}\,dt$$

$$[2\sqrt{x}]_0^{20} = \sqrt{10}t \Rightarrow 2\sqrt{20} = \sqrt{10}t$$

$$t = 2\sqrt{2}\text{ s}$$

45. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water, so that half of it is in water. The fundamental frequency of the air column is now **[JEE Main 2016 (Offline)]**

(a) $\frac{f}{2}$ (b) $\frac{3f}{4}$ (c) $2f$ (d) f

Exp. **(*d*)**

For open ends, fundamental frequency f in air, we have

$$\frac{\lambda}{2} = l$$

$$\Rightarrow \lambda = 2l$$

$$v = f\lambda$$

$$\Rightarrow f = \frac{v}{\lambda} = \frac{v}{2l} \quad \text{...(i)}$$

When a pipe is dipped vertically in water, so that half of it is in water, we have

$$\frac{\lambda}{4} = \frac{l}{2}$$

$$\Rightarrow \lambda = 2l \Rightarrow v = f'\lambda$$

$$\Rightarrow f' = \frac{v}{\lambda} = \frac{v}{2l} = f \quad \text{...(ii)}$$

Thus, the fundamental frequency of the air column is now,

$$f = f'$$

46. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90 s using a wrist watch of 1s resolution. The accuracy in the determination of g is **[JEE Main 2015]**

(a) 2% (b) 3% (c) 1% (d) 5%

Exp. **(b)**

Given time period $T = 2\pi\sqrt{L/g}$
Thus, changes can be expressed as

$$\pm\frac{2\Delta T}{T} = \pm\frac{\Delta L}{L} \pm \frac{\Delta g}{g}$$

According to the question, we can write

$$\frac{\Delta L}{L} = \frac{0.1\,\text{cm}}{20.0\,\text{cm}} = \frac{1}{200}$$

Again time period

$$T = \frac{90}{100}\text{ s and } \Delta T = \frac{1}{100}\text{s}$$

$$\Rightarrow \quad \frac{\Delta T}{T} = \frac{1}{90}$$

Now,

$$\because \quad T = 2\pi\sqrt{\frac{L}{g}}$$

$$\because \quad g = 4\pi^2\frac{L}{T^2}$$

$$\therefore \quad \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

$$\text{or} \quad \frac{\Delta g}{g}\times 100\% = \left(\frac{\Delta L}{L}\right)\times 100\% + \left(\frac{2\Delta T}{T}\right)\times 100\%$$

$$= \left(\frac{1}{200}\times 100\right)\% + 2\times\frac{1}{90}\times 100\%$$

$$\simeq 2.72\% \simeq 3\%$$

Thus, accuracy in the determination of g is approx 3%.

47. For a simple pendulum, a graph is plotted between its Kinetic Energy (KE) and Potential Energy (PE) against its displacement *d*. Which one of the following represents these correctly? (graphs are schematic and not drawn to scale)

[JEE Main 2015]

Exp. **(b)**

During oscillation, motion of a simple pendulum KE is maximum of mean position where PE is minimum. At extreme position, KE is minimum and PE is maximum. Thus, correct graph is depicted in option (b).

48. A train is moving on a straight track with speed $20\,\text{ms}^{-1}$. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is close to (speed of sound $= 320\,\text{ms}^{-1}$)

[JEE Main 2015]

(a) 6% (b) 12% (c) 18% (d) 24%

Exp. **(b)**

Apparent frequency heard by the person before crossing the train.

$$f_1 = \left(\frac{c}{c - v_s}\right)f_0 = \left(\frac{320}{320 - 20}\right)1000$$

Similarly, apparent frequency heard, after crossing the trains

$$f_2 = \left(\frac{c}{c + v_s}\right)f_0 = \left(\frac{320}{320 + 20}\right)1000$$

[c = speed of sound]

$$\Delta f = f_1 - f_2 = \left(\frac{2\,cv_s}{c^2 - v_s^2}\right)f_0$$

$$\text{or} \quad \frac{\Delta f}{f_0}\times 100 = \left(\frac{2cv_s}{c^2 - v_s^2}\right)\times 100$$

$$= \frac{2\times 320\times 20}{300\times 340}\times 100$$

$$= \frac{2\times 32\times 20}{3\times 34}$$

$$= 12.54\% = 12\%$$

49. A particle moves with simple harmonic motion in a straight line. In first t sec, after starting from rest, it travels a distance a and in next t sec, it travels $2a$ in same direction, then **[JEE Main 2014]**

(a) amplitude of motion is $3a$
(b) time period of oscillations is 5π
(c) amplitude of motion is $4a$
(d) time period of oscillations is 6π

Exp. **(d)**

In SHM, a particle starts from rest, we have

$$x = A\cos\omega t, \text{ at } t = 0, x = A$$

When $t = t$, then $x = A - a$

When $t = 2t$, then $x = A - 3a$

So, $A - a = A\cos\omega t$

$$A - 3a = A\cos 2\omega t$$

As, $\cos 2\omega\tau = 2\cos^2\omega t - 1$

$$\Rightarrow \quad \frac{A-3a}{A} = 2\left(\frac{A-a}{A}\right)^2 - 1$$

$$\Rightarrow \quad \frac{A-3a}{A} = \frac{2A^2 + 2a^2 - 4Aa - A^2}{A^2}$$

$$A^2 - 3aA = A^2 + 2a^2 - 4Aa$$

$$a^2 = 2aA \Rightarrow A = 2a$$

Now, $A - a = A\cos\omega t$

$$\Rightarrow \quad \cos\omega t = 1/2$$

$$\frac{2\pi}{T} = \frac{\pi}{3}$$

$$T = 6\pi$$

50. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s. **[JEE Main 2014]**

(a) 12 (b) 8 (c) 6 (d) 4

Exp. **(c)**

For closed organ pipe $= \dfrac{(2n+1)v}{4l}$ $(n = 0, 1, 2, \ldots)$

$$\frac{(2n+1)v}{4l} < 1250$$

$$(2n+1) < 1250 \times \frac{4 \times 0.85}{340}$$

$$(2n+1) < 12.5$$

$$\Rightarrow \quad 2n < 11.50$$

$$\Rightarrow \quad n < 5.25$$

So, $n = 0, 1, 2, 3, \ldots, 5$

So, we have 6 possibilities.

Alternate Method

In closed organ pipe, fundamental node

i.e., $\dfrac{\lambda}{4} = 0.85 \Rightarrow \lambda = 4 \times 0.85$

As we know, $\nu = \dfrac{c}{\lambda} = \dfrac{340}{4 \times 0.85} = 100$ Hz

$\therefore$ Possible frequencies = 10 Hz, 300 Hz, 500 Hz, 700 Hz, 900 Hz, 1100 Hz below 1250 Hz.

51. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s, it will decrease to α times its original magnitude, where α equals

(a) 0.7 (b) 0.81 **[JEE Main 2013]**

(c) 0.729 (d) 0.6

Exp. **(c)**

Amplitude of damped oscillator is given by

$$A = A_0 e^{-\frac{bt}{2m}}$$

After 5 s, $0.9\,A_0 = A_0 e^{-\frac{b(5)}{2m}}$

$$\Rightarrow \quad 0.9 = e^{-\frac{b(5)}{2m}} \quad \ldots(i)$$

After, 10 s, $A = A_0 e^{-b\frac{(15)}{2m}}$

$$\Rightarrow \quad A = A_0 \left(e^{-\frac{5b}{2m}}\right)^3 \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$A = 0.729\,A_0$$

Hence, $\alpha = 0.729$

52. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross-sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is p_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency **[JEE Main 2013]**

(a) $\frac{1}{2\pi}\frac{A_\gamma p_0}{V_0 M}$ (b) $\frac{1}{2\pi}\frac{V_0 M p_0}{A^2\gamma}$

(c) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma p_0}{MV_0}}$ (d) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A_\gamma p_0}}$

Exp. **(c)**

Pressure applied by piston, $\frac{Mg}{A} = p_0$

$$Mg = p_0 A \quad \ldots(i)$$

As no exchange of heat, so process is adiabatic.

$$p_0 V_0^\gamma = pV^\gamma,\ p_0 A x_0^\gamma = pA(x_0 - x)^\gamma$$

$$p = \frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma}$$

Let piston be displaced by x.

$$Mg - \left(\frac{p_0 x_0^\gamma}{(x_0 - x)^\gamma}\right) A = f_{\text{restoring}}$$

$$p_0 A\left(1 - \frac{x_0^\gamma}{(x_0 - x)^\gamma}\right) = F_{\text{restoring}} \quad [\because x_0 - x \approx x_0]$$

$$F = -\frac{\gamma p_0 Ax}{x_0}$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{\gamma p_0 A}{x_0 M}} = \frac{1}{2\pi}\sqrt{\frac{\gamma p_0 A^2}{MV_0}}$$

53. If a simple pendulum has significant amplitude (upto a factor of $1/e$ of original) only in the period between $t = 0$ s to $t = \tau$ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity with b as the constant of proportionality, the average life time of the pendulum (assuming damping is small) in seconds is **[AIEEE 2012]**

(a) $\frac{0.693}{b}$ (b) b (c) $\frac{1}{b}$ (d) $\frac{2}{b}$

Exp. **(d)**

For damped harmonic motion,

$$ma = -kx - mbv$$

or $\quad ma + mbv + kx = 0$

Solution of above equation is

$$x = A_0 e^{-\frac{bt}{2}} \sin\omega t; \quad \text{with } \omega^2 = \frac{k}{m} - \frac{b^2}{4}$$

where, amplitude drops exponentially with time.

i.e., $\quad A_\tau = A_0 e^{-\frac{b\tau}{2}}$

Average time τ is that duration when amplitude drops by 63%, *i.e.,* becomes A_0/e.

Thus, $\quad A_\tau = \frac{A_0}{e} = A_0 e^{-\frac{b\tau}{2}}$

or $\quad \frac{b\tau}{2} = 1$ or $\tau = \frac{2}{b}$

54. A cylindrical tube open at both ends, has a fundamental frequency f, in air. The tube is dipped vertically in water so that half of it is in water. The fundamental frequency of the air-column is now **[AIEEE 2012]**

(a) f (b) $\frac{f}{2}$ (c) $\frac{3f}{4}$ (d) $2f$

Exp. **(d)**

Initially for open organ pipe, fundamental frequency,

$$\nu_o = \frac{v}{2l} = f \quad \text{[given]}$$

But when it is half dipped in water, then it becomes closed organ pipe of length $\frac{l}{2}$. In this case, fundamental frequency,

$$\nu_c = \frac{v}{4l'} = \frac{v}{4\frac{l}{2}} = \frac{v}{2l} = f$$

55. **Statement I** Two longitudinal waves given by equations—$y_1(x,t) = 2a\sin(\omega t - kx)$ and $y_2(x,t) = a\sin(2\omega t - 2kx)$ will have equal intensity.

Statement II Intensity of waves of given frequency in same medium is proportional to the square of amplitude only. **[AIEEE 2011]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not correct explanation of Statement I

Exp. **(d)**

As Intensity, $I = \frac{1}{2}\rho\omega^2 A^2 v$

Here, ρ = density of medium, A = amplitude,

ω = angular frequency

and v = velocity of wave.

$\therefore$ Intensity depend upon amplitude, frequency as well as velocity of wave.

Also, $I_1 = I_2$

56. A travelling wave represented by $y = A\sin(\omega t - kx)$ is superimposed on another wave represented by $y = A\sin(\omega t + kx)$. The resultant is **[AIEEE 2011]**

(a) a standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$

(b) a wave travelling along $+x$ direction

(c) a wave travelling along $-x$ direction

(d) a standing wave having nodes at $x = \frac{n\lambda}{2}$; $n = 0, 1, 2$

Exp. **(a)**

By superposition principle,

$y = y_1 + y_2 = A\sin(\omega t - kx) + A\sin(\omega t + kx)$

$y = 2A\sin\omega t\cos kx$

Clearly, it is equation of standing wave for position of nodes $y = 0$.

i.e., $x = (2n + 1)\frac{\lambda}{4} \Rightarrow \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2, 3$

57. A wooden cube (density of wood d) of side l floats in a liquid of density ρ with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period, T. Then, T is equal to **[AIEEE 2011]**

(a) $2\pi\sqrt{\frac{l\rho}{(\rho - d)g}}$ (b) $2\pi\sqrt{\frac{ld}{\rho g}}$

(c) $2\pi\sqrt{\frac{l\rho}{dg}}$ (d) $2\pi\sqrt{\frac{ld}{(\rho - d)g}}$

Exp. **(b)**

Let at any instant, cube be at a depth x from the equilibrium position, then

Net force acting on the cube = Upthrust on the portion of length x

$\therefore \quad F = -\rho l^2 xg = -\rho l^2 g\, x$ ...(i)

$F \propto -x$ (SHM)

Negative sign shows that force is opposite to x. Hence, equation of SHM

$F = -kx$ ...(ii)

Comparing Eqs. (i) and (ii), we get

$k = \rho l^2 g$

$\therefore \quad T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{l^3 d}{\rho l^2 g}} = 2\pi\sqrt{\frac{ld}{\rho g}}$

58. If a spring of stiffness k is cut into two parts A and B of length $l_A : l_B = 2 : 3$, then the stiffness of spring A is given by **[AIEEE 2011]**

(a) $\frac{5}{2}k$ (b) $\frac{3k}{5}$ (c) $\frac{2k}{5}$ (d) k

Exp. **(a)**

As $F = -kx \Rightarrow k \propto \frac{1}{x} \propto \frac{1}{l}$

For spring, $k \propto \frac{1}{l}$

$\therefore \quad \frac{k_A}{k} = \frac{l}{l_A} \Rightarrow k_A = \frac{l_A + l_B}{l_A}k = \frac{5}{2}k$

59. The transverse displacement $y(x,t)$ of a wave on a string is given b $y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)}$.

This represents a **[AIEEE 2011]**

(a) wave moving in $-x$ direction with speed $\sqrt{\frac{b}{a}}$

(b) standing wave of frequency $\sqrt{b}$

(c) standing wave of frequency $\frac{1}{\sqrt{b}}$

(d) wave moving in $+x$ direction with speed $\sqrt{\frac{a}{b}}$

Exp. **(a)**

$y(x,t) = e^{-(ax^2 + bt^2 + 2\sqrt{ab}\,xt)} = e^{-(\sqrt{a}\,x + \sqrt{b}\,t)^2}$

It is a function of type $y = f(\omega t + kx)$

So, $y(x,t)$ represents wave travelling along $-x$ direction.

$\therefore$ Speed of wave $= \frac{\omega}{k} = \frac{\sqrt{b}}{\sqrt{a}} = \sqrt{\frac{b}{a}}$

60. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance X_0 $(X_0 > A)$. If the maximum separation between them is

$(X_0 + A)$, the phase difference between their motion is **[AIEEE 2011]**

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{2}$

Exp. **(a)**

Let $x_1 = A\sin(\omega t + \phi_1)$ and $x_2 = A\sin(\omega t + \phi_2)$

$$x_2 - x_1 = A[\sin(\omega t + \phi_2) - \sin(\omega t - \phi_1)]$$

$$= 2A\cos\left(\frac{2\omega t + \phi_1 + \phi_2}{2}\right)\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$$

The resultant motion can be treated as a simple harmonic motion with amplitude $2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right)$.

Given, maximum distance between the particles $= X_0 + A$

$\therefore$ Amplitude of resultant SHM $= X_0 + A - X_0 = A$

$\therefore \quad 2A\sin\left(\frac{\phi_2 - \phi_1}{2}\right) = A$

$\Rightarrow$ Phase difference, $\phi_2 - \phi_1 = \pi/3$

61. A mass M, attached to a horizontal spring, executes SHM with amplitude A_1. When the mass M passes through its mean position, then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is **[AIEEE 2011]**

(a) $\frac{M+m}{M}$ (b) $\left(\frac{M}{M+m}\right)^{1/2}$

(c) $\left(\frac{M+m}{M}\right)^{1/2}$ (d) $\frac{M}{M+m}$

Exp. **(c)**

At mean position, $F_{net} = 0$

$\therefore$ By conservation of linear momentum,

$$Mv_1 = (M + m)v_2$$

$$M\omega_1 A_1 = (M + m)\omega_2 A_2$$

But angular velocity,

$$\omega_1 = \sqrt{\frac{k}{M}}$$

and $$\omega_2 = \sqrt{\frac{k}{M+m}}$$

On solving, $$\frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

62. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, then D is **[AIEEE 2010]**

(a) $\frac{b^2}{2a}$ (b) $\frac{b^2}{12a}$ (c) $\frac{b^2}{4a}$ (d) $\frac{b^2}{6a}$

Exp. **(c)**

$$U(x) = \frac{a}{x^{12}} - \frac{b}{x^6} \Rightarrow U(x = \infty) = 0$$

As, $$F = -\frac{dU}{dx} = -\left[\frac{12a}{x^{13}} + \frac{6b}{x^7}\right]$$

At equilibrium, $F = 0$

$\therefore \quad x^6 = \frac{2a}{b}$

$\therefore \quad U_{\text{at equilibrium}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)} = \frac{-b^2}{4a}$

$\therefore \quad D = [U(x = \infty) - U_{\text{at equilibrium}}] = \frac{b^2}{4a}$

63. The equation of a wave on a string of linear mass density $0.04\ \text{kg m}^{-1}$ is given by

$$y = 0.02\ (\text{m})\sin\left[2\pi\left(\frac{t}{0.04\ (\text{s})} - \frac{x}{0.50\ (\text{m})}\right)\right].$$

The tension in the string is **[AIEEE 2010]**

(a) 4.0 N (b) 12.5 N (c) 0.5 N (d) 6.25 N

Exp. **(d)**

As $v = \sqrt{\frac{T}{\mu}}$

So, $T = \mu v^2 = \mu\frac{\omega^2}{k^2} = 0.04\frac{(2\pi/0.004)^2}{(2\pi/0.50)^2} = 6.25\ \text{N}$

64. If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then which of the following does not change with time? **[AIEEE 2009]**

(a) $a^2T^2 + 4\pi^2v^2$ (b) $\frac{aT}{x}$

(c) $aT + 2\pi v$ (d) $\frac{aT}{v}$

Exp. (b)

$$\frac{aT}{x} = \frac{\omega^2 xT}{x} = \frac{4\pi^2}{T^2} \times T = \frac{4\pi^2}{T} = \text{constant}$$

Hence, aT/x does not change with time.

65. A motor cycle starts from rest and accelerates along a straight path at 2 ms^{-2}. At the starting point of the motor cycle, there is a stationary electric siren. How far has the motor cycle gone when the driver hears the frequency of the siren at 94% of its value when the motor cycle was at rest? (Speed of sound = 330 ms^{-1}) **[AIEEE 2009]**

(a) 49 m (b) 98 m (c) 147 m (d) 196 m

Exp. (b)

For motor cycle, $u = 0, a = 2\ ms^{-2}$

Observer is in motion and source is at rest, then apparent frequency,

$$n' = n\frac{v - v_o}{v + v_s} \Rightarrow \frac{94}{100}n = n\frac{330 - v_o}{330}$$

$$\Rightarrow \quad 330 - v_o = \frac{330 \times 94}{100}$$

$$\Rightarrow \quad v_o = 330 - \frac{94 \times 33}{10} = \frac{33 \times 6}{10}\ ms^{-1}$$

$$s = \frac{v^2 - u^2}{2a} = \frac{9 \times 33 \times 33}{100} = \frac{9 \times 1089}{100} \approx 98\ m$$

66. Three sound waves of equal amplitudes have frequencies $(\nu - 1)$, ν, $(\nu + 1)$. They superpose to give beat. The number of beats produced per second will be **[AIEEE 2009]**

(a) 4 (b) 3 (c) 2 (d) 1

Exp. (c)

Maximum number of beats = $\nu + 1 - (\nu - 1) = 2$

67. The speed of sound in oxygen (O_2) at a certain temperature is 460 ms^{-1}. The speed of sound in helium (He) at the same temperature will be (assume both gases to be ideal) **[AIEEE 2008]**

(a) 460 ms^{-1} (b) 500 ms^{-1}
(c) 650 ms^{-1} (d) 1420 ms^{-1}

Exp. (d)

Speed of sound is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v_{O_2} = \sqrt{\frac{\frac{7}{5}RT}{32}} \quad \text{and} \quad v_{He} = \sqrt{\frac{\frac{5}{3}RT}{4}}$$

$$\therefore \quad \frac{v_{O_2}}{v_{He}} = \sqrt{\frac{7 \times 3 \times 4}{5 \times 32 \times 5}}$$

$$\text{or} \quad v_{He} = 460 \times \sqrt{\frac{5 \times 32 \times 5}{7 \times 3 \times 4}} \approx 1420\ ms^{-1}$$

68. While measuring the speed of sound by performing a resonance column experiment, a student gets the first resonance condition at a column length of 18 cm during winter. Repeating the same experiment during summer, she measures the column length to be x cm for the second resonance. Then, **[AIEEE 2008]**

(a) $18 > x$ (b) $x > 54$
(c) $54 > x > 36$ (d) $36 > x > 18$

Exp. (b)

$l_1 = 18$ cm

$$f = \frac{v_1}{4l_1} \quad \text{[for first resonances]}$$

$$f = \frac{3v_2}{4l_2} \quad \text{[for second resonances]}$$

where, $l_2 = x$ according to given situation and also $v_1 < v_2$ as during summer temperature would be higher.

$$\Rightarrow \quad \frac{3v_2}{4l_2} = \frac{v_1}{4l_1} \quad \text{or } l_2 = 3l_1 \times \frac{v_2}{v_1}$$

$\Rightarrow$ $x = 54 \times$ (A quantity greater than 1)

So, $x > 54$

69. A wave travelling along the x-axis is described by the equation $y(x,t) = 0.005\cos(\alpha x - \beta t)$. If the wavelength and the time period of the wave are 0.08 m and 2.0 s respectively, then α and β in appropriate units are **[AIEEE 2008]**

(a) $\alpha = 25.00\pi, \beta = \pi$ (b) $\alpha = \frac{0.08}{\pi}, \beta = \frac{2.0}{\pi}$
(c) $\alpha = \frac{0.04}{\pi}, \beta = \frac{1.0}{\pi}$ (d) $\alpha = 12.50\pi, \beta = \frac{\pi}{2.0}$

Exp. (a)

As general equation is $y(x, t) = a\cos(kx - \omega t)$

where, $k = \frac{2\pi}{\lambda}$ and $\omega = \frac{2\pi}{T}$

$\therefore \quad y(x, t) = 0.005\cos(\alpha x - \beta t)$

$\Rightarrow \quad \frac{2\pi}{\lambda} = \alpha$ and $\frac{2\pi}{T} = \beta$

So, $\alpha = \frac{2\pi}{0.08} = 25\pi \text{ m}^{-1}$ and $\beta = \frac{2\pi}{2} = \pi \text{ s}^{-1}$

70. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as $a = A\cos(\omega t + \delta)$, then **[AIEEE 2007]**

(a) $A = x_0, \delta = -\frac{\pi}{4}$ (b) $A = x_0\omega^2, \delta = \frac{\pi}{4}$

(c) $A = x_0\omega^2, \delta = -\frac{\pi}{4}$ (d) $A = x_0\omega^2, \delta = \frac{3\pi}{4}$

Exp. **(d)**

$$x = x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$$

Acceleration, $a = \frac{d^2x}{dt^2} = -\omega^2 x_0 \cos\left(\omega t - \frac{\pi}{4}\right)$

$$= \omega^2 x_0 \cos\left(\omega t + \frac{3\pi}{4}\right)$$

So, $A = \omega^2 x_0$ and $\delta = \frac{3\pi}{4}$

71. A sound absorber attenuates the sound level by 20 db. The intensity decreases by a factor of **[AIEEE 2007]**

(a) 1000 (b) 10000 (c) 10 (d) 100

Exp. **(d)**

Let intensity of sound be I and I'.

Loudness of sound initially,

$$\beta_1 = 10 \log\left(\frac{I}{I_0}\right)$$

Later, $\beta_2 = 10 \log\left(\frac{I'}{I_0}\right)$

Given, $\beta_1 - \beta_2 = 20$

$\therefore \quad 20 = 10 \log\left(\frac{I}{I'}\right)$ or $I' = \frac{I}{100}$

Therefore, intensity decreases by a factor of 100.

72. Two springs of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of the mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes **[AIEEE 2007]**

(a) $\frac{f}{2}$ (b) $\frac{f}{4}$ (c) $4f$ (d) $2f$

Exp. **(d)**

$$f = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}} \text{ and } f' = \frac{1}{2\pi}\sqrt{\frac{4k_1 + 4k_2}{m}}$$

$$\Rightarrow \quad f' = \frac{1}{2\pi} \cdot 2\sqrt{\frac{k_1 + k_2}{m}} = 2f$$

73. A particle of mass m executes simple harmonic motion with amplitude a and frequency ν. The average kinetic energy during its motion from the position of equilibrium to the end is **[AIEEE 2007]**

(a) $\pi^2 ma^2\nu^2$ (b) $\frac{1}{4}ma^2\nu^2$

(c) $4\pi^2 ma^2\nu^2$ (d) $2\pi^2 ma^2\nu^2$

Exp. **(a)**

Average kinetic energy of particle

$$= \frac{1}{4}ma^2\omega^2 \quad [\text{as } \omega = 2\pi\nu]$$

$$= \frac{1}{4}ma^2(2\pi\nu)^2 = \pi^2\nu^2 ma^2$$

74. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos \pi t$ metre. The time at which the maximum speed first occurs is **[AIEEE 2007]**

(a) 0.5 s (b) 0.75 s (c) 0.125 s (d) 0.25 s

Exp. **(a)**

$$x = (2 \times 10^{-2}) \cos \pi t$$

Here, $a = 2 \times 10^{-2}$ m $= 2$ cm

At $t = 0$, $x = 2$ cm, *i.e.*, the object is at positive extreme, so to acquire maximum speed (*i.e.*, to reach mean position), it takes $\frac{1}{4}$th of time period.

$\therefore$ Required time $= \frac{T}{4}$

As $x = a\cos\omega t$

where, $\omega = \frac{2\pi}{T} = \pi \Rightarrow T = 2$ s

So, required time $= \frac{T}{4} = \frac{2}{4} = 0.5$ s

75. A string is stretched between fixed points separated by 75.0 cm. It is observed to have resonant frequencies of 420 Hz and 315 Hz. There are no other resonant frequencies between these two. Then, the lowest resonant frequency for this string is **[AIEEE 2006]**

(a) 105 Hz (b) 1.05 Hz (c) 1050 Hz (d) 10.5 Hz

Exp. **(a)**

For string fixed at both the ends, resonant frequency are given by $f = \frac{nv}{2L}$, where symbols have their usual meanings. It is given that 315 Hz and 420 Hz are two consecutive resonant frequencies, let these be nth and $(n+1)$th harmonics.

Then, $$315 = \frac{nv}{2L} \quad \ldots(i)$$

and $$420 = \frac{(n+1)v}{2L} \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{315}{420} = \frac{n}{n+1} \quad \text{or } n = 3$$

From Eq. (i), lowest resonant frequency

$$f_0 = \frac{v}{2L} = \frac{315}{3} = 105 \text{ Hz}$$

76. A whistle producing sound waves of frequencies 9500 Hz and above is approaching a stationary person with speed v ms^{-1}. The velocity of sound in air is 300 ms^{-1}. If the person can hear frequencies upto a maximum of 10000 Hz, the maximum value of v upto which he can hear the whistle is **[AIEEE 2006]**

(a) $15\sqrt{2}$ ms^{-1} (b) $15/\sqrt{2}$ ms^{-1}
(c) 15 ms^{-1} (d) 30 ms^{-1}

Exp. **(c)**

Given, velocity of sound in air = 300 m/s

If a source of sound is moving towards a stationary listener, the frequency heard by the listener would be different from the actual frequency of the source, this apparent frequency is given by $f_{app} = \left(\frac{v_{\text{sound in air}}}{v_{\text{sound in air}} \pm v_{\text{source}}}\right)$, where symbols have their usual meanings.

In the denominator, +ve sign would be taken when source is receding away from the listener, while –ve sign would be taken when source is approaching the listener.

Let v be the maximum value of source velocity for which the person is able to hear the sound, then

$$10000 = f_{app} = \left(\frac{300}{300 - v}\right) \times 9500$$

or $$v = 15 \text{ ms}^{-1}$$

77. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 ms^{-1}. The period of oscillation is **[AIEEE 2006]**

(a) 0.01 s (b) 10 s (c) 0.1 s (d) 100 s

Exp. **(a)**

The maximum velocity of a particle performing SHM is given by $v = A\omega$, where A is the amplitude and ω is the angular frequency of oscillation.

$\therefore$ $$4.4 = (7 \times 10^{-3}) \times 2\pi/T$$

or $$T = \frac{7 \times 10^{-3}}{4.4} \times \frac{2 \times 22}{7} = 0.01 \text{ s}$$

78. Starting from the origin, a body oscillates simple harmonically with a period of 2 s. After what time will its kinetic energy be 75% of the total energy? **[AIEEE 2006]**

(a) $\frac{1}{6}$ s (b) $\frac{1}{4}$ s
(c) $\frac{1}{3}$ s (d) $\frac{1}{12}$ s

Exp. **(a)**

KE of a body undergoing SHM is given by

$$\text{KE} = \frac{1}{2} m\omega^2 A^2 \cos^2 \omega t \text{ and } \text{KE}_{max} = \frac{m\omega^2 A^2}{2}$$

[symbols represent standard quantities]

From given information, $\text{KE} = (\text{KE}_{max}) \times \frac{75}{100}$

$$\Rightarrow \quad \frac{m\omega^2 A^2}{2} \cos^2 \omega t = \frac{m\omega^2 A^2}{2} \times \frac{3}{4}$$

or $$\cos \omega t = \pm \frac{\sqrt{3}}{2} \quad \text{or} \quad \omega t = \frac{\pi}{6}$$

or $$\frac{2\pi}{T} \times t = \frac{\pi}{6} \quad \text{or} \quad t = \frac{T}{12} = \frac{1}{6} \text{ s}$$

79. The function $\sin^2(\omega t)$ represents

(a) a periodic but not simple harmonic motion with a period $2\pi/\omega$ **[AIEEE 2005]**

(b) a periodic but not simple harmonic motion with a period π/ω
(c) a simple harmonic motion with a period $2\pi/\omega$
(d) a simple harmonic motion with a period π/ω

Exp. **(b)**

Here, $y = \sin^2 \omega t$

$$\frac{dy}{dt} = 2\omega \sin \omega t \cos \omega t = \omega \sin 2\omega t$$

$$\frac{d^2y}{dt^2} = 2\omega^2 \cos 2\omega t$$

For SHM, $\frac{d^2y}{dt^2} \propto -y$

Hence, function is not SHM but periodic. From the y-t graph, time period is $T = \pi/\omega$. $\left[\because \frac{2\pi}{T} = 2\omega\right]$

80. The intensity of gamma radiation from a given source is I. On passing through 36 mm of lead, it is reduced to $I/8$. The thickness of lead, which will reduce the intensity to $I/2$ will be **[AIEEE 2005]**

(a) 6 mm (b) 9 mm (c) 18 mm (d) 12 mm

Exp. **(d)**

As intesity, $I' = Ie^{-\mu x}$

Taking log on both sides, we get

$$-\mu x = \log \frac{I'}{I} \quad \text{or} \quad -\mu \cdot 36 = \log \frac{I}{8I} \quad \ldots\text{(i)}$$

and $$-\mu x' = \log \frac{I}{2I} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{36}{x'} = \frac{3 \log\left(\frac{1}{2}\right)}{\log \frac{1}{2}} \quad \text{or} \quad x' = 12 \text{ mm}$$

81. Two simple harmonic motions are represented by the equations $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$ and $y_2 = 0.1 \cos \pi t$. The phase difference of the velocity of particle 1 with respect to the velocity of particle 2 is **[AIEEE 2005]**

(a) $\frac{-\pi}{6}$ (b) $\frac{\pi}{3}$
(c) $\frac{-\pi}{3}$ (d) $\frac{\pi}{6}$

Exp. **(a)**

Given, $y_1 = 0.1 \sin\left(100\pi t + \frac{\pi}{3}\right)$

$\Rightarrow$ Velocity of particle 1,

$$\frac{dy_1}{dt} = v_1 = 0.1 \times 100\pi \cos\left(100\pi t + \frac{\pi}{3}\right)$$

or $$v_1 = 10\pi \sin\left(100\pi t + \frac{\pi}{3} + \frac{\pi}{2}\right)$$

or $$v_1 = 10\pi \sin\left(100\pi t + \frac{5\pi}{6}\right)$$

and $y_2 = 0.1 \cos \pi t$

$\Rightarrow$ Velocity of particle 2,

$$\frac{dy_2}{dt} = v_2 = -0.1 \sin \pi t$$

or $$v_2 = 0.1 \sin(\pi t + \pi)$$

Hence, phase difference

$$\Delta\phi = \phi_1 - \phi_2 = \left(100\pi t + \frac{5\pi}{6}\right) - (\pi t + \pi)$$

$$= \frac{5\pi}{6} - \pi = -\frac{\pi}{6} \quad [\text{at } t = 0]$$

82. When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard.

If the frequency of fork 1 is 200 Hz, then what will be the original frequency of fork 2? **[AIEEE 2005]**

(a) 200 Hz (b) 202 Hz (c) 196 Hz (d) 204 Hz

Exp. **(b)**

The frequency of fork 2 = 200 ± 4
= 196 or 204 Hz

Since, on attaching the tape on the prong of fork 2, its frequency decreases but now the number of beats per second is 6 *i.e.*, the frequency difference now increases.

It is possible only when before attaching the tape, the frequency of fork 2 is less than the frequency of tuning fork 1. Hence, the frequency of fork 2 is 196 Hz.

83. If a simple harmonic motion is represented by $\frac{d^2x}{dt^2}+\alpha x=0$, its time period is **[AIEEE 2005]**

(a) $\frac{2\pi}{\alpha}$ (b) $\frac{2\pi}{\sqrt{\alpha}}$ (c) $2\pi\alpha$ (d) $2\pi\sqrt{\alpha}$

Exp. **(b)**

$$\frac{d^2x}{dt^2}=-\alpha x \quad \ldots(i)$$

We know that, for SHM

$$a=\frac{d^2x}{dt^2}=-\omega^2 x \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\omega^2=\alpha \text{ or } \omega=\sqrt{\alpha}$$

$$\Rightarrow \quad \frac{2\pi}{T}=\sqrt{\alpha}$$

or $$T=\frac{2\pi}{\sqrt{\alpha}}$$

84. The bob of a simple pendulum is a spherical hollow ball filled with water. A plugged hole near the bottom of the oscillating bob gets suddenly unplugged. During observation, till water is coming out, the time period of oscillation would **[AIEEE 2005]**

(a) first increase and then decrease to the original value
(b) first decrease and then increase to the original value
(c) remain unchanged
(d) increase towards a saturation value

Exp. **(a)**

Spherical hollow ball filled with water — Spherical hollow ball half filled with water

$$T=2\pi\sqrt{\frac{l}{g}}$$

As water is coming out, CG of the system goes down, effective length increase.

$$T_1=2\pi\sqrt{\frac{l+\Delta l}{g}}, T_1>T_2$$

So, time period also increases but upto half empty. As water is coming out after half empty, CG goes up.

Effective length decreases and for complete hollow sphere, CG is also at the initial position, so effective length as well as time period decreases and same as initial at last.

Spherical hollow ball

$$T_2=2\pi\sqrt{\frac{l-\Delta l}{g}}$$

and $T_1<T_2$

Hence, time period first increases and then decreases to the original value.

85. An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. What is the percentage increase in the apparent frequency? **[AIEEE 2005]**

(a) Zero (b) 0.5% (c) 5% (d) 20%

Exp. **(d)**

$$v_o=\frac{v}{5} \quad \Rightarrow \quad v_o=\frac{320}{5}=64\text{ ms}^{-1}$$

When observer moves towards the stationary source, then

$$n'=\left(\frac{v+v_o}{v}\right)n \quad \Rightarrow n'=\left(\frac{320+64}{320}\right)n$$

or $$n'=\left(\frac{384}{320}\right)n \text{ or } \frac{n'}{n}=\frac{384}{320}$$

Hence, percentage increase

$$\left(\frac{n'-n}{n}\right)=\left(\frac{384-320}{320}\times 100\right)$$

$$=\left(\frac{64}{320}\times 100\right)=20\%$$

86. The bob of a simple pendulum executes simple harmonic motion in water with a period t, while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is $(4/3)\times 1000\text{ kgm}^{-3}$. What relationship between t and t_0 is true? **[AIEEE 2004]**

(a) $t=t_0$ (b) $t=\frac{t_0}{2}$ (c) $t=2t_0$ (d) $t=4t_0$

Exp. **(c)**

The time period of simple pendulum in air,

$$T = t_0 = 2\pi\sqrt{\left(\frac{l}{g}\right)} \quad \ldots(i)$$

Here, l being the length of simple pendulum,
In water, effective weight of bob,

w' = Weight of bob in air – Upthrust

$$\Rightarrow \quad \rho V g_{\text{eff}} = mg - m'g$$
$$= \rho V g - \rho' V g = (\rho - \rho')Vg$$

where, ρ' = density of water,
ρ = density of bob.

$$\therefore \quad g_{\text{eff}} = \left(\frac{\rho - \rho'}{\rho}\right)g = \left(1 - \frac{\rho'}{\rho}\right)g$$

$$\therefore \quad t = 2\pi\sqrt{\left[\frac{l}{\left(1 - \frac{\rho'}{\rho}\right)g}\right]} \quad \ldots(ii)$$

$$\text{Thus, } \frac{t}{t_0} = \sqrt{\left[\frac{1}{\left(1 - \frac{\rho'}{\rho}\right)}\right]} = \sqrt{\left(\frac{1}{1 - \frac{1000}{(4/3) \times 1000}}\right)}$$

$$= \sqrt{\left(\frac{4}{4-3}\right)} = 2 \text{ or } t = 2t_0$$

87. A particle at the end of a spring executes simple harmonic motion with a period t_1, while the corresponding period for another spring is t_2. If the period of oscillation with the two springs in series is T, then **[AIEEE 2004]**

(a) $T = t_1 + t_2$ (b) $T^2 = t_1^2 + t_2^2$
(c) $T^{-1} = t_1^{-1} + t_2^{-1}$ (d) $T^{-2} = t_1^{-2} + t_2^{-2}$

Exp. **(b)**

Time period of spring,

$$T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$

Here, k being the force constant of spring.
For first spring,

$$t_1 = 2\pi\sqrt{\left(\frac{m}{k_1}\right)} \quad \ldots(i)$$

For second spring,

$$t_2 = 2\pi\sqrt{\left(\frac{m}{k_2}\right)} \quad \ldots(ii)$$

In series, f is same.

$$\therefore \quad x = x_1 + x_2$$

$$\frac{-F}{k} = \frac{-F}{k_1} - \frac{F}{k_2} \Rightarrow k = \frac{k_1 k_2}{k_1 + k_2}$$

So, the effective force constant in their series combination is

$$k = \frac{k_1 k_2}{k_1 + k_2}$$

Time period of combination,

$$T = 2\pi\sqrt{\left[\frac{m(k_1 + k_2)}{k_1 k_2}\right]}$$

$$\Rightarrow \quad T^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2} \quad \ldots(iii)$$

From Eqs. (i) and (ii), we obtain

$$t_1^2 + t_2^2 = 4\pi^2\left(\frac{m}{k_1} + \frac{m}{k_2}\right)$$

or $$t_1^2 + t_2^2 = 4\pi^2 m\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

or $$t_1^2 + t_2^2 = \frac{4\pi^2 m(k_1 + k_2)}{k_1 k_2}$$

$$\Rightarrow \quad t_1^2 + t_2^2 = T^2 \quad \text{[from Eq. (iii)]}$$

88. The total energy of a particle, executing simple harmonic motion is **[AIEEE 2004]**

(a) $\propto x$ (b) $\propto x^2$
(c) independent of x (d) $\propto x^{1/2}$

where, x is the displacement from the mean position.

Exp. **(c)**

In simple harmonic motion, when a particle is displaced to a position from its mean position, then its kinetic energy gets converted into potential energy.
Hence, total energy of a particle remains constant or the total energy in simple harmonic motion does not depend on displacement x.

89. The displacement y of a particle in a medium can be expressed as

$$y = 10^{-6}\sin\left(100t + 20x + \frac{\pi}{4}\right)\text{m}$$

where, t is in second and x in metre. The speed of the wave is **[AIEEE 2004]**

(a) $2000\ \text{ms}^{-1}$ (b) $5\ \text{ms}^{-1}$
(c) $20\ \text{ms}^{-1}$ (d) $5\pi\ \text{ms}^{-1}$

Exp. **(b)**

As given $y = 10^{-6} \sin\left(100t + 20x + \frac{\pi}{4}\right)$ ...(i)

Comparing it with

$y = a \sin(\omega t + kx + \phi)$, we obtain ...(ii)

$\omega = 100 \text{ rad s}^{-1}$, $k = 20 \text{ m}^{-1}$

$\therefore \quad v = \frac{\omega}{k} = \frac{100}{20} = 5 \text{ ms}^{-1}$

90. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force $F(t)$ proportional to $\cos\omega t (\omega \neq \omega_0)$ is applied to the oscillator. The time displacement of the oscillator will be proportional to **[AIEEE 2004]**

(a) $\frac{m}{\omega_0^2 - \omega^2}$ (b) $\frac{1}{m(\omega_0^2 - \omega^2)}$

(c) $\frac{1}{m(\omega_0^2 + \omega^2)}$ (d) $\frac{m}{\omega_0^2 + \omega^2}$

Exp. **(b)**

Initial angular velocity of particle $= \omega_0$
and at any instant t, angular velocity $= \omega$

Therefore, for a displacement x, the resultant acceleration

$f = (\omega_0^2 - \omega^2)x$ ...(i)

External force, $F = m(\omega_0^2 - \omega^2)x$ ...(ii)

Since, $F \propto \cos\omega t$ [given]

From Eq. (ii), we get

$m(\omega_0^2 - \omega^2)x \propto \cos\omega t$ ...(iii)

Now, from equation of simple harmonic motion,

$x = A \sin(\omega t + \phi)$ ...(iv)

At $t = 0$; $x = A$

$\therefore \quad A = A \sin(0 + \phi) \Rightarrow \phi = \frac{\pi}{2}$

$\therefore \quad x = A \sin\left(\omega t + \frac{\pi}{2}\right) = A \cos\omega t$ ...(v)

Hence, from Eqs. (iii) and (v), we get

$m(\omega_0^2 - \omega^2)A \cos\omega t \propto \cos\omega t$

$\Rightarrow \quad A \propto \frac{1}{m(\omega_0^2 - \omega^2)}$

91. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then **[AIEEE 2004]**

(a) $\omega_1 = \omega_2$ (b) $\omega_1 > \omega_2$

(c) $\omega_1 < \omega_2$, when damping is small and $\omega_1 > \omega_2$, when damping is large

(d) $\omega_1 < \omega_2$

Exp. **(a)**

For amplitude of oscillation and energy to be maximum, frequency of force must be equal to the initial frequency and this is only possible in case of resonance. In resonance state, $\omega_1 = \omega_2$.

92. A mass M is suspended from a spring of negligible mass. The spring is pulled a little and then released so that the mass executes SHM of time period T. If the mass is increased by m, the time period becomes $5T/3$, then the ratio of m/M is **[AIEEE 2003]**

(a) $\frac{3}{5}$ (b) $\frac{25}{9}$ (c) $\frac{16}{9}$ (d) $\frac{5}{3}$

Exp. **(c)**

As time period, $T = 2\pi\sqrt{\frac{M}{k}}$ ...(i)

$T' = 2\pi\sqrt{\frac{M+m}{k}}$

$\Rightarrow \quad \frac{5T}{3} = 2\pi\sqrt{\frac{M+m}{k}}$ ...(ii)

Dividing Eq. (i) by Eq. (ii), we have

$\frac{3}{5} = \sqrt{\frac{M}{M+m}}$ or $\frac{9}{25} = \frac{M}{M+m}$

or $9M + 9m = 25M$ or $16M = 9m$

or $\frac{m}{M} = \frac{16}{9}$

93. Two particles A and B of equal masses are suspended from two massless springs of spring constants k_1 and k_2, respectively. If the maximum velocities, during oscillations are equal, the ratio of amplitudes of A and B is **[AIEEE 2003]**

(a) $\sqrt{\frac{k_1}{k_2}}$ (b) $\frac{k_1}{k_2}$ (c) $\sqrt{\frac{k_2}{k_1}}$ (d) $\frac{k_2}{k_1}$

Exp. **(c)**

$$v_{max} = a\omega = a\frac{2\pi}{T} = \frac{2\pi a}{2\pi\sqrt{\frac{m}{k}}} = a\sqrt{\frac{k}{m}}$$

Hence, $\frac{v_{max_1}}{v_{max_2}} = \frac{a_1}{a_2}\sqrt{\frac{k_1}{k_2}}$

$\because v_{max_1} = v_{max_2}$ [given]

$\Rightarrow \quad \frac{a_1}{a_2} = \sqrt{\frac{k_2}{k_1}}$

94. The length of a simple pendulum executing simple harmonic motion is increased by 21%. The percentage increase in the time period of the pendulum of increased length is **[AIEEE 2003]**

(a) 11% (b) 21% (c) 42% (d) 10%

Exp. **(d)**

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$l = \left(\frac{g}{4\pi^2}\right)T^2$$

$\therefore \quad$ % Change $= x + y + \frac{xy}{100}$

Vaild only for two variables in terms of percentage.

$x \rightarrow$ % change in first variable

$x \rightarrow$ % change in second variable

% Increase in length $= x + x + \frac{x^2}{100}$

$$21 = 2x + \frac{x^2}{100}$$

On solving, $x = 10\%$ [by cross check method]

95. The displacement y of a wave travelling in the x-direction is given by

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right)\text{metre}$$

where, x is expressed in metres and t in seconds. The speed of the wave-motion, in ms^{-1} is **[AIEEE 2003]**

(a) 300 (b) 600
(c) 1200 (d) 200

Exp. **(a)**

The given equation of wave,

$$y = 10^{-4}\sin\left(600t - 2x + \frac{\pi}{3}\right) \quad \ldots(i)$$

Standard equation of wave

$$y = a\sin(\omega t - kx + \phi) \quad \ldots(ii)$$

Now, comparing Eqs. (i) and (ii), we get

$\omega = 600\text{ rad s}^{-1}$ and $k = 2\text{ m}^{-1}$

$\therefore$ Velocity of wave $= \frac{\omega}{k} = \frac{600}{2} = 300\text{ ms}^{-1}$

96. A metal wire of linear mass density of 9.8 g /m is stretched with a tension of 10 kg-wt between two rigid supports 1 m apart. The wire passes at its middle point between the poles of a permanent magnet and it vibrates in resonance when carrying an alternating current of frequency n. The frequency n of the alternating source is **[AIEEE 2003]**

(a) 50 Hz (b) 100 Hz (c) 200 Hz (d) 25 Hz

Exp. **(a)**

The wire will vibrate with the same frequency as that of source. This can be considered as an example of forced vibration.

$T = 10 \times 9.8\text{ N} = 98\text{ N}, \; m = 9.8 \times 10^{-3}\text{ kg m}^{-1}$

Frequency of wire, $f = \frac{1}{2L}\sqrt{\left(\frac{T}{m}\right)}$

$$= \frac{1}{2 \times 1}\sqrt{\left(\frac{98}{9.8 \times 10^{-3}}\right)} = 50\text{ Hz}$$

97. A tuning fork of known frequency 256 Hz makes 5 beats per second with the vibrating string of a piano. The beat frequency decreases to 2 beats per second when the tension in the piano string is slightly increased. The frequency of the piano string before increasing the tension was **[AIEEE 2003]**

(a) (256 + 2) Hz (b) (256 – 2) Hz
(c) (256 – 5) Hz (d) (256 + 5) Hz

Exp. **(c)** $f_1 = 256\text{ Hz}$

For tuning fork, where $f_2 - f_1 = \pm 5$

f_2 = frequency of piano

$f_2 = (256 + 5)$ Hz or $(256 - 5)$ Hz

When tension is increased, the beat frequency decreases to 2 beats per second.

If we assume that the frequency of piano string is 261 Hz, then on increasing tension, frequency, more than 261 Hz. But it is given that beat frequency decreases to 2, therefore 261 Hz is not possible.

Hence, 251 Hz *i.e.*, (256 – 5) was the frequency of piano string before increasing tension.

98. A body executes simple harmonic motion. The potential energy (PE), the kinetic energy (KE) and total energy (TE) are measured as function of displacement x. Which of the following statements is true? **[AIEEE 2003]**

(a) KE is maximum when $x = 0$

(b) TE is zero when $x = 0$

(c) KE is maximum when x is maximum

(d) PE is maximum when $x = 0$

Exp. **(a)**

At $x = 0$, kinetic energy is maximum and potential energy is minimum.

$\because$ Kinetic energy, $K = \frac{1}{2} m\omega^2 (a^2 - x^2)$

K_{max} when $x = 0$

$$K_{max} = \frac{1}{2} m\omega^2 a^2$$

and potential energy,

$$U = \frac{1}{2} m\omega^2 x^2,$$

at $x = 0$, $U = 0$

99. The displacement of a particle varies according to the relation $x = 4(\cos \pi t + \sin \pi t)$. The amplitude of the particle is **[AIEEE 2003]**

(a) -4 (b) 4 (c) $4\sqrt{2}$ (d) 8

Exp. **(c)**

$$x = 4(\cos \pi t + \sin \pi t)$$

$$= \frac{4}{\sqrt{2}} \times \sqrt{2}[\cos \pi t + \sin \pi t]$$

$$x = 4\sqrt{2}\left[\sin \pi t \cdot \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \cos \pi t\right]$$

$$x = 4\sqrt{2}\left[\sin \pi t + \cos \frac{\pi}{4} + \cos \pi t \sin \frac{\pi}{4}\right]$$

$$x = 4\sqrt{2} \sin\left[\pi t + \frac{\pi}{4}\right]$$

So, amplitude $= 4\sqrt{2}$

100. A child swinging on a swing in sitting position, stands up, then the time period of the swing will **[AIEEE 2002]**

(a) increase (b) decrease

(c) remain same

(d) increase, if the child is long and decrease, if the child is short

Exp. **(b)**

As the child stands up, the effective length of pendulum decreases due to the reason that the centre of gravity rises up. Hence, according to $T = 2\pi\sqrt{\frac{l}{g}}$, T will decrease.

101. In a simple harmonic oscillator, at the mean position **[AIEEE 2002]**

(a) kinetic energy is minimum, potential energy is maximum

(b) both kinetic and potential energies are maximum

(c) kinetic energy is maximum, potential energy is minimum

(d) both kinetic and potential energies are minimum

Exp. **(c)**

Kinetic energy of particle of mass m in SHM at any point $= \frac{1}{2} m\omega^2 (a^2 - x^2)$

and potential energy $= \frac{1}{2} m\omega^2 x^2$

where, a is amplitude of particle and x is the distance from mean position.

So, at mean position, $x = 0$

$\therefore$ KE $= \frac{1}{2} m\omega^2 a^2$ [maximum]

PE $= 0$ [minimum]

102. When temperature increases, the frequency of a tuning fork **[AIEEE 2002]**

(a) increases (b) decreases

(c) remains same

(d) increases or decreases depending on the material

Exp. **(b)**

As, $f = \frac{1}{2l}\sqrt{\frac{T}{m}}$

When temperature increases, then length increases and frequency $\left(f \propto \frac{1}{l}\right)$ decreases.

103. Length of a string tied to two rigid supports is 40 cm. Maximum length (wavelength in cm) of a stationary wave produced on it, is **[AIEEE 2002]**

(a) 20 (b) 80 (c) 40 (d) 120

Exp. **(b)**

When the string is plucked in the middle, it vibrates in one loop with nodes at fixed ends and an antinode in the middle.

So that, $\frac{\lambda_1}{2} = l$

or $\lambda_1 = 2 \times 40$

$= 80$ cm

104. If a spring has time period T and is cut into n equal parts, then the time period of each part will be **[AIEEE 2002]**

(a) $T\sqrt{n}$ (b) $\frac{T}{\sqrt{n}}$

(c) nT (d) T

Exp. **(b)**

As we know that spring constant of spring is inversely proportional to length of spring, so new spring constant for each part is given by $k' = nk$, where nk is the spring constant of whole spring.

From the theory of spring pendulum, we know that time period of spring pendulum is inversely proportional to square root of spring constant *i.e.*,

$$T \propto \frac{1}{\sqrt{k}} \quad \text{and} \quad T' \propto \frac{1}{\sqrt{nk}}$$

So, $T' = \frac{T}{\sqrt{n}}$

105. A wave $y = a \sin(\omega t - kx)$ on a string meets with another wave producing a node at $x = 0$. Then, the equation of the unknown wave is **[AIEEE 2002]**

(a) $y = a \sin(\omega t + kx)$ (b) $y = -a \sin(\omega t + kx)$

(c) $y = a \sin(\omega t - kx)$ (d) $y = -a \sin(\omega t - kx)$

Exp. **(b)**

Equation of a wave,

$$y = a \sin(\omega t - kx) \quad \ldots(\text{i})$$

Let equations of another wave may be

$$y = a \sin(\omega t + kx) \quad \ldots(\text{ii})$$

and $$y = -a \sin(\omega t + kx) \quad \ldots(\text{iii})$$

If Eq. (i) propagates with Eq. (ii), then we get

$$y = 2a \cos kx \sin \omega t \quad \ldots(\text{iv})$$

If Eq. (i), propagates with Eq. (iii), then we get

$$y = -2a \sin kx \cos \omega t \quad \ldots(\text{v})$$

After putting $x = 0$ in Eqs. (iv) and (v) respectively, we get

$$y = 2a \sin \omega t \quad \text{and} \quad y = 0$$

Hence, Eq. (iii) is an equation of unknown wave.

106. Tube A has both ends open while tube B has one end closed, otherwise they are identical. The ratio of fundamental frequency of tubes A and B is **[AIEEE 2002]**

(a) 1 : 2 (b) 1 : 4 (c) 2 : 1 (d) 4 : 1

Exp. **(c)**

Let the tubes A and B have equal length called as l. Since, tube A is opened at both the ends, therefore, its fundamental frequency,

$$n_A = \frac{v}{2l} \quad \ldots(\text{i})$$

Since, tube B is closed at one end, therefore its fundamental frequency, $n_B = \frac{v}{4l}$. $\ldots$(ii)

From Eqs. (i) and (ii), we get

$$\frac{n_A}{n_B} = \frac{v/2l}{v/4l} = \frac{4}{2} = 2 : 1$$

107. A tuning fork arrangement (pair) produces 4 beats per second with one fork of frequency 288 cps. A little wax is placed on the unknown fork and then it produces 2 beats per second. The frequency of the unknown fork is **[AIEEE 2002]**

(a) 286 cps (b) 292 cps

(c) 294 cps (d) 288 cps

Exp. **(b)**

The tuning fork of frequency 288 Hz is producing 4 beats per second with the unknown tuning fork *i.e.*, the frequency difference between them is 4. Therefore, the frequency of unknown tuning fork.

$$= 288 \pm 4 = 292 \text{ or } 284$$

On placing a little wax on unknown tuning fork, its frequency decreases but now the number of beats produced per second is 2 *i.e.*, the frequency difference now decreases.

It is possible only when before placing the wax, the frequency of unknown fork is greater than the frequency of given tuning fork. Hence, the frequency of unknown tuning fork = 292 Hz.

11

Electrostatics

1. The bob of a simple pendulum has mass 2g and a charge of 5.0 μC. It is at rest in a uniform horizontal electric field of intensity 2000 V/m. At equilibrium, the angle that the pendulum makes with the vertical is (take $g = 10\,\text{m/s}^2$) **[JEE Main 2019, 8 April Shift-I]**

(a) $\tan^{-1}(2.0)$ (b) $\tan^{-1}(0.2)$
(c) $\tan^{-1}(5.0)$ (d) $\tan^{-1}(0.5)$

Exp. **(*d*)**

Forces on the bob are as shown

For equilibrium,

$$T\cos\theta = mg \quad \ldots(i)$$

and $$T\sin\theta = qE \quad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{qE}{mg}$$

Here, $q = 5\mu C = 5 \times 10^{-6}$ C, $E = 2000$ V/m,

$m = 2g = 2 \times 10^{-3}$ kg, $g = 10$ ms^{-2}

$$\therefore \quad \tan\theta = \frac{5 \times 10^{-6} \times 2000}{2 \times 10^{-3} \times 10} = \frac{1}{2} = 0.5$$

So, the angle made by the string of the pendulum with the vertical is

$$\theta = \tan^{-1}(0.5)$$

2. A solid conducting sphere, having a charge Q, is surrounded by an uncharged conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of the hollow shell be V. If the shell is now given a charge of $-4Q$, the new potential difference between the same two surfaces is **[JEE Main 2019, 8 April Shift-I]**

(a) $-2V$ (b) $2V$ (c) $4V$ (d) V

Exp. **(*d*)**

Initially when uncharged shell encloses charge Q, charge distribution due to induction will be as shown,

The potential on surface of inner shell is

$$V_A = \frac{kQ}{a} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots(i)$$

where, k = proportionality constant.

Potential on surface of outer shell is

$$V_B = \frac{kQ}{b} + \frac{k(-Q)}{b} + \frac{kQ}{b} \quad \ldots(ii)$$

Then, potential difference is

$$\Delta V_{AB} = V_A - V_B = kQ\left(\frac{1}{a} - \frac{1}{b}\right)$$

Given, $$\Delta V_{AB} = V$$

So, $$kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \quad \ldots(iii)$$

Finally after giving charge d– 4Q to outer shell, potential difference will be

$$\Delta V_{AB} = V_A - V_B$$
$$= \left(\frac{kQ}{a} + \frac{k(-4Q)}{b}\right) - \left(\frac{kQ}{b} + \frac{k(-4Q)}{b}\right)$$
$$= kQ\left(\frac{1}{a} - \frac{1}{b}\right) = V \qquad \text{[from Eq. (iii)]}$$

Hence, we obtain that potential difference does not depend on the charge of outer sphere, hence potential difference remains same.

3. Voltage rating of a parallel plate capacitor is 500 V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4}\,\text{m}^2$. What is the dielectric constant, if the capacitance is 15 pF? (Take, $\varepsilon_0 = 8.86 \times 10^{-12}\,\text{C}^2/\text{N-m}^2$)

[JEE Main 2019, 8 April Shift-I]

(a) 3.8 (b) 8.5 (c) 4.5 (d) 6.2

Exp. **(b)**

As we know, capacitance of a capacitor filled with dielectric medium,

$$C = \frac{\varepsilon_0 KA}{d} \qquad \ldots\text{(i)}$$

and potential difference between plates is

$$E = \frac{V}{d} \Rightarrow d = \frac{V}{E} \qquad \ldots\text{(ii)}$$

So, by combining both Eqs. (i) and (ii), we get

$$K = \frac{CV}{\varepsilon_0 AE} \qquad \ldots\text{(iii)}$$

Given, $C = 15\text{pF} = 15 \times 10^{-12}$ F,

$V = 500$ V, $E = 10^6$ Vm^{-1},

$A = 10^{-4}\text{m}^2$

and $\varepsilon_0 = 8.85 \times 10^{-12}\ \text{C}^2\text{N}^{-1}\text{m}^{-2}$

Substituting values in Eq. (iii), we get

$$K = \frac{15 \times 10^{-12} \times 500}{8.85 \times 10^{-12} \times 10^{-4} \times 10^6} = 8.47 \approx 8.5$$

4. The electric field in a region is given by $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}$, where E is in NC^{-1} and x is in metres. The values of constants are $A = 20$ SI unit and $B = 10$ SI unit. If the potential at $x = 1$ is V_1 and that at $x = -5$ is V_2, then $V_1 - V_2$ is

[JEE Main 2019, 8 April Shift-II]

(a) -48 V (b) -520 V (c) 180 V (d) 320 V

Exp. **(c)**

Given, $\mathbf{E} = (Ax + B)\hat{\mathbf{i}}\ \text{NC}^{-1}$

The relation between electric field and potential is given as $dV = -\mathbf{E} \cdot d\mathbf{x}$

Integrating both sides within the specified limits, we get

$$\therefore \int_1^2 dV = V_2 - V_1 = -\int_{x_1}^{x_2} \mathbf{E} \cdot d\mathbf{x}$$

$$\Rightarrow \quad V_1 - V_2 = \int_{x_1}^{x_2} \mathbf{E} \cdot d\mathbf{x}$$

$$= \int_{x_1}^{x_2} (Ax + B)\hat{\mathbf{i}} \cdot (dx\,\hat{\mathbf{i}}) = \int_{x_1}^{x_2} (Ax + B) \cdot dx$$

Here, $A = 20$ SI unit, $B = 10$ SI unit,

$$x_1 = 1 \text{ and } x_2 = -5 \Rightarrow V_1 - V_2 = \int_1^{-5} (20x + 10) \cdot dx$$

$$= \left[\frac{20x^2}{2} + 10x\right]_1^{-5} = 10\,[x^2 + x]_1^{-5}$$

$$= 10\,[(-5)^2 + (-5) - (1)^2 - (1)]$$

$$= 10\,(25 - 5 - 2) = 180\text{ V}$$

5. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is

[JEE Main 2019, 8 April Shift-II]

(a) $\sqrt{\frac{2qE}{md}}$ (b) $2\sqrt{\frac{qE}{md}}$ (c) $\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

Exp. **(a)**

> **Key Idea** When an electric dipole is placed in an electric field **E** at some angle θ, then two forces equal in magnitude but opposite in direction acts on the +ve and –ve charges, respectively. These forces forms a couple which exert a torque, which is given as
>
> $$\tau = \mathbf{p} \times \mathbf{E}$$
>
> where, **p** is dipole moment.

∴ Torque on the dipole can also be given as

$$\tau = I\alpha = -pE\sin\theta$$

where, I is the moment of inertia and α is angular acceleration. For small angles, $\sin\theta \approx \theta$

$$\therefore \quad \alpha = -\left(\frac{pE}{I}\right)\theta \quad \ldots(i)$$

Moment of inertia of the given system is

$$I = m\left(\frac{d}{2}\right)^2 + m\left(\frac{d}{2}\right)^2$$

$$= \frac{2md^2}{4} = \frac{md^2}{2}$$

Substituting the value of I in Eq. (i), we get

$$\Rightarrow \quad \alpha = -\left(\frac{2pE}{md^2}\right)\cdot\theta \quad \ldots(ii)$$

The above equation is similar to the equation for a system executing angular SHM.

Comparing Eq. (ii), with the general equation of angular SHM, i.e.

$$\alpha = -\omega^2\theta$$

where, ω is the angular frequency, we get

$$\omega^2 = \frac{2pE}{md^2} \quad \text{or} \quad \omega = \sqrt{\frac{2pE}{md^2}}$$

As, $\quad p = qd$

$$\therefore \quad \omega = \sqrt{\frac{2qdE}{md^2}} = \sqrt{\frac{2qE}{md}}$$

6. A positive point charge is released from rest at a distance r_0 from a positive line charge with uniform density. The speed (v) of the point charge, as a function of instantaneous distance r from line charge, is proportional to **[JEE Main 2019, 8 April Shift-II]**

(a) $v \propto \left(\frac{r}{r_0}\right)$ (b) $v \propto e^{+\frac{r}{r_0}}$

(c) $v \propto \ln\left(\frac{r}{r_0}\right)$ (d) $v \propto \sqrt{\ln\left(\frac{r}{r_0}\right)}$

Exp. **(*d*)**

For a positive line charge or charged wire with uniform density λ, electric field at distance x is

$$E = \frac{2k\lambda}{x} = \frac{\lambda}{2\pi\varepsilon_0 x} \quad \ldots(i)$$

So, force on charge q which is at a distance r_0 due to this line charge is $F = qE = \frac{2kq\lambda}{x}$ ...(ii)

[using Eq. (i)]

Now, work done when charge is pushed by field by a small displacement dx is

$$dW = F\cdot dx = \frac{2kq\lambda}{x}\cdot dx \quad \text{[using Eq. (ii)]}$$

∴ Total work done by field of wire in taking charge q from distance r_0 to distance r will be

$$W = \int_{r_0}^{r} dW = \int_{r_0}^{r} \frac{2kq\lambda}{x}\cdot dx$$

$$= 2kq\lambda[\log x]_{r_0}^{r} = 2kq\lambda(\log r - \log r_0)$$

$$= 2kq\lambda\log\left|\frac{r}{r_0}\right| \quad \ldots(iii)$$

As we know, from work-kinetic energy theorem,

$$K_{\text{final}} - K_{\text{initial}} = W$$

$$\Rightarrow \quad \frac{1}{2}mv^2 - 0 = 2kq \times \log\left|\frac{r}{r_0}\right| \quad \text{[using Eq. (iii)]}$$

$$\Rightarrow \quad v = \left(\frac{4kq\lambda}{m}\log\left|\frac{r}{r_0}\right|\right)^{\frac{1}{2}}$$

$$\therefore \quad v \propto \left(\log\left|\frac{r}{r_0}\right|\right)^{\frac{1}{2}}$$

7. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate +4μC charge. The potential difference developed across the capacitor is **[JEE Main 2019, 8 April Shift-II]**

(a) 1 V (b) 5 V

(c) 2 V (d) 3 V

Exp. **(*a*)**

Net value of charge on plates of capacitor after steady state is reached is

$$q_{net} = \frac{q_2 - q_1}{2}$$

where, q_2 and q_1 are the charges given to plates. (Note that this formula is valid for any polarity of charge.)

Here, $q_2 = 4\mu C, q_1 = 2\mu C$

$\therefore$ Charge of capacitor is

$$q = \Delta q_{net} = \frac{4-2}{2} = 1\mu C$$

Potential difference between capacitor plates is

$$V = \frac{Q}{C} = \frac{1\mu C}{1\mu F} = 1V$$

8. A system of three charges are placed as shown in the figure

If $D >> d$, the potential energy of the system is best given by **[JEE Main 2019, 9 April Shift-I]**

(a) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} + \frac{2qQd}{D^2}\right]$ (b) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d} + \frac{qQd}{D^2}\right]$

(c) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{2D^2}\right]$ (d) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d} - \frac{qQd}{D^2}\right]$

Exp. **(d)**

The system of two charges, i.e. $+q$ and $-q$ that are separated by distance d can be considered as a dipole. Thus, the charge Q would be at D distance from the centre of an electric dipole on its axial line.

So, the total potential energy of the system will be due to two components.

(1) Potential energy of dipole's own system

$$(PE)_1 = \frac{Kq_1q_2}{d} = -\frac{Kq^2}{d} \quad \ldots(i)$$

(2) Potential energy of charge Q and dipole system

$$(PE)_2 = -\frac{KQq}{D^2}\cdot d \quad \ldots(ii)$$

Hence, total potential energy of the system

$$(PE)_{total} = (PE)_1 + (PE)_2 = -\frac{Kq^2}{d} - \frac{KQq}{D^2}\cdot d$$

$$\Rightarrow \quad (PE)_{total} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{q^2}{d} + \frac{Qqd}{D^2}\right]$$

9. A capacitor with capacitance $5\mu F$ is charged to $5\ \mu C$. If the plates are pulled apart to reduce the capacitance to $2\ \mu F$, how much work is done? **[JEE Main 2019, 9 April Shift-I]**

(a) 6.25×10^{-6} J (b) 2.16×10^{-6} J

(c) 2.55×10^{-6} J (d) 3.75×10^{-6} J

Exp. **(d)**

Potential energy stored in a capacitor is

$$U = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

So, initial energy of the capacitor, $U_i = \frac{1}{2}Q^2/C_1$

Final energy of the capacitor, $U_f = \frac{1}{2}Q^2/C_2$

As we know, work done, $W = \Delta U = U_f - U_i$

$$= \frac{1}{2}Q^2\left[\frac{1}{C_2} - \frac{1}{C_1}\right]$$

Here, $Q = 5\mu C = 5\times10^{-6}$ C,

$C_1 = 5\mu F = 5\times10^{-6}$F,

$C_2 = 2\mu F = 2\times10^{-6}$ F

$$\Rightarrow \quad \Delta U = \frac{1}{2}\times(5\times10^{-6})^2\left[\frac{1}{2\times10^{-6}} - \frac{1}{5\times10^{-6}}\right]$$

$$= \frac{1}{2}\times\frac{5\times5\times10^{-12}}{10^{-6}}\times\frac{3}{10}$$

$$= \frac{25\times3}{20}\times10^{-6}\text{ J}$$

$$\Rightarrow \quad \Delta U = 3.75\times10^{-6}\text{ J}$$

$\therefore$ Work done in reducing the capacitance from $5\mu F$ to $2\ \mu F$ by pulling plates of capacitor apart is 3.75×10^{-6} J.

10. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is **[JEE Main 2019, 9 April Shift-II]**

(a) $\frac{(n+1)V}{(K+n)}$ (b) $\frac{nV}{K+n}$ (c) V (d) $\frac{V}{K+n}$

Exp. (a)

Key Idea When a dielectric slab is inserted to fill the space between the plates, then the charge on the capacitor plates remains same. However, the capacitance increase, i.e. $C = KC_0$, where C_0 is the capacitance of the capacitor without slab and K is the dielectric constant of slab.

When parallel combination is fully charged, charge on the combination is

$$Q = C_{eq}V = C(1 + n)V$$

When battery is removed and a dielectric slab is placed between two plates of first capacitor, then charge on the system remains same. Now, equivalent capacitance after insertion of dielectric is

$$C_{eq} = KC + nC = (n + K)C$$

If potential value after insertion of dielectric is V', then charge on system is

$$Q' = C_{eq}V' = (n + K)CV'$$

As $Q = Q'$, we have

$$C(1 + n)V = (n + K)CV'$$

$$\therefore \quad V' = \frac{(1 + n)V}{(n + K)}$$

11. Figure shows charge (q) *versus* voltage (V) graph for series and parallel combination of two given capacitors. The capacitances are **[JEE Main 2019, 10 April Shift-I]**

(a) 60μF and 40μF (b) 50μF and 30μF
(c) 20μF and 30μF (d) 40μF and 10μF

Exp. (d)

Key Idea For a capacitor,

$$Q = CV \Rightarrow C = Q / V$$

Hence, slope of the Q-V curve should represent the capacitance.

In the given figure,

Slope of OA > Slope of OB

Since, we know that,

net capacitance of parallel combination > net capacitance of series combination

$\therefore$ Parallel combination's capacitance,

$$C_P = C_1 + C_2 = \frac{500\,\mu C}{10\text{ V}} = 50\mu F \quad \text{... (i)}$$

Series combination's capacitance,

$$C_S = \frac{C_1C_2}{C_1 + C_2} = \frac{80\mu C}{10V} = 8\,\mu F \quad \text{... (ii)}$$

or $\quad C_1C_2 = 8 \times (C_1 + C_2) = 8 \times 50\,\mu F$
$\quad = 400\,\mu F$ [using Eq. (i)] ...(iii)

From Eqs. (i) and (iii), we get

$C_1 = 50 - C_2$

and $\quad C_1C_2 = 400$

$\Rightarrow \quad C_2(50 - C_2) = 400$

$\Rightarrow \quad 50C_2 - C_2^2 = 400$

or $\quad C_2^2 - 50C_2 + 400 = 0$

$$\Rightarrow C_2 = \frac{+50 \pm \sqrt{2500 - 1600}}{2} = \frac{+50 \pm 30}{2}$$

$\Rightarrow C_2 = +40\,\mu F$ or $+10\,\mu F$

Also, $\quad C_1 = 50 - C_2$

$\Rightarrow C_1 = +10\,\mu F$ or $+40\,\mu F$

Hence, capacitance of two given capacitors is 10 μF and 40 μF.

12. A current of 5 A passes through a copper conductor (resistivity $= 1.7 \times 10^{-8}\ \Omega$-m) of radius of cross-section 5 mm. Find the mobility of the charges, if their drift velocity is 1.1×10^{-3} m/s. **[JEE Main 2019, 10 April Shift-I]**

(a) $1.5\ m^2/V\text{-}s$ (b) $1.3\ m^2/V\text{-}s$
(c) $1.0\ m^2/V\text{-}s$ (d) $1.8\ m^2/V\text{-}s$

Exp. (c)

Given, $\quad I = 5A$,

$\rho = 1.7 \times 10^{-8}\ \Omega$-m,

$r = 5\,\text{mm} = 5 \times 10^{-3}$ m,

$v_d = 1.1 \times 10^{-3}$ m/s

Mobility of charges in a conductor is given by

$$\mu = \frac{v_d}{E} \quad \ldots \text{(i)}$$

and resistivity is given by

$$\rho = \frac{E}{J} = \frac{E}{I/A} \quad (\because J = \sigma E = \frac{1}{\rho} \times E)$$

$$\Rightarrow \quad \rho = \frac{EA}{I}$$

or $$E = \frac{\rho I}{A} \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\mu = \frac{v_d A}{\rho I}$$

Substituting the given values, we get

$$= \frac{1.1 \times 10^{-3} \times \pi \times (5 \times 10^{-3})^2}{1.7 \times 10^{-8} \times 5}$$

$$= \frac{86.35 \times 10^{-9}}{8.5 \times 10^{-8}} = 10.1 \times 10^{-1}$$

$$\Rightarrow \quad \mu \approx 1\ \text{m}^2/\text{V-s}$$

13. In free space, a particle A of charge $1\,\mu$C is held fixed at a point P. Another particle B of the same charge and mass $4\,\mu$g is kept at a distance of 1 mm from P. If B is released, then its velocity at a distance of 9 mm from P is $\left[\text{Take}, \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ \text{N-m}^2\text{C}^{-2}\right]$

[JEE Main 2019, 10 April Shift-II]

(a) 1.5×10^2 m/s (b) 3.0×10^4 m/s

(c) 1.0 m/s (d) 2.0×10^3 m/s

Exp. **(d)**

Given situation is shown in the figure below,

When charged particle B is released due to mutual repulsion, it moves away from A. In this process, potential energy of system of charges reduces and this change of potential energy appears as kinetic energy of B.

Now, potential energy of system of charges at separation of 1 mm is

$$U_1 = \frac{Kq_1q_2}{r}$$

Here, $q_1 = q_2 = 1 \times 10^{-6}$ C

$r = 1\,\text{mm} = 1 \times 10^{-3}$ m

$$\therefore\ U_1 = \frac{9 \times 10^9 \times 1 \times 10^{-6} \times 1 \times 10^{-6}}{1 \times 10^{-3}} = 9\ \text{J}$$

Potential energy of given system of charges at separation of 9 mm is

$$U_2 = \frac{Kq_1q_2}{r} = \frac{9 \times 10^9 \times (1 \times 10^{-6})^2}{9 \times 10^{-3}} = 1\ \text{J}$$

By energy conservation,

Change in potential energy of system of A and B

= Kinetic energy of charged particle B

$$\Rightarrow \quad U_1 - U_2 = \frac{1}{2} m_B v_B^2$$

where, m_B = mass of particle $B = 4\,\mu$g

$= 4 \times 10^{-6} \times 10^{-3}$ kg $= 4 \times 10^{-9}$ kg

and v_B = velocity of particle B at separation of 9 mm

$$\Rightarrow \quad 9 - 1 = \frac{1}{2} \times 4 \times 10^{-9} \times v_B^2$$

$$\Rightarrow \quad v_B^2 = 4 \times 10^9 \Rightarrow v_B = 2 \times 10^3\ \text{ms}^{-1}$$

14. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by

[JEE Main 2019, 10 April Shift-II]

(a) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 + \left(\dfrac{qE}{m}\right)^2}}}$ (b) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2 - \dfrac{q^2E^2}{m^2}}}}$

(c) $2\pi\sqrt{\dfrac{L}{\left(g + \dfrac{qE}{m}\right)}}$ (d) $2\pi\sqrt{\dfrac{L}{\left(g - \dfrac{qE}{m}\right)}}$

Exp. **(a)**

When pendulum is oscillating between capacitor plates, it is subjected to two forces;

(i) Weight downwards = mg

(ii) Electrostatic force acting horizontally = qE

So, net acceleration of pendulum bob is resultant of accelerations produced by these two perpendicular forces.

Net acceleration is, $a_{\text{net}} = \sqrt{a_1^2 + a_2^2}$

$$= \sqrt{g^2 + \left(\frac{qE}{m}\right)^2}$$

So, time period of oscillations of pendulum is

$$T = 2\pi\sqrt{\frac{l}{a_{\text{net}}}} = 2\pi\sqrt{\frac{L}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$$

15. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b and carries charge Q. At its centre is a dipole $\mathbf{p}$ as shown.

In this case, **[JEE Main 2019, 12 April Shift-I]**

(a) surface charge density on the inner surface is uniform and equal to $\dfrac{\left(\frac{Q}{2}\right)}{4\pi a^2}$

(b) electric field outside the shell is the same as that of a point charge at the centre of the shell

(c) surface charge density on the outer surface depends on $|\mathbf{p}|$

(d) surface charge density on the inner surface of the shell is zero everywhere

Exp. **(b)**

Electric charge distribution at inner and outer surface of spherical shell due to the electric dipole can be shown as below

Here, we need to consider two different factors

(i) charge on the spherical shell is $+Q$ which will be distributed on its outer surface as shown in figure.

(ii) Electric dipole will create non-uniform electric field inside the shell which will distribute the charges on inner surface as shown in figure. But its net contribution to the outer side of the shell will be zero as net charge of a dipole is zero.

$\therefore$ Net charge on outer surface of shell will be $+Q$.

Hence, using (ii), option (a) is incorrect as field inside shell is not uniform. Option (b) is correct, as net charge on outer surface is $+Q$ even in the presence of dipole.

Option (c) is incorrect, as surface charge density at outer surface is uniform $\left(= \frac{Q}{A} = \frac{Q}{4\pi b^2}\right)$.

Option (d) is incorrect, as surface charge density at inner surface is non-zero.

So, option (b) is correct.

Alternate Solution Using Gauss' law at outer surface, let charge on dipole is q,

$$\phi = \frac{\Sigma q}{\varepsilon_0} = E \cdot A \text{ or } E = \frac{1}{A\varepsilon_0}\Sigma q$$

$$= \frac{(+Q + q - q)}{A\varepsilon_0} = \frac{Q}{A\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \text{constant}$$

16. A point dipole $\mathbf{p} = -p_0\,\hat{\mathbf{x}}$ is kept at the origin. The potential and electric field due to this dipole on the Y-axis at a distance d are, respectively [Take, $V = 0$ at infinity]

[JEE Main 2019, 12 April Shift-I]

(a) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (b) $0, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

(c) $0, \frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$ (d) $\frac{|\mathbf{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\mathbf{p}}{4\pi\varepsilon_0 d^3}$

Exp. **(b)**

The given problem can be shown as clearly potential difference at point P due to dipole is

$V = V_{AP} + V_{BP}$ (scalar addition)

$\Rightarrow \quad V = \frac{k(-q)}{AP} + \frac{k(q)}{BP}$ ...(i)

Here, $AP = BP = \sqrt{a^2 + r^2}$

$\therefore \quad V = -\frac{kq}{\sqrt{a^2+r^2}} + \frac{kq}{\sqrt{a^2+r^2}} = 0$ ...(ii)

Now, electric field at any point on Y-axis, i.e. equatorial line of the dipole can be given by

$\mathbf{E} = -\frac{k\,\mathbf{p}}{r^3}$ (standard expression)

$\Rightarrow \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{r^3}$

Given, $r = d$

$\therefore \quad \mathbf{E} = -\frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}}{d^3}$ ...(iii)

From Eqs. (ii) and (iii), correct option is (b).

Alternate Solution

Electric field at any point at θ angle from axial line of dipole is given by

$$\mathbf{E} = -\frac{k\mathbf{p}}{r^3}\sqrt{3\cos^2\theta + 1}$$

Here, $\theta = 90^\circ \Rightarrow \cos\theta = \cos 90^\circ = 0$ and $r = d$

$\therefore \quad \mathbf{E} = -\frac{k\mathbf{p}}{d^3} = -\frac{\mathbf{p}}{4\pi\varepsilon_0 d^3}$

17. Two identical parallel plate capacitors of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in Fig. I, and the second one is filled as shown in Fig. II. If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :

[JEE Main 2019, 12 April Shift-I]

(a) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(b) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$

(c) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

(d) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

Exp. **(d)**

Key Idea A capacitor filled with dielectrics can be treated/compared as series/parallel combinations of capacitor having individual dielectric. e.g.

and

Case I

Capacitance in the equivalent circuit are

$$C_1 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_1 = \frac{\varepsilon_0 A}{3d} K_1$$

$$C_2 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_2 = \frac{\varepsilon_0 A}{3d} K_2$$

and $$C_3 = \frac{\varepsilon_0\left(\frac{A}{3}\right)}{d} K_3 = \frac{\varepsilon_0 A}{3d} K_3$$

So, equivalent capacitance,

$$C_{\mathrm{I}} = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_0 A}{3d} K_1 + \frac{\varepsilon_0 A}{3d} K_2 + \frac{\varepsilon_0 A}{3d} K_3$$

$$C_{\mathrm{I}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \qquad \ldots\text{(i)}$$

Case II

Capacitance of equivalent circuit are

$$C_1 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} \cdot K_1 = \frac{3\varepsilon_0 A}{d} K_1$$

$$C_2 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_2 = \frac{3\varepsilon_0 A}{d} K_2$$

and $$C_3 = \frac{\varepsilon_0 A}{\left(\frac{d}{3}\right)} K_3 = \frac{3\varepsilon_0 A}{d} K_3$$

So, equivalent capacitance,

$$\frac{1}{C_{\mathrm{II}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$= \frac{d}{3\varepsilon_0 AK_1} + \frac{d}{3\varepsilon_0 AK_2} + \frac{d}{3\varepsilon_0 AK_3}$$

$$\Rightarrow \frac{1}{C_{\mathrm{II}}} = \frac{d}{3\varepsilon_0 A}\left[\frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3}\right]$$

$$= \frac{d}{3\varepsilon_0 A}\left[\frac{K_2K_3 + K_1K_3 + K_1K_2}{K_1K_2K_3}\right]$$

$$C_{\mathrm{II}} = \frac{3\varepsilon_0 A}{d}\left[\frac{K_1\,K_2\,K_3}{K_1\,K_2 + K_2\,K_3 + K_3\,K_1}\right] \qquad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{C_{\mathrm{I}}}{C_{\mathrm{II}}} = \frac{\varepsilon_0 A}{3d}(K_1 + K_2 + K_3) \times \frac{d(K_1K_2 + K_2K_3 + K_3K_1)}{3\varepsilon_0 A(K_1K_2K_3)}$$

$$= \frac{(K_1 + K_2 + K_3)(K_1K_2 + K_2K_3 + K_3K_1)}{9K_1K_2K_3}$$

Now, energy stored in capacitor, $E = \frac{1}{2}CV^2$

$$\Rightarrow \qquad E \propto C$$

$$\therefore \qquad \frac{E_{\mathrm{I}}}{E_{\mathrm{II}}} = \frac{C_{\mathrm{I}}}{C_{\mathrm{II}}}$$

18. Let a total charge $2Q$ be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of $-Q$ each, are placed on diametrically opposite points, at equal distance a, from the centre. If A and B do not experience any force, then **[JEE Main 2019, 12 April Shift-II]**

(a) $a = 8^{-1/4} R$ (b) $a = \dfrac{3R}{2^{1/4}}$

(c) $a = 2^{-1/4} R$ (d) $a = R/\sqrt{3}$

Exp. **(a)**

Key Idea Force on A is zero only when repulsion of A and B = attraction of positive charge distribution of radius a and charge A.

In given charge distribution, let r is radius of a shell of thickness dr.

Charge dQ present in shell of thickness dr = Volume of shell × Volumetric charge density

$\Rightarrow \quad dQ = (4\pi r^2 \times dr) \times (kr) = 4\pi k r^3\, dr$

Total charge in sphere is

$$2Q = \int_0^R dQ = \int_0^R 4\pi k r^3\, dr$$

$$\Rightarrow \quad 2Q = 4\pi k \left[\frac{r^4}{4}\right]_0^R$$

$$\Rightarrow \quad k = \frac{2Q}{\pi R^4}$$

Now, using Gauss' law, electric field on the surface of sphere of radius a is

$$E\int_0^a dA = \frac{1}{\varepsilon_0}\cdot\int_0^a (kr\, 4\pi r^2\, dr)$$

$$\Rightarrow \quad E\cdot 4\pi a^2 = \frac{1}{\varepsilon_0}\cdot 4\pi k\left(\frac{a^4}{4}\right)$$

$$\Rightarrow \quad E = \frac{ka^2}{4\varepsilon_0} = \frac{2Qa^2}{4\pi\varepsilon_0 R^4}$$

Force of attraction on charge A (or B) due to this field is

$$F_1 = QE = \frac{2Q^2a^2}{4\pi\varepsilon_0 R^4}$$

Force of repulsion on charge A due to B is

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2a)^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4a^2}$$

If charge A(or B) does not feel any force, then $F_1 = F_2$

$$\Rightarrow \quad \frac{2Q^2a^2}{4\pi\varepsilon_0 R^4} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q^2}{4a^2} \Rightarrow 8a^4 = R^4$$

$$\Rightarrow \quad a = 8^{-1/4} R$$

19. In the given circuit, the charge on 4 μF capacitor will be **[JEE Main 2019, 12 April Shift-II]**

(a) 5.4 μC (b) 9.6 μC (c) 13.4 μC (d) 24 μC

Exp. **(d)**

Given circuit is

In parallel, $C_{eq} = 5 + 1 = 6\,\mu F$

and in series, $C'_{eq} = \dfrac{6 \times 4}{6 + 4} = 2.4\,\mu F$

This is equivalent to

So, potential difference across upper branch

$= 10\,V$

Using $Q = C \times V$, charge delivered to upper branch is

$$Q = C'_{eq} \cdot V$$
$$= 2.4\,\mu F \times 10V = 24\,\mu C$$

As we know, in series connection, same charge is shared by capacitors, so charge on $4\,\mu F$ capacitor and $6\,\mu F$ capacitor would be same,

i.e. $Q'_{4\mu F} = 24\mu C$

Alternate Solution

The circuit obtained,

This can be further simplified as, $2.4\,\mu F$ and $3\,\mu F$ are in parallel.

So, net capacitance,

$$C_{net} = 2.4 + 3 = 5.4\,\mu F$$

Net charge flow through circuit,

$$Q = C_{net} V = 5.4 \times 10 = 54\,\mu C$$

$\therefore$ This charge will be distributed in the ratio of capacitance in the two branches AB and CD as

$$\frac{Q_1}{Q_2} = \frac{2.4}{3} = \frac{4}{5}$$

$\Rightarrow$ $9x = 54\mu C$ or $x = 6\,\mu C$

$\therefore$ Charge on $4\,\mu F$ capacitor is

$$= 4 \times 6\,\mu C = 24\,\mu C$$

20. Three charges $+Q, q, +Q$ are placed respectively at distance 0, $\dfrac{d}{2}$ and d from the origin on the X-axis. If the net force experienced by $+Q$ placed at $x = 0$ is zero, then value of q is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\dfrac{+Q}{2}$ (b) $\dfrac{+Q}{4}$

(c) $\dfrac{-Q}{2}$ (d) $\dfrac{-Q}{4}$

Exp. **(d)**

The given condition is shown in the figure given below,

Then, according to the Coulomb's law, the electrostatic force between two charges q_1 and q_2 such that the distance between them is (r) given as,

$$F = \frac{1 \cdot q_1 q_2}{4\pi\varepsilon_0 \cdot r^2}$$

$\therefore$ Net force on charge 'Q' placed at origin i.e. at $x = 0$ in accordance with the principle of superposition can be given as

$$F_{net} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

Since, it has been given that, $F_{net} = 0$.

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times q}{\left(\frac{d}{2}\right)^2} = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \times Q}{(d)^2}$$

or $q = -\dfrac{Q}{4}$

21. A parallel plate capacitor is made of two square plates of side 'a' separated by a distance d ($d << a$). The lower triangular portions is filled with a dielectric of dielectric constant k, as shown in the figure. Capacitance of this capacitor is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\frac{K\varepsilon_0 a^2}{d}\ln K$ (b) $\frac{K\varepsilon_0 a^2}{d(K-1)}\ln K$

(c) $\frac{K\varepsilon_0 a^2}{2d(K+1)}$ (d) $\frac{1}{2}\cdot\frac{K\varepsilon_0 a^2}{d}$

Exp. **(b)**

Let's consider a strip of thickness 'dx' at a distance of 'x' from the left end as shown in figure. From the figure, ΔABC and ΔADE are similar triangles,

$$\Rightarrow \quad \frac{y}{x} = \frac{d}{a} \Rightarrow y = \left(\frac{d}{a}\right)x \quad \text{...(i)}$$

We know that, the capacitance of parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$

$$C_1 = \frac{\varepsilon_0(adx)}{(d-y)} \text{ and } C_2 = \frac{K\varepsilon_0(adx)}{y}$$

Here, two capacitor are placed in series with variable thickness, therefore

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2}$$

$$\Rightarrow \quad C_{eq} = \frac{K\varepsilon_0 adx}{Kd + (1-K)y} \quad \text{...(ii)}$$

Now, integrate it from 0 to a

$$C = \int_0^a \frac{K\varepsilon_0 adx}{Kd + (1-K)y}$$

Using Eq. (i), $y = \left(\frac{d}{a}\right)x$, we get

$$C = \varepsilon_0 a\int_0^a \frac{dx}{d + \left(\frac{1}{K} - 1\right)\frac{d}{a}x}$$

$$\Rightarrow C = \frac{\varepsilon_0 a}{\left(\frac{1-K}{K}\right)\frac{d}{a}}\ln\left[\frac{1}{K}\right] \Rightarrow C = \frac{\varepsilon_0 a^2 K \ln K}{(K-1)d}$$

22. For a uniformly charged ring of radius R, the electric field on its axis has the largest magnitude at a distance h from its centre. Then, value of h is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\frac{R}{\sqrt{2}}$ (b) $R\sqrt{2}$ (c) R (d) $\frac{R}{\sqrt{5}}$

Exp. **(a)**

Electric field at a distance 'h' from the centre of uniformly charged ring of total charge q (say) on its axis is given as,

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{qh}{(h^2+R^2)^{3/2}}$$

For the magnitude to be maximum, then

$$\frac{dE}{dh} = 0$$

$$\Rightarrow \quad \frac{dE}{dh} = \frac{q}{4\pi\varepsilon_0}\left[\frac{(h^2+R^2)^{3/2} - h[3/2(h^2+R^2)^{1/2}\,2h]}{(h^2+R^2)^3}\right] = 0$$

$$\Rightarrow \quad 0 = \frac{(h^2+R^2)^{3/2} - 3h^2(h^2+R^2)^{1/2}}{(h^2+R^2)^3}$$

$$\Rightarrow \quad (h^2+R^2)^{3/2} = 3h^2(h+R^2)^{1/2}$$

$$\Rightarrow \quad 3h^2 = (h^2+R^2)$$

$$3h^2 - h^2 = R^2$$

$$2h^2 = R^2$$

$$\Rightarrow \quad h = \pm\frac{R}{\sqrt{2}}$$

$\therefore$ At $\frac{R}{\sqrt{2}}$, the value of electric field associated with a charged ring on its axis has the maximum value.

23. A parallel plate capacitor with square plates is filled with four dielectrics of dielectric constants K_1, K_2, K_3, K_4 arranged as shown in the figure. The effective dielectric constant K will be: **[JEE Main 2019, 9 Jan Shift-II]**

(a) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{2(K_1 + K_2 + K_3 + K_4)}$

(b) $K = \dfrac{(K_1 + K_2)(K_3 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(c) $K = \dfrac{(K_1 + K_3)(K_2 + K_4)}{K_1 + K_2 + K_3 + K_4}$

(d) $K = \dfrac{(K_1 + K_4)(K_2 + K_3)}{2(K_1 + K_2 + K_3 + K_4)}$

Exp. **(*)**

This capacitor system can be converted into two parts as shown in the figure

where C_1, C_2, C_3 and C_4 are capacitance of the capacitor having dielectric constants K_1, K_2, K_3 and K_4 respectively.

Here, $C_1 = \dfrac{K_1\varepsilon_0 A/2}{d/2} = \dfrac{K_1\varepsilon_0 A}{d}$

Similarly, $C_2 = \dfrac{K_2\varepsilon_0 A}{d}, C_3 = \dfrac{K_3\varepsilon_0 A}{d}$ and $C_4 = \dfrac{K_4\varepsilon_0 A}{d}$

Since, equivalent capacitance in series combination is

$$C_{eq} = \frac{C_1 \cdot C_2}{C_1 + C_2}$$

Here, C_1, C_2 and C_3, C_4 are in series combination.

$$\therefore \quad (C_{eq})_{12} = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{\dfrac{K_1\varepsilon_0 A}{d} \cdot \dfrac{K_2\varepsilon_0 A}{d}}{\dfrac{K_1\varepsilon_0 A}{d} + \dfrac{K_2\varepsilon_0 A}{d}}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d}$$

Similarly, $(C_{eq})_{34} = \dfrac{K_3 \cdot K_4}{K_3 + K_4} \cdot \dfrac{\varepsilon_0 A}{d}$

Now, $(C_{eq})_{12}$ and $(C_{eq})_{34}$ are in parallel combination.

$$\therefore \quad C_{net} = (C_{eq})_{12} + (C_{eq})_{34}$$

$$= \frac{K_1 \cdot K_2}{K_1 + K_2} \cdot \frac{\varepsilon_0 A}{d} + \frac{K_3 \cdot K_4}{K_3 + K_4} \cdot \frac{\varepsilon_0 A}{d}$$

$$\Rightarrow \quad C_{net} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d} \quad \ldots\text{(i)}$$

If K is effective dielectric constant, then

$$C_{net} = \frac{K\varepsilon_0 A}{d} \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii),

$$\frac{K\varepsilon_0 A}{d} = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)\frac{\varepsilon_0 A}{d}$$

or $$K = \left(\frac{K_1 \cdot K_2}{K_1 + K_2} + \frac{K_3 \cdot K_4}{K_3 + K_4}\right)$$

24. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \dfrac{A}{r^2} e^{\frac{-2r}{a}}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is

[JEE Main 2019, 9 Jan Shift-II]

(a) $a \log\left(\dfrac{1}{1 - \dfrac{Q}{2\pi aA}}\right)$ (b) $a \log\left(1 - \dfrac{Q}{2\pi aA}\right)$

(c) $\dfrac{a}{2} \log\left(1 - \dfrac{Q}{2\pi aA}\right)$ (d) $\dfrac{a}{2} \log\left(\dfrac{1}{1 - \dfrac{Q}{2\pi aA}}\right)$

Exp. **(*d*)**

Here, volume charge density, $\rho(r) = \dfrac{A}{r^2} \cdot e^{-\frac{2r}{a}}$

where, a and A are constant.

Let a spherical region of small element of radius r.

If Q is total charge distribution upto radius R, then

$$Q = \int_0^R \rho \cdot dV = \int_0^R \frac{A}{r^2} e^{-2r/a}(4\pi r^2\, dr)$$

(From figure, we observe $dV = A \cdot dr = 4\pi r^2 \cdot dr$)

$$= 4\pi A \int_0^R e^{-2r/a} dr = 4\pi A\left(\frac{e^{-2r/a}}{-2/a}\right)_0^R$$

$$= 4\pi A \times \left(\frac{-a}{2}\right)(e^{-2R/a} - e^0)$$

$$= 2\pi A(-a)\,[e^{-2R/a} - 1]$$

or $$Q = 2\pi a\,A\,(1 - e^{-2R/a})$$

or $$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi aA}}\right)$$

25. Two point charges $q_1\,(\sqrt{10}\,\mu\text{C})$ and $q_2\,(-25\,\mu\text{C})$ are placed on the x-axis at $x = 1$ m and $x = 4$ m, respectively. The electric field (in V/m) at a point $y = 3$ m on Y-axis is $\left(\text{Take}, \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ \text{N-m}^2\text{C}^{-2}\right)$

[JEE Main 2019, 9 Jan Shift-II]

(a) $(63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}})\times10^2$ (b) $(81\,\hat{\mathbf{i}} - 81\hat{\mathbf{j}})\times10^2$

(c) $(-81\,\hat{\mathbf{i}} + 81\,\hat{\mathbf{j}})\times10^2$ (d) $(-63\,\hat{\mathbf{i}} + 27\hat{\mathbf{j}})\times10^2$

Exp. **(a)**

Here, $q_1 = \sqrt{10}\,\mu\text{C} = \sqrt{10}\times10^{-6}$ C

$q_2 = -25\,\mu\text{C} = -25\times10^{-6}$ C

Let $\mathbf{E}_1$ and $\mathbf{E}_2$ are the values of electric field due to q_1 and q_2 respectively.

Here, $E_1 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{AC^2} = \frac{1}{4\pi\varepsilon_0}\times\frac{\sqrt{10}\times10^{-6}}{(1^2 + 3^2)}$

$= 9\times10^9\times\sqrt{10}\times10^{-7} = 9\sqrt{10}\times10^2$

$\therefore \mathbf{E}_1 = 9\sqrt{10}\times10^2[\cos\theta_1(-\hat{\mathbf{i}}) + \sin\theta_1\hat{\mathbf{j}}]$

From ΔOAC,

$\sin\theta_1 = \frac{3}{\sqrt{10}}$

and $\cos\theta_1 = \frac{1}{\sqrt{10}}$

$\therefore\quad E_1 = 9\sqrt{10}\times10^2\left[\frac{1}{\sqrt{10}}(-\hat{\mathbf{i}}) + \frac{3}{\sqrt{10}}\hat{\mathbf{j}}\right]$

$= 9\times10^2\,[-\hat{\mathbf{i}} + 3\hat{\mathbf{j}}]$

$= (-9\,\hat{\mathbf{i}} + 27\hat{\mathbf{j}})\times10^2$ V/m

and $E_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{-25\times10^{-6}}{(4^2 + 3^2)} = 9\times10^3$ V/m

From ΔOBC,

$\sin\theta_2 = \frac{3}{5}$

$\cos\theta_2 = \frac{4}{5}$

$\therefore\quad \mathbf{E}_2 = 9\times10^3[\cos\theta_2\hat{\mathbf{i}} - \sin\theta_2\hat{\mathbf{j}}]$

$\mathbf{E}_2 = 9\times10^3\left[\frac{4}{5}\hat{\mathbf{i}} - \frac{3}{5}\hat{\mathbf{j}}\right]$

$= (72\,\hat{\mathbf{i}} - 54\,\hat{\mathbf{j}})\times10^2$

$\therefore\quad \mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = (63\,\hat{\mathbf{i}} - 27\hat{\mathbf{j}})\times10^2$ V/m

26. A charge Q is distributed over three concentric spherical shells of radii a, b, c $(a < b < c)$ such that their surface charge densities are equal to one another.

The total potential at a point at distance r from their common centre, where $r < a$ would be

[JEE Main 2019, 10 Jan Shift-I]

(a) $\frac{Q(a^2 + b^2 + c^2)}{4\pi\varepsilon_0(a^3 + b^3 + c^3)}$ (b) $\frac{Q(a + b + c)}{4\pi\varepsilon_0(a^2 + b^2 + c^2)}$

(c) $\frac{Q}{4\pi\varepsilon_0(a + b + c)}$ (d) $\frac{Q}{12\,\pi\,\varepsilon_0}\cdot\frac{ab + bc + ca}{abc}$

Exp. **(b)**

Given charge distribution is shown in the figure below

Given surface charge densities of each shell are same.

$\therefore\quad \sigma_a = \sigma_b = \sigma_c$ ...(i)

As, surface charge density of shell of radius 'r' and having charge 'Q' is given as $\sigma = \dfrac{Q}{4\pi r^2}$

So relation (i) can be rewritten as

$$\frac{Q_a}{4\pi a^2} = \frac{Q_b}{4\pi b^2} = \frac{Q_c}{4\pi c^2}$$

$$\Rightarrow \quad Q_a : Q_b : Q_c = a^2 : b^2 : c^2$$

where Q_a, Q_b and Q_c are charges on shell of radius a, b and c, respectively.

Also, $\quad Q_a + Q_b + Q_c = Q$

Hence, $\quad Q_a = \dfrac{a^2}{a^2 + b^2 + c^2} \cdot Q$

$$Q_b = \frac{b^2}{a^2 + b^2 + c^2} \cdot Q \Rightarrow Q_c = \frac{c^2}{a^2 + b^2 + c^2} \cdot Q$$

As we know for charged spherical shell with charge Q of radius 'R', the potential at a point 'P' at distance r such that $r < R$ is,

$$V_P = \frac{kQ}{R}.$$

$\therefore$ potential at point P at a distance 'r' = Potential due to Q_a + Potential due to Q_b + Potential due to Q_c

$$= \frac{kQ_a}{a} + \frac{kQ_b}{b} + \frac{kQ_c}{c}$$

Substituting values of Q_a, Q_b and Q_c, we get

$$V = \frac{Q\,(a + b + c)}{4\pi\varepsilon_0\,(a^2 + b^2 + c^2)}.$$

27. A parallel plate capacitor is of area 6 cm^2 and a separation 3 mm. The gap is filled with three dielectric materials of equal thickness (see figure) with dielectric constants $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$. The dielectric constant of a material which give same capacitance when fully inserted in above capacitor, would be **[JEE Main 2019, 10 Jan Shift-I]**

(a) 4 (b) 36 (c) 12 (d) 14

Exp. **(c)**

In the given arrangement, capacitor can be viewed as three-different capacitors connected in parallel as shown below,

So, equivalent capacity of the system is

$$C_{eq} = C_1 + C_2 + C_3$$

$$\Rightarrow \frac{K\varepsilon_0 A}{d} = \frac{K_1\varepsilon_0 A/3}{d} + \frac{K_2\varepsilon_0 A/3}{d} + \frac{K_3\varepsilon_0 A/3}{d}$$

$$\Rightarrow \quad K = \frac{K_1}{3} + \frac{K_2}{3} + \frac{K_3}{3}$$

Here $K_1 = 10$, $K_2 = 12$ and $K_3 = 14$

So, $\quad K = \dfrac{10 + 12 + 14}{3} \Rightarrow K = 12$

28. Two electric dipoles, A, B with respective dipole moments $\mathbf{d}_A = -4qa\,\hat{\mathbf{i}}$ and $\mathbf{d}_B = -2\,qa\,\hat{\mathbf{i}}$ are placed on the X-axis with a separation R, as shown in the figure

The distance from A at which both of them produce the same potential is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\dfrac{\sqrt{2}\,R}{\sqrt{2}+1}$ (b) $\dfrac{\sqrt{2}\,R}{\sqrt{2}-1}$ (c) $\dfrac{R}{\sqrt{2}+1}$ (d) $\dfrac{R}{\sqrt{2}-1}$

Exp. **(a)**

Key Idea As, dipole moments points in same direction

So, potential of both dipoles can be same at some point between A and B.

Let potentials are same at P, distant x from B as shown below

Then, $$\frac{4qa}{(R-x)^2} = \frac{2qa}{(x)^2}$$

$$2x^2 = (R-x)^2$$

$$\Rightarrow \quad \sqrt{2}\,x = R - x$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}+1}$$

Distance from A is

$$\Rightarrow \quad R - x = R - \frac{R}{\sqrt{2}+1} = \frac{\sqrt{2}R}{\sqrt{2}+1}$$

29. Charges $-q$ and $+q$ located at A and B, respectively, constitute an electric dipole. Distance $AB = 2a$, O is the mid point of the dipole and OP is perpendicular to AB. A charge Q is placed at P, where $OP = y$ and $y >> 2a$. The charge Q experiences an electrostatic force F.

If Q is now moved along the equatorial line to P' such that $OP' = \left(\frac{y}{3}\right)$, the force on Q will be close to $\left(\frac{y}{3} >> 2a\right)$

[JEE Main 2019, 10 Jan Shift-II]

(a) $\frac{F}{3}$ (b) $3F$ (c) $9F$ (d) $27F$

Exp. **(*d*)**

Electric field on the equatorial line of a dipole at any point, which is at distance r from the centre is given by

$$E = \frac{2kP}{(r^2 + a^2)^{3/2}} \quad \ldots \text{(i)}$$

where, P is the dipole moment of the charges.

In first case $r = y$

$$\Rightarrow \quad E_1 = \frac{2kP}{(y^2 + a^2)^{3/2}}$$

Here, $y^2 >> a^2$

$$\Rightarrow \quad y^2 + a^2 \approx y^2$$

or $$E_1 = \frac{2kP}{y^3} \quad \ldots \text{(ii)}$$

So, force on the charge in its position at P will be

$$F = QE_1 = \frac{2kPQ}{y^3} \quad \ldots \text{(iii)}$$

In second case $r = y/3$

From Eq. (i), electric field at point P' will be

$$E_2 = \frac{2kP}{\left[\left(\frac{y}{3}\right)^2 + a^2\right]^{3/2}}$$

Again, $$\frac{y}{3} >> a$$

$$\Rightarrow \quad \left(\frac{y}{3}\right)^2 + a^2 \approx \left(\frac{y}{3}\right)^2$$

$$\Rightarrow \quad E_2 = \frac{2kP}{(y/3)^3} \Rightarrow E_2 = 27 \times \frac{2kP}{y^3}$$

Force on charge in this position,

$$F' = QE_2 = 27 \times \frac{2kPQ}{y^3} \quad \ldots \text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$F' = 27\,F$$

30. Four equal point charges Q each are placed in the xy-plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system (in joule) will be

[JEE Main 2019, 10 Jan Shift-II]

(a) $\frac{Q^2}{4\pi\varepsilon_0}$ (b) $\frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{3}}\right)$

(c) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (d) $\frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right)$

Exp. **(d)**

The four charges are shown in the figure below

Electric potential at origin (0, 0) due to these charges can be found by scalar addition of electric potentials due to each charge.

$$\therefore\ V = \frac{KQ}{r_1} + \frac{KQ}{r_2} + \frac{KQ}{r_3} + \frac{KQ}{r_4} \quad \ldots \text{(i)}$$

$$\Rightarrow V = KQ\left[\frac{1}{2} + \frac{1}{2} + \frac{1}{\sqrt{20}} + \frac{1}{\sqrt{20}}\right] = KQ\left[1 + \frac{1}{\sqrt{5}}\right]$$

$$\Rightarrow V = KQ\frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ volt} \quad \ldots \text{(ii)}$$

Now, if another charge Q is placed at origin, then work done to get the charge at origin

$$W = QV \quad \ldots \text{(iii)}$$

By putting value of V from Eq. (ii) in Eq. (iii), we get

$$W = KQ^2\frac{(\sqrt{5}+1)}{\sqrt{5}} \text{ joule}$$

or $$W = \frac{Q^2}{4\pi\varepsilon_0}\left(1 + \frac{1}{\sqrt{5}}\right) \text{ joule}$$

31. A parallel plate capacitor having capacitance 12 pF is charged by a battery to a potential difference of 10 V between its plates. The charging battery is now disconnected and a porcelain slab of dielectric constant 6.5 is slipped between the plates. The work done by the capacitor on the slab is

[JEE Main 2019, 10 Jan Shift-II]

(a) 560 pJ (b) 508 pJ
(c) 692 pJ (d) 600 pJ

Exp. **(b)**

Energy stored in a charged capacitor is given by

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\cdot\frac{Q^2}{C} \quad \ldots \text{(i)}$$

Here, $C = 12 \times 10^{-12}$F and $V = 10$ V.

$$\Rightarrow \quad U = \frac{1}{2} \times 12 \times 10^{-12} \times 100$$

$$U = 6 \times 10^{-10} \text{ J} \quad \ldots \text{(ii)}$$

After insertion of slab, capacitance will be

$C' = KC$ and final energy,

$$U' = \frac{1}{2}\cdot\frac{Q^2}{C'} = \frac{1}{2}\frac{Q^2}{KC}$$

$$\Rightarrow \quad U' = \frac{1}{K}U = \frac{1}{6.5} \times 6 \times 10^{-10}\text{J} \quad \ldots \text{(iv)}$$

($\because$ given, $K = 6.5$)

So, energy dissipated in the process will be equal to work done on the slab, i.e.

$$\Delta U = U - U' = \left(1 - \frac{1}{6.5}\right) \times 6 \times 10^{-10}\text{J}$$

$$\Rightarrow \quad \Delta U = \frac{5.5}{6.5} \times 6 \times 10^{-10} \text{ J}$$

$$\cong 5.08 \times 10^{-10} \text{ J or 508 pJ}$$

32. Three charges Q, $+q$ and $+q$ are placed at the vertices of a right angle isosceles triangle as shown below. The net electrostatic energy of the configuration is zero, if the value of Q is

[JEE Main 2019, 11 Jan Shift-I]

(a) $-2q$ (b) $\frac{-q}{1+\sqrt{2}}$ (c) $+q$ (d) $\frac{-\sqrt{2}q}{\sqrt{2}+1}$

Exp. **(d)**

Electrostatic energy between two charges q_1 and q_2 such that the distance between them is r is given as

$$U = \frac{K q_1 q_2}{r}$$

In accordance to the principle of superposition, total energy of the charge system as shown in the figure below is

$$U = \frac{Kq^2}{a} + \frac{KQq}{a} + \frac{KQq}{\sqrt{2}a}$$

It is given that, $U = 0$

$$\therefore \quad \frac{Kq}{a}\left[q + Q + \frac{Q}{\sqrt{2}}\right] = 0$$

$$\Rightarrow \quad Q = \frac{-\sqrt{2} \times q}{(\sqrt{2} + 1)}$$

33. In the figure shown below, the charge on the left plate of the $10\,\mu F$ capacitor is $-30\,\mu C$. The charge on the right plate of the $6\,\mu F$ capacitor is **[JEE Main 2019, 11 Jan Shift-I]**

(a) $+12\,\mu C$ (b) $+18\,\mu C$
(c) $-12\,\mu C$ (d) $-18\,\mu C$

Exp. **(b)**

Applying the concept of charge conservation on isolated plates of $10\,\mu F$, $6\,\mu F$ and $4\,\mu F$. Since, $6\,\mu F$ and $4\,\mu F$ are in parallel, so total charge on this combination will be $30\,\mu C$

$\therefore$ Charge on $6\,\mu F$, capacitor

$$= \left(\frac{C_1}{C_1 + C_2}\right) q = \frac{6}{6+4} \times 30 = 18\,\mu C$$

Since, the charge has been asked on the right plate of the capacitor. Thus, it would be $+18\,\mu C$.

Alternate Solution

Let charge on $6\,\mu F$ capacitor is $q\,\mu C$.

Now, V at $6\,\mu F = V$ at $4\,\mu F$

$$\therefore \quad \frac{q}{6\,\mu F} = \frac{30 - q}{4\,\mu F} \qquad (\because V = q/C)$$

$$\Rightarrow \quad 4q = -6q + 180$$

$$\Rightarrow \quad q = 18\,\mu C$$

34. The given graph shows variation (with distance r from centre) of **[JEE Main 2019, 11 Jan Shift-I]**

(a) electric field of a uniformly charged spherical shell
(b) potential of a uniformly charged spherical shell
(c) electric field of a uniformly charged sphere
(d) potential of a uniformly charged sphere

Exp. **(b)**

For a uniformly charged spherical shell, electric potential inside it is given by

$V_{inside} = V_{surface} = kq / r_0 =$ constant,

(where r_0 = radius of the shell).

and electric potential outside the shell at a distance r is

$$V_{outside} = \frac{kq}{r}$$

$$\Rightarrow \quad V \propto 1/r$$

$\therefore$ The given graph represents the variation of r and potential of a uniformly charged spherical shell.

35. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C-m. What is the potential energy of the electric dipole? **[JEE Main 2019, 11 Jan Shift-II]**

(a) -9×10^{-20} J (b) -10×10^{-29} J
(c) -20×10^{-18} J (d) -7×10^{-27} J

Exp. **(d)**

Given, $\mathbf{E} = 1000$ V/m

$\theta = 45°$

and $\mathbf{p} = 10^{-29}$ C-m

We know that, electric potential energy stored in an electric dipole kept in uniform electric field is given by the relation

$$U = -\mathbf{p} \cdot \mathbf{E} = -pE\cos\theta$$

$$= -10^{-29} \times 1000 \times \cos 45°$$

$$\Rightarrow \quad U \approx -7 \times 10^{-27}\ \text{J}$$

36. Seven capacitors, each of capacitance 2 μF are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)$μF.

Which of the combinations shown in figures below will achieve the desired value?

[JEE Main 2019, 11 Jan Shift-II]

Exp. **(c)**

Here, effective capacitance $= \left(\frac{6}{13}\right)$ μF

In the given options some capacitors are joined in parallel and some are in series.

For parallel combination,

$C_P = C_1 + C_2 + C_3 + ...$

For series combination, $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}...$

Here, capacity of each capacitor is 2 μF.

∴ Three capacitors must be in parallel to get 6 μF. Now, we consider the combinations with three capacitors in parallel (as shown in the figure below)

$$\therefore \quad \frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

$$= \frac{1}{6} + 2 = \frac{13}{6} \text{ or } C_{eq} = \frac{6}{13}\mu F$$

37. In the figure shown, after the switch '*S*' is turned from position '*A*' to position '*B*', the energy dissipated in the circuit in terms of capacitance '*C*' and total charge '*Q*' is

[JEE Main 2019, 12 Jan Shift-I]

(a) $\frac{3}{4} \cdot \frac{Q^2}{C}$ (b) $\frac{5}{8} \cdot \frac{Q^2}{C}$

(c) $\frac{1}{8} \cdot \frac{Q^2}{C}$ (d) $\frac{3}{8} \cdot \frac{Q^2}{C}$

Exp. **(d)**

In position '*A*' of switch, we have a capacitor joined with battery.

So, energy stored in position, $U_1 = \frac{1}{2} C \varepsilon^2$

When switch is turned to position *B*, we have a charged capacitor joined to an uncharged capacitor.

Common potential in steady state will be

$$V = \frac{\text{total charge}}{\text{total capacity}} = \frac{C\varepsilon}{4C} = \frac{\varepsilon}{4}$$

Now, energy stored will be

$$U_2 = \frac{1}{2}(C_{eq})(V_{common})^2$$

$$= \frac{1}{2} 4C \times \left(\frac{\varepsilon}{4}\right)^2 = \frac{1}{8} C\varepsilon^2$$

So, energy dissipated is

$$\Delta U = U_1 - U_2$$

$$= \frac{1}{2} C\varepsilon^2 - \frac{1}{8} C\varepsilon^2$$

$$= \frac{3}{8} C\varepsilon^2 = \frac{3}{8} C\left(\frac{Q}{C}\right)^2 = \frac{3}{8} \cdot \frac{Q^2}{C}$$

38. There is uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed $v[R(t)]$ of the distribution as a function of its instantaneous radius $R(t)$ is

[JEE Main 2019, 12 Jan Shift-I]

(a)

(b)

(c)

(d)

Exp. **(c)**

> **Key Idea** As, electrostatic force is conserved in nature so, total energy of charge distribution remains constant in absence of any external interaction

Let radius of distribution at some instant t is R.
At $t = 0$, radius is given R_0

Now by conservation of energy, we have

$$0 + \frac{kQ^2}{2R_0} = \frac{1}{2}mv^2 + \frac{kQ^2}{2R}$$

($\because$ The distribution starts from rest, so, initial kinetic energy is zero.)

Differentiating this equation with respect to R, we get

$$\frac{1}{2}m2v\frac{dv}{dR} - \frac{kQ^2}{2R^2} = 0 \text{ or } \frac{dv}{dR} = \frac{kQ^2}{2mvR^2}$$

Here, $\frac{dv}{dR}$ = slope of v *versus* R graph. It decreases with increasing v and R.
Also, slope $\longrightarrow 0$ as $R \longrightarrow \infty$.

From above conclusions, we can see that the best suited graph is given in option (c).

39. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle as shown in the figure. **[JEE Main 2019, 12 Jan Shift-I]**

(a) $\sqrt{3}\,ql\,\frac{\hat{\mathbf{j}} - \hat{\mathbf{i}}}{\sqrt{2}}$ (b) $2ql\,\hat{\mathbf{j}}$

(c) $-\sqrt{3}\,ql\,\hat{\mathbf{j}}$ (d) $(ql)\,\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}$

Exp. **(c)**

Given system is equivalent to two dipoles inclined at 60° to each other as shown in the figure below

Now, magnitude of resultant of these dipole moments is

$$p_{\text{net}} = \sqrt{p^2 + p^2 + 2p \cdot p\cos 60°}$$
$$= \sqrt{3}p = \sqrt{3}ql$$

As, resultant is directed along negative y-direction $p_{\text{net}} = -\sqrt{3}p\hat{\mathbf{j}} = -\sqrt{3}ql\,\hat{\mathbf{j}}$

40. The charge on a capacitor plate in a circuit as a function of time is shown in the figure.

[JEE Main 2019, 12 Jan Shift-II]

What is the value of current at $t = 4$ s?

(a) $2\,\mu A$ (b) $1.5\,\mu A$
(c) Zero (d) $3\,\mu A$

Exp. **(c)**

As we know, current, $I = \dfrac{dq}{dt}$

= Slope of q *versus* t graph

= Zero at $t = 4$s; (as graph is a line parallel to time axis at $t = 4$s)

41. A parallel plate capacitor with plates of area $1\ m^2$ each, are at a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge on each plate is $\left(\text{Take}, \varepsilon_0 = 8.85 \times 10^{-12}\ \dfrac{C^2}{N-m^2}\right)$

[JEE Main 2019, 12 Jan Shift-II]

(a) 9.85×10^{-10} C (b) 8.85×10^{-10} C
(c) 7.85×10^{-10} C (d) 6.85×10^{-10} C

Exp. **(b)**

If Q = charge on each plate, then

$$Q = CV = \frac{\varepsilon_0 A}{d} \cdot Ed$$

$$= \varepsilon_0 AE$$

Here, $A = 1\,m^2$, $E = 100$ N/C

and $\varepsilon_0 = 8.85 \times 10^{-12}\ \dfrac{C^2}{N\text{-}m^2}$

So, by substituting given values, we get

$$Q = 8.85 \times 10^{-12} \times 1 \times 100$$

$$= 8.85 \times 10^{-10}\ C$$

42. In the circuit shown, find C if the effective capacitance of the whole circuit is to be $0.5\,\mu F$. All values in the circuit are in μF.

[JEE Main 2019, 12 Jan Shift-II]

(a) $\dfrac{6}{5}\mu F$ (b) $4\,\mu F$ (c) $\dfrac{7}{10}\mu F$ (d) $\dfrac{7}{11}\mu F$

Exp. **(d)**

Key Idea Two capacitors C_1 and C_2, if connected in series, then their equivalent capacitance is

$$C_{eq} = \frac{C_1 \times C_2}{C_1 + C_2}$$

If they are connected in parallel, then their equivalent capacitance is, $C_{eq} = C_1 + C_2$.

We simplify given circuit as

So, $C_{AB} = \dfrac{C \times 7/3}{C + 7/3} = \dfrac{1}{2}$ (given)

$\Rightarrow \dfrac{7C}{3} = \dfrac{C}{2} + \dfrac{7}{6} \Rightarrow \left(\dfrac{14-3}{6}\right)C = \dfrac{7}{6}$

$\Rightarrow C = \dfrac{7}{11}\mu F$

43. Three concentric metal shells A, B and C of respective radii a, b and $c\,(a < b < c)$ have surface charge densities $+\sigma, -\sigma$ and $+\sigma$, respectively. The potential of shell B is

[JEE Main 2018]

(a) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a}+c\right]$ (b) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b}+c\right]$

(c) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b}+a\right]$ (d) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c}+a\right]$

Exp. **(*b*)**

Potential of B = Potential due to charge on A + Potential due to charge on B + Potential due to charge on C.

$$\therefore \quad V_B = \frac{k(Q_A+Q_B)}{b}+\frac{kQ_C}{c}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\sigma 4\pi a^2}{b}-\frac{\sigma 4\pi b^2}{b}+\frac{\sigma 4\pi c^2}{c}\right]$$

$$= \frac{\sigma\varepsilon}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+\frac{c^2}{c}\right)=\frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$$

$$V_B = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2-b^2}{b}+c\right)$$

44. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20 V. If a dielectric material of dielectric constant $K=\frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be **[JEE Main 2018]**

(a) 1.2 nC (b) 0.3 nC

(c) 2.4 nC (d) 0.9 nC

Exp. **(*a*)**

Magnitude of induced charge is given by

$$Q' = (K-1)CV_0 = \left(\frac{5}{3}-1\right)90\times10^{-12}\times 20$$

$$= 1.2\times10^{-9}\text{C} \Rightarrow Q' = 1.2\text{nC}$$

45. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V. The minimum number of capacitors required to achieve this is **[JEE Main 2017 (Offline)]**

(a) 16 (b) 24 (c) 32 (d) 2

Exp. **(*c*)**

As each capacitors cannot withstand more than 300 V, so there should be four capacitors in each row became in this condition 1 kV i.e. 1000 V will be divided by 4 (i.e. 250 not more than 300 V).

Now, equivalent capacitance of one row

$$= \frac{1}{4}\times 1\,\mu\text{F} = 0.25\mu\text{F}$$

[$\because$ in series combination, $C_{eq} = c/n$]

Now, we need equivalent of 2μF, so let we need n such rows

$$\therefore \quad n\times 0.25 = 2\mu\text{F}$$

[$\because$ in parallel combination $C_{eq} = nc$]

$$n = \frac{2}{0.25} = 8$$

$\therefore$ Total number of capacitors = number of rows × number of capacitors in each row

$= 8\times 4 = 32$

46. An electric dipole has a fixed dipole moment **p**, which makes angle θ with respect to X-axis. When subjected to an electric field $\mathbf{E}_1 = E\hat{\mathbf{i}}$, it experiences a torque $\mathbf{T}_1 = \tau\hat{\mathbf{k}}$. When subjected to another electric field $\mathbf{E}_2 = \sqrt{3}E_1\hat{\mathbf{j}}$, it experiences a torque $\mathbf{T}_2 = -\mathbf{T}_1$. The angle θ is **[JEE Main 2017 (Offline)]**

(a) 45° (b) 60° (c) 90° (d) 30°

Exp. **(*b*)**

Torque applied on a dipole $\tau = pE\sin\theta$

where, θ = angle between axis of dipole and electric field.

For electric field $E_1 = E\hat{\mathbf{i}}$

it means field is directed along positive X direction, so angle between dipole and field will remain θ, therefore torque in this direction

$$E_1 = pE_1\sin\theta$$

In electric field $E_2 = \sqrt{3}\, E\hat{j}$,

it means field is directed along positive Y-axis, so angle between dipole and field will be $90° - \theta$.

Torque in this direction $\tau_2 = pE \sin(90° - \theta)$

$= p\sqrt{3}\, E_1 \cos\theta$

According to question $\tau_2 = -\tau_1 \Rightarrow |\tau_2| = |\tau_1|$

$\therefore \quad pE_1 \sin\theta = p\sqrt{3}\, E_1 \cos\theta$

$\tan\theta = \sqrt{3} \Rightarrow \tan\theta = \tan 60°$

$\therefore \quad \theta = 60°$

47. The region between two concentric spheres of radii a and b, respectively (see the figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is **[JEE Main 2016 (Offline)]**

(a) $\frac{Q}{2\pi a^2}$ (b) $\frac{Q}{2\pi(b^2 - a^2)}$

(c) $\frac{2Q}{\pi(a^2 - b^2)}$ (d) $\frac{2Q}{\pi a^2}$

Exp. **(a)**

As, Gaussian surface at distance r from centre,

$$\frac{Q + \int_a^r \frac{A}{r} 4\pi r^2 dr}{\varepsilon_0} = E 4\pi r^2$$

$$E\, 4\pi\varepsilon_0 r^2 = Q + A\frac{4\pi}{r^2}\left(\frac{r^2 - a^2}{2}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{r^2} + A\,2\pi\left(\frac{r^2 - a^2}{r^2}\right)\right]$$

$$E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2} + A2\pi - \frac{A2\pi a^2}{r^2}\right)$$

$$E = \frac{1}{4\pi\varepsilon_0} \times A \times 2\pi$$

At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant is

As, $\quad Q = 2\pi A a^2$

i.e. $\quad A = \frac{Q}{2\pi a^2}$

48. A combination of capacitors is set-up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9μF capacitors), at a point distance 30 m from it, would equal to **[JEE Main 2016 (Offline)]**

(a) 240 N/C (b) 360 N/C

(c) 420 N/C (d) 480 N/C

Exp. **(c)**

Resultant circuit,

As, charge on 3μF = 3μF × 8V = 24μC

$\therefore$ Charge on 4μF = Charge on 12μF = 24μC

Charge on 3μF = 3μF × 2V = 6μC

Charge on 9μF = 9μF × 2V = 18μC

Charge on 4μF + Charge on 9μF

= (24 + 18)μC = 42μC

$\therefore$ Electric field at a point distant 30 m

$$= \frac{9 \times 10^3 \times 42 \times 10^{-6}}{30 \times 30} = 420 \text{N/C}$$

49. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in (figures are schematic and not drawn to scale) **[JEE Main 2015]**

(a)

(b)

(c)

(d)

Exp. **(*a*)**

Field lines should originate from positive charge and terminate to negative charge. Thus, (b) and (c) are not possible.

Electric field lines cannot form corners as shown in (d).

Thus, correct option is (a).

50. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere, the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3, and R_4 respectively. Then, **[JEE Main 2015]**

(a) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

(b) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

(c) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

(d) $2R < R_4$

Exp. **(*c, d*)**

Potential at the surface of the charged sphere

Charged sphere

$$V_0 = \frac{KQ}{R}$$

$$V = \frac{KQ}{r}, r \geq R$$

$$= \frac{KQ}{2R^3}(3R^2 - r^2); \quad r \leq R$$

$$V_{\text{centre}} = V_c = \frac{KQ}{2R^3} \times 3R^2$$

$$= \frac{3KQ}{2R} = \frac{3V_0}{2}$$

$\Rightarrow \quad R_1 = 0$

As potential decreases for outside points.

Thus, according to the question, we can write

$$V_{R_2} = \frac{5V_0}{4} = \frac{KQ}{2R^3}(3R^2 - R_2^2)$$

$$\frac{5V_0}{4} = \frac{V_0}{2R^2}(3R^2 - R_2^2)$$

or $\quad \frac{5}{2} = 3 - \left(\frac{R_2}{R}\right)^2$

or $\quad \left(\frac{R_2}{R}\right)^2 = 3 - \frac{5}{2} = \frac{1}{2}$

or $\quad R_2 = \frac{R}{\sqrt{2}}$

Similarly,

$$V_{R_3} = \frac{3V_0}{4} \Rightarrow \frac{KQ}{R_3} = \frac{3}{4} \times \frac{KQ}{R}$$

or $\quad R_3 = \frac{4}{3}R$

$$V_{R4} = \frac{KQ}{R_4} = \frac{V_0}{4} \Rightarrow \frac{KQ}{R_4} = \frac{1}{4} \times \frac{KQ}{R}$$

or $\quad R_4 = 4R$

51. In the given circuit, charge Q_2 on the $2\,\mu F$ capacitor changes as C is varied from $1\,\mu F$ to $3\mu F$. Q_2 as a function of C is given properly by (figures are drawn schematically and are not to scale) **[JEE Main 2015]**

Exp. **(*b*)**

Assume negative terminal of the battery as grounded (0 V).

Suppose, potential of point x is V.

From the circuit diagram, we can write

$$Q_C = Q_1 + Q_2$$

or $\quad C(E - V) = 1 \times V + 2 \times V$

or $\quad V[C + 3] = CE$

or $$V = \frac{CE}{3+C}$$

$\therefore$ $$Q_2 = C_2(V) = \frac{2CE}{3+C} = \frac{2E}{1+3/C}$$

As C_1 varied from 1 μF to 3 μF, charge increases with decreasing slope.

Note As $C \to \infty, Q_2 \to 2E = \text{constant}$

52. Assume that an electric field $\mathbf{E} = 30x^2\,\mathbf{i}$ exists in space. Then, the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A is the potential at $x = 2$ m, is **[JEE Main 2014]**

(a) 120 J (b) –120 J (c) –80 J (d) 80 J

Exp. **(c)**

As we know, potential difference $V_A - V_O$ is

$$dV = -Edx$$

$$\int_{V_O}^{V_A} dV = -\int_0^2 30x^2 dx$$

$$V_A - V_O = -30 \times \left[\frac{x^3}{3}\right]_0^2$$

$$= -10 \times [2^3 - (0)^3]$$

$$= -10 \times 8 = -80 \text{ J}$$

53. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm with a dielectric of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to **[JEE Main 2014]**

(a) 6×10^{-7} C/m^2 (b) 3×10^{-7} C/m^2
(c) 3×10^4 C/m^2 (d) 6×10^4 C/m^2

Exp. **(a)**

Electric field inside dielectric, $\frac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

$$\Rightarrow \quad \sigma = 2.2 \times 8.58 \times 10^{-12} \times 3 \times 10^4$$

$$= 6.6 \times 8.85 \times 10^{-8}$$

$$= 5.841 \times 10^{-7}$$

$$= 6 \times 10^{-7} \text{ C/m}^2$$

54. Two capacitors C_1 and C_2 are charged to 120 V and 200 V, respectively. It is found that by connecting them together, the potential on each one can be made zero. Then, **[JEE Main 2013]**

(a) $5C_1 = 3C_2$ (b) $3C_1 = 5C_2$
(c) $3C_1 + 5C_2 = 0$ (d) $9C_1 = 4C_2$

Exp. **(b)**

For potential to be made zero after connection, the charge of both capacitors are equal.

$\therefore$ $q_1 = q_2 \Rightarrow C_1V_1 = C_2V_2$

$\Rightarrow$ $120\,C_1 = 200\,C_2 \Rightarrow 3C_1 = 5C_2$

55. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at distance L from the end A is **[JEE Main 2013]**

(a) $\frac{Q}{8\pi\varepsilon_0 L}$ (b) $\frac{3Q}{4\pi\varepsilon_0 L}$
(c) $\frac{Q}{4\pi\varepsilon_0 L \ln 2}$ (d) $\frac{Q \ln 2}{4\pi\varepsilon_0 L}$

Exp. **(d)**

Let an element of length dx, charge dq, at distance x from point O.

$$dV = k\frac{dq}{x}$$

where, $dq = \frac{Q}{L}dx$

$\therefore$ $$V = \int_L^{2L} \frac{kdq}{x} = \int_L^{2L} \frac{\left(\frac{Q}{L}\right)dx}{x}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\int_L^{2L}\left(\frac{1}{x}\right)dx$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e x]_L^{2L}$$

$$= \frac{Q}{4\pi\varepsilon_0 L}[\log_e 2L - \log_e L]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\left[\log_e^2 \frac{2L}{L}\right]$$

$$= \frac{Q}{4\pi\varepsilon_0 L}\ln(2)$$

56. Two charges each equal to q, are kept at $x = -a$ and $x = a$ on the x-axis. A particle of mass m and charge $q_0 = q/2$ is placed at the origin. If charge q_0 is given, a small displacement ($y << a$) along the y-axis, the net force acting on the particle is proportional to **[JEE Main 2013]**

(a) y (b) $-y$ (c) $1/y$ (d) $-1/y$

Exp. **(b)**

$$F_{net} = 2F\cos\theta$$

$$\therefore \quad F_{net} = -\frac{2kq\left(\frac{q}{2}\right)}{(\sqrt{y^2+a^2})^2}\cdot\frac{y}{\sqrt{y^2+a^2}}$$

[negative sign indictate the net force is towards the mean position]

$$F_{net} = -\frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \Rightarrow \frac{kq^2y}{a^3} \propto -y$$

57. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as function of distance from the centre. The graph which would correspond to the above will be **[AIEEE 2012]**

(a)

(b)

(c)

(d)

Exp. **(c)**

Electric field inside the uniformly charged sphere varies linearly, $E = \frac{kQ}{R^3}\cdot r, (r \le R)$, while outside the sphere, it varies as inverse square of distance, $E = \frac{kQ}{r^2}$; $(r \ge R)$ which is correctly represented in option (c).

58. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2012]**

An insulating solid sphere of radius R has a uniform positive charge density ρ. As a result of this uniform charge distribution, there is a finite value of electric potential at the centre of the sphere, at the surface of the sphere and also at a point outside the sphere. The electric potential at infinite is zero.

Statement I When a charge q is taken from the centre of the surface of the sphere, its potential energy changes by $\frac{q\rho}{3\varepsilon_0}$.

Statement II The electric field at a distance r ($r < R$) from the centre of the sphere is $\frac{\rho r}{3\varepsilon_0}$.

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II the correct explanation for Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Exp. **(a)**

Statement I is dimensionally wrong while from Gauss' law,

$$E(4\pi r^2) = \frac{\rho\cdot\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0}$$

gives Statement II is correct.

59. Combination of two identical capacitors, a resistor R and a DC voltage source of voltage 6 V is used in an experiment on C-R circuit. It is found that for a parallel combination of the capacitor, the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 s. For series combination, the time needed for reducing the voltage of the fully charged series combination by half is **[AIEEE 2011]**

(a) 20 s (b) 10 s
(c) 5 s (d) 2.5 s

Exp. **(d)**

If C_e is the effective capacitance, then

$$V_C = \frac{1}{2}V_0, \quad \frac{q}{C_e} = \frac{q_0}{2C_e}$$

As $\quad q = q_0\,(1 - e^{\frac{-t}{RC_e}})$

$\Rightarrow \quad q_0\,(1 - e^{-t/RC_e}) = \frac{q_0}{2}$

$\Rightarrow \quad t = RC_e \ln 2$

For parallel grouping,

$$C_e = 2C$$

$\therefore \quad t_2 = 2\,RC \ln 2$

For series grouping,

$$C_e = \frac{C}{2}$$

$\therefore \quad t_1 = \frac{RC}{2}\ln 2$

$\therefore \quad \frac{t_2}{t_1} = \frac{1}{4}$

$\Rightarrow \quad t_2 = 2.5\text{ s}$

60. An electric charge $+q$ moves with velocity $\mathbf{v} = 3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$, in an electromagnetic field given by $\mathbf{E} = 3\mathbf{i} + \mathbf{j} + 2\mathbf{k}$, $\mathbf{B} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$. The y component of the force experienced by $+q$ is **[AIEEE 2011]**

(a) $2q$ (b) $11q$
(c) $5q$ (d) $3q$

Exp. **(b)**

By Lorentz force, net force

$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

$= q\,[(3\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + (3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (\mathbf{i} + \mathbf{j} - 3\mathbf{k})]$

$= q\,[-10\mathbf{i} + 11\mathbf{j} + \mathbf{k}]$

$\therefore \quad F_y = 11q$

61. Two positive charges of magnitude q are placed at the ends of a side 1 of a square of side $2a$. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is **[AIEEE 2011]**

(a) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{1}{\sqrt{5}}\right)$ (b) zero
(c) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 + \frac{1}{\sqrt{5}}\right)$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1 - \frac{2}{\sqrt{5}}\right)$

Exp. **(a)**

Initial potential energy of the system,

$$U_i = \frac{2kqQ}{a} + \frac{2k(-q)Q}{\sqrt{5}a}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

Final potential energy of the system,

$$U_f = 0$$

By conservation of energy,

Gain in KE = Loss in PE

$$K = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2qQ}{a}\left[1 - \frac{1}{\sqrt{5}}\right]$$

62. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d, (d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result, charges approach each other with a velocity v. Then, as a function of distance x between them, is **[AIEEE 2011]**

(a) $v \propto x^{-1}$ (b) $v \propto x^{1/2}$ (c) $v \propto x$ (d) $v \propto x^{-1/2}$

Exp. **(d)**

At any instant,

$$T\cos\theta = mg \quad \ldots(i)$$

$$T\sin\theta = F_e = \frac{kq^2}{x^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{kq^2}{x^2} = mg\tan\theta$$

$$\Rightarrow \quad q^2 = \frac{mg}{k} \cdot \frac{x}{2l} \cdot x^2 \qquad \left[\because \tan\theta \approx \frac{x}{2l}\right]$$

$$\Rightarrow \quad q^2 = \frac{mg}{2kl}x^3 \quad \ldots(iii)$$

$$\Rightarrow \quad 2q\frac{dq}{dt} = \frac{3mg}{2kl}x^2\frac{dx}{dt}$$

$$\Rightarrow \quad 2\left(\frac{mg}{2kl}x^3\right)^{1/2}\frac{dq}{dt} = \frac{3mg}{2kl}x^2 v$$

$$\left[\because q = \left(\frac{mg}{2kl}x^3\right)^{1/2}\right]$$

$$\Rightarrow vx^{1/2} = \text{constant} \Rightarrow v \propto x^{-1/2}$$

63. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is **[AIEEE 2011]**

(a) $\frac{\pi}{4}\sqrt{LC}$ (b) $2\pi\sqrt{LC}$ (c) $\sqrt{LC}$ (d) $\pi\sqrt{LC}$

Exp. **(a)**

As initially charge is maximum.

$$\therefore \quad q = q_0\cos\omega t$$

$$\Rightarrow \quad i = \frac{dq}{dt} = -\omega q_0\sin\omega t$$

Given, energy stored in inductor $\left(U = \frac{1}{2}Li^2\right)$ is equal to energy stored in capacitor $\left(U = \frac{q^2}{2C}\right)$.

$$\therefore \quad \frac{1}{2}Li^2 = \frac{q^2}{2C}$$

$$\Rightarrow \quad \frac{1}{2}L(\omega q_0\sin\omega t)^2 = \frac{(q_0\cos\omega t)^2}{2C}$$

But $\quad \omega = \frac{1}{\sqrt{LC}}$

$$\Rightarrow \quad \tan\omega t = 1$$

$$\Rightarrow \quad \omega t = \frac{\pi}{4}$$

$$\Rightarrow \quad t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC}$$

64. A resistor R and $2\,\mu F$ capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed (take $\log_{10} 2.5 = 0.4$) **[AIEEE 2011]**

(a) $1.7 \times 10^5\,\Omega$ (b) $2.7 \times 10^6\,\Omega$

(c) $3.3 \times 10^7\,\Omega$ (d) $1.3 \times 10^4\,\Omega$

Exp. **(b)**

Neon bulb is filled with gas, so its resistance is infinite, hence no current flows through it.

Now, $V_c = E(1 - e^{-t/RC})$

$\Rightarrow \quad 120 = 200(1 - e^{-t/RC})$

$\Rightarrow \quad e^{-t/RC} = \frac{2}{5} \Rightarrow t = RC \ \ln 2.5$

$\Rightarrow \quad R = \frac{t}{C\ln 2.5} = \frac{t}{2.303C \log 2.5}$

$= 2.7 \times 10^6 \Omega$

65. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$, where r is the distance from the centre and a, b are constants. Then, the charge density inside the ball is **[AIEEE 2011]**

(a) $-6a\,\varepsilon_0 r$ (b) $-24\,\pi a\,\varepsilon_0$

(c) $-6a\,\varepsilon_0$ (d) $-24\pi a\,\varepsilon_0 r$

Exp. **(c)**

Electric field, $E = -\frac{d\phi}{dr} = -2ar$

By Gauss' theorem,

$$E(4\pi r^2) = \frac{q}{\varepsilon_0} \quad \Rightarrow \quad q = -8\pi\,\varepsilon_0\, ar^3$$

$$\rho = \frac{dq}{dV} = \frac{dq}{dr} \times \frac{dr}{dV}$$

$$= (-24\pi\varepsilon_0\, ar^2)\left(\frac{1}{4\pi r^2}\right) = -6\varepsilon_0 a$$

66. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then, the ratio t_1 / t_2 will be **[AIEEE 2010]**

(a) 1 (b) 1 / 2

(c) 1 / 4 (d) 2

Exp. **(c)**

Energy stored in capacitor,

$$U = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2C}(q_0 e^{-t/\tau})^2$$

$$= \frac{q_0^2}{2C}e^{-2t/\tau} \qquad [\text{where, } \tau = CR]$$

$$U = U_i e^{-2t/\tau}$$

$$\frac{1}{2}U_i = U_i e^{-2t_1/\tau}$$

$$\frac{1}{2} = e^{-2t_1/\tau} \Rightarrow t_1 = \frac{\tau}{2}\ln 2$$

Now, $q = q_0 e^{-t/\tau}$

$$\frac{1}{4}q_0 = q_0 e^{-t_2/\tau}$$

$$t_2 = \tau \ln 4 = 2\tau \ln 2$$

$\therefore \quad \frac{t_1}{t_2} = \frac{1}{4}$

67. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field E at the centre O is **[AIEEE 2010]**

(a) $\frac{q}{4\pi^2\varepsilon_0 r^2}\mathbf{j}$ (b) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\mathbf{j}$

(c) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\mathbf{j}$ (d) $\frac{q}{2\pi^2\varepsilon_0 r^2}\mathbf{j}$

Exp. **(c)**

Linear charge density, $\lambda = \left(\frac{q}{\pi r}\right)$

$$E = \int dE\sin\theta\,(-\mathbf{j}) = \int_0^{\pi}\int \frac{K \cdot dq}{r^2}\sin\theta\,(-\mathbf{j})$$

$$E = \frac{K}{r^2}\int_0^{\pi}\frac{qr}{\pi r}d\theta\sin\theta\,(-\mathbf{j})$$

$$= \frac{K}{r^2}\frac{q}{\pi}\int_0^{\pi}\sin\theta\, d\theta\,(-\mathbf{j})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}[-\cos\theta]_0^{\pi}(-\hat{\mathbf{j}})$$

$$= \frac{q}{2\pi^2\varepsilon_0 r^2}(-\mathbf{j})$$

68. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r)=\rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto $r=R$ and $\rho(r)=0$ for $r>R$, where r is the distance from the origin. The electric field at a distance $r, (r<R)$ from the origin is given by **[AIEEE 2010]**

(a) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (b) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

(c) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (d) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

Exp. **(b)**

Apply Shell theorem, the total charge upto distance r can be calculated as followed

$$dq = 4\pi r^2 \cdot dr \cdot \rho$$
$$= 4\pi r^2 \cdot dr \cdot \rho_0\left[\frac{5}{4}-\frac{r}{R}\right]$$
$$= 4\pi\rho_0\left[\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right]$$
$$\int dq = q = 4\pi\rho_0 \int_0^r\left(\frac{5}{4}r^2 dr - \frac{r^3}{R}dr\right)$$
$$q = 4\pi\rho_0\left[\frac{5}{4}\frac{r^3}{3} - \frac{1}{R}\frac{r^4}{4}\right]$$

As electric field intensity,

$$E = \frac{kq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\cdot 4\pi\rho_0\left[\frac{5}{4}\left(\frac{r^3}{3}\right) - \frac{r^4}{4R}\right]$$
$$E = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

69. The question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I For a charged particle moving from point P to point Q, the net work done by an electrostatic field on the particle is independent of the path connecting point P to point Q.

Statement II The net work done by a conservative force on an object moving along a closed loop is zero.

(a) Statement I is true, Statement II is false

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(d) Statement I is false, Statement II is true

Exp. **(b)**

Work done by conservative force does not depend on the path. Electrostatic force is a conservative force.

70. Let $\rho(r)=\frac{Q}{\pi R^4}r$ be the charge density distribution for a solid sphere of radius R and total charge Q. For a point P inside the sphere at distance r_1 from the centre of the sphere, the magnitude of electric field is **[AIEEE 2009]**

(a) zero (b) $\frac{Q}{4\pi\varepsilon_0 r_1^2}$

(c) $\frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$ (d) $\frac{Qr_1^2}{3\pi\varepsilon_0 R^4}$

Exp. **(c)**

$$E4\pi r_1^2 = \frac{\int_0^{r_1}\frac{Q}{\pi R^4}r4\pi r^2\,dr}{\varepsilon_0}$$
$$\Rightarrow\ E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

71. Two points P and Q are maintained at the potentials of 10 V and -4 V, respectively. The work done in moving 100 electrons from P to Q is **[AIEEE 2009]**

(a) -19×10^{-17} J (b) 9.60×10^{-17} J

(c) -2.24×10^{-16} J (d) 2.24×10^{-16} J

Exp. **(d)**

$$W = QdV$$
$$= Q(V_q - V_p)$$
$$= -100\times(1.6\times10^{-19})\times(-4-10)$$
$$= +100\times1.6\times10^{-19}\times14$$
$$= +2.24\times10^{-16}\text{ J}$$

72. A charge Q is placed at each of the opposite corners of a square. A charge q is placed at each of the other two corners. If the net electrical force on Q is zero, then $\frac{Q}{q}$ equals **[AIEEE 2009]**

(a) $-2\sqrt{2}$ (b) -1 (c) 1 (d) $-\frac{1}{\sqrt{2}}$

Exp. **(a)**

Three forces F_{41}, F_{42} and F_{43} acting on Q as shown.

Resultant of $F_{41} + F_{43} = \sqrt{2}F_{each}$

$$= \sqrt{2}\frac{1}{4\pi\varepsilon_0}\frac{Qq}{d^2}$$

Resultant on Q becomes zero only when q charges are of negative nature.

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{Q \times Q}{(\sqrt{2}d)^2}$$

$$\Rightarrow \quad \sqrt{2}\frac{dQ}{d^2} = \frac{Q \times Q}{2d^2} \Rightarrow \sqrt{2} \times q = \frac{Q \times Q}{2}$$

$$\therefore \quad q = \frac{Q}{2\sqrt{2}}$$

or $$\frac{Q}{q} = -2\sqrt{2}$$

73. A parallel plate capacitor with air between the plates has a capacitance of 9 pF. The separation between its plates is d. The space between the plate is now filled with two dielectrics. One of the dielectrics has dielectric constant $K_1 = 3$ and thickness $d/3$ while the other one has dielectric constant $K_2 = 6$ and thickness $2d/3$. Capacitance of the capacitor is now **[AIEEE 2008]**

(a) 1.8 pF (b) 45 pF
(c) 40.5 pF (d) 20.25 pF

Exp. **(c)**

As both capacitors are connected in series.

$$\therefore \quad C = \frac{C_1C_2}{C_1 + C_2} \quad \ldots(i)$$

where, $$C_1 = \frac{K_1\varepsilon_0 A}{d/3} \quad \ldots(ii)$$

and $$C_2 = \frac{K_2\varepsilon_0 A}{2d/3} \quad \ldots(iii)$$

It is given that, $\frac{\varepsilon_0 A}{d} = 9$ pF

On substituting Eqs. (ii) and (iii) in Eq. (i), we get the result $C = 40.5$ pF.

➡ **Directions** *(Q.nos. 74 to 75) are based on the following paragraph.*

Consider a block of conducting material of resistivity ρ shown in the figure. Current I enters at A and leaves from D. We apply superposition principle to find voltage ΔV developed between B and C. The calculation is done in the following steps

(a) Take current I entering from A and assume it to spread over a hemispherical surface on the block.

(b) Calculate field $E(r)$ at distance r from A by using Ohm's law $= \rho J$, where J is the current per unit area at r.

(c) From the r dependence of $E(r)$, obtain the potential $V(r)$ at r.

(d) Repeat steps (i), (ii) and (iii) for current I leaving D and superpose results for A and D.

74. ΔV measured between B and C is **[AIEEE 2008]**

(a) $\frac{\rho I}{\pi a} - \frac{\rho I}{\pi(a+b)}$ (b) $\frac{\rho I}{a} - \frac{\rho I}{(a+b)}$

(c) $\frac{\rho I}{2\pi a} - \frac{\rho I}{2\pi(a+b)}$ (d) $\frac{\rho I}{2\pi(a-b)}$

Exp. **(c)**

Electric field at a distance r from A is

$$E = \rho \times \frac{I}{2\pi r^2}$$

$$\Rightarrow \quad \int dV = -\int E\,dr$$

$$\Rightarrow \quad V_C - V_B = \Delta V = -\int_a^{a+b} \frac{\rho I}{2\pi} \times \frac{dr}{r^2}$$

or $$\Delta V = \frac{\rho I}{2\pi}\left[\frac{1}{a} - \frac{1}{a+b}\right]$$

75. For current entering at A, the electric field at a distance r from A is **[AIEEE 2008]**

(a) $\frac{\rho I}{8\pi r^2}$ (b) $\frac{\rho I}{r^2}$ (c) $\frac{\rho I}{2\pi r^2}$ (d) $\frac{\rho I}{4\pi r^2}$

Exp. **(c)**

As $E = \frac{V}{l} = \frac{IR}{l} = \frac{l}{t}\frac{e}{A} = \rho J$

[$\because$ current density, $J = I/A$]

From $E = \rho J = \frac{\rho I}{2\pi r^2}$

76. A thin spherical shell of radius R has charge Q spread uniformly over its surface. Which of the following graphs most closely represents the electric field $E(r)$ produced by the shell in the range $0 \le r < x$, where r is the distance from the centre of the shell? **[AIEEE 2008]**

Exp. **(a)**

For uniformly charged spherical shell,

$$E = 0, \qquad r < R$$

$$= \frac{Q}{4\pi\varepsilon_0 r^2}, \quad r \ge R$$

77. The potential at a point x (measured in μm) due to some charges situated on the x-axis is given by $V(x) = 20/(x^2 - 4)$ volt.

The electric field E at $x = 4$ μm is given by **[AIEEE 2007]**

(a) $\frac{5}{3}$ V/μm and in the –ve x-direction

(b) $\frac{5}{3}$ V/μm and in the +ve x-direction

(c) $\frac{10}{9}$ V/μm and in the –ve x-direction

(d) $\frac{10}{9}$ V/μm and in the +ve x-direction

Exp. **(d)**

$$E = -\frac{\partial V}{\partial x}\mathbf{i} - \frac{\partial V}{\partial y}\mathbf{j} - \frac{\partial V}{\partial z}\mathbf{k}$$

$$\Rightarrow \quad E_x = -\frac{\partial V}{\partial x} = -\frac{d}{dx}\left[\frac{20}{x^2 - 4}\right]$$

$$= \frac{40x}{(x^2 - 4)^2}$$

$$\Rightarrow \quad E_x \text{ at } x = 4\,\mu\text{m} = \frac{10}{9}\ \text{V/}\mu\text{m}$$

and is along positive x-direction.

78. Charges are placed on the vertices of a square as shown. Let **E** be the electric field and V be the potential at the centre. If the charges on A and B are interchanged with those on D and C respectively, then **[AIEEE 2007]**

(a) **E** remains unchanged, V changes

(b) both **E** and V change

(c) **E** and V remain unchanged

(d) **E** changes, V remains unchanged

Exp. **(d)**

Direction of **E** reverses while magnitude remains same and V remains unchanged.

79. An electric charge 10^{-3} μC is placed at the origin (0, 0) of xy-coordinate system. Two points A and B are situated at $(\sqrt{2}, \sqrt{2})$ and (2, 0) respectively. The potential difference between the points A and B will be **[AIEEE 2007]**

(a) 9 V (b) zero (c) 2 V (d) 4.5 V

Exp. **(b)**

Potential at A due to charge at O,

$$V_A = \frac{1}{4\pi\varepsilon_0}\frac{(10^{-3})}{OA} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{\sqrt{(\sqrt{2})^2+(\sqrt{2})^2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{2}$$

Potential at B due to charge at O,

$$V_B = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{OB} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{(10^{-3})}{2}$$

So, $V_A - V_B = 0$

80. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then reinserted. The net work done by the system in this process is **[AIEEE 2007]**

(a) $\frac{1}{2}(K-1)CV^2$ (b) $CV^2(K-1)/K$

(c) $(K-1)CV^2$ (d) zero

Exp. **(d)**

On introduction and removal and again on introduction, the capacity and potential remain same. So, net work done by the system in this process,

$$W = U_f - U_i$$
$$= \frac{1}{2}CV^2 - \frac{1}{2}CV^2 = 0$$

81. A battery is used to charge a parallel plate capacitor till the potential difference between the plates becomes equal to the electromotive force of the battery. The ratio of the energy stored in the capacitor and the work done by the battery will be **[AIEEE 2007]**

(a) 1 (b) 2 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

Exp. **(d)**

Ratio of energy stored in the capacitor and the work done by the battery $= \dfrac{\frac{1}{2}qV}{qV} = \dfrac{1}{2}$

82. Two spherical conductors A and B of radii 1 mm and 2 mm are separated by a distance of 5 cm and are uniformly charged. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitude of the electric fields at the surfaces of spheres A and B is **[AIEEE 2006]**

(a) 4 : 1 (b) 1 : 2

(c) 2 : 1 (d) 1 : 4

Exp. **(c)**

When the two conducting spheres are connected by a conducting wire, charge will flow from one sphere (having higher potential) to other (having lower potential) till both acquire the same potential.

Therefore, $E = \dfrac{V}{r} \Rightarrow \dfrac{E_1}{E_2} = \dfrac{r_2}{r_1} = \dfrac{2}{1} = 2:1$

83. Two insulating plates are both uniformly charged in such a way that the potential difference between them is $V_2 - V_1 = 20$ V (*i.e.*, plate 2 is at a higher potential). The plates are separated by $d = 0.1$ m and can be treated as infinitely large. An electron is released from rest on the inner surface of plate 1. What is its speed when it hits plate 2?

$(e = 1.6\times10^{-19}\text{ C}, m_0 = 9.11\times10^{-31}\text{ kg})$

[AIEEE 2006]

(a) 2.65×10^6 ms^{-1} (b) 7.02×10^{12} ms^{-1}

(c) 1.87×10^6 ms^{-1} (d) 32×10^{-19} ms^{-1}

Exp. **(a)**

Since, $V_2 > V_1$, so electric field will point from plate 2 to plate 1.

The electron will experience an electric force, opposite to the direction of electric field and hence move towards the plate 2.

Use work-energy theorem to find speed of electron when it strikes the plate 2.

$$\therefore \quad \frac{m_e v^2}{2} - 0 = e(V_2 - V_1)$$

where, v is the required speed.

$$\Rightarrow \quad \frac{9.11 \times 10^{-31}}{2} v^2 = 1.6 \times 10^{-19} \times 20$$

or $$v = \sqrt{\frac{1.6 \times 10^{-19} \times 40}{9.11 \times 10^{-31}}}$$

$$= 2.65 \times 10^6 \text{ ms}^{-1}$$

84. An electric dipole is placed at an angle of 30° to a non-uniform electric field. The dipole will experience **[AIEEE 2006]**

(a) a translational force only in the direction of the field

(b) a translational force only in a direction normal to the direction of the field

(c) a torque as well as a translational force

(d) a torque only

Exp. **(c)**

In a non-uniform electric field, the dipole may experience both non-zero torque as well as translational force.

For example, as shown in figure

$F_1 \neq F_2$ as E is non-uniform.

Torque would also be non-zero.

85. A charged ball B hangs from a silk thread S, which makes an angle θ with a large charged conducting sheet P, as shown in the figure. The surface charge density σ of the sheet is proportional to **[AIEEE 2005]**

(a) $\cos\theta$ (b) $\cot\theta$ (c) $\sin\theta$ (d) $\tan\theta$

Exp. **(d)**

Electric field due to a charged conducting sheet of surface charge density σ is given by $E = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$.

where, ε_0 is the permittivity in vacuum and ε_r is the relative permittivity of medium.

Here, electrostatic force on B,

$$QE = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r}$$

FBD of B is shown in figure.

In equilibrium, $T \cos\theta = mg$

and $$T \sin\theta = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r}$$

Thus, $$\tan\theta = \frac{Q\sigma}{\varepsilon_0 \varepsilon_r mg} \Rightarrow \tan\theta \propto \sigma$$

86. Two point charges $+8q$ and $-2q$ are located at $x = 0$ and $x = L$, respectively. The location of a point on the x-axis at which the net electric field due to these two point charges is zero, is **[AIEEE 2005]**

(a) $2L$ (b) $\frac{L}{4}$ (c) $8L$ (d) $4L$

Exp. **(a)**

Suppose that a point B, where net electric field is zero due to charges $8q$ and $-2q$.

$$\mathbf{E}_{BO} = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{8q}{a^2}\,\mathbf{i}$$

$$\mathbf{E}_{BA} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{+2q}{(a+L)^2}\,\mathbf{i}$$

According to condition,

$$\mathbf{E}_{BO} + \mathbf{E}_{BA} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{8q}{a^2} = \frac{1}{4\pi\varepsilon_0}\frac{2q}{(a+L)^2}$$

or $$\frac{2}{a} = \frac{1}{a+L}$$

or $$2a + 2L = a$$

or $$2L = -a$$

Thus, at distance $2L$ from origin, net electric field will be zero.

87. Two thin wire rings each having a radius R are placed at a distance d apart with their axes coinciding. The charges on the two rings are $+q$ and $-q$. The potential difference between the centres of the two rings is **[AIEEE 2005]**

(a) $\frac{qR}{4\pi\varepsilon_0 d^2}$ (b) $\frac{q}{2\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

(c) zero (d) $\frac{q}{4\pi\varepsilon_0}\left[\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right]$

Exp. **(b)**

V_A = Potential due to charge $+q$ on ring A
+ Potential due to charge $-q$ on ring B

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R} - \frac{q}{d_1}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right)$$

$[\because d_1 = \sqrt{R^2+d^2}]$ ...(i)

Similarly, $V_B = \frac{1}{4\pi\varepsilon_0}\left(-\frac{q}{R} + \frac{q}{\sqrt{R^2+d^2}}\right)$

Potential difference, $V_A - V_B$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{R} - \frac{q}{\sqrt{R^2+d^2}}\right) - \frac{1}{4\pi\varepsilon_0}\left(\frac{-q}{R} + \frac{q}{\sqrt{R^2+d^2}}\right)$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{R} + \frac{1}{4\pi\varepsilon_0}\frac{q}{R} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2+d^2}} - \frac{1}{4\pi\varepsilon_0}\frac{q}{\sqrt{R^2+d^2}}$$

$$= \frac{q}{2\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{\sqrt{R^2+d^2}}\right)$$

88. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is **[AIEEE 2005]**

(a) $(n-1)C$ (b) $(n+1)C$

(c) C (d) nC

Exp. **(a)**

Each plate is taking part in the formation of two capacitors except the plates at the ends.

These capacitors are in parallel and n plates form $(n-1)$ capacitors.

Thus, equivalent capacitance between A and $B = (n-1)C$

89. A fully charged capacitor has a capacitance C. It is discharged through a small coil of resistance wire embedded in a thermally insulated block of specific heat capacity s and mass m. If the temperature of the block is raised by ΔT, the potential difference V across the capacitance is **[AIEEE 2005]**

(a) $\sqrt{\frac{2mC\Delta T}{s}}$ (b) $\frac{mC\Delta T}{s}$

(c) $\frac{ms\Delta T}{C}$ (d) $\sqrt{\frac{2ms\Delta T}{C}}$

Exp. **(d)**

$$E = \left(\frac{1}{2}\right) CV^2 \quad \ldots(i)$$

The energy stored in capacitor is lost in form of heat energy.

$$\therefore \quad H = ms\,\Delta T \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$ms\,\Delta T = \left(\frac{1}{2}\right) CV^2 \quad \text{or} \quad V = \sqrt{\frac{2ms\,\Delta T}{C}}$$

90. Two spherical conductors B and C having equal radii and carrying equal charges in them repel each other with a force F when kept apart at some distance. A third spherical conductor having same radius as that of B but uncharged, is brought in contact with B, then brought in contact with C and finally removed away from both. The new force of repulsion between B and C is **[AIEEE 2004]**

(a) $\frac{F}{4}$ (b) $\frac{3F}{4}$ (c) $\frac{F}{8}$ (d) $\frac{3F}{8}$

Exp. **(d)**

Let the spherical conductors B and C have same charge as q. The electric force between them is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}$$

Here, r being the distance between them.

When third uncharged conductor A is brought in contact with B, then charge on each conductor

$$q_A = q_B = \frac{q_A + q_B}{2} = \frac{0+q}{2} = \frac{q}{2}$$

When this conductor A is now brought in contact with C, then charge on each conductor

$$q_A = q_C = \frac{q_A + q_C}{2} = \frac{(q/2)+q}{2} = \frac{3q}{4}$$

Hence, electric force acting between B and C is

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{q_B q_C}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{(q/2)(3q/4)}{r^2}$$

$$= \frac{3}{8}\left[\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2}\right] = \frac{3F}{8}$$

91. A charged particle q is shot towards another charged particle Q which is fixed with a speed v. It approaches Q upto a closest distance r and then returns. If q was given a speed $2v$, the closest distance of approach would be

(a) r (b) $2r$ **[AIEEE 2004]**

(c) $\frac{r}{2}$ (d) $\frac{r}{4}$

Exp. **(d)**

Let a particle of charge q having velocity v approaches Q upto a closest distance r and if the velocity becomes $2v$, the closest distance will be r'.

The law of conservation of energy yields, kinetic energy of particle = electric potential energy between them at closest distance of approach.

or $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r}$$

or $$\frac{1}{2}mv^2 = \frac{kQq}{r} \quad \ldots(i)$$

$$\left[\because k = \text{constant} = \frac{1}{4\pi\varepsilon_0}\right]$$

and $$\frac{1}{2}m(2v)^2 = \frac{kQq}{r'} \quad \ldots(ii)$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{\frac{1}{2}mv^2}{\frac{1}{2}m(2v)^2} = \frac{\frac{kQq}{r}}{\frac{kQq}{r'}}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{r'}{r}$$

$$\Rightarrow \quad r' = \frac{r}{4}$$

92. Four charges equal to $-Q$ are placed at the four corners of a square and a charge q is at its centre. If the system is in equilibrium, the value of q is **[AIEEE 2004]**

(a) $-\frac{Q}{4}(1+2\sqrt{2})$ (b) $\frac{Q}{4}(1+2\sqrt{2})$

(c) $-\frac{Q}{2}(1+2\sqrt{2})$ (d) $\frac{Q}{2}(1+2\sqrt{2})$

Exp. **(b)**

The system is in equilibrium means the force experienced by each charge is zero. It is clear that charge placed at centre would be in equilibrium for any value of q, so we are considering the equilibrium of charge placed at any corner.

$$F_{CD} + F_{CA}\cos 45° + F_{CO}\cos 45° = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-Q)(-Q)}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)(-Q)}{(\sqrt{2}\,a)^2}\times\frac{1}{\sqrt{2}} + \frac{1}{4\pi\varepsilon_0}\frac{(-Q)q}{(\sqrt{2}\,a/2)^2}\times\frac{1}{\sqrt{2}} = 0$$

or $$\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{a^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{2a^2}\cdot\frac{1}{\sqrt{2}} - \frac{1}{4\pi\varepsilon_0}\frac{2Qq}{a^2}\times\frac{1}{\sqrt{2}} = 0$$

or $$Q + \frac{Q}{2\sqrt{2}} - \sqrt{2}q = 0$$

or $$2\sqrt{2}Q + Q - 4q = 0$$

or $$4q = (2\sqrt{2}+1)Q$$

or $$q = (2\sqrt{2}+1)\frac{Q}{4}$$

93. If the electric flux entering and leaving an enclosed surface respectively is ϕ_1 and ϕ_2, the electric charge inside the surface will be **[AIEEE 2003]**

(a) $(\phi_2 - \phi_1)\varepsilon_0$ (b) $\dfrac{(\phi_1 + \phi_2)}{\varepsilon_0}$

(c) $\dfrac{(\phi_2 - \phi_1)}{\varepsilon_0}$ (d) $(\phi_1 + \phi_2)\varepsilon_0$

Exp. **(a)**

From Gauss' law, $\dfrac{\text{Charge enclosed}}{\varepsilon_0} = \text{Net flux}$

$$\Rightarrow \frac{q}{\varepsilon_0} = \phi_2 - \phi_1 \text{ or } q = (\phi_2 - \phi_1)\varepsilon_0$$

94. A sheet of aluminium foil of negligible thickness is introduced between the plates of a capacitor. The capacitance of the capacitor **[AIEEE 2003]**

(a) decreases
(b) remains unchanged
(c) becomes infinite
(d) increases

Exp. **(b)**

Aluminium is a metal, so when we insert an aluminium foil, equal and opposite charges appear on its two surfaces. Since, it is of negligible thickness, it will not affect the capacitance.

Alternate Solution

From the formula,

$$C = \frac{\varepsilon_0 A}{d - t + \dfrac{t}{K}}$$

Here, $K = \infty$ and $t \to 0$

So, $$C = \frac{\varepsilon_0 A}{d + 0} = \frac{\varepsilon_0 A}{d} = C_0$$

95. A thin spherical conducting shell of radius R has a charge q. Another charge Q is placed at the centre of the shell. The electrostatic potential at a point P at a distance $R/2$ from the centre of the shell is **[AIEEE 2003]**

(a) $\dfrac{2Q}{4\pi\varepsilon_0 R}$ (b) $\dfrac{2Q}{4\pi\varepsilon_0 R} - \dfrac{2q}{4\pi\varepsilon_0 R}$

(c) $\dfrac{2Q}{4\pi\varepsilon_0 R} + \dfrac{q}{4\pi\varepsilon_0 R}$ (d) $\dfrac{(q+Q)}{4\pi\varepsilon_0}\dfrac{2}{R}$

Exp. **(c)**

At P due to shell, potential

$$V_1 = \frac{q}{4\pi\varepsilon_0 R}$$

At P due to Q, potential

$$V_2 = \frac{Q}{4\pi\varepsilon_0 \dfrac{R}{2}} = \frac{2Q}{4\pi\varepsilon_0 R}$$

$\therefore$ Net potential at P,

$$V = V_1 + V_2 = \frac{q}{4\pi\varepsilon_0 R} + \frac{2Q}{4\pi\varepsilon_0 R}$$

96. The work done in placing a charge of 8×10^{-18} C on a condenser of capacity $100\,\mu$F is **[AIEEE 2003]**

(a) 16×10^{-32} J (b) 3.1×10^{-26} J

(c) 4×10^{-10} J (d) 32×10^{-32} J

Exp. **(*d*)**

$$W = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2} \times \frac{(8 \times 10^{-18})^2}{100 \times 10^{-6}}$$

$$= \frac{1}{2} \times \frac{64 \times 10^{-36}}{100 \times 10^{-6}} = 32 \times 10^{-32} \text{ J}$$

97. Three charges $-q_1, +q_2$ and $-q_3$ are placed as shown in the figure. The x-component of the force on $-q_1$ is proportional to **[AIEEE 2003]**

(a) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\cos\theta$ (b) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta$

(c) $\frac{q_2}{b^2} + \frac{q_3}{a^2}\cos\theta$ (d) $\frac{q_2}{b^2} - \frac{q_3}{a^2}\sin\theta$

Exp. **(*b*)**

Force on $-q_1$,

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{b^2}\mathbf{i} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{a^2}[\sin\theta\mathbf{i} - \cos\theta\mathbf{j}]$$

From above, x' component of force is

$$F_x = \frac{q_1}{4\pi\varepsilon_0}\left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

$$F_x \propto \left[\frac{q_2}{b^2} + \frac{q_3}{a^2}\sin\theta\right]$$

98. A charged particle q is placed at the centre O of cube of length L ($ABCDEFGH$). Another same charge q is placed at a distance L from O. Then, the electric flux through $ABCD$ is **[AIEEE 2002]**

(a) $\frac{q}{4\pi\varepsilon_0 L}$ (b) $\frac{q}{2\pi\varepsilon_0 L}$

(c) $\frac{q}{2\pi\varepsilon_0 L}$ (d) None of these

Exp. **(*d*)**

Electric flux for any surface is defined as

$$\phi = \int \mathbf{E} \cdot d\mathbf{s}$$

As flux on the cube $= \frac{q}{\varepsilon_0}$

Flux on each face $= \frac{1}{6}\frac{q}{\varepsilon_0}$

So, flux on the face $ABCD = \frac{1}{6}\frac{q}{\varepsilon_0}$

The options (a), (c) and (d) are dimensionally incorrect, so they cannot be answers.

99. If there are n capacitors in parallel connected to V volt source, then the energy stored is equal to **[AIEEE 2002]**

(a) CV (b) $\frac{1}{2}nCV^2$

(c) CV^2 (d) $\frac{1}{2n}CV^2$

Exp. **(*b*)**

Energy stored by any system of capacitors

$$= \frac{1}{2}C_{net}V^2$$

where, V is source voltage.

Thus, n capacitors are connected in parallel.

So, $C_{net} = nC$

$\therefore$ $E_{net} = \frac{1}{2}nCV^2$

100. If a charge q is placed at the centre of the line joining two equal charges Q such that the system is in equilibrium, then the value of q is **[AIEEE 2002]**

(a) $\frac{Q}{2}$ (b) $-\frac{Q}{2}$

(c) $\frac{Q}{4}$ (d) $-\frac{Q}{4}$

Exp. **(d)**

Let charge q be placed at mid-point of line AB as shown below.

Also, $AB = x$ [say]

$\therefore$ $AC = \frac{x}{2}$,

$BC = \frac{x}{2}$

For the system to be in equilibrium,

$$F_{Qq} + F_{QQ} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\frac{Qq}{(x/2)^2} + \frac{1}{4\pi\varepsilon_0}\frac{QQ}{x^2} = 0$$

or $$q = -\frac{Q}{4}$$

101. Capacitance (in F) of a spherical conductor having radius 1 m, is **[AIEEE 2002]**

(a) 1.1×10^{-10} (b) 10^{-6}

(c) 9×10^{-9} (d) 10^{-3}

Exp. **(a)**

Capacitance of spherical conductor $= 4\pi\varepsilon_0 a$

where, a is radius of conductor.

Therefore, $C = \frac{1}{9 \times 10^9} \times 1$

$$= \frac{1}{9} \times 10^{-9} = 0.11 \times 10^{-9}\ \text{F} = 1.1 \times 10^{-10}\ \text{F}$$

102. On moving a charge of 20 C by 2 cm, 2 J of work is done, then the potential difference between the points is **[AIEEE 2002]**

(a) 0.1 V (b) 8 V (c) 2 V (d) 0.5 V

Exp. **(a)**

Potential difference between two points in an electric field is $V_A - V_B = \frac{W}{q_0}$

where, W is work done by moving charge q_0 from point A to B.

Here, $W = 2$ J, $q_0 = 20$ C

So, $V_A - V_B = \frac{2}{20} = 0.1$ V

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12

Current Electricity

1. A 200 Ω resistor has a certain colour code. If one replaces the red colour by green in the code, the new resistance will be

(a) 100 Ω (b) 400 Ω (c) 300 Ω (d) 500 Ω

[JEE Main 2019, 8 April Shift-I]

Exp. **(*d*)**

Given, resistance is 200 Ω.= 20×10^1 Ω

So, colour scheme will be red, black and brown.

Significant figure of red band is 2 and for green is 5. When red (2) is replaced with green (5), new resistance will be

200 ohm ⟶ 500 ohm.

2. For the circuit shown with $R_1 = 1.0\,\Omega$, $R_2 = 2.0\Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points *a* and *b* is approximately (in volt)

[JEE Main 2019, 8 April Shift-I]

(a) 2.7 (b) 2.3
(c) 3.7 (d) 3.3

Exp. **(*d*)**

Given circuit is

Above circuit can be viewed as

This is a parallel combination of three cells or in other words, a parallel grouping of three cells with internal resistances.

$$\text{So, } V_{ab} = E_{eq} = \frac{I_{eq}}{r_{eq}} = \frac{\dfrac{E_1}{r_1} + \dfrac{E_2}{r_2} + \dfrac{E_3}{r_3}}{\dfrac{1}{r_1} + \dfrac{1}{r_2} + \dfrac{1}{r_3}}$$

$$= \frac{\dfrac{2}{2} + \dfrac{4}{2} + \dfrac{4}{2}}{\dfrac{1}{2} + \dfrac{1}{2} + \dfrac{1}{2}} = \frac{10}{3}\text{ V} \approx 3.3\text{ V}$$

3. In the figure shown, what is the current (in ampere) drawn from the battery? You are given :

$R_1 = 15\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 20\,\Omega$, $R_4 = 5\,\Omega$, $R_5 = 25\,\Omega$, $R_6 = 30\,\Omega$, $E = 15$ V

[JEE Main 2019, 8 April Shift-II]

(a) 13/24 (b) 7/18
(c) 20/3 (d) 9/32

Exp. **(d)**

Given circuit is redrawn and can be simplified as

In series,
$R_{eq} = R_3+R_4+R_5$
$= 20+5+25$
$= 50\Omega$

$\Downarrow$

In parallel,

$$R'_{eq} = \frac{R_{eq}\times R_2}{R_{eq}+R_2} = \frac{50\times 10}{50+10} = \frac{50}{6}\ \Omega$$

$\Downarrow$

In series,

$$R''_{eq} = R_1+R'_{eq}+R_6 = 30 + \frac{50}{6} + 15 = \frac{320}{6} = \frac{160}{3}\Omega$$

So, current drawn through cell is

$$i = \frac{\text{Voltage}}{\text{Net resistance of the circuit}} = \frac{V}{R''_{eq}} = \frac{15}{(160/3)} = \frac{9}{32}\text{A}$$

4. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when

[JEE Main 2019, 8 April Shift-II]

(a) $R = 2r$ (b) $R = r$
(c) $R = 0.001\,r$ (d) $R = 1000\,r$

Exp. **(b)**

Given circuit is shown in the figure below

Net current, $I = \dfrac{E}{R+r}$ ...(i)

Power across R is given as

$$P = I^2R = \left(\frac{E}{R+r}\right)^2 \cdot R \quad \text{[using Eq. (i)]}$$

For the maximum power,

$$\frac{dP}{dR} = 0$$

$$\Rightarrow \quad \frac{dP}{dR} = \frac{d}{dR}\left(\left(\frac{E}{R+r}\right)^2 \cdot R\right)$$

$$= E^2\frac{d}{dR}\left(\frac{R}{(R+r)^2}\right)$$

$$= E^2\left[\frac{(R+r)^2 \times 1 - 2R\times(R+r)}{(R+r)^4}\right] = 0$$

$$\Rightarrow \quad (R+r)^2 = 2R(R+r)$$

or $R + r = 2R$

$\Rightarrow$ $r = R$

$\therefore$ The power delivered by the cell to the external resistance is maximum when $R = r$.

Alternate Solution

From maximum power theorem, power dissipated will be maximum when internal resistance of source will be equals to external load resistance, i.e. $r = R$.

5. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is $r = 0.01\ \Omega/\text{cm}$. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be

[JEE Main 2019, 8 April Shift-II]

(a) 0.20 V (b) 0.75 V
(c) 0.25 V (d) 0.50 V

Exp. **(c)**

In given potentiometer, resistance per unit length is $x = 0.01\ \Omega\text{cm}^{-1}$.

Length of potentiometer wire is $L = 400$ cm

Net resistance of the wire AB is

R_{AB} = resistance per unit length × length of AB

$= 0.01 \times 400$

$\Rightarrow \quad R_{AB} = 4\ \Omega$

Net internal resistance of the cells connected in series,

$r = 0.5 + 0.5 = 1\Omega$

∴ Current in given potentiometer circuit is

$$I = \frac{\text{Net emf}}{\text{Total resistance}} = \frac{\text{Net emf}}{r + R + R_{AB}}$$

$$= \frac{3}{1 + 1 + 4} = 0.5\text{ A}$$

Reading of voltmeter when the jockey is at 50 cm (l') from one end A,

$V = IR = I(xl')$

$= 0.5 \times 0.01 \times 50 = 0.25$ V

6. A wire of resistance R is bent to form a square $ABCD$ as shown in the figure. The effective resistance between E and C is [E is mid-point of arm CD]

[JEE Main 2019, 9 April Shift-I]

(a) $\frac{7}{64}R$ (b) $\frac{3}{4}R$
(c) R (d) $\frac{1}{16}R$

Exp. **(a)**

Let the length of each side of square $ABCD$ is a.

∴ Resistance per unit length of each side $= \frac{R}{4a}$

Now, $\quad R_1 = \frac{R}{4a}[EC] = \frac{R}{4a} \times \frac{a}{2} = \frac{R}{8}$

Similarly, $R_2 = \frac{R}{4a}[EDABC] = \frac{R}{4a} \times \frac{7a}{2} = \frac{7R}{8}$

Now, effective resistance between E and C is the equivalent resistance of R_1 and R_2 that are connected in parallel as shown below.

$$R_{EC} = \frac{R_1R_2}{R_1 + R_2}$$

$$= \frac{(R/8) \times (7R/8)}{(R/8) + (7R/8)}$$

$$= \frac{7R^2}{64} \times \frac{8}{8R} = \frac{7}{64}R$$

7. Determine the charge on the capacitor in the following circuit

[JEE Main 2019, 9 April Shift-I]

(a) 2 μC (b) 200μC
(c) 10 μC (d) 60 μC

Exp. **(b)**

Given circuit is

To find charge on capacitor, we need to determine voltage across it. In steady state, capacitor will acts as open circuit and circuit can be reduced as

In series, $R_{eq} = 2\,\Omega + 10\,\Omega = 12\,\Omega$

$\Downarrow$

In parallel, $R_{eq} = \dfrac{4 \times 12}{4 + 12} = 3\,\Omega$

In series, $R_{eq} = 6\,\Omega + 3\,\Omega = 9\,\Omega$

So, current in steady state, $I = \dfrac{V}{R} = \dfrac{72}{9} = 8\text{A}$

Now, by using current division, at point P, current in 6 Ω branch is

$$\frac{72 - V_P}{6\,\Omega} = 8\text{A}$$

$$\Rightarrow \qquad V_P = 72 - 48 = 24\text{ V}$$

Current in 4 Ω branch is,

$$i_2 = \frac{V_P - 0}{4}$$

$$= \frac{24 - 0}{4} = 6\,\Omega$$

So, current in 2 Ω resistance is

$$I_1 = 8 - I_2 \qquad [\because I = I_1 + I_2]$$

$$= 8 - 6 = 2\text{A}$$

∴ Potential difference across 10 Ω resistor is

$$V_{QG} = 2\text{A} \times 10\,\Omega = 20\text{ V}$$

Same potential difference will be applicable over the capacitor (parallel combination).

So, charge stored in the capacitor will be

$$Q = CV = 10 \times 10^{-6} \times 20$$

$$\Rightarrow \qquad Q = 2 \times 10^{-4}\text{ C} = 200\,\mu\text{C}$$

8. In a conductor, if the number of conduction electrons per unit volume is 8.5×10^{28} m^{-3} and mean free time is 25 fs (femto second), it's approximate resistivity is (Take, $m_e = 9.1 \times 10^{-31}$ kg)

[JEE Main 2019, 9 April Shift-II]

(a) 10^{-7} Ω-m (b) 10^{-5} Ω-m
(c) 10^{-6} Ω-m (d) 10^{-8} Ω-m

Exp. **(d)**

Resistivity of a conductor is

$$\rho = \frac{m_e}{ne^2\tau} \qquad \ldots \text{(i)}$$

where, m_e = mass of electron = 9.1×10^{-31} kg,

n = free charge density = 8.5×10^{28} m^{-3},

τ = mean free time = 25 fs = 25×10^{-15} s

and e = charge of electron = 1.6×10^{-19} C

Substituting values in Eq. (i), we get

$$\rho = \frac{9.1 \times 10^{-31}}{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 25 \times 10^{-15}}$$

$$= \frac{9.1 \times 10^{-6}}{8.5 \times 2.56 \times 25} = 0.016 \times 10^{-6}$$

$$= 1.6 \times 10^{-8}\ \Omega\text{-m} \simeq 10^{-8}\ \Omega\text{-m}$$

9. A metal wire of resistance 3 Ω is elongated to make a uniform wire of double its previous length. This new wire is now bent and the ends joined to make a circle. If two points on this circle make an angle 60° at the centre, the equivalent resistance between these two points will be **[JEE Main 2019, 9 April Shift-II]**

(a) $\frac{7}{2}\Omega$ (b) $\frac{5}{2}\Omega$

(c) $\frac{12}{5}\Omega$ (d) $\frac{5}{3}\Omega$

Exp. **(d)**

Initial resistance of wire is 3 Ω. Let its length is l and area is A.

Then, $R_{\text{initial}} = \rho\frac{l}{A} = 3\ \Omega$ … (i)

When wire is stretched twice its length, then its area becomes A', on equating volume, we have

$$Al = A'2l \Rightarrow A' = \frac{A}{2}$$

So, after stretching, resistance of wire will be

$R' = R_{\text{final}} = \rho\frac{l'}{A'} = 4\rho\frac{l}{A} = 12\ \Omega$ [using Eq. (i)]

Now, this wire is made into a circle and connected across two points A and B (making 60° angle at centre) as

Now, above arrangement is a combination of two resistances in parallel,

$$R_1 = \frac{60 \times R'}{360}$$

$$= \frac{1}{6} \times 12 = 2\ \Omega$$

and $$R_2 = \frac{300}{360} \times R'$$

$$= \frac{5}{6} \times 12 = 10\ \Omega$$

Since, R_1 and R_2 are connected in parallel.

So, $$R_{AB} = \frac{R_1R_2}{R_1 + R_2} = \frac{10 \times 2}{12} = \frac{5}{3}\ \Omega$$

10. In an experiment, the resistance of a material is plotted as a function of temperature (in some range). As shown in the figure, it is a straight line.

[JEE Main 2019, 10 April Shift-I]

One may conclude that

(a) $R(T) = R_0 e^{-T^2/T_0^2}$ (b) $R(T) = R_0 e^{T^2/T_0^2}$

(c) $R(T) = R_0 e^{-T_0^2/T^2}$ (d) $R(T) = \frac{R_0}{T^2}$

Exp. **(c)**

From the given graph,

We can say that, $\ln R(T) \propto -\frac{1}{T^2}$

Negative sign implies that the slope of the graph is negative.

or $\ln R(T) = \text{constant}\left(-\frac{1}{T^2}\right)$

$$\Rightarrow \quad R(T) = \frac{\exp(\text{const.})}{\exp\left(\frac{1}{T^2}\right)}$$

$$\Rightarrow \quad R(T) = R_0 \exp\left(-\frac{T_0^2}{T^2}\right)$$

Alternate Solution

From graph, $$\frac{\frac{1}{T^2}}{\frac{1}{T_0^2}} + \frac{\ln R(T)}{\ln R(T_0)} = 1$$

$$\Rightarrow \quad \ln R(T) = [\ln R(T_0)] \cdot \left[1 - \frac{T_0^2}{T^2}\right]$$

or $$R(T) = R_0 \exp\left(\frac{-T_0^2}{T^2}\right)$$

11. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure

[JEE Main 2019, 10 April Shift-I]

S. No.	R (Ω)	l (cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

(a) 3 (b) 2 (c) 1 (d) 4

Exp. **(d)**

Unknown resistance 'X' in meter bridge experiment is given by

$$X = \left(\frac{100 - l}{l}\right) R$$

Case (1) When $R = 1000\,\Omega$ and $l = 60$ cm, then

$$X = \frac{(100 - 60)}{60} \times 1000 = \frac{40 \times 1000}{60}$$

$$\Rightarrow \quad X = \frac{2000}{3}\,\Omega \approx 667\,\Omega$$

Case (2) When $R = 100\,\Omega$ and $l = 13$ cm, then

$$X = \left(\frac{100 - 13}{13}\right) \times 100 = \frac{100 \times 87}{13}$$

$$= \frac{8700}{13}\,\Omega \approx 669\,\Omega$$

Case (3) When $R = 10\,\Omega$ and $l = 1.5$ cm, then

$$X = \left(\frac{100 - 1.5}{1.5}\right) \times 10$$

$$= \frac{98.5}{1.5} \times 10 = \frac{9850}{15}\,\Omega \approx 656\,\Omega$$

Case (4) When $R = 1\,\Omega$ and $l = 1.0$ cm, then

$$X = \left(\frac{100 - 1}{1}\right) \times 1$$

$$\therefore \quad X = 99\,\Omega$$

Thus, from the above cases, it can be concluded that, value calculated in case (4) is inconsistent.

12. In the given circuit, an ideal voltmeter connected across the 10 Ω resistance reads 2 V. The internal resistance r, of each cell is

[JEE Main 2019, 10 April Shift-I]

(a) 1.5 Ω (b) 0.5 Ω (c) 1 Ω (d) 0 Ω

Exp. **(b)**

For the given circuit

Given, $V_{AB} = 2\text{V}$

∴ Current in circuit,

$$I = I_1 + I_2$$

$$= \frac{2}{15} + \frac{2}{10} \qquad [\because V = IR \text{ or } I = V/R]$$

$$= \frac{4 + 6}{30} = \frac{1}{3}\text{A} \qquad \ldots \text{(i)}$$

Also, voltage drop across $(r + r)$ resistors is

= voltage of the cell – voltage drop across AB

$$= 3 - 2 = 1\text{V}$$

Using $V = IR$ over the entire circuit

$$\Rightarrow \quad 1 = I(2 + 2r)$$

$$= \frac{1}{3}(2 + 2r) \qquad \text{[using Eq. (i)]}$$

$$\Rightarrow \quad 3 = 2 + 2r \quad \text{or} \quad 2r = 1\,\Omega$$

$$\text{or} \quad r = \frac{1}{2}\,\Omega = 0.5\,\Omega$$

Alternate Solution

Equivalent resistance between A and B is

$$\frac{1}{R_{AB}} = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$$

$$\Rightarrow \quad R_{AB} = 6\,\Omega$$

$\therefore$ Equivalent resistance of the entire circuit is, R_{eq} $= 6\,\Omega + 2\,\Omega + 2r = 8 + 2r$

Now, current passing through the circuit is given as, $I = \dfrac{E_{net}}{R + r_{eq}} = \dfrac{E_{net}}{R_{eq}}$

where, R is external resistance, r_{eq} is net internal resistance and E_{net} is the emf of the cells.

Here, $E_{net} = 1.5 + 1.5 = 3\,V$

$r_{eq} = r + r = 2r \Rightarrow I = \dfrac{3}{8 + 2r}$

Also, reading of the voltmeter, $V = 2V = I \cdot R_{AB}$

$2 = \left(\dfrac{3}{8 + 2r}\right) \times 6 \Rightarrow 8 + 2r = 9$ or $r = \dfrac{1}{2} = 0.5\,\Omega$

13. Space between two concentric conducting spheres of radii a and $b\,(b > a)$ is filled with a medium of resistivity ρ. The resistance between the two spheres (in ohm) will be

[JEE Main 2019, 10 April Shift-II]

(a) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$ (b) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$

(c) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a} - \dfrac{1}{b}\right)$ (d) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a} + \dfrac{1}{b}\right)$

Exp. **(b)**

> **Key Idea** Resistance between surface of inner shell and a circumferential point of outer shell can be formed by finding resistance of a thin (differentially thin) shell in between these two shells. Then, this result can be integrated (summed up) to get resistance of the complete arrangement.

For a elemental shell of radius x and thickness dx,

Resistance, $dR = \rho\dfrac{l}{A}$

$\Rightarrow \quad dR = \rho\dfrac{dx}{4\pi x^2}$

So, resistance of complete arrangement is

$$R = \int_a^b dR = \int_a^b \rho\frac{dx}{4\pi x^2} = \frac{\rho}{4\pi}\int_a^b x^{-2}dx$$

$$\Rightarrow \quad R = \frac{\rho}{4\pi}\left(\frac{x^{-1}}{-1}\right)_a^b = \frac{\rho}{4\pi}\left(-\frac{1}{x}\right)_a^b$$

$$= \frac{\rho}{4\pi}\left(-\frac{1}{b} - \left(-\frac{1}{a}\right)\right) = \frac{\rho}{4\pi}\left(\frac{1}{a} - \frac{1}{b}\right)\text{ohm}$$

14. To verify Ohm's law, a student connects the voltmeter across the battery as shown in the figure. The measured voltage is plotted as a function of the current and the following graph is obtained

[JEE Main 2019, 12 April Shift-I]

If V_0 is almost zero, then identify the correct statement.

(a) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(b) The value of the resistance R is 1.5 Ω

(c) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA

(d) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

Exp. **(a)**

Given circuit in a series combination of internal resistance of cell (r) and external resistance R.

∴ Effective resistance in the circuit,

$$R_{eff} = r + R$$

∴ Current in the circuit,

$$I = \frac{E}{R + r} \text{ or } E = IR + Ir$$

Voltage difference across resistance R is V, so

$$E = V + Ir \quad \ldots(i)$$

Now, from graph at $I = 0$, $V = 1.5$ V

From Eq. (i) at $I = 0$, $E = V = 1.5$ V ...(ii)

At $I = 1000$ mA (or 1 A), $V = 0$

From Eq. (i) at $I = 1$ A and $V = 0$

$$\Rightarrow \quad E = I \times r = r \quad \ldots(iii)$$

From Eqs. (ii) and (iii), we can get

$$r = E = V = 1.5\text{V}$$

$$\therefore \quad r = 1.5\,\Omega$$

15. The resistive network shown below is connected to a DC source of 16 V. The power consumed by the network is 4 W. The value of R is **[JEE Main 2019, 12 April Shift-I]**

(a) $6\,\Omega$ (b) $8\,\Omega$
(c) $1\,\Omega$ (d) $16\,\Omega$

Exp. **(b)**

Given circuit is

Equivalent resistance of part A,

$$R_A = \frac{4R \times 4R}{4R + 4R} = 2R$$

Equivalent resistance of part B,

$$R_B = \frac{6R \times 12R}{6R + 12R}$$

$$= \frac{72}{18}R = 4R$$

∴ Equivalent circuit is

∴ Total resistance of the given network is

$$R_s = 2R + R + 4R + R = 8R$$

As we know, power of the circuit,

$$P = \frac{E^2}{R_s} = \frac{(16)^2}{8R} = \frac{16 \times 16}{8R} \quad \ldots(i)$$

According to question, power consumed by the network, $P = 4$ W

From Eq. (i), we get

$$\therefore \quad \frac{16 \times 16}{8R} = 4 \Rightarrow R = \frac{16 \times 16}{8 \times 4} = 8\Omega$$

16. Drift speed of electrons, when 1.5 A of current flows in a copper wire of cross-section 5 mm^2 is v. If the electron density in copper is $9 \times 10^{28}/m^3$, the value of v (in mm/s) is close to (Take, charge of electron to be $= 1.6 \times 10^{-19}$ C) **[JEE Main 2019, 9 Jan Shift-I]**

(a) 0.02 (b) 0.2 (c) 2 (d) 3

Exp. **(a)**

Relation between current (I) flowing through a conducting wire and drift velocity of electrons (v_d) is given as

$$I = neAv_d$$

where, n is the electron density and A is the area of cross-section of wire.

$$\Rightarrow \quad v_d = \frac{I}{neA}$$

Substituting the given values, we get

$$v = \frac{1.5}{9 \times 10^{28} \times 1.6 \times 10^{-19} \times 5 \times 10^{-6}}$$

$$v = \frac{1.5 \times 10^{-3}}{72} \text{ m/s} = 0.2 \times 10^{-4} \text{ m/s}$$

or $\quad v = 0.02$ mm/s

17. A copper wire is stretched to make it 0.5% longer. The percentage change in its electrical resistance, if its volume remains unchanged is **[JEE Main 2019, 9 Jan Shift-I]**

(a) 2.0% (b) 1.0% (c) 0.5% (d) 2.5%

Exp. **(b)**

Electrical resistance of wire of length 'l', area of cross-section 'A' and resistivity 'ρ' is given as

$$R = \rho\frac{l}{A} \quad \text{...(i)}$$

Since we know, volume of the wire is

$$V = A \times l \quad \text{...(ii)}$$

∴ From Eqs. (i) and (ii), we get

$$R = \rho\frac{l^2}{V} \quad \text{...(iii)}$$

As, the length has been increased to 0.5%.

∴ New length of the wire,

$$l' = l + 0.5\% \text{ of } l$$

$$= l + 0.005\, l = 1.005\, l$$

But V and ρ remains unchanged.

So, new resistance, $R' = \dfrac{\rho[(1.005)\, l]^2}{V}$ ...(iv)

Dividing Eq. (iv) and Eq. (iii), we get

$$\frac{R'}{R} = (1.005)^2$$

$$\Rightarrow \% \text{ change in the resistance} = \left(\frac{R'}{R} - 1\right) \times 100$$

$$= [(1.005)^2 - 1] \times 100 = 1.0025\% \simeq 1\%$$

18. A resistance is shown in the figure. Its value and tolerance are given respectively by **[JEE Main 2019, 9 Jan Shift-I]**

(a) 270 Ω, 5% (b) 27 k Ω, 20%
(c) 27 k Ω, 10% (d) 270 k Ω, 10%

Exp. **(c)**

The value of a carbon resistor is given as,

$$R = AB \times C \pm D\% \quad \text{... (i)}$$

where, color band A and B (first two colors) indicate the first two significant figures of resistance in ohms, C (third color) indicates the decimal multiplier and D (forth color) indicates the tolerance in % as per the indicated value.

The table for color code of carbon resistor is given below,

Color Codes	Values (A, B)	Multiplier (C)	Tolerance (D) (%)
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Grey	8	10^8	
White	9	10^9	
Gold			± 5%
Silver			± 10%
No colour			± 20%

∴ Comparing the colors given in the question and the table and writing in the manner of Eq. (i), we get

$$R = 27 \times 10^3\ \Omega, \pm 10\%$$

$$= 27\ \text{k}\Omega, \pm 10\%$$

19. When the switch S in the circuit shown is closed, then the value of current i will be **[JEE Main 2019, 9 Jan Shift-I]**

(a) 4A (b) 3A (c) 2A (d) 5A

Exp. **(d)**

When the switch 'S' is closed the circuit, hence formed is given in the figure below,

Then, according to Kirchhoff's current law, which states that the sum of all the currents directed towards a point in a circuit is equal to the sum of all the currents directed away from that point.

Since, in the above circuit, that point is 'C'

$\therefore \quad i_1 + i_2 = i$

$\Rightarrow \quad \dfrac{V_A - V_C}{2} + \dfrac{V_B - V_C}{4} = \dfrac{V_C - V}{2}$

($\because$ using Ohm's law, $V = iR$)

or $\quad \dfrac{20 - V_C}{2} + \dfrac{10 - V_C}{4} = \dfrac{V_C - 0}{2}$

$\Rightarrow \quad 20 - V_C + (10 - V_C)2 = V_C$

$40 = V_C + 3V_C$

$40 = 4V_C$

or $\quad V_C = 10$ V

$\therefore$ The current, $i = \dfrac{V_C}{2} = \dfrac{10}{2} = 5$A

20. In the given circuit, the internal resistance of the 18 V cell is negligible. If $R_1 = 400\,\Omega$, $R_3 = 100\,\Omega$ and $R_4 = 500\,\Omega$ and the reading of an ideal voltmeter across R_4 is 5 V, then the value of R_2 will be **[JEE Main 2019, 9 Jan Shift-II]**

(a) $550\,\Omega$ (b) $230\,\Omega$ (c) $300\,\Omega$ (d) $450\,\Omega$

Exp. **(c)**

According to question, the voltage across R_4 is 5 volt, then the current across it

According to Ohm's law,

$\Rightarrow \quad V = IR \quad \Rightarrow \quad 5 = I_1 \times R_4$

$\Rightarrow \quad 5 = I_1 \times 500$

$I_1 = \dfrac{5}{500} = \dfrac{1}{100}$ A

The potential difference across series combination of R_3 and R_4

$\Rightarrow \quad V_2 = (R_3 + R_4)I = 600 \times \dfrac{1}{100} = 6$ V

So, potential difference (across R_1)

$V_1 = 18 - 6 = 12$ V

Current through R_1 is,

$I = \dfrac{V_1}{R_1} = \dfrac{12}{400} = \dfrac{3}{100}$ A

So current through R_2 is,

$I_2 = I - I_1 = \dfrac{3}{100} - \dfrac{1}{100}$ A $= \dfrac{2}{100}$ A

Now, from $V = IR$, we have,

$R_2 = \dfrac{V_2}{I_2} = \dfrac{6}{(2/100)} = 300\Omega.$

21. A carbon resistance has a following color code. What is the value of the resistance? **[JEE Main 2019, 9 Jan Shift-II]**

(a) $5.3\,\text{M}\Omega \pm 5\%$ (b) $64\,\text{k}\Omega \pm 10\%$
(c) $6.4\,\text{M}\Omega \pm 5\%$ (d) $530\,\text{k}\Omega \pm 5\%$

Exp. **(d)**

The value of a carbon resistor is given as

$R = AB \times C \pm D\%$ ...(i)

where, colour band A and B (first two colour) indicates the first two significant figures of resistance in Ohms.

C (third colour) indicate the decimal multiplies and D (fourth colour) indicates the tolerance in % as per the indicated value.

The table for colour code,

Colour code	Values (AB)	Multiplier (C)	Tolerance
Black	0	10^0	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Voilet	7	10^7	
Grey	8	10^8	

White	9	10^9	
Gold	-	-	± 5%
Silver	-	-	± 10%
No colour	-	-	± 20%

∵ Comparing the colours given in the question and table and writing in the manner of Eq. (i), we get

$$R = 53 \times 10^4 \pm 5\% \ \Omega$$

or $$R = 530 \times 10^3 \ \Omega \pm 5\%$$

or $$R = 530 \text{ k}\Omega \pm 5\%$$

22. A potentiometer wire AB having length L and resistance $12r$ is joined to a cell D of EMF ε and internal resistance r. A cell C having emf $\frac{\varepsilon}{2}$ and internal resistance $3r$ is connected. The length AJ at which the galvanometer as shown in figure shows no deflection is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $\frac{5}{12}L$ (b) $\frac{11}{12}L$
(c) $\frac{13}{24}L$ (d) $\frac{11}{24}L$

Exp. **(c)**

Given, length of potentiometer wire (AB) = L

Resistance of potentiometer wire (AB) = 12 r

EMF of cell D of potentiometer = ε

Internal resistance of cell 'D' = r

EMF of cell 'C' = $\frac{\varepsilon}{2}$

Internal resistance of cell 'C' = $3r$

Current in potentiometer wire

$$i = \frac{\text{EMF of cell of potentiometer}}{\text{Total resistance of potentiometer circuit}}$$

$$\Rightarrow i = \frac{\varepsilon}{r + 12r} = \frac{\varepsilon}{13r}$$

Potential drop across the balance length AJ of potentiometer wire is

$$V_{AJ} = i \times R_{AJ}$$

$\Rightarrow V_{AJ} = i$ (Resistance per unit length of potentiometer wire × length AJ)

$$\Rightarrow \quad V_{AJ} = i\left(\frac{12r}{L} \times x\right)$$

where, x = balance length AJ.

As null point occurs at J so potential drop across balance length AJ = EMF of the cell 'C'.

$$\Rightarrow \quad V_{AJ} = \frac{\varepsilon}{2}$$

$$\Rightarrow \quad i\left(\frac{12r}{L} \times x\right) = \frac{\varepsilon}{2}$$

$$\Rightarrow \quad \frac{\varepsilon}{13r} \times \frac{12r}{L} \times x = \frac{\varepsilon}{2} \Rightarrow x = \frac{13}{24}L$$

23. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is **[JEE Main 2019, 10 Jan Shift-I]**

(a) 0.4 mA (b) 63 mA
(c) 20 mA (d) 100 mA

Exp. **(c)**

Colour code of carbon resistance is shown in the figure below

So, resistance value of resistor using colour code is

$$R = 502 \times 10 = 502 \times 10^2 \ \Omega$$

Here, we must know that for given carbon resistor first three colours give value of resistance and fourth colour gives multiplier value.

Now using power, $P = i^2R$

we get, $i = \sqrt{\frac{P}{R}} = \sqrt{\frac{2}{50.2 \times 10^2}}$

$\simeq 20 \times 10^{-3}$ A $= 20$ mA.

24. In the given circuit, the cells have zero internal resistance. The currents (in Ampere) passing through resistances R_1 and R_2 respectively are

[JEE Main 2019, 10 Jan Shift-I]

(a) 0.5, 0 (b) 1, 2 (c) 2, 2 (d) 0, 1

***Exp.* (a)**

By Kirchhoff's loop rule in the given loop *ABEFA*, we get

$$10 - (I_1 + I_2)R_1 = 0$$

$$\Rightarrow \quad 10 - (I_1 + I_2)20 = 0$$

or $$I_1 + I_2 = \frac{1}{2} \quad \ldots(i)$$

and from loop *BCDEB*, we get

$$10 - (I_1 + I_2)R_1 - I_2R_2 = 0$$

$$\Rightarrow \quad 10 - (I_1 + I_2 + I_2)20 = 0$$

$$\Rightarrow \quad I_1 + 2I_2 = \frac{1}{2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$I_2 = 0$$

and $$I_1 = 0.5 \text{ A}$$

25. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is

[JEE Main 2019, 10 Jan Shift-I]

(a) 12 Ω (b) 8 Ω
(c) 2 Ω (d) 4 Ω

***Exp.* (d)**

Resistance of each arm of equilateral triangle will be

$$R = \frac{18}{3} = 6\ \Omega$$

So we have following combination will be

Equivalent resistance is

$$\therefore \quad R_{AB} = \frac{12 \times 6}{12 + 6} = \frac{12 \times 6}{18} = 4\ \Omega$$

26. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11 V is connected across it is

[JEE Main 2019, 10 Jan Shift-II]

(a) 11×10^{-4} W (b) 11×10^{-5} W
(c) 11×10^{5} W (d) 11×10^{-3} W

***Exp.* (b)**

Power dissipated by any resistor R, when I current flows through it is,

$$P = I^2R \quad \ldots(i)$$

Given $I = 2$ mA $= 2 \times 10^{-3}$ A and $P = 4.4$ W

Using Eq. (i), we get

$$4.4 = (2 \times 10^{-3})^2 \times R$$

or $$R = \frac{.4}{4 \times 10^{-6}} \cdot \Omega \quad \ldots(ii)$$

When this resistance R is connected with 11 V supply then power dissipated is

$$P = \frac{V^2}{R}$$

or $$P = \frac{(11)^2}{4.4} \times 4 \times 10^{-6}$$

[∵ Using Eq. (ii)]

$$\Rightarrow \quad P = \frac{11 \times 11 \times 4 \times 10^{-6}}{44 \times 10^{-1}} \text{ W}$$

or $$P = 11 \times 10^{-5} \text{ W}$$

27. The actual value of resistance R, shown in the figure is 30 Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is **[JEE Main 2019, 10 Jan Shift-II]**

(a) 600 Ω (b) 570 Ω (c) 350 Ω (d) 35 Ω

Exp. **(b)**

Measured value of R = 5% less than actual value of R.

Actual values of $R = 30\ \Omega$

So, measured value of R is

$$R' = 30 - (5\% \text{ of } 30) = 30 - \frac{5}{100} \times 30$$

$$\Rightarrow \quad R' = 28.5\ \Omega \quad \ldots \text{(i)}$$

Now, let us assume that internal resistance of voltmeter R_V. Replacing voltmeter with its internal resistance, we get following circuit.

It is clear that the measured value, R' should be equal to parallel combination of R and R_V. Mathematically,

$$R' = \frac{RR_V}{R + R_V} = 28.5\ \Omega$$

Given, $R = 30\ \Omega$

$$\Rightarrow \quad \frac{30R_V}{30 + R_V} = 28.5$$

$$\Rightarrow \quad 30R_V = (28.5 \times 30) + 28.5\, R_V$$

$$\Rightarrow \quad 1.5R_V = 28.5 \times 30$$

$$\Rightarrow \quad R_V = \frac{28.5 \times 30}{1.5} = 19 \times 30$$

or $R_V = 570\ \Omega$

28. The Wheatstone bridge shown in figure here, gets balanced when the carbon resistor is used as R_1 has the color code (orange, red, brown). The resistors R_2 and R_4 are 80 Ω and 40 Ω, respectively.

Assuming that the color code for the carbon resistors gives their accurate values, the color code for the carbon resistor is used as R_3 would be **[JEE Main 2019, 10 Jan Shift-II]**

(a) brown, blue, black (b) brown, blue, brown
(c) grey, black, brown (d) red, green, brown

Exp. **(b)**

The value of R_1 (orange, red, brown)

$\Rightarrow \quad = 32 \times 10 = 320\ \Omega$

Given, $R_2 = 80\ \Omega$ and $R_4 = 40\ \Omega$

In balanced Wheatstone bridge condition,

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow R_3 = R_4 \times \frac{R_1}{R_2}$$

$$\Rightarrow \quad R_3 = \frac{40 \times 320}{80}$$

or $R_3 = 160\ \Omega = 16 \times 10^1$

Comparing the value of R_3 with the colours assigned for the carbon resistor, we get

$$R_3 = \underset{\text{Brown}}{16} \times \underset{\text{Blue}}{10}^{\underset{\text{Brown}}{1}}$$

29. The resistance of the meter bridge AB in given figure is 4 Ω. With a cell of emf $\varepsilon = 0.5$ V and rheostat resistance $R_h = 2\ \Omega$. The null point is obtained at some point J. When the cell is replaced by another one of emf $\varepsilon = \varepsilon_2$, the same null point J is found for $R_h = 6\ \Omega$. The emf ε_2 is **[JEE Main 2019, 11 Jan Shift-I]**

(a) 0.6 V (b) 0.3 V
(c) 0.5 V (d) 0.4 V

Exp. **(b)**

Let length of null point 'J' be 'x' and length of the potentiometer wire be 'L'.

In first case, current in the circuit

$$I_1 = \frac{6}{4+2} = 1\text{ A}$$

$\therefore$ Potential gradient $= I \times R = \dfrac{1 \times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{1 \times 4}{L} \times x = \varepsilon_1$$

Given, $\varepsilon_1 = 0.5 = \dfrac{4x}{L}$ or $\dfrac{x}{L} = \dfrac{1}{8}$ ...(i)

In second case, current in the circuit

$$I_2 = \frac{6}{4+6} = 0.6\text{ A}$$

$\therefore$ Potential gradient $= \dfrac{0.6 \times 4}{L}$

$\Rightarrow$ Potential difference in part 'AJ'

$$= \frac{0.6 \times 4}{L} \times x = \varepsilon_2$$

$\Rightarrow$ $\varepsilon_2 = \dfrac{0.6 \times 4}{L} \times \dfrac{L}{8}$ [using Eq. (i)]

$\Rightarrow$ $\varepsilon_2 = 0.3\text{ V}$

30. Two equal resistances when connected in series to a battery consume electric power of 60 W. If these resistances are now connected in parallel combination to the same battery, the electric power consumed will be

[JEE Main 2019, 11 Jan Shift-I]

(a) 60 W (b) 30 W (c) 240 W (d) 120 W

Exp. **(c)**

Let P_1 and P_2 be the individual electric powers of the two resistances, respectively.

In series combination, power is

$$P_0 = \frac{P_1 P_2}{P_1 + P_2} = 60\text{W}$$

Since, the resistances are equal and the current through each resistor in series combination is also same. Then,

$$P_1 = P_2 = 120\text{ W}$$

In parallel combination, power is

$$P = P_1 + P_2 = 120 + 120$$
$$= 240\text{ W}$$

Alternate Solution

Let R be the resistance.

$\therefore$ Net resistance in series $= R + R = 2R$

$$P = \frac{V^2}{2R} = 60\text{ W}$$

$\Rightarrow$ $\dfrac{V^2}{R} = 120\text{ W}$

New resistance in parallel $= \dfrac{R \times R}{R + R} = R/2$

$$P' = \frac{V^2}{R/2} = 2\left(\frac{V^2}{R}\right) = 240\text{ W}$$

31. In a Wheatstone bridge (see figure), resistances P and Q are approximately equal. When $R = 400\,\Omega$, the bridge is balanced. On interchanging P and Q, the value of R for balance is $405\,\Omega$. The value of X is close to **[JEE Main 2019, 11 Jan Shift-I]**

(a) 404.5 Ω (b) 401.5 Ω
(c) 402.5 Ω (d) 403.5 Ω

Exp. **(c)**

For a balanced Wheatstone bridge,

$$\frac{P}{R} = \frac{Q}{X}$$

In first case when $R = 400\ \Omega$, the balancing equation will be

$$\frac{P}{R} = \frac{Q}{X}$$

$\Rightarrow$ $\dfrac{P}{400\ \Omega} = \dfrac{Q}{X}$

$\Rightarrow$ $P = \dfrac{400 \times Q}{X}$ ...(i)

In second case, P and Q are interchanged and $R = 405\ \Omega$

$\therefore$ $\dfrac{Q}{R} = \dfrac{P}{X}$

$\Rightarrow$ $\dfrac{Q}{405} = \dfrac{P}{X}$ ...(ii)

Substituting the value of P from Eq. (i) in Eq. (ii), we get

$$\frac{Q}{405} = \frac{Q \times 400}{X^2}$$

$$\Rightarrow \quad X^2 = 400 \times 405$$

$$\Rightarrow \quad X = \sqrt{400 \times 405} = 402.5$$

The value of X is close to 402.5 Ω.

32. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt is

[JEE Main 2019, 11 Jan Shift-II]

(a) 100 Ω (b) 125 Ω
(c) 120 Ω (d) 80 Ω

Exp. **(*d*)**

Resistance of galvanometer, $(G) = 20\,\Omega$
Number of divisions on both side = 30
Figure of merit = 0.005 ampere/division

Voltmeter

∴ Full scale deflection current (I_g)
= Number of divisions × figure of merit

$$\Rightarrow \quad I_g = 30 \times 0.005 = 0.15\text{ A}$$

Now, for measuring 15V by this galvanometer, let we use a resistance of R ohm in series with the galvanometer.

∴ Effective resistance of voltmeter, $R_{eff} = R + 20$

As, maximum potential measured by voltmeter.

$$V = I_g \cdot R_{eff} \text{ or } 15 = 0.15(R + 20)$$

or $R + 20 = 100$ or $R = 80\,\Omega$

33. In the experimental set up of meter bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10 Ω resistor is connected in series with R_1, the null point shifts by 10 cm.

The resistance that should be connected in parallel with $(R_1 + 10)\,\Omega$ such that the null point shifts back to its initial position is

[JEE Main 2019, 11 Jan Shift-II]

(a) 60 Ω (b) 20 Ω (c) 30 Ω (d) 40 Ω

Exp. **(*a*)**

For meter bridge, if balancing length is l cm, then in first case, $\frac{R_1}{R_2} = \frac{l}{(100 - l)}$

It is given that, $l = 40$ cm;

So; $$\frac{R_1}{40} = \frac{R_2}{100 - 40}$$

or $$\frac{R_1}{R_2} = \frac{2}{3} \quad \ldots(i)$$

In second case, $R_1' = R_1 + 10$, and balancing length is now 50 cm then

$$\frac{R_1 + 10}{50} = \frac{R_2}{(100 - 50)}$$

or $$R_1 + 10 = R_2 \quad \ldots(ii)$$

Substituting value of R_2 from (ii) to (i) we get,

or $$\frac{R_1}{10 + R_1} = \frac{2}{3}$$

$$\Rightarrow \quad 3R_1 = 20 + 2R_1 \text{ or } R_1 = 20\Omega$$

$$\Rightarrow \quad R_2 = 30\Omega$$

Let us assume the parallel connected resistance is x.

Then equivalent resistance is $\frac{x(R_1 + 10)}{x + R_1 + 100}$

So, this combination should be again equal to R_1.

$$\frac{(R_1 + 10)x}{R_1 + 10 + x} = R_1 \quad \Rightarrow \quad \frac{30x}{30 + x} = 20$$

or $$30x = 600 + 20x$$

or $$x = 60\Omega$$

34. In the circuit shown, the potential difference between A and B is

[JEE Main 2019, 11 Jan Shift-II]

(a) 3 V (b) 1 V
(c) 6 V (d) 2 V

Exp. **(*d*)**

In the given circuit, let's assume currents in the arms are i_1, i_2 and i_3, respectively.

Now, $i_1 = \frac{V_1}{R_1} = \frac{1}{1} = 1\text{ A}$

Similarly, $i_2 = \frac{2}{1} = 2\text{ A}$

and $i_3 = \frac{3}{1} = 3\text{ A}$

Total current in the arm *DA* is

$$i = i_1 + i_2 + i_3 = 6\text{A}$$

As all three resistors between *D* and *C* are in parallel.

∴ Equivalent resistance between terminals *D* and *C* is

$$\frac{1}{R_{DC}} = \left(\frac{1}{1} + \frac{1}{1} + \frac{1}{1}\right)$$

∴ $R_{DC} = \frac{1}{3}\Omega$

So, potential difference across *D* and *C* is

$$V_{DC} = iR_{DC} = 6 \times \frac{1}{3}$$

⇒ $V_{DC} = 2\text{ V}$

Now,

$$V_{AD} \text{ and } V_{CB} = 0$$

(In case of open circuits, $I = 0$)

So, $V_{AB} = V_{AD} + V_{DC} + V_{CB} = V_{DC}$

So, $V_{AB} = 2\text{V}$

35. The galvanometer deflection, when key K_1 is closed but K_2 is open equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$.

The resistance of the galvanometer is given by (neglect the internal resistance of battery) : **[JEE Main 2019, 12 Jan Shift-I]**

(a) 22 Ω (b) 5 Ω (c) 25 Ω (d) 12 Ω

Exp. **(*a*)**

For a galvanometer, $i_g \propto \theta$ or $i_g = C\theta$

where, I_g = current through coil of galvanometer,

θ = angle of deflection of coil

and *C* is the constant of proportionality.

Now, K_1 is closed and K_2 is opened, circuit resistance is

$$R_{eq} = R_1 + R_g$$

where, R_g = galvanometer resistance.

Hence, galvanometer current is

i.e. $i_g = \frac{V}{R_1 + R_g}$

where, V = supply voltage.

As deflection is given θ_0, we have

$$i_g = \frac{V}{R_1 + R_g} = C\theta_0 \quad \ldots(i)$$

When both keys K_1 and K_2 are closed, circuit resistance.

$$R_{eq} = R_1 + \left(\frac{R_2 \times R_g}{R_2 + R_g}\right)$$

Current through galvanometer will be

$$i_{g_2} = \frac{V}{\left(R_1 + \frac{R_2 R_g}{R_2 + R_g}\right)} \times \frac{R_2}{(R_2 + R_g)}$$

$$= \frac{VR_2}{R_1R_2 + R_1R_g + R_2R_g} = C \cdot \frac{\theta_0}{5} \quad \ldots(ii)$$

Now, dividing Eq. (i) by Eq. (ii), we get

$$\frac{R_1R_2 + R_1R_g + R_2R_g}{R_2(R_1 + R_g)} = 5$$

Substituting $R_1 = 220\ \Omega$ and $R_2 = 5\ \Omega$, we get

$$\frac{1100 + 220\,R_g + 5\,R_g}{5\,(220 + R_g)} = 5$$

$$1100 + 225\,R_g = 5500 + 25R_g$$

$\Rightarrow \quad 200\,R_g = 4400$

$\Rightarrow \quad R_g = \dfrac{4400}{200} = 22\,\Omega$

$\therefore \quad R_g = 22\,\Omega$

36. Two electric bulbs rated at 25 W, 220 V and 100 W, 220 V are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then

[JEE Main 2019, 12 Jan Shift-I]

(a) $P_1 = 16\,\text{W}, P_2 = 4\text{W}$ (b) $P_1 = 4\,\text{W}, P_2 = 16\text{W}$
(c) $P_1 = 9\,\text{W}, P_2 = 16\text{W}$ (d) $P_1 = 16\,\text{W}, P_2 = 9\text{W}$

Exp. **(a)**

Resistance of a bulb of power P and with a voltage source V is given by

$$R = \frac{V^2}{P}$$

Resistance of the given two bulbs are

$$R_1 = \frac{V^2}{P_1} = \frac{(220)^2}{25} \text{ and } R_2 = \frac{V^2}{P_2} = \frac{(220)^2}{100}$$

Since, bulbs are connected in series. This means same amount of current flows through them.

$\therefore$ Current in circuit is

$$i = \frac{V}{R_{\text{total}}} = \frac{220}{\dfrac{(220)^2}{25} + \dfrac{(220)^2}{100}} = \frac{1}{11}\text{A}$$

Power drawn by bulbs are respectively,

$$P_1 = i^2 R_1 = \left(\frac{1}{11}\right)^2 \times \frac{220 \times 220}{25} = 16\text{ W.}$$

and $P_2 = i^2 R_2 = \left(\dfrac{1}{11}\right)^2 \times \dfrac{220 \times 220}{100} = 4\text{ W.}$

37. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5 Ω. The value of R to give a potential difference of 5 mV across 10 cm of potentiometer wire is

[JEE Main 2019, 12 Jan Shift-I]

(a) 395 Ω (b) 495 Ω
(c) 490 Ω (d) 480 Ω

Exp. **(a)**

Given, potential difference of 5 mV is across 10 m length of potentiometer wire. So potential drop per unit length is

$$= \frac{5 \times 10^{-3}}{10 \times 10^{-2}} = 5 \times 10^{-2}\left(\frac{\text{V}}{\text{m}}\right)$$

Hence, potential drop across 1 m length of potentiometer wire is

$$V_{AB} = 5 \times 10^{-2}\left(\frac{\text{V}}{\text{m}}\right) \times 1$$

$$= 5 \times 10^{-2}\text{ V}$$

Now, potential drop that must occurs across resistance R is

$$V_R = 4 - 5 \times 10^{-2} = \frac{395}{100}\text{ V}$$

Now, circuit current is $i = \dfrac{V}{R_{\text{total}}} = \dfrac{4}{R + 5}$

Hence, for resistance R, using $V_R = iR$, we get

$$\frac{395}{100} = \frac{4}{R + 5} \times R$$

$$395\,(R + 5) = 400R$$

$$395 \times 5 = (400 - 395)R$$

$$\Rightarrow \quad R = 395\,\Omega$$

38. In a meter bridge, the wire of length 1m has a non-uniform cross-section such that the variation $\dfrac{dR}{dl}$ of its resistance R with length l is $\dfrac{dR}{dl} \propto \dfrac{1}{\sqrt{l}}$. Two equal resistance are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?

[JEE Main 2019, 12 Jan Shift-I]

(a) 0.3 m (b) 0.25 m (c) 0.2 m (d) 0.35 m

Exp. **(b)**

As, galvanometer shows zero deflection.

This means, the meter bridge is balanced.

$$\frac{R'}{R_{AP}} = \frac{R'}{R_{PB}} \Rightarrow R_{AP} = R_{PB} \qquad \ldots\text{(i)}$$

Now, for meter bridge wire, $\dfrac{dR}{dl} = \dfrac{k}{\sqrt{l}}$

where, 'k' is the constant of proportionality.

$\Rightarrow \quad dR = \frac{k}{\sqrt{l}} dl$

Integrating both sides, we get

$\Rightarrow \quad R = \int \frac{k}{\sqrt{l}} dl$

So, $\quad R_{AP} = \int_0^l \frac{k}{\sqrt{l}} dl = k(2\sqrt{l})\Big|_0^l = 2k\sqrt{l}$

and $\quad R_{PB} = \int_l^1 \frac{k}{\sqrt{l}} dl = 2k(\sqrt{l})\Big|_l^1$

$= 2k(\sqrt{1} - \sqrt{l})$

$= 2k(1 - \sqrt{l})$

Substituting values of R_{AP} and R_{PB} in Eq. (i), we get

$R_{AP} = R_{PB}$

$\Rightarrow \quad 2k\sqrt{l} = 2k(1 - \sqrt{l})$

$\Rightarrow \quad \sqrt{l} = \frac{1}{2}$ or $l = \frac{1}{4} = 0.25$ m

39. In the given circuit diagram, the currents $I_1 = -0.3$ A, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2, I_3 and I_6 respectively, are

[JEE Main 2019, 12 Jan Shift-II]

(a) 1.1 A, 0.4 A, 0.4 A (b) 1.1 A, – 0.4 A, 0.4 A
(c) 0.4 A, 1.1 A, 0.4 A (d) – 0.4 A, 0.4 A, 1.1 A

Exp. **(a)**

Given circuit with currents as shown in the figure below, [In the question I_1 = 0.3 A is given, due to it we change the direction of I_1, in this figure]

From Kirchoff's junction rule, $\Sigma I = 0$

At junction S, $\quad I_4 = I_5 + I_3$

$\Rightarrow \quad 0.8 = 0.4 + I_3$

$\Rightarrow \quad I_3 = 0.4$ A

At junction P, $\quad I_5 = I_6$

$\Rightarrow \quad I_6 = 0.4$ A

At junction Q, $\quad I_2 = I_1 + I_3 + I_6 = 0.3 + 0.4 + 0.4 = 1.1$ A

40. A galvanometer whose resistance is 50 Ω, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of

[JEE Main 2019, 12 Jan Shift-II]

(a) 250 Ω (b) 6200 Ω (c) 200 Ω (d) 6250 Ω

Exp. **(c)**

Current for deflection of pointer by

1 division = 4×10^{-4} A

So, current for full-scale deflection = I_g

= Number of divisions × Current for 1 division

$\Rightarrow \quad I_g = 25 \times 4 \times 10^{-4} = 1 \times 10^{-2}$ A

Now, let a resistance of R is put in series with galvanometer to make it a voltmeter of range 2.5 V.

Then, $I_g(R + G) = V_{AB}$

$1 \times 10^{-2}(R + 50) = 2.5$

[∵ Given, $G = 50\,\Omega$, $V_{AB} = 2.5$ V]

$\therefore \quad R = 250 - 50 = 200\,\Omega$

41. Two batteries with emf 12 V and 13 V are connected in parallel across a load resistor of 10 Ω. The internal resistances of the two batteries are 1 Ω and 2 Ω, respectively. The voltage across the load lies between

[JEE Main 2018]

(a) 11.6 V and 11.7 V (b) 11.5 V and 11.6 V
(c) 11.4 V and 11.5 V (d) 11.7 V and 11.8 V

Exp. **(b)**

For parallel combination of cells,

$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}}$$

$$\therefore \quad E_{eq} = \frac{\frac{12}{1} + \frac{13}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{37}{3}\text{ V}$$

Potential drop across 10 Ω resistance,

$$V = \left(\frac{E}{R_{total}}\right) \times 10$$

$$= \frac{37/3}{\left(10 + \frac{2}{3}\right)} \times 10 = 11.56\text{ V}$$

$$\therefore \quad V = 11.56\text{ V}$$

Alternate Solution

Applying KVL, in loop *ABCFA*,

$$-12 + 10(I_1 + I_2) + 1 \times I_1 = 0$$

$$\Rightarrow \quad 12 = 11I_1 + 10I_2 \quad \ldots\text{(i)}$$

Similarly,

In loop *ABDEA*,

$$-13 + 10(I_1 + I_2) + 2 \times I_2 = 0$$

$$\Rightarrow \quad 13 = 10I_1 + 12I_2 \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$I_1 = \frac{7}{16}\text{A}, \; I_2 = \frac{23}{32}\text{A}$$

∴ Voltage drop across 10 Ω resistance is

$$V = 10\left(\frac{7}{16} + \frac{23}{32}\right) = 11.56\text{ V}$$

42. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[JEE Main 2018]**

(a) 1 Ω (b) 1.5 Ω (c) 2 Ω (d) 2.5 Ω

Exp. **(*b*)**

With only the cell,

On balancing, $E = 52 \times x$ …(i)

where, x is the potential gradient of the wire.

When the cell is shunted,

Similarly, on balancing,

$$V = E - \frac{Er}{(R + r)} = 40 \times x \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$\frac{E}{V} = \frac{1}{1 - \frac{r}{R + r}} = \frac{52}{40}$$

$$\Rightarrow \quad \frac{E}{V} = \frac{R + r}{R} = \frac{52}{40} \Rightarrow \frac{5 + r}{5} = \frac{52}{40}$$

$$\Rightarrow \quad r = \frac{3}{2}\,\Omega \quad r = 1.5\,\Omega$$

43. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances? **[JEE Main 2018]**

(a) 990 Ω (b) 505 Ω (c) 550 Ω (d) 910 Ω

Exp. **(*c*)**

We have, $X + Y = 1000\,\Omega$

Initially, $\frac{X}{l} = \frac{1000 - X}{100 - l}$ …(i)

When X and Y are interchanged, then

$$\frac{1000 - X}{l - 10} = \frac{X}{100 - (l - 10)}$$

or $$\frac{1000 - X}{l - 10} = \frac{X}{110 - l} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{100 - l}{l} = \frac{l - 10}{110 - l}$$

$$(100 - l)(110 - l) = (l - 10)l$$

$$11000 - 100l - 110l + l^2 = l^2 - 10l$$

$\Rightarrow$ $11000 = 200l$

$\therefore$ $l = 55$ cm

Substituting the value of l in Eq. (i), we get

$$\frac{X}{55} = \frac{1000 - 55}{100 - 55} \Rightarrow 20X = 11000$$

$\therefore$ $X = 550\ \Omega$

44. Which of the following statements is false? **[JEE Main 2017 (Offline)]**

(a) In a balanced Wheatstone bridge, if the cell and the galvanometer are exchanged, the null point is disturbed

(b) A rheostat can be used as a potential divider

(c) Kirchhoff's second law represents energy conservation

(d) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude

Exp. **(a)**

In a balanced Wheatstone bridge, there is no effect on position of null point, if we exchange the battery and galvanometer. So, option (a) is incorrect.

45. In the given circuit diagram, when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be **[JEE Main 2017 (Offline)]**

(a) $CE\dfrac{r_1}{(r_2 + r)}$ (b) $CE\dfrac{r_2}{(r + r_2)}$

(c) $CE\dfrac{r_1}{(r_1 + r)}$ (d) CE

Exp. **(b)**

In steady state, no current flows through the capacitor. So, resistance r_p becomes ineffective.

So, the current in circuit,

$$I = \frac{E}{r + r_2 \text{ (Total Resistance)}}$$

$\because$ Potential drop across capacitor = Potential drop across r_2 $= Ir_2 = \dfrac{Er_2}{r + r_2}$

$\therefore$ Stored charge of capacitor,

$$Q = CV = CE\frac{r_2}{r + r_2}$$

46. In the below circuit, the current in each resistance is **[JEE Main 2017 (Offline)]**

(a) 0.25 A (b) 0.5 A

(c) 0 A (d) 1 A

Exp. **(c)**

Each resistance is converted with two cells combined in opposite direction, so potential drop across each resistor is zero. Hence, the current through each of resistor is zero.

47. The temperature dependence of resistances of Cu and undoped Si in the temperature range 300-400 K, is best described by **[JEE Main 2016 (Offline)]**

(a) linear increase for Cu, linear increase for Si

(b) linear increase for Cu, exponential increase for Si

(c) linear increase for Cu, exponential decrease for Si

(d) linear decrease for Cu, linear decrease for Si

Exp. **(c)**

As, we know Cu is a conductor, so increase in temperature, resistance will increase. Then, Si is semiconductor, so with increase in temperature, resistance will decrease.

48. When 5V potential difference is applied across a wire of length 0.1m, the drift speed of electrons is $2.5\times10^{-4}\,ms^{-1}$. If the electron density in the wire is $8\times10^{28}\,m^{-3}$ the resistivity of the material is close to **[JEE Main 2015]**

(a) $1.6\times10^{-8}\,\Omega m$ (b) $1.6\times10^{-7}\,\Omega m$

(c) $1.6\times10^{-6}\,\Omega m$ (d) $1.6\times10^{-5}\,\Omega m$

Exp. **(*d*)**

According to the question,

$$v_d = 2.5\times10^{-4}\,m/s$$

$$\Rightarrow \quad n = 8\times10^{28}/m^3$$

We know that

$$J = nev_d \quad \text{or} \quad I = nev_d A$$

where, symbols have their usual meaning.

$$\Rightarrow \quad \frac{V}{R} = nev_d A$$

or $\dfrac{V}{\dfrac{\rho L}{A}} = nev_d A$ or $\dfrac{V}{\rho L} = nev_d$

or $\rho = \dfrac{V}{nev_d L}$

$$= \frac{5}{8\times10^{28}\times1.6\times10^{-19}\times2.5\times10^{-4}\times0.1}$$

or $\rho = 1.6\times10^{-5}\,\Omega m$

49. In the circuit shown below, the current in the 1Ω resistor is **[JEE Main 2015]**

(a) 1.3 A, from P to Q (b) 0 A

(c) 0.13 A, from Q to P (d) 0.13 A, from P to Q

Exp. **(*c*)**

Connect point Q to ground and apply KCL.

Consider the grounded circuit as shown below.

Applying KCL of point Q we can write

Incoming current at Q = outgoing current from Q

$$\Rightarrow \quad \frac{V+6}{3} + \frac{V}{1} = \frac{9-V}{5}$$

or $V\left[\dfrac{1}{3}+\dfrac{1}{5}+1\right] = \dfrac{9}{5} - 2$

or $V\left[\dfrac{5+3+15}{15}\right] = \dfrac{9-10}{5}$

or $V\left[\dfrac{23}{15}\right] = \dfrac{-1}{5}$

or $V = \dfrac{-3}{23} = -0.13\,V$

Thus, current in the 1 Ω resistance is 0.13 A, from Q to P.

50. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1 kW. The voltage of the electric mains is 220 V. Then, the minimum current should be **[JEE Main 2014]**

(a) 8 A (b) 10 A (c) 12 A (d) 14 A

Exp. **(*c*)**

Total power (P) consumed

$= (15\times40) + (5\times100)$

$+ (5\times80) + (1\times1000) = 2500$ W

As, we know that

$$P = VI$$

$$\Rightarrow \quad I = \frac{2500}{200} = \frac{125}{11} = 11.3\ A$$

Hence, minimum capacity should be 12 A.

51. The supply voltage to room is 120 V. The resistance of the lead wires is 6 Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **[JEE Main 2013]**

(a) Zero (b) 2.9 V (c) 13.3 V (d) 10.04 V

Exp. (*d*)

$$P = \frac{V^2}{R}$$

Resistance of the bulb,

$$R = \frac{120 \times 120}{60} = 240\ \Omega$$

$$R_{eq} = 240 + 6 = 246\ \Omega$$

$$\Rightarrow \quad I_1 = \frac{V}{R_{eq}} = \frac{120}{246}$$

$$V_1 = I_1 R_1 = \frac{120}{246} \times 240$$

$$= 117.073\ \text{V}$$

Resistance of the heater

$$= \frac{V^2}{P} = \frac{120 \times 120}{240} = 60\ \Omega$$

As bulb and heater are connected in parallel.

Net resistance $= \frac{240 \times 60}{300} = 48\ \Omega$

Total resistance, $R_2 = 48 + 6 = 54\ \Omega$

Total current, $I_2 = V/R_2 = 120/54$

Potential across heater = Potential across bulb

$$V_2 = \frac{120}{54} \times 48 = 106.66\ \text{V}$$

$$V_1 - V_2 = 117.073 - 106.66 = 10.04\ \text{V}$$

52. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Higher the range, greater is the resistance of ammeter.

Statement II To increase the range of ammeter, additional shunt needs to be used across it. **[JEE Main 2013]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

(c) Statement I is true, Statement II is false

(d) Statement I is false, Statement II is true

Exp. (*d*)

To increase the range of ammeter, resistance should be decreased (so additional shunt is connected in parallel).

53. Two electric bulbs marked 25 W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse? **[AIEEE 2012]**

(a) Both (b) 100 W

(c) 25 W (d) None of these

Exp. (*c*)

As the rated power of 25 W is less than 100 W, it implies that 25 W bulb has higher resistance. As in series connection, current through both the bulbs is same but heating in 25 W bulb is more than that of 100 W bulb. So, 25 W bulb will get fused.

54. Resistance of a given wire is obtained by measuring the current flowing in it and the voltage difference applied across it. If the percentage errors in the measurement of the current and the voltage difference are 3% each, then error in the value of resistance of the wire is

(a) 6% (b) zero (c) 1% (d) 3%

Exp. (*a*)

From Ohm's law, $R = V/I$

By error method, $\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$

$$= 3\% + 3\% = 6\%$$

55. If 400 Ω of resistance is made by adding four 100 Ω resistance of tolerance 5%, then the tolerance of the combination is **[AIEEE 2011]**

(a) 20% (b) 5% (c) 10% (d) 15%

Exp. (*b*)

Resistance of combination, $R_e = 4R$

$$\Rightarrow \quad \frac{\Delta R_e}{R_e} = \frac{\Delta R}{R} = \frac{5 \times 100}{100} = 5\%$$

56. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are $4\times10^{-7}\ \Omega$-m and $8\times10^{-7}\ \text{m}^2$, respectively. The potential gradient will be equal to **[AIEEE 2011]**

(a) 0.2 V/m (b) 1 V/m (c) 0.3 V/m (d) 0.1 V/m

Exp. (*d*)

Potential gradient of a potentiometer, $K = \frac{V}{l}$

$$K = \frac{I\rho}{A} = \frac{0.2\times4\times10^{-7}}{8\times10^{-7}}$$

$$= 0.1 \text{ V/m}$$

57. If a wire is stretched to make it 0.1% longer, its resistance will **[AIEEE 2011]**

(a) increase by 0.2% (b) decrease by 0.2%

(c) decrease by 0.05% (d) increase by 0.05%

Exp. (*d*)

$$R = \frac{\rho l}{A} = \frac{\rho l^2}{V} \qquad [\because V = \text{volume}]$$

$$\therefore \qquad \frac{\Delta R}{R} = 2\frac{\Delta l}{l} = +\ 0.2\%$$

58. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are **[AIEEE 2010]**

(a) $I_1 = -I_2 = \frac{Blv}{R}, I = \frac{2Blv}{R}$

(b) $I_1 = I_2 = \frac{Blv}{3R}, I = \frac{2Blv}{3R}$

(c) $I_1 = I_2 = I = \frac{Blv}{R}$

(d) $I_1 = I_2 = \frac{Blv}{6R}, I = \frac{Blv}{3R}$

Exp. (*b*)

A moving conductor is equivalent to a battery of emf = vBl [motion emf]

Equivalent circuit, $I = I_1 + I_2$ ...(i)

Applying Kirchhoff's law,

$$I_1R + IR - vBl = 0 \qquad \text{...(ii)}$$

$$I_2R + IR - vBl = 0 \qquad \text{...(iii)}$$

Adding Eqs. (ii) and (iii), we get

$$I_1R + IR - VBl + I_2R + IR - vBl = 0$$

$$\Rightarrow \quad I_1R + I_2R + 2IR - 2vBl = 0$$

$$\Rightarrow \quad (I_1 + I_2)R + 2IR - 2vBl = 0$$

$$\Rightarrow \quad 2IR + IR = 2vBl \qquad \text{[From Eq. (i)]}$$

$$\Rightarrow \quad I = \frac{2vBl}{3R}$$

Subtracting Eq.(iii) from Eq.(ii), we get

$$I_1R + IR - VBl = 0$$

$$-I_2R + -IR - +VBl = -0$$

$$I_1R - I_2R = 0$$

$$\Rightarrow \quad R(I_1 - I_2) = 0$$

$$\Rightarrow \quad I_1 - I_2 = 0 \qquad [\because R \neq 0]$$

$$\Rightarrow \quad I_1 = I_2 \qquad \text{...(iv)}$$

From Eq. (i), we get

$$I = 2I_1 \qquad [\because I_1 = I_2]$$

$$\Rightarrow \quad 2I_1 = I$$

$$\Rightarrow \quad I_1 = \frac{I}{2} = \frac{2vBl}{2\times3R} = \frac{vBl}{3R}$$

$$\therefore \quad I_1 = I_2 = \frac{vBl}{3R}$$

59. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is **[AIEEE 2010]**

(a) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

(b) $\frac{V}{R_2}$ at $t=0$ and $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=\infty$

(c) $\frac{V}{R_2}$ at $t=0$ and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at $t=\infty$

(d) $\frac{V(R_1+R_2)}{R_1R_2}$ at $t=0$ and $\frac{V}{R_2}$ at $t=\infty$

Exp. **(b)**

At $t=0$, inductor behaves like an infinite resistance.

So, at $t=0$, $I=\frac{V}{R_2}$

and at $t=\infty$, inductor behaves like a conducting wire.

$$\therefore \quad I=\frac{V}{R_{eq}}=\frac{V(R_1+R_2)}{R_1R_2}$$

60. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly **[AIEEE 2010]**

(a) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$ (b) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(c) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$ (d) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

Exp. **(d)**

Let R_0 be the initial resistance of both conductors.

$\therefore$ At temperature θ, their resistances will be

$$R_1=R_0(1+\alpha_1\theta)$$

and $$R_2=R_0(1+\alpha_2\theta)$$

For series combination,

$$R_s=R_1+R_2$$

$$R_{s0}(1+\alpha_s\theta)=R_0(1+\alpha_1\theta)+R_0(1+\alpha_2\theta)$$

where, $R_{s0}=R_0+R_0=2R_0$

$\therefore \quad 2R_0(1+\alpha_s\theta)=2R_0+R_0\theta(\alpha_1+\alpha_2)$

or $$\alpha_s=\frac{\alpha_1+\alpha_2}{2}$$

For parallel combination,

$$R_p=\frac{R_1R_2}{R_1+R_2}$$

$$R_{p0}(1+\alpha_p\theta)=\frac{R_0(1+\alpha_1\theta)R_0(1+\alpha_2\theta)}{R_0(1+\alpha_1\theta)+R_0(1+\alpha_2\theta)}$$

where, $$R_{p0}=\frac{R_0R_0}{R_0+R_0}=\frac{R_0}{2}$$

$$\therefore \quad \frac{R_0}{2}(1+\alpha_p\theta)=\frac{R_0^2(1+\alpha_1\theta+\alpha_2\theta+\alpha_1\alpha_2\theta^2)}{R_0(2+\alpha_1\theta+\alpha_2\theta)}$$

As α_1 and α_2 are small quantities.

$\therefore$ $\alpha_1\alpha_2$ is negligible.

or $$\alpha_p=\frac{\alpha_1+\alpha_2}{2+(\alpha_1+\alpha_2)\theta}$$

$$=\frac{\alpha_1+\alpha_2}{2}\left[1-\left(\frac{\alpha_1+\alpha_2}{2}\right)\theta\right]$$

As $(\alpha_1+\alpha_2)^2$ is negligible.

$$\therefore \quad \alpha_p=\frac{\alpha_1+\alpha_2}{2}$$

61. This question contains Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements. **[AIEEE 2009]**

Statement I The temperature dependence of resistance is usually given as $R=R_0(1+\alpha\Delta t)$. The resistance of a wire changes from $100\,\Omega$ to $150\,\Omega$ when its temperature is increased from 27°C to 227°C. This implies that $\alpha=2.5\times10^{-3}/°C$.

Statement II $R=R_i(1+\alpha\Delta T)$ is valid only when the change in the temperature ΔT is small and $\Delta R=(R-R_0)<<R_0$.

(a) Statement I is true, Statement II is false

(b) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(c) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

(d) Statement I is false, Statement II is true

Exp. **(d)**

$$R=R_t\,(1+\alpha\,\Delta t)$$

Given, $R_t=100\,\Omega$, $R=150\,\Omega$, $\Delta t=200°$ C

$\therefore \quad 150=100\,(1+\alpha\,200)$

$\Rightarrow \quad 0.5=200\,\alpha$

$\Rightarrow \quad \alpha=\frac{0.5}{200}=\frac{1}{400}$

$=25\times10^{-4}/°C$

62. Shown in the figure adjacent is a meter-bridge set up with null deflection in the galvanometer. The value of the unknown resistor R is **[AIEEE 2008]**

(a) 13.75 Ω (b) 220 Ω (c) 110 Ω (d) 55 Ω

Exp. **(b)**

When bridge is balanced, ratio of resistance in opposite arms is equal.

$$\frac{P}{Q} = \frac{R}{S}$$

Given, $P = 55\ \Omega, Q = R$

As $R \propto l$

$R \propto l_1$, where $l_1 = 20$ cm

$S \propto l_2$, where $l_2 = 100 - 20 = 80$ cm

$$\frac{R}{S} = \frac{l_1}{l_2} = \frac{20}{80}$$

$$\therefore \quad \frac{55}{R} = \frac{20}{80} \quad \text{or} \quad R = 220\ \Omega$$

63. A 5 V battery with internal resistance 2 Ω and a 2 V battery with internal resistance 1 Ω are connected to a 10 Ω resistor as shown in the figure.

The current in the 10 Ω resistor is **[AIEEE 2008]**

(a) 0.27 A, P_2 to P_1
(b) 0.03 A, P_1 to P_2
(c) 0.03 A, P_2 to P_1
(d) 0.27 A, P_1 to P_2

Exp. **(c)**

Let potential at P_1 be 0 V and potential at P_2 be V_0.

Now apply KCL at P_2,

$$\Sigma I = 0$$

$$\frac{V_0 - 5}{2} + \frac{V_0 - 0}{10} + \frac{V_0 - (-2)}{1} = 0$$

or $$V_0 = \frac{5}{16}$$

So, current through 10 Ω resistor is $\frac{V_0}{10}$ from P_2 to P_1.

64. The resistance of a wire is 5 Ω at 50°C and 6 Ω at 100°C. The resistance of the wire at 0°C will be **[AIEEE 2007]**

(a) 2 Ω (b) 1 Ω (c) 4 Ω (d) 3 Ω

Exp. **(c)**

From $R_t = R_0(1 + \alpha t)$

$\therefore$ $5 = R_0(1 + 50\alpha)$ ...(i)

and $6 = R_0(1 + 100\alpha)$ ...(ii)

$$\therefore \quad \frac{5}{6} = \frac{1 + 50\alpha}{1 + 100\alpha}$$

or $$\alpha = \frac{1}{200}$$

Putting the value of α in Eq. (i), we get

$$5 = R_0\left(1 + 50 \times \frac{1}{200}\right) \quad \text{or} \quad R_0 = 4\ \Omega$$

65. A material B has twice the specific resistance of A. A circular wire made of B has twice the diameter of a wire made of A. Then, for the two wires to have the same resistance, the ratio l_B/l_A of their respective lengths must be **[AIEEE 2006]**

(a) 1 (b) 1/2 (c) 1/4 (d) 2

Exp. **(d)**

Let (ρ_A, l_A, r_A, A_A) and (ρ_B, l_B, r_B, A_B) be specific resistances, lengths, radii and areas of wires A and B, respectively.

Resistance of $A = R_A = \frac{\rho_A l_A}{A_A} = \frac{\rho_A l_A}{\pi r_A^2}$

Resistance of $B = R_B = \frac{\rho_B l_B}{A_B} = \frac{\rho_B l_B}{\pi r_B^2}$

From given information,

$$\rho_B = 2\rho_A$$

$$r_B = 2r_A$$

and $$R_A = R_B$$

$\therefore$ $$\frac{\rho_A l_A}{\pi r_A^2} = \frac{\rho_B l_B}{\pi r_B^2}$$

or $$\frac{\rho_A l_A}{\pi r_A^2} = \frac{2\rho_A \times l_B}{\pi (2r_A)^2}$$

or $$\frac{l_B}{l_A} = \frac{2}{1} = 2:1$$

66. An electric bulb is rated 220 V-100 W. The power consumed by it when operated on 110 V will be **[AIEEE 2006]**

(a) 75 W (b) 40 W
(c) 25 W (d) 50 W

Exp. **(c)**

Resistance of electric bulb, $R = \frac{V^2}{P}$, where subscripts denote for rated parameters.

$\Rightarrow$ $$R = \frac{(220)^2}{100}$$

Power consumed at 110 V,

$$P_{\text{consumed}} = \frac{V^2}{R}$$

$\therefore$ $$P_{\text{consumed}} = \frac{(110)^2}{(220)^2/100} = 25 \text{ W}$$

67. The resistance of a bulb filament is 100 Ω at a temperature of 100°C. If its temperature coefficient of resistance is 0.005 per °C, its resistance will become 200 Ω at a temperature of **[AIEEE 2006]**

(a) 300°C (b) 400°C
(c) 500°C (d) 200°C

Exp. **(b)**

Let resistance of bulb filament be R_0 at 0°C, then from expression,

$$R_\theta = R_0[1 + \alpha\Delta\theta]$$

We have, $100 = R_0[1 + 0.005 \times 100]$

and $200 = R_0[1 + 0.005 \times x]$

where, x is temperature in °C at which resistance become 200 Ω.

Dividing the above two equations, we get

$$\frac{200}{100} = \frac{1 + 0.005x}{1 + 0.005 \times 100}$$

$\Rightarrow$ $x = 400°C$

68. In a Wheatstone's bridge, three resistances P, Q and R are connected in the three arms and the fourth arm is formed by two resistances S_1 and S_2 connected in parallel. The condition for the bridge to be balanced will be **[AIEEE 2006]**

(a) $\frac{P}{Q} = \frac{2R}{S_1 + S_2}$ (b) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$
(c) $\frac{P}{Q} = \frac{R(S_1 + S_2)}{2S_1 S_2}$ (d) $\frac{P}{Q} = \frac{R}{S_1 + S_2}$

Exp. **(b)**

For balanced Wheatstone's bridge,

$$\frac{P}{Q} = \frac{R}{S}$$

Here, $$S = S_1 \,||\, S_2 = \frac{S_1 S_2}{S_1 + S_2}$$

$\Rightarrow$ $$\frac{P}{Q} = \frac{R(S_1 + S_2)}{S_1 S_2}$$

69. A thermocouple is made from two metals, Antimony and Bismuth. If one junction of the couple is kept hot and the other is kept cold, then an electric current will **[AIEEE 2006]**

(a) flow from Antimony to Bismuth at the hot junction
(b) flow from Bismuth to Antimony at the cold junction
(c) not flow through the thermocouple
(d) flow from Antimony to Bismuth at the cold junction

Exp. **(d)**

When a circuit is made up of any two metals in thermoelectric series, the current flows across the cold junction from the later occuring metal in the series to the one occuring earlier.

In thermoelectric series, Bismuth comes earlier than Antimony. So, at cold junction, current flows from Antimony to Bismuth and at hot junction, it flows from Bismuth to Antimony.

70. The Kirchhoff's first law ($\Sigma i = 0$) and second law ($\Sigma iR = \Sigma E$) where, the symbols have their usual meanings, are respectively based on **[AIEEE 2006]**

(a) conservation of charge, conservation of momentum
(b) conservation of energy, conservation of charge
(c) conservation of momentum, conservation of charge
(d) conservation of charge, conservation of energy

Exp. (d)

Kirchhoff's Ist law or KCL states that the algebraic sum of current meeting at any junction is equal to zero. In other words, we can say that "The sum of all the currents directed towards a junction in a circuit is equal to the sum of all the currents directed away from that junction."

Thus, no charge has been accumulated at any junction *i.e.*, charge is conserved and hence we can say that KCL ($\Sigma i = 0$) is based on conservation of charge.

Kirchhoff's IInd law or KVL states that algebraic sum of changes in potential around any closed resistor loop must be zero. In other words, "Around any closed loop, voltage drops are equal to voltage rises".

No energy is gained or lost in circulating a charge around a loop, thus we can say that KVL is based on conservation of energy.

71. The current I drawn from the 5 V source will be **[AIEEE 2006]**

(a) 0.33 A (b) 0.5 A
(c) 0.67 A (d) 0.17 A

Exp. (b)

The given circuit can be redrawn as

Which is a balanced Wheatstone's bridge and hence no current flows in the middle resistor, so equivalent circuit would be as shown below.

⇒

⇒

$$I = \frac{V}{R} = \frac{5}{10} = 0.5 \text{ A}$$

72. Two voltameters, one of copper and another of silver, are joined in parallel. When a total charge q flows through the voltameters, equal amount of metals are deposited. If the electrochemical equivalents of copper and silver are z_1 and z_2 respectively, the charge which flows through the silver voltameter is **[AIEEE 2005]**

(a) $\dfrac{q}{1+\dfrac{z_1}{z_2}}$ (b) $\dfrac{q}{1+\dfrac{z_2}{z_1}}$

(c) $q\dfrac{z_1}{z_2}$ (d) $q\dfrac{z_2}{z_1}$

Exp. **(b)**

According to Faraday's law, as m constant.

We know, $m = zq$

$\Rightarrow \quad z \propto \frac{1}{q}$ or $\frac{z_2}{z_1} = \frac{q_1}{q_2}$

Total charge, $q = q_1 + q_2 \Rightarrow \frac{q}{q_2} = \frac{q_1}{q_2} + 1$

or $\quad q_2 = \frac{q}{\left(1 + \frac{q_1}{q_2}\right)}$

or $\quad q_2 = \frac{q}{\left(1 + \frac{z_2}{z_1}\right)}$

73. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be **[AIEEE 2005]**

(a) 200 Ω (b) 100 Ω (c) 500 Ω (d) 1000 Ω

Exp. **(b)**

The galvanometer shows zero deflection *i.e.*, current through XY is zero

As a result potential drop across R is 2 V. Circuit can be redrawn as

$$I = \frac{12}{500 + R}$$

Voltage across R, $V = IR$

$\Rightarrow \quad 2 = \frac{12}{500 + R} \times R$

or $\quad 1000 + 2R = 12R$

or $\quad R = 100\ \Omega$

74. Two sources of equal emf are connected to an external resistance R. The internal resistances of the two sources are R_1 and R_2, $(R_2 > R_1)$. If the potential difference across the source having internal resistance R_2 is zero, then **[AIEEE 2005]**

(a) $R = \frac{R_2 \times (R_1 + R_2)}{(R_2 - R_1)}$ (b) $R = R_2 - R_1$

(c) $R = \frac{R_1 R_2}{(R_1 + R_2)}$ (d) $R = \frac{R_1 R_2}{(R_2 - R_1)}$

Exp. **(b)**

As R_1, R_2 and R in series.

$$R_{eq} = R_1 + R_2 + R$$

$\therefore$ Net current, $I = \frac{2E}{R_1 + R_2 + R}$

According to the question,

$$-(V_A - V_B) = E - IR_2$$

$\therefore \quad 0 = E - IR_2$

or $\quad E = IR_2$

or $\quad E = \frac{2E}{R_1 + R_2 + R} R_2$

or $\quad R_1 + R_2 + R = 2R_2$

or $\quad R = R_2 - R_1$

75. A heater coil is cut into two equal parts and only one part is now used in the heater. The heat generated will now be **[AIEEE 2005]**

(a) doubled (b) four times

(c) one-fourth (d) halved

Exp. **(a)**

Is heat generated

$$H_1 = \frac{V^2}{R}t \quad \text{and} \quad H_2 = \frac{V^2}{(R/2)}t$$

$$\therefore \quad \frac{H_2}{H_1} = 2 \quad \text{or} \quad H_2 = 2H_1$$

76. An energy source will supply a constant current into the load, if its internal resistance is **[AIEEE 2005]**

(a) equal to the resistance of the load
(b) very large as compared to the load resistance
(c) zero
(d) non-zero but less than the resistance of the load

Exp. **(c)**

$$I = \frac{E}{R + r}$$

$$I = \frac{E}{R} = \text{constant}$$

where, R = external resistance

r = internal resistance = 0

77. In a potentiometer experiment, the balancing with a cell is at length 240 cm. On shunting the cell with a resistance of 2 Ω, the balancing length becomes 120 cm. The internal resistance of the cell is **[AIEEE 2005]**

(a) 1 Ω (b) 0.5 Ω (c) 4 Ω (d) 2 Ω

Exp. **(d)**

The internal resistance of the cell,

$$r = \left(\frac{l_1 - l_2}{l_2}\right)R = \left(\frac{240 - 120}{120}\right) \times 2 = 2\ \Omega$$

78. The resistance of hot tungsten filament is about 10 times the cold resistance. What will be the resistance of 100 W and 200 V lamp, when not in use? **[AIEEE 2005]**

(a) 40 Ω (b) 20 Ω (c) 400 Ω (d) 200 Ω

Exp. **(a)**

$$P = V^2/R$$

$$\therefore \quad R_{hot} = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400\ \Omega$$

$$R_{cold} = \frac{400}{10} = 40\ \Omega$$

79. The total current supplied to the circuit by the battery is **[AIEEE 2004]**

(a) 1 A (b) 2 A
(c) 4 A (d) 6 A

Exp. **(c)**

The equivalent circuit can be drawn as 6 Ω and 2 Ω are in parallel.

$$R' = \frac{2 \times 6}{8} = 1.5\ \Omega$$

⇓

As 1.5 Ω and 1.5 Ω are in series.

$$R'' = 1.5 + 1.5 = 3\ \Omega$$

As 3 Ω and 3 Ω are in parallel.

$$R_{eq} = \frac{3 \times 3}{6} = 1.5\ \Omega$$

Hence, current supplied by the battery is

$$I = \frac{V}{R_{eq}} = \frac{6}{1.5} = 4\ \text{A}$$

80. The resistance of the series combination of two resistances is S. When they are joined in parallel, the total resistance is P. If $S = nP$, then the minimum possible value of n is **[AIEEE 2004]**

(a) 4 (b) 3
(c) 2 (d) 1

Exp. **(a)**

Let resistances be R_1 and R_2. Then,

$$S = R_1 + R_2$$

and $$P = \frac{R_1R_2}{R_1 + R_2}$$

$$\therefore \quad (R_1 + R_2) = \frac{n \times R_1R_2}{R_1 + R_2} \qquad [\because S = nP]$$

or $$(R_1 + R_2)^2 = nR_1R_2$$

$$\Rightarrow \quad n = \left[\frac{R_1^2 + R_2^2 + 2R_1R_2}{R_1R_2}\right]$$

$$= \left[\frac{R_1}{R_2} + \frac{R_2}{R_1} + 2\right]$$

We know,

Arithmetic Mean ≥ Geometric Mean

$$\frac{\frac{R_1}{R_2} + \frac{R_2}{R_1}}{2} \geq \sqrt{\frac{R_1}{R_2} \times \frac{R_2}{R_1}}$$

$$\Rightarrow \quad \frac{R_1}{R_2} + \frac{R_2}{R_1} \geq 2$$

So, n (minimum value) $= 2 + 2 = 4$

81. In a meter bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If $X < Y$, then where will be the new position of the null point from the same end, if one decides to balance a resistance of $4X$ against Y? **[AIEEE 2004]**

(a) 50 cm (b) 80 cm
(c) 40 cm (d) 70 cm

Exp. **(a)**

Meter bridge is an arrangement which works on Wheatstone's principle, so the balancing condition is

$$\frac{R}{S} = \frac{l_1}{l_2}$$

where, $l_2 = 100 - l_1$

Case I $R = X, S = Y, l_1 = 20$ cm,

$l_2 = 100 - 20 = 80$ cm

$$\therefore \quad \frac{X}{Y} = \frac{20}{80} \qquad \ldots(i)$$

Case II Let the position of null point be obtained at a distance l from same end.

$\therefore$ $R = 4X, \quad S = Y,$

$l_1 = l, l_2 = 100 - l$

So, from Eq. (i), we get

$$\frac{4X}{Y} = \frac{l}{100 - l}$$

$$\frac{X}{Y} = \frac{l}{4(100 - l)} \qquad \ldots(ii)$$

Therefore, from Eqs. (i) and (ii), we get

$$\frac{l}{4(100 - l)} = \frac{20}{80} \Rightarrow \frac{l}{4(100 - l)} = \frac{1}{4}$$

or $l = 100 - l$

or $2l = 100$

Hence, $l = 50$ cm

82. The thermistors are usually made of **[AIEEE 2004]**

(a) metals with low temperature coefficient of resistivity
(b) metals with high temperature coefficient of resistivity
(c) metal oxides with high temperature coefficient of resistivity
(d) semiconducting materials having low temperature coefficient of resistivity

Exp. **(c)**

Thermistors are the resistors made up of semiconductors whose resistance decreases with the increase in temperature. This implies that they have negative and high temperature coefficient of resistivity.

They are usually made of metal oxides with high temperature coefficient of resistivity.

83. Time taken by a 836 W heater to heat 1 L of water from 10°C to 40°C is **[AIEEE 2004]**

(a) 50 s (b) 100 s (c) 150 s (d) 200 s

Exp. **(c)**

Let time taken in boiling the water by the heater be t second. Then,

$$Q = ms\,\Delta T$$

$$\Rightarrow \quad \frac{Pt}{J} = ms\,\Delta T$$

$$\therefore \quad \frac{836}{4.2}t = 1 \times 1000(40 - 10)$$

or $$\frac{836}{4.2}t = 1000 \times 30$$

or $$t = \frac{1000 \times 30 \times 4.2}{836} = 150 \text{ s}$$

84. The thermo-emf of a thermocouple varies with the temperature θ of the hot junction as $E = a\theta + b\theta^2$ in volts, where the ratio a/b is 700°C. If the cold junction is kept at 0°C, then the neutral temperature is **[AIEEE 2004]**

(a) 700°C
(b) 350°C
(c) 1400°C
(d) no neutral temperature is possible for this thermocouple

Exp. **(d)**

$$E = a\theta + b\theta^2 \quad \text{[given]}$$

For neutral temperature (θ_n),

$$\frac{dE}{d\theta} = 0$$

$$\Rightarrow \quad a + 2b\theta_n = 0$$

or $$\theta_n = -\frac{a}{2b}$$

$$\therefore \quad \theta_n = -\frac{700}{2} \quad \left[\because \frac{a}{b} = 700°\text{C}\right]$$

$$= -350°\text{C} < 0°\text{C}$$

But neutral temperature can never be negative (less than zero) *i.e.*, $\theta_n \not< 0°$C.

Hence, no neutral temperature is possible for this thermocouple.

85. The electrochemical equivalent of metal is 3.3×10^{-7} kg C^{-1}. The mass of the metal liberated at the cathode when a 3A current is passed for 2 s, will be **[AIEEE 2004]**

(a) 19.8×10^{-7} kg (b) 9.9×10^{-7} kg
(c) 6.6×10^{-7} kg (d) 1.1×10^{-7} kg

Exp. **(a)**

Mass of substance liberated at cathode

$$m = zIt$$

where, z = electro-chemical equivalent

$$= 3.3 \times 10^{-7} \text{ kg C}^{-1}$$

I = current flowing = 3 A

t = 2 s

$$\therefore \quad m = 3.3 \times 10^{-7} \times 3 \times 2$$

$$= 19.8 \times 10^{-7} \text{ kg}$$

86. The length of a wire of a potentiometer is 100 cm and the emf of its stand and cell is E volt. It is employed to measure the emf of a battery whose internal resistance is 0.5 Ω. If the balance point is obtained at l = 30 cm from the positive end, the emf of the battery is **[AIEEE 2003]**

(a) $\frac{30E}{100.5}$ (b) $\frac{30E}{100 - 0.5}$

(c) $\frac{30(E - 0.5I)}{100}$, where I is the current in the potentiometer wire

(d) $\frac{30E}{100}$

Exp. **(d)**

$$V \propto l$$

$$\therefore \quad \frac{V}{E} = \frac{l}{L}$$

where, l = balance point distance

L = length of potentiometer wire

or $$V = \frac{l}{L}E$$

or $$V = \frac{30 \times E}{100} = \frac{30}{100}E$$

87. The thermo-emf of a thermocouple is 25 V/°C at room temperature. A galvanometer of 40 Ω resistance, capable of detecting current as low as 10^{-5} A, is connected with the thermocouple. The smallest temperature difference that can be detected by this system is **[AIEEE 2003]**

(a) 16°C (b) 12°C (c) 8°C (d) 20°C

Exp. **(a)**

Thermo-emf of thermocouple = 25 μV/°C

Let θ be the smallest temperature difference. Therefore, after connecting the thermocouple with the galvanometer, thermo-emf

$$E = (25\mu\text{V/°C}) \times \theta(°\text{C})$$

$$= 25\theta \times 10^{-6} \text{ V}$$

Potential drop developed across the galvanometer = $IR = 10^{-5} \times 40 = 4 \times 10^{-4}$ V

$\therefore \quad 4 \times 10^{-4} = 25\theta \times 10^{-6}$

or $\quad \theta = \frac{4}{25} \times 10^{2} = 16°C$

88. The negative Zn pole of Daniell cell, sending a constant current through a circuit, decreases in mass by 0.13 g in 30 min. If the electrochemical equivalent of Zn and Cu are 32.5 and 31.5 respectively, the increase in the mass of the positive Cu pole in this time is **[AIEEE 2003]**

(a) 0.180 g (b) 0.141 g
(c) 0.126 g (d) 0.242 g

Exp. **(c)**

According to Faraday's law of electrolysis,

$$m = zit \Rightarrow \frac{m_{Zn}}{m_{Cu}} = \frac{z_{Zn}}{z_{Cu}}$$

$$\therefore \quad \frac{0.13}{m_{Cu}} = \frac{32.5}{31.5} \text{ or } m_{Cu} = 0.126 \text{ g}$$

89. A 3 V battery with negligible internal resistance is connected in a circuit as shown in the figure. **[AIEEE 2003]**
The current I in the circuit will be

(a) 1 A (b) 1.5 A (c) 2 A (d) $\frac{1}{3}$ A

Exp. **(b)**

Resistance in the arms AC and BC are in series.

$\therefore \quad R' = 3 + 3 = 6\ \Omega$

Now, R' and 3 Ω are in parallel.

$$\therefore \quad R_{eq} = \frac{6 \times 3}{6 + 3} = 2\ \Omega$$

Now, $\quad V = IR \quad \Rightarrow I = \frac{3}{2} = 1.5$ A

90. The length of a given cylindrical wire is increased by 100%. Due to the consequent decrease in diameter, the change in the resistance of the wire will be **[AIEEE 2003]**

(a) 200% (b) 100% (c) 50% (d) 300%

Exp. **(d)**

Given, $\quad l' = l + 100\% l = 2l$

Initial volume = Final volume

i.e., $\quad \pi r^2 l = \pi r'^2 l'$

or $\quad r'^2 = \frac{r^2 l}{l'} = r^2 \times \frac{l}{2l}$

or $\quad r'^2 = \frac{r^2}{2}$

$$\therefore \quad R' = \rho \frac{l'}{A'} = \rho \frac{2l}{\pi r'^2} \qquad \left[\because R = \frac{\rho l}{A}\right]$$

$$= \frac{\rho \cdot 4l}{\pi r^2} = 4R$$

Thus, $\quad \Delta R = R' - R = 4R - R = 3R$

$$\therefore \quad \%\Delta R = \frac{3R}{R} \times 100\% = 300\%$$

Alternate Solution

By the shortcut formula,

$$\%\text{ Change} = x + y + \frac{xy}{100}$$

Valid only for 2 variables in terms of percentage.

As $\quad R \propto l^2\%$

Increase in resistance

$$= 100 + 100 + \frac{100 \times 100}{100} = 300\%$$

91. A 220 V–1000 W bulb is connected across a 110 V mains supply. The power consumed will be **[AIEEE 2003]**

(a) 750 W (b) 500 W (c) 250 W (d) 1000 W

Exp. **(c)**

$$R = \frac{V^2}{P} = \frac{(220)^2}{1000}$$

where, V and P are denoting rated voltage and power, respectively.

$$\therefore \quad P_{consumed} = \frac{V_{applied}^2}{R}$$

$$= \frac{110 \times 110}{220 \times 220} \times 1000$$

$$= 250 \text{ W}$$

92. A wire when connected to 220 V mains supply has power dissipation P_1. Now, the wire is cut into two equal pieces which are connected in parallel to the same supply. Power dissipation in this case is P_2. Then, $P_2 : P_1$ is **[AIEEE 2002]**

(a) 1 (b) 4 (c) 2 (d) 3

Exp. **(b)**

***Case* I**

Using the formula, $P = \frac{V^2}{R}$ ...(i)

where, R is resistance of wire, V is voltage across wire and P is power dissipation in wire

and $R = \frac{\rho l}{A}$ ...(ii)

From Eqs. (i) and (ii), we have

$$P_1 = \frac{V^2}{\rho l/A}$$

or $P_1 = \frac{V^2}{\rho l} \cdot A$ ...(iii)

***Case* II**

Let R_2 be net resistance of two wires in parallel, then

$$R_2 = \frac{R \times R}{R + R} = \frac{R}{2}$$

where, R is the resistance of half wire.

$$\Rightarrow \quad R_2 = \frac{\rho \cdot \left(\frac{l}{2}\right)}{A \cdot 2} = \frac{\rho l}{4A}$$

or $P_2 = \frac{V^2}{\rho l} \cdot 4A$ ...(iv)

Hence, from Eqs. (iii) and (iv), we get

$$\frac{P_1}{P_2} = \frac{1}{4} \Rightarrow \frac{P_2}{P_1} = \frac{4}{1}$$

93. If in the circuit, power dissipation is 150 W, then R is **[AIEEE 2002]**

(a) 2 Ω (b) 6 Ω
(c) 5 Ω (d) 4 Ω

Exp. **(b)**

$$P = \frac{V^2}{R_{net}}$$

$$\therefore \quad 150 = \frac{(15)^2}{\frac{R \cdot 2}{2 + R}} \Rightarrow R = 6\ \Omega$$

94. If θ_i is the inversion temperature, θ_n is the neutral temperature and θ_c is the temperature of the cold junction, then **[AIEEE 2002]**

(a) $\theta_i + \theta_c = \theta_n$
(b) $\theta_i - \theta_c = 2\theta_n$
(c) $\frac{\theta_i + \theta_c}{2} = \theta_n$
(d) $\theta_c - \theta_i = 2\theta_n$

Exp. **(c)**

In Seebeck (thermoelectric) effect, the temperature of hot junction at which the thermo-emf is maximum is called the neutral temperature (θ_n) and the temperature at which thermo-emf changes its sign is called inversion temperature (θ_i).

If θ_c is the temperature of cold junction, then these three are related by the expression

$$\theta_n - \theta_c = \theta_i - \theta_n$$

or $\theta_n = \frac{\theta_i + \theta_c}{2}$

95. The mass of a product liberated on anode in an electrochemical cell depends on **[AIEEE 2002]**

(a) $(It)^{1/2}$ (b) It (c) $\frac{I}{t}$ (d) $I^2 t$

(where, t is the time period for which the current is passed)

Exp. **(b)**

In electrochemical cell, the anode produces the charge q which depends on It.

As $m = zIt$

13

Magnetic Effect *of* Current

1. A thin strip 10 cm long is on an U-shaped wire of negligible resistance and it is connected to a spring of spring constant $0.5\,\text{Nm}^{-1}$ (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillations it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10 Ω and air drag negligible, N will be close to

[JEE Main 2019, 8 April Shift-I]

(a) 1000 (b) 50000 (c) 5000 (d) 10000

Exp. **(c)**

There are two forces on slider.

Spring force = kx

where, k = spring constant.

As the slider is kept in a uniform magnetic field $B = 0.1$ T, hence it will experience a force, i.e.

Magnetic force = Bil

where, l = length of the strip.

Now, using

$$F_{\text{net}} = ma$$

We have,

$$(-kx) + (-Bil) = ma$$

$$\Rightarrow \quad -kx - Bil - ma = 0$$

$$\Rightarrow \quad -kx - \frac{B^2 l^2}{R} \cdot v - ma = 0 \qquad \left[\because i = \frac{Blv}{R}\right]$$

and acceleration, $a = \dfrac{d^2x}{dt^2}$

Hence, the modified equation becomes

$$\Rightarrow \quad \frac{md^2x}{dt^2} + \frac{B^2l^2}{R}\left(\frac{dx}{dt}\right) + kx = 0$$

This is the equation of damped simple harmonic motion.

So, amplitude of oscillation varies with time as

$$A = A_0 e^{-\frac{B^2l^2}{2Rm}t}$$

Now, when amplitude is $\dfrac{A_0}{e}$, then

$$\frac{A_0}{e} = \frac{A_0}{e^{\frac{B^2l^2}{2Rm}t}} \qquad \text{(as given)}$$

$$\Rightarrow \quad \left(\frac{B^2l^2}{2Rm}\right)t = 1 \quad \text{or} \quad t = \frac{2Rm}{B^2l^2}$$

According to the question, magnetic field $B = 0.1$ T, mass of strip $m = 50 \times 10^{-3}$ kg, resistance $R = 10\,\Omega$, $l = 10$ cm $= 10 \times 10^{-2}$ m

$$\therefore t = \frac{2Rm}{B^2 l^2} = \frac{2 \times 10 \times 50 \times 10^{-3}}{(0.1)^2 \times (10 \times 10^{-2})^2}$$

$$= \frac{1}{10^{-4}} = 10000 \text{ s}$$

Given, spring constant, $k = 0.5\ \text{Nm}^{-1}$

Also, time period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{50 \times 10^{-3}}{0.5}} = \frac{2\pi}{\sqrt{10}} \approx 2\text{s}$$

So, number of oscillations is

$$N = \frac{t}{T} = \frac{10000}{2} = 5000$$

2. A circular coil having N turns and radius r carries a current I. It is held in the XZ-plane in a magnetic field $B\hat{\mathbf{i}}$. The torque on the coil due to the magnetic field (in N-m) is

[JEE Main 2019, 8 April Shift-I]

(a) $\frac{Br^2 I}{\pi N}$ (b) $B\pi r^2 IN$ (c) $\frac{B\pi r^2 I}{N}$ (d) Zero

Exp. **(b)**

According to the question, the situation can be drawn as

Let the current I is flowing in anti-clockwise direction, then the magnetic moment of the coil is

$$m = NIA$$

where, N = number of turns in coil

and A = area of each coil $= \pi r^2$.

Its direction is perpendicular to the area of coil and is along Y-axis.

Then, torque on the current coil is

$$\tau = \mathbf{m} \times \mathbf{B} = mB\sin 90° = NIAB$$
$$= NI\pi r^2 B (\text{N-m})$$

3. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing through their mid-point P, at angle $\theta = 45°$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole) **[JEE Main 2019, 8 April Shift-II]**

(a) $\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (b) 0

(c) $\sqrt{2}\left(\frac{\mu_0}{4\pi}\right)\frac{M}{\left(\frac{d}{2}\right)^3} \times qv$ (d) $\left(\frac{\mu_0}{4\pi}\right)\frac{2M}{\left(\frac{d}{2}\right)^3} \times qv$

Exp. **(b)**

Let $2l_1$ and $2l_2$ be the length of dipole X and Y, respectively.

For dipole X, point P lies on its axial line. So, magnetic field strength at P due to X is

$$\mathbf{B}_X = \frac{\mu_0}{4\pi} \cdot \frac{2Mr}{(r^2 - l_1^2)^2}, \text{ along } OP$$

Here, $r = \frac{d}{2}$

Also, $d >> l_1$

$$\Rightarrow \quad |\mathbf{B}_X| = \frac{\mu_0}{4\pi} \cdot \frac{2M(d/2)}{(d/2)^4} = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$$

Similarly, for dipole Y, point P lies on its equatorial line. So, magnetic field strength at P due to Y is

$$\mathbf{B}_Y = \frac{\mu_0}{4\pi} \cdot \frac{2M}{(r^2 + l_2^2)^{3/2}},$$

(along a line perpendicular to $O'P$)

Here, $r = \frac{d}{2}$

Also, $d >> l_2$

$\Rightarrow$ $$|\mathbf{B}_Y| = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$$

Thus, the resultant magnetic field due to X and Y at P is

$$\mathbf{B}_{net} = \mathbf{B}_X + \mathbf{B}_Y$$

Since, $|\mathbf{B}_Y| = |\mathbf{B}_X|$

Thus, the resultant magnetic field ($\mathbf{B}_{net}$) at P will be at 45° with the horizontal.

This means, direction of $\mathbf{B}_{net}$ and velocity of the charged particle is same.

$\therefore$ Force on the charged particle moving with velocity v in the presence of magnetic field which is

$$\mathbf{B} = q(\mathbf{v} \times \mathbf{B}) = q\,|\mathbf{v}||\mathbf{B}|\sin\theta$$

where, θ is the angle between $\mathbf{B}$ and $\mathbf{v}$.

According to the above analysis, we get

$\theta = 0$

$\therefore$ $F = 0$

Thus, magnitude of force on the particle at that instant is zero.

4. Two very long, straight and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure.

These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be **[JEE Main 2019, 8 April Shift-II]**

(a) zero (b) $\frac{+\mu_0 I}{\pi d}(\hat{\mathbf{z}})$

(c) $-\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$ (d) $\frac{\mu_0 I}{2\pi d}(\hat{\mathbf{x}} + \hat{\mathbf{y}})$

Exp. **(a)**

Magnetic field due to an infinitely long straight wire at point P is given as

$$|\mathbf{B}| = \frac{\mu_0 I}{2\pi r}$$

Thus, in the given situation, magnetic field due to wire 1 at point P is

$$\mathbf{B}_1 = \frac{\mu_0}{2\pi}\frac{I}{d}, \otimes$$

Similarly, magnetic field due to wire 2 at point P is

$$\mathbf{B}_2 = \frac{\mu_0}{2\pi}\frac{I}{d}, \odot$$

Resultant field at point P is

$$\mathbf{B}_{net} = \mathbf{B}_1 + \mathbf{B}_2$$

Since, $|\mathbf{B}_1| = |\mathbf{B}_2|$, but they are opposite in direction.

Thus, $\mathbf{B}_{net} = 0$

$\therefore$ Net magnetic field at point P will be zero.

5. A moving coil galvanometer has resistance 50 Ω and it indicates full deflection at 4 mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to **[JEE Main 2019, 9 April Shift-I]**

(a) 40 V (b) 10 V (c) 15 V (d) 20 V

Exp. **(d)**

Given, resistance of galvanometer, $R_g = 50\,\Omega$

Current, $I_g = 4\text{ mA} = 4 \times 10^{-3}$ A

Resistance used in converting a galvanometer in voltmeter, $R = 5\text{ k}\Omega = 5 \times 10^3\,\Omega$

Voltmeter

∴ Maximum current in galvanometer is

$$I_g = \frac{E}{R + R_g}$$

$$\therefore \quad E = I_g(R + R_g)$$
$$= 4 \times 10^{-3} \times (5 \times 10^3 + 50)$$
$$= 5050 \times 4 \times 10^{-3}$$
$$= 20.2 \text{ V} \simeq 20 \text{ V}$$

6. A rectangular coil (dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3A in the clockwise direction, is kept centred at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is **[JEE Main 2019, 9 April Shift-I]**

(a) 0.27 N-m (b) 0.38 N-m
(c) 0.42 N-m (d) 0.55 N-m

Exp. **(a)**

Given,

Area of the rectangular coil,

$$A = 5 \text{ cm} \times 2.5 \text{ cm}$$
$$\Rightarrow \quad A = 12.5 \text{ cm}^2 = 12.5 \times 10^{-4} \text{ m}^2$$

Number of turns, $N = 100$ turns

Current through the coil, $I = 3$ A

Magnetic field applied, $B = 1$ T

Angle between the magnetic field and area vector of the coil, $\theta = 45°$

As we know that, when a coil is tilted by an angle θ in the presence of some external magnetic field, then the net torque experienced by the coil is,

$$\tau = \mathbf{M} \times \mathbf{B} = NI(\mathbf{A} \times \mathbf{B}) = NIAB \sin\theta$$

Substituting the given values, we get

$$\tau = 100 \times 3 \times 12.5 \times 10^{-4} \times 1 \times \sin 45°$$
$$\tau = 0.707 \times 100 \times 3 \times 12.5 \times 10^{-4} \text{ N-m}$$
$$= 2.651 \times 10^{-1} \text{ N-m} \approx 0.27 \text{ N-m}$$

7. A rigid square loop of side a and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to the wire will be **[JEE Main 2019, 9 April Shift-I]**

(a) repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$
(b) attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$
(c) zero
(d) repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

Exp. **(d)**

> **Key Idea** Net force experienced by two wires separated by same distance is attractive, if current flow in them in same direction. However, this force is repulsive in nature, if current in them flows in opposite direction.

Force on a wire 1 in which current I_1 is flowing due to another wire 2 which are separated by a distance r is given as

$$\mathbf{F} = I_1(l \times \mathbf{B}_2)$$

or $$F = \frac{\mu_0 I_1 I_2}{2\pi r} \cdot l \sin\theta \qquad \left[\because \mathbf{B}_2 = \frac{\mu_0 I_2}{2\pi r}\right]$$

Thus, the given square loop can be drawn as shown below

$$F_{AB} = \frac{\mu_0 I_1}{2\pi a} \cdot I_2 a$$

(away from wire PQ)

$$F_{BC} = F_{AD} = 0 \qquad [\because \theta = 0°]$$
$$F_{CD} = \frac{\mu_0 I_1}{2\pi(2a)} \cdot I_2 a$$
$$= \frac{\mu_0 I_1}{4\pi a} \cdot I_2 a \qquad \text{(towards the wire } PQ\text{)}$$

$$\therefore \quad F_{net} = F_{AB} - F_{CD}$$
$$= \frac{\mu_0 I_1 I_2}{2\pi} - \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(away from wire)}$$
$$= \frac{\mu_0 I_1 I_2}{4\pi} \qquad \text{(repulsive in nature)}$$

8. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0-0.5 A? **[JEE Main 2019, 9 April Shift-II]**

(a) 0.2 ohm (b) 0.5 ohm
(c) 0.002 ohm (d) 0.02 ohm

Exp. **(a)**

> **Key Idea** An ammeter is a type of galvanometer with a shunt connected in parallel to the galvanometer.

Ammeter circuit is shown in the figure below

So, $I_g G = (I - I_g) S$

Here, $I_g = 0.002$ A,

$I = 0.5$ A,

$G = 50\ \Omega$

So, shunt resistance required is

$$S = \frac{I_g G}{I - I_g} = \frac{0.002 \times 50}{(0.5 - 0.002)} \approx 0.2\ \Omega$$

9. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in tesla) is approximately **[JEE Main 2019, 9 April Shift-II]**

(a) 10^{-3} (b) 10^{-4} (c) 10^{-1} (d) 10^{-2}

Exp. **(a)**

In a moving coil galvanometer in equilibrium, torque on coil due to current is balanced by torque of torsion band.

As, torque on coil,

$$\tau = \mathbf{M} \times \mathbf{B} = NIAB \sin\alpha$$

where, B = magnetic field strength,

I = current,

N = number of turns of coil

Since, plane of the coil is parallel to the field.

$\therefore \quad \alpha = 90^o \Rightarrow \tau = NIBA$

Torque of torsion band, $T = k\theta$

where, k = torsion constant of torsion band

and θ = deflection of coil in radians or angle of twist of restoring torque.

$$\therefore \quad BINA = k\theta \text{ or } B = \frac{k\theta}{INA} \quad \ldots \text{(i)}$$

Here, $k = 10^{-6}$ N - m/rad,

$I = 1 \times 10^{-3}$ A,

$N = 175$,

$A = 1\ cm^2 = 1 \times 10^{-4}\ m^2$

$\theta = 1^o = \frac{\pi}{180}$ rad

Substituting values in Eq. (i), we get

$$B = \frac{10^{-6} \times 22}{1 \times 10^{-3} \times 175 \times 7 \times 180 \times 10^{-4}}$$

$$= 0.998 \times 10^{-3}$$

$$\simeq 10^{-3}\ T$$

10. A uniformly charged ring of radius $3a$ and total charge q is placed in xy-plane centred at origin. A point charge q is moving towards the ring along the Z-axis and has speed v at $z = 4a$. The minimum value of v such that it crosses the origin is **[JEE Main 2019, 10 April Shift-I]**

(a) $\sqrt{\frac{2}{m}}\left(\frac{1}{5}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (b) $\sqrt{\frac{2}{m}}\left(\frac{4}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

(c) $\sqrt{\frac{2}{m}}\left(\frac{1}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$ (d) $\sqrt{\frac{2}{m}}\left(\frac{2}{15}\frac{q^2}{4\pi\varepsilon_0 a}\right)^{1/2}$

Exp. **(d)**

Potential at any point at distance x from the centre of the ring is given by

$$V_p = \frac{Kq}{\sqrt{R^2 + x^2}}$$

Given, $R = 3a$ and $x = 4a$

$$\therefore \quad V_P = \frac{Kq}{\sqrt{9a^2 + 16a^2}} = \frac{Kq}{5a} \quad \ldots \text{(i)}$$

At centre, $x = 0$

So, potential at centre is

$$V_C = \frac{Kq}{R} = \frac{Kq}{3a} \quad \ldots \text{(ii)}$$

Now, energy required to get this charge from $x = 4a$ to the centre is

$$\Delta U = q\ \Delta V = q\ [V_C - V_P] = q\left[\frac{Kq}{3a} - \frac{Kq}{5a}\right]$$

$$= \frac{Kq^2}{a}\left[\frac{1}{3} - \frac{1}{5}\right]$$

$$\Delta U = \frac{2}{15}\frac{Kq^2}{a} \quad \ldots \text{(iii)}$$

This energy must be equal to (or less than) the kinetic energy of the charge, i.e.

$$\frac{1}{2}mv^2 \geq \frac{2}{15}\frac{Kq^2}{a}$$

So, minimum energy required is

$$\frac{1}{2}mv^2 = \frac{2}{15} \times \frac{1}{4\pi\varepsilon_0} \times \frac{q^2}{a} \quad (\text{put } K = 1/4\pi\varepsilon_0)$$

$\therefore$ Minimum velocity,

$$v^2 = \frac{2}{m} \times \frac{2}{4\pi\varepsilon_0} \times \frac{q^2}{15a} \text{ or } v = \sqrt{\frac{2}{m}} \times \sqrt{\frac{2q^2}{4\pi\varepsilon_0 a \times 15}}$$

11. Two wires A and B are carrying currents I_1 and I_2 as shown in the figure. The separation between them is d. A third wire C carrying a current I is to be kept parallel to them at a distance x from A such that the net force acting on it is zero. The possible values of x are **[JEE Main 2019, 10 April Shift-I]**

(a) $x = \left(\frac{I_2}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

(b) $x = \left(\frac{I_1}{I_1 - I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 + I_2}\right)d$

(c) $x = \left(\frac{I_1}{I_1 + I_2}\right)d$ and $x = \left(\frac{I_2}{I_1 - I_2}\right)d$

(d) $x = \pm\frac{I_1 d}{(I_1 - I_2)}$

Exp. **(*d*)**

Net force on the third wire, carrying current I in the following first case is

$$F_{13} + F_{23} = 0$$

Using thumb rule, direction of **B** at inside region of wires A and B will be same.

$$\therefore \frac{\mu_0 I_1 I}{2\pi x} + \frac{\mu_0 I_2 I}{2\pi(d - x)} = 0$$

$$\Rightarrow \quad \frac{I_1}{x} + \frac{I_2}{d - x} = 0 \Rightarrow \frac{I_1}{x} = \frac{I_2}{x - d}$$

or $\quad (x - d)\, I_1 = x\, I_2 \Rightarrow x\,(I_1 - I_2) = dI_1$

$$\Rightarrow \quad x = \frac{I_1}{(I_1 - I_2)} \cdot d \quad \ldots \text{(i)}$$

Second case of balanced force can be as shown

Using thumb rule, directions of **B** at any point on wires A and B will be opposite, so net force,

$$\frac{\mu_0 I_1 I}{2\pi x} - \frac{\mu_0 I_2 I}{2\pi(d + x)} = 0$$

or $\quad \frac{I_1}{x} - \frac{I_2}{(d + x)} = 0$

$$\Rightarrow \quad \frac{I_1}{x} = \frac{I_2}{d + x}$$

$$\Rightarrow \quad (d + x)\, I_1 = x\, I_2$$

$$\Rightarrow \quad (I_2 - I_1)\, x = dI_1$$

$$\Rightarrow \quad x = -\frac{I_1}{(I_1 - I_2)} \cdot d \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), it is clear that

$$x = \pm\frac{I_1}{(I_1 - I_2)}d$$

12. A proton, an electron and a helium nucleus, have the same energy. They are in circular orbits in a plane due to magnetic field perpendicular to the plane.

Let r_p, r_e and r_{He} be their respective radii, then **[JEE Main 2019, 10 April Shift-I]**

(a) $r_e < r_p = r_{He}$ (b) $r_e > r_p = r_{He}$
(c) $r_e < r_p < r_{He}$ (d) $r_e > r_p > r_{He}$

Exp. **(c)**

When a moving charged particle is placed in a magnetic field **B**.

Then, the net magnetic force acting on it is

$$\mathbf{F}_m = q(\mathbf{v} \times \mathbf{B})$$

$$\mathbf{F}_m = q\, vB\sin\theta$$

Here, $\theta = 90°$

$$\mathbf{F}_m = q\, vB$$

Also, due to this net force, the particle transverses a circular path, whose necessary centripetal force is being provided by F_m, i.e.

$$\frac{mv^2}{r} = q\,vB \Rightarrow r = \frac{mv^2}{qvB} = \frac{mv}{qB} \Rightarrow r \propto m$$

So, for electron,

$$r_e = \frac{m_e v}{eB} \text{ or } r_e \propto m_e$$

For proton,

$$r_p = \frac{m_p v}{eB} \text{ or } r_p \propto m_p$$

For He-particle,

$$r_{He} = \frac{4m_p v}{2eB} = \frac{2m_p v}{eB}$$

Clearly, $r_{He} > r_p$ $(\because r_{He} = 2r_p)$

and we know that, $m_p > m_e$ $\left[\begin{array}{l}\because m_p \approx 10^{-27}\text{kg}, \\ m_e \approx 10^{-31}\text{ kg}\end{array}\right]$

$\therefore$ $r_p > r_e \Rightarrow r_{He} > r_p > r_e$

13. A moving coil galvanometer allows a full scale current of 10^{-4} A. A series resistance of 2 MΩ is required to convert the above galvanometer into a voltmeter of range 0-5 V. Therefore, the value of shunt resistance required to convert the above galvanometer into an ammeter of range 0.10 mA is **[JEE Main 2019, 10 April Shift-I]**

(a) 100 Ω (b) 500 Ω
(c) 200 Ω (d) 10 Ω

Exp. **(*)**

Given data,

$$I = 10^{-4}\text{ A},$$

$$R_S = 2\text{ M}\Omega = 2 \times 10^6\,\Omega,$$

$$V_{max} = 5\text{ V}$$

Let internal resistance of galvanometer is R_G.

Voltmeter

Then,

$$I \times R_S + I \times R_G = V_{max}$$

$$\Rightarrow \quad 2 \times 10^6 \times 10^{-4} + 10^{-4} \times R_G = 5$$

$$\Rightarrow \quad 10^{-4}\, R_G = 5 - 200 = -195$$

or $R_G = -195 \times 10^4\,\Omega$

Resistance cannot be negative.

$\therefore$ No option is correct.

14. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop (in A-m) will be **[JEE Main 2019, 10 April Shift-II]**

(a) $\frac{4m}{\pi}$ (b) $\frac{3m}{\pi}$ (c) $\frac{2m}{\pi}$ (d) $\frac{m}{\pi}$

Exp. **(a)**

> **Key Idea** Magnetic dipole moment of a current carrying loop is
> $m = IA$ (A - m^2)
> where, I = current in loop and A = area of loop.

Let the given square loop has side a, then its magnetic dipole moment will be

$$m = Ia^2$$

When square is converted into a circular loop of radius r,

Then, wire length will be same in both area,

$$\Rightarrow \quad 4a = 2\pi r \quad \Rightarrow \quad r = \frac{4a}{2\pi} = \frac{2a}{\pi}$$

Hence, area of circular loop formed is, $A' = \pi r^2$

$$= \pi\left(\frac{2a}{\pi}\right)^2 = \frac{4a^2}{\pi}$$

Magnitude of magnetic dipole moment of circular loop will be

$$m' = IA' = I\frac{4a^2}{\pi}$$

Ratio of magnetic dipole moments of both shapes is,

$$\frac{m'}{m} = \frac{I \cdot \frac{4a^2}{\pi}}{Ia^2} = \frac{4}{\pi} \Rightarrow m' = \frac{4m}{\pi}(\text{A - m})$$

15. The magnitude of the magnetic field at the centre of an equilateral triangular loop of side 1 m which is carrying a current of 10 A is [Take, $\mu_0 = 4\pi \times 10^{-7}\,\text{NA}^{-2}$]

[JEE Main 2019, 10 April Shift-II]

(a) 9μT (b) 1μT (c) 3μT (d) 18μT

Exp. **(d)**

For a current carrying wire, from result obtained by Biot-Savart's law, magnetic field at a distance r is given by

$$B = \frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Now, in given case,

Due to symmetry of arrangement, net field at centre of triangle is

B_{net} = Sum of fields of all wires (sides)

$$= 3 \times \frac{\mu_0 i}{4\pi r}(\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = 60°$

$\therefore \sin\theta_1 = \sin\theta_2 = \frac{\sqrt{3}}{2}, i = 10\text{A}, \frac{\mu_0}{4\pi} = 10^{-7}\text{NA}^{-2}$

and $r = \frac{1}{3} \times$ altitude

$$= \frac{1}{3} \times \frac{\sqrt{3}}{2} \times \text{sides length} = \frac{1}{2\sqrt{3}} \times 1 = \frac{1}{2\sqrt{3}}\text{m}$$

So,

$$B_{net} = \frac{3 \times 10^{-7} \times 10 \times 2\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2\sqrt{3}}\right)} = 18 \times 10^{-6}\ \text{T}$$

$\Rightarrow B_{net} = 18\,\mu\text{T}$

16. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2$).

[JEE Main 2019, 12 April Shift-I]

(a) 2×10^{-6} C (b) 3×10^{-5} C

(c) 4×10^{-5} C (d) 7×10^{-6} C

Exp. **(b)**

Given, $\mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2$,

$\omega = 40\pi$ rad/s

$B_{\text{at centre}} = 3.8 \times 10^{-9}$ T

and $R = 10$ cm $= 0.1$ m

Now, we know that, magnetic field at the centre of a current carrying ring is given by

$$B = \frac{\mu_0 I}{2r} \quad \ldots(\text{i})$$

Here, I can be determined by flow of charge per rotation, i.e.

$$I = \frac{Q}{T} \quad \ldots(\text{ii})$$

Here, $T = \frac{2\pi}{\omega}$

$$\Rightarrow \quad I = \frac{Q\omega}{2\pi} \quad \ldots(\text{iii})$$

By putting value of I from Eq. (iii) to Eq. (i), we get

$$B = \frac{\mu_0 Q\omega}{2r \times 2\pi} \text{ or } Q = \frac{2Br \times 2\pi}{\mu_0\omega}$$

$$= \frac{2 \times 3.8 \times 10^{-9} \times 0.1 \times 2\pi}{4\pi \times 10^{-7} \times 40\pi}$$

$$= \frac{2 \times 3.8 \times 0.1}{2 \times 40\pi} \times 10^{-2}$$

$$= 0.003022 \times 10^{-2} \text{ C}$$

$$= 3.022 \times 10^{-5} \text{ C}$$

or $\quad Q = 3 \times 10^{-5}$ C

17. A magnetic compass needle oscillates 30 times per minute at a place, where the dip is 45° and 40 times per minute, where the dip is 30°. If B_1 and B_2 are respectively, the total magnetic field due to the earth at the two places, then the ratio $\frac{B_1}{B_2}$ is best given by

[JEE Main 2019, 12 April Shift-I]

(a) 1.8 T (b) 0.7 T
(c) 3.6 T (d) 2.2 T

Exp. **(b)**

Given, at first place, angle of dip, $\theta_1 = 45^\circ$

Time period, $T_1 = \frac{60}{30} = 2$ s

At second place, angle of dip, $\theta_2 = 30^\circ$

Time period, $T_2 = \frac{60}{40} = \frac{3}{2}$ s

Now, at first place,

$$B_{H_1} = B_1 \cos\theta_1 = B_1 \cos 45^\circ = \frac{B_1}{\sqrt{2}} \quad \ldots(i)$$

and at second place,

$$B_{H_2} = B_2 \cos\theta_2 = B_2 \cos 30^\circ = \frac{\sqrt{3}}{2} B_2 \quad \ldots(ii)$$

Also, time period of a magnetic needle is given by

$$T = 2\pi \sqrt{\frac{I}{MB_H}} \quad \ldots(iii)$$

$$\therefore \quad T \propto \sqrt{\frac{1}{B_H}} \text{ or } \frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}} \quad \ldots(iv)$$

By putting the values from Eqs. (i) and (ii) into Eq. (iv), we get

$$\frac{2}{\frac{3}{2}} = \sqrt{\frac{\sqrt{3}\frac{B_2}{2}}{\frac{B_1}{\sqrt{2}}}} \text{ or } \left(\frac{4}{3}\right)^2 = \frac{\sqrt{3} \times \sqrt{2}\, B_2}{2B_1}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{\sqrt{3} \times \sqrt{2}}{2} \times \frac{9}{16}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9\sqrt{3}}{16\sqrt{2}}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = \frac{9 \times 1.732}{16 \times 1.414} = \frac{15.588}{22.624}$$

$$\Rightarrow \quad \frac{B_1}{B_2} = 0.689 \approx 0.7 \text{ T}$$

18. A galvanometer of resistance 100 Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges of 0-2 V, 0-10 V and 0-20 V. The appropriate circuit to do so is **[JEE Main 2019, 12 April Shift-I]**

Exp. **(c)**

Given, divisions in scale of galvanometer,

$$n = 50$$

Sensitivity of galvanometer,

$$\frac{I_g}{n} = 20\,\mu\text{A / division}$$

∴ Current in galvanometer,

$$I_g = \frac{I_g}{n} \times n = 20\mu\text{A} \times 50$$

$$\Rightarrow \quad I_g = 1000\,\mu\text{A} = 1 \text{ mA}$$

Now, for R, it should be converted into 2V voltmeter.

$\therefore \quad V_1 = I_g\ (R_1 + G)$

$2 = 10^{-3}(R_1 + 100)$

$\Rightarrow \quad 2000 = R_1 + 100$

$\therefore \quad R_1 = 1900\,\Omega$ ...(i)

For R_2, it should be converted into 10V voltmeter.

$\therefore \quad V_2 = I_g\ [(R_1 + R_2) + G]$

$\Rightarrow \quad 10 = 10^{-3}\ [(R_1 + R_2) + 100]$

$\Rightarrow \quad 10000 = R_1 + R_2 + 100 = 2000 + R_2$

$\therefore \quad R_2 = 8000\,\Omega$ ...(ii)

For R_3, it should be converted into 20V voltmeter.

$\therefore \quad V_3 = I_g[(R_1 + R_2 + R_3 + G]$

$\Rightarrow \quad 20 = 10^{-3}[1900 + 8000 + R_3 + 100]$

$\Rightarrow \quad 20000 = R_3 + 10000$

$\therefore \quad R_3 = 10000\ \Omega$ ...(iii)

From Eqs. (i), (ii) and (iii), it is clear that option (c) is correct.

19. An electron moving along the X-axis with an initial energy of 100 eV, enters a region of magnetic field $\mathbf{B} = (1.5 \times 10^{-3}\,\text{T})\,\hat{\mathbf{k}}$ at S (see figure). The field extends between $x = 0$ and $x = 2$ cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is

(Take, electron's charge $= 1.6 \times 10^{-19}$ C, mass of electron $= 9.1 \times 10^{-31}$ kg)

[JEE Main 2019, 12 April Shift-II]

(a) 11.65 cm (b) 12.87 cm
(c) 1.22 cm (d) 2.25 cm

Exp. **(*)**

When electron enters the region of magnetic field, it experiences a Lorentz force which rotates electron in a circular path of radius R.

So, Lorentz force acts like a centripetal force and we have

$$\frac{mv^2}{R} = Bqv$$

where, m = mass of electron,

q = charge of electron, v = speed of electron,

R = radius of path,

and B = magnetic field intensity.

Radius of path of electron,

$$R = \frac{mv}{Bq}$$

Now, from geometry of given arrangement, comparing values of $\tan\theta$, we have

$$\tan\theta = \frac{L}{R} = \frac{d}{D} \Rightarrow d = \frac{LD}{R} = \frac{Bq\ LD}{mv}$$

$$\Rightarrow \quad d = \frac{BqLD}{\sqrt{2mk}} \qquad [\because mv = \sqrt{2mk}]$$

where, k = kinetic energy of electron

Here, $B = 1.5 \times 10^{-3}$ T,

$q = 1.6 \times 10^{-19}$ C, $L = 2 \times 10^{-2}$ m, $D = 6 \times 10^{-2}$ m,

$m = 9.1 \times 10^{-31}$ kg, $k = 100 \times 1.6 \times 10^{-19}$ J

So,

$$d = \frac{(1.5 \times 10^{-3} \times 1.6 \times 10^{-19} \times 2 \times 10^{-2} \times 6 \times 10^{-2})}{\sqrt{(2 \times 9.1 \times 10^{-31} \times 100 \times 1.6 \times 10^{-19})}}$$

$$= \frac{28.8 \times 10^{-26}}{\sqrt{29.12 \times 10^{-48}}} = \frac{28.8 \times 10^{-26}}{5.39 \times 10^{-24}} = 5.34 \times 10^{-2}\text{m}$$

$= 5.34$ cm

No option is matching.

20. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to $I_0 (I_0 > I_g)$ by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to V $(V = GI_0)$ by connecting a series resistance R_V to it. Then, **[JEE Main 2019, 12 April Shift-II]**

(a) $R_A R_V = G^2 \left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{(I_0 - I_g)}\right)^2$

(b) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

(c) $R_A R_V = G^2 \left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$

(d) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$

Exp. **(b)**

To use galvanometer as an ammeter, a low resistance in parallel is used.

In ammeter, if I_g = full scale deflection current, then equating potential drops across points marked AB, we have

$$V_{AB} = I_g G = (I_0 - I_g) R_A$$

$$\Rightarrow \quad R_A = \frac{I_g G}{I_0 - I_g} \quad \ldots(i)$$

Here, G = resistance of galvanometer coil.

When a galvanometer is used as a voltmeter, a high resistance (R_V) in series is used.

Equating potential across point AB,

$$V_{AB} = (G + R_V) I_g$$

But $\quad V_{AB} = I_0 G \quad$ (given)

So, $\quad I_0 G = (G + R_V) I_g$

$$\Rightarrow \quad R_V = \frac{(I_0 - I_g) G}{I_g} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we have

$$\frac{R_A}{R_V} = \frac{\left(\frac{I_g G}{I_0 - I_g}\right)}{\frac{(I_0 - I_g) G}{I_g}} = \frac{I_g^2}{(I_0 - I_g)^2}$$

and $\quad R_A \times R_V = \frac{I_g G}{(I_0 - I_g)} \times \frac{(I_0 - I_g) G}{I_g} = G^2$

21. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A (See figure). (Take, $\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2}) **[JEE Main 2019, 12 April Shift-II]**

(a) 2.0×10^{-5} T (b) 1.5×10^{-5} T

(c) 3.0×10^{-5} T (d) 2.5×10^{-5} T

Exp. **(b)**

The given figure can be drawn as shown below

By Biot-Savart's law, magnetic field due to a wire segment at point P is

$$B = \frac{\mu_0 I}{4\pi d} (\sin\theta_1 + \sin\theta_2)$$

Here, $\theta_1 = \theta_2 = \theta$ (say)

Then, $B = \dfrac{\mu_0 I}{4\pi d} \times 2\sin\theta$ ...(i)

From given data,

$I = 5A$,

$\mu = 4\pi \times 10^{-7}\ NA^{-2}$

$d = \sqrt{5^2 - 3^2} = \sqrt{16} = 4\ cm$

$\sin\theta = \dfrac{3}{5}$

On substituting these values in Eq. (i), we get

$$B = \frac{\mu_0 I}{4\pi d} \times 2\sin\theta$$

$$= \frac{4\pi \times 10^{-7} \times 5}{4\pi \times 4 \times 10^{-2}} \times 2 \times \left(\frac{3}{5}\right)$$

$$= \frac{5 \times 2 \times 3}{4 \times 5} \times 10^{-5}$$

$$= 1.5 \times 10^{-5}\ T$$

22. A bar magnet is demagnetised by inserting it inside a solenoid of length 0.2 m, 100 turns and carrying a current of 5.2 A. The coercivity of the bar magnet is

[JEE Main 2019, 9 Jan Shift-I]

(a) 1200 A/m (b) 285 A/m

(c) 2600A/m (d) 520A/m

Exp. **(c)**

Coercivity of a bar magnet is the value of magnetic field intensity (H) that is needed to reduce magnetisation to zero.

Since, for a solenoid magnetic induction is given as,

$$B = \mu_0 nI \quad ...(i)$$

where, n is the number of turns (N) per unit, length (l) and I is the current.

Also, $B = \mu_0 H$ ...(ii)

$\therefore$ From Eqs. (i) and (ii), we get

$$\mu_0 nI = \mu_0 H$$

or $H = nI = \dfrac{N}{l} I$

Substituting the given values, we get

$$H = \frac{100}{0.2} \times 5.2$$

$$= 2600\ A/m$$

Thus, the value of coercivity of the bar magnet is 2600 A/m.

23. An infinitely long current-carrying wire and a small current-carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is $d (d >> a)$. If the loop applies a force F on the wire, then

[JEE Main 2019, 9 Jan Shift-I]

(a) $F \propto \left(\dfrac{a^2}{d^3}\right)$ (b) $F = 0$

(c) $F \propto \left(\dfrac{a}{d}\right)$ (d) $F \propto \left(\dfrac{a}{d}\right)^2$

Exp. **(d)**

In the given condition, the current-carrying loop is at a very large distance from the long current-carrying conducting wire. Thus, it can be considered as a dipole (a magnet with north pole facing in upward direction and south in the downward direction). Suppose the effective length of this dipole be 'l'. Thus, the top view of the condition can be shown in the figure given below.

Now, the net force on the loop (i.e. at the two poles) due to the wire is given as,

$$F_{net} = 2F\cos\theta = 2mB\cos\theta$$

where, m is the pole strength.

From the figure, we have

$$\cos\theta = \frac{\frac{l}{2}}{\sqrt{d^2 + \frac{l^2}{4}}}$$

$$\Rightarrow \qquad F_{\text{net}} = \frac{2mBI}{2\sqrt{d^2 + \frac{l^2}{4}}} \qquad \ldots\text{(i)}$$

Since, the magnetic moment of a loop of radius r is

$$M = IA = I\pi r^2 = ml \qquad \ldots\text{(ii)}$$

and magnetic field due to a straight infinitely long current-carrying conductor at a distance x is

$$B = \frac{\mu_0 I'}{2\pi x} \qquad \ldots\text{(iii)}$$

$\therefore$ Using Eqs. (ii) and (iii), rewriting Eq. (i), we get

$$F_{\text{net}} = \frac{I\pi a^2 \mu_0 I'}{2\pi\left(\sqrt{d^2 + \frac{l^2}{4}}\right)^2}$$

$$\Rightarrow \qquad F_{\text{net}} \propto \frac{a^2}{\left(d^2 + \frac{l^2}{4}\right)}$$

l can be neglected as the loop is kept at very large distance.

or $$F_{\text{net}} \propto \left(\frac{a}{d}\right)^2$$

24. A current loop, having two circular arcs joined by two radial lines as shown in the figure. It carries a current of 10 A. The magnetic field at point O will be close to

[JEE Main 2019, 9 Jan Shift-I]

(a) 1.0×10^{-5} T (b) 1.0×10^{-7} T

(c) 1.5×10^{-7} T (d) 1.5×10^{-5} T

Exp. **(a)**

Key Idea When a point 'P' lies on the axial position of current-carrying conductor, then magnetic field at P is always zero.

From the given figure as shown below.

The magnetic field at point 'O' due to wires PQ and RS will be zero.

Magnetic field due to arc QR at point 'O' will be

$$B_1 = \frac{\theta}{2\pi}\left(\frac{\mu_0 i}{2a}\right)$$

Here, $\theta = 45° = \frac{\pi}{4}$ rad, $i = 10$ A

and $a = 3$ cm $= 3 \times 10^{-2}$ m

$$\Rightarrow \qquad B_1 = \frac{\pi}{2\pi \times 4}\left(\frac{\mu_0 \times 10}{2 \times 3 \times 10^{-2}}\right)$$

$$= \frac{\mu_0 \times 5}{2 \times 12 \times 10^{-2}} = \frac{5 \times \mu_0 \times 10^2}{24}$$

Direction of field B_1 will be coming out of the plane of figure.

Similarly, field at point 'O' due to arc SP will be

$$B_2 = \frac{\pi}{4}\left(\frac{1}{2\pi}\right)\left[\frac{\mu_0 \times 10}{2 \times (2+3) \times 10^{-2}}\right]$$

$$= \frac{\mu_0 \times 5}{2 \times 20 \times 10^{-2}}$$

$$= \frac{\mu_0}{2 \times 4 \times 10^{-2}} = \frac{\mu_0 \times 10^2}{8}$$

Direction of B_2 is going into the plane of the figure.

$\therefore$ The resultant field at O is

$$B = B_1 - B_2 = \frac{1}{2}\left(\frac{5 \times \mu_0}{12 \times 10^{-2}} - \frac{\mu_0}{4 \times 10^{-2}}\right)$$

$$= \frac{1}{2}\left(\frac{5\mu_0 - 3\mu_0}{12 \times 10^{-2}}\right) = \frac{1}{2}\left(\frac{2\mu_0}{12 \times 10^{-2}}\right)$$

$$= \frac{4\pi \times 10^{-7}}{12 \times 10^{-2}} \cong 1 \times 10^{-5}\ \text{T}$$

25. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Take, charge of electron $= 1.6 \times 10^{-19}$ C)

[JEE Main 2019, 9 Jan Shift-II]

(a) 1.6×10^{-19} kg (b) 1.6×10^{-27} kg

(c) 9.1×10^{-31} kg (d) 2.0×10^{-24} kg

Exp. **(d)**

According to given condition, when a particle having charge same as electron move in a magnetic field on circular path, then the force always acts towards the centre and perpendicular to the velocity.

Here, $B = 0.5$ T

R = radius of circular path = 5 cm

Now, the magnetic force is

$$F_m = q(v \times B) = q\ vB \sin 90°$$

$$F_m = qvB \quad \ldots\text{(i)}$$

When the electric field applied, then the particle moves in a straight path, then this is the case of velocity selector.

Here, the electric force on charge,

$$F_e = qE \quad \ldots\text{(ii)}$$

In velocity selector,

$$F_m = F_e$$

$$\Rightarrow \quad qvB = qE \quad \ldots\text{(iii)}$$

Initially particle moves under the magnetic field, So the radius of circular path taken by the particle is

$$R = \frac{mv}{qB} \quad \ldots\text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$m = \frac{qB^2R}{E}$$

$$m = \frac{1.6 \times 10^{-19} \times 0.25 \times 0.5 \times 10^{-2}}{10^2}$$

$$m = 2 \times 10^{-24}\ \text{kg}$$

26. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the centre of the loop (B_L) to that at the centre of the coil (B_C), i.e. $\frac{B_L}{B_C}$ will be

[JEE Main 2019, 9 Jan Shift-I]

(a) $\frac{1}{N}$ (b) N (c) $\frac{1}{N^2}$ (d) N^2

Exp. **(c)**

Let consider the length of first wire is L, then according to question, if radius of loop formed is R_1, then,

For wire 1,

$$L = 2\pi R_1 \Rightarrow R_1 = \frac{L}{2\pi}$$

The magnetic field due to this loop at its centre is

$$B_L = \frac{\mu_0 I}{2R_1} = \frac{\mu_0 I}{2L} \times 2\pi \quad \ldots\text{(i)}$$

Now, several wire is made into a coil of N- turns.

(R_2=radius of coil having N loops)

Thin, $L = N(2\pi R_2) \Rightarrow R_2 = \frac{L}{2\pi N}$

The magnetic field due to this circular coil of N-turns is

$$B_C = \left(\frac{\mu_0 I}{2R_2}\right)N = N \cdot \frac{\mu_0 I \cdot (2\pi N)}{2L} \quad \ldots(ii)$$

Using Eqs. (i) and (ii), the ratio of $\frac{B_L}{B_C}$ is :

$$\Rightarrow \quad \frac{B_L}{B_C} = \frac{\frac{\mu_0 I}{2R_1}}{N\frac{\mu_0 I}{2R_2}}$$

$$= \frac{\frac{\mu_0 I}{2L} \cdot (2\pi)}{\frac{\mu_0 I}{2L} \cdot (2\pi)N^2} = \frac{1}{N^2}$$

27. An insulating thin rod of length l has a linear charge density $\rho(x) = \rho_0 \frac{x}{l}$ on it. The rod is rotated about an axis passing through the origin $(x = 0)$ and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is **[JEE Main 2019, 10 Jan Shift-I]**

(a) $n\rho l^3$ (b) $\pi n \rho l^3$

(c) $\frac{\pi}{3} n \rho l^3$ (d) $\frac{\pi}{4} n \rho l^3$

Exp. **(d)**

> **Key Idea** A rotating charge constitutes a current. Hence, a rotating charged rod behaves like a current carrying coil. If charge q rotates with a frequency n, then equivalent current is $I = qn$ and magnetic moment associated with this current is $M = IA$ where, A = area of coil or area swept by rotating rod.

Let dq be the charge on dx length of rod at a distance x from origin as shown in the figure below.

The magnetic moment dm of this portion dx is given as

$$dm = (dl)\, A$$

$$dm = ndqA \qquad [\because I = qn,\ \therefore dl = n \cdot dq]$$

$$= n\rho\, dx\, A$$

where, ρ = charge density of rod $= \rho_0 \frac{x}{l}$.

So, $$dm = \frac{n\rho_0 x\, dx\, \pi x^2}{l}$$

$$= \frac{\pi n \rho_0}{l} \cdot x^3 \cdot dx$$

Total magnetic moment associated with rotating rod is sum of all the magnetic moments of such differentiable elements of rod.

So, magnetic moment associated with complete rod is

$$M = \int_{x=0}^{x=l} dm = \int_0^l \frac{\pi n \rho_0}{l} \cdot x^3\, dx$$

$$= \frac{\pi n \rho_0}{l} \cdot \int_0^l x^3\, dx = \frac{\pi n \rho_0}{l}\left[\frac{x^4}{4}\right]_0^l = \frac{\pi n \rho_0 l^3}{4}$$

at $x = l$

$\rho = \rho_0$

$$\therefore \quad M = \frac{\pi}{4} n \rho l^3$$

28. A solid metal cube of edge length 2 cm is moving in a positive Y-direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive Z-direction. The potential difference between the two faces of the cube perpendicular to the X-axis is **[JEE Main 2019, 10 Jan Shift-I]**

(a) 2 mV (b) 12 mV

(c) 6 mV (d) 1 mV

Exp. **(b)**

Potential difference between opposite faces of cube is

$$V = \text{induced emf} = B\, l\, v$$

where,

B = magnetic field = 0.1 T,

l = distance between opposite faces of cube $= 2\text{ cm} = 2 \times 10^{-2}$ m and

v = speed of cube = 6 ms^{-1}.

Hence, $V = 0.1 \times 2 \times 10^{-2} \times 6 = 12$ mV

29. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then

[JEE Main 2019, 10 Jan Shift-II]

(a) $T_h = 0.5\, T_c$ (b) $T_h = T_c$
(c) $T_h = 2\, T_c$ (d) $T_h = 1.5\, T_c$

Exp. **(b)**

The time-period of oscillations made by a magnet of magnetic moment M, moment of inertia I, placed in a magnetic field is given by

$$T = 2\pi\sqrt{\frac{I}{MB}} \quad \ldots \text{(i)}$$

For the hoop, let us assume its moment of inertia I_h and magnetic moment M_h, then its time period will be

$$T_h = 2\pi\sqrt{\frac{I_h}{M_h B}} \quad \ldots \text{(ii)}$$

Similarly, for solid cylinder, time period is,

$$T_c = 2\pi\sqrt{\frac{I_c}{M_c B}} \quad \ldots \text{(iii)}$$

Dividing Eq. (ii) by Eq. (iii), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{I_h M_c}{M_h I_c}} \quad \ldots \text{(iv)}$$

Now, it is given that,

$$M_h = 2M_c$$

and we know that, moment of inertia of hoop $I_h = mR^2$ and moment of inertia of solid cylinder $I_c = \frac{1}{2} mR^2$

Substituting these values in Eq. (iv), we get

$$\frac{T_h}{T_c} = \sqrt{\frac{mR^2 \times M_c}{\frac{1}{2} mR^2 \times 2M_c}} = 1$$

$$\Rightarrow \quad T_h = T_c$$

30. In an experiment, electrons are accelerated, from rest by applying a voltage of 500 V. Calculate the radius of the path, if a magnetic field 100 mT is then applied.
(Take, charge of the electron $= 1.6 \times 10^{-19}$ C and mass of the electron $= 9.1 \times 10^{-31}$ kg)

[JEE Main 2019, 11 Jan Shift-I]

(a) 7.5×10^{-2} m
(b) 7.5×10^{-4} m
(c) 7.5×10^{-3} m
(d) 7.5 m

Exp. **(b)**

During the circular motion of accelerated electron in the presence of magnetic field, its radius is given by

$$r = \frac{mv}{Be} = \frac{\sqrt{2meV}}{eB}$$

where, v is velocity and V is voltage.

After substituting the given values, we get

$$= \frac{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 500}}{1.6 \times 10^{-19} \times 100 \times 10^{-3}}$$

$$= 10\left[\frac{2 \times 9.1 \times 500}{1.6} \times 10^{-12}\right]^{1/2}$$

$$r = 7.5 \times 10^{-4} \text{ m}$$

31. In the circuit shown,

The switch S_1 is closed at time $t = 0$ and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behaviour of the current I as a function of time 't' is given by

[JEE Main 2019, 11 Jan Shift-I]

Exp. **(b)**

Initially in the given *RL* circuit with a source, when S_1 is closed and S_2 is open at $t \le t_0$.

$$I_1 = \frac{V}{R}\left[1 - \exp\left(\frac{-R}{L}t\right)\right]$$

In this case, inductor *L* is charging.

When switch S_2 is closed and S_1 is open (after $t > t_0$), the inductor will be discharged through resistor.

In this case ($t > t_0$),

$$I_2 = \frac{V}{R}\exp\left[-\frac{R}{L}(t - t_0)\right]$$

Thus, the variation of *I* with *t* approximately is shown below

32. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^3 A/m is applied. Its magnetic susceptibility is

[JEE Main 2019, 11 Jan Shift-II]

(a) 3.3×10^{-4} (b) 3.3×10^{-2}

(c) 4.3×10^{-2} (d) 2.3×10^{-2}

Exp. **(a)**

Given, side of cube = 1cm = 10^{-2} m

$\therefore$ Volume, $V = 10^{-6}\text{m}^3$

Dipole moment, $\mathbf{M} = 20 \times 10^{-6}$ J/T

Applied magnetic intensity, $\mathbf{H} = 60 \times 10^3$ A/m

Intensity of magnetisation

$$I = \frac{\mathbf{M}}{V} = \frac{20 \times 10^{-6}}{10^{-6}} = 20 \text{ A/m}$$

Now, magnetic susceptibility χ is

$$\chi = \frac{\text{Intensity of magnetisation}}{\text{Applied magnetic intensity}} = \frac{I}{H}$$

$$= \frac{20}{60 \times 10^3}$$

$$\Rightarrow \quad \chi = \frac{1}{3} \times 10^{-3} = 3.3 \times 10^{-4}$$

33. A particle of mass *m* and charge *q* is in an electric and magnetic field is given by

$$\mathbf{E} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}},\ \mathbf{B} = 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}}.$$

The charged particle is shifted from the origin to the point $P(x = 1; y = 1)$ along a straight path. The magnitude of the total work done is **[JEE Main 2019, 11 Jan Shift-II]**

(a) $(0.35)\,q$ (b) $(0.15)\,q$

(c) $(2.5)\,q$ (d) $5\,q$

Exp. **(d)**

Here,

$$\mathbf{E} = 2\hat{\mathbf{i}} + 3\hat{\mathbf{j}},\ \mathbf{B} = 4\hat{\mathbf{j}} + 6\hat{\mathbf{k}},$$

q = charge on a particle.

Initial position, $r_1 = (0,0)$

Final position, $r_2 = (1, 1)$

Net force experienced by charge particle inside electromagnetic field is

$$\mathbf{F}_{net} = q\mathbf{E} + q(\mathbf{v} \times \mathbf{B}) = q(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$

[here, $\mathbf{v} \times \mathbf{B} = 0$]

$$= (2q\,\hat{\mathbf{i}} + 3q\,\hat{\mathbf{j}})$$

$$\therefore \quad dW = \mathbf{F}_{net} \cdot d\mathbf{r}$$

$$\Rightarrow \quad \int dW = \int_{r_1}^{r_2} (2q\,\hat{\mathbf{i}} + 3q\,\hat{\mathbf{j}}) \cdot (dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}})$$

[here $d\mathbf{r} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}}$]

$$\Rightarrow \quad W = 2q\int_0^1 dx + 3q\int_0^1 dy$$

or $\quad W = 2q + 3q$ or $W = 5q$

34. The region between $y = 0$ and $y = d$ contains a magnetic field $\mathbf{B} = B\hat{\mathbf{k}}$. A particle of mass m and charge q enters the region with a velocity $\mathbf{v} = v\hat{\mathbf{i}}$. If $d = \frac{mv}{2qB}$ then the acceleration of the charged particle at the point of its emergence at the other side is

[JEE Main 2019, 11 Jan Shift-II]

(a) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{\mathbf{i}} + \frac{1}{2}\hat{\mathbf{j}}\right)$ (b) $\frac{qvB}{m}\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$

(c) $\frac{qvB}{m}\left(\frac{-\hat{\mathbf{j}} + \hat{\mathbf{i}}}{\sqrt{2}}\right)$ (d) $\frac{qvB}{m}\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)$

Exp. **(*)**

Situation given in question is shown below;

Path taken by particle of charge 'q' and mass 'm' is a circle of radius r where,

$$r = \frac{mv}{Bq}$$

Here final velocity

$$\mathbf{v}_f = \mathbf{v}_x\hat{\mathbf{i}} + \mathbf{v}_{yf}(-\hat{\mathbf{j}})$$

$$= v\cos 60\,\hat{\mathbf{i}} - v\sin 60°\,\hat{\mathbf{j}}$$

$$= v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

So, change of velocity of charged particle is

$$\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$$

$$= v\left(\frac{1}{2}\hat{\mathbf{i}} - \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right) - v\hat{\mathbf{i}}$$

$$= -v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)$$

If t = time taken by charged particle to cross region of magnetic field then,

$$t = \frac{\text{distance } OP}{\text{speed in direction } OP}$$

$$= \frac{r \times \frac{\sqrt{3}}{2}}{v} = \frac{\frac{mv}{Bq} \times \frac{\sqrt{3}}{2}}{v} = \frac{\sqrt{3}m}{2Bq}$$

So, acceleration of charged particle at the point its emergence is;

$$\text{Acceleration } \mathbf{a} = \frac{\Delta\mathbf{v}}{\Delta t} = \frac{-v\left(\frac{1}{2}\hat{\mathbf{i}} + \frac{\sqrt{3}}{2}\hat{\mathbf{j}}\right)}{\frac{\sqrt{3}}{2}\frac{m}{Bq}}$$

$$= \frac{-Bqv}{m}\left(\frac{\hat{\mathbf{i}}}{\sqrt{3}} + \hat{\mathbf{j}}\right)\text{ms}^{-2}$$

None of the option given matches with answer.

35. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the X-axis, while segments PS and QN are parallel to the Y-axis. If $OP = OQ = 4$ cm and the magnitude of the magnetic field at O is 10^{-4} T and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be (Take, $\mu_0 = 4\pi \times 10^{-7}\,\text{NA}^{-2}$)

[JEE Main 2019, 12 Jan Shift-I]

(a) 40 A, perpendicular out of the page
(b) 20 A, perpendicular into the page
(c) 20 A, perpendicular out of the page
(d) 40 A, perpendicular into the page

Exp. **(b)**

There is no magnetic field along axis of a current-carrying wire.

Also, magnetic field near one of end of an infinitely long wire is $\frac{\mu_0 I}{4\pi r}$ tesla.

Hence, magnetic field due to segments *LP* and *MQ* at '*O*' is zero. Using right hand rule, we can check that magnetic field due to segments *PS* and *QN* at '*O*' is in same direction perpendicularly into the plane of paper.

Hence, $B_O = B_{PS} + B_{QN}$

$$= \frac{\mu_0 i}{4\pi r} + \frac{\mu_0 i}{4\pi r} = \frac{\mu_0 i}{2\pi r}$$

So, $$i = \frac{2\pi r B_0}{\mu_0}$$

Here, $r = OP = OQ = 4$ cm and $B_0 = 10^{-4}$ T.

Substituting values, we get

$$\Rightarrow \quad i = \frac{2\pi \times 4 \times 10^{-2} \times 10^{-4}}{4\pi \times 10^{-7}}$$

$\Rightarrow i = 20$ A, Also, magnetic field points perpendicular into the plane of paper.

36. A proton and an α-particle (with their masses in the ratio of 1 : 4 and charges in the ratio of 1 : 2) are accelerated from rest through a potential difference *V*. If a uniform magnetic field *B* is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be

[JEE Main 2019, 12 Jan Shift-I]

(a) $1:\sqrt{2}$ (b) $1:\sqrt{3}$ (c) $1:3$ (d) $1:2$

Exp. **(a)**

Radius of path of charged particle *q* in a uniform magnetic field *B* of mass '*m*' moving with velocity *v* is

$$r = \frac{mv}{Bq} = \frac{m\sqrt{(2qV/m)}}{Bq}$$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{\sqrt{q}}$ so, required ratio is

$$\Rightarrow \quad \frac{r_p}{r_\alpha} = \sqrt{\frac{m_p}{m_\alpha}} \times \sqrt{\frac{q_\alpha}{q_p}}$$

$$= \sqrt{\frac{1}{4}} \times \sqrt{\frac{2}{1}} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

37. A paramagnetic material has 10^{28} atoms/m^3. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4}. Its susceptibility at 300 K is

[JEE Main 2019, 12 Jan Shift-II]

(a) 3.726×10^{-4} (b) 3.672×10^{-4}
(c) 2.672×10^{-4} (d) 3.267×10^{-4}

Exp. **(d)**

From Curie's law for paramagnetic substance, we have

Magnetic susceptiblity $\chi \propto \frac{1}{T}$

$$\therefore \quad \frac{\chi_2}{\chi_1} = \frac{T_1}{T_2} \Rightarrow \chi_2 = \frac{\chi_1 \cdot T_1}{T_2}$$

$$\Rightarrow \chi_2 = \frac{2.8 \times 10^{-4} \times 350}{300} = 3.267 \times 10^{-4}$$

38. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively, in a uniform magnetic field *B*. The relation between r_e, r_p, r_α is

[JEE Main 2018]

(a) $r_e > r_p = r_\alpha$ (b) $r_e < r_p = r_\alpha$
(c) $r_e < r_p < r_\alpha$ (d) $r_e < r_\alpha < r_p$

Exp. **(b)**

From $Bqv = \frac{mv^2}{r}$, we have

$$r = \frac{mv}{Bq} = \frac{\sqrt{2mK}}{Bq}$$

where, *K* is the kinetic energy.

As, kinetic energies of particles are same;

$$r \propto \frac{\sqrt{m}}{q}$$

$$\Rightarrow \quad r_e : r_p : r_\alpha = \frac{\sqrt{m_e}}{e} : \frac{\sqrt{m_p}}{e} : \frac{\sqrt{4m_p}}{2e}$$

Clearly, $r_p = r_\alpha$ and r_e is least $[\because m_e < m_p]$

So, $r_p = r_\alpha > r_e$

39. The dipole moment of a circular loop carrying a current *I*, is *m* and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is

[JEE Main 2018]

(a) 2 (b) $\sqrt{3}$ (c) $\sqrt{2}$ (d) $\frac{1}{\sqrt{2}}$

Exp. **(c)**

As $m = IA$, so to change dipole moment (current is kept constant), we have to change radius of loop.

Initially, $m = I\pi R^2$ and $B_1 = \frac{\mu_0 I}{2R_1}$

Finally, $m' = 2m = I\pi R_2^2$

$\Rightarrow$ $2I\pi R_1^2 = I\pi R_2^2$ or $R_2 = \sqrt{2}R_1$

So, $B_2 = \frac{\mu_0 I}{2(R_2)} = \frac{\mu_0 I}{2\sqrt{2}R_1}$

Hence, ratio $\frac{B_1}{B_2} = \frac{\left(\frac{\mu_0 I}{2R_1}\right)}{\left(\frac{\mu_0 I}{2\sqrt{2}R_1}\right)} = \sqrt{2}$

$\therefore$ Ratio $\frac{B_1}{B_2} = \sqrt{2}$

40. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into a voltmeter of range 0-10 V is **[JEE Main 2017 (Offline)]**

(a) 2.045×10^3 Ω (b) 2.535×10^3 Ω

(c) 4.005×10^3 Ω (d) 1.985×10^3 Ω

Exp. **(*d*)**

Suppose a resistance R_s is connected in series with galvanometer to convert it into voltmeter.

$$I_g (G + R_s) = V \Rightarrow R = \frac{V}{I_g} - G$$

$\Rightarrow$ $R = 1985 = 1.985$ kΩ

or $R = 1.985 \times 10^3$ Ω

41. A magnetic needle of magnetic moment 6.7×10^{-2} Am^2 and moment of inertia 7.5×10^{-6} kg m^2 is performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is **[JEE Main 2017 (Offline)]**

(a) 8.89 s (b) 6.98 s

(c) 8.76 s (d) 6.65 s

Exp. **(*d*)**

Time period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{MB}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 0.01}} = 0.665 \text{ s}$$

Hence, time for 10 oscillations is $t = 6.65$ s.

42. Two identical wires A and B, each of length l, carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side a. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is **[JEE Main 2016 (Offline)]**

(a) $\frac{\pi^2}{8}$ (b) $\frac{\pi^2}{16\sqrt{2}}$ (c) $\frac{\pi^2}{16}$ (d) $\frac{\pi^2}{8\sqrt{2}}$

Exp. **(*d*)**

Magnetic field in case of circle of radius R, we have

$$B_A = \frac{\mu_0 I}{2R}$$

As, $2\pi R = l$, (where l is length of a wire)

$$R = \frac{l}{2\pi}$$

$$\Rightarrow \quad B_A = \frac{\mu_0 I}{2 \times \frac{l}{2\pi}} = \frac{\mu_0 I \pi}{l} \quad \text{...(i)}$$

Magnetic field in case of square of side a, we get

$$B_B = 4 \times \frac{\mu_0}{4\pi} \times \frac{I}{\left(\frac{a}{2}\right)}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow \qquad B_B = \frac{4/\mu_0}{\pi a\sqrt{2}} = \frac{\mu_0 2\sqrt{2}I}{a\pi}$$

As, $\quad 4a = l,\ a = \frac{l}{4} \Rightarrow B_B = \frac{8\sqrt{2}\,\mu_0 I}{\pi l}$ ... (ii)

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$$

43. Hysteresis loops for two magnetic materials A and B are as given below

These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then, it is proper to use **[JEE Main 2016 (Offline)]**

(a) A for electric generators and transformers
(b) A for electromagnets and B for electric generators
(c) A for transformers and B for electric generators
(d) B for electromagnets and transformers

Exp. **(*a*)**

Area of hysteresis loop is proportional to the net energy absorbed per unit volume by the material, as it is taken over a complete cycle of magnetisation.

For electromagnets and transformers, energy loss should be low.

i.e. thin hysteresis curves.

Also, $|B| \to 0$ when $H = 0$ and $|H|$ should be small when $B \to 0$.

44. A galvanometer having a coil resistance of 100 Ω gives a full scale deflection when a current of 1 mA is passed through it. The value of the resistance which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10 A, is **[JEE Main 2016 (Offline)]**

(a) 0.01 Ω (b) 2 Ω (c) 0.1 Ω (d) 3 Ω

Exp. **(*a*)**

Maximum voltage that can be applied across the galvanometer coil = 100 Ω × 10^{-3} A = 0.1 V.

If R_s is the shunt resistance, then

$$R_s \times 10\text{A} = 0.1\text{ V}$$

$$\Rightarrow \qquad R_s = 0.01\ \Omega$$

45. Two coaxial solenoids of different radii carry current I in the same direction. Let $\mathbf{F}_1$ be the magnetic force on the inner solenoid due to the outer one and $\mathbf{F}_2$ be the magnetic force on the outer solenoid due to the inner one. Then, **[JEE Main 2015]**

(a) $\mathbf{F}_1 = \mathbf{F}_2 = 0$
(b) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2$ is radially outwards
(c) $\mathbf{F}_1$ is radially inwards and $\mathbf{F}_2 = 0$
(d) $\mathbf{F}_1$ is radially outwards and $\mathbf{F}_2 = 0$

Exp. **(*a*)**

Consider the two coaxial solenoids. Due to one of the solenoids magnetic field at the centre of the other can be assumed to be constant.

Due to symmetry, forces on upper and lower part of a solenoid will be equal and opposite and hence resultant is zero.

Therefore, $F_1 = F_2 = 0$

46. Two long current carrying thin wires, both with current I, are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle θ with the vertical. If wires have mass λ per unit length then, the value of I is

(g = gravitational acceleration)

[JEE Main 2015]

(a) $\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$ (b) $2\sin\theta\sqrt{\dfrac{\pi\lambda gL}{\mu_0\cos\theta}}$

(c) $2\sqrt{\dfrac{\pi gL}{\mu_0}\tan\theta}$ (d) $\sqrt{\dfrac{\pi\lambda gL}{\mu_0}\tan\theta}$

Exp. **(*b*)**

Consider free body diagram of the wire.

As the wires are in equilibrium, they must carry current in opposite direction.

Here, $F_B = \dfrac{\mu_0 I^2 l}{2\pi d}$, where l is length of each wire and d is separation between wires.

From figure, $d = 2L\sin\theta$

$$T = \cos\theta = mg = \lambda lg$$

(in vertical direction)…(i)

$$T\sin\theta = F_B = \frac{\mu_0 I^2 l}{4\pi L\sin\theta}$$

(in horizontal direction)…(ii)

From Eqs. (i) and (ii),

$$\frac{T\sin\theta}{T\cos\theta} = \frac{\mu_0 I^2 l}{4\pi L\sin\theta\times\lambda lg}$$

$$\therefore \quad I = \sqrt{\frac{4\pi\lambda Lg\sin^2\theta}{\mu_0\cos\theta}} = 2\sin\theta\sqrt{\frac{\pi\lambda Lg}{\mu_0\cos\theta}}$$

47. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is placed in different orientations as shown in the figures below. **[JEE Main 2015]**

If there is a uniform magnetic field of 0.3 T in the positive z-direction in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium? **[JEE Main 2015]**

(a) (a) and (b) respectively
(b) (a) and (c) respectively
(c) (b) and (d) respectively
(d) (b) and (c) respectively

Exp. **(*c*)**

Since **B** is uniform only torque acts on a current carrying loop.

As, $\tau = \mathbf{M}\times\mathbf{B} \Rightarrow |\tau| = |\mathbf{M}|\,|\mathbf{B}|\sin\theta$

For orientation shown in (b) $\theta = 0^0$, $\tau = 0$ (stable equilibrium)

and for (d) $\theta = \pi$, $\tau = 0$ (unstable equilibrium)

48. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-a_z$ direction (see figure). For a field $\mathrm{B} = 3.0\times10^{-4}\,e^{-0.2x}a_y$ T, find the power required to move the conductor at constant speed to $x = 2.0$ m, $y = -0$ in 5×10^{-3} s. Assume parallel motion along the x-axis. **[JEE Main 2014]**

(a) 1.57 W (b) 2.97 W (c) 14.85 W (d) 29.7 W

Exp. **(*b*)**

Force exerted on a current carrying conductor,

$$F_{ext} = B(x)\,IL$$

$$\text{Average power} = \frac{\text{Work done}}{\text{Time taken}}$$

$$P = \frac{1}{t}\int_0^2 F_{ext}\cdot dx = \frac{1}{t}\int_0^2 B(x)ILdx$$

$$= \frac{1}{5\times10^{-3}}\int_0^2 3\times10^{-4}e^{-0.2x}\times10\times3dx$$

$$= 9\,[1-e^{-0.4}]$$

$$= 9\left[1-\frac{1}{e^{0.4}}\right] = 2.967 \approx 2.97 \text{ W}$$

49. The coercivity of a small magnet where the ferromagnet gets demagnetised is 3×10^3 Am^{-1}. The current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetised when inside the solenoid is **[JEE Main 2014]**

(a) 30 mA (b) 60 mA (c) 3 A (d) 6 A

Exp. **(c)**

For solenoid, the magnetic field needed to be magnetised, the magnet is given by

$$B = \mu_0 nI$$

where, n is number of turns per unit length = N/l

and $N = 100$, $l = 10\text{ cm} = \frac{10}{100}\text{ m} = 0.1\text{ m}$

$$\Rightarrow \quad 3\times10^3 = \frac{100}{0.1}\times I \Rightarrow I = 3\text{A}$$

50. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am^2 and 1.00 Am^2, respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0 cm. The value of the resultant horizontal magnetic induction at the mid-point O of the line joining their centres is close to (Horizontal component of the earth's magnetic induction is 3.6×10^{-5} Wb/m^2) **[JEE Main 2013]**

(a) 3.6×10^{-5} Wb/m^2 (b) 2.56×10^{-4} Wb/m^2
(c) 3.50×10^{-4} Wb/m^2 (d) 5.80×10^{-4} Wb/m^2

Exp. **(b)**

Net magnetic field, $B_{net} = B_1 + B_2 + B_H$

$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}(1.2 + 1)}{(0.1)^3} + 3.6\times10^{-5}$$

$$= 2.56\times10^{-4}\text{ Wb/m}^2$$

51. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is **[JEE Main 2013]**

(a) 9.2×10^{-11} Wb (b) 6×10^{-11} Wb
(c) 3.3×10^{-11} Wb (d) 6.6×10^{-9} Wb

Exp. **(a)**

The magnetic field due to the bigger circular loop is

$$B = \frac{\mu_0 IR^2}{2(x^2 + R^2)^{3/2}}$$

Given, $I = 2$ A, $R = 20\text{ cm} = 20\times10^{-2}$ m,

$x = 15\text{ cm} = 15\times10^{-2}$ m

$$B = \frac{\mu_0\times2\times(20\times10^{-2})^2}{[(0.2)^2 + (0.15)^2]^{3/2}}$$

Flux linked, $\phi = BA$

where, A is area of small circular loop.

$$A = \pi r^2 = \pi\times(0.3\times10^{-2})^2$$

$$\therefore \quad \phi = \frac{\mu_0\times2\times(20\times10^{-2})^2}{2[(0.2)^2 + (0.15)^2]^{3/2}}\times\pi\times(0.3\times10^{-2})^2$$

$$\phi = 9.2\times10^{-11}\text{ Wb}$$

52. Proton, deuteron and alpha particles of same kinetic energy are moving in circular trajectories in a constant magnetic field. The radii of proton, deuteron and alpha particle are respectively r_p, r_d and r_α. Which one of the following relation is correct? **[AIEEE 2012]**

(a) $r_\alpha = r_p = r_d$ (b) $r_\alpha = r_p < r_d$
(c) $r_\alpha > r_d > r_p$ (d) $r_\alpha = r_d > r_p$

Exp. **(b)**

For charged particle in magnetic field, radius

$$r = \frac{mv}{qB} = \frac{\sqrt{2Km}}{qB}$$

$$\left[\text{as } mv = p \Rightarrow K = \frac{1}{2}mv^2 = \frac{m^2v^2}{2m} = p^2/2m \quad p = \sqrt{2\,Km}\right]$$

$\Rightarrow \quad r \propto \frac{\sqrt{m}}{q}$

or $\quad m_d = 2m_p$

and $\quad q_d = q_p$;

$m_\alpha = 4m_p$

and $\quad q_\alpha = 2q_p$

$\Rightarrow \quad r_p : r_d : r_\alpha = \frac{\sqrt{m_p}}{q_p} : \frac{\sqrt{2m_p}}{q_p} : \frac{\sqrt{4m_p}}{2q_p}$

$= 1 : \sqrt{2} : 1$

$\Rightarrow \quad r_\alpha = r_p < r_d$

53. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation, a magnetic field of induction B is obtained at the centre of the disc.

If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc, then the variation of the magnetic induction at the centre of the disc will be represented by the figure **[AIEEE 2012]**

Exp. **(a)**

Taking an elemental ring of radius r and thickness dr, we obtain magnetic field at the centre of the ring,

$$dB = \frac{\mu_0}{2r}\,dI = \frac{\mu_0}{2r}\cdot\frac{dq}{T} = \frac{\mu_0}{2r}\cdot\frac{\frac{Q}{\pi R^2}\cdot 2\pi r\,dr}{\frac{2\pi}{\omega}}$$

$$= \frac{\mu_0 Q\omega}{2\pi R^2}\,dr$$

So, net magnetic field at the centre of disk,

$$B = \int_0^R dB = \frac{\mu_0\omega Q}{2\pi R^2}\int_0^R dr = \frac{\mu_0\omega Q}{2\pi R}$$

i.e., $\quad B \propto \frac{1}{R}$

54. A horizontal straight wire 20 m long extending from East to West is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb/m^2. The instantaneous value of the emf induced in the wire will be **[AIEEE 2011]**

(a) 6.0 mV (b) 3 mV (c) 4.5 mV (d) 1.5 mV

Exp. **(b)**

Induced emf, $e = B_H\,lv = 0.30\times10^{-4}\times20\times5.0$

$= 3\text{mV}$

55. A thin circular disk of radius R is uniformly charged with density $\sigma > 0$ per unit area. The disk rotates about its axis with a uniform angular speed ω. The magnetic moment of the disk is **[AIEEE 2011]**

(a) $2\pi R^4\sigma\omega$ (b) $\pi R^4\sigma\omega$ (c) $\frac{\pi R^4}{2}\sigma\omega$ (d) $\frac{\pi R^4}{4}\sigma\omega$

Exp. **(d)**

Let us consider the disc to be made up of large number of concentric elementary rings.

Consider one such ring of radius x and thickness dx. Charge on this elementary ring,

$$dq = \sigma \times 2\pi x dx = 2\pi\sigma x dx$$

Current associated with this elementary ring,

$$dI = \frac{dq}{dt} = dq \times f = \sigma\omega x dx$$

[$\because f$ is frequency and $\omega = 2\pi f$]

Magnetic moment of this elementary ring,

$$dM = dI\,\pi x^2 = \pi\sigma\omega\, x^3 dx$$

$\therefore$ Magnetic moment of the entire disc,

$$M = \int_0^R dM = \pi\sigma\omega\int_0^R x^3\, dx = \frac{1}{4}\pi R^4\,\sigma\omega$$

56. A boat is moving due East in a region where the earth's magnetic field is 5.0×10^{-5} $NA^{-1}\,m^{-1}$ due North and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is **[AIEEE 2011]**

(a) 0.75 mV (b) 0.50 mV (c) 0.15 mV (d) 1 mV

Exp. **(c)**

Induced emf, $e = B_H\, l v = 5.0 \times 10^{-5} \times 2 \times 1.50$

$= 0.15 \times 10^{-3}$ V = 0.15 mV

57. A current I flows in an infinitely long wire with cross-section in the form of a semi-circular ring of radius R. The magnitude of the magnetic induction along its axis is **[AIEEE 2011]**

(a) $\frac{\mu_0 I}{2\pi^2 R}$ (b) $\frac{\mu_0 I}{2\pi R}$ (c) $\frac{\mu_0 I}{4\pi R}$ (d) $\frac{\mu_0 I}{\pi^2 R}$

Exp. **(d)**

Consider the wire to be made up of large number of thin wires of infinite length. Consider such wire of thickness dl subtending an angle $d\theta$ at centre.

Current through this wire,

$$dI = \frac{d\theta}{\pi} I,$$

$\therefore$ Magnetic field at centre due to this portion,

$$dB = \frac{\mu_0}{4\pi}\cdot\frac{2dI}{R} = \frac{\mu_0 I}{2\pi^2 R}\,d\theta$$

Net magnetic field at the centre,

$$B = \int_{-\pi/2}^{\pi/2} dB\cos\theta = \frac{\mu_0 I}{2\pi^2 R}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{\mu_0 I}{\pi^2 R}$$

58. Two long parallel wires are at a distance $2d$ apart. They carry steady equal current flowing out of the plane of the paper as shown. The variation of the magnetic field along the line xx' is given by **[AIEEE 2010]**

(a)

(b)

(c)

(d)

Exp. **(a)**

The magnetic field in between because of each will be in opposite direction.

$$\therefore \quad B_{\text{in between}} = \frac{\mu_0 i}{2\pi x}\mathbf{j} - \frac{\mu_0 i}{2\pi(2d - x)}(-\mathbf{j})$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{2d - x}\right](\mathbf{j})$$

At $x = d$, $B_{\text{in between}} = 0$

For $x < d$, $B_{\text{in between}} = (\mathbf{j})$

For $x > d$, $B_{\text{in between}} = (-\mathbf{j})$

Towards x, net magnetic field will add up and direction will be $(-\mathbf{j})$.

Towards x', net magnetic field will add up and direction will be $(\mathbf{j})$.

59. The magnitude of the magnetic field (B) due to loop $ABCD$ at the origin (O) is **[AIEEE 2009]**

(a) zero (b) $\frac{\mu_0 I(b-a)}{24ab}$

(c) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$ (d) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

Exp. **(b)**

Net magnetic field due to loop $ABCD$ at O is

$$B = B_{AB} + B_{BC} + B_{CD} + B_{OA}$$

$$= 0 + \frac{\mu_0 I}{4\pi a}\times\frac{\pi}{6} + 0 - \frac{\mu_0 I}{\pi b}\times\frac{\pi}{6}$$

$$= \frac{\mu_0 I}{24a} - \frac{\mu_0 I}{24b} = \frac{\mu_0 I}{24ab}(b-a)$$

60. Due to the presence of the current I_1 at the origin in the figure, **[AIEEE 2009]**

(a) the forces on AB and DC are zero

(b) the forces on AD and BC are zero

(c) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(d) the magnitude of the net force on the loop is given by $\frac{\mu_0 I I_1}{24ab}(b-a)$

Exp. **(b)**

The forces on AD and BC are zero because magnetic field due to a straight wire on AD and BC is parallel to elementary length of the loop.

61. Relative permittivity and permeability of a material are ε_r and μ_r, respectively. Which of the following values of these quantities are allowed for a diamagnetic material? **[AIEEE 2008]**

(a) $\varepsilon_r = 0.5, \mu_r = 1.5$ (b) $\varepsilon_r = 1.5, \mu_r = 0.5$

(c) $\varepsilon_r = 0.5, \mu_r = 0.5$ (d) $\varepsilon_r = 1.5, \mu_r = 1.5$

Exp. **(b)**

For diamagnetic material, $0 < \mu_r < 1$ and for any material, $\varepsilon_r > 1$.

62. A horizontal overhead powerline is at a height of 4 m from the ground and carries a current of 100 A from East to West. The magnetic field directly below it on the ground is (take $\mu_0 = 4\pi\times10^{-7}$ T mA^{-1}) **[AIEEE 2008]**

(a) 2.5×10^{-7} T, Southward

(b) 5×10^{-6} T, Northward

(c) 5×10^{-6} T, Southward

(d) 2.5×10^{-7} T, Northward

Exp. **(c)**

$$B = \frac{\mu_0 I}{2\pi R}$$

Direction is given by right hand palm rule No. 1.

$$B = \frac{2\times10^{-7}\times100}{4}\text{ T, Southward}$$

$$= 5\times10^{-6}\text{ T, Southward}$$

63. Two identical conducting wires AOB and COD are placed at right angles to each other. The wire AOB carries an electric current I_1 and COD carries a current I_2. The magnetic field on a point lying at a distance d

from O, in a direction perpendicular to the plane of the wires AOB and COD, will be given by

[AIEEE 2007]

(a) $\frac{\mu_0}{2\pi}\left(\frac{I_1+I_2}{d}\right)^{1/2}$ (b) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)^{1/2}$

(c) $\frac{\mu_0}{2\pi d}(I_1+I_2)$ (d) $\frac{\mu_0}{2\pi d}(I_1^2+I_2^2)$

Exp. **(b)**

The magnetic field inductions at a point P, at a distance d from O in a direction perpendicular to the plane of the wires due to currents through AOB and COD are perpendicular to each other, is

$$B=\sqrt{B_1^2+B_2^2}=\left[\left(\frac{\mu_0}{4\pi}\frac{2I_1}{d}\right)^2+\left(\frac{\mu_0}{4\pi}\frac{2I_2}{d}\right)^2\right]^{1/2}$$

$$=\frac{\mu_0}{2\pi d}\sqrt{(I_1^2+I_2^2)}$$

64. A long straight wire of radius a carries a steady current I. The current is uniformly distributed across its cross-section. The ratio of the magnetic field at $\frac{a}{2}$ and $2a$ is

[AIEEE 2007]

(a) 1/4 (b) 4 (c) 1 (d) 1/2

Exp. **(c)**

Current density, $J=\frac{I}{\pi a^2}$

From Ampere's circuital law,

$$\oint B\cdot dl=\mu_0\cdot I_{\text{enclosed}}$$

For $r<a$,

$$B\times 2\pi r=\mu_0\times J\times \pi r^2$$

$$\Rightarrow \quad B=\frac{\mu_0 I}{\pi a^2}\times\frac{r}{2}$$

At $r=\frac{a}{2}$, $\quad B_1=\frac{\mu_0 I}{4\pi a}$

For $r>a$, $B\times 2\pi r=\mu_0 I$

$$\Rightarrow \quad B=\frac{\mu_0 I}{2\pi r}$$

At $r=2a$, $\quad B_2=\frac{\mu_0 I}{4\pi a}$

$$\therefore \quad \frac{B_1}{B_2}=1$$

65. A charged particle with charge q enters a region of constant, uniform and mutually orthogonal fields $\mathbf{E}$ and $\mathbf{B}$ with a velocity $\mathbf{v}$ perpendicular to both $\mathbf{E}$ and $\mathbf{B}$ and comes out without any change in magnitude or direction of $\mathbf{v}$. Then, **[AIEEE 2007]**

(a) $\mathbf{v}=\mathbf{E}\times\frac{\mathbf{B}}{B^2}$

(b) $\mathbf{v}=\mathbf{B}\times\frac{\mathbf{E}}{B^2}$

(c) $\mathbf{v}=\mathbf{E}\times\frac{\mathbf{B}}{E^2}$

(d) $\mathbf{v}=\mathbf{B}\times\frac{\mathbf{E}}{E^2}$

Exp. **(a)**

As $\mathbf{v}$ of charged particle is remaining constant, it means force acting on charged particle is zero.

$$f=q\mathbf{E}+q(\mathbf{v}\times\mathbf{B})=0$$

$$q\mathbf{E}-q(\mathbf{v}\times\mathbf{B})=0$$

So, $\quad q(\mathbf{v}\times\mathbf{B})=q\mathbf{E}$

$$\Rightarrow \quad \mathbf{v}\times\mathbf{B}=\mathbf{E}$$

$$\Rightarrow \quad \mathbf{v}=\frac{\mathbf{E}\times\mathbf{B}}{B^2}$$

66. A current I flows along the length of an infinitely long, straight, thin walled pipe. Then, **[AIEEE 2007]**

(a) the magnetic field is zero only on the axis of the pipe

(b) the magnetic field is different at different points inside the pipe

(c) the magnetic field at any point inside the pipe is zero

(d) the magnetic field at all points inside the pipe is the same but not zero

Exp. **(c)**

By using Ampere's circuital law, the magnetic field at any point inside the pipe is zero.

67. A charged particle moves through a magnetic field perpendicular to its direction. Then, **[AIEEE 2007]**

(a) the momentum changes but the kinetic energy is constant
(b) both momentum and kinetic energy of the particle are not constant
(c) both momentum and kinetic energy of the particle are constant
(d) kinetic energy changes but the momentum is constant

Exp. **(a)**

In case of motion of a charged particle perpendicular to the motion, *i.e.*, displacement, the work done

$$W = \int \mathbf{F}\cdot d\mathbf{s} = \int F ds \cos\theta = 0 \qquad [\text{as } \theta = 90°]$$

and by work-energy theorem, $W = \Delta KE$, the kinetic energy and hence speed v remains constant. But **v** changes, so momentum changes.

68. Needles N_1, N_2 and N_3 are made of a ferromagnetic, a paramagnetic and a diamagnetic substance respectively. A magnet when brought close to them will **[AIEEE 2006]**

(a) attract N_1 and N_2 strongly but repel N_3
(b) attract N_1 strongly, N_2 weakly and repel N_3 weakly
(c) attract N_1 strongly, but repel N_2 and N_3 weakly
(d) attract all three of them

Exp. **(b)**

Ferromagnetic substances have strong tendency to get magnetised (induced magnetic moment) in the same direction as that of applied magnetic field, so magnet attracts N_1 strongly.

Paramagnetic substances get weakly magnetised (magnetic moment induced is small) in the same direction as that of applied magnetic field, so magnet attracts N_2 weakly.

Diamagnetic substances also get weakly magnetised when placed in an external magnetic field but in opposite direction and hence N_3 is weakly repelled by magnet.

69. In a region, steady and uniform electric and magnetic fields are present. These two fields are parallel to each other. A charged particle is released from rest in this region. The path of the particle will be a **[AIEEE 2006]**

(a) helix (b) straight line
(c) ellipse (d) circle

Exp. **(b)**

Let **E** and **B** be along x-axis. When a charged particle is released from rest, it will experience an electric force along the direction of electric field or opposite to the direction of electric field depending on the nature of charge.

Due to this force, it acquires some velocity along x-axis. Due to this motion of charge, magnetic force cannot have non-zero value because angle between **v** and **B** would be either 0° or 180°.

So, only electric force is acting on particle and hence it will move along a straight line.

70. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milliampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance in Ohms needed to be connected in series with the coil will be **[AIEEE 2005]**

(a) 10^3 (b) 10^5 (c) 99995 (d) 9995

Exp. **(d)**

Full scale deflection current $= \frac{150}{10}$ mA = 15 mA

Full scale deflection voltage $= \frac{150}{2}$ mV = 75 mV

Galvanometer resistance, $G = \frac{75\text{ mV}}{15\text{ mA}} = 5\ \Omega$

Required full scale deflection voltage,

$$V = 1 \times 150 = 150\text{ V}$$

Let resistance to be connected in series be R.

$\Rightarrow \quad V = I_g(R + G)$

$\therefore \quad 150 = 15 \times 10^{-3}\ (R + 5)$

or $\quad 10^4 = R + 5$

or $\quad R = 10000 - 5 = 9995$

71. Two thin, long, parallel wires, separated by a distance d carry a current of I ampere in the same direction. They will **[AIEEE 2005]**

(a) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(b) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d)}$

(c) attract each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

(d) repel each other with a force of $\frac{\mu_0 I^2}{(2\pi d^2)}$

Exp. **(a)**

The force per unit length between the two wires is

$$\frac{F}{l} = \frac{\mu_0}{4\pi} \cdot \frac{2I^2}{d} = \frac{\mu_0 I^2}{2\pi d}$$

The force will be attractive as current directions in both are same.

72. Two concentric coils each of radius equal to 2π cm are placed at right angles to each other. 3 A and 4A are the currents flowing in each coil respectively. The magnetic induction in Wb/m^2 at the centre of the coils will be ($\mu_0 = 4\pi \times 10^{-7}$ Wb/Am) **[AIEEE 2005]**

(a) 12×10^{-5} (b) 10^{-5}

(c) 5×10^{-5} (d) 7×10^{-5}

Exp. **(c)**

Magnetic field or magnetic induction at P,

$$B_P = \frac{\mu_0 I_2}{2R} = \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.02\pi} = 4 \times 10^{-5} \text{ Wb/m}^2$$

and $$B_Q = \frac{\mu_0 I_1}{2R} = \frac{4\pi \times 10^{-7} \times 3}{2 \times 0.02\pi}$$

$$= 3 \times 10^{-5} \text{ Wb/m}^2$$

$$\therefore \quad B = \sqrt{B_P^2 + B_Q^2}$$

$$= \sqrt{(4 \times 10^{-5})^2 + (3 \times 10^{-5})^2}$$

$$= 5 \times 10^{-5} \text{ Wb/m}^2$$

73. A magnetic needle is kept in a non-uniform magnetic field. It experiences **[AIEEE 2005]**

(a) a torque but not a force

(b) neither a force nor a torque

(c) a force and a torque

(d) a force but not a torque

Exp. **(c)**

Magnetic needle is placed in non-uniform magnetic field. It experiences force and torque both due to unequal forces acting on poles.

74. A charged particle of mass m and charge q travels on a circular path of radius r that is perpendicular to a magnetic field B. The time taken by the particle to complete one revolution is **[AIEEE 2005]**

(a) $\frac{2\pi mq}{B}$ (b) $\frac{2\pi q^2 B}{m}$ (c) $\frac{2\pi qB}{m}$ (d) $\frac{2\pi m}{qB}$

Exp. **(d)**

Magnetic force, $F = qvB$ [$\because \theta = 90°$] ...(i)

Centripetal force, $F = \frac{mv^2}{r}$ ...(ii)

From Eqs. (i) and (ii), we have

$$\frac{mv^2}{r} = qvB \quad \text{or} \quad r = \frac{mv}{qB}$$

The time taken by the particle to complete one revolution,

$$T = \frac{2\pi r}{v} = \frac{2\pi mv}{vqB} = \frac{2\pi m}{qB}$$

75. A current I ampere flows along an infinitely long straight thin walled tube, then the magnetic induction at any point inside the tube is **[AIEEE 2004]**

(a) infinite (b) zero

(c) $\frac{\mu_0}{4\pi} \cdot \frac{2I}{r}$ T (d) $\frac{2I}{r}$ T

Exp. (b)

Let R be the radius of a long thin cylindrical shell. To calculate the magnetic induction at a distance r, $(r < R)$ from the axis of cylinder, a circular shell of radius r is shown.

Since, no current is enclosed in the circle, so from Ampere's circuital law, magnetic induction is zero at every point of circle. Hence, the magnetic induction at very point inside the infinitely long straight thin walled tube (cylindrical) is zero.

76. A long wire carries a steady current. It is bent into a circle of one turn and the magnetic field at the centre of the coil is B. It is then bent into a circular loop of n turns. The magnetic field at the centre of the coil will be **[AIEEE 2004]**

(a) nB (b) n^2B
(c) $2nB$ (d) $2n^2B$

Exp. (b)

The magnetic field at the centre of circular coil is

$$B = \frac{\mu_0 I}{2r}$$

where, r = radius of circle $= \frac{l}{2\pi}$ $[\because l = 2\pi r]$

$$\therefore \quad B = \frac{\mu_0 I}{2} \times \frac{2\pi}{l} = \frac{\mu_0 I \pi}{l} \quad \ldots(i)$$

When wire of length l bents into a circular loops of n turns, then

$$l = n \times 2\pi r'$$

$$\Rightarrow \quad r' = \frac{l}{n \times 2\pi}$$

Thus, new magnetic field,

$$B' = \frac{\mu_0 n I}{2r'}$$

$$= \frac{\mu_0 n I}{2} \times \frac{n \times 2\pi}{l}$$

$$= \frac{\mu_0 I \pi}{l} \times n^2 = n^2 B$$

[from Eq. (i)]

77. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop? **[AIEEE 2004]**

(a) 250 μT (b) 150 μT
(c) 125 μT (d) 75 μT

Exp. (a)

The magnetic field at a point on the axis of a circular loop at a distance x from the centre is

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}} \quad \ldots(i)$$

Given, $B = 54\ \mu T$, $x = 4$ cm, $R = 3$ cm

Putting the given values in Eq. (i), we get

$$54 = \frac{\mu_0 I \times (3)^2}{2(3^2 + 4^2)^{3/2}}$$

$$\Rightarrow \quad 54 = \frac{9\mu_0 I}{2(25)^{3/2}} = \frac{9\mu_0 I}{2 \times (5)^3}$$

$$\therefore \quad \mu_0 I = \frac{54 \times 2 \times 125}{9}$$

$$\Rightarrow \quad \mu_0 I = 1500\ \mu T\text{-cm} \quad \ldots(ii)$$

Now, putting $x = 0$ in Eq. (i), magnetic field at the centre of loop is

$$B = \frac{\mu_0 I R^2}{2R^3}$$

$$= \frac{\mu_0 I}{2R} = \frac{1500}{2 \times 3} \quad \text{[from Eq. (ii)]}$$

$$= 250\ \mu T$$

78. Two long conductors, separated by a distance d carry currents I_1 and I_2 in the same direction. They exert a force F on each other. Now, the current in one of them is increased to two times and its direction is reversed. The distance is also increased to $3d$. The new value of the force between them is **[AIEEE 2004]**

(a) $-2F$ (b) $\frac{F}{3}$
(c) $-\frac{2F}{3}$ (d) $-\frac{F}{3}$

Exp. (c)

Force acting between two current carrying conductors

$$F = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d} l \quad \ldots(i)$$

where d = distance between the conductors

l = length of small conductor

Again, $$F' = \frac{\mu_0}{2\pi}\frac{(-2I_1)(I_2)}{(3d)} \cdot l$$

$$= -\frac{\mu_0}{2\pi}\frac{2I_1I_2}{3d} \cdot l \quad \ldots\text{(ii)}$$

Thus, from Eqs. (i) and (ii), we have

$$\frac{F'}{F} = -\frac{2}{3} \Rightarrow F' = -\frac{2}{3}F$$

79. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2 s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be **[AIEEE 2004]**

(a) 2 s (b) $\frac{2}{3}$ s (c) $2\sqrt{3}$ s (d) $\frac{2}{\sqrt{3}}$ s

Exp. **(b)**

The time period of oscillations of magnet,

$$T = 2\pi\sqrt{\left(\frac{I}{MH}\right)} \quad \ldots\text{(i)}$$

where, I = moment of inertia of magnet

$= \frac{mL^2}{12}$ [m, being the mass of magnet]

M = Pole strength $\times L$

and H = horizontal component of the earth's magnetic field.

When the three equal parts of magnet are placed on one another with their like poles together, then

$$I' = \frac{1}{12}\left(\frac{m}{3}\right)\times\left(\frac{L}{3}\right)^2 \times 3 = \frac{1}{12}\frac{mL^2}{9} = \frac{I}{9}$$

and M' = Pole strength $\times \frac{L}{3} \times 3 = M$

Hence, $T' = 2\pi\sqrt{\left(\frac{I/9}{MH}\right)}$ or $T' = \frac{1}{3} \times T$

or $T' = \frac{2}{3}$ s

80. The materials suitable for making electromagnets should have **[AIEEE 2004]**

(a) high retentivity and high coercivity
(b) low retentivity and low coercivity
(c) high retentivity and low coercivity
(d) low retentivity and high coercivity

Exp. **(c)**

Electromagnets are made of soft iron. The soft iron has high retentivity and low coercivity.

81. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now, it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T'/T is **[AIEEE 2003]**

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 2 (d) $\frac{1}{4}$

Exp. **(b)**

When magnet is divided into two equal parts, the magnetic dipole moment

$$M' = \text{Pole strength} \times \frac{l}{2} = \frac{M}{2}$$

[$\because$ pole strength remains same]

Also, the mass of magnet becomes half *i.e.*,

$$m' = \frac{m}{2}$$

Moment of inertia of magnet, $I = \frac{ml^2}{12}$

New moment of inertia,

$$I' = \frac{1}{12}\left(\frac{m}{2}\right)\left(\frac{l}{2}\right)^2 = \frac{ml^2}{12 \times 8}$$

$\therefore$ $I' = \frac{I}{8}$

Now, $T = 2\pi\sqrt{\left(\frac{I}{MB}\right)}$

$$T' = 2\pi\sqrt{\left(\frac{I'}{M'B}\right)} = 2\pi\sqrt{\left(\frac{I/8}{MB/2}\right)}$$

$\therefore$ $T' = \frac{T}{2} \Rightarrow \frac{T'}{T} = \frac{1}{2}$

82. A magnetic needle lying parallel to a magnetic field requires W unit of work to turn it through 60°. The torque needed to maintain the needle in this position will be **[AIEEE 2003]**

(a) $\sqrt{3}W$ (b) W
(c) $\left(\frac{\sqrt{3}}{2}\right)W$ (d) $2W$

Exp. **(a)**

$W = MB(1 - \cos\theta)$

$\Rightarrow \quad W = MB(1 - \cos 60°) \quad [\because \theta = 60°]$

or $\quad W = \frac{MB}{2}$ or $MB = 2W$

Torque, $\quad \tau = MB \sin 60°$

$= \frac{MB\sqrt{3}}{2} = \frac{2W\sqrt{3}}{2} = W\sqrt{3}$

83. The magnetic lines of force inside a bar magnet

(a) are from North-pole to South-pole of the magnet
(b) do not exist
(c) depend upon the area of cross-section of the bar magnet
(d) are from South-pole to North-pole of the magnet

Exp. **(d)**

Inside bar magnet, lines of force are from South to North.

84. Curie temperature is the temperature above which **[AIEEE 2003]**

(a) a ferromagnetic material becomes para-magnetic
(b) a paramagnetic material becomes dia-magnetic
(c) a ferromagnetic material becomes dia-magnetic
(d) a paramagnetic material becomes ferro-magnetic

Exp. **(a)**

At Curie temperature, a ferromagnetic substance transits into paramagnetic substance.

85. A particle of mass M and charge Q moving with velocity **v** describes a circular path of radius R when subjected to a uniform transverse magnetic field of induction B. The work done by the field when the particle completes one full circle is **[AIEEE 2003]**

(a) $\left(\frac{Mv^2}{R}\right)2\pi R$ (b) zero
(c) $BQ2\pi R$ (d) $BQv2\pi R$

Exp. **(b)**

When particle describes circular path in a magnetic field, its velocity is always perpendicular to the magnetic force.

Power, $\quad P = \mathbf{F}\cdot\mathbf{v} = Fv\cos\theta$

Here, $\quad \theta = 90°$

$\therefore \quad P = 0$

But $\quad P = \frac{W}{t} \quad \Rightarrow \quad W = P\cdot t$

Hence, work done, $W = 0$ [everywhere]

86. A particle of charge -16×10^{-18} C moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis and an electric field of magnitude 10^4 Vm^{-1} is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is **[AIEEE 2003]**

(a) 10^3 Wb/m^2 (b) 10^5 Wb/m^2
(c) 10^{16} Wb/m^2 (d) 10^{-3} Wb/m^2

Exp. **(a)**

The force on a particle is

So, $\quad \mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$ or $\mathbf{F} = \mathbf{F}_e + \mathbf{F}_m$

$\therefore \quad \mathbf{F}_e = q\mathbf{E} = -16\times10^{-18}\times10^4(-\mathbf{j})$

$= 16\times10^{-14}\,\mathbf{k}$

and $\quad \mathbf{F}_m = -16\times10^{-18}(10\mathbf{i}\times B\mathbf{j})$

$= -16\times10^{-17}\times B(+\mathbf{k})$

$= -16\times10^{-17}B\mathbf{k}$

Since, particle will continue to move along $+x$-axis, so resultant force is equal to zero.

$\mathbf{F}_e + \mathbf{F}_m = 0$

$\Rightarrow \quad 16\times10^{-14} = 16\times10^{-17}B$

$\Rightarrow \quad B = \frac{16\times10^{-14}}{16\times10^{-17}} = 10^3$

or $\quad B = 10^3$ Wb/m^2

87. An ammeter reads upto 1 A. Its internal resistance is 0.81 Ω. To increase the range to 10 A, the value of the required shunt is **[AIEEE 2003]**

(a) 0.03 Ω (b) 0.3 Ω (c) 0.9 Ω (d) 0.09 Ω

Exp. **(d)**

To increase the range of ammeter, we have to connect a small resistance in parallel (shunt), let its value be R.

Apply KCL at junction to divide the current.

Voltage across R = Voltage across ammeter

i.e. , $9R = 0.81 \times 1$

or $R = \frac{0.81}{9} = 0.09\ \Omega$

88. If an ammeter is to be used in place of a voltmeter, then we must connect with the ammeter a **[AIEEE 2002]**

(a) low resistance in parallel
(b) high resistance in parallel
(c) high resistance in series
(d) low resistance in series

Exp. **(c)**

A voltmeter is a high resistance device and is always connected in parallel with the circuit. While an ammeter is a low resistance device and is always connected in series with the circuit.

So, to use ammeter in place of voltmeter, a high resistance must be connected in series with the ammeter to make its resistance high.

89. If in a circular coil A of radius R, current I is flowing and in another coil B of radius $2R$ a current $2I$ is flowing, then the ratio of the magnetic fields, B_A and B_B produced by them will be **[AIEEE 2002]**

(a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 4

Exp. **(a)**

Magnetic field in circular coil A is

$$B_A = \frac{\mu_0 NI}{2R}$$

where, R is radius and I is current flowing in coil.

Similarly, $$B_B = \frac{\mu_0 N(2I)}{2 \cdot (2R)} = \frac{\mu_0 NI}{2R}$$

$$\therefore \quad \frac{B_A}{B_B} = 1$$

90. If an electron and a proton having same momenta enter perpendicularly to a magnetic field, then **[AIEEE 2002]**

(a) curved path of electron and proton will be same (ignoring the sense of revolution)
(b) they will move undeflected
(c) curved path of electron is more curved than that of proton
(d) path of proton is more curved

Exp. **(a)**

Since, momenta are same and masses of electron and proton are different, so they will attain different velocities and hence experience different forces. But radius of circular path is depending on momenta, so both will be moving on same trajectory (curved path).

91. The time period of a charged particle undergoing a circular motion in a uniform magnetic field is independent of its

(a) speed
(b) mass **[AIEEE 2002]**
(c) charge
(d) magnetic induction

Exp. **(a)**

In a circular motion of a uniform magnetic field, the necessary centripetal force to the charged particle is provided by the magnetic force.

i.e., $$\frac{mv^2}{r} = qvB$$

or $$r = \frac{mv}{qB}$$

Thus, the time period T is

$$T = \frac{2\pi r}{v}$$

$$= \frac{2\pi}{v}\left(\frac{mv}{qB}\right) = \frac{2\pi m}{qB}$$

So, T is independent of its speed.

92. At a specific instant, emission of radioactive compound is deflected in a magnetic field. The compound can emit **[AIEEE 2002]**

(i) electrons (ii) protons
(iii) He^{2+} (iv) neutrons

The emission at the instant can be

(a) (i), (ii), (iii)
(b) (i), (ii), (iii), (iv)
(c) (iv)
(d) (ii), (iii)

Exp. **(a)**

Electrons, protons and helium atoms are deflected in magnetic field, so the compound can emit electrons, protons and He^{2+}.

93. Wires 1 and 2 carrying currents I_1 and I_2 respectively are inclined at an angle θ to each other. What is the force on a small element dl of wire 2 at a distance r from wire 1 (as shown in figure) due to the magnetic field of wire 1? **[AIEEE 2002]**

(a) $\frac{\mu_0}{2\pi r} I_1 I_2\, dl \tan\theta$ (b) $\frac{\mu_0}{2\pi r} I_1 I_2\, dl \sin\theta$

(c) $\frac{\mu_0}{2\pi r} I_1 I_2\, dl \cos\theta$ (d) $\frac{\mu_0}{4\pi r} I_1 I_2\, dl \sin\theta$

Exp. **(c)**

The component $dl \cos\theta$ of element dl is parallel to the length of the wire 1. Hence, force on this elemental component

$$F = \frac{\mu_0}{4\pi} \cdot \frac{2 I_1 I_2}{r} (dl \cos\theta)$$

$$= \frac{\mu_0 I_1 I_2\, dl \cos\theta}{2\pi r}$$

14

Electromagnetic Induction *and* AC

1. A 20 H inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (Joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor, is **[JEE Main 2019, 8 April Shift-I]**

(a) $\frac{2}{\ln 2}$ (b) $\frac{1}{2}\ln 2$ (c) $2\ln 2$ (d) $\ln 2$

Exp. **(c)**

Given circuit is a series *L-R* circuit

In an *L - R* circuit, current increases is

$$i = \frac{E}{R}(1 - e^{-\frac{R}{L}\cdot t})$$

Now, energy stored in inductor is

$$U_L = \frac{1}{2}Li^2$$

where, L = self inductance of the coil and energy dissipated by resistor is

$$U_R = i^2R$$

Given, rate of energy stored in inductor is equal to the rate of energy dissipation in resistor. So, after differentiating, we get

$$iL\frac{di}{dt} = i^2R \Rightarrow \frac{di}{dt} = \frac{R}{L}i$$

$$\Rightarrow \quad \frac{E}{R}\cdot\frac{R}{L}e^{-\frac{R}{L}t} = \frac{R}{L}\cdot\frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\Rightarrow \quad 2e^{-\frac{R}{L}t} = 1 \Rightarrow e^{-\frac{R}{L}t} = \frac{1}{2}$$

Taking log on both sides, we have

$$\Rightarrow \quad \frac{-R}{L}t = \ln\left(\frac{1}{2}\right) \Rightarrow \frac{R}{L}t = \ln 2$$

$$\Rightarrow \quad t = \frac{L}{R}\ln 2 = \frac{20}{10}\ln 2 \Rightarrow t = 2\ln 2$$

2. An alternating voltage $V(t) = 220\sin 100\pi t$ volt is applied to a purely resistive load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is **[JEE Main 2019, 8 April Shift-I]**

(a) 5 ms (b) 2.2 ms (c) 7.2 ms (d) 3.3 ms

Exp. **(d)**

In an AC resistive circuit, current and voltage are in phase.

So, $$I = \frac{V}{R} \Rightarrow I = \frac{220}{50}\sin(100\pi t) \quad \ldots(i)$$

∴ Time period of one complete cycle of current is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{100\pi} = \frac{1}{50}\text{ s}$$

So, current reaches its maximum value at

$$t_1 = \frac{T}{4} = \frac{1}{200}\text{ s}$$

When current is half of its maximum value, then from Eq. (i), we have

$$I = \frac{I_{max}}{2} = I_{max}\sin(100\pi t_2)$$

$$\Rightarrow \sin(100\pi t_2) = \frac{1}{2} \Rightarrow 100\pi t_2 = \frac{5\pi}{6}$$

So, instantaneous time at which current is half of maximum value is $t_2 = \frac{1}{120}$ s

Hence, time duration in which current reaches half of its maximum value after reaching maximum value is

$$\Delta t = t_2 - t_1 = \frac{1}{120} - \frac{1}{200} = \frac{1}{300}\text{ s} = 3.3\text{ ms}$$

3. A circuit connected to an AC source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this? **[JEE Main 2019, 8 April Shift-II]**

(a) RC circuit with $R = 1\text{ k}\Omega$ and $C = 1\,\mu\text{F}$
(b) RL circuit with $R = 1\text{ k}\Omega$ and $L = 1\text{ mH}$
(c) RC circuit with $R = 1\text{ k}\Omega$ and $C = 10\,\mu\text{F}$
(d) RL circuit with $R = 1\text{ k}\Omega$ and $L = 10\text{ mH}$

Exp. **(c)**

Given, phase difference, $\phi = \frac{\pi}{4}$

As we know, for R-L or R-C circuit,

$$\tan\phi = \frac{\text{Capacitive reactance } (X_C) \text{ or inductive reactance } (X_L)}{\text{Resistance } (R)}$$

$$\tan\frac{\pi}{4} = \frac{X_C \text{ or } X_L}{R}$$

$$1 = \frac{X_C \text{ or } X_L}{R}$$

$$\Rightarrow \quad R = X_C \text{ or } X_L$$

Also, given $e = e_0\sin(100t)$

Comparing the above equation with general equation of emf, i.e. $e = e_0\sin\omega t$, we get

$$\omega = 100\text{ rad/s} = 10^2\text{ rad/s}$$

Now, checking option wise,

For R-C circuit, with

$R = 1\text{k}\Omega = 10^3\ \Omega$ and $C = 1\mu\text{F} = 10^{-6}\text{F}$

So, $X_C = \frac{1}{\omega C} = \frac{1}{10^2 \times 10^{-6}} = 10^4\ \Omega \Rightarrow R \neq X_C$

For R - L circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega$$

and $L = 1\text{ mH} = 10^{-3}\text{H}$

So, $X_L = \omega L = 10^2 \times 10^{-3} = 10^{-1}\Omega \Rightarrow R \neq X_L$

For R - C circuit, with

$$R = 1\text{k}\Omega = 10^3\,\Omega$$

and $C = 10\mu\text{F} = 10 \times 10^{-6}\text{F} = 10^{-5}\text{F}$

So, $X_C = \frac{1}{10^2 \times 10^{-5}} = 10^3\,\Omega \Rightarrow R = C$

For R - L circuit, with

$R = 1\text{k}\Omega = 10^3\,\Omega$

and $L = 10\ \text{mH} = 10 \times 10^{-3}\text{H} = 10^{-2}\text{H}$

$X_L = 10^2 \times 10^{-2} = 1\,\Omega \Rightarrow R \neq X_L$

Alternate Solution

Since, $\tan\frac{\pi}{4} = 1 = \frac{X_C \text{ or } X_L}{R}$

$\therefore$ For R-C circuit, we have

$$1 = \frac{1}{C\omega R} \text{ or } \omega = \frac{1}{CR} \quad \ldots\text{(i)}$$

Similarly, for R-L circuit, we have

$$1 = \frac{\omega L}{R} \Rightarrow \omega = \frac{R}{L} \quad \ldots\text{(ii)}$$

It is given in the question that, $\omega = 100$ rad/s

Thus, again by substituting the given values of R, C or L option wise in the respective Eqs. (i) and (ii), we get that only for option (c),

$$\omega = \frac{1}{CR} = \frac{1}{10 \times 10^{-6} \times 10^3}$$

or $\omega = 100$ rad/s

4. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to **[JEE Main 2019, 9 April Shift-I]**

(a) $1/L$
(b) L^2
(c) L
(d) $1/L^2$

Exp. **(a)**

Self inductance L_{sol} of a solenoid is given by

$$L_{sol} = \mu_0 n^2 \pi r^2 L$$

(Here, $n = N/L$ and L = length of solenoid)

or $$L_{sol} = \frac{\mu_0 N^2 \pi r^2}{L}$$

Clearly, $$L_{sol} \propto \frac{1}{L}$$

($\because$ All other parameters are fixed)

Note We can determine expression of L as follows

$$\phi = NBA = L_{sol} I$$

But for a solenoid, $B = \mu_0 nI$, $A = \pi r^2$

$\therefore$ $$L_{sol} I = \mu_0 n I \pi r^2 N$$

or $$L_{sol} = \mu_0 n^2 \pi r^2 L = \mu_0 \frac{N^2}{L} \pi r^2$$

5. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ $(k > 0)$, as a function of time $(t \geq 0)$. Counter clockwise current is taken to be positive. A circular conducting coil of radius $2R$ is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by **[JEE Main 2019, 9 April Shift-II]**

Exp. **(d)**

Magnetic flux associated with the outer coil is

$$\phi_{outer} = \mu_0 \pi NR \cdot I = \mu_0 N\pi R(kte^{-\alpha t})$$
$$= Cte^{-\alpha t}$$

where,

$$C = \mu_0 N\pi Rk = \text{constant}$$

Induced emf,

$$e = \frac{-d\phi_{outer}}{dt} = Ce^{-\alpha t} + (-\alpha C t e^{-\alpha t})$$
$$= Ce^{-\alpha t}(1 - \alpha t)$$

$\therefore$ Induced current, $I = \dfrac{e}{\text{Resistance}}$

$\Rightarrow$ At $t = 0$, $I = -$ ve

$\therefore$ The correct graph representing this condition is given in option (d).

Alternate Solution

Given solenoid is shown below as,

At $t = 0$, current in solenoid

$$= I(t = 0) = k(0)e^{-\alpha \cdot 0} = 0$$

Graph of $e^{\alpha t}$ and kt *versus* time can be shown as,

As, $$I = \frac{kt}{e^{\alpha t}};$$

Initially, $kt > e^{\alpha t}$

So, current is increasing in magnitude.

Finally, after a short time $kt < e^{\alpha t}$. So, current is decreasing in magnitude.

But in both cases, it remains positive or counter clockwise. So, current induced is at first anti-clockwise (following Lenz's law) and then it becomes clockwise and finally reduces to zero as $t \longrightarrow \infty$.

So, correct graph of induced current is

6. Two coils P and Q are separated by some distance. When a current of 3 A flows through coil P, a magnetic flux of 10^{-3} Wb passes through Q. No current is passed through Q. When no current passes through P and a current of 2 A passes through Q, the flux through P is

[JEE Main 2019, 9 April Shift-II]

(a) 6.67×10^{-3} Wb (b) 6.67×10^{-4} Wb

(c) 3.67×10^{-3} Wb (d) 3.67×10^{-4} Wb

Exp. **(b)**

As, coefficient of mutual induction is same for both coils

$\therefore \quad M_{PQ} = M_{QP}$

$$\Rightarrow \quad \frac{N_P\phi_{PQ}}{I_Q} = \frac{N_Q\phi_{QP}}{I_P} \quad \ldots \text{(i)}$$

Here, $N_P = N_Q = 1$,

$\phi_{PQ} = ?$, $\phi_{QP} = 10^{-3}$ Wb

$I_Q = 2\text{A}$, $I_P = 3\text{A}$

Substituting values in Eq (i), we get

$$\phi_{PQ} = \frac{I_Q \cdot \phi_{QP}}{I_P} = \frac{2}{3}\times10^{-3}$$

$$= 0.667\times10^{-3} = 6.67\times10^{-4} \text{ Wb}$$

7. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10 A, then the input voltage and current in the primary coil are

[JEE Main 2019, 10 April Shift-I]

(a) 440 V and 5 A (b) 220 V and 20 A

(c) 220 V and 10 A (d) 440 V and 20 A

Exp. **(a)**

Given,

Number of turns in primary, $N_1 = 300$

Number of turns in secondary, $N_2 = 150$

Output power, $P_2 = 2.2 \text{ kW} = 2.2\times10^3$ W

Current in secondary coil, $I_2 = 10$ A

Output power, $P_2 = I_2V_2$

$$\Rightarrow \quad V_2 = \frac{P_2}{I_2} = \frac{2.2\times10^3}{10} = 220\text{V} \quad \ldots \text{(i)}$$

We know that,

$$\frac{N_1}{N_2} = \frac{\text{Input voltage}}{\text{Output voltage}} = \frac{V_1}{V_2} \Rightarrow V_1 = \left(\frac{N_1}{N_2}\right)V_2$$

$$\Rightarrow \quad V_1 = \left(\frac{300}{150}\right)\times(220\text{ V}) \quad \text{[using Eq. (i)]}$$

$$V_1 = 440 \text{ V} \quad \ldots \text{(ii)}$$

Again, $\dfrac{V_1}{V_2} = \dfrac{I_2}{I_1}$

$$\Rightarrow I_1 = \left(\frac{V_2}{V_1}\right)I_2 = \frac{220}{440}\times10 \quad \text{[using Eqs. (i) and (ii)]}$$

$$\Rightarrow \quad I_1 = 5\text{A}$$

8. A coil of self inductance 10 mH and resistance $0.1\,\Omega$ is connected through a switch to a battery of internal resistance $0.9\,\Omega$. After the switch is closed, the time taken for the current to attain 80% of the saturation value is [Take, $\ln 5 = 1.6$]

[JEE Main 2019, 10 April Shift-II]

(a) 0.002 s (b) 0.324 s

(c) 0.103 s (d) 0.016 s

Exp. **(d)**

> **Key Idea** In an L-R circuit, current during charging of inductor is given by
>
> $$i = i_0(1 - e^{-\frac{R}{L}\cdot t})$$
>
> where, i_0 = saturation current.

In given circuit,

Inductance of circuit is

$L = 10\text{ mH} = 10\times10^{-3}$ H

Resistance of circuit is

$R = (R_s + r) = 0.1 + 0.9 = 1\Omega$

Now, from

$$i = i_0(1 - e^{-\frac{R}{L}t}) \quad \text{...(i)}$$

Given, $i = 80\%$ of i_0

$\Rightarrow \quad i = \frac{80 i_0}{100} = 0.8 i_0$

Substituting the value of i in Eq. (i), we get

$$0.8 = 1 - e^{-\frac{R}{L}t}$$

$\Rightarrow \quad e^{-\frac{R}{L}t} = 0.2$

$\Rightarrow \quad e^{\frac{R}{L}t} = 5$

$\Rightarrow \ln \quad (e)^{\frac{R}{L}t} = \ln 5 \Rightarrow \frac{R}{L}t = \ln 5$

$\Rightarrow \quad t = \frac{L}{R} \cdot \ln(5) = \frac{10 \times 10^{-3}}{1} \times \ln(5)$

$= 10 \times 10^{-3} \times 1.6$

$= 1.6 \times 10^{-2}$ s $= 0.016$ s

9. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop.
If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to

[JEE Main 2019, 12 April Shift-I]

(a) 60μ A (b) 170μ A
(c) 150μ A (d) 115μ A

Exp. **(b)**

Induced emf in the conductor of length L moving with velocity of 1 cm/s in the magnetic field of 1T is given by

$$V = BLv \quad \text{...(i)}$$

If equivalent resistance of the circuit is R_{eq}, then current in the loop will be

$$i = \frac{V}{R_{eq}} = \frac{BLv}{R_{eq}} \quad \text{...(ii)}$$

Now, given network is a balanced Wheatstone bridge $\left(\frac{P}{Q} = \frac{R}{S}\right)$.

So, equivalent resistance of the Wheatstone bridge is

$$R_W = \frac{2 \times 4}{2 + 4} = \frac{8}{6} = \frac{4}{3}\Omega$$

Again, resistance of conductor is 1.7 Ω.

So, effective resistance will be

$R_{eq} = \frac{4}{3} + 1.7 = \frac{4}{3} + \frac{17}{10}$,

$R_{eq} = \frac{40 + 51}{30} = \frac{91}{30} \simeq 3\Omega$

By putting given values of R_{eq}, B and v in Eq. (ii), we have

$$i = \frac{(1)(5 \times 10^{-2}) \times 10^{-2}}{3}$$

[here, $L = 5 \times 10^{-2}$ m, $v = 1$ cm/s $= 10^{-2}$ m/s]

$$i = \frac{5 \times 10^{-4}}{3} = 1.67 \times 10^{-4} \text{A}$$

$$i = 167\ \mu\text{A} \approx 170\ \mu\text{A}$$

10. Consider the L-R circuit shown in the figure. If the switch S is closed at $t = 0$, then the amount of charge that passes through the battery between $t = 0$ and $t = \frac{L}{R}$ is

[JEE Main 2019, 12 April Shift-II]

(a) $\frac{2.7EL}{R^2}$ (b) $\frac{EL}{2.7R^2}$
(c) $\frac{7.3EL}{R^2}$ (d) $\frac{EL}{7.3R^2}$

Exp. **(b)**

In an *L-R* circuit, current during charging is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where, $I_0 = \frac{E}{R}$ = saturation current.

So, we have $\frac{dq}{dt} = I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$$\Rightarrow \quad dq = I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

So, charge q that passes through battery from time $t = 0$ to $t = \frac{L}{R}$ is obtained by integrating the above equation within the specified limits, i.e.

$$q = \int_0^Q dq = \int_{t=0}^{t=\frac{L}{R}} I_0\left(1 - e^{-\frac{R}{L}t}\right)dt$$

$$= I_0\left[\left(t - \frac{1}{\left(-\frac{R}{L}\right)} \cdot e^{-\frac{R}{L}t}\right)\right]_0^{\frac{L}{R}}$$

$$= \frac{E}{R}\left[\left\{\frac{L}{R} + \frac{L}{Re'}\right\} - \left\{0 + \frac{L}{R}\right\}\right]$$

$$= \frac{E}{R} \times \frac{L}{Re} = \frac{EL}{R^2 e}$$

$$\Rightarrow \quad Q = \frac{EL}{2.7\,R^2} \qquad [\because e \approx 2.72]$$

11. A conducting circular loop is made of a thin wire has area $3.5 \times 10^{-3}\,\text{m}^2$ and resistance 10 Ω. It is placed perpendicular to a time dependent magnetic field $B(t) = (0.4\text{T})\sin(0.5\pi t)$. The field is uniform in space. Then the net charge flowing through the loop during $t = 0$ s and $t = 10$ ms is close to **[JEE Main 2019, 9 Jan Shift-I]**

(a) 6 mC (b) 21 mC
(c) 7 mC (d) 14 mC

Exp. **(d)**

Since, the magnetic field is dependent on time, so the net charge flowing through the loop will be given as

$$Q = \frac{\text{change in magnetic flux, } \Delta\phi_B}{\text{resistance, } R}$$

As, $\Delta\phi_B = BA = BA\cos\theta$

where, A is the surface area of the loop and 'θ' is an angle between B and A.

Here, $\theta = 0 \Rightarrow \Delta\phi_B = BA$

∴ For the time interval, $t = 0$ ms to $t = 10$ ms,

$$Q = \frac{\Delta\phi_B}{R} = \frac{A}{R}(B_{f\text{ at }0.01\text{s}} - B_{i\text{at }0\text{s}})$$

Substituting the given values, we get

$$= \frac{3.5 \times 10^{-3}}{10}[0.4\sin(0.5\pi) - 0.4\sin 0]$$

$$= 3.5 \times 10^{-4}(0.4\sin\pi/2)$$

$$= 1.4 \times 10^{-4}\text{C} = 14\text{ mC}$$

12. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60 Ω) is driven by an AC source of 24 V/50 Hz. The energy dissipated in the circuit in 60 s is **[JEE Main 2019, 9 Jan Shift-II]**

(a) 3.39×10^3 J (b) 5.65×10^2 J
(c) 2.26×10^3 J (d) 5.17×10^2 J

Exp. **(d)**

The given series *R-L-C* circuit is shown in the figure below.

Here,

V_R = potential across resistance (R)

V_L = potential across inductor (L) and

V_C = potential across capacitor (C).

Impedance of this series circuit is,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \qquad \ldots(\text{i})$$

$$\because \quad X_L = \omega L$$

$$= (2\pi f)(L)$$

$$= 2\pi \times 50 \times 20 \times 10^{-3}\ \Omega$$

$$X_L = 6.28\,\Omega \qquad \ldots(\text{ii})$$

and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

$$= \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}} = \frac{250}{3\pi}\Omega \qquad \ldots(\text{iii})$$

and $X_L - X_C = \left(6.28 - \frac{250}{3\pi}\right) = -20.23\ \Omega \qquad \ldots(\text{iv})$

RMS value of current in circuit is,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{24}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I_{rms} = \frac{24}{\sqrt{60^2 + (-20.23)^2}} = \frac{24}{63.18}$$

$$I_{rms} = 0.379 \text{ A}$$

Therefore, energy dissipated is

$$= I_{rms}^2 \times R \times t$$

$$E = (0.379)^2 \times 60 \times 60$$

or $= 517.10 = 5.17 \times 10^2$ J

13. A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V by the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be

[JEE Main 2019, 9 Jan Shift-II]

(a) 45 A (b) 50 A (c) 25 A (d) 35 A

Exp. **(a)**

For a transformer, there are two circuits which have N_p and N_s (number of coil turns), I_p and I_S (currents) respectively as shown below.

Here, input voltage, $V_p = 2300$ V

Number of turns in primary coil, $N_P = 4000$

Output voltage, $V_S = 230$ volt

Output power, $P_S = V_S \cdot I_S$

Input power, $P_P = V_P I_P$

∴ The efficiency of the transformer is

$$\eta = \frac{\text{Output (secondary) power}}{\text{Input (primary) power}}$$

$$\Rightarrow \quad \eta = \frac{V_S \cdot I_S}{V_P \cdot I_P} \times 100$$

$$\Rightarrow \quad \eta = \frac{(230)(I_S)}{(2300)(5)} \times 100$$

$$90 = \frac{230\, I_S}{(2300) \times 5} \times 100$$

$$\Rightarrow \quad I_S = 45 \text{ A}$$

14. A magnet of total magnetic moment $10^{-2}\hat{\mathbf{i}}$ A-m^2 is placed in a time varying magnetic field, $B\hat{\mathbf{i}}(\cos\omega t)$, where $B = 1$ T and $\omega = 0.125$ rad/s. The work done for reversing the direction of the magnetic moment at $t = 1$ s is **[JEE Main 2019, 10 Jan Shift-I]**

(a) 0.01 J (b) 0.007 J (c) 0.014 J (d) 0.028 J

Exp. **(c)**

Work done in reversing dipole is

$$W = 2\,MB$$

where, M = magnetic dipole moment

$= 10^{-2}$ A-m^2

and B = external field

$= B\cos\omega t = 1 \times \cos(0.125 \times 1)$

$= \cos(7^\circ) = 0.992$

Substituting these values, we get

$$W = 2 \times 10^{-2} \times 0.992$$

$$= 0.0198 \text{ J}$$

which is nearest to 0.014 J.

15. At some location on earth, the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 A-m is suspended from its mid point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is

[JEE Main 2019, 10 Jan Shift-II]

(a) 6.5×10^{-5} N (b) 3.6×10^{-5} N

(c) 1.3×10^{-5} N (d) 1.8×10^{-5} N

Exp. **(a)**

Without applied forces, (in equilibrium position) the needle will stay in the resultant magnetic field of earth. Hence, the dip 'θ' at this place is 45° (given).

We know that, horizontal and vertical components of earth's magnetic field (B_H and B_V) are related as

$$\frac{B_V}{B_H} = \tan\theta$$

Here, $\theta = 45^\circ$ and $B_H = 18 \times 10^{-6}$ T

$\Rightarrow \quad B_V = B_H \tan 45^\circ$

$\Rightarrow \quad B_V = B_H = 18 \times 10^{-6}$ T $\quad (\because \tan 45^\circ = 1)$

Now, when the external force F is applied, so as to keep the needle stays in horizontal position is shown below,

Taking torque at point P, we get

$$mB_V \times 2l = Fl$$

$\therefore \quad F = 2 \times mB_V$

Substituting the given values, we get

$$= 2 \times 1.8 \times 18 \times 10^{-6}$$

$$= 6.48 \times 10^{-5} = 6.5 \times 10^{-5} \text{ N}$$

16. The self-induced emf of a coil is 25 V. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is

[JEE Main 2019, 10 Jan Shift-II]

(a) 437.5 J (b) 740 J (c) 637.5 J (d) 540 J

Exp. **(a)**

Energy stored in an inductor of inductance L and current I is given by

$$E = \frac{1}{2}LI^2$$

When current is being changed from I_1 to I_2, change in energy will be

$$\Delta E = E_2 - E_1 = \frac{1}{2}LI_2^2 - \frac{1}{2}LI_1^2 \quad \ldots(i)$$

As I_1 and I_2 are given, we need to find value of L.

Now, induced emf in a coil is

$$\varepsilon = L\frac{dI}{dt}$$

Here, $\varepsilon = 25$ V,

$dI = I_2 - I_1 = (25 - 10) = 15$ A and $dt = 1$ s

$\Rightarrow \quad 25 = L \times \frac{15}{1}$ or $L = \frac{25}{15} = 5/3$ H

Putting values of L, I_1 and I_2 in Eq. (i), we get

$$\Delta E = \frac{1}{2} \times \frac{5}{3} \times [25^2 - 10^2] = \frac{1}{2} \times \frac{5}{3} \times 525$$

$$\Delta E = 437.5 \text{ J}$$

17. There are two long coaxial solenoids of same length l. The inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2, respectively. The ratio of mutual inductance to the self-inductance of the inner coil is **[JEE Main 2019, 11 Jan Shift-I]**

(a) $\frac{n_2}{n_1} \cdot \frac{r_1}{r_2}$ (b) $\frac{n_2}{n_1} \cdot \frac{r_2^2}{r_1^2}$ (c) $\frac{n_2}{n_1}$ (d) $\frac{n_1}{n_2}$

Exp. **(c)**

Mutual inductance for a coaxial solenoid of radius r_1 and r_2 and number of turns n_1 and n_2, respectively is given as,

$M = \mu_0 n_1 n_2 \pi r_1^2 l$ (for internal coil of radius r_1)

Self inductance for the internal coil,

$$L = \mu_0 n_1^2 \pi r_1^2 l$$

$$\therefore \quad \frac{M}{L} = \frac{n_1 n_2}{n_1^2} = \frac{n_2}{n_1}$$

18. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self-inductance of the coil

[JEE Main 2019, 11 Jan Shift-II]

(a) increases by a factor of 3
(b) decreases by a factor of $9\sqrt{3}$
(c) increases by a factor of 27
(d) decreases by a factor of 9

Exp. **(c)**

Self-inductance of a coil is given by the relation

$$L = \mu_0 n^2 A \cdot l$$

where, n is number of turns per unit length. Shape of the wooden frame is equilateral triangle.

$\therefore$ Area of equilateral triangle,

$$A = \frac{\sqrt{3}}{4}a^2$$

(where, a is side of equilateral triangle)

∴ Self-inductance,

$$L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] . l$$

Here, $l = 3a \times N$ (where, N is total turns)

$$\therefore \quad L = \mu_0 n^2 \left[\frac{\sqrt{3}}{4} a^2\right] \times 3aN \text{ or } L \propto a^3$$

When each side of frame is increased by a factor 3 keeping the number of turns per unit length of the frame constant.

Then, $a' = 3a$

∴ $L' \propto (a')^3$

or $L' \propto (3a)^3$

or $L' \propto 27a^3$

or $L' = 27L$

So, self-inductance will becomes 27 times.

19. In the figure shown, a circuit contains two identical resistors with resistance $R = 5\,\Omega$ and an inductance with $L = 2$ mH. An ideal battery of 15 V is connected in the circuit.

[JEE Main 2019, 12 Jan Shift-I]

What will be the current through the battery long after the switch is closed?

(a) 6 A
(b) 3 A
(c) 5.5 A
(d) 7.5 A

Exp. **(a)**

After a sufficiently long time, in steady state, resistance offered by inductor is zero. So, circuit is reduced to

∴ Current in circuit is

$$I = \frac{E}{R_{eq}} = \frac{15}{\left(\frac{5 \times 5}{5 + 5}\right)} = \frac{15 \times 2}{5} = 6 \text{ A}$$

20. A 10 m long horizontal wire extends from North-East to South-West. It is falling with a speed of $5.0\,\text{ms}^{-1}$ at right angles to the horizontal component of the earth's magnetic field of 0.3×10^{-4} Wb/ m^2. The value of the induced emf in wire is

[JEE Main 2019, 12 Jan Shift-II]

(a) 1.5×10^{-3} V (b) 1.1×10^{-3} V
(c) 0.3×10^{-3} V (d) 2.5×10^{-3} V

Exp. **(a)**

Wire falls perpendicularly to horizontal component of earth's magnetic field, so induced electromotive force (ε) = Blv

Substituting the given values, we get

$$\varepsilon = 0.3 \times 10^{-4} \times 10 \times 5 = 1.5 \times 10^{-3} \text{ V}$$

21.

In the above circuit, $C = \frac{\sqrt{3}}{2}\mu$F, $R_2 = 20\,\Omega$, $L = \frac{\sqrt{3}}{10}$H and $R_1 = 10\,\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path is I_2. The voltage of AC source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is

[JEE Main 2019, 12 Jan Shift-II]

(a) 30° (b) 60°
(c) 0° (d) 90°

Exp. **(a)**

Phase difference between I_2 and V, i.e. $C-R_2$ circuit is given by

$$\tan\phi = \frac{X_C}{R_2} \Rightarrow \tan\phi = \frac{1}{C\omega R_2}$$

Substituting the given values, we get

$$\tan\phi = \frac{1}{\frac{\sqrt{3}}{2}\times 10^{-6}\times 100\times 20} = \frac{10^3}{\sqrt{3}}$$

$\therefore \phi_1$, is nearly 90°.

Phase difference between I_1 and V, i.e. in $L-R_1$ circuit is given by

$$\tan\phi_2 = -\frac{X_L}{R_1} = -\frac{L\omega}{R}$$

Substituting the given values, we get

$$\tan\phi_2 = -\frac{\frac{\sqrt{3}}{10}\times 100}{10} = -\sqrt{3}$$

As, $\tan\phi_2 = -\sqrt{3}$

$\therefore$ $\phi_2 = 120°$

Now, phase difference between I_1 and I_2 is

$$\Delta\phi = \phi_2 - \phi_1 = 120° - 90° = 30°$$

22. In an AC circuit, the instantaneous emf and current are given by

$$e = 100\sin 30t,\ i = 20\sin\left(30t - \frac{\pi}{4}\right)$$

In one cycle of AC, the average power consumed by the circuit and the wattless current are, respectively **[JEE Main 2018]**

(a) 50 , 10 (b) $\frac{1000}{\sqrt{2}}$, 10 (c) $\frac{50}{\sqrt{2}}$, 0 (d) 50 , 0

Exp. **(b)**

Given, $e = 100\sin 30t$

and $i = 20\sin\left(30t - \frac{\pi}{4}\right)$

$\therefore$ Average power ,

$$P_{av} = V_{rms}I_{rms}\cos\phi$$
$$= \frac{100}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\cos\frac{\pi}{4}$$
$$= \frac{1000}{\sqrt{2}}\text{ watt}$$

Wattless current is,

$$I = I_{rms}\sin\phi$$
$$= \frac{20}{\sqrt{2}}\times\sin\frac{\pi}{4} = \frac{20}{2} = 10\text{ A}$$

$\therefore$ $P_{av} = \frac{1000}{\sqrt{2}}$ watt

and $I_{wattless} = 10$ A

23. For an R-L-C circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$, the current exhibits resonance. The quality factor, Q is given by **[JEE Main 2018]**

(a) $\frac{\omega_0 L}{R}$ (b) $\frac{\omega_0 R}{L}$ (c) $\frac{R}{\omega_0 C}$ (d) $\frac{CR}{\omega_0}$

Exp. **(a)**

Sharpness of resonance of a resonant L-C-R circuit is determined by the ratio of resonant frequency with the selectivity of circuit. This ratio is also called ''Quality Factor'' or Q-factor.

$$Q\text{-factor} = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

24. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is **[JEE Main 2017 (Offline)]**

(a) 225 Wb (b) 250 Wb (c) 275 Wb (d) 200 Wb

Exp. **(b)**

Induced constant, $I = \frac{e}{R}$

Here, e = induced emf $= \frac{d\phi}{dt}$

$$I = \frac{I}{R} = \left(\frac{d\phi}{dt}\right)\cdot\frac{1}{R}$$
$$d\phi = IRdt$$
$$\phi = \int IRdt$$

$\therefore$ Here, R is constant

$\therefore$ $\phi = R\int Idt$

$\int I\cdot dt$ = Area under $I-t$ graph

$$= \frac{1}{2}\times 10\times 0.5 = 2.5$$

$\therefore$ $\phi = R\times 2.5 = 100\times 2.5 = 250$ Wb

25. An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220 V (rms), 50 Hz AC supply, the series inductor needed for it to work is close to

[JEE Main 2016 (Offline)]

(a) 80 H (b) 0.08 H
(c) 0.044 H (d) 0.065 H

Exp. **(d)**

Given, $I = 10\,\text{A}$, $V = 80\,\text{V}$,

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega \quad \text{and} \quad \omega = 50\,\text{Hz}$$

For AC circuit, we have

$$I = \frac{V}{\sqrt{8^2 + X_L^2}} \Rightarrow 10 = \frac{220}{\sqrt{64 + X_L^2}}$$

$$\Rightarrow \sqrt{64 + X_L^2} = 22$$

Squaring on both sides, we get

$$64 + X_L^2 = 484$$

$$\Rightarrow X_L^2 = 484 - 64 = 420$$

$$X_L = \sqrt{420} \Rightarrow 2\pi \times \omega L = \sqrt{420}$$

Series inductor on an arc lamp,

$$L = \frac{\sqrt{420}}{(2\pi \times 50)} = 0.065\,\text{H}$$

26. An inductor ($L = 0.03\,\text{H}$) and a resistor ($R = 0.15\,\text{k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1\,\text{ms}$, the current in the circuit will be ($e^5 \cong 150$)

[JEE Main 2015]

(a) 100 mA (b) 67 mA
(c) 6.7 mA (d) 0.67mA

Exp. **(d)**

After long time inductor behaves as short-circuit.

At $t = 0$, the inductor behaves as short-circuited. The current

$$I_0 = \frac{E_0}{R} = \frac{15\,\text{V}}{0.15\,\text{k}\Omega} = 100\ \text{mA}$$

As K_2 is closed, current through the inductor starts decay, which is given at any time t as

$$I = I_0\, e^{\frac{-tR}{L}} = (100\ \text{mA})\, e^{\frac{-t \times 15000}{3}}$$

At $t = 1\,\text{ms}$

$$I = (100\ \text{mA})\, e^{-\frac{1\times10^{-3}\times15\times10^{3}}{3}}$$

$$I = (100\,\text{mA})\, e^{-5} = 0.6737\ \text{mA}$$

or $I = 0.67\ \text{mA}$

27. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit, the capacitor is charged to Q_0 and then connected to the L and R as shown below.

If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L, then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

[JEE Main 2015]

Exp. **(a)**

Considert the *LCR* circuit at any time *t*

Now, applying KVL

We have $\frac{q}{C} - iR - \frac{Ldi}{dt} = 0$

As current is decreasing with time we can write

$i = -\frac{dq}{dt}$

$\Rightarrow \quad \frac{q}{C} + \frac{dq}{dt}R + \frac{Ld^2q}{dt^2} = 0$

or $\quad \frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{q}{LC} = 0$

This equation is equivalent to that of a damped oscillator

Thus, we can write the solution as

$Q_{max}(t) = Q_0 \cdot e^{-Rt/2L}$ or $Q^2_{max} = Q_0^2 e^{-\frac{Rt}{L}}$

As $L_1 > L_2$ damping is faster for L_2

Aliter Inductance is inertia of circuit. It means inductance opposes the flow of charge, more inductance means decay of charge is slow.

In option (a), in a given time to, $Q_1^2 > Q_2^2$.

So, $L_1 > L_2$. Hence, option (a) is correct.

28. In the circuit shown here, the point C is kept connected to point A till the current flowing through the circuit becomes constant. Afterward, suddenly point C is disconnected from point A and connected to point B at time $t = 0$. Ratio of the voltage across resistance and the inductor at $t = L/R$ will be equal to **[JEE Main 2014]**

(a) $\frac{e}{1-e}$ (b) 1 (c) −1 (d) $\frac{1-e}{e}$

Exp. **(c)**

After connecting C to B hanging the switch, the circuit will act like *L-R* discharging circuit.

Applying Kirchhoff's loop equation,

$V_R + V_L = 0 \Rightarrow V_R = -V_L$

$\therefore \quad \frac{V_R}{V_L} = -1$

29. A metallic rod of length l is tied to a string of length $2l$ and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field B in the region, the emf induced across the ends of the rod is **[JEE Main 2013]**

(a) $\frac{2B\omega l^3}{2}$ (b) $\frac{3B\omega l^3}{2}$ (c) $\frac{4B\omega l^2}{2}$ (d) $\frac{5B\omega l^2}{2}$

Exp. **(d)**

$\because$ Induced emf is rate of change of magnetic flux.

$e = \int Bv\,dx \qquad [\because v = \omega x]$

$\Rightarrow \quad e = \int_{2l}^{3l} (\omega x) \cdot B \cdot dx$

Using $\int x^n dx = \frac{x^{n+1}}{n+1}$

$\Rightarrow \quad e = B\omega\left[\frac{x^2}{2}\right]_{2l}^{3l}$

$e = B\omega\frac{[(3l)^2 - (2l)^2]}{2} = \frac{5Bl^2\omega}{2}$

30. In an L-C-R circuit as shown below, both switches are open initially. Now, switch S_1 is closed and S_2 kept open (q is charge on the capacitor and $\tau = RC$ is capacitance time constant). Which of the following statement is correct? **[JEE Main 2013]**

(a) Work done by the battery is half of the energy dissipated in the resistor
(b) At $t = \tau$, $q = CV/2$
(c) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(d) At $t = \frac{\tau}{2}$, $q = CV(1 - e^{-1})$

Exp. **(c)**

For charging capacitor, q is given as

$$q = q_0(1 - e^{-t/\tau}) = CV(1 - e^{-t/\tau})$$

At $t = 2\tau$,

$$q = CV(1 - e^{\frac{-2\tau}{\tau}}) \Rightarrow q = CV(1 - e^{-2})$$

31. The figure shows an experimental plot discharging of a capacitor in an R-C circuit. The time constant τ of this circuit lies between

(a) 150 s and 200 s (b) 0 and 50 s
(c) 50 s and 100 s (d) 100 s and 150 s

Exp. **(d)**

Time constant τ is the duration when the value of potential drops by 63% of its initial maximum value (*i.e.*, V_0/e). Here, 37% of 25 V = 9.25 V which lies between 100 s to 150 s in the graph.

32. A coil is suspended in a uniform magnetic field with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil, it starts oscillating; it is very difficult to stop. But, if an aluminium plate is placed near to the coil, it stops. This is due to **[AIEEE 2012]**

(a) development of air current when the plate is placed
(b) induction of electrical charge on the plate
(c) shielding of magnetic lines of force as aluminium is a paramagnetic material
(d) electromagnetic induction in the aluminium plate giving rise to electromagnetic damping

Exp. **(d)**

According to Lenz's law, electromagnetic induction takes place in the aluminium plate due to which eddy current is developed which oppose the motion or vibrations of coil.

This causes loss in energy which results in damping of oscillatory motion of the coil.

33. In a series L-C-R circuit $R = 200\,\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz, respectively. On taking out the capacitance from the circuit, the current lags behind the voltage by 30°. On taking out the inductor from the circuit, the current leads the voltage by 30°. The power dissipated in the L-C-R circuit is **[AIEEE 2010]**

(a) 305 W (b) 210 W (c) zero (d) 242 W

Exp. **(d)**

The given circuit is under resonance as $X_L = X_C$.
[$\because$ same phase change in L-R and C-R circuits]
Hence, power dissipated in the circuit is

$$P = \frac{V^2}{R} = 242 \text{ W}$$

34. An inductor of inductance $L = 400$ mH and resistors of resistances $R_1 = 4\,\Omega$ and $R_2 = 2\,\Omega$ are connected to battery of emf 12 V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at $t = 0$. The potential drop across L as a function of time is **[AIEEE 2009]**

(a) $6e^{-5t}$ V (b) $\frac{12}{t}e^{-3t}$ V

(c) $6(1-e^{-t/0.2})$V (d) $12e^{-5t}$ V

Exp. **(d)**

$$I_1 = \frac{E}{R_1} = \frac{12}{4} = 3\text{A}$$

$\because$ Potential drop $= E - I_2 R_2$

$$I_2 = I_0(1-e^{-T/t_c})$$

[current as a function of time]

$\Rightarrow$ $$I_0 = \frac{E}{R_2} = \frac{12}{2} = 6\text{ A}$$

and $$t_c = \frac{L}{R_2} = \frac{400\times10^{-3}}{2} = 0.2$$

$$I_2 = 6(1-e^{-t})$$

Potential drop across

$$L = E - R_2 I_2$$
$$= 12 - 2\times 6(1-e^{-5t})$$
$$= 12e^{-5t}$$

35. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\text{ cm}^2$ and length $= 20$ cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is ($\mu_0 = 4\pi\times10^{-7}$ TmA^{-1})

[AIEEE 2008]

(a) $2.4\pi\times10^{-5}$ H (b) $4.8\pi\times10^{-4}$ H

(c) $4.8\pi\times10^{-5}$ H (d) $2.4\pi\times10^{-4}$ H

Exp. **(d)**

$$M = \frac{\mu_0 N_1 \times N_2 \times A}{l}$$

where, $N_1 = 300$ turns, $N_2 = 400$turns,

$A = 10\text{ cm}^2 = 10\times10^{-4}\text{m}^2$

and $l = 20\text{ cm} = 20\times10^{-2}$ m

Substituting the values in the given formula, we get

$$M = 2.4\pi\times10^{-4}\text{ H.}$$

36. In an AC circuit, the voltage applied is $E = E_0 \sin\omega t$. The resulting current in the circuit is $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$. The power consumption in the circuit is given by

(a) $P = \frac{E_0 I_0}{\sqrt{2}}$ (b) $P =$ zero

[AIEEE 2007]

(c) $P = \frac{E_0 I_0}{2}$ (d) $P = \sqrt{2}E_0 I_0$

Exp. **(b)**

For given circuit, current is lagging the voltage by $\frac{\pi}{2}$, so circuit is purely inductive and there is no power consumption in the circuit. The work done by battery is stored as magnetic energy in the inductor.

37. An ideal coil of 10 H is connected in series with a resistance of 5 Ω and a battery of 5 V. After 2 s, the connection is made, the current flowing (in ampere) in the circuit is

(a) $(1-e)$ (b) e **[AIEEE 2007]**

(c) e^{-1} (d) $(1-e^{-1})$

Exp. **(d)**

Rise of current in *L-R* circuit is given by

$$I = I_0(1-e^{-t/\tau})$$

where, $$I_0 = \frac{E}{R} = \frac{5}{5} = 1\text{ A}$$

Now, $$\tau = \frac{L}{R} = \frac{10}{5} = 2\text{ s}$$

After 2s *i.e.*, at $t = 2$s,

Rise of current, $I = (1-e^{-1})$A

38. An inductor ($L = 100$ mH), a resistor ($R = 100\,\Omega$) and a battery ($E = 100$ V) are initially connected in series as shown in the figure. After a long time, the battery is disconnected after short

circuiting the points A and B. The current in the circuit 1 millisecond after the short circuit is **[AIEEE 2006]**

(a) $1/e$ A (b) e A (c) 0.1 A (d) 1 A

Exp. **(a)**

This is a combined example of growth and decay of current in an L-R circuit.

The current through circuit just before shorting the battery,

$$I_0 = \frac{E}{R} = \frac{100}{100} = 1 \text{ A}$$

[as inductor would be shorted in steady state]

After this decay of current starts in the circuit, according to the equation,

$$I = I_0 e^{-t/\tau}$$

where, $\tau = L/R$

$$I = 1 \times e^{-(1\times10^{-3})/(100\times10^{-3}/100)} = \left(\frac{1}{e}\right) \text{A}$$

[$\because t$ = 1 millisecond = 1×10^{-3} s and $L = 100 \times 10^{-3}$ H]

39. In an AC generator, a coil with N turns, all of the same area A and total resistance R, rotates with frequency ω in a magnetic field B. The maximum value of emf generated in the coil is **[AIEEE 2006]**

(a) $NABR\omega$ (b) NAB (c) $NABR$ (d) $NAB\omega$

Exp. **(d)**

The emf generated would be maximum when flux (cutting) would be maximum *i.e.,* angle between area vector of coil and magnetic field is 0°. The emf generated is given by (as a function of time)

$$e = NBA\omega \cos \omega t$$

$$\Rightarrow e_{max} = NAB\omega \quad [\because \cos \omega t = \cos\theta = 1 \Rightarrow \theta = 0°]$$

40. In a series resonant L-C-R circuit, the voltage across R is 100 V and $R = 1\ \text{k}\Omega$ with $C = 2\ \mu\text{F}$. The resonant frequency ω is 200 rad/s. At resonance, the voltage across L is **[AIEEE 2006]**

(a) 2.5×10^{-2} V (b) 40 V

(c) 250 V (d) 4×10^{-3} V

Exp. **(c)**

At resonance, $\omega L = 1/\omega C$

Current flowing through the circuit,

$$I = \frac{V_R}{R} = \frac{100}{1000} = 0.1 \text{ A}$$

So, voltage across L is given by $V_L = IX_L = I\omega L$.

But $\omega L = 1/\omega C$

$$\therefore \quad V_L = \frac{I}{\omega C} = V_C = \frac{0.1}{200 \times 2 \times 10^{-6}} = 250 \text{ V}$$

41. The flux linked with a coil at any instant t is given by $\phi = 10t^2 - 50t + 250$. The induced emf at $t = 3$ s is **[AIEEE 2006]**

(a) –190 V (b) –10 V (c) 10 V (d) 190 V

Exp. **(b)**

$$\phi = 10t^2 - 50t + 250$$

From Faraday's law of electromagnetic induction,

$e = -d\phi/dt$

$$\therefore \quad e = -[10 \times 2t - 50]$$

$$\therefore \quad e|_{t=3\text{s}} = -[10 \times 6 - 50] = -10 \text{ V}$$

42. A long solenoid has 200 turns per cm and carries a current I. The magnetic field at its centre is 6.28×10^{-2} Wb/m^2. Another long solenoid has 100 turns per cm and it carries a current $I/3$. The value of the magnetic field at its centre is **[AIEEE 2006]**

(a) 1.05×10^{-2} Wb/m^2 (b) 1.05×10^{-5} Wb/m^2

(c) 1.05×10^{-3} Wb/m^2 (d) 1.05×10^{-4} Wb/m^2

Exp. **(a)**

Magnetic field due to a long solenoid is given by

$$B = \mu_0 nI$$

From given data,

$$6.28 \times 10^{-2} = \mu_0 \times 200 \times 10^2 \times I \quad \ldots\text{(i)}$$

and $$B = \mu_0 \times 100 \times 10^2 \times \left(\frac{I}{3}\right) \quad \ldots\text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$B \approx 1.05 \times 10^{-2} \text{ Wb / m}^2$$

43. A coil of inductance 300 mH and resistance 2 Ω is connected to a source of voltage 2 V. The current reaches half of its steady state value in **[AIEEE 2005]**

(a) 0.05 s (b) 0.1 s (c) 0.15 s (d) 0.3 s

Exp. **(b)**

The current at any instant is given by

$$I = I_0(1 - e^{-Rt/L})$$

$$\Rightarrow \quad \frac{I_0}{2} = I_0(1 - e^{-Rt/L})$$

or $$\frac{1}{2} = (1 - e^{-Rt/L})$$

or $$e^{-Rt/L} = \frac{1}{2}$$

or $$\frac{Rt}{L} = \ln 2$$

$$\therefore \quad t = \frac{L}{R}\ln 2 = \frac{300 \times 10^{-3}}{2} \times 0.693$$

$$= 150 \times 0.693 \times 10^{-3} = 0.10395 \text{ s} = 0.1 \text{ s}$$

44. The self-inductance of the motor of an electric fan is 10 H. In order to impart maximum power at 50 Hz, it should be connected to a capacitance of **[AIEEE 2005]**

(a) 4 μF (b) 8 μF (c) 1 μF (d) 2 μF

Exp. **(c)**

Given, $L = 10$ H, $f = 50$ Hz

For maximum power,

$$X_C = X_L \quad [\because \text{resonance condition}]$$

or $$\frac{1}{\omega C} = \omega L \quad \text{or} \quad C = \frac{1}{\omega^2 L}$$

$$\therefore \quad C = \frac{1}{4\pi^2 \times 50 \times 50 \times 10}$$

or $$C = 0.1 \times 10^{-5} \text{ F} = 1 \mu\text{F}$$

45. A circuit has a resistance of 12 Ω and an impedance of 15 Ω. The power factor of the circuit will be **[AIEEE 2005]**

(a) 0.8 (b) 0.4 (c) 1.25 (d) 0.125

Exp. **(a)**

$$\text{Power factor} = \cos\phi = \frac{R}{Z} = \frac{12}{15} = \frac{4}{5} = 0.8$$

46. The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit ? **[AIEEE 2005]**

(a) C alone (b) R, L (c) L, C (d) L alone

Exp. **(c)**

(a) In a circuit having C alone, the voltage lags the current by $\frac{\pi}{2}$.

(b) In circuit containing R and L, the voltage leads the current by $\frac{\pi}{2}$.

(c) In L-C circuit, the phase difference between current and voltage can have any value between 0 to $\frac{\pi}{2}$ depending on the values of L and C.

(d) In a circuit containing L alone, the voltage leads the current by $\pi/2$.

47. One conducting U-tube can slide inside another as shown in figure, maintaining electrical contacts between the tubes. The magnetic field B is perpendicular to the plane of the figure. If each tube moves towards the other at a constant speed v, then the emf induced in the circuit in terms of B, l and v, where l is the width of each tube, will be **[AIEEE 2005]**

(a) Blv (b) $-Blv$ (c) zero (d) $2\,Blv$

Exp. **(d)**

$$\text{Relative velocity} = v - (-v) = 2v = \frac{dl}{dt}$$

Now, $$e = \frac{d\phi}{dt} \Rightarrow e = \frac{Bl\,dl}{dt} \quad [\because \phi = BA]$$

Induced emf, $$e = 2Blv \quad \left[\because \frac{dl}{dt} = 2v\right]$$

48. Alternating current cannot be measured by DC ammeter because **[AIEEE 2004]**

(a) AC cannot pass through DC ammeter

(b) AC changes direction

(c) average value of current for complete cycle is zero

(d) DC ammeter will get damaged

Exp. **(c)**

The full cycle of alternating current consists of two half cycles. For one half, current is positive and for second half, current is negative.

Therefore, for an AC cycle, the net value of current average out to zero. While the DC ammeter, read the average value. Hence, the alternating current cannot be measured by DC ammeter.

49. In an L-C-R series AC circuit, the voltage across each of the components L, C and R is 50 V. The voltage across the L-C combination will be **[AIEEE 2004]**

(a) 50 V (b) $50\sqrt{2}$ V (c) 100 V (d) zero

Exp. **(d)**

In an L-C-R series AC circuit, the voltage across inductor L leads the current by 90° and the voltage across capacitor C lags behind the current by 90°. $[\because V = V_L \sim V_C = 50 - 50 = 0]$

Hence, the voltage across L-C combination will be zero.

50. A coil having n turns and resistance $R\,\Omega$ is connected with a galvanometer of resistance $4R\,\Omega$. This combination is moved for time t seconds from a magnetic field W_1 weber to W_2 weber. The induced current in the circuit is **[AIEEE 2004]**

(a) $\dfrac{W_2 - W_1}{5Rnt}$ (b) $-\dfrac{n(W_2 - W_1)}{5Rt}$

(c) $-\dfrac{(W_2 - W_1)}{Rnt}$ (d) $-\dfrac{n(W_2 - W_1)}{Rt}$

Exp. **(b)**

The rate of change of flux or emf induced in the coil is $e = -n\dfrac{d\phi}{dt}$.

$\therefore$ Induced current, $I = \dfrac{e}{R'} = -\dfrac{n}{R'}\dfrac{d\phi}{dt}$ ...(i)

Given, $R' = R + 4R = 5R, d\phi = W_2 - W_1, dt = t$

[here, W_1 and W_2 are flux associated with one turn]

Putting the given values in Eq. (i), we get

$$I = -\frac{n}{5R}\frac{(W_2 - W_1)}{t}$$

51. In a uniform magnetic field of induction B, a wire in the form of semi-circle of radius r rotates about the diameter of the circle with angular frequency ω. If the total resistance of the circuit is R, the mean power generated per period of rotation is **[AIEEE 2004]**

(a) $\dfrac{B\pi r^2\omega}{2R}$ (b) $\dfrac{(B\pi r^2\omega)^2}{8R}$

(c) $\dfrac{(B\pi r\omega)^2}{2R}$ (d) $\dfrac{(B\pi r\omega^2)^2}{8R}$

Exp. **(b)**

The flux associated with coil of area A and magnetic induction B is

$$\phi = BA\cos\theta = \frac{1}{2}B\pi r^2\cos\omega t \qquad \left[\because A = \frac{1}{2}\pi r^2\right]$$

$$\therefore \quad e_{\text{induced}} = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{1}{2}B\pi r^2\cos\omega t\right)$$

$$= \frac{1}{2}B\pi r^2\omega\sin\omega t$$

$$\therefore \text{ Power, } P = \frac{e^2_{\text{induced}}}{R} \qquad [\because P = V^2/R]$$

$$= \frac{B^2\pi^2 r^4\omega^2\sin^2\omega t}{4R}$$

Hence, $P_{\text{mean}} = \langle P \rangle$

$$= \frac{B^2\pi^2 r^4\omega^2}{4R}\cdot\frac{1}{2} \quad \left[\because \langle\sin^2\omega t\rangle = \frac{1}{2}\right]$$

$$= \frac{(B\pi r^2\omega)^2}{8R}$$

52. In an L-C-R circuit, capacitance is changed from C to $2C$. For the resonant frequency to remains unchanged, the inductance should be changed from L to **[AIEEE 2004]**

(a) $4L$ (b) $2L$ (c) $L/2$ (d) $L/4$

Exp. **(c)**

In the condition of resonance,

$$X_L = X_C \text{ or } \omega L = \frac{1}{\omega C} \qquad \text{...(i)}$$

Since, resonance frequency remains unchanged, so

$$\sqrt{LC} = \text{constant or } LC = \text{constant}$$

$$\therefore \quad L_1C_1 = L_2C_2$$

or $\quad L \times C = L_2 \times 2C$ or $L_2 = L/2$

53. A metal conductor of length 1 m rotates vertically about one of its ends at angular velocity 5 rad/s. If the horizontal component of earth's magnetic field is 0.2×10^{-4} T, then the emf developed between the two ends of the conductor is **[AIEEE 2004]**

(a) 5 μV (b) 50 μV (c) 5 mV (d) 50 mV

Exp. **(b)**

The emf induced between ends of conductor

$$e = \frac{1}{2} B\omega L^2 = \frac{1}{2} \times 0.2 \times 10^{-4} \times 5 \times (1)^2$$
$$= 0.5 \times 10^{-4} \text{ V}$$
$$= 5 \times 10^{-5} \text{ V} = 50\,\mu\text{V}$$

54. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon **[AIEEE 2003]**

(a) the rates at which currents are changing in the two coils
(b) relative position and orientation of the two coils
(c) the materials of the wires of the coils
(d) the currents in the two coils

Exp. **(b)**

Mutual inductance M between two coils is given two coils is given by $M = \mu_0 n_1 n_2 \pi r_1^2 L$

where, n_1, n_2 are number of turns, r_1 is the radius of, coil and L is the length. From the above formula, it is clear that mutual inductance depends on distance between the coils and geometry (πr^2 = area) of two coils.

55. When the current changes from +2 A to –2 A in 0.05 s, an emf of 8 V is induced in a coil. The coefficient of self-induction of the coil is **[AIEEE 2003]**

(a) 0.2 H (b) 0.4 H (c) 0.8 H (d) 0.1 H

Exp. **(d)**

$$e = -L\frac{dI}{dt} = -L\frac{(-2-2)}{0.05} \Rightarrow 8 = L\frac{(4)}{0.05}$$
$$\therefore \quad L = \frac{8 \times 0.05}{4} = 0.1 \text{ H}$$

56. In an oscillating L-C circuit, the maximum charge on the capacitor is Q. The charge on the capacitor when the energy is stored equally between the electric and magnetic fields is **[AIEEE 2003]**

(a) $\frac{Q}{2}$ (b) $\frac{Q}{\sqrt{3}}$ (c) $\frac{Q}{\sqrt{2}}$ (d) Q

Exp. **(c)**

In an L-C circuit, the energy oscillates between inductor (in the magnetic field) and capacitor (in the electric field).

$U_{E_{max}}$ [Maximum energy stored in capacitor]

$$= \frac{Q^2}{2C}$$

$U_{B_{max}}$ [Maximum energy stored in inductor]

$$= \frac{Li^2}{2}$$

where, I is the current at this time.

For the given instant, $U_E = U_B$

i.e., $$\frac{q^2}{2C} = \frac{LI^2}{2} \quad \ldots(i)$$

From energy conservation,

$$U_E + U_B = U_{E_{max}} = U_{B_{max}}$$
$$\frac{q^2}{2C} + \frac{1}{2}LI^2 = \frac{Q^2}{2C} \Rightarrow \frac{2q^2}{2C} = \frac{Q^2}{2C} \quad \text{[from Eq. (i)]}$$

or $$q = \frac{Q}{\sqrt{2}}$$

57. The core of any transformer is laminated so as to **[AIEEE 2003]**

(a) reduce the energy loss due to eddy currents
(b) make it light weight
(c) make it robust and strong
(d) increase the secondary voltage

Exp. **(a)**

The core of transformer is laminated to reduce energy loss due to eddy currents because induction is reduced by laminating.

58. The inductance between A and D is

(a) 3.66 H (b) 9 H **[AIEEE 2002]**
(c) 0.66 H (d) 1 H

Exp. **(d)**

Here, inductors are in parallel.

$\therefore \quad \frac{1}{L} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ or $L = 1$ H

59. If a current is passed through a spring, then the spring will **[AIEEE 2002]**

(a) expand (b) compress
(c) remain same (d) None of these

Exp. **(b)**

Due to flow of current in same direction in two adjacent sides, an attractive magnetic force will be produced due to which spring will get compressed.

60. The power factor of an AC circuit having resistance R and inductance L (connected in series) and an angular velocity ω is **[AIEEE 2002]**

(a) $\frac{R}{\omega L}$ (b) $\frac{R}{(R^2 + \omega^2 L^2)^{1/2}}$
(c) $\frac{\omega L}{R}$ (d) $\frac{R}{(R^2 - \omega^2 L^2)^{1/2}}$

Exp. **(b)**

From the relation, $\tan\phi = \frac{\omega L}{R}$

Power factor, $\cos\phi = \frac{1}{\sqrt{1 + \tan^2\phi}}$

$$\Rightarrow \cos\phi = \frac{R}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$

$$= \frac{1}{\sqrt{1 + \tan^2\phi}} = \frac{1}{\sqrt{1 + (\omega L/R)^2}} = \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

61. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4 A, then that in the secondary is **[AIEEE 2002]**

(a) 4 A (b) 2 A (c) 6 A (d) 10 A

Exp. **(b)**

Given, $I_p = 4$ A, $N_p = 140$ and $N_s = 280$

From the formula, $\frac{I_p}{I_s} = \frac{N_s}{N_p}$ or $\frac{4}{I_s} = \frac{280}{140}$

So, $I_s = 2$ A

62. A conducting square loop of side L and resistance R moves in its plane with a uniform velocity v perpendicular to one of its sides. A magnetic induction B constant in time and space, pointing perpendicular and into the plane at the loop exists everywhere with half the loop outside the field, as shown in figure. The induced emf is **[AIEEE 2002]**

(a) zero (b) RvB
(c) $\frac{vBL}{R}$ (d) vBL

Exp. **(d)**

As the side BC is outside the field, no emf is induced across BC. Since, AB and CD are not cutting any flux, the emf induced across these two sides will also be zero.

The side AD is cutting the flux and emf induced across this side is BvL with corner A at higher potential.

[according to Lorentz force, force on the charges will be towards A and on – ve charges will be towards D]

15

Electromagnetic Waves

1. A plane electromagnetic wave travels in free space along the x-direction. The electric field component of the wave at a particular point of space and time is $E = 6\text{Vm}^{-1}$ along y-direction. Its corresponding magnetic field component, B would be

(a) 2×10^{-8} T along z - direction
(b) 6×10^{-8} T along x - direction
(c) 6×10^{-8} T along z - direction
(d) 2×10^{-8} T along y - direction

[JEE Main 2019, 8 April Shift-I]

Exp. **(a)**

> **Key Idea** For an electromagnetic wave, ratio of magnitudes of electric and magnetic field is
> $$\frac{E}{B} = c$$
> where, c is the speed of electromagnetic wave in vacuum.

Given, $E = 6\text{ V/m}, c = 3\times10^8\text{ ms}^{-1}$

So, $$B = \frac{E}{c} = \frac{6}{3\times10^8} = 2\times10^{-8}\text{T}$$

Also, direction of propagation of electromagnetic wave is given by

$$\hat{\mathbf{n}} = \mathbf{E}\times\mathbf{B}$$

Here, $\hat{\mathbf{n}} = \hat{\mathbf{i}}$ and $\mathbf{E}$ = Unit vector of electric field $(\hat{\mathbf{j}})$

$\mathbf{B}$ = unit vector of magnetic field.

$\Rightarrow$ $\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\mathbf{B}$

$\therefore$ $\mathbf{B} = \hat{\mathbf{k}}$

Hence, magnetic field components

$$\mathbf{B} = 2\times10^{-8}\hat{\mathbf{k}}\,\text{T} = 2\times10^{-8}\,\text{T}$$

(along z-direction)

2. The magnetic field of an electromagnetic wave is given by

$\mathbf{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15}t)$
$(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{Wbm}^{-2}$

The associated electric field will be

[JEE Main 2019, 8 April Shift-II]

(a) $\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z - 6\times10^{15}t)$
$(-2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\text{Vm}^{-1}$

(b) $\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z - 6\times10^{15}t)$
$(2\hat{\mathbf{j}} + \hat{\mathbf{i}})\,\text{Vm}^{-1}$

(c) $\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)$
$(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\,\text{Vm}^{-1}$

(d) $\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)$
$(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\text{Vm}^{-1}$

Exp. **(c)**

(c) Given, $\mathbf{B} = 1.6\times10^{-6}\cos(2\times10^7 z + 6\times10^{15}t)$
$(2\hat{\mathbf{i}} + \hat{\mathbf{j}})\,\text{Wbm}^{-2}$

From the given equation, it can be said that the electromagnetic wave is propagating negative z-direction, i.e. $-\hat{\mathbf{k}}$.

Equation of associated electric field will be

$$\mathbf{E} = (|\mathbf{B}|c)\cos(kz + \omega t)\cdot\hat{\mathbf{n}}$$

where, $\hat{\mathbf{n}}$ = a vector perpendicular to $\mathbf{B}$.

So,

$$|\mathbf{E}| = |\mathbf{B}|\cdot c$$
$$= 1.6\times10^{-6}\times3\times10^8 = 4.8\times10^2\text{ V/m}$$

Since, we know that for an electromagnetic wave, $\mathbf{E}$ and $\mathbf{B}$ are mutually perpendicular to each other.

So, $\mathbf{E}\cdot\mathbf{B} = 0$

From the given options, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = 0$$

Also, when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\cdot\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\cdot(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = 0$$

But, we also know that the direction of propagation of electromagnetic wave is perpendicular to both **E** and **B**, i.e. it is in the direction of $\mathbf{E}\times\mathbf{B}$.

Again, when $\hat{\mathbf{n}} = \hat{\mathbf{i}} - 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\times(\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) = -\hat{\mathbf{k}}$$

and when $\hat{\mathbf{n}} = -\hat{\mathbf{i}} + 2\hat{\mathbf{j}}$

$$\mathbf{E}\times\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}})\times(-\hat{\mathbf{i}} + 2\hat{\mathbf{j}}) = \hat{\mathbf{k}}$$

But, it is been given in the question that the direction of propagation of wave is in $-\hat{\mathbf{k}}$.

Thus, associated electric field will be

$$\mathbf{E} = 4.8\times10^2\cos(2\times10^7 z + 6\times10^{15}t)(\hat{\mathbf{i}} - 2\hat{\mathbf{j}})\ \text{Vm}^{-1}$$

3. The magnetic field of a plane electromagnetic wave is given by

$$\mathbf{B} = B_0[\cos(kz - \omega t)]\hat{\mathbf{i}} + B_1\cos(kz + \omega t)\hat{\mathbf{j}}$$

where, $B_0 = 3\times10^{-5}$ T and $B_1 = 2\times10^{-6}$ T. The rms value of the force experienced by a stationary charge $Q = 10^{-4}$ C at $z = 0$ is closest to **[JEE Main 2019, 9 April Shift-I]**

(a) 0.1 N (b) 3×10^{-2} N

(c) 0.6 N (d) 0.9 N

Exp. **(c)**

Given, magnetic field of an electromagnetic wave is

$$\mathbf{B} = B_0[\cos(kz - \omega t]\,\hat{\mathbf{i}} + B_1[\cos(kz + \omega t]\,\hat{\mathbf{j}}$$

Here, $B_0 = 3\times10^{-5}$ T and $B_1 = 2\times10^{-6}$ T

Also, stationary charge, $Q = 10^{-4}$C at $z = 0$

As charge is released from the rest at $z = 0$, in this condition.

Maximum electric field, $E_0 = cB_0$ and $E_1 = cB_1$

So, $E_0 = c\times3\times10^{-5}$

and $E_1 = c\times2\times10^{-6}$

Now,the direction of electric field of an electromagnetic wave is perpendicular to **B** and to the direction of propagation of wave ($\mathbf{E}\times\mathbf{B}$) which is $\hat{\mathbf{k}}$.

So, for E_0,

$$\mathbf{E}_0\times\mathbf{B}_0 = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_0\times\hat{\mathbf{i}} = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_0 = -\hat{\mathbf{j}}$$

Similarly,for E_1 $\quad \mathbf{E}_1\times\mathbf{B}_1 = \mathbf{k}$

$$\Rightarrow \quad \mathbf{E}_1\times\hat{\mathbf{j}} = \hat{\mathbf{k}}$$

$$\Rightarrow \quad \mathbf{E}_1 = \hat{\mathbf{i}}$$

$$\therefore \mathbf{E}_0 = c\times3\times10^{-5}(-\hat{\mathbf{j}})\text{NC}^{-1}$$

$$\mathbf{E} = c\times2\times10^{-6}(+\hat{\mathbf{i}})\text{NC}^{-1}$$

$\therefore$ Maximum force experienced by stationary charge is

$$\mathbf{F}_{max} = Q\mathbf{E} = Q(\mathbf{E}_0 + \mathbf{E}_1)$$

$$= Q\times c\,[-3\times10^{-5}\hat{\mathbf{j}} + 2\times10^{-6}\hat{\mathbf{i}}]$$

$\Rightarrow$

$$|\mathbf{F}_{max}| = 10^{-4}\times3\times10^8\times\sqrt{(3\times10^{-5})^2 + (2\times10^{-6})^2}$$

$$= 3\times10^4\times10^{-6}\sqrt{900 + 4}$$

$$= 3\times10^{-2}\times\sqrt{904} \approx 0.9\text{ N}$$

$\therefore$ rms value of experienced force is

$$F_{rms} = \frac{F_{max}}{\sqrt{2}} = \frac{0.9}{\sqrt{2}} = 0.707\times0.9$$

$$= 0.6363\text{ N} \approx 0.6\text{ N}$$

4. The electric field of a plane electromagnetic wave is given by

$$\mathbf{E} = E_0\hat{\mathbf{i}}\cos(kz)\cos(\omega t)$$

The corresponding magnetic field **B** is then given by **[JEE Main 2019, 10 April Shift-I]**

(a) $\mathbf{B} = \frac{E_0}{C}\hat{\mathbf{j}}\sin(kz)\sin(\omega t)$

(b) $\mathbf{B} = \frac{E_0}{C}\hat{\mathbf{j}}\sin(kz)\cos(\omega t)$

(c) $\mathbf{B} = \frac{E_0}{C}\hat{\mathbf{k}}\sin(kz)\cos(\omega t)$

(d) $\mathbf{B} = \frac{E_0}{C}\hat{\mathbf{j}}\cos(kz)\sin(\omega t)$

Exp. **(a)**

> **Key Idea** For an electromagnetic wave, its electric field vector (**E**) and magnetic field vector (**B**) is mutually perpendicular to each other and also to its direction of propagation.

We know that, $\mathbf{E}\times\mathbf{B}$ represents direction of propagation of an electromagnetic wave

$$\Rightarrow \quad (\mathbf{E}\times\mathbf{B})\,||\,v$$

$\therefore$ From the given electric field, we can state that direction of propagation is along Z-axis and direction of **E** is along X-axis.

Thus, from the above discussion, direction of **B** must be Y-axis.

From Maxwell's equation,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Here, $$\frac{\partial \mathbf{E}}{\partial Z} = -\frac{\partial B}{\partial t} \quad \ldots \text{(i)}$$

and $$B_0 = E_0 / C \quad \ldots \text{(ii)}$$

Given, $\mathbf{E} = E_0\, \hat{\mathbf{i}} \cos kz \cos \omega t$

$$\Rightarrow \quad \frac{-\partial \mathbf{E}}{\partial Z} = kE_0 \sin kz \cos \omega t$$

$\therefore$ Using Eq. (i), we get

$$\frac{\partial \mathbf{B}}{\partial t} = kE_0 \sin kz \cos \omega t$$

Integrating both sides of the above equation w.r.t. t, we get

$$\Rightarrow \quad \mathbf{B} = \frac{k}{\omega} E_0 \sin kz \sin \omega t$$

$$= \frac{E_0}{C} \sin kz \sin \omega t$$

$$\Rightarrow \quad \mathbf{B} = \frac{E_0}{C} \sin(kz) \sin(\omega t)\, \hat{\mathbf{j}}$$

5. An electromagnetic wave is represented by the electric field $\mathbf{E} = E_0\, \hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$. Taking unit vectors in x, y and z- directions to be $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$, the direction of propagation $\hat{\mathbf{s}}$, is

[JEE Main 2019, 12 April Shift-I]

(a) $\hat{\mathbf{s}} = \dfrac{3\hat{\mathbf{i}} - 4\hat{\mathbf{j}}}{5}$ (b) $\hat{\mathbf{s}} = \dfrac{-4\hat{\mathbf{k}} + 3\hat{\mathbf{j}}}{5}$

(c) $\hat{\mathbf{s}} = \left(\dfrac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}\right)$ (d) $\hat{\mathbf{s}} = \dfrac{4\hat{\mathbf{j}} - 3\hat{\mathbf{k}}}{5}$

Exp. (c)

Key Idea Standard expression of electromagnetic wave is given by

$$\mathbf{E} = E_0 \hat{\mathbf{n}}\, [\sin(\omega t - \mathbf{k} \cdot \hat{\mathbf{r}})] \quad \ldots \text{(i)}$$

Here, $\mathbf{k}$ is the propagation vector.

Direction of propagation in this case is $\hat{\mathbf{k}}$.

Given expression of electromagnetic wave,

$$\mathbf{E} = E_0 \hat{\mathbf{n}} \sin[\omega t + (6y - 8z)]$$

$$\Rightarrow \quad \mathbf{E} = E_0 \hat{\mathbf{n}} \sin[\omega t - (8z - 6y)] \quad \ldots \text{(ii)}$$

Comparing Eq. (ii) with Eq. (i), we get

$$\mathbf{k} \cdot \hat{\mathbf{r}} = 8z - 6y \quad \ldots \text{(iii)}$$

Here, $$\hat{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$$

and $$\mathbf{k} = k_x\hat{\mathbf{i}} + k_y\hat{\mathbf{j}} + k_z\hat{\mathbf{k}}$$

$\therefore$ $$\mathbf{k} \cdot \hat{\mathbf{r}} = xk_x + yk_y + zk_z \quad \ldots \text{(iv)}$$

From Eqs. (iii) and (iv), we get

$$xk_x = \text{zero} \Rightarrow k_x = 0$$

$$yk_y = -6y \Rightarrow k_y = -6$$

$$zk_z = 8z \Rightarrow k_z = 8$$

Hence, $\mathbf{k} = -6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}$

So, direction of propagation,

$$\hat{\mathbf{s}} = \hat{\mathbf{k}} = \frac{\mathbf{k}}{|\mathbf{k}|} = \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}}$$

$$= \frac{-6\hat{\mathbf{j}} + 8\hat{\mathbf{k}}}{10} = \frac{-3\hat{\mathbf{j}} + 4\hat{\mathbf{k}}}{5}$$

6. A plane electromagnetic wave having a frequency $\nu = 23.9$ GHz propagates along the positive z-direction in free space. The peak value of the electric field is 60 V/m. Which among the following is the acceptable magnetic field component in the electromagnetic wave?

[JEE Main 2019, 12 April Shift-II]

(a) $\mathbf{B} = 2 \times 10^7 \sin(0.5 \times 10^3 z + 1.5 \times 10^{11} t)\, \hat{\mathbf{i}}$

(b) $\mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\, \hat{\mathbf{i}}$

(c) $\mathbf{B} = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)\, \hat{\mathbf{k}}$

(d) $\mathbf{B} = 2 \times 10^{-7} \sin(1.5 \times 10^2 x + 0.5 \times 10^{11} t)\, \hat{\mathbf{j}}$

Exp. (b)

In an electromagnetic wave, magnetic field and electric field are perpendicular to each other and both are also perpendicular to the direction of propagation of wave.

Now, given direction of propagation is along z-direction. So, magnetic field is in either x or y direction. Also, angular wave number for wave is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi\nu}{c}$$

$$= \frac{2\pi \times 23.9 \times 10^9}{3 \times 10^8}$$

$$\approx 0.5 \times 10^3 \text{ m}^{-1}$$

and angular frequency ω for wave is

$$\omega = 2\pi\nu = 2\pi \times 23.9 \times 10^9 \text{ Hz}$$

$$= 1.5 \times 10^{11} \text{ Hz}$$

Magnitude of magnetic field is

$$B_0 = \frac{E_0}{c} = \frac{60}{3 \times 10^8} = 2 \times 10^{-7} \text{T}$$

As the general equation of magnetic field of an electromagnetic wave propagating in + z- direction is given as,

$$\mathbf{B} = B_0 \sin(kz - \omega t)\hat{\mathbf{i}} \text{ or } \hat{\mathbf{j}}$$

Thus, substituting the values of B_0, k and ω, we get

$\Rightarrow \mathbf{B} = 2 \times 10^{-7} \sin(0.5 \times 10^3 z - 1.5 \times 10^{11} t)\hat{\mathbf{i}}$ or $\hat{\mathbf{j}}$

7. A plane electromagnetic wave of frequency 50 MHz travels in free space along the positive x - direction. At a particular point in space and time, $\mathbf{E} = 6.3\,\hat{\mathbf{j}}$ V/m. The corresponding magnetic field $\mathbf{B}$, at that point will be **[JEE Main 2019, 9 Jan Shift-I]**

(a) $18.9 \times 10^8\,\hat{\mathbf{k}}$ T (b) $6.3 \times 10^{-8}\,\hat{\mathbf{k}}$ T
(c) $18.9 \times 10^{-8}\,\hat{\mathbf{k}}$ T (d) $2.1 \times 10^{-8}\,\hat{\mathbf{k}}$ T

Exp. **(d)**

Since, the magnitude of electric field vector and magnetic field vector of electromagnetic wave is related as,

$$\frac{|\mathbf{E}|}{|\mathbf{B}|} = c$$

where, c is the speed of light in vacuum.

Here, $|\mathbf{E}| = 6.3$ and $c = 3 \times 10^8$ m/s.

$$\Rightarrow \quad |\mathbf{B}| = \frac{|\mathbf{E}|}{c} = \frac{6.3}{3 \times 10^8}$$

$$= 2.1 \times 10^{-8}$$

Now, as we know for a EM wave, **E** and **B** are perpendicular to each other as well as to direction of propagation of wave ($\mathbf{E} \times \mathbf{B}$).

As here, direction of **E** is + y-direction i.e. $\hat{\mathbf{j}}$ and direction of propagation of wave is in + x-direction i.e., $\hat{\mathbf{i}}$

$\therefore$ If **B** has to be perpendicular to **E** and direction of propagation, then it must in + z-direction i.e., $\hat{\mathbf{k}}$.

Thus, magnetic field, $\mathbf{B} = 2.1 \times 10^{-8}\,\hat{\mathbf{k}}$ T.

8. In free space, the energy of electromagnetic wave in electric field is U_E and in magnetic field is U_B. Then **[JEE Main 2019, 9 Jan Shift-II]**

(a) $U_E = U_B$ (b) $U_E > U_B$
(c) $U_E < U_B$ (d) $U_E = \frac{U_B}{2}$

Exp. **(a)**

Energy density of an electomagnetic wave in electric field,

$$U_E = \frac{1}{2}\varepsilon_0 \cdot E^2 \quad \ldots\text{(i)}$$

Energy density of an electromagnetic wave in magnetic field,

$$U_B = \frac{B^2}{2\mu_0} \quad \ldots\text{(ii)}$$

where, E = electric field,
B = magnetic field,
ε_0 = permittivity of medium and
μ_0 = magnetic permeability of medium.

From the theory of electro-magnetic waves, the relation between μ_0 and ε_0 is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \quad \ldots\text{(iii)}$$

where, c = velocity of light $= 3 \times 10^8$ m/s

and $$\frac{E}{B} = c \quad \ldots\text{(iv)}$$

Dividing Eq. (i) by Eq. (ii), we get

$$\frac{U_E}{U_B} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{1}{2}B^2 \times \frac{1}{\mu_0}} = \frac{\mu_0 \varepsilon_0 E^2}{B^2} \quad \ldots\text{(v)}$$

Using Eqs. (iii), (iv) and (v), we get

$$\frac{U_E}{U_B} = \frac{c^2}{c^2} = 1$$

Therefore, $U_E = U_B$

9. If the magnetic field of a plane electromagnetic wave is given by

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right],$$

then the maximum electric field associated with it is (Take, the speed of light $= 3 \times 10^8$ m/s) **[JEE Main 2019, 10 Jan Shift-I]**

(a) 6×10^4 N/C
(b) 4×10^4 N/C
(c) 3×10^4 N/C
(d) 4.5×10^4 N/C

Exp. **(c)**

Given, instantaneous value of magnetic field

$$B = 100 \times 10^{-6} \sin\left[2\pi \times 2 \times 10^{15}\left(t - \frac{x}{c}\right)\right]$$

and speed of light, $c = 3 \times 10^8$ ms^{-1}

For an electromagnetic wave,

$$E_{max} = B_{max} \times c$$

where, E_{max} = maximum value of the electric field.

We get,

$$E_{max} = 100 \times 10^{-6} \times 3 \times 10^8$$

$$= 3 \times 10^4 \ \frac{\text{N}}{\text{C}}$$

10. The electric field of a plane polarised electromagnetic wave in free space at time $t = 0$ is given by an expression.

$\mathbf{E}(x, y) = 10\hat{\mathbf{j}}\cos[(6x + 8z)]$

The magnetic field $\mathbf{B}(x, z, t)$ is given by (where, c is the velocity of light)

[JEE Main 2019, 10 Jan Shift-II]

(a) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z + 10ct)]$

(b) $\frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

(c) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x - 8z + 10ct)]$

(d) $\frac{1}{c}(6\hat{\mathbf{k}} + 8\hat{\mathbf{i}})\cos[(6x + 8z - 10ct)]$

Exp. **(b)**

We are given electric field as

$$\mathbf{E} = 10\,\hat{\mathbf{j}}\cos(6x + 8z) \quad \ldots \text{(i)}$$

where, phase angle is independent of time, i.e., phase angle at $t = 0$ is $\phi = 6x + 8z$.

Phase angle for $\mathbf{B}$ will also be $6x + 8z$ because for an electromagnetic wave $\mathbf{E}$ and $\mathbf{B}$ oscillate in same phase.

Thus, direction of wave propagation

$$= \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{\sqrt{6^2 + 8^2}} = \frac{6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}}{10} \quad \ldots \text{(ii)}$$

Let magnetic field vector,

$\mathbf{B} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}}$, then direction of wave propagation is given by

$$\frac{\mathbf{E} \times \mathbf{B}}{|\mathbf{E}'||\mathbf{B}|} = \frac{10\hat{\mathbf{j}} \times (a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + d\hat{\mathbf{k}})}{10 \times (a^2 + b^2 + d^2)^{1/2}}$$

$$= \frac{-10a\hat{\mathbf{k}} + 10d\hat{\mathbf{i}}}{10(a^2 + b^2 + d^2)^{1/2}} \quad \ldots \text{(iii)}$$

As, $\quad |\mathbf{B}| = \frac{|\mathbf{E}|}{c} = \frac{10}{c}$

We get, $|\mathbf{B}| = \sqrt{a^2 + b^2 + d^2} = 10/c$

By putting this value in Eqs. (iii) and (ii), we get direction of propagation

$$\Rightarrow \quad \frac{c10(d\,\hat{\mathbf{i}} - a\hat{\mathbf{k}})}{10 \times 10} = \frac{6\hat{\mathbf{i}} \times 8\hat{\mathbf{k}}}{10}$$

$$\Rightarrow \quad d = 6/c \text{ and } a = -8/c$$

Hence, $\quad \mathbf{B} = \frac{6}{c}\hat{\mathbf{k}} - \frac{8}{c}\hat{\mathbf{i}} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})$

As the general equation of magnetic field of an EM wave propagating in positive y-direction is given as,

$$B = B_0 \cos(Ry - \omega t))$$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos(6x + 8z - 10ct)$$

Alternate Solution Given, electric field is $E(x, y)$, i.e. electric field is in xy-plane which is given as

$\because \mathbf{E} = 10\hat{\mathbf{j}}\cos(6x + 8z)$

Since, the magnetic field given is $\mathbf{B}(x, z, t)$, this means $\mathbf{B}$ is in xz-plane.

$\therefore$ Propagation of wave is in y-direction.

[$\because$ for an electromagnetic wave, $\mathbf{E} \perp \mathbf{B} \perp$ propagation direction]

As Poynting vector suggests that $\mathbf{E} \times \mathbf{B}$ is parallel to $(6\hat{\mathbf{i}} + 8\hat{\mathbf{k}})$.

Let $\quad \mathbf{B} = (x\hat{\mathbf{i}} + z\hat{\mathbf{k}})$,

then $\quad \mathbf{E} \times \mathbf{B} = \hat{\mathbf{j}} \times (x\hat{\mathbf{i}} + z\hat{\mathbf{k}}) = 6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}$

or $\quad -x\hat{\mathbf{k}} + z\hat{\mathbf{i}} = 6\hat{\mathbf{i}} + 8\hat{\mathbf{k}}$

or $\quad x = -8$ and $z = 6$

$$\therefore \quad \mathbf{B} = \frac{1}{c}(6\hat{\mathbf{k}} - 8\hat{\mathbf{i}})\cos(6x + 8z - 10ct)$$

$$\left[\because \frac{|\mathbf{E}|}{|\mathbf{B}|} = c\right]$$

11. An electromagnetic wave of intensity 50 Wm^{-2} enters in a medium of refractive index 'n' without any loss. The ratio of the magnitudes of electric fields and the ratio of the magnitudes of magnetic fields of the wave before and after entering into the medium are respectively, given by

[JEE Main 2019, 11 Jan Shift-I]

(a) $\left(\frac{1}{\sqrt{n}}, \sqrt{n}\right)$ (b) $(\sqrt{n}, \sqrt{n})$

(c) $\left(\frac{1}{\sqrt{n}}, \frac{1}{\sqrt{n}}\right)$ (d) $\left(\sqrt{n}, \frac{1}{\sqrt{n}}\right)$

Exp. **(d)**

In the free space, the speed of electromagnetic wave is given as,

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{E_0}{B_0} \quad \ldots \text{(i)}$$

where, E_0 and B_0 are the amplitudes of varying electric and magnetic fields, respectively.

Now, when it enters in a medium of refractive index 'n', its speed is given as,

$$v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{K\varepsilon_0\mu}} = \frac{E}{B} \quad \ldots(ii)$$

where, K is dielectric strength of the medium.

Using Eqs. (i) and (ii), we get

$$\frac{v}{c} = \frac{1}{\sqrt{K}} \quad \ldots(iii)$$

($\because$ For a transparent medium, $\mu_0 \approx \mu$)

Also, refractive index of medium is 'n' and is given as

$$\frac{c}{v} = n \quad \text{or} \quad \frac{v}{c} = \frac{1}{n} \quad \ldots(iv)$$

$\therefore$ From Eqs. (iii) and (iv), we get

$$n = \sqrt{K} \quad \text{or} \quad K = n^2 \quad \ldots(v)$$

The intensity of a EM wave is given as,

$$I = \frac{1}{2}\varepsilon_0 E_0^2 c$$

and in the medium, it is given as $I' = \frac{1}{2}K\varepsilon_0 E^2 v$

It is given that, $I = I'$

$$\Rightarrow \quad \frac{1}{2}\varepsilon_0 E_0^2 c = \frac{1}{2}K\varepsilon_0 E^2 v$$

$$\text{or} \quad \left(\frac{E_0}{E}\right)^2 = \frac{Kv}{c} \quad \ldots(vi)$$

From Eqs. (iv), (v) and (vi), we get

$$\left(\frac{E_0}{E}\right)^2 = \frac{n^2}{n} = n \quad \text{or} \quad E_0 / E = \sqrt{n}$$

Similarly, $\frac{1}{2}\cdot\frac{B_0^2}{\mu_0}c = \frac{1}{2}\cdot\frac{B^2}{\mu_0}v$

$$\Rightarrow \quad \frac{B_0}{B} = \frac{1}{\sqrt{n}}$$

Alternate Solution

We know that,

$$\left(\frac{E_0}{B_0}\right)_{\text{air / vacuum}} = c \quad \text{and} \quad \left(\frac{E}{B}\right)_{\text{medium}} = v$$

Also, $n = \frac{c}{v}$

$$\frac{E_0 / B_0}{E / B} = \frac{c}{v} = n$$

$$\Rightarrow \quad \frac{E_0 / E}{B_0 / B} = n$$

This is possible only if $\frac{E_0}{E} = \sqrt{n}$ and $\frac{B_0}{B} = \frac{1}{\sqrt{n}}$.

12. A 27 mW laser beam has a cross-sectional area of 10 mm^2. The magnitude of the maximum electric field in this electromagnetic wave is given by [Take, permittivity of space, $\varepsilon_0 = 9\times10^{-12}$ SI units and speed of light, $c = 3\times10^8$ m/s]

[JEE Main 2019, 11 Jan Shift-II]

(a) 1 kV/m (b) 0.7 kV/m
(c) 2 kV/m (d) 1.4 kV/m

Exp. **(*d*)**

Given,

Power of laser beam

$$(P) = 27\text{mW} = 27\times10^{-3}\text{ W}$$

Area of cross-section (A)

$$= 10\text{ m m}^2 = 10\times10^{-6}\text{m}^2$$

Permittivity of free space (ε_0) = 9×10^{-12} SI unit

Speed of light (c) = 3×10^8 m/s

Intensity of electromagnetic wave is given by the relation

$$I = \frac{1}{2}nc\varepsilon_0 E^2$$

where, n is refractive index, for air $n = 1$.

$$\therefore \quad I = \frac{1}{2}c\cdot\varepsilon_0 E^2 \quad \ldots(i)$$

$$\text{Also,} \quad I = \frac{P}{A} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{1}{2}c\,\varepsilon_0 E^2 = \frac{P}{A}$$

$$\text{or} \quad E^2 = \frac{2P}{Ac\,\varepsilon_0}$$

$$\text{or} \quad E = \sqrt{\frac{2\times27\times10^{-3}}{10\times10^{-6}\times3\times10^8\times9\times10^{-12}}}$$

$$\simeq 1.4\times10^3\text{ V/m}$$

$$= 1.4\text{ k V/m}$$

13. A light wave is incident normally on a glass slab of refractive index 1.5. If 4% of light gets reflected and the amplitude of the electric field of the incident light is 30 V/m, then the amplitude of the electric field for the wave propogating in the glass medium will be

[JEE Main 2019, 12 Jan Shift-I]

(a) 30 V/m (b) 6 V/m
(c) 10 V/m (d) 24 V/m

Exp. **(d)**

Energy of a light wave $\propto$ Intensity of the light wave

Since, intensity $= \varepsilon v A^2$

where, ε is the permittivity of the medium in which light is travelling with velocity v and A is its amplitude.

Since, only 4% of the energy of the light gets reflected.

$\therefore$ 96% of the energy of the light is transmitted.

$E_{\text{transmitted}}(E_t) = 96\%$ of $E_{\text{incident}}(E_i)$

$$\varepsilon_0 \varepsilon_r v A_t^2 = \frac{96}{100} \times \varepsilon_0 \times c \times A_1^0$$

$$A_t^2 = \frac{96}{100} \cdot \frac{\varepsilon_0}{\varepsilon_r} \cdot \frac{c}{v} A_i^2$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v^2}{c^2} \cdot \frac{c}{v} A_i^2 \qquad \left[\because \sqrt{\frac{\varepsilon_0}{\varepsilon_r}} = \frac{v}{c}\right]$$

$$A_t^2 = \frac{96}{100} \cdot \frac{v}{c} \cdot A_i^2$$

$$= \frac{96}{100} \times \frac{1}{3/2} \times 30^2 \qquad \left[\because \mu = \frac{c}{v} = 1.5\right]$$

$$A_t = \sqrt{\frac{64}{100} \times 30^2}$$

$$A_t = 24 \text{ V/m}$$

14. The mean intensity of radiation on the surface of the sun is about 10^8 W / m^2. The rms value of the corresponding magnetic field is closest to **[JEE Main 2019, 12 Jan Shift-II]**

(a) 1 T (b) 10^2 T (c) 10^{-4} T (d) 10^{-2} T

Exp. **(c)**

Mean radiation intensity is

$$I = \varepsilon_0 c E_{\text{rms}}^2$$

$$= \varepsilon_0 c (c B_{\text{rms}})^2 \qquad \left[\because \frac{E_{\text{rms}}}{B_{\text{rms}}} = c\right]$$

$$= \varepsilon_0 c^3 B_{\text{rms}}^2$$

$$\Rightarrow \qquad B_{\text{rms}} = \sqrt{\frac{I}{\varepsilon_0 c^3}}$$

Substituting the given values, we get

$$= \sqrt{\left(\frac{10^8}{8.85 \times 10^{-12} \times (3 \times 10^8)^3}\right)}$$

$$= \sqrt{\left(\frac{10^8}{8.85 \times 27 \times 10^{12}}\right)}$$

$$\approx \sqrt{(10^{-8})} \approx 10^{-4} \text{ T}$$

15. An EM wave from air enters a medium. The electric fields are $\mathbf{E}_1 = E_{01}\hat{\mathbf{x}} \cos\left[2\pi\nu\left(\frac{z}{c} - t\right)\right]$ in air and $\mathbf{E}_2 = E_{02}\hat{\mathbf{x}} \cos[k(2z - ct)]$ in medium, where the wave number k and frequency ν refer to their values in air. The medium is non-magnetic.

If ε_{r_1} and ε_{r_2} refer to relative permittivities of air and medium respectively, which of the following options is correct? **[JEE Main 2018]**

(a) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 4$ (b) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = 2$

(c) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$ (d) $\frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{2}$

Exp. **(c)**

Speed of progressive wave is given by, $v = \frac{\omega}{k}$

As electric field in air is,

$$E_1 = E_{01}\mathbf{x} \cos\left(\frac{2\pi\nu z}{c} - 2\pi\nu t\right)$$

$\therefore$ Speed in air $= \dfrac{2\pi\nu}{\left(\dfrac{2\pi\nu}{c}\right)} = c$

Also, $$c = \frac{1}{\sqrt{\mu_0 \varepsilon_{r_1} \varepsilon_0}} \qquad \ldots(\text{i})$$

In medium,

$$E_2 = E_{02} x \cos(2kz - kct)$$

$\therefore$ Speed in medium $= \dfrac{kc}{2k} = \dfrac{c}{2}$

Also, $$\frac{c}{2} = \frac{1}{\sqrt{\mu_0 \varepsilon_{r_2} \varepsilon_0}} \qquad \ldots(\text{ii})$$

As medium is non-magnetic medium,

$$\mu_{\text{medium}} = \mu_{\text{air}}$$

On dividing Eq. (i) by Eq. (ii), we have

$$2 = \sqrt{\frac{\varepsilon_{r_2}}{\varepsilon_{r_1}}}$$

$$\Rightarrow \qquad \frac{\varepsilon_{r_1}}{\varepsilon_{r_2}} = \frac{1}{4}$$

16. Arrange the following electromagnetic radiations per quantum in the order of increasing energy.

A. Blue light B. Yellow light
C. X-ray D. Radio wave

[JEE Main 2016 (Offline)]

(a) D, B, A, C (b) A, B, D, C
(c) C, A, B, D (d) B, A, D, C

Exp. **(a)**

As, we know energy liberated, $E = \frac{hc}{\lambda}$

i.e. $E \propto \frac{1}{\lambda}$

So, lesser the wavelength, than greater will be energy liberated by electromagnetic radiations per quantum.

As, order of wavelength is given by

X-ray, VIBGYOR, **Radio waves**
(C) (A) (B) (D)

$\therefore$ Order of electromagnetic radiations per quantum.

$\Rightarrow$ D < B < A < C

17. Match List I (Electromagnetic wave type) with List II (Its association/application) and select the correct option from the choices given below the lists. **[JEE Main 2014]**

	List I		List II
A.	Infrared waves	1.	To treat muscular strain
B.	Radio waves	2.	For broadcasting
C.	X-rays	3.	To detect fracture of bones
D.	Ultraviolet	5.	Absorbed by the ozone layer of the atmosphere

Codes

	A	B	C	D		A	B	C	D
(a)	4	3	2	1	(b)	1	2	4	3
(c)	3	2	1	4	(d)	1	2	3	4

Exp. **(d)**

(a) Infrared rays are used to treat muscular strain.
(b) Radiowaves are used for broadcasting purposes.
(c) X-rays are used to detect fracture of bones.
(d) Ultraviolet rays are absorbed by ozone.

18. During the propagation of electromagnetic waves in a medium, **[JEE Main 2014]**

(a) electric energy density is double of the magnetic energy density
(b) electric energy density is half of the magnetic energy density
(c) electric energy density is equal to the magnetic energy density
(d) Both electric and magnetic energy densities are zero

Exp. **(c)**

Both the energy densities are equal i.e., is equally divided between electric and magnetic field.

19. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is **[JEE Main 2013]**

(a) 3 V/m (b) 6 V/m
(c) 9 V/m (d) 12 V/m

Exp. **(b)**

In electromagnetic wave,

$E = B \times c = Bc\sin\theta$

$\theta = 90°, \sin\theta = 1$

$\therefore \quad E = Bc$

Given, $B = 20\,\text{nT} = 20 \times 10^{-9}\,\text{T}$

$c = 3 \times 10^8$ m/s

$\therefore \quad E = 20 \times 10^{-9} \times 3 \times 10^8$

$\Rightarrow \quad E = 6$ V/m

20. An electromagnetic wave in vacuum has the electric and magnetic fields **E** and **B**, which are always perpendicular to each other. The direction of polarisation is given by **X** and that of wave propagation by **k**. Then, **[AIEEE 2012]**

(a) $\mathbf{X} \parallel \mathbf{B}$ and $\mathbf{k} \parallel \mathbf{B} \times \mathbf{E}$
(b) $\mathbf{X} \parallel \mathbf{E}$ and $\mathbf{k} \parallel \mathbf{E} \times \mathbf{B}$
(c) $\mathbf{X} \parallel \mathbf{B}$ and $\mathbf{k} \parallel \mathbf{E} \times \mathbf{B}$
(d) $\mathbf{X} \parallel \mathbf{E}$ and $\mathbf{k} \parallel \mathbf{B} \times \mathbf{E}$

Exp. **(b)**

In electromagnetic wave, the direction of propagation of wave, electric field and magnetic field are mutually perpendicular, *i.e.,* wave propagates perpendicular to **E** and **B** or along **E** × **B**. While polarisation of wave takes place parallel to electric field vector.

21. The rms value of the electric field of the light coming from the sun is 720 NC^{-1}. The average total energy density of the electromagnetic wave is **[AIEEE 2006]**

(a) 4.58×10^{-6} Jm^{-3} (b) 6.37×10^{-9} Jm^{-3}

(c) 81.35×10^{-12} Jm^{-3} (d) 3.3×10^{-3} Jm^{-3}

Exp. **(a)**

Total average energy $= \varepsilon_0 E_{rms}^2$

$$= 8.85 \times 10^{-12} \times (720)^2$$

$$= 4.58 \times 10^{-6}\ Jm^{-3}$$

22. An electromagnetic wave of frequency $\nu = 3.0$ MHz passes from vacuum into a dielectric medium with permittivity $\varepsilon = 4.0$. Then, **[AIEEE 2004]**

(a) wavelength is doubled and the frequency remains unchanged

(b) wavelength is doubled and frequency becomes half

(c) wavelength is halved and frequency remains unchanged

(d) wavelength and frequency both remain unchanged

Exp. **(c)**

In vacuum, $\varepsilon_0 = 1$

In medium, $\varepsilon = 4$

$\therefore$ Refractive index, $\mu = \sqrt{\dfrac{\varepsilon}{\varepsilon_0}} = \sqrt{\dfrac{4}{1}} = 2$

Wavelength, $\lambda' = \dfrac{\lambda}{\mu} = \dfrac{\lambda}{2}$

and wave velocity, $v = \dfrac{c}{\mu} = \dfrac{c}{2}$ $\left[\because \mu = \dfrac{c}{v}\right]$

Hence, it is clear that wavelength and velocity will become half but frequency remains unchanged when the wave is passing through any medium.

23. A radiation of energy E falls normally on a perfectly reflecting surface. The momentum transferred to the surface is **[AIEEE 2004]**

(a) $\dfrac{E}{c}$ (b) $\dfrac{2E}{c}$ (c) Ec (d) $\dfrac{E}{c^2}$

Exp. **(b)**

Initial momentum of surface, $p_i = E/c$

where, c = velocity of light (constant).

Since, the surface is perfectly reflecting, so the same momentum will be reflected completely. Final momentum,

$$p_f = E/c \qquad \text{[negative value]}$$

$\therefore$ Change in momentum,

$$\Delta p = p_f - p_i = -\frac{E}{c} - \frac{E}{c} = -\frac{2E}{c}$$

Thus, momentum transferred to the surface is

$$\Delta p' = |\Delta p| = \frac{2E}{c}$$

24. Which of the following radiations has the least wavelength? **[AIEEE 2003]**

(a) γ-rays (b) β-rays (c) α-rays (d) X-rays

Exp. **(a)**

The spectrum of electromagnetic waves stretches over an infinite range of wavelengths. from 10^{-12}m to 10^6 m, least wavelength being of gamma rays from 10^{-10} m to 10^{-14} m (γ has highest frequency).

25. Which of the following are not electromagnetic waves? **[AIEEE 2002]**

(a) cosmic rays (b) γ-rays

(c) β-rays (d) X-rays

Exp. **(c)**

Cosmic rays, γ-rays and X-rays are part of electromagnetic spectrum, while β-rays are emitted by radioactive elements. Hence, β-rays is not electromagnetic waves.

26. Electromagnetic waves are transverse in nature is evident by **[AIEEE 2002]**

(a) polarisation (b) interference

(c) reflection (d) diffraction

Exp. **(a)**

Polarisation, using polariser and analyser, we get change in intensity of light at different rotations, hence we conclude that light waves are transverse in nature.

16

Optics

1. In an interference experiment, the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be

(a) 2 (b) 18 (c) 4 (d) 9

[JEE Main 2019, 8 April Shift-I]

Exp. **(c)**

In Young's double slit experiment, ratio of maxima and minima intensity is given by

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}}\right)^2 = \left(\frac{\sqrt{I_1/I_2} + 1}{\sqrt{I_1/I_2} - 1}\right)^2$$

As, intensity $(I) \propto [\text{amplitude } (a)]^2$

$$\therefore \quad \frac{I_1}{I_2} = \left(\frac{a_1}{a_2}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$$

So, $$\frac{I_{max}}{I_{min}} = \left(\frac{\frac{1}{3} + 1}{\frac{1}{3} - 1}\right)^2 = 4 : 1$$

2. An upright object is placed at a distance of 40 cm in front of a convergent lens of focal length 20 cm. A convergent mirror of focal length 10 cm is placed at a distance of 60 cm on the other side of the lens. The position and size of the final image will be

[JEE Main 2019, 8 April Shift-I]

(a) 20 cm from the convergent mirror, same size as the object

(b) 40 cm from the convergent mirror, same size as the object

(c) 40 cm from the convergent lens, twice the size of the object

(d) 20 cm from the convergent mirror,twice size of the object

Exp. **(*)**

In given system of lens and mirror, position of object O in front of lens is at a distance $2f$.

i.e. $u = 2f = 40$ cm

So, image (I_1) formed is real, inverted and at a distance, $v = 2f = 2 \times 20 = 40$ cm, (behind lens)

magnification, $m_1 = \frac{v}{u} = \frac{40}{40} = 1$

Thus, size of image is same as that of object.

This image (I_1) acts like a real object for mirror.

As object distance for mirror is

$$u = C = 2f = -20\text{ cm}$$

where, C = centre of curvature.

So, image (I_2) formed by mirror is at $2f$.

$\therefore$ For mirror $v = 2f = 2(-10)$

$$= -20\text{ cm}$$

Magnification, $m_2 = -\frac{v}{u} = -\frac{(-20)}{(-20)} = -1$

Thus, image size is same as that of object.

The image I_2 formed by the mirror will act like an object for lens.

As the object is at $2f$ distance from lens, so image (I_3) will be formed at a distance $2f$ or 40 cm. Thus, magnification,

$$m_3 = \frac{v}{u} = \frac{40}{40} = 1$$

So, final magnification,

$$m = m_1 \times m_2 \times m_3 = -1$$

Hence, final image (I_3) is real, inverted of same size as that of object and coinciding with object.

3. In figure, the optical fibre is $l = 2$ m long and has a diameter of $d = 20\,\mu$m. If a ray of light is incident on one end of the fibre at angle $\theta_1 = 40°$, the number of reflections it makes before emerging from the other end is close to (refractive index of fibre is 1.31 and $\sin 40° = 0.64$) **[JEE Main 2019, 8 April Shift-I]**

(a) 55000 (b) 66000 (c) 45000 (d) 57000

Exp. **(d)**

Total internal reflection occurs through given glass rod as shown in figure.

From Snell's law, $n_1 \sin i = n_2 \sin r$

where, $n_1 = 1$, $n_2 = 1.31$ and $i = 40°$

So, we get

$$1\sin 40° = 1.31\sin r \Rightarrow \sin r = \frac{0.64}{1.31} = 0.49 \approx 0.5$$

So, $r = 30°$

From ΔOAB, $\theta = 90 - r = 60°$

Now, $\tan\theta = \frac{x}{20\,\mu\text{m}}$

$\Rightarrow$ $x = 20\sqrt{3}\,\mu$m $[\because \tan 60° = \sqrt{3}]$

One reflection occurs in $20\sqrt{3}\,\mu$m.

$\therefore$ Total number of reflections occurring in 2m

$$= n = \frac{2\text{m}}{20\sqrt{3}\,\mu\text{m}} = \frac{2}{20\sqrt{3} \times 10^{-6}}$$

$$= 57735 \text{ reflections} \approx 57000 \text{ reflections}$$

4. Calculate the limit of resolution of a telescope objective having a diameter of 200 cm, if it has to detect light of wavelength 500 nm coming from a star. **[JEE Main 2019, 8 April Shift-II]**

(a) 610×10^{-9} rad (b) 305×10^{-9} rad

(c) 457.5×10^{-9} rad (d) 152.5×10^{-9} rad

Exp. **(b)**

For a telescope, limit of resolution is given by $\Delta\theta = \frac{1.22\lambda}{D}$

Here, $\lambda = 500\,\text{nm} = 500 \times 10^{-9}$ m,

$D = 200\,\text{cm} = 200 \times 10^{-2}$ m

So, limit of resolution is $\Delta\theta = \dfrac{1.22 \times 500 \times 10^{-9}}{200 \times 10^{-2}}$

$= 305 \times 10^{-9}$ rad

5. A convex lens (of focal length 20 cm) and a concave mirror, having their principal axes along the same lines, are kept 80 cm apart from each other. The concave mirror is to the right of the convex lens. When an object is kept at a distance of 30 cm to the left of the convex lens, its image remains at the same position even if the concave mirror is removed. The maximum distance of the object for which this concave mirror, by itself would produce a virtual image would be **[JEE Main 2019, 8 April Shift-II]**

(a) 25 cm (b) 20 cm
(c) 10 cm (d) 30 cm

Exp. **(c)**

The given situation can be drawn as shown below

For lens formula, $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

Substituting given values, we get

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{20} \Rightarrow v = 60\,\text{cm}$$

So, this image is at a distance of

$80 - 60 = 20$ cm from the mirror.

As, the image formed by the mirror coincides with image formed by the lens.

This condition is only possible, if any object that has been placed in front of concave mirror is at centre of curvature, i.e. at $2f$.

So, radius of curvature of mirror is $R = 20$ cm

$\therefore$ Focal length of mirror, $f = \dfrac{R}{2} = 10$ cm

As, for virtual image, the object is to kept between pole and focus of the mirror.

$\therefore$ The maximum distance of the object for which this concave mirror by itself produce a virtual image would be 10 cm.

6. A concave mirror for face viewing has focal length of 0.4 m. The distance at which you hold the mirror from your face in order to see your image upright with a magnification of 5 is **[JEE Main 2019, 9 April Shift-I]**

(a) 0.16 m (b) 1.60 m (c) 0.32 m (d) 0.24 m

Exp. **(c)**

Given, focal length of concave mirror,

$$f = -0.4\,\text{m}$$

Magnification = 5

We know that, magnification produced by a mirror, $m = -\dfrac{\text{image distance}}{\text{object distance}}$

$$\Rightarrow \quad \frac{v}{u} = -5 \text{ or } v = -5u$$

Using mirror formula, $\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

Substituting the given values in the above equation, we get

$$\frac{1}{-5u} + \frac{1}{u} = -\frac{1}{0.4} \Rightarrow \frac{4}{5u} = -\frac{1}{0.4}$$

$$\Rightarrow \quad u = -\frac{1.6}{5} = -0.32\,\text{m}$$

Alternate Solution

Magnification produced by a mirror can also be given as

$$m = \frac{f}{f-u}$$

Substituting the given values, we get

$$5 = \frac{-0.4}{-0.4-u}$$

or $\quad u = -0.32$ m

7. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gets shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be **[JEE Main 2019, 9 April Shift-I]**

(a) $\frac{2nD\lambda}{a(\mu-1)}$ (b) $\frac{2D\lambda}{a(\mu-1)}$

(c) $\frac{D\lambda}{a(\mu-1)}$ (d) $\frac{nD\lambda}{a(\mu-1)}$

Exp. **(*)**

Path difference introduced by a slab of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

Position of the fringe is $x = \frac{\Delta D}{d} = \frac{(\mu-1)t\,D}{d}$

Also, fringe width is given by

$$\beta = \frac{\lambda D}{d}$$

According to the question,

$$n\beta = x$$

$$\Rightarrow \quad \frac{n\lambda D}{d} = (\mu-1)t\frac{D}{d}$$

$$\Rightarrow \quad n\lambda = (\mu-1)t \text{ or } t = \frac{n\lambda}{(\mu-1)}$$

Alternate Solution

Path difference, $\Delta = \frac{xd}{D} = (\mu-1)t$

or $$t = \frac{xd}{(\mu-1)D}$$

Also, $$\beta = \frac{\lambda D}{d} \Rightarrow \frac{d}{D} = \frac{\lambda}{\beta}$$

$$\therefore \quad t = \frac{x\lambda}{(\mu-1)\beta} = \frac{n\lambda D \times \lambda}{d(\mu-1)\beta} \qquad [\because x = n\beta]$$

$$\Rightarrow \quad t = \frac{n\lambda}{(\mu-1)}$$

So, no option is correct.

8. A thin convex lens L (refractive index = 1.5) is placed on a plane mirror M. When a pin is placed at A, such that $OA = 18$ cm, its real inverted image is formed at A itself, as shown in figure. When a liquid of refractive index μ_l is put between the lens and the mirror, the pin has to be moved to A', such that $OA' = 27$ cm, to get its inverted real image at A' itself. The value of μ_l will be

[JEE Main 2019, 9 April Shift-II]

(a) $\sqrt{3}$ (b) $\sqrt{2}$ (c) $\frac{4}{3}$ (d) $\frac{3}{2}$

Exp. **(c)**

Light from plane mirror is reflected back on it's path, so that image of A coincides with A itself.

This would happen when rays refracted by the convex lens falls normally on the plane mirror, i.e. the refracted rays form a beam parallel to principal axis of the lens. Hence, the object would then be considered at the focus of convex lens.

$\therefore$ Focal length of curvature of convex lens is,

$$f_1 = 18 \text{ cm}$$

With liquid between lens and mirror, image is again coincides with object, so the second measurement is focal length of combination of liquid lens and convex lens.

$$\therefore \quad \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f_{eq}} \Rightarrow \frac{1}{18} + \frac{1}{f_2} = \frac{1}{27} \Rightarrow f_2 = -54 \text{ cm}$$

For convex lens by lens maker's formula, we have

$$\frac{1}{f} = (\mu-1)\left(\frac{2}{R}\right)$$

$$\Rightarrow \quad \frac{1}{18} = 0.5 \times \frac{2}{R} \Rightarrow R = 18 \text{ cm}$$

and for plano-convex liquid lens, we have

$$\frac{1}{f} = (\mu_l - 1)\left(\frac{-1}{R}\right)$$

$$\Rightarrow \quad -\frac{1}{54} = (\mu_l - 1)\left(\frac{-1}{18}\right)$$

$$\Rightarrow \quad \mu_l = 1 + \frac{1}{3} = \frac{4}{3}$$

9. A convex lens of focal length 20 cm produces images of the same magnification 2 when an object is kept at two distances x_1 and x_2 $(x_1 > x_2)$ from the lens. The ratio of x_1 and x_2 is **[JEE Main 2019, 9 April Shift-II]**

(a) 5 : 3 (b) 2 : 1
(c) 4 : 3 (d) 3 : 1

Exp. **(d)**

Since, image formed by a convex lens can be real or virtual in nature.

Thus, $m = +2$ or -2.

First let us take image to be real in nature, then

$m = -2 = \frac{v}{u}$, where v is image distance and u is object distance.

$\Rightarrow \quad v = -2x_1$ [Taking $u = x_1$]

Now, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{-2x_1} - \frac{1}{x_1} = \frac{1}{20} \Rightarrow \frac{-3}{2x_1} = \frac{1}{20}$$

$$\Rightarrow \quad x_1 = -30 \text{ cm}$$

Now, let us take image to be virtual in nature, then

$m = 2 = \frac{v}{u} \Rightarrow v = 2x_2$ [Taking $u = x_2$]

Again, by using lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{2x_2} - \frac{1}{x_2} = \frac{1}{20}$$

$$\Rightarrow \quad \frac{-1}{2x_2} = \frac{1}{20}$$

$$\Rightarrow \quad x_2 = -10 \text{ cm}$$

So, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{-30}{-10} = 3:1$$

Alternate Solution

Magnification for a lens can also be written as

$$m = \left(\frac{f}{f+u}\right)$$

When $m = -2$ (for real image)

$$-2 = \frac{f}{f + x_1}$$

$$\Rightarrow \quad -2f + (-2)x_1 = f \text{ or } x_1 = -\frac{3f}{2}$$

Similarly, when $m = +2$ (for virtual image)

$$+2 = \frac{f}{f + x_2} \Rightarrow 2f + 2x_2 = f$$

or $\quad x_2 = \frac{-f}{2}$

Now, the ratio of x_1 and x_2 is

$$\frac{x_1}{x_2} = \frac{\frac{-3f}{2}}{-f/2} = \frac{3}{1}$$

10. Diameter of the objective lens of a telescope is 250 cm. For light of wavelength 600 nm coming from a distant object, the limit of resolution of the telescope is close to **[JEE Main 2019, 9 April Shift-II]**

(a) 3.0×10^{-7} rad (b) 2.0×10^{-7} rad
(c) 1.5×10^{-7} rad (d) 4.5×10^{-7} rad

Exp. **(a)**

Limit of resolution for a telescope from Rayleigh's criteria is

$$\theta_R = \frac{1.22\,\lambda}{D}$$

Here, $D = 250 \text{ cm} = 250 \times 10^{-2}$ m

and $\lambda = 600 \text{ nm} = 600 \times 10^{-9}$ m

So, limit of resolution is

$$\theta_R = \frac{1.22 \times 600 \times 10^{-9}}{250 \times 10^{-2}} = 2.93 \times 10^{-7} \text{ rad}$$

$$\approx 3 \times 10^{-7} \text{rad}$$

11. A ray of light AO in vacuum is incident on a glass slab at angle 60° and refracted at angle 30° along OB as shown in the figure.

The optical path length of light ray from A to B is **[JEE Main 2019, 10 April Shift-I]**

(a) $\frac{2\sqrt{3}}{a} + 2b$ (b) $2a + \frac{2b}{3}$
(c) $2a + 2b$ (d) $2a + \frac{2b}{\sqrt{3}}$

Exp. **(c)**

From the figure,

$$\cos 60° = \frac{a}{AO}$$

$$\Rightarrow \quad AO = \frac{a}{\cos 60°} = 2a \quad \text{...(i)}$$

and $\quad \cos 30° = \dfrac{b}{BO}$

or $\quad BO = \dfrac{b}{\cos 30°} = \dfrac{2}{\sqrt{3}} b \quad \text{... (ii)}$

Optical path length of light ray

$$= AO + \mu\,(BO) \quad \text{... (iii)}$$

Here, μ can be determined using Snell's law, i.e.

$$\mu = \frac{\sin 60°}{\sin 30°} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3} \quad \text{... (iv)}$$

Substituting the values from Eqs. (i), (ii) and (iv) in Eq. (iii), we get

$$\therefore \quad \text{Optical path} = 2a + (\sqrt{3} \times \frac{2}{\sqrt{3}} b)$$

$$= 2a + 2b$$

12. One plano-convex and one plano-concave lens of same radius of curvature R but of different materials are joined side by side as shown in the figure. If the refractive index of the material of 1 is μ_1 and that of 2 is μ_2, then the focal length of the combination is

[JEE Main 2019, 10 April Shift-I]

(a) $\dfrac{2R}{\mu_1 - \mu_2}$ (b) $\dfrac{R}{2-(\mu_1 - \mu_2)}$

(c) $\dfrac{R}{2(\mu_1 - \mu_2)}$ (d) $\dfrac{R}{\mu_1 - \mu_2}$

Exp. **(d)**

> **Key Idea** Focal length of two lenses in contact is given as
> $$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$
> where, f_1 and f_2 are the focal lengths of the respective lenses.

Focal length of a lens is given as

$$\frac{1}{f} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$\therefore$ Focal length of plano-convex lens, i.e. lens 1 ,

$$R_1 = \infty \text{ and } R = -R$$

$$\Rightarrow \quad \frac{1}{f_1} = \frac{\mu_1 - 1}{R}$$

or $\quad f_1 = \dfrac{R}{(\mu_1 - 1)} \quad \text{... (i)}$

Similarly, focal length of plano-concave lens, i.e. lens 2,

$$R_1 = -R \text{ and } R_2 = \infty$$

$$\Rightarrow \quad \frac{1}{f_2} = -\frac{(\mu_2 - 1)}{R}$$

or $\quad f_2 = \dfrac{-R}{(\mu_2 - 1)} \quad \text{... (ii)}$

From Eqs. (i) and (ii), net focal length is

$$\frac{1}{f} = \frac{\mu_1 - 1}{R} - \frac{\mu_2 - 1}{R}$$

$$= \frac{\mu_1 - \mu_2}{R} \Rightarrow f = \frac{R}{\mu_1 - \mu_2}$$

13. In a Young's double slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be

[JEE Main 2019, 10 April Shift-II]

(a) 4 : 1 (b) 25 : 9 (c) 9 : 1 (d) $(\sqrt{3}+1)^4 : 16$

Exp. **(c)**

> **Key Idea** Amplitude of light is directly proportional to area through which light is passing.
> For same length of slits,
> amplitude $\propto$ (width)$^{1/2}$
> Also, intensity $\propto$ (amplitude)^2

In YDSE, ratio of intensities of maxima and minima is given by

$$\frac{I_{max}}{I_{min}} = \frac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2}$$

where, I_1 and I_2 are the intensities obtained from two slits, respectively.

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{(a_1 + a_2)^2}{(a_1 - a_2)^2}$$

where, a_1 and a_2 are light amplitudes from slits 1 and 2, respectively.

$$\Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{(\sqrt{W_1} + \sqrt{W_2})^2}{(\sqrt{W_1} - \sqrt{W_2})^2}$$

where, W_1 and W_2 are the widths of slits, respectively.

Here, $\left(\frac{W_1}{W_2}\right) = \left(\frac{a_1}{a_2}\right)^2 = \frac{4}{1} \Rightarrow \sqrt{\frac{W_1}{W_2}} = \frac{2}{1}$

So, we have

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{W_1} + \sqrt{W_2}}{\sqrt{W_1} - \sqrt{W_2}}\right)^2 = \left(\frac{\sqrt{\frac{W_1}{W_2}} + 1}{\sqrt{\frac{W_1}{W_2}} - 1}\right)$$

$$= \left(\frac{2+1}{2-1}\right)^2 = 9:1$$

14. The graph shows how the magnification m produced by a thin lens varies with image distance v. What is the focal length of the lens used? **[JEE Main 2019, 10 April Shift-II]**

(a) $\frac{b^2c}{a}$ (b) $\frac{b^2}{ac}$ (c) $\frac{a}{c}$ (d) $\frac{b}{c}$

Exp. **(d)**

First we find relation of magnification m and focal length f. By lens equation,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad 1 - \frac{v}{u} = \frac{v}{f}$$

$$\Rightarrow \quad 1 - m = \frac{v}{f} \qquad \left[\because \frac{v}{u} = m\right]$$

$$\Rightarrow \quad m = 1 - \frac{v}{f} \qquad \ldots(i)$$

Now, from given graph and from Eq. (i),

At $v = a$, magnification is

$$m_1 = 1 - \frac{a}{f} \qquad \ldots(ii)$$

At $v = a + b$, magnification is

$$m_2 = 1 - \frac{a+b}{f} \qquad \ldots(iii)$$

From graph, we can also say that

$$m_2 - m_1 = c \qquad \ldots(iv)$$

So, from Eqs. (ii), (iii) and (iv), we have

$$\left(1 - \frac{(a+b)}{f}\right) - \left(1 - \frac{a}{f}\right) = c$$

$$\Rightarrow \quad \frac{a - (a+b)}{f} = c$$

$$\Rightarrow \quad f = -\frac{b}{c} \text{ or } |f| = \frac{b}{c}$$

15. The value of numerical aperture of the objective lens of a microscope is 1.25. If light of wavelength 5000Å is used, the minimum separation between two points, to be seen as distinct, will be **[JEE Main 2019, 12 April Shift-I]**

(a) $0.24\,\mu m$ (b) $0.38\,\mu m$
(c) $0.12\,\mu m$ (d) $0.48\,\mu m$

Exp. **(a)**

> **Key Idea** Numerical Aperture (NA) of the microscope is given by
>
> $$NA = \frac{0.61\lambda}{d}$$
>
> Here, d = minimum separation between two points to be seen as distinct and λ = wavelength of light.

Given, $\lambda = 5000Å = 5000 \times 10^{-10}$ m and

$NA = 1.25$

Now, $d = \frac{0.61\lambda}{NA} = \frac{0.61 \times 5000 \times 10^{-10}}{1.25}$

or $d = \frac{3.05}{1.25} \times 10^{-7}$ m

$= 2.4 \times 10^{-7}$ m

or $d = 0.24\,\mu m$

16. A concave mirror has radius of curvature of 40 cm. It is at the bottom of a glass that has water filled up to 5 cm (see figure).

If a small particle is floating on the surface of water, its image as seen, from directly above the glass, is at a distance d from the surface of water. The value of d is close to [Refractive index of water = 1.33]

[JEE Main 2019, 12 April Shift-I]

(a) 6.7 cm (b) 13.4 cm
(c) 8.8 cm (d) 11.7 cm

Exp. **(c)**

In the given case, $u = -5$ cm

Focal length, $f = \frac{-R}{2} = \frac{-40}{2} = -20$ cm

Now, using mirror formula,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{20} + \frac{1}{5} = +\frac{3}{20}$$

$$\Rightarrow \quad v = +\frac{20}{3} \text{ cm}$$

For the light getting refracted at water surface, this image will act as an object.

So, distance of object,

$$d = 5 \text{ cm} + \frac{20}{3} \text{ cm} = \frac{35}{3} \text{ cm}$$

(below the surface). Let's assume final image at distance d after refraction.

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad d' = d\left(\frac{\mu_2}{\mu_1}\right) = \left(\frac{35}{3}\text{cm}\right)\left(\frac{1}{\frac{4}{3}}\right)$$

$$= \frac{35}{3} \times \frac{3}{4} \text{ cm} = \frac{35}{4} \text{ cm}$$

$$= 8.75 \text{ cm} \approx 8.8 \text{ cm}$$

17. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used)

[JEE Main 2019, 12 April Shift-I]

(a) $\frac{2\lambda}{(\mu - 1)}$ (b) $\frac{\lambda}{2(\mu - 1)}$
(c) $\frac{\lambda}{(\mu - 1)}$ (d) $\frac{\lambda}{(2\mu - 1)}$

Exp. **(c)**

As we know,

Path difference introduced by thin film,

$$\Delta = (\mu - 1)t \quad \ldots(i)$$

and if fringe pattern shifts by one frings width, then path difference,

$$\Delta = 1 \times \lambda = \lambda \quad \ldots(ii)$$

So, from Eqs. (i) and (ii), we get

$$(\mu - 1)t = \lambda$$

$$\Rightarrow \quad t = \frac{\lambda}{\mu - 1}$$

Alternate Solution

Path difference introduced by the thin film of thickness t and refractive index μ is given by

$$\Delta = (\mu - 1)t$$

$\therefore$ Position of the fringe is

$$x = \frac{\Delta D}{d} = \frac{(\mu - 1)t\,D}{d} \quad \ldots(i)$$

Fringe width of one fringe is given by

$$\beta = \frac{\lambda D}{d} \quad \ldots(ii)$$

Given that $x = \beta$, so from Eqs. (i) and (ii), we get

$$\Rightarrow \quad \frac{(\mu - 1)tD}{d} = \frac{\lambda D}{d}$$

$$\Rightarrow \quad (\mu - 1)t = \lambda \text{ or } t = \frac{\lambda}{(\mu - 1)}$$

18. A transparent cube of side d, made of a material of refractive index μ_2, is immersed in a liquid of refractive index $\mu_1 (\mu_1 < \mu_2)$. A ray is incident on the face AB at an angle θ (shown in the figure). Total internal reflection takes place at point E on the face BC. **[JEE Main 2019, 12 April Shift-II]**

Then, θ must satisfy

(a) $\theta < \sin^{-1}\frac{\mu_1}{\mu_2}$ (b) $\theta > \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$

(c) $\theta < \sin^{-1}\sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$ (d) $\theta > \sin^{-1}\frac{\mu_1}{\mu_2}$

Exp. **(c)**

> **Key Idea** The critical angle is defined as the angle of incidence that provides an angle of refraction of 90°.
>
> So, $\theta_c = \sin^{-1}\frac{\mu_2}{\mu_1}$

For total internal reflection, angle of incidence (i) at medium interface must be greater than critical angle (C).

where,

$$\sin C = \frac{\mu_1}{\mu_2} \quad \ldots(i)$$

Now, in given arrangement, at point D,

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \text{(Snell's law)}$$

$$\Rightarrow \quad \frac{\sin\theta}{\sin(90° - C)} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{\sin\theta}{\cos C} = \frac{\mu_2}{\mu_1}$$

$$\Rightarrow \quad \sin\theta = \frac{\mu_2}{\mu_1} \cdot \cos C = \frac{\mu_2}{\mu_1}\sqrt{1 - \sin^2 C}$$

[from Eq. (i)]

$$= \frac{\mu_2}{\mu_1}\sqrt{1 - \frac{\mu_1^2}{\mu_2^2}} = \sqrt{\frac{\mu_2^2}{\mu_1^2} - 1}$$

$$\Rightarrow \quad \theta = \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

For TIR at E, $i > C$

$$\Rightarrow \quad \theta < \sin^{-1}\sqrt{\left(\frac{\mu_2^2}{\mu_1^2} - 1\right)}$$

19. A system of three polarisers P_1, P_2, P_3 is set up such that the pass axis of P_3 is crossed with respect to that of P_1. The pass axis of P_2 is inclined at 60° to the pass axis of P_3. When a beam of unpolarised light of intensity I_0 is incident on P_1, the intensity of light transmitted by the three polarisers is I. The ratio (I_0 / I) equals (nearly) **[JEE Main 2019, 12 April Shift-II]**

(a) 5.33 (b) 16.00

(c) 10.67 (d) 1.80

Exp. **(c)**

When unpolarised light pass through polaroid P_1, intensity obtained is

$$I_1 = \frac{I_0}{2}$$

where, I_0 = intensity of incident light.

Now, this transmitted light is polarised and it pass through polariser P_2. So, intensity I_2 transmitted is obtained by Malus law.

$$\Rightarrow \qquad I_2 = I_1 \cos^2\theta$$

As angle of pass axis of P_1 and P_3 is 90° and angle of pass axis of P_2 and P_3 is 60°, so angle between pass axis of P_1 and P_2 is (90°– 60°) = 30°. So,

$$I_2 = \frac{I_0}{2}\cos^2 30° = \frac{I_0}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{8} I_0$$

When this light pass through third polariser P_3, intensity I transmitted is again obtained by Malus law.

$$\text{So,} \qquad I = I_2 \cos^2 60° = \left(\frac{3}{8} I_0\right)\cos^2 60°$$

$$= \frac{3}{8} I_0 \times \left(\frac{1}{2}\right)^2 = \frac{3}{32} I_0$$

$$\text{So, ratio } \frac{I_0}{I} = \frac{32}{3} = 10.67$$

20. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio **[JEE Main 2019, 9 Jan Shift-I]**

(a) 16 : 9 (b) 5 : 3 (c) 25 : 9 (d) 4 : 1

Exp. **(c)**

Let the intensity of two coherent sources be I_1 and I_2, respectively.

It is given that,

$$\frac{\text{maximum intensity}}{\text{minimum intensity}} = \frac{I_{max}}{I_{min}} = \frac{16}{1}$$

Since, we know,

$$I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2 \text{ and } I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$\therefore$ We can write, $\dfrac{I_{max}}{I_{min}} = \dfrac{(\sqrt{I_1} + \sqrt{I_2})^2}{(\sqrt{I_1} - \sqrt{I_2})^2} = 16$

$$\Rightarrow \quad \frac{\sqrt{I_1} + \sqrt{I_2}}{\sqrt{I_1} - \sqrt{I_2}} = \frac{\sqrt{16}}{1} = \frac{4}{1}$$

$$\sqrt{I_1} + \sqrt{I_2} = 4\sqrt{I_1} - 4\sqrt{I_2}$$

$$\Rightarrow \quad 5\sqrt{I_2} = 3\sqrt{I_1}, \quad \frac{\sqrt{I_1}}{\sqrt{I_2}} = \frac{5}{3}$$

Squaring both the sides, we get $\dfrac{I_1}{I_2} = \dfrac{25}{9}$

21. Consider a tank made of glass (refractive index is 1.5) with a thick bottom. It is filled with a liquid of refractive index μ. A student finds that, irrespective of what the incident angle i (see figure) is for a beam of light entering the liquid, the light reflected from the liquid glass interface is never completely polarised.

For this to happen, the minimum value of μ is **[JEE Main 2019, 9 Jan Shift-I]**

(a) $\dfrac{3}{\sqrt{5}}$ (b) $\dfrac{5}{\sqrt{3}}$

(c) $\dfrac{4}{3}$ (d) $\sqrt{\dfrac{5}{3}}$

Exp. **(a)**

> **Key Idea** When a beam of unpolarised light is reflected from a transparent medium of refractive index μ, then the reflected light is completely plane polarised at a certain angle of incidence i_B, which is known as Brewster's angle.

In the given condition, the light reflected irrespective of an angle of incidence is never completely polarised. So, $i_C > i_B$

where, i_C is the critical angle.

$$\Rightarrow \qquad \sin i_C < \sin i_B \qquad \text{...(i)}$$

From Brewster's law, we know that

$$\tan i_B = {}^w\mu_g = \frac{\mu_{glass}}{\mu_{water}} = \frac{1.5}{\mu} \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{1}{\mu} < \frac{1.5}{\sqrt{(1.5)^2 + (\mu)^2}}$$

$$\Rightarrow \qquad \sqrt{(1.5)^2 + \mu^2} < 1.5\,\mu$$

$$\mu^2 + (1.5)^2 < (1.5\,\mu)^2 \text{ or } \mu < \frac{3}{\sqrt{5}}$$

$\therefore$ The minimum value of μ should be $\dfrac{3}{\sqrt{5}}$.

22. A convex lens is put 10 cm from a light source and it makes a sharp image on a screen, kept 10 cm from the lens. Now, a glass block (refractive index is 1.5) of 1.5 cm thickness is placed in contact with the light source.

To get the sharp image again, the screen is shifted by a distance d. Then, d is

[JEE Main 2019, 9 Jan Shift-I]

(a) 0
(b) 1.1 cm away from the lens
(c) 0.55 cm away from the lens
(d) 0.55 cm towards the lens

Exp. **(c)**

Initially, when a light source (i.e. an object) is placed at 10 cm from the convex mirror and an image is form on the screen as shown in the figure below,

Since, $u = v$ which can only be possible in the situation when the object is place at '2f' of the lens.

So, that the image can form at '2f' only on the other side of the lens.

Thus, the distance from the optical centre (O) of the lens and $2f$ is $2 \times$ focal length (f) = object distance

$\Rightarrow \quad 2 \times f = 10$ or $f = 5$ cm

Now, when a glass block is placed in contact with the light source i.e., object, then the situation is shown in the figure given below

Then due to the block, the position of the object in front of the lens would now be shifted due to refraction of the light source rays through the block.

The shift in the position of the object is given as

$$x = \left(1 - \frac{1}{\mu}\right)t$$

where, μ is the refractive index of the block and t is its thickness.

$$\Rightarrow \quad x = \left(1 - \frac{1}{1.5}\right)1.5 = \left(1 - \frac{2}{3}\right)\frac{3}{2}$$

$$= \frac{1}{3} \times \frac{3}{2} = \frac{1}{2} = 0.5 \text{ cm}$$

$\therefore$ The new object distance of the light source in front of the lens will be

$u' = 10 - 0.5 = 9.5$ cm

Since, the focal length of the lens is 5 cm.

Therefore, the image distance of the light source now can be given as,

$$\frac{1}{v'} = \frac{1}{f} + \frac{1}{u'} \quad \text{(using lens formula)}$$

Substituting the values, we get

$$\frac{1}{v'} = \frac{1}{5} + \left(\frac{1}{-9.5}\right)$$

$$= \frac{+9.5 - 5}{47.5} = \frac{4.5}{47.5}$$

or $\quad v' = 10.55$ cm

$\therefore$ The value of $d = v' - v = 10.55 - 10$

$= 0.55$ cm, away from the lens

Focal length in the above question can be calculated by using lens formula i.e. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

23. The plane mirrors (M_1 and M_2) are inclined to each other such that a ray of light incident on mirror M_1 and parallel to the mirror M_2 is reflected from mirror M_2 parallel to the mirror M_1. The angle between the two mirror is

[JEE Main 2019, 9 Jan Shift-II]

(a) 45° (b) 75° (c) 90° (d) 60°

Exp. **(d)**

The given condition is shown in the figure given below, where two plane mirror inclined to each other such that a ray of light incident on the first mirror (M_1) and parallel to the second mirror

(M_2) is finally reflected from second mirror (M_2) parallel to the first mirror.

where, PQ = incident ray parallel to the mirror M_2,
QR = reflected ray from the mirror M_1,
RS = reflected ray from the mirror M_2 which is parallel to the M_1 and
θ = angle between M_1 and M_2.
According to geometry,
$\angle PAS = \angle PQM_1 = \theta$ (angle on same line)
$\angle AQN_1$ = angle of incident = $90 - \theta$
$\angle N_1QR$ = angle of reflection = $(90 - \theta)$.
Therefore, for triangle ΔORQ, (according to geometry)

$$\angle\theta + \angle\theta + \angle ORQ = 180°$$
$$\angle ORQ = 180° - 2\theta \quad \ldots(i)$$

For normal N_2, angle of incident = angle of reflection

$$= 2\theta - 90° \quad \ldots(ii)$$

Therefore, for the triangle ΔRAQ

$$\Rightarrow \quad 4\theta - 180° + 180° - 2\theta + \theta = 180$$
$$3\theta = 180°$$
$$\theta = 60°$$

24. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ n-m is incident on the slits. The total number of bright fringes that are observed in the angular range $-30° \le \theta \le 30°$ is **[JEE Main 2019, 9 Jan Shift-II]**

(a) 320 (b) 321 (c) 640 (d) 641

Exp. **(d)**

In Young's double slit experiment, the condition of bright fringe and dark fringe are,
for bright fringes (maxima), path difference = $n\lambda$

$$d\sin\theta = n\lambda$$

for dark fringes (minima),
path-difference = $(2n - 1)\dfrac{\lambda}{2}$

$$d\sin\theta = (2n - 1)\frac{\lambda}{2}$$

For the given question, distance between slits (d) = 0.320 mm
Wavelength of light used (λ) = 500 nm
Angular range for bright fringe (θ) = – 30° to 30°
Hence, for bright-fringe,

$$n\lambda = d\sin\theta$$
$$n = \frac{d\sin\theta}{\lambda}$$
$$= \frac{0.320 \times 10^{-3} \times \sin 30°}{500 \times 10^{-9}}$$
$$n_{max} = 320$$

$\therefore$ Total number of maxima between the two lines are

$$n = (n_{max} \times 2) + 1$$

Here, $n = (320 \times 2) + 1$
$n = 641$

25. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\dfrac{1}{40}$ rad by using light of wavelength λ_1. When the light of the wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 n-m to 740 n-m), their values are **[JEE Main 2019, 10 Jan Shift-I]**

(a) 380 n-m, 525 n-m
(b) 400 n-m, 500 n-m
(c) 380 n-m, 500 n-m
(d) 625 n-m, 500 n-m

Exp. **(d)**

Key Idea In a YDSE, path difference between 2 rays, reaching at some common point P located at angular position 'θ' as shown in the figure below is

For a bright fringe at same angular position 'θ', both of the rays from slits S_1 and S_2 are in phase. Hence, path difference is an integral multiple of wavelength of light used.

i.e., $\Delta L = n\lambda$

or $d\theta = n\lambda \Rightarrow \lambda = \frac{d\theta}{n}$

Here, $\theta = \frac{1}{40}$ rad, $d = 0.1$ mm

Hence, $\lambda = \frac{0.1}{40n}$ mm $= \frac{0.1 \times 10^{-3}\text{ m}}{40n}$

$= \frac{0.1 \times 10^{-3} \times 10^{9}}{40n}$ nm

$\Rightarrow \lambda = \frac{2500}{n}$ nm

So, with light of wavelength λ_1 we have

$$\lambda_1 = \frac{2500}{n_1} \text{ (nm)}$$

and with light of wavelength λ_2, we have

$$\lambda_2 = \frac{2500}{n_2} \text{ (nm)}$$

Now, choosing different integral values for n_1 and n_2, (i.e., $n_1, n_2 = 1, 2, 3 \ldots$ etc) we find that for

$$n_1 = 4, \lambda_1 = \frac{2500}{4} = 625 \text{ nm}$$

and for $n_2 = 5$,

$$\lambda_2 = \frac{2500}{5} = 500 \text{ nm}$$

These values lie in given interval 500 nm to 625 nm.

26. A plano-convex lens of refractive index μ_1 and focal length f_1 is kept in contact with another plano-concave lens of refractive index μ_2 and focal length f_2. If the radius of curvature of their spherical faces is R each and $f_1 = 2f_2$, then μ_1 and μ_2 are related as

[JEE Main 2019, 10 Jan Shift-I]

(a) $3\mu_2 - 2\mu_1 = 1$ (b) $2\mu_2 - \mu_1 = 1$
(c) $2\mu_1 - \mu_2 = 1$ (d) $\mu_1 + \mu_2 = 3$

Exp. **(c)**

Given, $f_1 = 2f_2$

$$\Rightarrow \frac{1}{|f_1|} = \frac{1}{|2f_2|} \quad \ldots(i)$$

Using lens Maker's formula, we get

$$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{R_1'} - \frac{1}{R_2'}\right)$$

$$\Rightarrow \left|(\mu_1 - 1)\left(\frac{1}{\infty} - \frac{1}{-R}\right)\right| = \left|\frac{1}{2}(\mu_2 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right)\right|$$

[∵ using Eq. (i)]

$$\Rightarrow \frac{\mu_1 - 1}{R} = \frac{\mu_2 - 1}{2R}$$

$$\Rightarrow 2\mu_1 - \mu_2 = 1$$

27. The eye can be regarded as a single refracting surface. The radius of curvature of this surface is equal to that of cornea (7.8 mm). This surface separates two media of refractive indices 1 and 1.34. Calculate the distance from the refracting surface at which a parallel beam of light will come to focus. **[JEE Main 2019, 10 Jan Shift-II]**

(a) 4.0 cm (b) 2 cm
(c) 3.1 cm (d) 1 cm

Exp. **(c)**

The given condition is shown in the figure below

where, a parallel beam of light is coming from air ($\mu = 1$) to a spherical surface (eye) of refractive index 1.34.

Radius of curvature of this surface is 7.8 mm.

From the image formation formula for spherical surface, i.e. relation between object, image and radius of curvature.

$$\frac{\mu_r}{v} - \frac{\mu_i}{u} = \frac{\mu_r - \mu_i}{R} \quad \ldots \text{(i)}$$

Given, $\mu_r = 1.34$, $\mu_i = 1$, $u = \infty$ (– ve) and $R = 7.8$

Substituting the given values, we get

$$\frac{1.34}{v} + \frac{1}{\infty} = \frac{1.34 - 1}{7.8}$$

or $$\frac{1.34}{v} = \frac{0.34}{7.8}$$

$\Rightarrow$ $$v = \frac{1.34 \times 7.8}{0.34} \text{ mm}$$

$\Rightarrow$ $$v = \frac{4}{3} \times 3 \times 7.8 \text{ mm}$$

($\because$ approximately $1.34 = 4/3$ and $0.34 = 1/3$)

$\Rightarrow$ $v = 31.2$ mm or 3.12 cm

28. Consider a Young's double slit experiment as shown in figure.

What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1)? **[JEE Main 2019, 10 Jan Shift-II]**

(a) $\frac{\lambda}{2(5-\sqrt{2})}$ (b) $\frac{\lambda}{(5-\sqrt{2})}$

(c) $\frac{\lambda}{2(\sqrt{5}-2)}$ (d) $\frac{\lambda}{(\sqrt{5}-2)}$

Exp. **(c)**

In the given case, figure for first minima will be as shown below

We know that condition for minima in Young's double slit experiment is path difference,

$\Delta x = (2n - 1)\lambda/2$

For first minima, $n = 1$

$\Rightarrow$ $$\Delta x = \lambda/2 \quad \ldots \text{(i)}$$

Path difference between the rays coming from virtual sources S_1 and S_2 at point 'P' will be

$$\Delta x = S_2P - S_1P \quad \ldots \text{(ii)}$$

From triangle S_1S_2P,

$$S_1P = 2d \quad \ldots \text{(iii)}$$

and $$(S_2P^2) = (S_1S_2)^2 + (S_1P)^2 = d^2 + (2d)^2$$

$\Rightarrow$ $$(S_2P^2) = 5d^2 \text{ or } S_2P = \sqrt{5}\,d \quad \ldots \text{(iv)}$$

Substituting the values from Eqs. (iii) and (iv) in Eq. (ii), we get

$$\Delta x = \sqrt{5}\,d - 2d \quad \ldots \text{(v)}$$

From Eqs. (i) and (v), we get

$$\sqrt{5}\,d - 2d = \lambda/2 \Rightarrow d = \frac{\lambda}{2(\sqrt{5}-2)}$$

29. An object is at a distance of 20 m from a convex lens of focal length 0.3 m. The lens forms an image of the object. If the object moves away from the lens at a speed of 5 m/s, the speed and direction of the image will be **[JEE Main 2019, 11 Jan Shift-I]**

(a) 3.22×10^{-3} m/s towards the lens

(b) 0.92×10^{-3} m/s away from the lens

(c) 2.26×10^{-3} m/s away from the lens

(d) 1.16×10^{-3} m/s towards the lens

Exp. **(d)**

Lens formula is given as

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \ldots \text{(i)}$$

$\Rightarrow$ $$\frac{1}{v} = \frac{1}{f} + \frac{1}{u}$$

$\Rightarrow$ $$\frac{uf}{u+f} = v$$

$\Rightarrow$ $$\frac{v}{u} = \frac{f}{u+f} \quad \ldots \text{(ii)}$$

Now, by differentiating Eq. (i), we get

$$0 = -\frac{1}{v^2} \cdot \frac{dv}{dt} + \frac{1}{u^2} \cdot \frac{du}{dt}$$

[$\because f$ (focal length of a lens is constant)]

or $$\frac{dv}{dt} = \frac{v^2}{u^2}\, du/dt$$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{f}{u+f}\right)^2 \frac{du}{dt} \qquad \text{[using Eq. (ii)]}$$

Given, $f = 0.3$ m, $u = -20$ m, $du/dt = 5$ m/s

$$\therefore \quad \frac{dv}{dt} = \left(\frac{0.3}{0.3-20}\right)^2 \times 5 = \left(\frac{3}{197}\right)^2 \times 5$$

$$= 1.16 \times 10^{-3} \text{ m/s}$$

Thus, the image is moved with a speed of 1.16×10^{-3} m/s towards the lens.

30. The variation of refractive index of a crown glass thin prism with wavelength of the incident light is shown. Which of the following graphs is the correct one, if D_m is the angle of minimum deviation ?

[JEE Main 2019, 11 Jan Shift-I]

Exp. **(c)**

For a crown glass thin prism i.e., prism with small angle, the angle of minimum deviation is given as,

$$D_m = (n-1)A$$

where, A is prism angle and n is refractive index.

$$\Rightarrow \quad D_m \propto n \qquad \ldots \text{(i)}$$

Since from the given graph, the value of 'n' decrease with the increase in 'λ'. Thus, from relation (i), we can say that, D_m will also decrease with the increase in 'λ'.

∴ Hence, option (c) is correct.

31. In a Young's double slit experiment, the path difference at a certain point on the screen between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a bright fringe is close to **[JEE Main 2019, 11 Jan Shift-I]**

(a) 0.80 (b) 0.74 (c) 0.94 (d) 0.85

Exp. **(d)**

Let intensity of each wave is I_0.

Then, intensity at the centre of bright fringe will be $4I_0$.

Given, path difference, $\Delta x = \lambda/8$

$$\therefore \quad \text{Phase difference, } \phi = \Delta x \times \frac{2\pi}{\lambda}$$

$$\Rightarrow \quad \phi = \frac{\lambda}{8} \times \frac{2\pi}{\lambda} \text{ or } \phi = \pi/4$$

Intensity of light at this point,

$$I' = I_0 + I_0 + 2I_0 \cos(\pi/4)$$

$$= 2I_0 + \sqrt{2}\, I_0 = 3.41\, I_0$$

$$\text{Now,} \quad \frac{I'}{4I_0} = \frac{3.41}{4} = 0.85$$

32. A monochromatic light is incident at a certain angle on an equilateral triangular prism and suffers minimum deviation. If the refractive index of the material of the prism is $\sqrt{3}$, then the angle of incidence is

[JEE Main 2019, 11 Jan Shift-II]

(a) 45° (b) 90° (c) 60° (d) 30°

Exp. **(c)**

Given, refractive index of material of prism $n = \sqrt{3}$, prism angle $A = 60°$

Method 1

$$\text{Using prism formula, } n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \quad \sqrt{3} = \frac{\sin\left(\frac{60+\delta}{2}\right)}{\sin 30°}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right)=\frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad \sin\left(\frac{60+\delta}{2}\right)=\sin 60^\circ \text{ or } \frac{60+\delta}{2}=60$$

or angle of minimum deviation $\delta = 60^\circ$

Incident angle, $i = \frac{60+\delta}{2} = 60^\circ$

Method 2 For minimum deviation, ray should pass symmetrically (i.e. parallel to the base of the equilateral prism)

$\Rightarrow$ From geometry of given figure, we have,

$r = 30^\circ$

Using Snell's law, $n = \frac{\sin i}{\sin r}$

$\sin i = n \sin r = \sqrt{3}\sin 30^\circ$

$$\Rightarrow \quad \sin i = \frac{\sqrt{3}}{2} \text{ or } i = 60^\circ$$

33. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μ-m and a width of 4.05 μ-m. The number of bright fringes between the first and the second diffraction minima is

[JEE Main 2019, 11 Jan Shift-II]

(a) 5 (b) 10
(c) 9 (d) 4

Exp. **(a)**

Here, wavelength of light used (λ) = 5303 Å.

Distance between two slit (d) = 19.44 μm

Width of single slit (a)= 4.05 μm

Here, angular width between first and second diffraction minima $\theta \simeq \frac{\lambda}{a}$

and angular width of a fringe due to double slit is

$$\theta' = \frac{\lambda}{d}$$

$\therefore$ Number of fringes between first and second diffraction minima, $n = \frac{\theta}{\theta'} = \frac{\frac{\lambda}{a}}{\frac{\lambda}{d}} = \frac{d}{a} = \frac{19.44}{4.05} = 4.81$

or $n \simeq 5$

$\therefore$ 5 interfering bright fringes lie between first and second diffraction minima.

34. A point source of light, S is placed at a distance L in front of the centre of plane mirror of width d which is hanging vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror, at a distance $2L$ as shown below.

The distance over which the man can see the image of the light source in the mirror is **[JEE Main 2019, 12 Jan Shift-I]**

(a) $\frac{d}{2}$ (b) d (c) $3d$ (d) $2d$

Exp. **(c)**

Light from mirror is reflected in a straight line and It is appear to come from its image formed at same distance (as that of source) behind the mirror as shown in the ray diagram below.

From ray diagram in similar triangles $\Delta S'CD$ and $\Delta S'OB$, we have

$$\frac{S'C}{S'O} = \frac{CD}{OB}$$

So, $$OB = \frac{CD \times S'O}{S'C} = \frac{\frac{d}{2} \times 3L}{L} = \frac{3d}{2}$$

Also, $OA = \frac{3d}{2}$

So, distance over which man can see the image S' is $\frac{3d}{2} + \frac{3d}{2} = 3d$

35. What is the position and nature of image formed by lens combination shown in figure? (where, f_1 and f_2 are focal lengths)

[JEE Main 2019, 12 Jan Shift-I]

(a) $\frac{20}{3}$ cm from point B at right, real
(b) 70 cm from point B at right, real
(c) 40 cm from point B at right, real
(d) 70 cm from point B at left, virtual

Exp. **(b)**

For lens A, object distance, $u = -20$ cm

Focal length, $f = +5$ cm

From lens formula, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

We have, $$\frac{1}{v} = \frac{1}{5} - \frac{1}{20}$$

$$\frac{1}{v} = \frac{20-5}{20 \times 5} = \frac{15}{100}$$

$$v = \frac{20}{3} \text{ cm}$$

For lens B, image of A is object for B.

$$\therefore \quad u = \frac{20}{3} - 2 = +\frac{14}{3} \text{ cm}$$

$$f = -5 \text{ cm}$$

Now, from lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\Rightarrow \quad \frac{1}{v} = \frac{1}{-5} + \frac{3}{14}$$

$$\frac{1}{v} = \frac{15-14}{5 \times 14}$$

$$v = 70 \text{ cm}$$

Hence, image is on right of lens B and is real in nature.

36. Formation of real image using a biconvex lens is shown below. If the whole set up is immersed in water without disturbing the object and the screen positions, what will one observe on the screen?

[JEE Main 2019, 12 Jan Shift-II]

(a) No change
(b) Magnified image
(c) Image disappears
(d) Erect real image

Exp. **(c)**

When given set up is immersed in water, focal length of lens increases,

Note
$$\frac{f_{\text{liquid}}}{f_{\text{air}}} = \frac{n_{ga} - 1}{n_{gl} - 1} = \frac{n_{ga} - 1}{\left(\frac{n_{ga}}{n_{la}} - 1\right)}$$

Now, $n_{ga} = \frac{3}{2}$ and $n_{la} = \frac{4}{3}$

$$\therefore f_{\text{liquid}} = f_{\text{air}} \left(\frac{\frac{1}{2}}{\frac{1}{8}}\right) = 4 f_{\text{air}}$$

As focal length increases there is no focussing of image on screen

∴ image will disappea

Actually a virtual image is formed on same side of object.

37. A plano-convex lens (focal length f_2, refractive index μ_2, radius of curvature R) fits exactly into a plano-concave lens (focal length f_1, refractive index μ_1, radius of curvature R). Their plane surfaces are parallel to each other. Then, the focal length of the combination will be

[JEE Main 2019, 12 Jan Shift-II]

(a) $f_1 - f_2$ (b) $\dfrac{R}{\mu_2 - \mu_1}$

(c) $f_1 + f_2$ (d) $\dfrac{2f_1 f_2}{f_1 + f_2}$

Exp. **(b)**

Given combination is as shown below

As lenses are in contact, equivalent focal length of combination is

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$$

Using lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Here, $$\frac{1}{f_1} = (\mu_1 - 1)\left(\frac{1}{-R} - \frac{1}{\infty}\right) = \frac{(1-\mu_1)}{R}$$

and $$\frac{1}{f_2} = (\mu_2 - 1)\left(\frac{1}{\infty} - \frac{1}{(-R)}\right) = \frac{(\mu_2 - 1)}{R}$$

∴ $$\frac{1}{f_{eq}} = \left(\frac{\mu_2 - 1}{R}\right) + \left(\frac{1-\mu_1}{R}\right)$$

$$= \frac{\mu_2 - 1 + 1 - \mu_1}{R} = \frac{\mu_2 - \mu_1}{R}$$

So, $$f_{eq} = \frac{R}{\mu_2 - \mu_1}.$$

38. Unpolarised light of intensity I passes through an ideal polariser A. Another identical polariser B is placed behind A. The intensity of light beyond B is found to be $\frac{I}{2}$. Now, another identical polariser C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polariser A and C is **[JEE Main 2018]**

(a) 0° (b) 30° (c) 45° (d) 60°

Exp. **(c)**

Using Malus's law, intensity available after $C = \frac{I}{2} \times \cos^2\alpha$

and intensity available after $B = \frac{I}{2}\cos^2\alpha \times \cos^2\beta$

$= \frac{I}{8}$ (given)

So, $\frac{I}{2} \times \cos^2\alpha \cdot \cos^2\beta = \frac{I}{8}$

⇒ $\cos^2\alpha \cdot \cos^2\beta = \frac{1}{4}$

This is satisfied with $\alpha = 45°$ and $\beta = 45°$

So, angle between A and C is 45°.

39. The angular width of the central maximum in a single slit diffraction pattern is 60°. The width of the slit is 1 μm. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what is slit separation distance? (i.e. distance between the centres of each slit.)

[JEE Main 2018]

(a) 25 μm (b) 50 μm (c) 75 μm (d) 100 μm

Exp. **(*a*)**

Angular width of diffraction pattern = 60°

For first minima,

$$\frac{a}{2}\sin\theta = \frac{\lambda}{2}, \text{ [here, } a = 10^{-6} \text{ m}, \theta = 30°]$$

$$\Rightarrow \quad \lambda = 10^{-6} \times \sin 30°$$

$$\Rightarrow \quad \lambda = \frac{10^{-6}}{2} \text{m}$$

Now, in case of interference caused by bringing second slit,

$\therefore$ Fringe width, $\beta = \frac{\lambda D}{d}$

$$\text{[here, } \lambda = \frac{10^{-6}}{2} \text{ m}, \beta = 1 \text{ cm} = \frac{1}{100} \text{ m},$$
$$d = ? \text{ and } D = 50 \text{ cm} = \frac{50}{100} \text{ m]}$$

So, $$d = \frac{\lambda D}{\beta} = \frac{10^{-6} \times 50}{2 \times \frac{1}{100} \times 100}$$
$$= 25 \times 10^{-6} \text{ m}$$

or $d = 25\,\mu\text{m}$

40. A diverging lens with magnitude of focal length 25 cm is placed at a distance of 15 cm from a converging lens of magnitude of focal length 20 cm. A beam of parallel light falls on the diverging lens. The final image formed is

[JEE Main 2017 (Offline)]

(a) virtual and at a distance of 40 cm from convergent lens
(b) real and at a distance of 40 cm from the divergent lens
(c) real and at a distance of 6 cm from the convergent lens
(d) real and at a distance of 40 cm from convergent lens

Exp. **(*d*)**

Focal length of diverging lens is 25 cm.

As the rays are coming parallel, so the image (I_1) will be formed at the focus of diverging lens i.e. at 25 cm towards left of diverging lens.

Now, the image (I_1) will work as object for converging lens.

For converging lens, distance of object u (i.e. distance of I_1) = – (25 + 15) = – 40 cm

$f = 20$ cm

$\therefore$ From len's formula $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$$\frac{1}{20} = \frac{1}{v} - \frac{1}{-40} \Rightarrow \frac{1}{v} = \frac{1}{20} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{1}{90} \Rightarrow v = 40 \text{ cm}$$

v is positive so image will be real and will form at right side of converging lens at 40 cm.

41. An observer looks at a distance tree of height 10 m with a telescope of magnifying power of 20. To the observer the tree appears

[JEE Main 2016 (Offline)]

(a) 10 times taller (b) 10 times nearer
(c) 20 times taller (d) 20 times nearer

Exp. **(*c*)**

Height of image depends upon the magnifying power to see a 20 times taller object, as the angular magnification should be 20 and we observe angular magnification. Option *(c)* would not be very correct as the telescope can be adjusted to form the image anywhere between infinity and least distance for distinct vision.

Suppose that the image is formed at infinity. Then, the observer will have to focus the eyes at infinity to observe the image. Hence, it is incorrect to say that the image will be appear nearer to the observer.

42. The box of a pin hole camera, of length L, has a hole of radius a. It is assumed that when the hole is illuminated by a parallel beam of light of wavelength λ the spread of the spot (obtained on the opposite wall of the camera) is the sum of its geometrical spread and the spread due to diffraction.

The spot would then have its minimum size (say $b_{\min}$) when **[JEE Main 2016 (Offline)]**

(a) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(b) $a = \sqrt{\lambda L}$ and $b_{\min} = \left(\frac{2\lambda^2}{L}\right)$

(c) $a = \sqrt{\lambda L}$ and $b_{\min} = \sqrt{4\lambda L}$

(d) $a = \frac{\lambda^2}{L}$ and $b_{\min} = \sqrt{4\lambda L}$

Exp. **(c)**

In diffraction, first minima, we have , $\sin\theta = \frac{\lambda}{a}$

So, size of a spot $b = 2a + \frac{2L\lambda}{a}$ ...(i)

Then, minimum size of a spot, we get

$$\frac{\partial b}{\partial a} = 0 \Rightarrow 1 - \frac{L\lambda}{a^2} = 0 \Rightarrow a = \sqrt{\lambda L} \quad \text{...(ii)}$$

So, $b_{\min} = 2\sqrt{\lambda L} + 2\sqrt{\lambda L}$

[by substituting the value of a from Eq. (ii) in Eq. (i)]

$= 4\sqrt{\lambda L}$

So, the radius of the spot, $b_{\min} = \frac{4}{2}\sqrt{\lambda L} = \sqrt{4\lambda L}$

43. In an experiment for determination of refractive index of glass of a prism by i-δ, plot, it was found that a ray incident at an angle 35° suffers a deviation of 40° and that it emerges at an angle 79°. In that case, which of the following is closest to the maximum possible value of the refractive index? **[JEE Main 2016 (Offline)]**

(a) 1.5 (b) 1.6 (c) 1.7 (d) 1.8

Exp. **(a)**

If μ is refractive index of material of prism, then from Snell's law

$$\mu = \frac{\sin i}{\sin r} = \frac{\sin(A + \delta_m)/2}{\sin A/2} \quad \text{...(i)}$$

where, A is angle of prism and δ_m is minimum deviation through prism.

Given, $i = 35°$, $\delta = 40°$, $e = 79°$.

So, angle of deviation by a glass prism,

$\delta = i + e - A \Rightarrow 40° = 35° + 79° - A$

i.e. Angle of prism $\Rightarrow A = 74°$.

Such that, $r_1 + r_2 = A = 74°$.

Let us put $\mu = 1.5$ in eq. (i), we get

$$1.5 = \frac{\sin\left(\frac{A + \delta_{\min}}{2}\right)}{\sin A/2}$$

$$\Rightarrow \quad 1.5 = \frac{\sin\left(\frac{74° + \delta_{\min}}{2}\right)}{\sin 37°}$$

$$\Rightarrow \quad 0.9 = \sin\left(37° + \frac{\delta_{\min}}{2}\right) \quad (\because \sin 37° \approx 0.6)$$

$$\sin 64° = \sin\left(37° + \frac{\delta_{\min}}{2}\right) \quad (\because \sin 64° = 0.9)$$

$$37° + \frac{\delta_{\min}}{2} = 64° \Rightarrow \delta_{\min} \approx 54°$$

This angle is greater than the 40° deviation angle already given. For greater μ, deviation will be even higher. Hence, μ of the given prism should be lesser than 1.5. Hence, the closest option will be 1.5.

44. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ, a ray incident at an angle θ, on the face AB would get transmitted through the face AC of the prism provided **[JEE Main 2015]**

(a) $\theta > \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(b) $\theta < \sin^{-1}\left[\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(c) $\theta > \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

(d) $\theta < \cos^{-1}\left[\mu \sin\left(A + \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right]$

Exp. **(a)**

The ray will get transmitted through face AC if $i_{AC} < i_C$

Consider the ray diagram as shown below.

A ray of light incident on face *AB* at an angle θ.

r_1 = Angle of refraction on face *AB*

r_2 = Angle of incidence at face *AC*

For transmission of light through face *AC*

$$i_{AC} < i_C \quad \text{or} \quad A - r_1 < i_C$$

or $\sin(A - r_1) < \sin i_C$ or $\sin(A - r_1) < \frac{1}{\mu}$

$$A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

or $$\sin r_1 > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

Now, applying Snell's law at the face *AB*

$$1 \times \sin\theta = \mu \sin r_1 \quad \text{or} \quad \sin r_1 = \frac{\sin\theta}{\mu}$$

$$\Rightarrow \quad \frac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\frac{1}{\mu}\right)\right]$$

or $$\theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\frac{1}{\mu}\right)\right\}\right]$$

45. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens principle leads us to conclude that as it travels, the light beam **[JEE Main 2015]**

(a) becomes narrower

(b) goes horizontally without any deflection

(c) bends downwards

(d) bends upwards

Exp. **(d)**

According to Snell's law,

$$\mu \sin\theta = \text{constant}$$

$$\therefore \quad \sin\theta \propto \frac{1}{\mu}$$

As μ increases, θ decreases.

Hence, beam will bend upward.

46. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is **[JEE Main 2015]**

(a) 1 μm (b) 30 μm

(c) 100 μm (d) 300 μm

Exp. **(b)**

We can write resolving angle of necked eye as

$$\theta = 1.22\,\frac{\lambda}{D}$$

where, *D* is the diameter of eye lens.

or $$\frac{Y}{25 \times 10^{-2}} = \frac{1.22 \times 500 \times 10^{-9}}{0.25 \times 2 \times 10^{-2}}$$

$$Y = 30 \times 10^{-6}\ \text{m} = 30\,\mu\text{m}$$

47. A thin convex lens made from crown glass $\left(\mu = \frac{3}{2}\right)$ has focal length *f*. When it is measured in two different liquids having refractive indices $\frac{4}{3}$ and $\frac{5}{3}$. It has the focal lengths f_1 and f_2, respectively. The correct relation between the focal lengths is

(a) $f_1 = f_2 < f$ **[JEE Main 2014]**

(b) $f_1 > f$ and f_2 becomes negative

(c) $f_2 > f$ and f_1 becomes negative

(d) f_1 and f_2 both become negative

It is based on lens maker's formula

i.e., $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Exp. **(b)**

According to lens maker's formula, when the lens in the air

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$$

Here, $\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}$

In case of liquid,where refractive indices are 4/3 and 5/3, we get

Focal lenght in first liquid,

$$\frac{1}{f_1} = \left(\frac{\mu_s}{\mu_{l_1}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\frac{1}{x}$$

$\Rightarrow F_1$ is positive.

Nature of lens is not change.

$$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f} \Rightarrow f_1 = 4f$$

Focal length in second liquid,

$$\frac{1}{f_2} = \left(\frac{\mu_s}{\mu_{l_2}} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{x}\right)$$

$\Rightarrow$ f_2 is negative.

Nature of lens change *i.e.,* convex behave as concave.

48. A green light is incident from the water to the air-water interface at the critical angle (θ). Select the correct statement.

[JEE Main 2014]

(a) The entire spectrum of visible light will come out of the water at an angle of 90° to the normal

(b) The spectrum of visible light whose frequency is less than that of green light will come out of the air medium

(c) The spectrum of visible light whose frequency is more than that of green light will come out to the air medium

(d) The entire spectrum of visible light will come out of the water at various angles to the normal

For total internal reflection of light take place, then following conditions must be obeyed.

(i) The ray must travel from denser to rarer medium.

(ii) Angle of incidence (θ) must be greater than or equal to critical angle (C) i.e.,

$$C = \sin^{-1}\left[\frac{\mu_{\text{rarer}}}{\mu_{\text{denser}}}\right]$$

Exp. **(b)**

Here, $\sin C = \frac{1}{n_{\text{water}}}$

and $n_{\text{water}} = a + \frac{b}{\lambda^2}$

If frequency is less, then λ is greater and hence RI $n_{(\text{water})}$ is less and therefore critical angle increases. So, they do not suffer reflection and come out at angle less than 90° of the air medium.

49. Two beams, *A* and *B*, of plane polarised light with mutually perpendicular planes of polarisation are seen through a polaroid. From the position, when the beam *A* has maximum intensity (and beam *B* has zero intensity), a rotation of polaroid through 30° makes the two beams appear equally bright. If the initial intensities of the two beams are I_A and I_B respectively, then I_A / I_B equals

[JEE Main 2014]

(a) 3 (b) 3/2

(c) 1 (d) 1/3

Exp. **(d)**

By law of Malus *i.e.,* $I = I_0 \cos^2 \theta$

Now, $I_{A'} = I_A \cos^2 30°$

$I_{B'} = I_B \cos^2 60°$

As, $I'_A = I'_B$

$I_A \cos^2 30° = I_B \cos^2 60°$

$$\Rightarrow \quad I_A \frac{3}{4} = I_B \frac{1}{4} \Rightarrow \frac{I_A}{I_B} = \frac{1}{3}$$

50. A beam of unpolarised light of intensity I_0 is passed through a polaroid A and then through another polaroid B which is oriented so that its principal plane makes an angle of 45° relative to that of A. The intensity of the emergent light is **[JEE Main 2013]**

(a) I_0 (b) $\frac{I_0}{2}$

(c) $\frac{I_0}{4}$ (d) $\frac{I_0}{8}$

Exp. **(c)**

Relation between intensities is

$$I_R = \left(\frac{I_0}{2}\right)\cos^2(45°) = \frac{I_0}{2} \times \frac{1}{2} = \frac{I_0}{4}$$

51. Two coherent point sources S_1 and S_2 are separated by a small distance d as shown. The fringes obtained on the screen will be **[JEE Main 2013]**

(a) points (b) straight lines

(c) semi-circle (d) concentric circles

Exp. **(d)**

For bright fringe,

$S_1P - S_2P = n\lambda$

So, fringes are concentric circles (centre of origin).

[∵ due to point source, wave fronts will be spherical]

52. Diameter of a plano-convex lens is 6 cm and thickness at the centre is 3 mm. If speed of light in material of lens is 2×10^8 m/s, the focal length of the lens is **[JEE Main 2013]**

(a) 15 cm (b) 20 cm

(c) 30 cm (d) 10 cm

Exp. **(c)**

$$n = \frac{3}{2} \qquad \left[\because n = \frac{c}{v}\right]$$

$$3^2 + (R - 3\text{ mm})^2 = R^2$$

$$\Rightarrow 3^2 + R^2 - 2R(3\text{ mm}) + (3\text{ mm})^2 = R^2$$

$$\Rightarrow \quad R \approx 15\text{ cm}$$

$$\therefore \quad \frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right)$$

$$\left[\because \frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ and } R_1 = \infty, R_2 = -15\right]$$

$$\Rightarrow \quad f = 30\text{ cm}$$

53. The graph between angle of deviation (δ) and angle of incidence (i) for a triangular prism is represented by **[JEE Main 2013]**

Exp. **(c)**

We know that the angle of deviation depends upon the angle of incidence. If we determine experimentally, the angles of deviation corresponding to different angles of incidence and then plot i (on x-axis) and δ (on y-axis), we get a curve as shown in figure.

Clearly, if angle of incidence is gradually increased from a small value, the angle of deviation first decreases, becomes minimum for a particular angle of incidence and then begins to increase.

54. In Young's double slit experiment, one of the slit is wider than other, so that amplitude of the light from one slit is double of that from other slit. If I_m is the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ, is given by **[AIEEE 2012]**

(a) $\frac{I_m}{9}(4+5\cos\phi)$

(b) $\frac{I_m}{3}\left(1+2\cos^2\frac{\phi}{2}\right)$

(c) $\frac{I_m}{5}\left(1+4\cos^2\frac{\phi}{2}\right)$

(d) $\frac{I_m}{9}\left(1+8\cos^2\frac{\phi}{2}\right)$

Exp. **(*d*)**

Given, $a_1 = 2a_2$

$\Rightarrow \quad I_1 = 4I_2 = 4I_0 \qquad [\because I \propto a^2]$

$\therefore \quad I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$= (3\sqrt{I_2})^2 = 9I_2 = 9I_0$

Now, $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$= 4I_0 + I_0 + 2\sqrt{4I_0 \cdot I_0}\cos\phi$

$= 5I_0 + 4I_0\cos\phi$

$= \frac{I_m}{9}(5+4\cos\phi) \qquad \left[\because I_0 = \frac{I_m}{9}\right]$

$= \frac{I_m}{9}[1+4(1+\cos\phi)]$

$= \frac{I_m}{9}(1+8\cos^2\phi/2)$

$\left[\because \cos\phi = 2\cos^2\frac{\phi}{2} - 1\right]$

55. An object 2.4 m infront of a lens forms a sharp image on a film 12 cm behind the lens. A glass plate 1 cm thick, of refractive index 1.50 is interposed between lens and film with its plane faces parallel to film. At what distance (from lens) should object shifted to be in sharp focus on film? **[AIEEE 2012]**

(a) 7.2 m (b) 2.4 m (c) 3.2 m (d) 5.6 m

Exp. **(*d*)**

Shift in image position due to glass plate,

$$S = \left(1-\frac{1}{\mu}\right)t = \left(1-\frac{1}{1.5}\right)\times 1\,\text{cm} = \frac{1}{3}\,\text{cm}$$

For focal length of the lens,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{1}{12} - \frac{1}{-240}$$

or $\quad \frac{1}{f} = \frac{20+1}{240}$

$\Rightarrow \quad f = \frac{240}{21}$ cm

Now, to get back image on the film, lens has to form image at $\left(12-\frac{1}{3}\right)$ cm $= \frac{35}{3}$ cm such that the glass plate will shift the image on the film.

As $\quad \frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{3}{35} - \frac{21}{240}$

$= \frac{48\times 3 - 7\times 21}{1680}$

$= -\frac{1}{560}$

$\Rightarrow \quad u = -5.6$ m

56. At two points P and Q on screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$, respectively. The ratio of intensities at P and Q will be **[AIEEE 2011]**

(a) 3 : 2 (b) 2 : 1

(c) $\sqrt{2}$: 1 (d) 4 : 1

Exp. **(*b*)**

Let I_0 be intensity of light emitted from the source, then

Resultant intensity

$$I = 4I_0\cos^2\phi/2$$

$\therefore \quad I_1 = 4I_0\cos^2 0/2 = 4I_0 \qquad [\because \phi = 0]$

Now, $\Delta x = \lambda/4$

$\therefore \quad \phi = \frac{2\pi}{\lambda}\times\Delta x = \frac{2\pi}{\lambda}\times\frac{\lambda}{4}$

$\Rightarrow \quad \phi = \frac{\pi}{2}$

and $\quad I_2 = 4I_0\cos^2\frac{\pi}{4} = 2I_0 \qquad \left[\because \phi = \frac{2\pi}{\lambda}\Delta x\right]$

$\therefore \quad I_1 : I_2 = 2 : 1$

57. A beaker contains water upto a height h_1 and kerosene of height h_2 above water so that the total height of (water + kerosene) is $(h_1 + h_2)$. Refractive index of water is μ_1 and that of kerosene is μ_2. The apparent shift in the position of the bottom of the beaker when viewed from above is **[AIEEE 2011]**

(a) $\left(1-\frac{1}{\mu_1}\right)h_2+\left(1-\frac{1}{\mu_2}\right)h_1$

(b) $\left(1+\frac{1}{\mu_1}\right)h_1+\left(1+\frac{1}{\mu_2}\right)h_2$

(c) $\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$

(d) $\left(1+\frac{1}{\mu_1}\right)h_2-\left(1+\frac{1}{\mu_2}\right)h_1$

Exp. **(c)**

We know, apparent shift, $\Delta h=\left(1-\frac{1}{\mu}\right)h$

$\therefore$ Apparent shift produced by water,

$$\Delta h_1=\left(1-\frac{1}{\mu_1}\right)h_1$$

and apparent shift produced by kerosene,

$$\Delta h_2=\left(1-\frac{1}{\mu_2}\right)h_2$$

$$\therefore\quad \Delta h=\Delta h_1+\Delta h_2=\left(1-\frac{1}{\mu_1}\right)h_1+\left(1-\frac{1}{\mu_2}\right)h_2$$

58. When monochromatic red light is used instead of blue light in a convex lens, its focal length will **[AIEEE 2011]**

(a) not depend on colour of light
(b) increase
(c) decrease
(d) remain same

Exp. **(b)**

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Also, by Cauchy's formula,

$$\mu=A+\frac{B}{\lambda^2}+\frac{C}{\lambda^4}+\ldots\quad\Rightarrow\quad \mu\propto\frac{1}{\lambda}$$

As $\quad \lambda_{blue}<\lambda_{red}\Rightarrow\mu_{blue}>\mu_{red}$

Hence, $\quad f_{red}>f_{blue}$

59. **Statement I** On viewing the clear, blue portion of the sky through a Calcite crystal, the intensity of transmitted light varies as the crystal is rotated.

Statement II The light coming from the sky is polarised due to scattering of sun light by particles in the atmosphere. The scattering is largest for blue light. **[AIEEE 2011]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Exp. **(c)**

Scattering for blue light is largest and it is polarised by scattering.

Also, for polarised light,

$$I=I_0\cos^2\theta$$

60. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement, the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case is I_2, then the ratio $\frac{I_1}{I_2}$ is **[AIEEE 2011]**

(a) 4 (b) 2 (c) 1 (d) 0.5

Exp. **(b)**

For coherent sources, $\quad I_1=4I_0\cos^2\frac{\phi}{2}=4I_0$

For incoherent sources, $\quad I_2=I_0+I_0=2I_0$

$$\therefore\quad \frac{I_1}{I_2}=2$$

61. Let the zx-plane be the boundary between two transparent media. Medium 1 in $z\geq 0$ has a refractive index of $\sqrt{2}$ and medium 2 with $z<0$ has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector

$\mathbf{A} = 6\sqrt{3}\,\mathbf{i} + 8\sqrt{3}\,\mathbf{j} - 10\,\mathbf{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is **[AIEEE 2011]**

(a) 45° (b) 60° (c) 75° (d) 30°

Exp. (*a*)

As refractive index for $z > 0$ and $z \le 0$ is different, *xy*-plane should be boundary between two media.

Angle of incidence,

$$\cos i = \left| \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} \right| = \frac{1}{2}$$

$\therefore \quad i = 60°$

From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\sqrt{3}}{\sqrt{2}}$

$$\sin r = \frac{\sqrt{2}}{\sqrt{3}} \times \sin 60° = \frac{\sqrt{2}}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{2}} = 45°$$

$\Rightarrow \quad r = 45°$

62. This question has a paragraph followed by two statements, Statement I and Statement II. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film. **[AIEEE 2011]**

Statement I When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement II The centre of the interference pattern is dark.

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

Exp. (*b*)

Both statements are correct but Statement II does not explain Statement I.

63. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.8 m behind the first car is overtaking the first car at a relative speed of 15m/s. The speed of the image of the second car as seen in the mirror of the first one is **[AIEEE 2011]**

(a) $\frac{1}{15}$ m/s (b) 10 m/s (c) 15 m/s (d) $\frac{1}{10}$ m/s

Exp. (*a*)

For the mirror, $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Differentiate this equation w.r.t. *t*, we get

$$-\frac{1}{u^2}\frac{du}{dt} - \frac{1}{v^2}\frac{dv}{dt} = 0$$

$$\Rightarrow \quad \frac{dv}{dt} = -\frac{v^2}{u^2}\left(\frac{du}{dt}\right)$$

But $\quad \frac{v}{u} = \frac{f}{u - f}$

$$\therefore \quad \frac{dv}{dt} = -\left(\frac{f}{u - f}\right)^2 \left(\frac{du}{dt}\right)$$

$$= \left(\frac{0.2}{-2.8 - 0.2}\right)^2 \times 15 = \frac{1}{15}\,\text{ms}^{-1}$$

64. As the beam enters the medium, it will **[AIEEE 2010]**

(a) diverge
(b) converge
(c) diverge near the axis and converge near the periphery
(d) travel as a cylindrical beam

Exp. (*b*)

As intensity is maximum at axis.

Therefore, μ will be maximum and speed will be minimum on the axis of the beam.

Hence, beam will converge.

65. The initial shape of the wavefront of the beam is **[AIEEE 2010]**

(a) convex
(b) concave
(c) convex near the axis and concave near the periphery
(d) planar

Exp. (*d*)

For a parallel cylindrical beam, wavefront will be planar.

66. The speed of light in the medium is **[AIEEE 2010]**

(a) minimum on the axis of the beam
(b) the same everywhere in the beam
(c) directly proportional to the intensity I
(d) maximum on the axis of the beam

Exp. **(c)**

The speed of light in the medium is directly proportional to the intensity I.

67. In an optics experiment, with the position of the object fixed, a student varies the position of a convex lens and for each position, the screen is adjusted to get a clear image of the object. A graph between the object distance u and the image distance v, from the lens, is plotted using the same scale for the two axes. A straight line passing through the origin and making an angle of 45° with the x-axis meets the experimental curve at P. The coordinates of P will be **[AIEEE 2009]**

(a) $(2f, 2f)$ (b) $\left(\frac{f}{2}, \frac{f}{2}\right)$ (c) (f, f) (d) $(4f, 4f)$

Exp. **(a)**

It is possible when object kept at centre of curvature.

For $u = v$

$u = 2f, v = 2f$

68. A mixture of light, consisting of wavelength 590 nm and an unknown wavelength, illuminates Young's double slit and gives rise to two overlapping interference patterns on the screen. The central maximum of both lights coincide. Further, it is observed that the third bright fringe of known light coincides with the 4th bright fringe of the unknown light. From this data, the wavelength of the unknown light is **[AIEEE 2009]**

(a) 393.4 nm (b) 885.0 nm
(c) 442.5 nm (d) 776.8 nm

Exp. **(c)**

$$3\lambda_1 = 4\lambda_2 \qquad \left[\because \frac{3D\lambda_1}{d} = 4\frac{D\lambda_2}{d}\right]$$

$$\Rightarrow \quad \lambda_2 = \frac{3}{4}\lambda_1 = \frac{3}{4} \times 590$$

$$= \frac{1770}{4} = 442.5 \text{ nm}$$

69. A transparent solid cylinder rod has a refractive index of $\frac{2}{\sqrt{3}}$. It is surrounded by air. A light ray is incident at the mid-point of one end of the rod as shown in the figure.

The incident angle θ for which the light ray grazes along the wall of the rod is **[AIEEE 2009]**

(a) $\sin^{-1}\left(\frac{1}{2}\right)$

(b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{2}{\sqrt{3}}\right)$

(d) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

Exp. **(d)**

$$\sin C = \frac{\sqrt{3}}{2} \qquad \ldots(i)$$

$$\sin r = \sin(90° - C) = \cos C = \frac{1}{2}$$

$$\frac{\sin\theta}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\sin\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$$

70. A student measures the focal length of a convex lens by putting an object pin at a distance u from the lens and measuring the distance v of the image pin. The graph between u and v plotted by the student should look like **[AIEEE 2008]**

(a)

(b)

(c)

(d)

Exp. **(*c*)**

Experimental observation at $v = \infty, u = -f$

At $u = \infty, v = f$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} = \text{constant}$$

So, (c) is the correct graph.

71. In a Young's double slit experiment, the intensity at a point where the path difference is $\frac{\lambda}{6}$ (λ being the wavelength of the light used) is I. If I_0 denotes the maximum intensity, then I/I_0 is equal to **[AIEEE 2007]**

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{2}$ (d) $\frac{3}{4}$

Exp. **(*d*)**

Phase difference $= \frac{2\pi}{\lambda} \times$ Path difference

i.e., $\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{6} = \frac{\pi}{3}$

As, $I = I_{max} \cos^2\left(\frac{\phi}{2}\right)$

or $\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right)$

or $\frac{I}{I_0} = \cos^2\left(\frac{\pi}{6}\right) = \frac{3}{4}$ $[\because I_{max} = I_0]$

72. Two lenses of power –15D and +5D are in contact with each other. The focal length of the combination is **[AIEEE 2007]**

(a) –20 cm (b) –10 cm (c) +20 cm (d) +10 cm

Exp. **(*b*)**

Power of a lens is reciprocal of its focal length. Power of combined lens is

$$P = P_1 + P_2$$
$$= -15 + 5$$
$$= -10\,\text{D}$$

$\therefore$ $f = \frac{1}{P} = \frac{100}{-10}$ cm or $f = -10$ cm

73. The refractive index of glass is 1.520 for red light and 1.525 for blue light. Let D_1 and D_2 be angles of minimum deviations for red and blue light respectively in a prism of this glass. Then, **[AIEEE 2006]**

(a) $D_1 < D_2$ (b) $D_1 = D_2$

(c) can be less than or greater than D_2 depending upon the angle of prism

(d) $D_1 > D_2$

Exp. **(*a*)**

$$D = (\mu - 1)\,A$$

For blue light, μ is greater than that for red light, so $D_2 > D_1$.

Aliter

$$D_1 = (1.520 - 1)A$$
$$D_2 = (1.525 - 1)A$$
$$\Rightarrow \quad \frac{D_1}{D_2} < 1$$
$$D_2 > D_1$$

74. A Young's double slit experiment uses a monochromatic source. The shape of the interference fringes formed on a screen is **[AIEEE 2005]**

(a) hyperbola (b) circle

(c) straight line (d) parabola

Exp. **(*d*)**

The shape of the interference fringes formed on a screen is parabola.

75. A fish looking up through the water sees the outside world, contained in a circular horizon. If the refractive index of water is 4/3 and the fish is 12 cm below the water surface, the radius of this circle (in cm) is **[AIEEE 2005]**

(a) $36\sqrt{7}$ (b) $\frac{36}{\sqrt{7}}$ (c) $36\sqrt{5}$ (d) $4\sqrt{5}$

Exp. **(*b*)**

The situation is shown in figure.

$$\sin\theta_C = \frac{1}{\mu} \Rightarrow \tan\theta_C = \frac{AB}{OA}$$

$$\therefore\ AB = OA\tan\theta_C = OA\frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} \times OA$$

$$= \frac{1/\mu}{\sqrt{1-1/\mu^2}} \times OA = \frac{OA}{\sqrt{\mu^2-1}}$$

or $$AB = \frac{OA}{\sqrt{\mu^2-1}} = \frac{12}{\sqrt{\left(\frac{4}{3}\right)^2-1}} = \frac{36}{\sqrt{7}}$$

76. Two point white dots are 1 mm apart on a black paper. They are viewed by eye of pupil diameter 3 mm. Approximately, what is the maximum distance at which these dots can be resolved by the eye? [Take wavelength of light = 500 nm] **[AIEEE 2005]**

(a) 5 m (b) 1 m (c) 6 m (d) 3 m

Exp. **(*a*)**

We know $\frac{y}{D} \geq 1.22\frac{\lambda}{d} \Rightarrow D \leq \frac{yd}{1.22\lambda}$

$$\therefore\ D = \frac{10^{-3}\times 3\times 10^{-3}}{1.22\times 5\times 10^{-7}} = 5\text{ m} \Rightarrow D_{max} = 5\text{ m}$$

77. A thin glass (refractive index 1.5) lens has optical power of –5 D in air. Its optical power in a liquid medium with refractive index 1.6 will be **[AIEEE 2005]**

(a) 1 D (b) –1 D (c) 25 D (d) –25 D

Exp. **(*a*)**

$$\frac{1}{f_a} = (\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots\text{(i)}$$

$$= (1.5-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

and $$\frac{1}{f_m} = \left(\frac{\mu_g-\mu_m}{\mu_m}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \ldots\text{(ii)}$$

$$\frac{1}{f_m} = \left(\frac{1.5}{1.6}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Thus, $$\frac{f_m}{f_a} = \frac{(1.5-1)}{\left(\frac{1.5}{1.6}-1\right)} = -8$$

or $$f_m = -8\times f_a = -8\times\frac{-1}{5} = 1.6\text{ m}$$

$$\left[\because\ f_a = \frac{1}{P} = -\frac{1}{5}\text{m}\right]$$

$$\therefore\quad P_m = \frac{\mu}{f_m} = \frac{1.6}{1.6} = 1\text{D}$$

78. When an unpolarised light of intensity I_0 is incident on a polarising sheet, the intensity of the light which does not get transmitted is **[AIEEE 2005]**

(a) $\frac{1}{2}I_0$ (b) $\frac{1}{4}I_0$ (c) zero (d) I_0

Exp. **(*a*)**

$$I = I_0\cos^2\theta$$

Intensity of polarised light = $I_0/2$

$\therefore$ Intensity of untransmitted light = $I_0 - \frac{I_0}{2} = \frac{I_0}{2}$

79. A light ray is incident perpendicular to one face of a 90° prism and is totally internally reflected at the glass-air interface. If the angle of reflection is 45°, we conclude that for the refractive index n as **[AIEEE 2004]**

(a) $n < \frac{1}{\sqrt{2}}$ (b) $n > \sqrt{2}$

(c) $n > \frac{1}{\sqrt{2}}$ (d) $n < \sqrt{2}$

Exp. **(*b*)**

For total internal reflection from glass-air interface, critical angle C must be less than angle of incidence.

i.e., $C < i$

or $C < 45°$ $[\because \angle i = 45°]$

But $n = \dfrac{1}{\sin C} \Rightarrow C = \sin^{-1}\left(\dfrac{1}{n}\right)$

or $\sin^{-1}\left(\dfrac{1}{n}\right) < 45°$ or $\dfrac{1}{n} < \sin 45°$

or $n > \dfrac{1}{\sin 45°}$ or $n > \dfrac{1}{(1/\sqrt{2})}$ or $n > \sqrt{2}$

80. A plano-convex lens of refractive index 1.5 and radius of curvature 30 cm is silvered at the curved surface. Now, this lens has been used to form the image of an object. At what distance from this lens, an object be placed in order to have a real image of the size of the object **[AIEEE 2004]**

(a) 20 cm (b) 30 cm

(c) 60 cm (d) 80 cm

Exp. **(*a*)**

A plano-convex lens behaves as a concave mirror, if its one surface (curved) is silvered. The rays refracted from plane surface are reflected from curved surface and again refract from plane surface. Therefore, in this lens two refractions and one reflection occur.

Let the focal length of silvered lens be F.

$$\frac{1}{F} = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_m}$$

$$= \frac{2}{f} + \frac{1}{f_m}$$

where, f = focal length of lens before silvering,

f_m = focal length of spherical mirror.

$$\frac{1}{F} = \frac{2}{f} + \frac{2}{R} \qquad [\because R = 2f_m] \ldots \text{(i)}$$

Now, $$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \ldots \text{(ii)}$$

Here, $R_1 = \infty, R_2 = 30$ cm

$$\therefore \quad \frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{30}\right)$$

or $$\frac{1}{f} = -\frac{0.5}{30} = -\frac{1}{60}$$

or $f = -60$ cm

Hence, from Eq. (i), we get

$$\frac{1}{F} = \frac{2}{60} + \frac{2}{30} = \frac{6}{60}$$

$$F = 10 \text{ cm}$$

Again given that,

Size of object = Size of image

i.e., $O = I$

$$\therefore \quad m = -\frac{v}{u} = \frac{I}{O}$$

$$\Rightarrow \quad \frac{v}{u} = -1$$

or $v = -u$

Thus, from lens formula,

$$\frac{1}{F} = \frac{1}{v} - \frac{1}{u}$$

$$\Rightarrow \quad \frac{1}{10} = \frac{1}{-u} - \frac{1}{u}$$

$$\frac{1}{10} = -\frac{2}{u}$$

$$\therefore \quad u = -20 \text{ cm}$$

Hence, to get a real image, object must be placed at a distance 20 cm on the left side of lens.

81. The angle of incidence at which reflected light is totally polarised for reflection from air to glass (refractive index n), is **[AIEEE 2004]**

(a) $\sin^{-1}(n)$

(b) $\sin^{-1}\left(\dfrac{1}{n}\right)$

(c) $\tan^{-1}\left(\dfrac{1}{n}\right)$

(d) $\tan^{-1}(n)$

Exp. **(*d*)**

The particular angle of incidence for which reflected light is totally polarised for reflection from air to glass, is called the angle of polarisation (i_p) or **Brewster's law.**

Accordingly, $n = \tan i_p$

$\Rightarrow \quad i_p = \tan^{-1}(n)$

where, n is refractive index of glass.

82. The maximum number of possible interference maxima for slit-separation equal to twice the wavelength in Young's double-slit experiment, is **[AIEEE 2004]**

(a) infinite (b) five
(c) three (d) zero

Exp. **(*b*)**

For possible interference maxima on the screen, the condition is

$$d \sin\theta = n\lambda \quad \ldots(i)$$

Given, d = slit-width = 2λ

$\therefore \quad 2\lambda \sin\theta = n\lambda$

or $\quad 2\sin\theta = n$

The maximum value of $\sin\theta$ is 1, hence

$$n = 2 \times 1 = 2$$

Thus, Eq. (i) must be satisfied by 5 integer values *i.e.*, –2, –1, 0, 1, 2. Hence, the maximum number of possible interference maxima is 5.

83. The image formed by an objective of a compound microscope is **[AIEEE 2003]**

(a) virtual and diminished
(b) real and diminished
(c) real and enlarged
(d) virtual and enlarged

Exp. **(*c*)**

Objective of compound microscope is a convex lens. Convex lens form real and enlarged image when an object is placed between its focus and lens.

84. The earth radiates in the infra-red region of the spectrum. The spectrum is correctly given by **[AIEEE 2003]**

(a) Rayleigh Jeans law
(b) Planck's law of radiation
(c) Stefan's law of radiation
(d) Wien's law

Exp. **(*a*)**

The spectrum is correctly given by Rayleigh Jeans law

85. To get three images of a single object, one should have two plane mirrors at an angle of **[AIEEE 2003]**

(a) 60° (b) 90°
(c) 120° (d) 30°

Exp. **(*b*)**

Number of images, $n = \dfrac{360°}{\theta} - 1$

where, θ is angle between mirrors.

$\therefore \quad 3 = \dfrac{360°}{\theta} - 1$ or $\theta = 90°$

86. To demonstrate the phenomenon of interference, we require two sources which emit radiations of **[AIEEE 2003]**

(a) nearly the same frequency
(b) the same frequency
(c) different wavelength
(d) the same frequency and having a definite phase relationship

Exp. **(*d*)**

Sustained interference is possible with coherent sources only *i.e.,* sources having same frequency and constant or zero phase difference.

87. If two mirrors are kept at 60° to each other, then the number of images formed by them is **[AIEEE 2002]**

(a) 5 (b) 6
(c) 7 (d) 8

Exp. **(*a*)**

Number of images, $n = \dfrac{360°}{\theta} - 1$

where, θ is angle between mirrors.

Thus, $\theta = 60°$ [given]

So, number of images, $n = \dfrac{360°}{60°} - 1 = 5$

88. Wavelength of light used in an optical instrument are $\lambda_1 = 4000$ Å and $\lambda_2 = 5000$ Å, then ratio of their respective resolving powers (corresponding to λ_1 and λ_2) is **[AIEEE 2002]**

(a) 16 : 25 (b) 9 : 1
(c) 4 : 5 (d) 5 : 4

Exp. (*d*)

Resolving power of an optical instrument is inversely proportional to λ *i.e.*,

$$RP \propto \frac{1}{\lambda}$$

$$\therefore \frac{\text{Resolving power at } \lambda_1}{\text{Resolving power at } \lambda_2} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{4000} = 5 : 4$$

89. An astronomical telescope has a large aperture to **[AIEEE 2002]**

(a) reduce spherical aberration
(b) have high resolution
(c) increase span of observation
(d) have low dispersion

Exp. (*b*)

Resolving power of telescope, $RP = \frac{d}{1.22\lambda}$

90. Which of the following is used in optical fibres ? **[AIEEE 2002]**

(a) Total internal reflection
(b) Scattering
(c) Diffraction
(d) Refraction

Exp. (*a*)

Optical fibres work on the principle of total internal reflection.

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17

Dual Nature *of* Radiation

1. Two particles move at right angle to each other. Their de-Broglie wavelengths are λ_1 and λ_2, respectively. The particles suffer perfectly inelastic collision. The de-Broglie wavelength λ of the final particle, is given by **[JEE Main 2019, 8 April Shift-I]**

(a) $\frac{1}{\lambda^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$ (b) $\lambda = \sqrt{\lambda_1 \lambda_2}$

(c) $\lambda = \frac{\lambda_1 + \lambda_2}{2}$ (d) $\frac{2}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$

Exp. **(a)**

Given, de-Broglie wavelengths for particles are λ_1 and λ_2.

So, $\lambda_1 = \frac{h}{p_1}$ and $\lambda_2 = \frac{h}{p_2}$

and momentum of particles are

$$p_1 = \frac{h}{\lambda_1} \text{ and } p_2 = \frac{h}{\lambda_2}$$

Given that, particles are moving perpendicular to each other and collide inelastically.

So, they move as a single particle.

So, by conservation of momentum and vector addition law, net momentum after collision,

$$p_{net} = \sqrt{p_1^2 + p_2^2 + 2p_1p_2\cos 90°} = \sqrt{p_1^2 + p_2^2}$$

Since, $p_1 = \frac{h}{\lambda_1}$ and $p_2 = \frac{h}{\lambda_2}$

So, $$p_{net} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \quad \ldots(i)$$

Let the de-Broglie wavelength after the collision is λ_{net}, then

$$p_{net} = \frac{h}{\lambda_{net}} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{h}{\lambda_{net}} = \sqrt{\frac{h^2}{\lambda_1^2} + \frac{h^2}{\lambda_2^2}} \Rightarrow \frac{1}{\lambda_{net}^2} = \frac{1}{\lambda_1^2} + \frac{1}{\lambda_2^2}$$

2. The electric field of light wave is given as $\mathbf{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}\ \text{NC}^{-1}$. This light falls on a metal plate of work function 2eV. The stopping potential of the photoelectrons is **[JEE Main 2019, 9 April Shift-I]**

Given, E (in eV) $= \frac{12375}{\lambda(\text{in Å})}$

(a) 0.48 V (b) 0.72 V

(c) 2.0 V (d) 2.48 V

Exp. **(a)**

Given,

$$\mathbf{E} = 10^{-3}\cos\left(\frac{2\pi x}{5\times10^{-7}} - 2\pi\times6\times10^{14}t\right)\hat{\mathbf{x}}\,\text{NC}^{-1}$$

By comparing it with the general equation of electric field of light, i.e.

$$E = E_0\cos(kx - \omega t)\,\hat{\mathbf{x}}, \text{ we get}$$

$$k = \frac{2\pi}{5\times10^{-7}} = 2\pi/\lambda$$

(from definition, $k = 2\pi/\lambda$)

$$\Rightarrow \quad \lambda = 5\times10^{-7}\text{ m} = 5000\,\text{Å} \quad \ldots\text{(i)}$$

Or

The value of λ can also be calculated as, after comparing the given equation of **E** with standard equation, we get

$$\omega = 6\times10^{14}\times2\pi$$

$$\Rightarrow \quad \nu = 6\times10^{14} \quad [\because 2\pi\nu = \omega]$$

As, $c = \nu\lambda$

$$\Rightarrow \quad \lambda = \frac{c}{\nu} = \frac{3\times10^{8}}{6\times10^{14}} = 5\times10^{-7}\text{ m} = 5000\,\text{Å}$$

According to Einstein's equation for photoelectric effect, i.e.

$$\frac{hc}{\lambda} - \phi = (\text{KE})_{max} = eV_0 \quad \ldots\text{(ii)}$$

For photon, substituting the given values,

$$E = \frac{hc}{\lambda} = \frac{12375\text{ eV}}{\lambda} \quad \text{[given]}$$

or $$\frac{hc}{\lambda} = \frac{12375}{5000}\text{ eV} \quad \text{[using Eq. (i)]} \ldots\text{(iii)}$$

Now, substituting the values from Eq. (iii) in Eq. (ii), we get

$$\frac{12375}{5000}\text{eV} - 2\text{eV} = eV_0$$

$$\Rightarrow \quad 2.475\text{ eV} - 2\text{eV} = eV_0$$

or $$V_0 = 2.475\text{V} - 2\text{V} = 0.475\text{ V} \Rightarrow V_0 \approx 0.48\text{ V}$$

3. A particle P is formed due to a completely inelastic collision of particles x and y having de-Broglie wavelengths λ_x and λ_y, respectively. If x and y were moving in opposite directions, then the de-Broglie wavelength of P is

[JEE Main 2019, 9 April Shift-II]

(a) $\lambda_x - \lambda_y$ (b) $\dfrac{\lambda_x\lambda_y}{\lambda_x - \lambda_y}$

(c) $\dfrac{\lambda_x\lambda_y}{\lambda_x + \lambda_y}$ (d) $\lambda_x + \lambda_y$

Exp. **(b)**

Initially,

We have, de-Broglie wavelengths associated with particles are

$$\lambda_x = \frac{h}{p_x} \text{ and } \lambda_y = \frac{h}{p_y}$$

$$\Rightarrow \quad p_x = \frac{h}{\lambda_x} \text{ and } p_y = \frac{h}{\lambda_y}$$

Finally, particles collided to form a single particle.

As we know that, linear momentum is conserved in collision.

So, $\mathbf{p}_p = |\mathbf{p}_x - \mathbf{p}_y| \Rightarrow \mathbf{p}_p = \left|\dfrac{h}{\lambda_x} - \dfrac{h}{\lambda_y}\right|$

So, de-Broglie wavelength of combined particle is

$$\lambda_p = \frac{h}{|\mathbf{p}_p|} = \frac{h}{\left|\dfrac{h}{\lambda_x} - \dfrac{h}{\lambda_y}\right|} = \frac{h}{\left|\dfrac{h\lambda_y - h\lambda_x}{\lambda_x\lambda_y}\right|}$$

$$= \frac{\lambda_x\lambda_y}{|\lambda_x - \lambda_y|}$$

4. 50 W/m^2 energy density of sunlight is normally incident on the surface of a solar panel. Some part of incident energy (25%) is reflected from the surface and the rest is absorbed. The force exerted on 1 m^2 surface area will be close to ($c = 3\times10^8$ m/s)

[JEE Main 2019, 9 April Shift-II]

(a) 20×10^{-8} N (b) 35×10^{-8} N

(c) 15×10^{-8} N (d) 10×10^{-8} N

Exp. **(a)**

Radiation pressure or momentum imparted per second per unit area when light falls is

$$p = \begin{cases} \dfrac{2I}{c} & ; \text{ for reflection of radiation} \\ \dfrac{I}{c} & ; \text{ for absorption of radiation} \end{cases}$$

where, I is the intensity of the light.

In given case, there is 25% reflection and 75% absorption, so radiation pressure = force per unit area

$$= \frac{25}{100} \times \frac{2I}{c} + \frac{75}{100} \times \frac{I}{c}$$

$$= \frac{1}{2} \times \frac{I}{c} + \frac{3}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{I}{c} = \frac{5}{4} \times \frac{50}{3 \times 10^8}$$

$$= 20.83 \times 10^{-8} \text{N} \approx 20 \times 10^{-8} \text{ N}$$

5. In a photoelectric effect experiment, the threshold wavelength of light is 380 nm. If the wavelength of incident light is 260 nm, the maximum kinetic energy of emitted electrons will be

Given, E (in eV) $= \dfrac{1237}{\lambda(\text{in nm})}$

[JEE Main 2019, 10 April Shift-I]

(a) 15.1 eV (b) 3.0 eV (c) 1.5 eV (d) 4.5 eV

Exp. **(c)**

Given, threshold wavelength, $\lambda_0 = 380$ nm

Wavelength of incident light, $\lambda = 260$ nm

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{max} = eV_0 = h\nu - h\nu_0 \quad \ldots(\text{i})$$

But $\quad h\nu = E = \dfrac{1237}{\lambda(\text{nm})}$ ev (Given)

$$\therefore \quad E_0 = \frac{1237}{\lambda_0(\text{nm})} \text{ eV} \quad \ldots(\text{ii})$$

From Eqs. (i) and (ii), we get

$$(\text{KE})_{max} = E - E_0 \left(\frac{1237}{\lambda} - \frac{1237}{\lambda_0}\right) \text{eV}$$

$$= 1237 \left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right] \text{eV } (\lambda \text{ in nm}) \quad \ldots(\text{iii})$$

By putting values of λ and λ_0 in Eq. (iii), we get

$$(\text{KE})_{max} = 1237 \left(\frac{1}{260} - \frac{1}{380}\right) \text{eV}$$

$$= 1237 \times \left[\frac{380 - 260}{380 \times 260}\right] \text{eV}$$

$$\Rightarrow \quad (\text{KE})_{max} = 1.5 \text{ eV}$$

6. Light is incident normally on a completely absorbing surface with an energy flux of 25 W cm^{-2}. If the surface has an area of 25 cm^2, the momentum transferred to the surface in 40 min time duration will be

[JEE Main 2019, 10 April Shift-II]

(a) 3.5×10^{-6} N-s (b) 6.3×10^{-4} N-s

(c) 1.4×10^{-6} N-s (d) 5.0×10^{-3} N-s

Exp. **(d)**

Radiation pressure over an absorbing surface is,

$$p = \frac{I}{c}$$

where, I = intensity or energy flux

and c = speed of light.

If A = area of surface, then force due to radiation on the surface is

$$F = p \times A = \frac{IA}{c}$$

If force F acts for a duration of Δt seconds, then momentum transferred to the surface is

$$\Delta p = F \times \Delta t = \frac{IA}{c} \times \Delta t$$

Here, $I = 25 \text{ W cm}^{-2}$, $A = 25 \text{ cm}^2$,

$c = 3 \times 10^8 \text{ ms}^{-1}$,

$\Delta t = 40 \text{ min} = 2400 \text{ s}$

So, momentum transferred to the surface,

$$\Delta p = \frac{25 \times 25 \times 2400}{3 \times 10^8}$$

$$= 5 \times 10^{-3} \text{ N-s}$$

7. A 2 mW laser operates at a wavelength of 500 nm. The number of photons that will be emitted per second is

[Given, Planck's constant $h = 6.6 \times 10^{-34}$ Js, speed of light $c = 3.0 \times 10^8$ m/s]

[JEE Main 2019, 10 April Shift-2]

(a) 1×10^{16} (b) 5×10^{15} (c) 1.5×10^{16} (d) 2×10^{16}

Exp. **(b)**

Power of laser is given as, $P = \dfrac{\text{Energy}}{\text{Time}}$

$$= \frac{\text{Number of photons emitted} \times \text{Energy of one photon}}{\text{Time}}$$

$$\Rightarrow \quad P = \frac{NE}{t} = \left(\frac{N}{t}\right) \cdot E$$

So, number of photons emitted per second

$$= \frac{N}{t} = \frac{P}{E}$$

$$= \frac{P}{hc/\lambda} = \frac{P\lambda}{hc} \quad \left[\because E = h\nu = \frac{hc}{\lambda}\right]$$

Here, $h = 6.6 \times 10^{-34}$ J-s, $\lambda = 500$ nm

$= 500 \times 10^{-9}$ m

$c = 3 \times 10^8 \text{ ms}^{-1}$

$$P = 2 \text{ mW} = 2 \times 10^{-3} \text{ W}$$

$$\therefore \quad \frac{N}{t} = \frac{2 \times 10^{-3} \times 500 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 5.56 \times 10^{15}$$

$$\approx 5 \times 10^{15} \text{ photons per second}$$

8. The stopping potential V_0 (in volt) as a function of frequency (ν) for a sodium emitter, is shown in the figure. The work function of sodium, from the data plotted in the figure, will be

(Take, Planck's constant (h) = 6.63×10^{-34} J-s, electron charge, e = 1.6×10^{-19} C]

[JEE Main 2019, 12 April Shift-I]

(a) 1.82 eV (b) 1.66 eV (c) 1.95 eV (d) 2.12 eV

Exp. **(b)**

Given, Planck's constant,

$$h = 6.63 \times 10^{-34} \text{ J-s},$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

and there is a graph between stopping potential and frequency.

We need to determine work function W.

Using Einstein's relation of photoelectric effect,

$$(\text{KE})_{max} = eV_0 = h\nu - h\nu_0 = h\nu - W \quad [\because W = h\nu_0]$$

or $\quad V_0 = \frac{h}{e}\nu - \frac{W}{e}$

From graph at $V_0 = 0$ and $\nu = 4 \times 10^{14}$ Hz

$$\therefore \quad 0 = \frac{6.63 \times 10^{-34}}{e} \times 4 \times 10^{14} - \frac{W}{e}$$

$$\Rightarrow \quad \frac{W}{e} = \frac{6.63 \times 10^{-34} \times 4 \times 10^{14}}{e} \text{ J}$$

or $\quad W = 6.63 \times 4 \times 10^{-20}$ J

or $\quad W = \frac{6.63 \times 4 \times 10^{-20}}{1.6 \times 10^{-19}} \text{eV} = 1.657 \text{ eV}$

$\therefore \quad W = 1.66$ eV

Alternate Solution

From graph, threshold frequency,

$$\nu_0 = 4 \times 10^{14} \text{ Hz (where, } V_0 = 0)$$

$\therefore$ Work function, $W = h\nu_0$

$$\Rightarrow \quad W = 6.63 \times 10^{-34} \times 4 \times 10^{14} \text{ J}$$

$$\Rightarrow \quad W = \frac{6.63 \times 4 \times 10^{-20}}{1.6 \times 10^{-19}} \text{ eV}$$

$$= 1.657 \text{ eV} \approx 1.66 \text{ eV}$$

9. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12} m, the minimum electron energy required is close to

[JEE Main 2019, 10 Jan Shift-I]

(a) 500 keV (b) 1 keV
(c) 100 keV (d) 25 keV

Exp. **(d)**

Given, resolution achieved in electron microscope is of the order of wavelength.

So, to resolve 7.5×10^{-12} m seperation wavelength associated with electrons is

$$\lambda = 7.5 \times 10^{-12} \text{m}$$

$\therefore$ Momentum of electrons required is

$$p = \frac{h}{\lambda}$$

or kinetic energy of electron must be

$$\text{KE} = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m}$$

Substituting the given values, we get

$$= \frac{\left(\frac{6.6 \times 10^{-34}}{7.5 \times 10^{-12}}\right)^2}{2 \times 9.1 \times 10^{-31}} \text{ J}$$

$$= \frac{(6.6 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31} \times (7.5 \times 10^{-12})^2 \times (1.6 \times 10^{-19})} \text{eV}$$

$$(\because 1 \text{ eV} = 1.6 \times 10^{-19} \text{ J})$$

$$= 26593.4 \simeq 26.6 \times 10^{3} \text{ eV} \simeq 26 \text{ keV}$$

which is nearest to 25 keV.

10. A metal plate of area 1×10^{-4} m^2 is illuminated by a radiation of intensity 16 m W/m^2. The work function of the metal is 5 eV. The energy of the incident photons is 10 eV and only 10% of it produces photoelectrons. The number of emitted photoelectrons per second and their maximum energy, respectively will be (Take, 1 eV $= 1.6\times10^{-19}$ J)

[JEE Main 2019, 10 Jan Shift-II]

(a) 10^{11} and 5 eV (b) 10^{12} and 5 eV
(c) 10^{10} and 5 eV (d) 10^{14} and 10 eV

Exp. **(a)**

We know that, intensity of a radiation I with energy 'E' incident on a plate per second per unit area is given as

$$\Rightarrow \quad I = \frac{dE}{dA\times dt} \quad \Rightarrow \frac{dE}{dt} = IdA \text{ or } IA$$

i.e., energy incident per unit time = IA

Substituting the given values, we get

$$\frac{dE}{dt} = 16\times10^{-3}\times1\times10^{-4}$$

$$\frac{dE}{dt} = 16\times10^{-7}\text{ W} \qquad \ldots(i)$$

Using Einstein's photoelectric equation, we can find kinetic energy of the incident radiation as

$$E = \frac{1}{2}mv^2 + \phi$$

(Here, ϕ is work function of metal)

or $\quad E = \text{KE} + \phi$

$$\text{KE} = E - \phi$$

$$= 10\text{ eV} - 5\text{ eV}$$

$$\Rightarrow \quad \text{KE} = 5\text{ eV} \qquad \ldots(ii)$$

Now, energy per unit time for incident photons will be

$$\because \quad E = Nh\nu$$

$$\therefore \quad \frac{dE}{dt} = h\nu\frac{dN}{dt} \text{ or } h\nu\dot{N} \qquad \ldots(iii)$$

From Eqs. (i) and (iii), we get

$$h\nu\dot{N} = 16\times10^{-7} \text{ or } E\dot{N} = 16\times10^{-7}$$

But $\quad E = 10$ eV, so

$$\dot{N}(10\times1.6\times10^{-19}) = 16\times10^{-7} \Rightarrow \dot{N} = 10^{12}$$

$\because$ Only 10% of incident photons emit electrons. So, emitted electrons per second are

$$\frac{10}{100}\times10^{12} = 10^{11}$$

11. If the de-Broglie wavelength of an electron is equal to 10^{-3} times, the wavelength of a photon of frequency 6×10^{14} Hz, then the speed of electron is equal to
(Take, speed of light $= 3\times10^8$ m/s,
Planck's constant $= 6.63\times10^{-34}$ J-s and
mass of electron $= 9.1\times10^{-31}$ kg)

[JEE Main 2019, 11 Jan Shift-I]

(a) 1.45×10^6 m/s (b) 1.8×10^6 m/s
(c) 1.1×10^6 m/s (d) 1.7×10^6 m/s

Exp. **(a)**

Wavelength of the given photon is given as,

$$\lambda_p = \frac{c}{\nu_p} = \frac{3\times10^8}{6\times10^{14}}\text{ m}$$

$$= 5\times10^{-7}\text{m} \qquad \ldots(i)$$

As, it is given that, de-Broglie wavelength of the electron is

$$\lambda_e = 10^{-3}\times\lambda_p \qquad [\because \text{using Eq. (i)}]$$

$$= 5\times10^{-10}\text{m}$$

Also, the de-Broglie wavelength of an electron is given as,

$$\lambda_e = \frac{h}{p} = \frac{h}{mv_e}$$

$$\Rightarrow \quad v_e = \frac{h}{\lambda_e m_e}$$

Substituting the given values, we get

$$= \frac{6.63\times10^{-34}}{5\times10^{-10}\times9.1\times10^{-31}}\text{ m/s}$$

$$= 1.45\times10^6\text{ m/s}$$

12. In a photoelectric experiment, the wavelength of the light incident on a metal is changed from 300 n-m to 400 n-m. The decrease in the stopping potential is close to $\left(\frac{hc}{e} = 1240\text{ n-mV}\right)$

[JEE Main 2019, 11 Jan Shift-II]

(a) 0.5 V (b) 2.0 V (c) 1.5 V (d) 1.0 V

Exp. **(d)**

Given, $\lambda_1 = 300$ nm

$$\lambda_2 = 400\text{ nm}$$

$$\frac{hc}{e} = 1240\text{ nm}$$

Using Einstein equation for photoelectric effect,

$$E = h\nu = \phi + eV_0 \quad \ldots(i)$$

(here, ϕ is work function of the metal and V_0 is stopping potential)

For λ_1 wavelength's wave,

$$E_1 = h\nu_1 = \phi + eV_{01}$$

or $$\frac{hc}{\lambda_1} = \phi + eV_{01} \quad \ldots(ii)$$

Similarly, $$\frac{hc}{\lambda_2} = \phi + eV_{02} \quad \ldots(iii)$$

From Eqs. (ii) and (iii), we get

$$hc\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = e(V_{01} - V_{02})$$

or $$\frac{hc}{e}\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = \Delta V$$

By using given values,

$$\Delta V = 1240\left[\frac{1}{300} - \frac{1}{400}\right]\frac{\text{nm -V}}{\text{nm}}$$

$$= 1240 \times \frac{1}{1200}\text{ V}$$

$$\Rightarrow \quad \Delta V = 1.03.\text{ V}$$

$$\approx 1\text{V}$$

13. A particle A of mass 'm' and charge 'q' is accelerated by a potential difference of 50 V. Another particle B of mass '$4m$' and charge 'q' is accelerated by a potential difference of 2500 V. The ratio of de-Broglie wavelengths $\frac{\lambda_A}{\lambda_B}$ is close to

[JEE Main 2019, 12 Jan Shift-I]

(a) 4.47 (b) 10.00
(c) 0.07 (d) 14.14

Exp. **(d)**

de-Broglie wavelength associated with a moving charged particle of charge q is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

where, V = accelerating potential.

Ratio of de-Broglie wavelength for particle A and B is,

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{m_B q_B V_B}}{\sqrt{m_A q_A V_A}}$$

$$= \sqrt{\frac{m_B}{m_A}} \cdot \sqrt{\frac{q_B}{q_A}} \cdot \sqrt{\frac{V_B}{V_A}}$$

Substituting the given values, we get

$$= \sqrt{\frac{4m}{m}} \cdot \sqrt{\frac{q}{q}} \cdot \sqrt{\frac{2500}{50}}$$

$$= 2 \times 1 \times 5 \times 1.414 = 14.14$$

14. In a Frank-Hertz experiment, an electron of energy 5.6 eV passes through mercury vapour and emerges with an energy 0.7 eV. The minimum wavelength of photons emitted by mercury atoms is close to

[JEE Main 2019, 12 Jan Shift-II]

(a) 250 nm (b) 2020 nm
(c) 1700 nm (d) 220 nm

Exp. **(a)**

Minimum wavelength occurs when mercury atom deexcites from highest energy level.

$\therefore$ Maximum possible energy absorbed by mercury atom = ΔE = 5.6 – 0.7 = 4.9 eV

Wavelength of photon emitted in deexcitation is

$$\lambda = \frac{hc}{E} \approx \frac{1240\text{ eVnm}}{4.9\text{ eV}} \approx 250\text{ nm}$$

Note Frank-Hertz experiment was the first electrical measurement to show quantum nature of atoms. In a vacuum tube energatic electrons are passed through thin mercury vapour film. It was discovered that when an electron collided with a mercury atom, it loses only a specific quantity (4.9 eV) of it's kinetic energy. This experiment shows existence of quantum energy levels.

15. When a certain photosensitive surface is illuminated with monochromatic light of frequency ν, the stopping potential for the photocurrent is $-V_0/2$. When the surface is illuminated by monochromatic light of frequency $\nu/2$, the stopping potential is $-V_0$. The threshold frequency for photoelectric emission is

[JEE Main 2019, 12 Jan Shift-II]

(a) $\frac{4}{3}\nu$ (b) 2ν
(c) $\frac{3\nu}{2}$ (d) $\frac{5\nu}{3}$

Exp. **(c)**

Relation between stopping potential and incident light's frequency is $eV_0 = hf - \phi_0$

where, V_0 is the stopping potential and ϕ_0 is the the work function of the photosensitive surface.

So, from given data, we have

$$-e\frac{V_0}{2} = h\nu - \phi_0 \quad \ldots(i)$$

and $$-eV_0 = \frac{h\nu}{2} - \phi_0 \quad \ldots(ii)$$

Subtracting Eqs. (i) from (ii), we have

$$-eV_0 - \left(-\frac{eV_0}{2}\right) = \frac{h\nu}{2} - h\nu$$

$$\Rightarrow \quad -\frac{eV_0}{2} = -\frac{h\nu}{2}$$

$$\Rightarrow \quad eV_0 = h\nu$$

Substituting this in Eq. (i), we get

$$-\frac{eV_0}{2} = eV_0 - \phi_0$$

$$\Rightarrow \quad -\left(\frac{3}{2}eV_0\right) = -\phi_0$$

or $$\frac{3}{2}h\nu = \phi_0$$

If threshold frequency is ν_0, then

$$h\nu_0 = \frac{3}{2}h\nu \Rightarrow \nu_0 = \frac{3}{2}\nu$$

16. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de-Broglie wavelength of the electron in the nth state and the ground state, respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the nth state to the ground state. For large n, (A, B are constants) **[JEE Main 2018]**

(a) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (b) $\Lambda_n \approx A + B\lambda_n^2$

(c) $\Lambda_n^2 \approx A + B\lambda_n^2$ (d) $\Lambda_n^2 \approx \lambda$

Exp. **(a)**

If wavelength of emitted photon in de-excitation is Λ_n;

Then, $$\frac{hc}{\Lambda_n} = E_n - E_g$$

$$\frac{hc}{\Lambda_n} = \frac{p_n^2}{2m} - \frac{p_g^2}{2m}, \quad \left[\because E = \frac{p^2}{2m}\right]$$

As energies are negative, we get

$$\frac{hc}{\Lambda_n} = \frac{p_g^2}{2m} - \frac{p_n^2}{2m}$$

$$\Rightarrow \quad = \frac{p_g^2}{2m}\left(1 - \left(\frac{p_n}{p_g}\right)^2\right)$$

$$= \frac{h^2}{2m\lambda_g^2}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)$$

$$[\because p \propto \lambda^{-1}, p = \frac{h}{\lambda}]$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 - \frac{\lambda_g^2}{\lambda_n^2}\right)^{-1}$$

$$\Rightarrow \quad \Lambda_n = \frac{2m\lambda_g^2 c}{h}\left(1 + \frac{\lambda_g^2}{\lambda_n^2}\right)$$

$$[\because (1-x)^{-n} = 1 + nx]$$

$$\Rightarrow \quad \Lambda_n \simeq A + \frac{B}{\lambda_n^2}$$

where, $A = \left[\frac{2mc\lambda_g^2}{h}\right]$ and $B = \left[\frac{2mc\lambda_g^4}{h}\right]$ are constants.

17. An electron beam is accelerated by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-rays in the spectrum, the variation of $\log \lambda_{min}$ with $\log V$ is correctly represented in **[JEE Main 2017 (Offline)]**

Exp. **(d)**

$$\lambda_{min} = \frac{hc}{eV}$$

$$\log(\lambda_{min}) = \log\left(\frac{hc}{e}\right) - \log V$$

$$y = c - mx$$

So, the required graph is given in option (d).

18. A particle A of mass m and initial velocity v collides with a particle B of mass $\frac{m}{2}$ which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is

[JEE Main 2017 (Offline)]

(a) $\frac{\lambda_A}{\lambda_B} = 2$ (b) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$ (c) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (d) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$

Exp. **(a)**

For elastic collision,

$$p_{\text{before collision}} = p_{\text{after collision}}.$$

$$mv = mv_A + \frac{m}{2}v_B$$

$$2v = 2v_A + v_B \quad \ldots \text{(i)}$$

Now, coefficient of restitution,

$$e = \frac{v_B - v_A}{u_A - v_B}$$

Here, $u_B = 0$ (Particle at rest) and for elastic collision $e = 1$

$$\therefore \quad 1 = \frac{v_B - v_A}{v} \Rightarrow v = v_B - v_A \quad \ldots \text{(ii)}$$

From Eqs. (i) and (ii), we get

$$v_A = \frac{v}{3} \quad \text{and} \quad v_B = \frac{4v}{3}$$

Hence, $$\frac{\lambda_A}{\lambda_B} = \frac{\left(\frac{h}{mv_A}\right)}{\left(\frac{h}{\frac{m}{2}.v_B}\right)} = \frac{v_B}{2v_A} = \frac{4/3}{2/3} = 2$$

19. Radiation of wavelength λ is incident on a photocell. The fastest emitted electron has speed v. If the wavelength is changed to $3\lambda/4$, the speed of the fastest emitted electron will be

(a) $> v\left(\frac{4}{3}\right)^{1/2}$ (b) $< v\left(\frac{4}{3}\right)^{1/2}$

(c) $= v\left(\frac{4}{3}\right)^{1/2}$ (d) $= v\left(\frac{3}{4}\right)^{1/2}$

Exp. **(a)**

According to the law of conservation of energy, i.e. Energy of a photon ($h\nu$) = Work function (ϕ) + Kinetic energy of the photoelectron $\left(\frac{1}{2}mv^2{}_{max}\right)$

According to Einstein's photoelectric emission of light,

$$E = (\text{KE})_{\text{max}} + \phi$$

As, $$\frac{hc}{\lambda} = (\text{KE})_{\text{max}} + \phi$$

If the wavelength of radiation is changed to $\frac{3\lambda}{4}$, then

$$\Rightarrow \quad \frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}(\text{KE})_{\text{max}} + \frac{\phi}{3}\right) + \phi$$

For fastest emitted electron,

$$(\text{KE})_{\text{max.}} = \frac{1}{2}mv'^2 + \phi$$

$$\Rightarrow \quad \frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

i.e. $$v' > v\left(\frac{4}{3}\right)^{1/2}$$

20. Match List I (fundamental experiment) with List II (its conclusion) and select the correct option from the choices given below the list.

[JEE Main 2015]

List I	List II
A. Franck-Hertz experiment	1. Particle nature of light
B. Photo-electric experiment	2. Discrete energy levels of atom
C. Davisson-Germer experiment	3. Wave nature of electron
	4. Structure of atom

	A	B	C		A	B	C
(a)	1	4	3	(b)	2	4	3
(c)	2	1	3	(d)	4	3	2

Exp. **(c)**

(A) Franck-Hertz experiments is associated with discrete energy levels of atom.

(B) Photo-electric experiment is associated with particle nature of light.

(C) Davisson-Germer experiment is associated with wave nature of electron.

21. The radiation corresponding to $3 \rightarrow 2$ transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T.

If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to

[JEE Main 2014]

(a) 1.8 eV (b) 1.1 eV (c) 0.8 eV (d) 1.6 eV

The problem is based on frequency dependence of photoelectric emission. When incident light with certain frequency (greater than on the threshold frequency) is focus on a metal surface, then some electrons are emitted from the metal with substantial initial speed.

Exp. **(b)**

When an electron moves in a circular path, then

Radius, $r = \frac{mv}{eB} \Rightarrow \frac{r^2e^2B^2}{2} = \frac{m^2v^2}{2}$

$$KE_{max} = \frac{(mv)^2}{2m} \Rightarrow \frac{r^2e^2B^2}{2m} = (KE)_{max}$$

Work function of the metal (W),

i.e., $W = h\nu - KE_{max}$

$$1.89 - \phi = \frac{r^2e^2B^2}{2m}\frac{1}{2}eV = \frac{r^2eB^2}{2m}eV$$

[$h\nu \rightarrow 1.89$ eV, for the transition on from third to second orbit of H-atom]

$$= \frac{100 \times 10^{-6} \times 1.6 \times 10^{-19} \times 9 \times 10^{-8}}{2 \times 9.1 \times 10^{-31}}$$

$$\phi = 1.89 - \frac{1.6 \times 9}{2 \times 9.1} = 1.89 - 0.79 = 1.1 \text{ eV}$$

22. The anode voltage of a photocells kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of photocell varies as follows

[JEE Main 2013]

Exp. **(d)**

As λ is increased, there will be a value of λ above which photoelectron will be cease to come out, so photocurrent will becomes zero.

23. This question has Statement I and Statement II. Of the four choices given the statements, choose the one that describes the two statements. **[AIEEE 2012]**

Statement I Davisson-Germer experiment established the wave nature of electrons.

Statement II If electrons have wave nature, they can interfere and show diffraction.

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement I is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Exp. **(c)**

Davisson and Germer experimentally established wave nature of electron by observing diffraction pattern while bombarding electrons on Ni-crystal.

24. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0, respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement II The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light. **[AIEEE 2011]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is false

Exp. **(c)**

$$K_{max} = eV_0 = h(\nu - \nu_0)$$

If $\nu' = 2\nu$

$$\therefore \quad K'_{max} = eV'_0 = h(2\nu - \nu_0)$$

$$= 2K_{max} + h\nu_0$$

$$\therefore \quad K'_{max} > 2K_{max} \text{ and } \nu'_0 > 2\nu_0$$

Aliter

From graph, we can predict that K_{max} and ν do not correspond linear graph from origin.

25. If a source of power 4 kW produces 10^{20}, photons/second, the radiation belong to a part of the spectrum called **[AIEEE 2010]**

(a) X-rays (b) ultraviolet rays
(c) microwaves (d) γ-rays

Exp. **(a)** As power = Energy × time

or $4 \times 10^3 = hf \times 10^{20}$ $\left[\because \frac{\text{energy}}{\text{time}} = \text{power}\right]$

$$f = \frac{4 \times 10^3}{10^{20} \times 6.023 \times 10^{-34}}$$

$$f = 6.64 \times 10^{16} \text{ Hz}$$

The obtained frequency lies in the band of X-rays.

26. The surface a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work function of the metal is ($hc = 1240$ eV-nm) **[AIEEE 2009]**

(a) 3.09 eV (b) 1.42 eV (c) 151 eV (d) 1.68 eV

Exp. **(b)**

$\because \quad KE_{max} = eV_0$

$\Rightarrow \quad \frac{1}{2}mv^2 = eV_0 = 1.68$ eV

$\Rightarrow \quad h\nu = \frac{hc}{\lambda} = \frac{1240 \text{ eV-nm}}{400 \text{ nm}} = 3.1$ eV

$\Rightarrow \quad 3.1 \text{ eV} = W_0 + 1.6 \text{ eV}$

[from Einstein equation, $E = W_0 + K_{max}$]

$W_0 = 1.42$ eV

➡ **Directions** *Q. Nos. 27 and 29 are based on the following paragraph.*

Wave property of electrons implies that they will show diffraction effects. Davisson and Germer demonstrated this by diffracting electrons from crystals. The law governing the diffraction from a crystal is obtained by requiring that electron waves reflected from the planes of atoms in a crystal interfere constructively (see figure). **[AIEEE 2008]**

27. Electrons accelerated by potential V are diffracted from a crystal. If $d = 1$ Å and $i = 30°$, V should be about ($h = 6.6 \times 10^{-34}$ J-s, $m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C) **[AIEEE 2008]**

(a) 2000 V (b) 50 V (c) 500 V (d) 1000 V

Exp. **(b)**

For constructive interference,

$$2d \cos i = n\lambda = \frac{h}{\sqrt{2meV}}$$

On substituting values, we get

$$V \simeq 50 \text{ V}$$

28. If a strong diffraction peak is observed when electrons are incident at an angle i from the normal to the crystal planes with distance d between them (see figure), de-Broglie wavelength λ_{dB} of electrons can be calculated by the relationship (n is an integer) **[AIEEE 2008]**

(a) $d \sin i = n\lambda_{dB}$ (b) $2d \cos i = n\lambda_{dB}$
(c) $2d \sin i = n\lambda_{dB}$ (d) $d \cos i = n\lambda_{dB}$

Exp. **(b)**

Expression is given by $2d \cos i = n\lambda_{dB}$.

29. In an experiment, electrons are made to pass through a narrow slit of width d comparable to their de-Broglie wavelength. They are detected on a screen at a distance D from the slit (see figure). **[AIEEE 2008]**

Which of the following graphs can be expected to represent the number of electrons N detected as a function of the detector position y ($y = 0$ corresponds to the middle of the slit) ?

Exp. **(*d*)**

As diffraction pattern has to be wider than slit width, so (d) is the correct option.

30. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is **[AIEEE 2007]**

(a) $\frac{\nu}{c}$ (b) $h\nu c$ (c) $\frac{h\nu}{c^2}$ (d) $\frac{h\nu}{c}$

Exp. **(*d*)**

The momentum of the photon, $p = \frac{h}{\lambda} = \frac{h\nu}{c}$

31. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV and the stopping potential for a radiation incident on this surface is 5 V. The incident radiation lies in **[AIEEE 2006]**

(a) ultra-violet region
(b) infrared region
(c) visible region
(d) X-ray region

Exp. **(*a*)**

$$h\nu_0 = 6.2\text{eV}, \quad eV_0 = 5\text{eV}$$

From Einstein's photoelectric equation

$$h\nu = h\nu_0 + eV_0$$
$$= 6.2 + 5 = 11.2 \text{ eV}$$

$$\Rightarrow \quad \frac{hc}{\lambda} = 11.2 \text{ eV}$$

or $$\lambda = \frac{hc}{112} = 1108.9 \text{ Å}$$

Which belongs to ultra-violet region.

32. The time taken by a photoelectron to come out after the photon strikes is approximately

(a) 10^{-4} s (b) 10^{-10} s **[AIEEE 2006]**
(c) 10^{-16} s (d) 10^{-1} s

Exp. **(*b*)**

The photoelectric effect is an instantaneous phenomenon (experimentally proved). It takes approximate time of the order of 10^{-10} s.

33. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows **[AIEEE 2006]**

(d) None of these

Exp. **(*d*)**

According to the photoelectric effect in a photocell, if a light of wavelength λ is incident on a cathode, then electrons are emitted, which constitute the photoelectric current.

Photocell is based on the principle of photoelectric effect. As the wavelength of light changes, there is no change in number of electrons emitted and hence, no change in current (plate current of photocell). Thus, the two wavelength of incident light and plate current are independent to each other.

Plate current depends on intensity of light used.

Hence option (d) is true.

34. A uniform electric field and a uniform magnetic field are acting along the same direction in a certain region. If an electron is projected along the direction of the fields with a certain velocity, then **[AIEEE 2005]**

(a) its velocity will decrease
(b) its velocity will increase
(c) it will turn towards right of direction of motion
(d) it will turn towards left of direction of motion

Exp. (*a*)

When **E**, **v** and **B** are all along same direction, then magnetic force experienced by electron is zero while electric force is acting opposite to velocity of electron, so velocity of electron will decrease.

$$\left[\begin{array}{ll} \because \text{Lorentz force} & = 0 \\ \therefore & \theta = 0 \\ \Rightarrow & qvB\sin\theta = 0 \end{array}\right]$$

35. A photocell is illuminated by a small bright source placed 1 m away. When the same source of light is placed 1/2 m away, the number of electrons emitted by photocathode would **[AIEEE 2004]**

(a) decrease by a factor of 4
(b) increase by a factor of 4
(c) decrease by a factor of 2
(d) increase by a factor of 2

Exp. (*b*)

We know intensity depends on source distance.

$$\therefore \quad \frac{I_2}{I_1} = \frac{(r_1)^2}{(r_2)^2} \qquad \left[\text{as } I \propto \frac{1}{r^2}\right]$$

$$\Rightarrow \quad \frac{I_2}{I_1} = \frac{(1)^2}{(1/2)^2}$$

$$I_2 = 4I_1$$

Now, since number of electrons emitted per second is directly proportional to intensity, so number of electrons emitted by photocathode would increase by a factor of 4.

36. If the kinetic energy of a free electron doubles, its de-Broglie wavelength changes by the factor **[AIEEE 2005]**

(a) $\frac{1}{2}$ (b) 2 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$

Exp. (*c*)

We know that, $\lambda = \frac{h}{mv}$

and $K = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m}$

$\Rightarrow \quad mv = \sqrt{2mK}$

Thus, $\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{K}}$

$$\Rightarrow \quad \frac{\lambda_2}{\lambda_1} = \frac{\sqrt{K_1}}{\sqrt{K_2}} = \frac{\sqrt{K_1}}{\sqrt{2K_1}} \qquad [\because K_2 = 2K_1]$$

or $\frac{\lambda_2}{\lambda_1} = \frac{1}{\sqrt{2}}$ or $\lambda_2 = \frac{\lambda_1}{\sqrt{2}}$

37. According to Einstein's photoelectric equation, the plot of the kinetic energy of the emitted photoelectrons from a metal *Versus* the frequency of the incident radiation gives a straight line whose slope **[AIEEE 2004]**

(a) depends on the nature of the metal used
(b) depends on the intensity of the radiation
(c) depend both on the intensity of the radiation and the metal used
(d) is the same for all metals and independent of the intensity of the radiation

Exp. (*d*)

Einstein's photoelectric equation is

$$KE_{max} = h\nu - \phi \qquad \ldots(i)$$

The equation of line is

$$y = mx + c \qquad \ldots(ii)$$

Comparing above two equations, we get

$$m = h, \quad c = -\phi$$

Hence, slope of graph is equal to Planck's constant (non-variable) and does not depend on intensity of radiation. But interception 'c' depends on work function 'ϕ'.

38. The work function of a substance is 4.0 eV. The longest wavelength of light that can cause photoelectron emission from this substance is approximately **[AIEEE 2004]**

(a) 540 nm (b) 400 nm
(c) 310 nm (d) 220 nm

Exp. **(c)**

$$\frac{hc}{\lambda_0} = \phi_0 \Rightarrow \lambda_{max} = \frac{hc}{\phi_0} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{4 \times 1.6 \times 10^{-19}}$$
$$= 310 \text{ nm}$$

39. A charged oil drop is suspended in uniform field of 3×10^4 Vm^{-1} so that it neither falls nor rises. The charge on the drop will be (take the mass of the charge $= 9.9 \times 10^{-15}$ kg and $g = 10$ ms^{-2}) **[AIEEE 2004]**

(a) 3.3×10^{-18} C (b) 3.2×10^{-18} C
(c) 1.6×10^{-18} C (d) 4.8×10^{-18} C

Exp. **(a)**

In steady state,

Electric force on drop = Weight of drop

i.e., $qE = mg$

$$\Rightarrow \quad q = \frac{mg}{E} = \frac{9.9 \times 10^{-15} \times 10}{3 \times 10^4}$$
$$= 3.3 \times 10^{-18} \text{ C}$$

40. Two identical, photocathodes receive light of frequencies f_1 and f_2. If the velocities of the photoelectrons (of mass m) coming out are respectively v_1 and v_2, then **[AIEEE 2003]**

(a) $v_1^2 - v_2^2 = \frac{2h}{m}(f_1 - f_2)$

(b) $v_1 + v_2 = \left[\frac{2h}{m}(f_1 + f_2)\right]^{1/2}$

(c) $v_1^2 + v_2^2 = \frac{2h}{m}(f_1 + f_2)$

(d) $v_1 - v_2 = \left[\frac{2h}{m}(f_1 - f_2)\right]^{1/2}$

Exp. **(a)**

We know, Einstein's equations

$$\underbrace{hf}_{\text{Energy of incident light}} = \underbrace{hf_0}_{\text{Work function}} + \underbrace{\frac{1}{2}mv^2}_{\text{Kinetic energy}}$$

$$\Rightarrow \quad v_1^2 = \frac{2hf_1}{m} - \frac{2hf_0}{m}$$

and $$v_2^2 = \frac{2hf_2}{m} - \frac{2hf_0}{m}$$

$$\therefore \quad v_1^2 - v_2^2 = \frac{2h}{m}[f_1 - f_2]$$

41. Formation of covalent bonds in compounds exhibits **[AIEEE 2002]**

(a) wave nature of electron
(b) particle nature of electron
(c) both wave and particle nature of electron
(d) None of the above

Exp. **(a)**

Formation of covalent bonds due to the wave nature of particles is done in compounds.

42. Sodium and copper have work functions 2.3 eV and 4.5 eV, respectively. Then, the ratio of the wavelengths is nearest to

(a) 1 : 2 (b) 4 : 1 **[AIEEE 2002]**
(c) 2 : 1 (d) 1 : 4

Exp. **(c)**

Work function, $W = \frac{hc}{\lambda}$

[here, they are interested in asking threshold wavelength]

where, h = Planck's constant,

c = velocity of light.

Therefore, $$\frac{W_{Na}}{W_{Cu}} = \frac{\lambda_{Cu}}{\lambda_{Na}}$$

or $$\frac{\lambda_{Na}}{\lambda_{Cu}} = \frac{W_{Cu}}{W_{Na}} = \frac{4.5}{2.3} = 2 \text{ (nearly)}$$

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18

Atoms *and* Nuclei

1. Radiation coming from transitions $n=2$ to $n=1$ of hydrogen atoms fall on He^+ ions in $n=1$ and $n=2$ states. The possible transition of helium ions as they absorb energy from the radiation is **[JEE Main 2019, 8 April Shift-I]**

(a) $n=2 \rightarrow n=3$ (b) $n=1 \rightarrow n=4$
(c) $n=2 \rightarrow n=5$ (d) $n=2 \rightarrow n=4$

Exp. **(d)**

De-excitation energy of hydrogen electron in transition $n=2$ to $n=1$ is

$$E = 13.6 \times \left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \text{eV} = 13.6\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= 10.2 \text{ eV}$$

Now, energy levels of helium ion's (He^+) electron are
(For helium, $Z=2$)

So, a photon of energy 10.2 eV can cause a transition $n=2$ to $n=4$ in a He^+ ion.

Alternate Solution

For He^+ ion, when in $n=1$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right) \Rightarrow \quad n=1$$

Thus, no transition takes place.

Similarly, when in $n=2$ state,

$$10.2 = 13.6 \times 2^2\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \Rightarrow n=4$$

2. A nucleus A, with a finite de-Broglie wavelength λ_A, undergoes spontaneous fission into two nuclei B and C of equal mass. B flies in the same directions as that of A, while C flies in the opposite direction with a velocity equal to half of that of B. The de-Broglie wavelengths λ_B and λ_C of B and C respectively **[JEE Main 2019, 8 April Shift-II]**

(a) $2\lambda_A, \lambda_A$ (b) $\frac{\lambda_A}{2}, \lambda_A$
(c) $\lambda_A, 2\lambda_A$ (d) $\lambda_A, \frac{\lambda_A}{2}$

Exp. **(b)**

Let m be the mass of nuclei B and C.

So, the given situation can be shown in the figure below

Now, according to the conservation of linear momentum,

Initial momentum = Final momentum

$$\Rightarrow \quad p_A = p_B + p_C \quad \text{or} \quad 2mv_A = mv_B + mv_C$$

$$2mv_A = mv_B - \frac{mv_B}{2}$$

$$\Rightarrow \quad 2v_A = \frac{1}{2}v_B \Rightarrow v_B = 4v_A \qquad \ldots\text{(i)}$$

$$\text{and} \quad v_C = \frac{v_B}{2} = 2v_A \qquad \ldots\text{(ii)}$$

So, momentum of B and C respectively, can now be given as

$p_B = m_B v_B = m4v_A = 2(2mv_A)$ [using Eq. (i)]

or $p_B = 2p_A$ ...(iii)

and $p_C = m_C v_C = m2v_A$ [using Eq. (ii)]

or $p_C = p_A$ ...(iv)

From the relation of de-Broglie wavelength,

i.e. $\lambda = \frac{h}{p}$

where, p is momentum and h is Planck's constant.

So, for A, $\lambda_A = \frac{h}{p_A}$ or $p_A = \frac{h}{\lambda_A}$ ...(v)

For B, $\lambda_B = \frac{h}{p_B} = \frac{h}{2p_A}$ [using Eq. (iii)]

From Eq. (v), λ_B can be written as

$$\lambda_B = \frac{h}{2 \times \frac{h}{\lambda_A}} = \frac{\lambda_A}{2}$$

Similarly, for C, $\lambda_C = \frac{h}{p_C} = \frac{h}{p_A}$ [using Eq. (iv)]

Similarly, from Eq. (v), λ_C can be written as

$$\lambda_C = \frac{h}{\frac{h}{\lambda_A}} = \lambda_A$$

3. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to

[JEE Main 2019, 8 April Shift-II]

(a) 5 (b) 2 (c) 0.1 (d) 1

Exp. **(d)**

Mass density of nuclear matter is a constant quantity for all elements. It does not depends on element's mass number or atomic radius.

$\therefore$ The ratio of mass densities of ^{40}Ca and ^{16}O is 1 : 1.

4. Taking the wavelength of first Balmer line in hydrogen spectrum (n = 3 to n = 2) as 660 nm, the wavelength of the 2nd Balmer line (n = 4 to n = 2) will be

[JEE Main 2019, 9 April Shift-I]

(a) 889.2 nm (b) 388.9 nm
(c) 642.7 nm (d) 488.9 nm

Exp. **(d)**

Expression for the energy of the hydrogenic electron states for atoms of atomic number Z is given by

$$E = h\nu = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right] \quad \text{(Here, } m < n\text{)}$$

or $$\frac{hc}{\lambda} = \frac{Z^2 me^4}{8h^2 E_0^2}\left[\frac{1}{m^2} - \frac{1}{n^2}\right]$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{m^2} - \frac{1}{n^2}\right) Z^2$$

For first case, λ = 660nm, m = 2 and n = 3

$$\therefore \quad \frac{1}{660\text{ nm}} \propto \left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] Z^2$$

$$\Rightarrow \quad \frac{1}{660\text{ nm}} \propto \left(\frac{1}{4} - \frac{1}{9}\right) Z^2 \text{ or } \frac{5}{36} Z^2 \quad \text{...(i)}$$

For second case, transition is from n = 4 to n = 2, i.e. m = 2 and n = 4

$$\therefore \quad \frac{1}{\lambda} \propto \left(\frac{1}{(2)^2} - \frac{1}{(4)^2}\right) Z^2$$

$$\Rightarrow \quad \frac{1}{\lambda} \propto \left(\frac{1}{4} - \frac{1}{16}\right) Z^2$$

or $$\frac{1}{\lambda} \propto \frac{3}{16} Z^2 \quad \text{...(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{\lambda}{660\text{nm}} = \frac{5}{36} \times \frac{16}{3}$$

$$\Rightarrow \quad \lambda = \frac{80}{108} \times 660\text{ nm}$$

$$= 488.9\text{ nm}$$

5. A He^+ ion is in its first excited state. Its ionisation energy is

[JEE Main 2019, 9 April Shift-II]

(a) 54.40 eV (b) 13.6 eV
(c) 48.36 eV (d) 6.04 eV

Exp. **(b)**

Energy of a hydrogen atom like ion by Bohr's model is $E_n = -13.6\frac{Z^2}{n^2}$

where, Z = atomic number

and n = principal quantum number.

For a He^+ ion in first excited state,

$n = 2, Z = 2$

$$\therefore \quad E_2 = -13.6 \times \frac{4}{4} = -13.6\text{ eV}$$

6. Two radioactive materials A and B have decay constants 10λ and λ, respectively. If initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be $1/e$ after a time

[JEE Main 2019, 10 April Shift-I]

(a) $\frac{1}{11\lambda}$ (b) $\frac{11}{10\lambda}$ (c) $\frac{1}{9\lambda}$ (d) $\frac{1}{10\lambda}$

Exp. **(c)**

Given, $\lambda_A = 10\lambda$ and $\lambda_B = \lambda$

Number of nuclei (at any instant) present in material is

$$N = N_0 e^{-\lambda t}$$

So, for materials A and B, we can write

$$\frac{N_A}{N_B} = \frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t} \quad \ldots \text{(i)}$$

Given, $$\frac{N_A}{N_B} = \frac{1}{e} \quad \ldots \text{(ii)}$$

Equating Eqs. (i) and (ii), we get

$$\frac{1}{e} = e^{-(\lambda_A - \lambda_B)t}$$

$$\Rightarrow \quad e^{-1} = e^{-(\lambda_A - \lambda_B)t}$$

Comparing the power of 'e' on both sides, we get

or $(\lambda_A - \lambda_B)t = 1$

$$\Rightarrow \quad t = \frac{1}{\lambda_A - \lambda_B}$$

By putting values of λ_A and λ_B in the above equation, we get

$$t = \frac{1}{10\lambda - \lambda} \Rightarrow t = \frac{1}{9\lambda}$$

7. In Li^{++}, electron in first Bohr orbit is excited to a level by a radiation of wavelength λ. When the ion gets de-excited to the ground state in all possible ways (including intermediate emissions), a total of six spectral lines are observed. What is the value of λ?
[Take, $h = 6.63 \times 10^{-34}$ Js; $c = 3 \times 10^8$ ms^{-1}]

[JEE Main 2019, 10 April Shift-II]

(a) 9.4 nm (b) 12.3 nm
(c) 10.8 nm (d) 11.4 nm

Exp. **(c)**

Number of spectral lines produced as an excited electron falls to ground state ($n = 1$) is,

$$N = \frac{n(n-1)}{2}$$

In given case, $N = 6$

$$\therefore 6 = \frac{n(n-1)}{2}$$

$$\Rightarrow \quad n = 4$$

So, L^{++} electron is in it's 3rd excited state.

Now, using the expression of energy of an electron in nth energy level,

$$E_n = -\frac{13.6Z^2}{n^2} \text{ eV}$$

where, Z is the atomic number.

$\therefore$ Energy levels of L^{++} electron are as shown

So, energy absorbed by electron from incident photon of wavelength λ is

$$\Delta E = \frac{hc}{\lambda}$$

$$\Rightarrow (13.6 \times 9 - 0.85 \times 9) = \frac{hc}{\lambda}$$

$$\Rightarrow \quad \lambda = \frac{hc}{9(13.6 - 0.85)}$$

$$\Rightarrow \quad \lambda = \frac{1240 \text{ eV-nm}}{9 \times 12.75 \text{ eV}} = 10.8 \text{ nm}$$

8. Two radioactive substances A and B have decay constants 5λ and λ, respectively. At $t = 0$, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be

[JEE Main 2019, 10 April Shift-II]

(a) $2/\lambda$ (b) $1/2\lambda$
(c) $1/4\lambda$ (d) $1/\lambda$

Exp. **(b)**

Number of active nuclei remained after time t in a sample of radioactive substance is given by

$$N = N_0 e^{-\lambda t}$$

where, N_0 = initial number of nuclei at $t = 0$ and λ = decay constant.

Here, at $t = 0$,

Number of nuclei in sample A and B are equal, i.e. $N_{0_A} = N_{0_B} = N_0$

Also, $\lambda_A = 5\lambda$ and $\lambda_B = \lambda$

So, after time t, number of active nuclei of A and B are

$$N_A = N_0 e^{-5\lambda t} \text{ and } N_B = N_0 e^{-\lambda t}$$

If $\frac{N_A}{N_B} = \frac{1}{e^2}$, then

$$\frac{N_A}{N_B} = \frac{N_0 e^{-5\lambda t}}{N_0 e^{-\lambda t}} = \frac{1}{e^2}$$

$$\Rightarrow \quad e^{(\lambda - 5\lambda)t} = e^{-2}$$

Comparing the power of e on both sides, we get

$$4\lambda t = 2$$

$$\Rightarrow \quad t = \frac{1}{2\lambda}$$

9. An excited He^+ ion emits two photons in succession, with wavelengths 108.5 nm and 30.4 nm, in making a transition to ground state. The quantum number n corresponding to its initial excited state is [for photon of wavelength λ, energy $E = \frac{1240 \text{ eV}}{\lambda \text{ (in nm)}}$]

[JEE Main 2019, 12 April Shift-I]

(a) $n = 4$ (b) $n = 5$
(c) $n = 7$ (d) $n = 6$

Exp. **(b)**

Change in energy in transition from n to m stage is given by ($n > m$),

$$E_n = -\frac{E_0 Z^2}{n^2}$$

Here, $Z = 2$

$$\Delta E_n = +13.6 \times 4\left[\frac{1}{m^2} - \frac{1}{n^2}\right] = \frac{hc}{\lambda} \quad \text{...(i)}$$

Let it start from n to m and then m to ground.

So, in first case,

$$13.6 \times 4 \times \left(\frac{1}{m^2} - \frac{1}{n^2}\right) = \frac{hc}{108.5 \text{ nm}} \quad \text{...(ii)}$$

and in second case,

$$13.6 \times 4 \times \left(\frac{1}{1^2} - \frac{1}{m^2}\right) = \frac{hc}{30.4 \text{ nm}}$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = \frac{1240 \text{ eV}}{30.4 \times 13.6 \times 4}$$

$$\left(\text{Given, } E = \frac{1240 \text{ eV}}{\lambda \text{(in nm)}}\right)$$

$$\Rightarrow \quad \left(1 - \frac{1}{m^2}\right) = 0.74980 \approx 0.75$$

or $\frac{1}{m^2} = 1 - 0.75 = 0.25$

$$\Rightarrow \quad m^2 = \frac{1}{0.25} = 4$$

Hence, $m = 2$

So, by putting the value of m in Eq. (ii), we get

$$13.6 \times 4 \times \left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{1240}{108.5} \text{eV}$$

$$\Rightarrow \quad \left(\frac{1}{4} - \frac{1}{n^2}\right) = \frac{1240}{108.5 \times 13.6 \times 4}$$

$$\Rightarrow \quad \frac{1}{4} - \frac{1}{n^2} = 0.21$$

or $\frac{1}{n^2} = 0.25 - 0.21 = 0.04$

$$n^2 = \frac{1}{0.04} = 25$$

$$\Rightarrow \quad n^2 = 25$$

$$\Rightarrow \quad n = 5$$

10. The electron in a hydrogen atom first jumps from the third excited state to the second excited state and subsequently to the first excited state. The ratio of the respective wavelengths λ_1 / λ_2 of the photons emitted in this process is

[JEE Main 2019, 12 April Shift-II]

(a) 20/7 (b) 27/5
(c) 7/5 (d) 9/7

Exp. **(a)**

Wavelength λ of emitted photon as an electron transits from an initial energy level n_i to some final energy level n_f is given by Balmer's formula,

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

where, R = Rydberg constant.

In transition from $n = 4$ to $n = 3$, we have

$$\frac{1}{\lambda_1} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= R\left(\frac{7}{9 \times 16}\right) \quad \text{...(i)}$$

In transition from $n = 3$ to $n = 2$, we have

$$\frac{1}{\lambda_2} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{5}{9 \times 4}\right) \quad \ldots(ii)$$

So, from Eqs. (i) and (ii), the ratio of $\frac{\lambda_1}{\lambda_2}$ is

$$\frac{\lambda_1}{\lambda_2} = \frac{\left(\frac{9 \times 16}{7R}\right)}{\left(\frac{9 \times 4}{5R}\right)} = \frac{20}{7}$$

11. Consider an electron in a hydrogen atom, revolving in its second excited state (having radius 4.65 Å). The de-Broglie wavelength of this electron is **[JEE Main 2019, 12 April Shift-II]**

(a) 3.5 Å (b) 6.6 Å
(c) 12.9 Å (d) 9.7 Å

Exp. **(d)**

By Bohr's IInd postulate, for revolving electron,

Angular momentum $= \frac{nh}{2\pi} \Rightarrow mvr_n = \frac{nh}{2\pi}$

$\Rightarrow$ Momentum of electron, $p = mv = \frac{nh}{2\pi r_n}$

de-Broglie wavelength associated with electron is

$$\lambda_n = \frac{h}{p} = \frac{2\pi r_n}{n}$$

Given, $n = 3, r_n = 4.65$ Å

$\therefore$ $\lambda_n = \frac{(2 \times \pi \times 4.65)}{3} \approx 9.7$ Å

12. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be **[JEE Main 2019, 12 April Shift-II]**

(a) 3 : 8 (b) 1 : 8 (c) 8 : 1 (d) 9 : 8

Exp. **(d)**

For substance A, half-life is 10 min, so it decays as

$$\underset{\text{(initial number of nuclei at } t=0)}{N_{0_A}} \xrightarrow{10\,\text{min}} \underset{\text{(Active nuclei remained after 10 min)}}{\frac{N_{0_A}}{2}} \xrightarrow{10\,\text{min}} \frac{N_{0_A}}{4} \xrightarrow{10\,\text{min}} \frac{N_{0_A}}{8}$$

$$\xrightarrow{10\,\text{min}} \underset{\text{nuclei remained after 60 min)}}{\frac{N_{0_A}}{16}} \xrightarrow{10\,\text{min}} \underset{\text{(number of active}}{\frac{N_{0_A}}{32}} \xrightarrow{10\,\text{min}} \frac{N_{0_A}}{64}$$

$\therefore$ For substance A, number of nuclei decayed in 60 min is

$$N_{0_A} - \frac{N_{0_A}}{64} = \frac{63N_{0_A}}{64}$$

Similarly, for substance B, half-life is 20 min, so it's decay scheme is

$$N_{0_B} \xrightarrow{20\,\text{min}} \frac{N_{0_B}}{2} \xrightarrow{20\,\text{min}} \frac{N_{0_B}}{4} \xrightarrow{20\,\text{min}} \frac{N_{0_B}}{8}$$

So, number of nuclei of B decayed in 60 min is

$$N_{0_B} - \frac{N_{0_B}}{8} = \frac{7}{8}N_{0_B}$$

Hence, ratio of decayed nuclei of A and B in 60 min is

$$\frac{\frac{63}{64}N_{0_A}}{\frac{7}{8}N_{0_B}} = \frac{9}{8} \quad [\because N_{0_A} = N_{0_B}]$$

Alternate Solution

Number of active nuclei remained after 60 min can also be calculated as

$$N' = \frac{N}{2^{T/t_{1/2}}}$$

where, $T = 60$ min

So, for nuclei A,

$$N'_{0_A} = \frac{N_{0_A}}{2^{\frac{60}{10}}} = \frac{N_{0_A}}{2^6} = \frac{N_{0_A}}{64}$$

Similarly, for nuclei B,

$$N'_{0_B} = \frac{N_{0_B}}{2^{\frac{60}{20}}} = \frac{N_{0_B}}{2^3} = \frac{N_{0_B}}{8}$$

13. Surface of certain metal is first illuminated with light of wavelength $\lambda_1 = 350$ n-m and then by light of wavelength $\lambda_2 = 540$ n-m. It is found that the maximum speed of the photoelectrons in the two cases differ by a factor of 2. The work function of the metal (in eV) is close to

(energy of photon $= \frac{1240}{\lambda(\text{in n-m})}$ eV)

[JEE Main 2019, 9 Jan Shift-I]

(a) 5.6 (b) 2.5 (c) 1.8 (d) 1.4

Exp. **(c)**

Let maximum speed of photo electrons in first case is v_1 and maximum speed of photo electrons in second case is v_2

Assumption I if we assume difference in maximum speed in two cases is 2 then $v_1 = v$ and $v_2 = 3v$

According to Einstein's photo electron equation

Energy of incident photon = work function + KE

i.e. $$\frac{hc}{\lambda} = \phi_0 + \frac{1}{2}mv^2$$

Where $hc = 1240$ eV, λ is wavelength of light incident, ϕ_0 is work function and v is speed of photo electrons.

When $\lambda_1 = 350$ nm

$$\therefore \quad \frac{hc}{350} = \phi_0 + \frac{1}{2}mv^2$$

or $$\frac{hc}{350} - \phi_0 = \frac{1}{2}mv^2 \quad \ldots(i)$$

when $\lambda_2 = 540$ nm

$$\therefore \quad \frac{hc}{540} = \phi_0 + \frac{1}{2}m(3v^2)$$

$$\therefore \quad \frac{hc}{540} - \phi_0 = (\frac{1}{2}mv^2) \times 9 \quad \ldots(ii)$$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{350} - \phi_0}{\frac{hc}{540} - \phi_0} = \frac{\frac{1}{2}mv^2}{(\frac{1}{2}mv^2) \times 9} = \frac{1}{9}$$

or $$9\left(\frac{hc}{350} - \phi_0\right) = \frac{hc}{540} - \phi_0$$

or $$8\phi_0 = hc\left[\frac{9}{350} - \frac{1}{540}\right]$$

or $$\phi_0 = \frac{1}{8} \times 1240\left[\frac{9 \times 540 - 350}{350 \times 540}\right] = 3.7 \text{ eV}.$$

No option given is correct.

Alternate Solution

Assumption II If we assume velocity of one is twice in factor with second then.

Let $v_1 = 2v$ and $v_2 = v$

We know that from Einstein's photoelectric equation, energy of incident radiation = work function + KE

or $$\frac{hc}{\lambda} = \phi + \frac{1}{2}mv^2$$

Let when $\lambda_1 = 350$ nm then $v_1 = 2v$

and when $\lambda_1 = 540$ nm then $v_2 = v$

$\therefore$ Above Eq. becomes

$$\frac{hc}{\lambda_1} = \phi + \frac{1}{2}m(2v^2)$$

or $$\frac{hc}{\lambda_1} - \phi = \frac{1}{2}m \times 4v^2 \quad \ldots(i)$$

and $$\frac{hc}{\lambda_2} = \phi + \frac{1}{2}mv^2$$

or $$\frac{hc}{\lambda_2} - \phi = \frac{1}{2}mv^2 \quad \ldots(ii)$$

Now, we divide Eq. (i) by Eq. (ii), we get

$$\frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} = \frac{\frac{1}{2}m \times 4v^2}{\frac{1}{2}mv^2} = 4$$

or $$\frac{hc}{\lambda_1} - \phi = \frac{4hc}{\lambda_2} - 4\phi$$

or $$\phi = \frac{1}{3}hc\left(\frac{4}{\lambda_2} - \frac{1}{\lambda_1}\right)$$

$$= \frac{1}{3} \times 1240\left(\frac{4 \times 350 - 540}{350 \times 540}\right)$$

or $$\phi = 1.8 \text{ eV}$$

According the assumption II, correct option is (c).

14. A sample of radioactive material A, that has an activity of 10 mCi (1 Ci $= 3.7 \times 10^{10}$ decays/s) has twice the number of nuclei as another sample of a different radioactive material B which has an activity of 20 mCi. The correct choices for half-lives of A and B would, then be respectively

[JEE Main 2019, 9 Jan Shift-I]

(a) 20 days and 10 days
(b) 5 days and 10 days
(c) 10 days and 40 days
(d) 20 days and 5 days

Exp. **(*d*)**

Activity of a radioactive material is given as

$$R = \lambda N$$

where, λ is the decay constant and N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = 10 \text{ mCi}$$

For substance B,

$$R_B = \lambda_B N_B = 20 \text{mCi} \quad \ldots(i)$$

As given in the question,

$$N_A = 2N_B$$

$$\Rightarrow \quad R_A = \lambda_A(2N_B) = 10 \text{ mCi} \quad \ldots(ii)$$

∴ Dividing Eq. (ii) and Eq.(i), we get

$$\frac{R_A}{R_B} = \frac{\lambda_A(2N_B)}{\lambda_B(N_B)}$$

$$= \frac{10}{20} \text{ or } \frac{\lambda_A}{\lambda_B} = \frac{1}{4} \quad \text{...(iii)}$$

As, half-life of a radioactive material is given as

$$T_{1/2} = \frac{0.693}{\lambda}$$

∴ For material A and B, we can write

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{\frac{0.693}{\lambda_A}}{\frac{0.693}{\lambda_B}} = \frac{\lambda_B}{\lambda_A}$$

Using Eq. (iii), we get

$$\frac{(T_{1/2})_A}{(T_{1/2})_B} = \frac{4}{1}$$

Hence, from the given options, only option (d) satisfies this ratio.

Therefore, $(T_{1/2})_A = 20$ days

and $(T_{1/2})_B = 5$ days

15. In given time $t = 0$, Activity of two radioactive substances A and B are equal. After time t, the ratio of their activities $\frac{R_B}{R_A}$ decreases according to e^{-3t}. If the half life of A is In 2, the half-life of B will be

[JEE Main 2019, 9 Jan Shift-II]

(a) $4 \ln 2$ (b) $\frac{\ln 2}{4}$

(c) $\frac{\ln 2}{2}$ (d) $2 \ln 2$

Exp. **(b)**

Activity of radioactive material is given as

$R = \lambda N$

where, λ is the decay constant N is the number of nuclei in the radioactive material.

For substance A,

$$R_A = \lambda_A N_A = \lambda_A N_{0A} \text{ (initially } N_A = N_{0A})$$

For substance B,

$$R_B = \lambda_B N_B = \lambda_B N_{0B} \text{ (initially } N_B = N_{0B})$$

At $t = 0$, activity is equal, therefore

$$\lambda_A N_{0A} = \lambda_B N_{0B} \quad \text{...(i)}$$

The half-life is given by

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{\ln 2}{\lambda}$$

So, for substance A,

$$(T_{1/2})_A = \frac{\ln 2}{\lambda_A} \Rightarrow \ln 2 = \frac{\ln 2}{\lambda_A}$$

$$\lambda_A = 1 \quad \text{...(ii)}$$

According to the given question,

at time t, $\frac{R_B}{R_A} = e^{-3t}$ ...(iii)

Using Eqs. (i), (ii) and (iii). we get

$$\frac{R_B}{R_A} = e^{-3t} = \frac{\lambda_B N_{0B} e^{-\lambda_B t}}{\lambda_A N_{0A} e^{-\lambda_A t}}$$

$$\Rightarrow \quad e^{-3t} = e^{(\lambda_A - \lambda_B)t}$$

$$-3 = \lambda_A - \lambda_B$$

$$\Rightarrow \quad \lambda_B = \lambda_A + 3$$

$$\lambda_B = 1 + 3 = 4 \quad \text{...(iv)}$$

The half-life of substance B is

$$(T_{1/2})_B = \frac{\ln 2}{\lambda_B} = \frac{\ln 2}{4}$$

16. The magnetic field associated with a light wave is given at the origin, by $B = B_0 [\sin(3.14 \times 10^7) ct + \sin(6.28 \times 10^7) ct]$.

If this light falls on a silver plate having a work function of 4.7 eV, what will be the maximum kinetic energy of the photoelectrons?

(Take, $c = 3 \times 10^8 \text{ ms}^{-1}$ and $h = 6.6 \times 10^{-34}$ J-s)

[JEE Main 2019, 9 Jan Shift-II]

(a) 7.72 eV (b) 6.82 eV

(c) 8.52 eV (d) 12.5 eV

Exp. **(a)**

According to question, the wave equation of the magnetic field which produce photoelectric effect

$B = B_0[(\sin(3.14 \times 10^7 ct) + \sin(6.28 \times 10^7 ct)]$

Here, the photoelectric effect produced by the angular frequency $(\omega) = 6.28 \times 10^7$ c

$$\Rightarrow \quad \omega = 6.28 \times 10^7 \times 3 \times 10^8$$

$$\omega = 2\pi \times 10^7 \times 3 \times 10^8 \text{ rad/s} \quad \text{...(i)}$$

Using Eqs. (i)

$$h\nu = \frac{h\omega}{2\pi} = \frac{h \times 2\pi \times 10^7 \times 3 \times 10^8}{2\pi}$$

$$h\nu = 12.4 \text{ eV}$$

Therefore, according to Einstein equation for photoelectric effect

$$E = h\nu = \phi + KE_{max}$$

$\Rightarrow \quad KE_{max} = E - \phi$

(where, ϕ = work-function = 4.7 eV)

$KE_{max} = 12.4 - 4.7 = 7.7$ eV

or $KE_{max} = 7.7$ eV

17. Using a nuclear counter, the count rate of emitted particles from a radioactive source is measured. At $t = 0$, it was 1600 counts per second and $t = 8$ s, it was 100 counts per second. The count rate observed as counts per second at $t = 6$ s is close to

[JEE Main 2019, 10 Jan Shift-I]

(a) 400 (b) 200 (c) 150 (d) 360

Exp. **(b)**

Here given, at $t = 0$, count rate or initial activity is $A_0 = 1600\ s^{-1}$.

At $t = 8$ s, count rate or activity is $A = 100\ s^{-1}$

So, decay scheme for given sample is

$$1600 \xrightarrow{T_{1/2}} 800 \xrightarrow{T_{1/2}} 400 \xrightarrow{T_{1/2}} 200 \xrightarrow{T_{1/2}} 100$$

So, $8\ s = 4T_{1/2}$

where, $T_{1/2}$ = Half-life time.

$\Rightarrow \quad T_{1/2} = 2$ s

$\therefore$ From above decay scheme, we see that activity after 6 s is 200 counts per second.

18. Consider the nuclear fission

$$Ne^{20} \longrightarrow 2He^4 + C^{12}$$

Given that the binding energy/nucleon of Ne^{20}, He^4 and C^{12} are respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement.

[JEE Main 2019, 10 Jan Shift-I]

(a) Energy of 3.6 MeV will be released.
(b) Energy of 12.4 MeV will be supplied.
(c) 8.3 MeV energy will be released.
(d) Energy of 11.9 MeV has to be supplied.

Exp. **(*)**

Energy absorbed or released in a nuclear reaction is given by;

ΔQ = Binding energy of products – Binding energy of reactants.

If energy is absorbed, ΔQ is negative and if it is positive then energy is released. Also, Binding energy = Binding energy per nucleon × Number of nucleons.

Here, binding energy of products

$= 2 \times (\text{B.E. of } He^4) + (\text{B.E. of } C^{12})$

$= 2\ (4 \times 7.07) + (12 \times 7.86)$

$= 150.88$ MeV

and binding energy of reactants

$= 20 \times 8.03 = 160.6$ MeV

So, $\Delta Q = (\text{B.E.})_{\text{Products}} - (\text{B.E.})_{\text{reactants}}$

$= 150.88 - 160.6 = -9.72$ MeV.

As ΔQ is negative

$\therefore$ energy of 9.72 Mev is absorbed in the reaction.

$\therefore$ No option is correct.

19. A hydrogen atom, initially in the ground state is excited by absorbing a photon of wavelength 980 Å. The radius of the atom in the excited state in terms of Bohr radius a_0 will be (Take $hc = 12500$ eV-Å)

[JEE Main 2019, 11 Jan Shift-I]

(a) $4a_0$ (b) $9a_0$ (c) $16a_0$ (d) $25a_0$

Exp. **(c)**

We know that net change in energy of a photon in a transition with wavelength λ is $\Delta E = hc / \lambda$.

Here, $hc = 12500$ eV Å and $\lambda = 980$ Å

$\Delta E = 12500 / 980 = 12.76$ eV

$\Rightarrow \quad E_n - E_1 = 12.76$ eV

Since, the energy associated with an electron in n^{th} Bohr's orbit is given as,

$$E_n = \frac{-13.6}{n^2} \text{ eV} \quad \ldots(i)$$

$$\Rightarrow \quad E_n = E_1 + 12.76 \text{ eV} \quad \ldots(ii)$$

$$= \frac{-13.6}{(1)^2} + 12.76 = -0.84$$

Putting this value in Eq. (i)

$$\Rightarrow \quad n^2 = \frac{-13.6}{-0.84} = 16 \Rightarrow n = 4$$

and radius of nth orbit, $r_n = n^2 a_0 \Rightarrow r_n = 16\, a_0$

20. In a hydrogen like atom, when an electron jumps from the M-shell to the L-shell, the wavelength of emitted radiation is λ. If an electron jumps from N-shell to the L-shell, the wavelength of emitted radiation will be

[JEE Main 2019, 11 Jan Shift-II]

(a) $\frac{27}{20}\lambda$ (b) $\frac{25}{16}\lambda$ (c) $\frac{20}{27}\lambda$ (d) $\frac{16}{25}\lambda$

Exp. **(c)**

For hydrogen or hydrogen like atoms, we know that

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] \quad \ldots(i)$$

where, R is Rydberg constant and Z is atomic number. When electron jumps from M - shell to the L - shell, then

$n_1 = 2$ (for L - shell)

$n_2 = 3$ (for M - shell)

$\therefore$ Eq (i) becomes

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5}{36}RZ^2] \quad \ldots(ii)$$

Now, electron jumps from N-shell to the L - shell, for this

$n_1 = 2$ (for L - shell)

$n_2 = 4$ (for N - shell)

$\therefore$ Eq (i) becomes

$$\frac{1}{\lambda'} = RZ^2\left[\frac{1}{2^2} - \frac{1}{4^2}\right] = \frac{3}{16}RZ^2 \quad \ldots(iii)$$

Now, we divide Eq (ii) by Eq (iii)

$$\frac{\lambda'}{\lambda} = \left(\frac{5}{36}RZ^2\right) \div \left(\frac{3}{16}RZ^2\right) = \frac{20}{27}$$

or $\lambda' = \frac{20}{27}\lambda$

21. A particle of mass m moves in a circular orbit in a central potential field $U(r) = \frac{1}{2}kr^2$. If Bohr's quantization conditions are applied, radii of possible orbitals and energy levels vary with quantum number n as

[JEE Main 2019, 12 Jan 2019, Shift-I]

(a) $r_n \propto n, E_n \propto n$ (b) $r_n \propto n^2, E_n \propto \frac{1}{n^2}$

(c) $r_n \propto \sqrt{n}, E_n \propto n$ (d) $r_n \propto \sqrt{n}, E_n \propto \frac{1}{n}$

Exp. **(c)**

As, for conservative fields $\mathbf{F} = -\left(\frac{d\mathbf{U}}{d\mathbf{r}}\right)$

$\therefore$ Magnitude of force on particle is

$$\Rightarrow \quad F = \frac{dU}{dr} = \frac{d}{dr}\left(\frac{1}{2}kr^2\right)$$

$$\Rightarrow \quad F = kr$$

This force is acting like centripetal force.

$$\therefore \quad \frac{mv^2}{r} = kr \quad \ldots(i)$$

So, for n^{th} orbit,

$$\Rightarrow \quad m^2v_n^2 = mkr_n^2$$

$$\Rightarrow \quad \frac{n^2h^2}{4\pi^2r^2} = mkr_n^2 \quad \left[\because v_n = \frac{nh}{2\pi mr}\right]$$

Therefore, $r_n^4 \propto n^2$

$$\Rightarrow \quad r_n^2 \propto n$$

So, $r_n \propto \sqrt{n}$

Energy of particle is

$$E_n = \text{PE} + \text{KE}$$

$$= \frac{1}{2}kr_n^2 + \frac{1}{2}mv_n^2$$

$$= \frac{1}{2}kr_n^2 + \frac{1}{2}kr_n^2 \quad \text{[using Eq. (i)]}$$

$$= kr_n^2$$

So, energy, $E_n \propto r_n^2$

$$\Rightarrow \quad E_n \propto n$$

22. In a radioactive decay chain, the initial nucleus is $^{232}_{90}Th$. At the end, there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus is $^{A}_{Z}X$, A and Z are given by

[JEE Main 2019, 12 Jan Shift-II]

(a) $A = 202; Z = 80$ (b) $A = 208; Z = 82$

(c) $A = 200; Z = 81$ (d) $A = 208; Z = 80$

Exp. **(b)**

An α-particle decay ($^{4}_{2}He$) reduces, mass number by 4 and atomic number by 2.

$\therefore$ Decay of 6α-particles results

$$^{232}_{90}Th \xrightarrow{6\,a} {}^{232-24}_{90-12}Y = {}^{208}_{78}Y$$

A β-decay does not produces any change in mass number but it increases atomic number by 1.

$\therefore$ Decay of 4β-particles results, $^{208}_{78}Y \xrightarrow{4\,b} {}^{208}_{82}X$

$\therefore$ In the end nucleus $A = 208$, $Z = 82$

23. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is **[JEE Main 2018]**

(a) $25\,\nu_L$ (b) $16\,\nu_L$ (c) $\frac{\nu_L}{16}$ (d) $\frac{\nu_L}{25}$

Exp. **(d)**

Series limit occurs in the transition $n_2 = \infty$ to $n_1 = 1$ in Lyman series and $n_2 = \infty$ to $n_1 = 5$ in pfund series. For Lyman series,

$$h\nu_L = 13.6 \quad \ldots(i)$$

In *p*fund series

$$h\nu_p = \frac{13.6}{5^2} \quad \ldots(ii)$$

From Eqs. (i) and (ii), we get

$$25h\nu_p = h\nu_L$$

$$\therefore \quad \nu_p = \frac{\nu_L}{25}$$

24. It is found that, if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is P_d; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively **[JEE Main 2018]**

(a) (.89, .28) (b) (.28, .89) (c) (0, 0) (d) (0, 1)

Exp. **(a)**

Neutron-Deuterium collision;

Momentum conservation gives;

$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_0 = v_1 + 2v_2 \quad \ldots(i)$$

Collision given is elastic .

So, coefficient of restitution, $e = 1$

$$\therefore \quad e = 1 = \frac{\text{Velocity of separation}}{\text{Velocity of approach}} \Rightarrow 1 = \frac{v_2 - v_1}{v_0 - 0}$$

$$\Rightarrow \quad v_0 = v_2 - v_1 \quad \ldots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2v_0 = 3v_2 \quad \Rightarrow \quad \frac{2v_0}{3} = v_2$$

So, from Eq. (i), we get

$$v_1 = v_0 - 2v_2 = v_0 - \frac{4v_0}{3} \quad \Rightarrow \quad v_1 = -\frac{v_0}{3}$$

Fractional loss of energy of neutron

$$= \left(\frac{-K_f + K_i}{K_i}\right)_{\text{for neutron}}$$

$$= \frac{-\frac{1}{2}mv_1^2 + \frac{1}{2}mv_0^2}{\frac{1}{2}mv_0^2} = \frac{-\frac{v_0^2}{9} + v_0^2}{v_0^2}$$

$$= \left(-\frac{1}{9} + 1\right) = \frac{8}{9} = 0.8\overline{8} = 0.89$$

Similarly, for neutron-carbon atom collision;

Momentum conservation gives;

$$v_0 = v_1 + 12v_2 \text{ and } e = 1 \quad \Rightarrow \quad v_0 = v_2 - v_1$$

So, $$v_1 = \frac{11}{13}v_0$$

$$\therefore \text{Loss of energy} = \left(-\frac{121}{169} + 1\right) = 0.28$$

So, $P_d = 0.89$ and $P_c = 0.28$

25. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths $r = \lambda_1/\lambda_2$ is given by **[JEE Main 2017 (Offline)]**

(a) $r = \frac{2}{3}$ (b) $r = \frac{3}{4}$

(c) $r = \frac{1}{3}$ (d) $r = \frac{4}{3}$

Exp. **(c)**

We have, $\lambda = \frac{hc}{\Delta E}$

$\therefore$ So, ratio of wave lengths

$$\frac{\lambda_1}{\lambda_2} = \frac{hc/\Delta E_1}{hc/\Delta E_2} = \frac{\Delta E_2}{\Delta E_1} = \frac{\left(\frac{4}{3}E - E\right)}{2E - E} = \frac{1}{3}$$

26. A radioactive nucleus A with a half-life T, decays into a nucleus B. At $t = 0$, there is no nucleus B. After sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by **[JEE Main 2017 (Offline)]**

(a) $t = T\dfrac{\log 1.3}{\log_e 2}$ (b) $t = T\log 1.3$

(c) $t = \dfrac{T}{\log 1.3}$ (d) $t = \dfrac{T\log_e 2}{2\log 1.3}$

Exp. **(a)**

Decay scheme is ,

Given, $\dfrac{N_B}{N_A} = 0.3 = \dfrac{3}{10} \Rightarrow \dfrac{N_B}{N_A} = \dfrac{30}{100}$

So, $N_0 = 100 + 30 = 130$ atoms

By using $N = N_0 e^{-\lambda t}$

We have, $100 = 130e^{-\lambda t}$

$\Rightarrow \dfrac{1}{1.3} = e^{-\lambda t}$

$\Rightarrow \log 1.3 = \lambda t$

If T is half-life, then $\lambda = \dfrac{\log_e 2}{T}$

$\Rightarrow \log 1.3 = \dfrac{\log_e 2}{T}\cdot t$

$\therefore t = \dfrac{T\cdot\log(1.3)}{\log_e 2}$

27. Half-lives of two radioactive elements A and B are 20 min and 40 min, respectively. Initially, the samples have equal number of nuclei. After 80 min, the ratio of decayed numbers of A and B nuclei will be **[JEE Main 2016 (Offline)]**

(a) 1 : 16 (b) 4 : 1

(c) 1 : 4 (d) 5 : 4

Exp. **(d)**

Given, 80 min = 4 half-lives of A = 2 half-lives of B.

Let the initial number of nuclei in each sample be N.

For radioactive element *A*,

N_A after 80 min $= \dfrac{N}{2^4}$

$\Rightarrow$ Number of A nuclides decayed

$= N - \dfrac{N}{16} = \dfrac{15}{16}N$

For radioactive element *B*,

N_B after 80 min. $= \dfrac{N}{2^2} \Rightarrow$ Number of B nuclides decayed $= N - \dfrac{N}{4} = \dfrac{3}{4}N$

$\therefore$ Ratio of decayed numbers of A and B nuclei will be

$$\frac{(15/16)N}{(3/4)N} = \frac{5}{4}$$

28. As an electron makes a transition from an excited state to the ground state of a hydrogen like atom/ion **[JEE Main 2015]**

(a) its kinetic energy increases but potential energy and total energy decrease

(b) kinetic energy, potential energy and total energy decrease

(c) kinetic energy decreases, potential energy increases but total energy remains same

(d) kinetic energy and total energy decrease but potential energy increases

Exp. **(a)**

As we know that kinetic energy of an electron is

$$\text{KE} \propto \left(\frac{Z}{n}\right)^2$$

When the electron makes transition from an excited state to the ground state, then n decreases and KE increases. We know that PE is lowest for ground state. As TE = – KE. TE also decreases.

29. Hydrogen $({}_1H^1)$, deuterium $({}_1H^2)$, singly ionised helium $({}_2He^4)^+$ and doubly ionised lithium $({}_3Li^8)^{++}$ all have one electron around the nucleus. Consider an electron transition form $n = 2$ to $n = 1$. If the wavelengths of emitted radiation are $\lambda_1, \lambda_2, \lambda_3$ and λ_4 respectively for four elements, then approximately which one of the following is correct? **[JEE Main 2014]**

(a) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (b) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$

(c) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (d) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

Exp. **(c)**

For hydrogen atom, we get

$$\frac{1}{\lambda} = R\,Z^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$\frac{1}{\lambda_1} = R(1)^2\left(\frac{3}{4}\right)$$

$$\Rightarrow \quad \frac{1}{\lambda_2} = R(1)^2\left(\frac{3}{4}\right)$$
$$\frac{1}{\lambda_3} = R(2)^2\left(\frac{3}{4}\right)$$
$$\frac{1}{\lambda_4} = R(3)^2\left(\frac{3}{4}\right)$$
$$\therefore \quad \frac{1}{\lambda_1} = \frac{1}{4\lambda_3} = \frac{1}{9\lambda_4} = \frac{1}{\lambda_2}$$

30. In a hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number $(n-1)$. If $n >> 1$, the frequency of radiation emitted is proportional to **[JEE Main 2013]**

(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^3/2}$ (d) $\frac{1}{n^3}$

Exp. **(d)**

$$\Delta E = h\nu$$

where, ΔE is energy of radiation, h is Planck's constant and ν is frequency.

$$\Rightarrow \quad \nu = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$
$$= \frac{k2n}{n^2(n-1)^2} \approx \frac{2k}{n^3} \propto \frac{1}{n^3}$$

31. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then, the number of spectral lines in the emission spectra will be **[JEE Main 2013]**

(a) 2 (b) 3 (c) 5 (d) 6

Exp. **(d)**

In emission spectrum, number of bright lines is given by

$$\frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

32. Assume that a neutron breaks into a proton and an electron. The energy released during this process is (mass of neutron $= 1.6725 \times 10^{-27}$ kg, mass of proton $= 1.6725 \times 10^{-27}$ kg, mass of electron $= 9 \times 10^{-31}$ kg) **[AIEEE 2012]**

(a) 0.73 MeV (b) 7.10 MeV
(c) 6.30 MeV (d) 5.4 MeV

Exp. **(a)**

According to given data, mass of neutron and proton are equal which do not permit the breaking up of neutron and proton.

But if we take standard mass of neutron as 1.6750×10^{-27} kg, then

Energy released = Mass defect $\times c^2$

$$= (m_n - m_p - m_e) \times c^2$$

[$\because$ energy released mass of reactant – mass of product $\times c^2$]

$$= \frac{(1.6750 \times 10^{-27} - 1.6725 \times 10^{-27} - 9 \times 10^{-31})}{1.66 \times 10^{-27}}$$

$\times$ 931.5 MeV [$\because$ 1amu = 931.5 MeV]

$$= \frac{16 \times 10^{-31}}{1.66 \times 10^{-27}} \times 931.5 \text{ MeV}$$
$$= \frac{1.6 \times 0.9315}{1.66} \text{ MeV}$$
$$\approx 0.9 \text{ MeV}$$

which is close to option (a).

33. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantisation, its energy will be given by (n is an integer) **[AIEEE 2012]**

(a) $\frac{(m_1 + m_2)^2 n^2 h^2}{2 m_1^2 m_2^2 r^2}$ (b) $\frac{n^2 h^2}{2(m_1 + m_2) r^2}$

(c) $\frac{2n^2 h^2}{(m_1 + m_2) r^2}$ (d) $\frac{(m_1 + m_2) n^2 h^2}{2 m_1 m_2 r^2}$

Exp. **(d)**

Rotational kinetic energy of the two body system rotating about their centre of mass is

$$\text{RKE} = \frac{1}{2}\mu\omega^2 r^2$$

where, $\mu = \frac{m_1 m_2}{m_1 + m_2}$ = reduced mass

and angular momentum, $L = \mu\omega r^2 = \frac{nh}{2\pi}$

$$\therefore \quad \text{RKE} = \frac{1}{2}\mu\omega^2 r^2 = \frac{1}{2}\mu \cdot \left(\frac{nh}{2\pi\mu r^2}\right)^2 r^2$$
$$= \frac{n^2 h^2}{8\pi^2 \mu r^2} = \frac{n^2 \hbar^2}{2\mu r^2}$$
$$= \frac{(m_1 + m_2) n^2 \hbar^2}{2 m_1 m_2 r^2}$$

34. After absorbing a slowly moving neutron of mass m_N (momentum ~ 0), a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1, (6m_1 = M + m_N)$, respectively. If the de-Broglie wavelength of the nucleus with mass m_1 is λ, then de-Broglie wavelength of the other nucleus will be **[AIEEE 2011]**

(a) 25λ (b) 5λ (c) $\lambda / 5$ (d) λ

Exp. **(d)**

de-Broglie wavelength, $\lambda = \frac{h}{mv} = \frac{h}{p}$

where, p = momentum

By conservation of momentum,

$$p_1 + p_2 = 0 \text{ or } p_1 = p_2 \Rightarrow \frac{h}{p_1} = \frac{h}{p_2}$$

$$\therefore \quad \lambda_1 = \lambda_2 = \lambda \qquad \left(\because \lambda = \frac{h}{p}\right)$$

35. **Statement I** A nucleus having energy E_1 decays be β^- emission to daughter nucleus having energy E_2, but β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement II To conserve energy and momentum in β-decay, atleast three particles must take part in the transformation. **[AIEEE 2011]**

(a) Statement I is false, Statement II is true
(b) Statement I is true, Statement II is false
(c) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I
(d) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

Exp. **(c)**

In particle situation, atleast three particles take place in transformation, so

Energy of β-particle + Energy of third particle

$$= E_1 - E_2$$

Hence, energy of β-particle $\le E_1 - E_2$

36. Energy required for the electron excitation in Li^{2+} from the first to the third Bohr orbit is

(a) 36.3 eV (b) 108.8 eV **[AIEEE 2011]**
(c) 122.4 eV (d) 12.1 eV

Exp. **(b)**

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \quad \left[\because \Delta E = Rhc\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\right]$$

$$= 13.6(3)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 108.8 \text{ eV}$$

37. The half-life of a radioactive substance is 20 min. The approximate time interval $(t_2 - t_1)$ between the time t_2 when 2/3 of it has decayed and time t_1 when 1/3 of it had decayed is **[AIEEE 2011]**

(a) 14 min (b) 20 min (c) 28 min (d) 7 min

Exp. **(b)**

$$N_1 = N_0 - \frac{1}{3}N_0 = \frac{2}{3}N_0$$

$$N_2 = N_0 - \frac{2}{3}N_0 = \frac{1}{3}N_0$$

$$\frac{N_1}{N_2} = \left(\frac{1}{2}\right)^n \Rightarrow n = 1$$

$\therefore \quad t_2 - t_1$ = one half-life = 20 min

➡ **Directions** *(Q. Nos. 38 to 39) are based on the following paragraph.*

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $M/2$ each. Speed of light is c.

38. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then, **[AIEEE 2010]**

(a) $E_2 = 2E_1$ (b) $E_1 > E_2$
(c) $E_2 > E_1$ (d) $E_1 = 2E_2$

Exp. **(c)**

After decay, the daughter nuclei will be more stable, hence binding energy per nucleon will be more than that of their parent nucleus.

39. The speed of daughter nuclei is **[AIEEE 2010]**

(a) $c\frac{\Delta m}{M + \Delta m}$ (b) $c\sqrt{\frac{2\Delta m}{M}}$
(c) $c\sqrt{\frac{\Delta m}{M}}$ (d) $c\sqrt{\frac{\Delta m}{M + \Delta m}}$

Exp. **(b)**

By conservation of momentum, $0 = \frac{M}{2}v_1 - \frac{M}{2}v_2$

$$v_1 = v_2 \quad \text{...(i)}$$

$$\Delta mc^2 = \frac{1}{2}\cdot\frac{M}{2}v_1^2 + \frac{1}{2}\cdot\frac{M}{2}\cdot v_2^2 \quad \text{...(ii)}$$

$$\Delta mc^2 = \frac{M}{2}v_1^2$$

$$\frac{2\Delta mc^2}{M} = v_1^2 \Rightarrow v_1 = c\sqrt{\frac{2\Delta m}{M}}$$

40. A radioactive nucleus (initial mass number A and atomic number Z) emits 3α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be **[AIEEE 2010]**

(a) $\frac{A-Z-8}{Z-4}$ (b) $\frac{A-Z-4}{Z-8}$

(c) $\frac{A-Z-12}{Z-4}$ (d) $\frac{A-Z-4}{Z-2}$

Exp. **(b)**

In positive beta decay, a proton is transformed into a neutron and a positron is emitted.

$$p^+ \longrightarrow n^0 + e^+$$

Number of neutrons initially was $A - Z$.

Number of neutrons after decay $(A - Z) - 3 \times 2$ (due to alpha particles) $+ 2 \times 1$ (due to positive beta decay)

The number of protons will reduce by 8.

[as 3×2 (due to alpha particles) + 2 (due to positive beta decay)]

Hence, atomic number reduces by 8.

So, the ratio of number of neutrons to that of protons $= \frac{A-Z-4}{Z-8}$

41. The below is a plot of binding energy per nucleon E_b, against the nuclear mass M; A, B, C, D, E, F correspond to different nuclei. Consider four reactions

(i) $A + B \rightarrow C + \varepsilon$ (ii) $C \rightarrow A + B + \varepsilon$

(iii) $D + E \rightarrow F + \varepsilon$ and (iv) $F \rightarrow D + E + \varepsilon$

where, ε is the energy released. In which reactions ε is positive? **[AIEEE 2009]**

(a) (i) and (iv) (b) (i) and (iii)

(c) (ii) and (iv) (d) (ii) and (iii)

Exp. **(a)**

1st reaction is fusion and 4th reaction is fission.

42. The transition from the state $n = 4$ to $n = 3$ in a hydrogen like atom results in ultraviolet radiation. Infrared radition will be obtained in the transition from **[AIEEE 2009]**

(a) $2 \rightarrow 1$ (b) $3 \rightarrow 2$ (c) $4 \rightarrow 2$ (d) $5 \rightarrow 4$

Exp. **(d)**

IR corresponds to least value of $\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

i.e., from Paschen, Brackett and Pfund series. Thus, the transition corresponds to $5 \rightarrow 4$.

➡ **Directions** *Q. No.* 43 *is Assertion-Reason type question. The question contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). The question also has four alternative choices, only one of which is correct answer. You have to select the correct choice.*

43. **Statement I** Energy is released when heavy nuclei undergo fission or light nuclei undergo fusion.

Statement II For heavy nuclei, binding energy per nucleon increases with increasing Z while for light nuclei, it decreases with increasing Z. **[AIEEE 2008]**

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I

(b) Statement I is true, Statement II is false

(c) Statement I is false, Statement II is true

(d) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

Exp. **(b)**

Statement I is correct and Statement II is wrong which can be directly concluded from binding energy/ nucleon curve.

44. Suppose an electron is attracted towards the origin by a force k/r, where k is a constant and r is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be r_n and the kinetic energy of the electron to be T_n. Then, which of the following is true? **[AIEEE 2008]**

(a) $T_n \propto \frac{1}{n^2}, r_n \propto n^2$ (b) T_n is independent of $n, r_n \propto n$

(c) $T_n \propto \frac{1}{n}, r_n \propto n$ (d) $T_n \propto \frac{1}{n}, r_n \propto n^2$

Exp. **(b)**

$$\frac{mv^2}{r_n} = \frac{k}{r_n} \quad \text{[given]}$$

$$mvr_n = \frac{nh}{2\pi} \quad \text{[from Bohr's theory]}$$

On solving, $r_n \propto n$ and T_n is independent of n.

45. The half-life period of a radioactive element X is same as the mean life time of another radioactive element Y. Initially they have the same number of atoms. Then, **[AIEEE 2007]**

(a) X will decay faster than Y

(b) Y will decay faster than X

(c) Y and X have same decay rate initially

(d) X and Y decay at same rate always

Exp. **(b)**

$$T_{1/2}(X) = \tau(Y)$$

$$\Rightarrow \quad \frac{0.693}{\lambda_X} = \frac{1}{\lambda_Y}$$

or $$\lambda_Y = \frac{\lambda_X}{0.693}$$

$$\Rightarrow \quad \lambda_Y > \lambda_X$$

So, Y will decay faster than X.

46. If M_o is the mass of an oxygen isotope ${}_8O^{17}$, M_p and M_n are the masses of a proton and a neutron respectively, the nuclear binding energy of the isotope is **[AIEEE 2007]**

(a) $(M_o - 8M_p)c^2$ (b) $(M_o - 8M_p - 9M_n)c^2$

(c) $M_o c^2$ (d) $(M_o - 17M_n)c^2$

Exp. **(b)**

Binding energy, BE = $(M_{\text{nucleus}} - M_{\text{nucleons}})c^2$

$= (M_o - 8M_p - 9M_n)c^2$

47. Which of the following transitions in hydrogen atoms emit photons of highest frequency? **[AIEEE 2007]**

(a) $n = 2$ to $n = 6$ (b) $n = 6$ to $n = 2$

(c) $n = 2$ to $n = 1$ (d) $n = 1$ to $n = 2$

Exp. **(c)**

Emission spectrum would rises when electron makes a jump from higher energy level to lower energy level.

Frequency of emitted photon is proportional to change in energy of two energy levels, *i.e.*,

$$\nu = RcZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

48. In gamma ray emission from a nucleus,

(a) both the neutron number and the proton number change **[AIEEE 2007]**

(b) there is no change in the proton number and the neutron number

(c) only the neutron number changes

(d) only the proton number changes

Exp. **(b)**

In gamma ray emission, the energy is released from nucleus, so that nucleus get stabilised (to get the minimum energy state).

49. An alpha nucleus of energy $(1/2)\,mv^2$ bombards a heavy nuclear target of charge Ze. Then, the distance of closest approach for the alpha nucleus will be proportional to **[AIEEE 2006]**

(a) v^2 (b) $\frac{1}{m}$ (c) $\frac{1}{v^4}$ (d) $\frac{1}{Ze}$

Exp. **(d)**

Since, nuclear target is heavy, it can be assumed safely that it will remain stationary and will not move due to the Coulombic interaction force.

At distance of closest approach, relative velocity of two particles is v.

Here, target is considered as stationary, so α-particle comes to rest instantaneously at distance be closest approach. Let required distance be r, then from work-energy theorem,

$$0 - \frac{mv^2}{2} = -\frac{1}{4\pi\varepsilon_0}\frac{Ze \times 2e}{r}$$

$$\Rightarrow \quad r \propto \frac{1}{m} \text{ or } r \propto \frac{1}{v^2} \text{ or } r \propto Ze^2$$

50. If the binding energy per nucleon in ${}_3^7Li$ and ${}_2^4He$ nuclei are 5.60 MeV and 7.06 MeV respectively, then in the reaction

$$p + {}_3^7Li \longrightarrow 2\,{}_2^4He$$

energy of proton must be **[AIEEE 2006]**

(a) 28.24 MeV (b) 17.28 MeV

(c) 1.46 MeV (d) 39.2 MeV

Exp. **(b)**

From the equation,

Energy of proton + $(7 \times 5.60) = 2 \times [4 \times 7.06]$

$\therefore$ Energy of proton = 17.28 MeV

51. The energy spectrum of β-particles [number $N(E)$ as a function of β-energy E] emitted from a radioactive source is **[AIEEE 2006]**

Exp. **(c)**

Energy spectrum of emitted β-particles from radioactive source is drawn as

52. When $^{7}_{3}Li$ nuclei are bombarded by protons, and the resultant nuclei are $^{8}_{4}Be$, the emitted particles will be **[AIEEE 2006]**

(a) alpha particles (b) beta particles
(c) gamma photons (d) neutrons

Exp. **(c)**

The nuclear reaction can be represented as

$$^{7}_{3}Li + ^{1}_{1}H \longrightarrow ^{8}_{4}Be + ^{A}_{Z}X$$

Applying conservation of atomic number (charge),

$$3 + 1 = 4 + Z$$

$\Rightarrow$ $Z = 0$

Applying conservation of atomic mass,

$$7 + 1 = 8 + A$$

$\Rightarrow$ $A = 0$

Thus, the emitted particles are γ-photons ($^{0}_{0}X$).

53. Starting with a sample of pure ^{66}Cu, 7/8 of it decays into Zn in 15 min. The corresponding half-life is **[AIEEE 2005]**

(a) 10 min (b) 15 min (c) 5 min (d) $7\frac{1}{2}$ min

Exp. **(c)**

$$N = N_0(1 - e^{-\lambda t}) \Rightarrow \frac{N_0 - N}{N_0} = e^{-\lambda t}$$

$\therefore$ $1/8 = e^{-\lambda t}$ or $8 = e^{\lambda t}$

or $3 \ln 2 = \lambda t$ $[\because \log_e m^n = n \log_e m]$

or $\lambda = \dfrac{3 \times 0.693}{15}$

Half-life period, $t_{1/2} = \dfrac{0.693}{3 \times 0.693} \times 15$

$$\left[\because t_{1/2} = \frac{0.693}{\lambda}\right]$$

= 5 min

54. If radius of the $^{27}_{13}Al$ nucleus is estimated to be 3.6 fm, then the radius of $^{125}_{52}Te$ nucleus be nearly **[AIEEE 2005]**

(a) 6 fm (b) 8 fm (c) 4 fm (d) 5 fm

Exp. **(a)** $R = R_0(A)^{1/3}$

$$\frac{R_{Al}}{R_{Te}} = \frac{R_0(A_{Al})^{1/3}}{R_0(A_{Te})^{1/3}} \text{ or } \frac{R_{Al}}{R_{Te}} = \frac{(A_{Al})^{1/3}}{(A_{Te})^{1/3}}$$

or $\dfrac{R_{Al}}{R_{Te}} = \dfrac{(27)^{1/3}}{(125)^{1/3}} = \dfrac{3}{5}$

or $R_{Te} = \dfrac{5}{3} \times 3.6$ or $R_{Te} = 6$ fm

55. The diagram shows the energy levels for an electron in a certain atom. Which transition shown represents the emission of a photon with the most energy? **[AIEEE 2005]**

(a) III (b) IV
(c) I (d) II

Exp. **(a)**

$$E = Rhc\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

$$E_{(4\to 3)} = Rhc\left[\frac{1}{3^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{7}{9\times 16}\right] = 0.05\,Rhc$$

$$E_{(4\to 2)} = Rhc\left[\frac{1}{2^2} - \frac{1}{4^2}\right]$$

$$= Rhc\left[\frac{3}{16}\right] = 0.2\,Rhc$$

$$E_{(2\to 1)} = Rhc\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right]$$

$$= Rhc\left[\frac{3}{4}\right] = 0.75\,Rhc$$

$$E_{(1\to 3)} = Rhc\left[\frac{1}{(3)^2} - \frac{1}{(1)^2}\right] = -\frac{8}{9}\,Rhc = -0.9\,Rhc$$

Thus, III transition gives most energy. I transition represents the absorption of energy.

56. A nuclear transformation is denoted by $X(n,\alpha)\to {}^7_3\text{Li}$. Which of the following is the nucleus of element X? **[AIEEE 2005]**

(a) ${}^{12}_{6}\text{C}$ (b) ${}^{10}_{5}\text{B}$ (c) ${}^{9}_{5}\text{B}$ (d) ${}^{11}_{4}\text{Be}$

Exp. **(b)**

$${}^A_Z X + {}^1_0 n \longrightarrow {}^7_3\text{Li} + {}^4_2\text{He}$$

It implies that, $A + 1 = 7 + 4 \Rightarrow A = 10$

and $Z + 0 = 3 + 2 \Rightarrow Z = 5$

Thus, it is Boron ${}^{10}_{5}\text{B}$.

57. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1. The ratio of their nuclear sizes will be **[AIEEE 2004]**

(a) $2^{1/3}:1$ (b) $1:3^{1/2}$ (c) $3^{1/2}:1$ (d) $1:2^{1/3}$

Exp. **(d)**

Law of conservation of momentum gives

$$m_1v_1 = m_2v_2 \Rightarrow \frac{m_1}{m_2} = \frac{v_2}{v_1}$$

But $m = \frac{4}{3}\pi r^3\rho$ or $m \propto r^3$

$$\therefore \quad \frac{m_1}{m_2} = \frac{r_1^3}{r_2^3} = \frac{v_2}{v_1}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \left(\frac{1}{2}\right)^{1/3}$$

or $r_1 : r_2 = 1 : 2^{1/3}$

58. The binding energy per nucleon of deuteron (${}^2_1\text{H}$) and helium nucleus (${}^4_2\text{He}$) is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is **[AIEEE 2004]**

(a) 13.9 MeV (b) 26.9 MeV
(c) 23.6 MeV (d) 19.2 MeV

Exp. **(c)**

As given ${}^2_1\text{H} + {}^2_1\text{H} \longrightarrow {}^4_2\text{He}$ + energy

The binding energy per nucleon of a deuteron(${}^2_1\text{H}$)

= 1.1 MeV

$\therefore$ Total binding energy of one deuteron nucleus

$= 2 \times 1.1$

$= 2.2$ MeV

The binding energy per nucleon of helium (${}^4_2\text{He}$)

= 7 MeV

$\therefore$ Total binding energy $= 4 \times 7 = 28$ MeV

Hence, energy released in the above process

$= 28 - 2 \times 2.2$

$= 28 - 4.4 = 23.6$ MeV

59. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of the closest approach is of the order of **[AIEEE 2004]**

(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12} cm (d) 10^{-15} cm

Exp. **(c)**

According to law of conservation of energy,

Kinetic energy of α-particle = Potential energy of α-particle at distance of closest approach

i.e., $$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$$

$$\therefore \quad 5\text{ MeV} = \frac{9\times 10^9 \times (2e)\times(92e)}{r}$$

$$\left[\because \frac{1}{2}mv^2 = 5\text{ MeV, given}\right]$$

$$\Rightarrow \quad r = \frac{9\times 10^9 \times 2\times 92\times(1.6\times 10^{-19})^2}{5\times 10^6 \times 1.6\times 10^{-19}}$$

or $r = 5.3\times 10^{-14}\text{ m} \approx 10^{-12}$ cm

60. The manifestation of band structure in solids is due to **[AIEEE 2004]**

(a) Heisenberg's uncertainty principle
(b) Pauli's exclusion principle
(c) Bohr's correspondence principle
(d) Boltzmann's law

Exp. **(b)**

According to Pauli's exclusion principle, the electronic configuration of number of subshells existing in a shell and number of electrons entering each subshell is found.

Hence, on the basis of Pauli's exclusion principle, the manifestation of band structure in solids can be explained.

61. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is **[AIEEE 2003]**

(a) $\frac{4u}{238}$ (b) $-\frac{4u}{234}$
(c) $\frac{4u}{234}$ (d) $-\frac{4u}{238}$

Exp. **(c)**

Apply conservation of linear momentum.

$$0 = 4u - 234v \qquad [\because Mu = m_1v_1 + m_2v_2]$$

$$\Rightarrow \qquad v = \frac{4u}{234}$$

The residual nucleus will recoil with a velocity of $\frac{4u}{234}$ unit.

Note If they will ask the recoil velocity, then answer remains same *i.e.*, $\frac{4u}{234}$ and not $-\frac{4u}{234}$ as the word 'recoil' itself is signifying the direction of motion of residual nucleus.

62. A radioactive sample at any instant has its disintegration rate 5000 disintegrations/min. After 5 min, the rate is 1250 disintegrations/min. Then, the decay constant (per minute) is **[AIEEE 2003]**

(a) $0.4 \ln 2$ (b) $0.2 \ln 2$ (c) $0.1 \ln 2$ (d) $0.8 \ln 2$

Exp. **(a)**

Given, $N_0\lambda = 5000$, $N\lambda = 1250$

$$N = N_0 e^{-\lambda t} = N_0 e^{-5\lambda}$$

where, λ is decay constant per minute.

$$N\lambda = N_0\lambda e^{-5\lambda}$$

$$\Rightarrow \qquad 1250 = N_0\lambda e^{-5\lambda}$$

$$\therefore \qquad \frac{N_0\lambda}{N_0\lambda e^{-5\lambda}} = \frac{5000}{1250} = 4$$

or $\qquad e^{5\lambda} = 4$

or $\qquad 5\lambda = 2 \log_e 2$

or $\qquad \lambda = 0.4 \ln 2$

63. A nucleus with $Z = 92$ emits the following in a sequence

$\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha; \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha.$

The Z of the resulting nucleus is **[AIEEE 2003]**

(a) 76 (b) 78
(c) 82 (d) 74

Exp. **(b)**

Since, 8 α-particles $4\beta^-$-particles and $2\beta^+$-particles are emitted, so new atomic number [$\alpha \rightarrow {}^4_2\text{He}$ and $\beta \rightarrow -{}^0_1 n$].

$$Z' = Z - 8 \times 2 + 4 \times 1 - 2 \times 1$$
$$= 92 - 16 + 4 - 2 = 92 - 14 = 78$$

64. Which of the following cannot be emitted by radioactive substances during their decay?

(a) Protons (b) Neutrinos **[AIEEE 2003]**
(c) Helium nuclei (d) Electrons

Exp. **(a)**

Protons cannot be emitted by radioactive substances during their decay.

65. In the nuclear fusion reaction,

$${}^2_1\text{H} + {}^3_1\text{H} \longrightarrow {}^4_2\text{He} + n$$

given that the repulsive potential energy between the two nuclei is 7.7×10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J/K] **[AIEEE 2003]**

(a) 10^7 K (b) 10^5 K
(c) 10^3 K (d) 10^9 K

Exp. (b)

$$\frac{3}{2}T = 7.7 \times 10^{-14} \text{ J}$$

or $$T = \frac{2 \times 7.7 \times 10^{-14}}{3 \times 1.38 \times 10^{-23}} = 3.7 \times 10^{9} \text{ K}$$

66. Which of the following atoms has the lowest ionisation potential? **[AIEEE 2003]**

(a) $^{14}_{7}N$ (b) $^{133}_{55}Cs$
(c) $^{40}_{18}Ar$ (d) $^{16}_{8}O$

Exp. (b) According to problem,

Since, ionisation potential is minimum energy required to eject the etectron from the outermest orbit. As $^{133}_{55}Cs$ has larger size among the four atoms given, thus electrons present in the outermost orbit will be away from the nucleus and the electrostatic force experienced by electrons due to nucleus will be minimum.

Therefore, the energy required to liberate electrons from outer orbit will be minimum in case of $^{133}_{55}Cs$.

67. The wavelengths involved in the spectrum of deuterium ($^{2}_{1}D$) are slightly different from that of hydrogen spectrum, because **[AIEEE 2003]**

(a) sizes of the two nuclei are different
(b) nuclear forces are different in the two cases
(c) masses of the two nuclei are different
(d) attraction between the electron and the nucleus is different in the two cases

Exp. (c)

$\because$ Masses of the two nuclei are different.

68. If the binding energy of the electron in a hydrogen atom is 13.6 eV, the energy required to remove the electron from the first excited state of Li^{2+} is **[AIEEE 2003]**

(a) 30.6 eV (b) 13.6 eV
(c) 3.4 eV (d) 122.4 eV

Exp. (a)

$$E = -Z^2 \frac{13.6}{n^2} \text{ eV}$$

For first excited state,

$$E_2 = -3^2 \times \frac{13.6}{4}$$

[$\because n = 2$, for Ist excited state]

$$= -30.6 \text{ eV}$$

Ionisation energy for first excited state of Li^{2+} is 30.6 eV.

69. If 13.6 eV energy is required to ionise the hydrogen atom, then the energy required to remove an electron from $n = 2$ is **[AIEEE 2002]**

(a) 10.2 eV (b) zero (c) 3.4 eV (d) 6.8 eV

Exp. (c)

Energy required to remove an electron from nth orbit is

$$E_n = -\frac{13.6}{n^2}$$

Here, $n = 2$

Therefore, $E_2 = -\frac{13.6}{2^2} = -3.4$ V

70. If N_0 is the original mass of the substance of half-life period $t_{1/2} = 5$ yr, then the amount of substance left after 15 yr, is **[AIEEE 2002]**

(a) $\frac{N_0}{8}$ (b) $\frac{N_0}{16}$ (c) $\frac{N_0}{2}$ (d) $\frac{N_0}{4}$

Exp. (a)

N_0 is the initial amount of substance and N is the amount left after decay.

Thus, $$N = N_0 \left(\frac{1}{2}\right)^n$$

$$n = \text{number of half-lives} = \frac{t}{t_{1/2}} = \frac{15}{5} = 3$$

Therefore, $$N = N_0 \left(\frac{1}{2}\right)^3 = \frac{N_0}{8}$$

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19

Electronic Devices

1. The reverse breakdown voltage of a Zener diode is 5.6 V in the given circuit.

The current I_z through the Zener is

(a) 10 mA (b) 17 mA
(c) 15 mA (d) 7 mA

[JEE Main 2019, 8 April Shift-I]

Exp. **(a)**

Given circuit is Zener diode circuit

where,potential drop across 800 Ω resistance = potential drop across Zener diode = 5.6 V

So, current, $i_2 = \frac{V}{R} = \frac{5.6}{800} = 7\text{mA}$

Now, potential drop across 200 Ω resistance

$= 9 - 5.6 = 3.4\text{ V}$

Current, $i_1 = \frac{V}{R} = \frac{3.4}{200} = 17\text{ mA}$

So, current, $i_z = i_1 - i_2 = 17 - 7 = 10\text{ mA}$

2. A common emitter amplifier circuit, built using an n-p-n transistor, is shown in the figure. Its DC current gain is 250, $R_C = 1\,\text{k}\Omega$ and $V_{CC} = 10$ V. What is the minimum base current for V_{CE} to reach saturation?

[JEE Main 2019, 8 April Shift-II]

(a) 40μA (b) 10μA
(c) 100μA (d) 7μA

Exp. **(a)**

For a common emitter *n-p-n* transistor, DC current gain is

$$\beta_{DC} = \frac{I_C}{I_B}$$

At saturation state, V_{CE} becomes zero.

$$\therefore \quad V_{CC} - I_C R_C = 0$$

$$I_C \approx \frac{V_{CC}}{R_C}$$

$$= \frac{10}{1000} = 10^{-2}\text{A}$$

Hence, saturation base current,

$$I_B = \frac{I_C}{\beta_{DC}} = \frac{10^{-2}}{250} = 40\mu\text{A}$$

3. An *n-p-n* transistor is used in common emitter configuration as an amplifier with 1 kΩ load resistance. Signal voltage of 10 mV is applied across the base-emitter. This produces a 3 mA change in the collector current and 15 μA change in the base current of the amplifier. The input resistance and voltage gain are **[JEE Main 2019, 9 April Shift-I]**

(a) 0.67 kΩ, 200 (b) 0.33 kΩ, 1.5
(c) 0.67 kΩ, 300 (d) 0.33 kΩ, 300

Exp. **(c)**

Given, load resistance, $R_L = 1\,\text{k}\Omega$

Input voltage, $V_{in} = 10\ \text{mV} = 10 \times 10^{-3}$ V

Base current, $\Delta I_B = 15\,\mu\text{A} = 15 \times 10^{-6}$ A

Collector current, $\Delta I_C = 3\,\text{mA}$

Input resistance,

$$R_{in} = \frac{V_{in}}{\Delta I_B} = \frac{10 \times 10^{-3}}{15 \times 10^{-6}} = 0.67\,\text{k}\Omega$$

and voltage gain $= \beta \times \dfrac{R_L}{R_{in}}$

$$= \frac{\Delta I_C \times R_L}{\Delta I_B \times R_{in}} = \left(\frac{3\,\text{mA}}{15\mu\text{A}}\right) \times \left(\frac{1\,\text{k}\Omega}{0.67\ \text{k}\Omega}\right)$$

$$= \left(\frac{3 \times 10^{-3}}{15 \times 10^{-6}}\right)\left(\frac{1 \times 10^{3}}{0.67 \times 10^{3}}\right)$$

$$= \frac{1000 \times 3 \times 3}{15 \times 2} = 300 \qquad (\because 0.67 \cong 2/3)$$

Alternate Solution

$$\therefore \text{Voltage gain} = \frac{V_{output}}{V_{input}}$$

$$= \frac{R_L \times \Delta I_C}{V_{in}}$$

$$= \frac{1 \times 10^{3} \times 3 \times 10^{-3}}{10 \times 10^{-3}} = 300$$

4. The logic gate equivalent to the given logic circuit is **[JEE Main 2019, 9 April Shift-II]**

(a) NOR (b) NAND
(c) OR (d) AND

Exp. **(c)**

Truth table for given combination of logic gates is

A	B	$Y_1 = \overline{A}$	$Y_2 = \overline{B}$	$Y = \overline{Y_1 \cdot Y_2}$
0	0	1	1	0
1	0	0	1	1
0	1	1	0	1
1	1	0	0	1

Output Y resembles output of an OR gate. So, given combination acts like an OR gate.

Alternate Solution

The given logic gate circuit can be drawn as shown below

Here, $Y = \overline{Y_1 \cdot Y_2} = \overline{\overline{A} \cdot \overline{B}}$

Using de-Morgan's theorem, i.e.

$$\overline{x \cdot y} = \overline{x} + \overline{y}$$

$\therefore Y = \overline{\overline{A}} + \overline{\overline{B}} = A + B \qquad [\because \overline{\overline{x}} = x]$

This represents the boolean expression for OR gate.

5. Four point charges $-q, +q, +q$ and $-q$ are placed on Y-axis at $y = -2d$, $y = -d$, $y = +d$ and $y = +2d$, respectively. The magnitude of the electric field E at a point on the X-axis at $x = D$, with $D >> d$, will behave as **[JEE Main 2019, 9 April Shift-II]**

(a) $E \propto \dfrac{1}{D}$ (b) $E \propto \dfrac{1}{D^3}$
(c) $E \propto \dfrac{1}{D^2}$ (d) $E \propto \dfrac{1}{D^4}$

Exp. **(*d*)**

Given charge distribution is as shown below

So, we can view above point charges as combination of pair of dipoles or a quadrupole.

By symmetry, the field components parallel to quadrupole cancels and the resultant perpendicular field is

$$E = \frac{2q}{4\pi\varepsilon_0}\left(\frac{1}{D^2} - \frac{D}{(D^2 + d^2)^{3/2}}\right)$$

$$= \frac{2q}{4\pi\varepsilon_0 D^2}\left(1 - \left(1 + \frac{d^2}{D^2}\right)^{-3/2}\right)$$

As, $$\left(1 + \frac{d^2}{D^2}\right)^{-\frac{3}{2}} \approx \left(1 - \frac{3}{2}\frac{d^2}{D^2}\right)$$

(using binomial expansion)

We have,

$$E = \frac{3qd^2}{4\pi\varepsilon_0 D^4}$$

$$\therefore \quad E \propto \frac{1}{D^4}$$

Note Dependence of field for a point charge is

$$E \propto \frac{1}{r^2}$$

For a dipole, it is

$$E \propto \frac{1}{r^3}$$

For a quadrupole, it is

$$E \propto \frac{1}{r^4} \text{ ..... etc.}$$

Alternate Solution The given distribution of charges can be shown as the figure below

Electric field at point P,

$$E = E_1\cos\theta_1 + E_1\cos\theta_1 - E_2\cos\theta_2 - E_2\cos\theta_2$$

$$= 2E_1\cos\theta_1 - 2E_2\cos\theta_2$$

$$= \frac{2kq}{(d^2 + D^2)}\cos\theta_1 - \frac{2kq}{(2d)^2 + D^2}\cos\theta_2$$

As, $$\cos\theta_1 = \frac{D}{(d^2 + D^2)^{1/2}}$$

Similarly, $$\cos\theta_2 = \frac{D}{[(2d)^2 + D^2]^{1/2}}$$

$$= 2kqD[(d^2 + D^2]^{-3/2} - (4d^2 + D^2)^{-3/2}]$$

$$= \frac{2kqD}{D^3}\left[\left(1 + \frac{d^2}{D^2}\right)^{-3/2} - \left(1 + \frac{4d^2}{D^2}\right)^{-3/2}\right]$$

As $D >> d$, then by applying binomial approximation, we get

$$= \frac{2kq}{D^2}\left[1 - \frac{3}{2}\frac{d^2}{D^2} - \left(1 - \frac{3}{2}\left(\frac{4d^2}{D^2}\right)\right)\right]$$

$$= \frac{2kq}{D^2}\left[\frac{9d^2}{2D^2}\right] = \frac{9kqd^2}{D^4}$$

$$\Rightarrow \quad E \propto \frac{1}{D^4}$$

6. An *n-p-n* transistor operates as a common emitter amplifier, with a power gain of 60 dB. The input circuit resistance is 100 Ω and the output load resistance is 10 kΩ. The common emitter current gain β is

[JEE Main 2019, 10 April Shift-I]

(a) 10^2 (b) 6×10^2 (c) 10^4 (d) 60

Exp. **(a)**

Given, $A_P = 60\,\text{dB}$ (in decibel)

Power gain in decibel can be given as

$$A_P = 10\log_{10}\left(\frac{\text{Output power}}{\text{Input power}}\right)$$

$$\Rightarrow 60 = 10\log_{10}\left(\frac{P_{out}}{P_{in}}\right) \Rightarrow \log_{10}\left(\frac{P_{out}}{P_{in}}\right) = 6$$

$$\Rightarrow \quad \frac{P_{out}}{P_{in}} = 10^6 = A_P \quad \ldots \text{(i)}$$

Also, given $R_{out} = 10\,\text{k}\Omega$, $R_{in} = 100\,\Omega$

$\therefore$ Power gain of a transistor is given by

$$A_P = \beta^2\left(\frac{R_{out}}{R_{in}}\right)$$

where, β is current gain.

$$\Rightarrow \beta^2 = A_P \times \frac{R_{in}}{R_{out}} = 10^6 \times \frac{100}{10 \times 10^3}$$

$$\Rightarrow \beta^2 = 10^4 \text{ or } \beta = 10^2$$

7. The figure represents a voltage regulator circuit using a Zener diode. The breakdown voltage of the Zener diode is 6 V and the load resistance is $R_L = 4\,\text{k}\Omega$. The series resistance of the circuit is $R_i = 1\,\text{k}\Omega$. If the battery voltage V_B varies from 8V to 16V, what are the minimum and maximum values of the current through Zener diode?

[JEE Main 2019, 10 April Shift-II]

(a) 1.5 mA, 8.5 mA (b) 1 mA, 8.5 mA
(c) 0.5 mA, 8.5 mA (d) 0.5 mA, 6 mA

Exp. **(c)**

In given voltage regulator circuit,

Zener breakdown voltage, $V_Z = 6$ V

So, across R_L, potential drop is always 6 V.

So, current through load resistance is

$$i_L = \frac{V_Z}{R_L} = \frac{6}{4 \times 10^3} = 1.5 \times 10^{-3}\text{ A}$$

Now, when $V_B = 8$ V

Potential drop across $R_1 = 8 - 6 = 2$ V

So, current through R_1 is $i_1 = \dfrac{V}{R_1} = \dfrac{2}{1 \times 10^3}$

$$= 2 \times 10^{-3}\text{ A}$$

So, current through Zener diode is

$$i_Z = i_1 - i_L = 2 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 0.5 \times 10^{-3}\text{ A} = 0.5\,\text{mA}$$

Similarly, when $V_B = 16$ V

$$V_{R_1} = 16 - 6 = 10\text{ V}$$

$$\therefore \quad i_1 = \frac{10}{1 \times 10^3} = 10 \times 10^{-3}\text{ A}$$

Hence,

$$i_Z = i_1 - i_L = 10 \times 10^{-3} - 1.5 \times 10^{-3}$$

$$= 8.5 \times 10^{-3}\text{ A} = 8.5\,\text{mA}$$

8. The truth table for the circuit given in the figure is **[JEE Main 2019, 12 April Shift-I]**

(a)

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	1

(b)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

Exp. **(c)**

Given circuit is

Let the intermediate state X of OR gate is shown in figure.

Clearly, $Y = \overline{AX}$ ...(i)

Here, $X = A + B$ ...(ii)

$\therefore$ $Y = \overline{A(A+B)} = \overline{AA + AB}$

$= \overline{A + AB}$ $(\because AA = A)$

$= \overline{A(1+B)}$

$\Rightarrow$ $Y = \overline{A}$ $(\because 1 + B = 1)$

So, truth table shown in option (c) is correct.

Alternate Solution We can solve it using truth table

A	B	$X = A + B$	$Y = \overline{AX}$
0	0	0	1
0	1	1	1
1	0	1	0
1	1	1	0

9. The transfer characteristic curve of a transistor, having input and output resistance 100 Ω and 100 kΩ respectively, is shown in the figure. The voltage and power gain, are respectively

[JEE Main 2019, 12 April Shift-I]

(a) $2.5\times10^4, 2.5\times10^6$ (b) $5\times10^4, 5\times10^6$

(c) $5\times10^4, 5\times10^5$ (d) $5\times10^4, 2.5\times10^6$

Exp. **(d)**

Given curve is between I_c and I_b as output and input currents, respectively.

So, it is transfer characteristics curve of a common emitter (CE) configuration.

In CE configuration,

Current gain, $\beta = \dfrac{I_{out}}{I_{in}} = \dfrac{I_c}{I_b}$ ...(i)

Voltage gain,

$$A_V = \frac{V_{out}}{V_{in}} = \frac{I_c \times R_{out}}{I_b \times R_{in}} = \beta \times \frac{R_{out}}{R_{in}} \quad ...(ii)$$

and power gain,

$$A_P = \frac{P_{out}}{P_{in}} = \frac{I_c^2 \times R_{out}}{I_b^2 \times R_{in}} = \beta^2 \times \frac{R_{out}}{R_{in}} \quad ...(iii)$$

Given, $R_{in} = 100\ \Omega$ and $R_{out} = 100\times10^3\ \Omega$

From Eq. (i), we get

$$\beta = \frac{5\text{ mA}}{100\ \mu\text{A}}\left(\text{or } \frac{10\text{ mA}}{200\ \mu\text{A}} \text{ or } \frac{15\text{ mA}}{300\ \mu\text{A}} \text{ or } \frac{20\text{ mA}}{400\ \mu\text{A}}\right)$$

$$\Rightarrow \beta = \frac{5\times10^{-3}}{100\times10^{-6}} = 50 \quad ...(iv)$$

From Eqs. (ii) and (iv), we get

Voltage gain, $A_V = \beta \times \dfrac{R_{out}}{R_{in}} = 50 \times \dfrac{100\times10^3}{100}$

$$\Rightarrow A_V = 50000 = 5\times10^4 \quad ...(v)$$

From Eqs. (iii) and (iv), we get

Power gain, $A_P = \beta^2 \times \dfrac{R_{out}}{R_{in}} = (50)^2 \times \dfrac{100\times10^3}{100}$

$= 2500\times1000$

$\Rightarrow A_P = 2.5\times10^6$

10. Figure shows a DC voltage regulator circuit, with a Zener diode of breakdown voltage = 6 V. If the unregulated input voltage varies between 10 V to 16 V, then what is the maximum Zener current?

[JEE Main 2019, 12 April Shift-II]

(a) 2.5 mA (b) 1.5 mA

(c) 7.5 mA (d) 3.5 mA

Exp. **(d)**

In given voltage regulator circuit breakdown of Zener occurs at 6 V.

After breakdown, voltage across load resistance (R_L = 4kΩ) is,

$$V = V_Z = 6V$$

∴ Load current after breakdown,

$$I_L = \frac{V_Z}{R_L} = \frac{6}{4000} = 1.5 \times 10^{-3} \text{ A}$$

When unregulated supply is of 16 V, potential drop occurring across series resistance (R_S = 2 kΩ) is;

$$V_S = V - V_Z = 16 - 6 = 10 \text{ V}$$

So, current across series resistance is

$$I_S = \frac{V_S}{R_S} = \frac{10}{2 \times 10^3} = 5 \times 10^{-3} \text{ A}$$

So, current across Zener diode is

$$I_Z = I_S - I_L = 5 \times 10^{-3} - 1.5 \times 10^{-3} = 3.5 \times 10^{-3} \text{ A}$$
$$= 3.5 \text{ mA}$$

11. Mobility of electrons in a semiconductor is defined as the ratio of their drift velocity to the applied electric field. If for an n - type semiconductor, the density of electrons is $10^{19} m^{-3}$ and their mobility is $1.6 m^2$ (V-s), then the resistivity of the semiconductor (since, it is an n-type semiconductor contribution of holes is ignored) is close to

[JEE Main 2019, 9 Jan Shift-I]

(a) 2 Ω-m (b) 0.2 Ω-m (c) 0.4 Ω-m (d) 4 Ω-m

***Exp.* (c)**

Since, it is an n-type semiconductor and concentration of the holes has been ignored. So, its conductivity is given as $\sigma = n_e e \mu_e$

where, n_e is the number density of electron, e is the charge on electron and μ_e is its mobility.

Substituting the given values, we get

$$\sigma = 10^{19} \times 1.6 \times 10^{-19} \times 1.6 = 2.56$$

As, resistivity, $\rho = \frac{1}{\sigma} = \frac{1}{2.56}$

or $\rho = 0.39 \simeq 0.4$ Ω-m

12. At 0.3V and 0.7 V, the diodes Ge and Si become conductor respectively. In given figure, if ends of diode Ge overturned, the change in potential V_0 will be

[JEE Main 2019, 9 Jan Shift-II]

(a) 0.2 V (b) 0.6V (c) 0.4 V (d) 0.8V

***Exp.* (c)**

Initially Ge and Si are both forward biased. So, current will effectively pass through Ge diode with a voltage drop of 0.3 V.

∴ Initial output voltage,

$$V_0 = 12 - 0.3 = 11.7 \text{ V}$$

If Ge is reversed biased, then only Si diode will work. In this condition, output voltage

$$V_0 = 12 - 0.7 = 11.3 \text{ V}$$

∴ Change in output voltage

$$= 11.7 - 11.3$$
$$= 0.4 \text{ V}$$

13. To get output '1' at R, for the given logic gate circuit, the input values must be

[JEE Main 2019, 10 Jan Shift-I]

(a) $X = 0, Y = 0$ (b) $X = 1, Y = 0$
(c) $X = 1, Y = 1$ (d) $X = 0, Y = 1$

Exp. **(b)**

The given circuit can be drawn as shown in the figure given below

Truth table for this given logic gate is

Given inputs in the options		A_1	A_2	A_3	A_4	R
X	Y					
0	0	1	1	1	1	0
1	0	0	1	0	0	1
1	1	0	0	1	1	0
0	1	1	0	1	1	0

So to get output $R = 1$, inputs must be $X = 1$ and $Y = 0$.

14. For the circuit shown below, the current through the Zener diode is

[JEE Main 2019, 10 Jan Shift-II]

(a) 14 mA (b) zero (c) 5 mA (d) 9 mA

Exp. **(d)**

In the circuit, let the current in branches is as shown in figure below

By Kirchhoff's node law,

$$I_1 = I_2 + I_3 \quad \ldots \text{(i)}$$

Now, when diode conducts, voltage difference between points A and B will be

$$V_{AB} = 120 - 50 = 70 \text{ V}$$

So, current $I_1 = \dfrac{V_{AB}}{5\,\text{k}\Omega} = \dfrac{70}{5 \times 10^3}$

$$I_1 = 14 \text{ mA} \quad \ldots \text{(ii)}$$

Since, diode and 10 kΩ resistor are in parallel combination, so voltage across 10 k Ω resistor will be 50 V only.

$$\Rightarrow \quad I_3 = \frac{50}{10\,\text{k}\Omega} = \frac{50}{10 \times 10^3}$$

$$\Rightarrow \quad I_3 = 5 \text{ mA} \quad \ldots \text{(iii)}$$

$\therefore$ From Eqs. (i), (ii) and (iii), we get

$$14 \text{ mA} = I_2 + 5 \text{ mA}$$

or current through diode,

$$I_2 = 14 \text{ mA} - 5\text{mA} = 9 \text{ mA}$$

15. In the given circuit, the current through zener diode is close to

[JEE Main 2019, 11 Jan Shift-I]

(a) 6.0 mA (b) 6.7 mA
(c) 0 (d) 4.0 mA

Exp. **(c)**

> **Key Idea** When the applied reverse voltage (V) reaches the breakdown voltage of the Zener diode, then only a large amount of current is flown through it, otherwise it is approximately zero.
>
> In the given situation, if we consider that Zener diode is at breakdown. Then, potential drop across 1500 Ω resistances will be 10 V. So potential drop at 500 Ω resistor will be 2 V.

$\therefore$ Current in $R_1 = \frac{2}{500} = 4\text{mA} = I_1$ (say)

Current in each

$$R_2 = \frac{10}{750} = \frac{2}{150} = 13.33 \text{ mA} = I_2 \text{ (say)}$$

$\Rightarrow$ $I_1 < I_2$ which is not possible.

So, Zener diode will never reach to its breakdown.

$\therefore$ Current flowing through a reverse biased Zener diode = 0.

16. The circuit shown below contains two ideal diodes, each with a forward resistance of 50 Ω. If the battery voltage is 6 V, the current through the 100 Ω resistance (in ampere) is

[JEE Main 2019, 11 Jan Shift-II]

(a) 0.027 (b) 0.020 (c) 0.030 (d) 0.036

Exp. **(b)**

In this circuit, D_1 is forward biased and D_2 is reversed biased.

Resistance of D_1 is 50Ω.

$\therefore$ Net resistance of the circuit,

$$R_{net} = 50 + 150 + 100 = 300\,\Omega$$

$\therefore$ Current through the 100Ω resistance

$$= \frac{V}{R_{net}} = \frac{6}{300} = 0.020\text{ A}$$

17. The output of the given logic circuit is

[JEE Main 2019, 12 Jan Shift-I]

(a) $A\overline{B}$ (b) $\overline{A}B$ (c) $AB + \overline{AB}$ (d) $A\overline{B} + \overline{A}B$

Exp. **(a)**

Truth table for given circuit is

A	B	Y_1	Y_2	Y_3	Y
0	0	1	1	1	0
1	0	1	1	0	1
0	1	1	1	1	0
1	1	0	1	1	0

This is the same output produced by $A \cdot \overline{B}$ gate or

So, given circuit is equivalent to Boolean expression $A \cdot \overline{B}$.

Alternate Solution

Using the Boolean algebra, output of the given logic circuit can be given as

Here $Y_2 = \overline{A \cdot (\overline{A \cdot B})}$

Using de-Morgan's principle,

$$\overline{x \cdot y} = \overline{x} + \overline{y} \text{ and } \overline{x + y} = \overline{x} \cdot \overline{y}$$

$\Rightarrow$ $Y_2 = \overline{A} + (\overline{\overline{A \cdot B}})$ $[\because \overline{\overline{x}} = x]$

$= \overline{A} + (A.B)$ ...(i)

Similarly, $Y_3 = B + \overline{A} + \overline{B} = 1 + \overline{A}$ $[\because x + \overline{x} = 1]$

$\Rightarrow$ $Y_3 = 1$ ...(ii) $[\because \overline{x} + 1 = 1]$

As, $Y = \overline{Y_2 \cdot Y_3}$

Using Eqs. (i) and (ii), we get

$$Y = (\overline{\overline{A} + A \cdot B})(1)$$
$$= (\overline{\overline{A} + A \cdot B}) + \overline{1} = \overline{\overline{A}} \cdot (\overline{A \cdot B}) + 0$$
$$= A \cdot (\overline{A} + \overline{B}) \qquad [\because x + 0 = x]$$
$$= A \cdot \overline{A} + A \cdot \overline{B} \qquad [\because x\overline{x} = 0]$$

18. In the figure, given that V_{BB} supply can vary from 0 to 5.0 V, $V_{CC} = 5$ V, $\beta_{DC} = 200$, $R_B = 100$ k Ω, $R_C = 1$ k Ω and $V_{BE} = 1.0$ V. The minimum base current and the input voltage at which the transistor will go to saturation, will be, respectively **[JEE Main 2019, 12 Jan Shift-II]**

(a) 25 μA and 2.8 V (b) 25 μA and 3.5 V
(c) 20 μA and 3.5 V (d) 20 μA and 2.8 V

***Exp.* (b)**

Transistor saturation occurs when $V_{CE} = 0$.

Now, for closed loop of collector and emitter by Kirchhoff's voltage rule, we have

$$V_{CE} = V_{CC} - I_C R_C \Rightarrow 0 = V_{CC} - I_C R_c$$
$$\Rightarrow \quad I_C = \frac{V_{CC}}{R_C} = \frac{5}{1 \times 10^3} = 5 \times 10^{-3} \text{ A}$$

Now, $\beta_{DC} = 200$ (given) $\Rightarrow \frac{I_C}{I_B} = \beta_{DC} = 200$

$$\Rightarrow \quad I_B = \frac{I_C}{200} = \frac{5 \times 10^{-3}}{200}$$
$$\Rightarrow \quad I_B = 2.5 \times 10^{-5} = 25\,\mu\text{A}$$

Now, we apply Kirchhoff's voltage rule in base-emitter closed loop, we get

$$V_{BB} = I_B R_B + V_{BE}$$

$$\Rightarrow V_{BB} = (25 \times 10^{-6} \times 100 \times 1000) + 1 = 3.5 \text{ V}$$

19. The reading of the ammeter for a silicon diode in the given circuit is **[JEE Main 2018]**

(a) 0 (b) 15 mA
(c) 11.5 mA (d) 13.5 mA

***Exp.* (c)**

Potential drop in a silicon diode in forward bias is around 0.7 V.

In given circuit, potential drop across 200 Ω resistor is

$$I = \frac{\Delta V_{net}}{R} = \frac{3 - 0.7}{200}$$
$$\Rightarrow \quad I = 0.0115 \text{ A} \quad \Rightarrow \quad I = 11.5 \text{ mA}$$

20. In a common emitter amplifier circuit using an *n-p-n* transistor, the phase difference between the input and the output voltages will be **[JEE Main 2017 (Offline)]**

(a) 90° (b) 135° (c) 180° (d) 45°

***Exp.* (c)**

In a CE *n-p-n* transistor amplifier output is 180° out of phase with input.

21. If *a, b, c* and *d* are inputs to a gate and *x* is its output, then, as per the following time graph, the gate is **[JEE Main 2016 (Offline)]**

(a) NOT (b) AND (c) OR (d) NAND

Exp. **(*c*)**

Output of OR gate is 0 when all inputs are 0 and output is 1 when atleast one of the inputs is 1.

Observing output *x* It is 0 when all inputs are 0 and it is 1 when atleast one of the inputs is 1.

∴ The gate is OR.

Alternative Solution

OR Gate

a	b	c	d	x
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1

22. Identify the semiconductor devices whose characteristics are as given below, in the order *(a),(b),(c),(d)*. **[JEE Main 2016 (Offline)]**

(a) Simple diode, Zener diode, Solar cell, Light dependent resistance
(b) Zener diode, Simple diode, Light dependent resistance, Solar cell
(c) Solar cell, Light dependent resistance, Zener diode, Simple diode
(d) Zener diode, Solar cell, Simple diode, Light dependent resistance

Exp. **(*a*)**

Zener diode works in breakdown region.

So, Simple diode → (*a*)

Zener diode → (*b*)

Solar cell → (*c*)

Light dependent resistance → (*d*)

23. For a common emitter configuration, if α and β have their usual meanings, the incorrect relationship between α and β is **[JEE Main 2016 (Offline)]**

(a) $\frac{1}{\alpha} = \frac{1}{\beta} + 1$ (b) $\alpha = \frac{\beta}{1-\beta}$

(c) $\alpha = \frac{\beta}{1+\beta}$ (d) $\alpha = \frac{\beta^2}{1+\beta^2}$

Exp. **(*a, c*)**

As, we know, In case of a common-emitter configuration, DC current gain, $\alpha = \frac{I_c}{I_e}$.

Where, I_c is collector current and I_e is emitter current

and AC current gain, $\beta = \frac{I_c}{I_b}$.

where, I_b is base current.

Also, $I_e = I_b + I_c$

Dividing whole equation by I_c, we get

$$\Rightarrow \frac{I_e}{I_c} = \frac{I_b}{I_c} + 1 \Rightarrow \frac{1}{\alpha} = \frac{1}{\beta} + 1 \Rightarrow \alpha = \frac{\beta}{1+\beta}$$

24. A red LED emits light at 0.1 W uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is **[JEE Main 2015]**

(a) 1.73 V/m (b) 2.45 V/m

(c) 5.48 V/m (d) 7.75 V/m

Exp. **(*b*)**

Consider the LED as a point source of light.

Let power of the LED is *P*.

Intensity at r from the source

$$I = \frac{P}{4\pi r^2} \quad \text{...(i)}$$

As we know that $I = \frac{1}{2}\varepsilon_0 E_0^2 c$ ...(ii)

From Eqs. (i) and (ii), we can write

$$\frac{P}{4\pi r^2} = \frac{1}{2}\varepsilon_0 E_0^2 c$$

or $$E_0^2 = \frac{2P}{4\pi\varepsilon_0 r^2 c} = \frac{2 \times 0.1 \times 9 \times 10^9}{1 \times 3 \times 10^8}$$

or $$E_0^2 = 6 \Rightarrow E_0 = \sqrt{6} = 2.45 \text{ V/m}$$

25. The forward biased diode connection is **[JEE Main 2014]**

(a)

(b)

(c)

(d)

Exp. **(a)**

For forward bias, p-side must be a higher potential than n-side.

So,

is forward biased.

26. The I-V characteristic of an LED is **[JEE Main 2013]**

(a)

(b)

(c)

(d)

Exp. **(a)**

To increase frequency of light emitted from LED (Light Emitting Diode) potential barrier to diode is increased, so for same value of current, higher value of voltage is required for higher frequency.

27. Truth table for system of four NAND gates as shown in figure is **[AIEEE 2012]**

(a)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(b)

A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

(c)

A	B	Y
0	0	1
0	1	1
1	0	0
1	1	0

(d)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Exp. **(a)**

$$Y = \overline{\overline{\{A\cdot(\overline{A\cdot B})\}}\cdot\overline{[B\cdot(\overline{A\cdot B})]}}$$

[using De-Morgan's rules]

$$= \overline{(\overline{A} + A\cdot B)\cdot(\overline{B} + A\cdot B)}$$

[break the line, change the sign]

$$= \overline{(\overline{A} + A\cdot B)} + \overline{(\overline{B} + (A\cdot B))}$$

$$= A\cdot(\overline{A\cdot B}) + B\cdot(\overline{A\cdot B})$$

$$= A\cdot(\overline{A} + \overline{B}) + B\cdot(\overline{A} + \overline{B})$$

$$= A\cdot\overline{B} + B\cdot\overline{A} \quad [\because A\cdot\overline{A} = B\cdot\overline{B} = 0]$$

A	B	$\overline{A}$	$\overline{B}$	$A\cdot\overline{B}$	$B\cdot\overline{A}$	Y
0	0	1	1	0	0	0
0	1	1	0	0	1	1
1	0	0	1	1	0	1
1	1	0	0	0	0	0

It is the truth table of XOR gate.

28. The output of an OR gate is connected to both the inputs of a NAND gate. The combination will serve as a **[AIEEE 2011]**

(a) OR gate (b) NOT gate
(c) NOR gate (d) AND gate

Exp. **(*c*)**

$y' = A + B$ and $y = \bar{y}' = \overline{A + B}$

i.e., output of a NOR gate.

29. The combination of gates shown below yields **[AIEEE 2010]**

(a) OR gate (b) NOT gate
(c) XOR gate (d) NAND gate

Exp. **(*a*)**

Truth table for given combination is

A	B	X
0	0	0
1	0	1
0	1	1
1	1	1

This comes out to be truth table of OR gate.

30. A *p-n* junction (*D*) shown in the figure can act as a rectifier. An alternating current source (*V*) is connected in the circuit. **[AIEEE 2009]**

Exp. **(*c*)**

Given figure is half wave rectifier.

[since, diode allows the signal to pass during positive half only *i.e.*, forward bias]

31. In the adjacent circuit, *A* and *B* represent two inputs and *C* represents the output.

The circuit represents **[AIEEE 2008]**

(a) NOR gate (b) AND gate
(c) NAND gate (d) OR gate

Exp. **(*d*)**

If we give the following inputs to *A* and *B*, then corresponding output is shown in truth table

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

The above table is similar to OR gate.

32. If in a *p-n* junction diode, a square input signal of 10 V is applied as shown. **[AIEEE 2007]**

Then, the output signal across R_L will be

Exp. **(*d*)**

During – ve cycle, diode will not allow the signal to pass.

Fig. (i)

Fig. (ii)

Fig. (iii)

For $V_i < 0$, the diode is reverse biased and hence offer infinite resistance, so circuit would be like as shown in Fig. (ii) and $V_o = 0$.

For $V_i > 0$, the diode is forward biased and circuit would be as shown in Fig. (iii) and $V_o = V_i$.

Hence, the option (d) is correct.

33. Carbon, silicon and germanium have four valence electrons each. At room temperature, which one of the following statements is most appropriate? **[AIEEE 2007]**

(a) The number of free conduction electrons is significant in C but small in Si and Ge

(b) The number of free conduction electrons is negligibly small in all the three

(c) The number of free electrons for conduction is significant in all the three

(d) The number of free electrons for conduction is significant only in Si and Ge but small is C

Exp. **(*d*)**

The number of free electrons for conduction is significant only in Si and Ge but small in C, as C is an impurity.

34. In a common-base mode of a transistor, the collector current is 5.488 mA for an emitter current of 5.60 mA. The value of the base current amplification factor (β) will be **[AIEEE 2006]**

(a) 49 (b) 50 (c) 51 (d) 48

Exp. **(*a*)**

$$\beta = I_C / I_B$$

and $$I_E = I_C + I_B$$

$$\therefore \quad \beta = \frac{I_C}{I_E - I_C} = \frac{5.488}{5.60 - 5.488} = 49$$

35. If the lattice constant of this semiconductor is decreased, then which of the following is correct? **[AIEEE 2006]**

(a) All E_c, E_g, E_v increase

(b) E_c and E_v increase but E_g decreases

(c) E_c and E_v decrease but E_g increases

(d) All E_c, E_g, E_v decrease

Exp. **(*c*)**

If lattice constant of semi-conductor is decreased, then E_c and E_v decrease but E_g increases.

36. A solid which is not transparent to visible light and whose conductivity increases with temperature is formed by **[AIEEE 2006]**

(a) ionic bonding

(b) covalent bonding

(c) van der Waals' bonding

(d) metallic bonding

Exp. **(*b*)**

Covalent bonding.

[they are simply semiconductors having 4th group]

37. If the ratio of the concentration of electrons to that of holes in a semiconductor is $\frac{7}{5}$ and the ratio of currents is $\frac{7}{4}$, then what is the ratio of their drift velocities? **[AIEEE 2006]**

(a) $\frac{5}{8}$ (b) $\frac{4}{5}$ (c) $\frac{5}{4}$ (d) $\frac{4}{7}$

Exp. **(*c*)**

$$I = n_e A v_d$$

$$\therefore \quad \frac{I_e}{I_h} = \frac{n_e \times (v_d)_e}{n_h \times (v_d)_h}$$

Here, $\frac{n_e}{n_h} = \frac{7}{5}, \quad \frac{I_e}{I_h} = \frac{7}{4}$

$$\therefore \quad \frac{7}{4} = \frac{7}{5} \times \frac{(v_d)_e}{(v_d)_h} \quad \text{or} \quad \frac{(v_d)_e}{(v_d)_h} = \frac{5}{7} \times \frac{7}{4} = \frac{5}{4}$$

38. The circuit has two oppositely connected ideal diodes in parallel. What is the current flowing in the circuit? **[AIEEE 2006]**

(a) 1.71 A (b) 2.00 A (c) 2.31 A (d) 1.33 A

Exp. **(b)**

In the given, circuit diode D_1 is reverse biased while D_2 is forward biased, so the circuit can be redrawn as

[∵ for ideal diodes, reverse biased means open and forward biased means short]

Apply KVL to get current following through the circuit

$$-12 + 4I + 2I = 0 \quad \text{or} \quad I = \frac{12}{6} = 2\text{ A}$$

39. In the following, which one of the diodes is reverse biased? **[AIEEE 2006]**

Exp. **(d)**

For forward bias, *p is* – ve, *n* is – ve.

For reverse biasing of an ideal diode, the potential of *n*-side should be higher than potential of *p*-side. Only option(d) is satisfying the criterion for reverse biasing.

40. The electrical conductivity of a semiconductor increases when electromagnetic radiation of wavelength shorter than 2480 nm, is incident on it. The band gap in (eV) for the semiconductor is **[AIEEE 2005]**

(a) 1.1 eV (b) 2.5 eV (c) 0.5 eV (d) 0.7 eV

Exp. **(c)**

$$E_g = h\nu = \frac{hc}{\lambda} = \left(\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2480 \times 10^{-9} \times 1.6 \times 10^{-19}}\right) \text{eV}$$

$[\because 1\text{ eV} = 1.6 \times 10^{-19}\text{ J}]$

$= 0.5$ eV

41. In a common base amplifier, the phase difference between the input signal voltage and output voltage is **[AIEEE 2005]**

(a) $\frac{\pi}{4}$ (b) π

(c) zero (d) $\frac{\pi}{2}$

Exp. **(c)**

In common base amplifier, the input signal is amplified but remain in phase with output signal.

42. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency in the ripple would be **[AIEEE 2005]**

(a) 50 Hz (b) 25 Hz (c) 100 Hz (d) 70.7 Hz

Exp. **(c)**

Given, $f = 50$ Hz

$$\therefore \quad T = \frac{1}{50}$$

For full wave rectifier, $T_1 = \frac{T}{2} = \frac{1}{100}$

$$\therefore \quad f_1 = 100\text{ Hz} \qquad \left[\because f = \frac{1}{T}\right]$$

43. When *n-p-n* transistor is used as an amplifier, **[AIEEE 2004]**

(a) electrons move from base to collector
(b) holes move from emitter to base
(c) electrons move from collector to base
(d) holes move from base to emitter

Exp. **(*d*)**

When forward bias is applied on *n-p-n* transistor, then it works as an amplifier. In forward biased *n-p-n* transistor, electrons move from emitter to base and holes move from base to emitter.

44. For a transistor amplifier in common emitter configuration for load impedance of $1\,\text{k}\Omega\,(h_{fe} = 50 \text{ and } h_{oe} = 25\,\mu\,\text{A/V})$, the current gain is **[AIEEE 2004]**

(a) – 5.2 (b) – 15.7
(c) – 24.8 (d) – 48.78

Exp. **(*d*)**

For a transistor amplifier in common emitter configuration, current gain

$$A_i = -\frac{h_{fe}}{1 + h_{oe} R_L}$$

where, h_{fe} and h_{oe} are hybrid parameters of a transistor.

$$\therefore \quad A_i = -\frac{50}{1 + 25 \times 10^{-6} \times 1 \times 10^3}$$

$$= -48.78$$

45. A piece of copper and another of germanium are cooled from room temperature to 77 K, the resistance of **[AIEEE 2004, 03]**

(a) each of them increases
(b) each of them decreases
(c) copper decreases and germanium increases
(d) copper increases and germanium decreases

Exp. **(*c*)**

We know that resistance of conductor is directly proportional to temperature (*i.e.*, $R \propto \Delta t$), while resistance of semiconductor is inversely proportional to temperature $\left(i.e., R \propto \frac{1}{\Delta t}\right)$.

Therefore, it is clear that resistance of conductor decreases with decrease in temperature or *vice-versa*, while in case of semi-conductor, resistance increases with decrease in temperature or *vice-versa.*

Since, copper is pure conductor and germanium is a semiconductor, hence due to decrease in temperature, resistance of conductor decreases while that of semi-conductor increases.

46. When *p-n* junction diode is forward biased, then **[AIEEE 2004]**

(a) the depletion region is reduced and barrier height is increased
(b) the depletion region is widened and barrier height is reduced
(c) both the depletion region and barrier height are reduced
(d) both the depletion region and barrier height are increased

Exp. **(*c*)**

When *p*-end of *p-n* junction is connected to positive terminal of battery and *n*-end to negative terminal of battery, then *p-n* junction is said to be in forward bias.

In forward bias, the more numbers of electrons go from *n*-region to *p*-region and more numbers of holes go from *p*-region to *n*-region.

Therefore, major current due to both types of carriers takes place through the junction, causing more recombination of electron hole pairs, thus causing reduction in height of depletion region and barrier potential.

47. The difference in the variation of resistance with temperature in a metal and a semiconductor arises essentially due to the difference in the **[AIEEE 2003]**

(a) crystal structure
(b) variation of the number of charge carriers with temperature
(c) type of bonding
(d) variation of scattering mechanism with temperature

Exp. **(*b*)**

Metal has number of free electrons while semiconductor has not at room temperature.

The difference in the variation of resistance with temperature in metal and semiconductor is caused due to difference in the variation of the number of charge carriers with temperature.

48. In the middle of the depletion layer of reverse biased *p-n* junction, the **[AIEEE 2003]**
(a) electric field is zero
(b) potential is maximum
(c) electric field is maximum
(d) potential is zero

Exp. (a)
Due to the reverse biasing, the width of depletion region increases and current flowing through the diode is almost zero. In this case, electric field is almost zero at the middle of the depletion region.

49. At absolute zero, Si acts as
(a) non-metal (b) metal
(c) insulator (d) None of these

Exp. (c)
At absolute zero, the energy band is large (> 3 eV) and hence Si acts as insulator or in other words, we can say that there is no free charge carriers.

50. The part of a transistor which is most heavily doped to produce large number of majority carriers is **[AIEEE 2002]**
(a) emitter (b) base
(c) collector
(d) can be any of the above three

Exp. (a)
Because emitter has largest size and maximum number of charge carriers.

51. The energy band gap is maximum in **[AIEEE 2002]**
(a) metals (b) superconductors
(c) insulators (d) semiconductors

Exp. (c)
Metals have minimum energy gap and insulators have maximum energy gap and semi-conductors have energy gap lying between insulators and metals.

52. By increasing the temperature, the specific resistance of a conductor and a semiconductor **[AIEEE 2002]**
(a) increases for both
(b) decreases for both
(c) increases, decreases respectively
(d) decreases, increases respectively

Exp. (c)
Increases, decreases respectively.
If we increase the temperature, the specific resistance (resistivity) of conductor increases while that of semiconductor decreases.

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Communication Systems

1. The wavelength of the carrier waves in a modern optical fibre communication network is close to

(a) 2400 nm (b) 1500 nm
(c) 600 nm (d) 900 nm

[JEE Main 2019, 8 April Shift-I]

Exp. **(b)**

In optical fibre communication network, the signals are transmitted by laser light operating in range of 1310 nm-1550 nm.
So, the closest value is 1500 nm.

2. In a line of sight radio communication, a distance of about 50 km is kept between the transmitting and receiving antennas. If the height of the receiving antenna is 70 m, then the minimum height of the transmitting antenna should be
(Radius of the earth $= 6.4 \times 10^6$ m)

[JEE Main 2019, 8 April Shift-II]

(a) 20 m (b) 32 m (c) 40 m (d) 51 m

Exp. **(b)**

Key Idea In line of sight communication, distance d between transmitting antenna and receiving antenna is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

Here in figure, h_R and h_T is the height of receiving and transmitting antenna, respectively.

Given, $d = 50$ km $= 50 \times 10^3$ m,

$h_R = 70$ m, $R = 6.4 \times 10^6$ m

Then, distance between transmitting and receiving antenna, i.e. $d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$

$$50 \times 10^3 = \sqrt{2R}(\sqrt{h_T} + \sqrt{h_R})$$

$$= \sqrt{2 \times 6.4 \times 10^6}(\sqrt{h_T} + \sqrt{70})$$

$$\Rightarrow \quad \sqrt{h_T} \approx \frac{50 \times 10^3}{3577.7} - 8.37$$

$$= 13.98 - 8.37 = 5.61$$

or $\quad h_T = 31.5 \approx 32$ m

3. A signal $A\cos\omega t$ is transmitted using $v_0 \sin\omega_0 t$ as carrier wave. The correct amplitude modulated (AM) signal is **[JEE Main 2019, 9 April Shift-I]**

(a) $(v_0 \sin\omega_0 t + A\cos\omega t$
(b) $(v_0 + A)\cos\omega t \sin\omega_0 t$
(c) $v_0 \sin[\omega_0(1 + 0.01A\sin\omega t)t]$
(d) $v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$

Exp. **(d)**

Given, modulating signal,

$$A_m = A\cos\omega t$$

Carrier wave, $A_c = v_0 \sin\omega_0 t$

In amplitude modulation, modulated wave is given by

$$Y_m = [A_0 + A_m]\sin\omega_0 t$$

where, A_0 is amplitude of the carrier wave (given as v_0)

$$\therefore Y_m = [v_0 + A\cos\omega t]\sin\omega_0 t$$
$$= v_0 \sin\omega_0 t + A\sin\omega_0 t\cos\omega t$$
$$= v_0 \sin\omega_0 t + \frac{A}{2}[\sin(\omega_0 + \omega)t + \sin(\omega_0 - \omega)t]$$
$$= v_0 \sin\omega_0 t + \frac{A}{2}\sin(\omega_0 - \omega)t + \frac{A}{2}\sin(\omega_0 + \omega)t$$

4. The physical sizes of the transmitter and receiver antenna in a communication system are **[JEE Main 2019, 9 April Shift-II]**

(a) proportional to carrier frequency
(b) inversely proportional to modulation frequency
(c) independent of both carrier and modulation frequency
(d) inversely proportional to carrier frequency

Exp. **(d)**

Size of antenna is directly proportional to the wavelength of the signal.

Also, the speed at which signal moves = carrier frequency × wavelength

$$\Rightarrow \quad f\lambda = c \Rightarrow \lambda \propto \frac{1}{f}$$

$\therefore$ Size of antenna $\propto \frac{1}{f}$.

Note Minimum size of the antenna is $\lambda / 4$.

5. A message signal of frequency 100 MHz and peak voltage 100 V is used to execute amplitude modulation on a carrier wave of frequency 300 GHz and peak voltage 400 V. The modulation index and difference between the two side band frequencies are **[JEE Main 2019, 10 April Shift-I]**

(a) $0.25 ; 1 \times 10^8$ Hz (b) $4 ; 1 \times 10^8$ Hz
(c) $0.25 ; 2 \times 10^8$ Hz (d) $4 ; 2 \times 10^8$ Hz

Exp. **(c)**

Given, $f_m = 100\,\text{MHz} = 100 \times 10^6$ Hz,

$$A_m = 100\,\text{V},$$

$$A_c = 400\,\text{V}$$

Range of frequency in case of amplitude modulation is $(f_c - f_m)$ to $(f_c + f_m)$.

$\therefore$ Bandwidth $= 2f_m = 2 \times 100 \times 10^6$ Hz

$$= 2 \times 10^8 \text{Hz}$$

and modulation index,

$$\text{MI} = \frac{A_m}{A_c} = \frac{100}{400} = 0.25$$

6. Given below in the left column are different modes of communication using the kinds of waves given in the right column. **[JEE Main 2019, 10 April Shift-I]**

A.	Optical fibre communication	P.	Ultrasound
B.	Radar	Q.	Infrared light
C.	Sonar	R.	Microwaves
D.	Mobile phones	S.	Radio waves

From the options given below, find the most appropriate match between entries in the left and the right column.

(a) A-Q, B-S, C-R, D-P
(b) A-S, B-Q, C-R, D-P
(c) A-Q, B-S, C-P, D-R
(d) A-R, B-P, C-S, D-Q

Exp. **(c)**

In optical fibre communication, infrared light is used to transmit information from one point to another.

RADAR (Radio detection and ranging) is a detection system that uses radio waves to determine range, angle or velocity of objects.

SONAR (Sound navigation and ranging) is also a detection system that uses ultrasound to detect under water objects, submarines, etc.

Mobile phone is a portable telephone that can make and receive calls, which make use of microwave.

$\therefore$ Correct sequence is

$$A \to Q,\ B \to S,\ C \to P \text{ and } D \to R$$

7. In an amplitude modulator circuit, the carrier wave is given by $C(t) = 4\sin(20000\,\pi t)$ while modulating signal is given by, $m(t) = 2\sin(2000\,\pi t)$. The values of modulation index and lower side band frequency are **[JEE Main 2019, 12 April Shift-II]**

(a) 0.5 and 10 kHz (b) 0.4 and 10 kHz
(c) 0.3 and 9 kHz (d) 0.5 and 9 kHz

Exp. **(d)**

Given, carrier wave,

$$C(t) = 4\sin(20000\,\pi t)$$

Modulating signal,

$$m(t) = 2\sin(2000\,\pi t)$$

So, carrier wave's amplitude and frequency are

$$A_c = 4\text{ V},\ \omega_c = 20000\,\pi$$
$$= 2\pi \times 10^4 \text{ rad/s}$$

$$\Rightarrow \quad f_c = \frac{\omega_c}{2\pi} = 10^4 \text{ Hz} = 10\text{ kHz}$$

and modulating signal's amplitude and frequency are

$$A_m = 2\text{ V},\ \omega_m = 2000\pi$$
$$= 2\pi \times 10^3 \text{ rad/s}$$

$$\Rightarrow \quad f_m = \frac{\omega_m}{2\pi} = 10^3 \text{ Hz} = 1\text{ kHz}$$

So, modulating index is

$$m = \frac{A_m}{A_c} = \frac{2}{4} = 0.5$$

and lower side band frequency is,

$$f_{LSB} = f_c - f_m = 10 - 1 = 9\text{ kHz}$$

8. In communication system, only one percent frequency of signal of wavelength 800 nm can be used as bandwidth. How many channal of 6MHz bandwidth can be broadcast this?
$(c = 3 \times 10^8 \text{ m/s},\ h = 6.6 \times 10^{-34}\text{ J-s})$

[JEE Main 2019, 9 Jan Shift-II]

(a) 3.75×10^6 (b) 3.86×10^6
(c) 6.25×10^5 (d) 4.87×10^5

Exp. **(c)**

Here,
Signal wavelength, $\lambda = 800\text{ nm} = 8 \times 10^{-7}$ m

Frequency of source is

As, $$f = \frac{c}{\lambda} = \frac{3 \times 10^8}{8 \times 10^{-7}} = 3.75 \times 10^{14} \text{ Hz}$$

$\therefore$ Total bandwidth used for communication
$= 1\%$ of 3.75×10^{14}

$$= 3.75 \times 10^{12} \text{ Hz} \quad \ldots(i)$$

So, number of channel for signals

$$= \frac{\text{total bandwidth available for communication}}{\text{bandwidth of TV signal}}$$

$$= \frac{3.75 \times 10^{12}}{6 \times 10^6} = 0.625 \times 10^6 = 6.25 \times 10^5$$

9. A TV transmission tower has a height of 140 m and the height of the receiving antenna is 40 m. What is the maximum distance upto which signals can be broadcasted from this tower in LOS (Line of Sight) mode? (Take, radius of earth $= 6.4 \times 10^6$ m).

[JEE Main 2019, 10 Jan Shift-I]

(a) 65 km (b) 80 km
(c) 40 km (d) 48 km

Exp. **(a)**

Maximum distance of transmission is given by

$$d = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$

where, h_T = height of transmitter, = 140 m,
h_R = height of receiver = 40 m and
R = radius of earth = 6.4×10^6 m.

Substituting values, we get

$$d = \sqrt{2 \times 6.4 \times 10^6}\ (\sqrt{140} + \sqrt{40})$$
$$= 65\text{ km}$$

10. The modulation frequency of an AM radio station is 250 kHz, which is 10% of the carrier wave. If another AM station approaches you for license, what broadcast frequency will you allot ? **[JEE Main 2019, 10 Jan Shift-II]**

(a) 2000 kHz (b) 2250 kHz
(c) 2900 kHz (d) 2750 kHz

Exp. **(a)**

For a given carrier wave of frequency f_c with modulation frequency f_m, the bandwidth is calculated by

$$f_{upper} = f_c + f_m$$
$$f_{lower} = f_c - f_m \quad \ldots(i)$$

To avoid overlapping of bandwidths, next broadcast frequencies can be

$$f_1 = f_c \pm 2f_m,\ f_2 = f_c \pm 3f_m$$

So, next immediate available broadcast frequency is

$$f_1 = f_c + 2f_m$$

and $$f_1' = f_c - 2f_m$$

Given, $f_m = 250$ kHz

and also that f_m is 10% of f_c, i.e. $f_c = 2500$ kHz.

So, $f_1 = 2500 + (2 \times 250) = 3000$ kHz

and $f'_1 = 2500 - (2 \times 250) = 2000$ kHz

11. An amplitude modulates signal is given by $v(t) = 10[1 + 0.3\cos(2.2 \times 10^4 t)]\sin(5.5 \times 10^5 t)$. Here, t is in seconds. The sideband frequencies (in kHz) are $\left(\text{Take}, \pi = \frac{22}{7}\right)$

[JEE Main 2019, 11 Jan Shift-I]

(a) 892.5 and 857.5 (b) 89.25 and 85.75
(c) 178.5 and 171.5 (d) 1785 and 1715

Exp. **(b)**

$$v(t) = 10\,[1 + 0.3\cos(2.2 \times 10^4 t)]$$
$$[\sin(5.5 \times 10^5 t)]$$

Upper band angular frequency

$$\omega_v = (2.2 \times 10^4 + 5.5 \times 10^5)\,\text{rad/s}$$
$$= 572 \times 10^3\ \text{rad/s}$$

Similarly, lower band angular frequency.

$$\omega_L = (5.5 \times 10^5 - 2.2 \times 10^4)\,\text{rad/s}$$
$$= 528 \times 10^3\ \text{rad/s}$$

$\therefore$ Side band frequency are,

$$f_U = \frac{\omega_U}{2\pi} = \frac{572}{2\pi}\,\text{kHz} \simeq 91\,\text{kHz}$$

and $$f_L = \frac{\omega_L}{2\pi} = \frac{528}{2\pi}\,\text{kHz} \simeq 84\,\text{kHz}$$

Thus, the approximate side band frequencies are 89.25 and 85.75.

12. An amplitude modulated signal is plotted below

Which one of the following best describes the above signal? **[JEE Main 2019, 11 Jan Shift-II]**

(a) $[1 + 9\sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)\text{V}$
(b) $[9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)\text{V}$
(c) $[9 + \sin(4\pi \times 10^4 t)]\sin(5\pi \times 10^5 t)\text{V}$
(d) $[9 + \sin(2.5\pi \times 10^5 t)]\sin(2\pi \times 10^4 t)\text{V}$

Exp. **(b)**

Equation of an amplitude modulated wave is given by the relation,

$$C_m = (A_c + A_m \sin\omega_m t) \cdot \sin\omega_c \cdot t \quad \ldots\text{(i)}$$

For the given graph,

maximum amplitude, $A_c + A_m = 10$ ...(ii)

and minimum amplitude, $A_c - A_m = 8$ ...(iii)

From Eqs. (ii) and (iii), we get

$$A_c = 9\text{V and } A_m = 1\text{V} \quad \ldots\text{(iv)}$$

$\because$ For angular frequency of message signal and carrier wave, we use a relation

$$\omega_c = \frac{2\pi}{T_c} = \frac{2\pi}{8 \times 10^{-6}}$$

(as from given graph, $T_c = 8 \times 10^{-6}$ s)

$$= 2.5\pi \times 10^5\,\text{s}^{-1} \quad \ldots\text{(v)}$$

and $$\omega_m = \frac{2\pi}{T_m} = \frac{2\pi}{100 \times 10^{-6}}$$

(as from given graph, $T_m = 100 \times 10^{-6}$s)

$$= 2\pi \times 10^4\,\text{s}^{-1}$$

When we put values of A_c, A_m, ω_c and ω_m in Eq. (i), we get

$$C_m = [9 + \sin(2\pi \times 10^4 t)]\sin(2.5\pi \times 10^5 t)\,\text{V}$$

13. A 100 V carrier wave is made to vary between 160 V and 40 V by a modulating signal. What is the modulation index?

[JEE Main 2019, 12 Jan Shift-I]

(a) 0.4 (b) 0.5
(c) 0.6 (d) 0.3

Exp. **(c)**

Modulation index is given by

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$
$$= \frac{160 - 40}{160 + 40} = \frac{120}{200} = 0.6$$

14. To double the covering range of a TV transmission tower, its height should be multiplied by **[JEE Main 2019, 12 Jan Shift-II]**

(a) $\sqrt{2}$ (b) 4 (c) 2 (d) $\frac{1}{\sqrt{2}}$

Exp. **(b)**

Range of TV transmitting tower $d = \sqrt{2hR}$

where, h is the height of the transmission tower.

when range is doubled,

$$\therefore \quad 2d = 2\sqrt{2hR} = \sqrt{2(4h)R}$$

So, height must be multiplied with 4.

15. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilised for transmission. How many telephonic channels can be transmitted simultaneously, if each channel requires a bandwidth of 5 kHz? **[JEE Main 2018]**

(a) 2×10^3 (b) 2×10^4
(c) 2×10^5 (d) 2×10^6

Exp. **(c)**

Only 10% of 10 GHz is utilised for transmission.

$\therefore$ Band available for transmission

$$= \frac{10}{100} \times 10 \times 10^9 \text{ Hz} = 10^9 \text{ Hz}$$

Now, if there are n channels each using 5 kHz then,

$$n \times 5 \times 10^3 = 10^9 \quad \Rightarrow \quad n = 2 \times 10^5$$

16. In amplitude modulation, sinusoidal carrier frequency used is denoted by ω_c and the signal frequency is denoted by ω_m. The bandwidth ($\Delta\omega_m$) of the signal is such that $\Delta\omega_m << \omega_c$. Which of the following frequencies is not contained in the modulated wave? **[JEE Main 2017 (Offline)]**

(a) ω_c (b) $\omega_m + \omega_c$ (c) $\omega_c - \omega_m$ (d) ω_m

Exp. **(d)**

Frequency spectrum of modulated wave is

Clearly, ω_m is not included in the spectrum.

17. Choose the correct statement: **[JEE Main 2016 (Offline)]**

(a) In amplitude modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(b) In amplitude modulation, the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(c) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.
(d) In frequency modulation, the amplitude of the high frequency carrier wave is made to vary in proportion to the frequency of the audio signal.

Exp. **(b)**

As, we know, an amplitude modulated wave, the bandwidth is twice the frequency of modulating signal. Therefore, amplitude modulation (AM), the frequency of the high frequency carrier wave is made to vary in proportion to the amplitude of the audio signal.

18. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2MHz. The frequencies of the resultant signal is/are **[JEE Main 2015]**

(a) 2 MHz only
(b) 2005 kHz and 1995 kHz
(c) 2005 kHz 2000 kHz and 1995 kHz
(d) 2000 kHz and 1995 kHz

Exp. **(c)**

Frequency associated with AM are

$$f_c - f_m, f, f_c + f_m$$

According to the question

$$f_c = 2 \text{ MHz} = 2000 \text{ kHz}$$

$$f_m = 5 \text{ kHz}$$

Thus, frequency of the resultant signal is/are carrier frequency $f_c = 2000$ kHz, LSB frequency $f_c - f_m = 2000$ kHz -5 kHz $= 1995$ kHz and USB frequency $f_c + f_m = 2005$ kHz

19. A diode detector is used to detect an amplitude modulated wave of 60% modulation by using a condenser of capacity 250 pF in parallel with a load resistance 100 kΩ. Find the maximum modulated frequency which could be detected by it. **[JEE Main 2013]**

(a) 10.62 MHz (b) 10.62 kHz
(c) 5.31 MHz (d) 5.31 kHz

Exp. **(b)**

$$\tau = RC = 100 \times 10^3 \times 250 \times 10^{-12} \text{ s}$$

$$= 2.5 \times 10^7 \times 10^{12} \text{ s}$$
$$= 2.5 \times 10^{-5} \text{ s}$$

The highest frequency which can be detected with tolerable distortion is

$$f = \frac{1}{2\pi m_a RC} \quad \text{[where, } m_a \text{ is modulation]}$$
$$= \frac{1}{2\pi \times 0.6 \times 2.5 \times 10^{-5}} \text{ Hz}$$
$$= \frac{100 \times 10^4}{25 \times 1.2\ \pi} \text{ Hz} = \frac{4}{1.2\ \pi} \times 10^4 \text{ Hz}$$
$$= 10.61 \text{ kHz}$$

This condition is obtained by applying the condition that rate of decay of capacitor voltage must be equal or less than the rate of decay modulated signal voltage for proper detection of modulated signal.

20. A radar has a power of 1 kW and is operating at a frequency of 10 GHz. It is located on a mountain top of height 500 m. The maximum distance upto which it can detect object located on the surface of the earth (Radius of earth $= 6.4 \times 10^6$ m) is **[AIEEE 2012]**

(a) 80 km (b) 16 km (c) 40 km (d) 64 km

Exp. **(a)**

Range of radar on earth surface (optical distance, for space wave, i.e., line of view).

$$\text{Range} = \sqrt{(R+h)^2 - R^2}$$
$$= \sqrt{2Rh + h^2} \simeq \sqrt{2Rh}$$
$$= \sqrt{2 \times 6400 \times \frac{1}{2}} \text{ km} = 80 \text{ km}$$

21. Which of the following four alternatives is not correct?

We need modulation **[AIEEE 2011]**

(a) to increase the selectivity

(b) to reduce the time lag between transmission and reception of the information signal

(c) to reduce the size of antenna

(d) to reduce the fractional band width, *i.e.,* the ratio of the signal band width to the centre frequency

Exp. **(b)**

Modulation does not change time lag between transmission and reception.

22. This question has Statement I and Statement II. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement I Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement II The state of ionosphere varies from hour to hour, day to day and season to season. **[AIEEE 2011]**

(a) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

(b) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I

(c) Statement I is false, Statement II is true

(d) Statement I is true, Statement II is false

Exp. **(b)**

In radio communication, sky wave refers to the propagation of radio waves reflected or refracted back towards earth from the ionosphere.

Since, it is not limited by the curvature of the earth, sky wave propagation can be used to communicate beyond horizon. Ionosphere is a region of upper atmosphere and induces the thermosphere and parts of mesosphere and exosphere. It is distinguished because it is ionised by solar radiation. It plays an important part in atmospheric electricity.

23. The logic circuit shown below has the input waveforms A and B as shown. Pick out the correct output waveform. **[AIEEE 2009]**

Exp. **(a)**

Truth Table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

24. Consider telecommunication through optical fibres. Which of the following statements is not true? **[AIEEE 2003]**

(a) Optical fibres can be of graded refractive index

(b) Optical fibres are subjected to electromagnetic interference from outside

(c) Optical fibres have extremely low transmission loss

(d) Optical fibres may have homogeneous core with a suitable cladding

Exp. **(b)**

Some of the characteristics of an optical fibre are as follows

(i) It works on the principle of total internal reflection.

(ii) It consists of core made up of glass/silica/plastic with refractive index n_1, whisch is surrounded by a glass or plastic cladding with refractive index $n_2, (n_2 > n_1)$. The refractive index of cladding can be either changing abruptly or gradually changing (graded index fibre).

(iii) There is a very little transmission loss through optical fibres.

(iv) There is no interference from stray electric and magnetic fields to the signals through optical fibres.

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Practical Physics

1. A screw gauge with a pitch of 0.5 mm and a circular scale with 50 divisions is used to measure the thickness of a thin sheet of aluminium. Before starting the measurement, it is found that when the two jaws of the screw gauge are brought in contact, the 45th division coincides with the main scale line and that the zero of the main scale is barely visible. What is the thickness of the sheet, if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line? **[JEE Main 2016 (Offline)]**

(a) 0.75 mm (b) 0.80 mm
(c) 0.70 mm (d) 0.50 mm

Exp. **(a)**

Given that the screw gauge has zero error.

So, least count of a screw gauge $= \frac{0.5}{50}$ mm.

Thickness of the sheet if the main scale reading is 0.5 mm and the 25th division coincides with the main scale line, we have

$$= 0.50 \text{ mm} + (25) \times \frac{0.5}{50} \text{ mm}$$

$$= 0.50 \text{ mm} + 0.25 \text{ mm} = 0.75 \text{ mm}$$

2. A student measured the length of a rod and wrote it as 3.50 cm. Which instrument did he use to measure it? **[JEE Main 2014]**

(a) A meter scale
(b) A vernier calliper where the 10 divisions in vernier scale matches with 9 divisions in main scale and main scale has 10 divisions in 1 cm
(c) A screw gauge having 100 divisions in the circular scale and pitch as 1 mm
(d) A screw gauge having 50 divisions in the circular scale and pitch as 1 mm

Exp. **(a)**

If student measures 3.50 cm, it means that there is an uncertainly of order 0.01 cm.

For vernier scale with 1 MSD = 1 mm

and 9 MSD = 10 VSD

LC of vernier calliper = 1MSD – 1VSD

$$= \frac{1}{10}\left(1 - \frac{9}{10}\right)$$

$$= \frac{1}{100} \text{ cm}$$

3. A screw gauge gives the following reading when used to measure the diameter of a wire.
Main scale reading 0 mm
Circular scale reading 52 divisions
Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.
The diameter of wire from the above data is

(a) 0.052 cm (b) 0.026 cm **[AIEEE 2011]**
(c) 0.005 cm (d) 0.52 cm

Exp. **(a)**

Diameter of wire,

$$d = \text{MSR} + (\text{CSR} \times \text{LC}) \qquad \left[\because \text{LC} = \frac{1}{100}\right]$$

$$= 0 + 52 \times \frac{1}{100} = 0.52 \text{ mm}$$

$$= 0.052 \text{ cm}$$

4. An experiment is performed to find the refractive index of glass using a travelling microscope. In this experiment, distances are measured by **[AIEEE 2008]**

(a) a vernier scale provided on the microscope
(b) a standard laboratory scale
(c) a meter scale provided on the microscope
(d) a screw gauge provided on the microscope

Exp. (*a*)

If an experiment is performed to find the refractive index of glass using a travelling microscope, distances are measured by a vernier scale which is provided on the microscope.

5. Two full turns of the circular scale of a screw gauge cover a distance of 1 mm on its main scale. The total number of divisions on the circular scale is 50. Further, it is found that the screw gauge has a zero error of – 0.03 mm. While measuring the diameter of a thin wire, a student notes the main scale reading of 3 mm and the number of circular scale divisions in line with the main scale as 35. The diameter of the wire is **[AIEEE 2008]**

(a) 3.32 mm (b) 3.72 mm
(c) 3.67 mm (d) 3.38 mm

Exp. (*d*)

Diameter = Main scale reading
+ Circular scale reading × LC
+ Zero error

$$z = 3 + 35 \times \frac{1}{2 \times 50} + 0.03$$

$$= 3.38 \text{ mm}$$

6. A working transistor with its three legs marked P, Q and R is tested using a multimeter. No conduction is found between P and Q. By connecting the common (negative) terminal of the multimeter to R and the other (positive) terminal to P or Q, some conduction is seen on the multimeter. Which of the following is true for the transistor? **[AIEEE 2008]**

(a) It is an n-p-n transistor with R as base
(b) It is a p-n-p transistor with R as collector
(c) It is a p-n-p transistor with R as emitter
(d) It is an n-p-n transistor with R as collector

Exp. (*a*)

Since, no conduction is found when multimeter is connected across P and Q, it means either both P and Q are n-type or p-type.

So, it means R is base. When R is connected to common terminal and conduction is seen when other terminal is connected to P or Q, so it means transistor is n-p-n with R as base.

Practice Set 1

Instructions

- This test consists of 30 questions to be completed in 3 hrs.
- Each question is allotted 4 marks for correct response.
- 1 marks will be deducted for indicating incorrect response of each question. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.
- There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted according as per instructions.

1. For the following diagram [used to measure the length of a small metal piece by using vernier callipers], determine the length of the metal piece. Least count of the vernier callipers is 0.1 mm.

(a) 18 mm (b) 15.7 mm
(c) 12.6 mm (d) None of these

2. Consider the situation shown in figure. The uniform 0.60 kN beam is hinged at P. Find the tension in the tie rope. Give your answer to two significant figures.

(a) 2280 N (b) 2.3 kN (c) 2.28 kN (d) 228 N

3. A 1200 kg car is coasting down a 30° hill as shown in figure. At a time when the car's speed is 12 m/s, the driver applies the brakes. What constant force F (parallel to the road) must result, if the car is to stop after travelling 100 m?

(a) 6.7 kN (b) 6.7 N
(c) 100 N (d) 20 N

4. A plane is travelling eastward at an air speed of 500 km/h. But a 90 km/h, wind is blowing southward. What is speed of the plane relative to the ground?

(a) 235 km/h (b) 158 km/h
(c) 258 km/h (d) 508 km/h

5. A skier starts from rest and slides 9.0 m down a slope in 3.0 s. In what time after starting will be skier acquire a speed of 24 m/s? Assume that the acceleration is constant.

(a) 12 s (b) 15 s (c) 2 s (d) 12.5 s

Note *All the Practice Sets (1-10) available for free online practice see detailed instructions back side of the title.*

6. As shown in figure, a uniform solid sphere rolls on a horizontal surface at 20 m/s and then rolls up the incline. If friction losses are negligible, what will be the value of h, where the ball stops?

(a) 20 m (b) 24.5 m (c) 29 m (d) 90 m

7. A wind tunnel is to be used with a 20 cm high model car to approximately reproduce the situation in which a 550 cm high car is moving at 15 m/s. What should be the wind speed in the tunnel?

(a) 2.41 km/s (b) 3.41 km/s
(c) 0.41 km/s (d) 5.41 km/s

➡ **Directions :** *Question Nos.* 8 *to* 10 *are based on following paragraph.*

Two identical blocks P and Q have mass m each. They are attached to two identical springs initially unstretched. Now, the left spring (along with P) is compressed by $\frac{A}{2}$ and the right spring (along with Q) is compressed by A. Both the blocks are released simultaneously. They collide perfectly inelastically. Initially time period of both the blocks was T.

8. The time period of oscillation of combined mass is

(a) $\frac{T}{\sqrt{2}}$ (b) $\sqrt{2}\,T$
(c) T (d) $\frac{T}{2}$

9. The amplitude of combined mass is

(a) $\frac{A}{4}$ (b) $\frac{A}{2}$ (c) $\frac{2A}{3}$ (d) $\frac{3A}{4}$

10. What is energy of oscillation of the combined mass?

(a) $\frac{1}{2}kA^2$ (b) $\frac{1}{4}kA^2$ (c) $\frac{1}{8}kA^2$ (d) $\frac{1}{16}kA^2$

11. When a force of 500 N pushes on a 25 kg box as shown in figure, the acceleration of the box up the incline is $0.75\ \text{m/s}^2$. The coefficient of kinetic friction between box and incline is

(a) 0.41 (b) 0.3 (c) 0.25 (d) 1.5

12. When a building is constructed at –10°C, a steel beam (cross-sectional area 45 cm^2) is put in place with its ends cemented in pillars. If the sealed ends cannot move, what will be the compressional force in the beam when the temperature is 25°C ? For this kind of steel, $\alpha = 1.1 \times 10^{-5}\ ^\circ\text{C}^{-1}$ and $Y = 2.0 \times 10^{11}\ \text{N/m}^2$.

(a) 2.5×10^3 N (b) 3.5×10^5 N
(c) 2.5×10^5 N (d) 3.5×10^3 N

➡ **Directions :** *Question.* 13 *is Assertion-Reason type questions. Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below.*

13. **Statement I** The speed of whirlwind in a tornado is alarmingly high.

Statement II If no external torque acts on a body, its angular velocity remains conserved.

(a) Statement I is true, Statement II is true; Statement II is not the correct explanation of Statement I
(b) Statement I is true, Statement II is false
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true; Statement II is the correct explanation of Statement I

14. A 2.0 mg droplet of liquid nitrogen is present in a 30 mL tube as it is sealed off at very low temperature. What will be the nitrogen pressure in the tube when it is warmed to 20°C? Express your answer in atmospheres. (M for nitrogen is 28 kg/K mol).

(a) 0.057 atm (b) 0.57 atm
(c) 5.7 atm (d) 0.0057 atm

15. A beverage cooler is in the shape of a cube, 42 cm on each inside edge. Its 3.0 cm thick walls are made of plastic ($k_T = 0.050$ W/m-K). When the outside temperature is 20°C, how much ice will melt inside the cooler each hour?

(a) 2.5 g (b) 2.5 kg
(c) 0.38 g (d) 0.38 kg

16. A steam engine operating between a boiler temperature of 220°C and a condenser temperature of 35.0°C delivers 8.00 HP. If its efficiency is 30.0 per cent of that for a Carnot engine operating between these temperature limits, how many calories are absorbed each second by the boiler?

(a) 12.7 kcal/s (b) 12.7 cal/s
(c) 2.54 cal/s (d) 2.54 kcal/s

17. In the Bohr model of the hydrogen atom, an electron ($q = -e$) circles a proton ($q' = e$) in an orbit of radius 5.3×10^{-11} m. The attraction of the proton for the electron furnishes the centripetal force needed to hold the electron in orbit. What is the electron's speed? The electron mass is 9.1×10^{-31} kg.

(a) 25.0×10^6 m/s (b) 22.5×10^3 m/s
(c) 2.2×10^6 m/s (d) 4.2×10^3 m/s

18. Two capacitors, 3.0 μF and 4.0 μF, are individually charged across a 6.0 V battery. After being disconnected from the battery, they are connected together with a negative plate of one attached to the positive plate of the other. What is the final charge on capacitors?

(a) 2.6 μC, 3.4 μC (b) 3.4 μC, 2.6 μC
(c) 2.6 μC, 2.6 μC (d) 3.4 μC, 3.4 μC

19. A voltmeter is to deflect full scale for a potential difference of 5.000 V across it and is to be made by connecting a resistor R_x in series with a galvanometer. The 80.00 Ω galvanometer deflects full scale for a potential of 20.00 mV across it. Find R_x.

(a) 12.2 kΩ (b) 20.54 kΩ
(c) 19.92 kΩ (d) None of these

20. The lights on a car are inadvertently left on. They dissipate 95.0 W. About how long will it take for the fully charged 12.0 V car battery to run down, if the battery is rated at 150 A-h?

(a) 18.9 h (b) 16.0 h
(c) 22.5 h (d) 29.0 h

21. At a certain place on the planet, the earth's magnetic field is 5.0×10^{-5} T, directed 40° below the horizontal. Find the force per metre of length on a horizontal wire that carries a current of 30 A northward.

(a) 2.45×10^{-4} N (b) 2.45×10^{-2} N
(c) 9.6×10^{-2} N (d) 9.6×10^{-4} N

22. A 5.0 μF capacitor is charged to a potential difference of 20 kV between plates. After being disconnected from the power source, it is connected across a 7.0 MΩ resistor to discharge. What is the initial discharge current and how long will it take for the capacitor voltage to decrease to 37% of the 20 kV?

(a) 20 s (b) 35 s
(c) 19 s (d) 14 s

23. How much electrical potential energy does a proton lose as it falls through a potential drop of 5 kV?

(a) -8×10^{-16} J (b) -2.4×10^{-16} J
(c) -1.5×10^{-14} J (d) 19.5×10^{-14} J

24. What is the minimum value of the refractive index for a 45.0° prism which is used to turn a beam of light by total internal reflection through a right angle?

(a) 1.67 (b) 1.5
(c) 1.41 (d) 2.0

25. A lens has a convex surface of radius 20 cm and a concave surface of radius 40 cm and is made of glass of refractive index 1.54. Compute the focal length of the lens

(a) 74 cm (b) 23 cm (c) 15 cm (d) 29 cm

26. Red light falls normally on a diffraction grating ruled 4000 lines/cm and the second order image is diffracted 34.0° from the normal. Compute the wavelength of the light.

(a) 229 nm (b) 137 nm
(c) 699 nm (d) 250 nm

27. When a hydrogen atom is bombarded, the atom may be raised into a higher energy state. As the excited electron falls back to the lower energy levels, light is emitted. What are the three longest wavelength spectral lines emitted by the hydrogen atom as it returns to the $n = 1$ state from higher energy states? Give your answers to three significant figures.

(a) 122 nm, 102 nm, 96.9 nm
(b) 122 nm, 112 nm, 96.9 nm
(c) 145 nm, 102 nm, 83.7 nm
(d) 145 nm, 112 nm, 83.7 nm

28. Given below are symbols for some logic gates.

The XOR gate and NOR gate respectively are

(a) 1 and 2 (b) 2 and 3
(c) 3 and 4 (d) 1 and 4

29. If an AM transmitter shows an antenna current of 8 A when only carrier is transmitted and 9 A when current is modulated using sine wave. Then, the modulation index is

(a) 65.8% (b) 72.9%
(c) 40.2% (d) 0

30. The bandwidth required for an FM signal, if modulating frequency is 2 kHz and maximum deviation is 10 kHz, is

(a) 32 kHz (b) 10 kHz
(c) 15 kHz (d) 40 kHz

Answers

1. (c)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (c)	**8.** (c)	**9.** (a)	**10.** (d)
11. (a)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (a)	**17.** (c)	**18.** (a)	**19.** (c)	**20.** (a)
21. (d)	**22.** (b)	**23.** (a)	**24.** (c)	**25.** (a)	**26.** (c)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (a)

Solutions

1. Reading of vernier calliper

= Main scale divisions + Least count × Vernier scale divisions

= MSD + LC × VSD

As 6th VSD is coinciding with one of the MSD and MSD is 12th, hence reading

$= 12 + 6 \times 0.1 = 12.6$ mm

2. The reaction forces acting on the beam are shown in figure, where the force exerted by the hinge is represented by its components, F_{RH} and F_{RV}. The torque equation about P as axis is

$$\left(\frac{3L}{4}\right)(F_T)(\sin 40°) - (L)(800\text{ N})(\sin 90°)$$
$$-\left(\frac{L}{2}\right)(600\text{ N})\ (\sin 90°) = 0$$

We take the axis at P, so that F_{RH} and F_{RV} do not appear in the torque equation. Solution of this equation yields $F_T = 2280$ N or to two significant figures $F_T = 2.3$ kN.

3. From work-energy theorem,

The change in total energy of the car (KE + PE) is equal to the work done on it by the breaking force F. This work is $Fs \cos 180°$ because F retards the car's motion. We have,

$$\frac{1}{2}m(v_f^2 - v_i^2) + mg(h_f - h_i) = Fs(-1)$$

where, $m = 1200$ kg

$v_f = 0$

$v_i = 12$ m/s

$h_f - h_i = (100\text{ m})\sin 30°$

$s = 100$ m

With these values, the equation yields

$F = 6.7$ kN

4. The plane's resultant velocity is the sum of two velocities, 500 km/h—East and 90 km/h— South. These component of velocities are shown in figure. The plane's resultant velocity is then

$$R = \sqrt{(500\text{ km/h})^2 + (90\text{ km/h})^2} = 508\text{ km/h}$$

The angle α is given by

$$\tan\alpha = \frac{90\text{ km/h}}{500\text{ km/h}} = 0.18$$

from which $\alpha = 10°$.

The plane's velocity relative to the ground is 508 km/h at 10° south to east.

5. We must find the skier's acceleration from the data concerning the 3.0 s trip. Taking the direction of motion as the $+x$-direction, we have $t = 3.0$ s, $v_{ix} = 0$ and $x = 9.0$ m.

Then, $x = v_{ix}t + \frac{1}{2}at^2$ gives

$$a = \frac{2x}{t^2} = \frac{18\text{ m}}{(3.0\text{ s})^2} = 2.0\text{ m/s}^2$$

We can now use this value of a for the longer trip, from the starting point to the place, where $v_{fx} = 24$ m/s. For this trip, $v_{ix} = 0$, $v_{fx} = 24$ m/s, $a = 2.0\text{ m/s}^2$. Then, from $v_f = v_i + at$,

$$t = \frac{v_{fx} - v_{ix}}{a} = \frac{24\text{ m/s}}{2.0\text{ m/s}^2} = 12\text{ s}$$

6. The rotational and translational KE of the sphere at the bottom will be changed to PE_G when it stops. Therefore, we write

$$\left(\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2\right)_{\text{start}} = (mgh)_{\text{end}}$$

But for a solid sphere, $I = \frac{2}{5}mr^2$. Also, $\omega = v/r$. The above equation becomes

$$\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}\right)(mr^2)\left(\frac{v}{r}\right)^2 = mgh$$

or $$\frac{1}{2}v^2 + \frac{1}{5}v^2 = (9.81\text{ m/s}^2)\,h$$

Using $v = 20$ m/s gives

$$h = 29\text{ m}$$

7. We want the Reynolds number N_R to be the same in both cases, so that the situation is similar in both the cases. That is, we want

$$N_R = \left(\frac{\rho vD}{\eta}\right)_{\text{tunnel}} = \left(\frac{\rho vD}{\eta}\right)_{\text{air}}$$

Both ρ and η are the same in the two cases, so we have

$$v_t D_t = v_a D_a \text{ from which } v_t = v_a\frac{D_a}{D_t}$$
$$= (15\text{ m/s})\left(\frac{550}{20}\right) = 0.41\text{ km/s}$$

8. They will collide at their mean positions because time period of both are same and that is $2\pi\sqrt{\frac{m}{k}}$.

After collision, combined mass is $2m$ and $k_{\text{eff}} = 2k$. Hence, time period remains unchanged.

$$T = 2\pi\sqrt{\frac{2m}{2k}} \Rightarrow T = 2\pi\sqrt{\frac{m}{k}}$$

9. From conservation of linear momentum, we can see that velocity of combined mass just after collision is $v = \frac{\omega A}{4}$.

Before collision Just after collision

Since, this is the velocity at mean position.

Hence, $v = \omega' A'$

$$\frac{\omega A}{4} = \omega' A'$$

or $A' = \frac{A}{4}$ because $\omega' = \omega = \sqrt{\frac{k}{m}}$ or $\sqrt{\frac{2k}{2m}}$

10. $E = \frac{1}{2}(2m)v^2 = \frac{1}{2}(2m)\left(\frac{\omega A}{4}\right)^2$

$$= \frac{m\omega^2 A^2}{16} = \frac{kA^2}{16} \quad \left[\because \omega^2 = \frac{k}{m} = \frac{2k}{2m}\right]$$

or $E = \frac{1}{2}(2k)\left(\frac{A}{4}\right)^2 = \frac{kA^2}{16}$

11. Since, the box moves up the incline, the friction force (which always acts to retard the motion) is directed down the incline.

Let us first find F_f by writing $\Sigma F_x = ma_x$. From figure (using $\sin 40° = 0.643$) $F - F_f - mg \sin\theta = ma$

$383\text{ N} - F_f - (0.64)(25)(9.81)\text{ N} = (25\text{ kg})(0.75\text{ m/s}^2)$

from which $F_f = 207$ N

We also need F_N. Writing $\Sigma F_y = ma_y = 0$ and using $\cos 40° = 0.766$, we get

$$F_N - 321\text{ N} - (0.77)(25)(9.81)\text{ N} = 0$$

or $F_N = 510$ N

Then, $\mu_k = \frac{F_f}{F_N} = \frac{207}{510} = 0.41$

12. $\frac{\Delta L}{L_0} = \alpha\,\Delta T = (1.1 \times 10^{-5}\ °\text{C}^{-1})(35°\text{C})$

$[\because \Delta L = L_0\,\alpha\,\Delta t]$ where, ΔL = Change in length and L_0 = original length = 3.85×10^{-4}

So, $F = YA\frac{\Delta L}{L_0}$ $\left[\because \text{Young's modulus, } Y = \frac{F/A}{\Delta L/L_0}\right]$

$= (2.0 \times 10^{11}\text{ N/m}^2)(45 \times 10^{-4}\text{ m}^2)(3.85 \times 10^{-4})$

$= 3.5 \times 10^5$ N

13. In a whirlwind in a tornado, the air from nearby regions gets concentrated in a small space thereby decreasing the value of its moment of inertia considerably. Since, $I\omega$ = constant, so due to decrease in moment of inertia of the air, its angular speed increases to a high value.

If no external torque acts, then

$\tau = 0$ or $\frac{dL}{dt} = 0$ or L = constant

or $I\omega$ = constant

As in the rotational motion, the moment of inertia of the body can change due to the change in position of the axis of rotation, the angular speed may not remain conserved.

14. We use $pV = \left(\frac{m}{M}\right)RT$ to find $p = \frac{mRT}{MV}$

$$= \frac{(2.0 \times 10^{-6}\text{ kg})(8314\text{ J/kmol-K})(293\text{ K})}{(28\text{ kg/kmol})(30 \times 10^{-6}\text{ m}^3)}$$

$= 5800\text{ N/m}^2$

$= (5800\text{ N/m}^2)\left(\frac{1.0\text{ N/atm}}{1.01 \times 10^5\text{ N/m}^2}\right)$

$= 0.057$ atm

15. We have with the ice inside at 0°C $\Delta Q/\Delta t = k_T A\,\Delta T/L$

$$\frac{\Delta Q}{\Delta t} = (0.050\text{ W/m-K})(6)(0.42\text{ m})^2\left(\frac{20°\text{C}}{0.030\text{ m}}\right)$$

$= 35.3\text{ J/s} = 8.43\text{ cal/s}$

In one hour, $\Delta Q = (60)^2(8.43) = 30.350$ cal. To melt 1.0 g of ice requires 80 cal, so the mass of ice melted in one hour is

$$m = \frac{30.350\text{ cal}}{80\text{ cal/g}} = 0.38\text{ kg}$$

16. Actual efficiency = (0.30) (Carnot efficiency)

$$= (0.30)\left(1 - \frac{308\text{ K}}{493\text{ K}}\right) = 0.113$$

$$\left[\begin{array}{l}\because \eta = 1 - \frac{T_2}{T_1} \text{ where, } \eta = \text{efficiency} \\ T_2 = \text{sink and } T_1 = \text{saurce.}\end{array}\right]$$

But the relation efficiency $= \frac{\text{Output work}}{\text{Input heat}}$

$\therefore$ Input heat/s $= \frac{\text{Output work/s}}{\text{Efficiency}}$

$$= \frac{(8.00\text{ HP})(746\text{ W/HP})\left(\frac{1.00\text{ cal/s}}{4.184\text{ W}}\right)}{0.113}$$

$= 12.7$ kcal/s

17. $F_E = k\frac{qq'}{r^2}$ $[as\ q \Rightarrow e \text{ and } q \to e]$

$$= (9.0 \times 10^9\text{ N-m}^2/\text{C}^2)\frac{(1.6 \times 10^{-19}\text{ C})^2}{(5.3 \times 10^{-11}\text{ m})^2}$$

$= 8.2 \times 10^{-8}$ N = 82 N

The force found is the centripetal force, mv^2/r. Therefore,

$$8.2 \times 10^{-8}\ \text{N} = \frac{mv^2}{r}$$

from which $$v = \sqrt{\frac{(8.2 \times 10^{-8}\ \text{N})(r)}{m}}$$

$$= \sqrt{\frac{(8.2 \times 10^{-8}\ \text{N})(5.3 \times 10^{-11}\text{m})}{9.1 \times 10^{-31}\ \text{kg}}}$$

$$= 2.2 \times 10^6\ \text{m/s}$$

Aliter Velocity in nth orbit, $v_n = \frac{c}{137}\left(\frac{1}{n}\right)$

$$\Rightarrow \quad v_1 = \frac{c}{137} \cdot \frac{1}{1} = 2.2 \times 10^6\ \text{m/s}$$

18. The situation is shown in figure. Before being connected, their charges are

$$q_3 = CV = (3.0 \times 10^{-6}\ \text{F})(6.0\ \text{V}) = 18\mu\text{C}$$

$$q_4 = CV = (4.0 \times 10^{-6}\ \text{F})(6.0\ \text{V}) = 24\mu\text{C}$$

These charges partly cancel when the capacitors are connected together. Their final charges are given by

$$q_3' + q_4' = q_4 - q_3 = 6.0\,\mu\text{C}$$

Also, the potentials across them are now the same, so that $V = q/C$ which gives

$$\frac{q_3'}{3.0 \times 10^{-6}\ \text{F}} = \frac{q_4'}{4.0 \times 10^{-6}\ \text{F}} \qquad [\because V_1 = V_2]$$

or $$q_3' = 0.75 q_4'$$

Substitution in the previous equation gives

$$0.75 q_4' + q_4' = 6.0\,\mu\text{C}$$

or $$q_4' = 3.4\,\mu\text{C}$$

Then, $$q_3' = 0.75 q_4' = 2.6\,\mu\text{C}$$

19. When the galvanometer is deflecting full scale, the current through it is

$$I = \frac{V}{R} = \frac{20.00 \times 10^{-3}\ \text{V}}{80.00\ \Omega} = 2.500 \times 10^{-4}\ \text{A}$$

When R_x is connected in series with the galvanometer, we wish I to be 2.500×10^{-4} A for a potential difference of 5.000 V across the combination. Hence, $V = IR$ becomes

$$5.000\ \text{V} = (2.500 \times 10^{-4}\ \text{A})(80.00\ \Omega + R_x)$$

from which $$R_x = 19.92\ \text{k}\Omega$$

20. As an approximation, assume the battery maintains 12.0 V until it goes dead. Its 150 A-h rating means it can supply the energy equivalent of a 150 A current that flows for 1.00 h (3600 s).

Total output energy

$$= (\text{power})(\text{time}) = (VI)t$$
$$= (12.0\ \text{V} \times 150\ \text{A})\ (3600\ \text{s})$$
$$= 6.48 \times 10^6\ \text{J}$$

The energy consumed by the lights in a time t is

Energy dissipated = (95 W)(t)

Equating these two energies and solving for t, we find $t = 6.82 \times 10^4$ s = 18.9 h

21. Nearly everywhere, the earth's field is directed Northward *i.e.,* the direction in which a compass needle points. Therefore, the situation is that shown in figure. The force on the wire is

$$F_M = (30\ \text{A})(L)(5.0 \times 10^{-5}\ \text{T}) \sin 40° \qquad [\because F = iBl \sin\theta]$$

So that $$\frac{F_M}{L} = 9.6 \times 10^{-4}\ \text{N}$$

The right hand rule indicates that the force is into the page, which is West.

22. The loop equation for the discharging capacitor is

$$V_c - IR = 0$$

where, V_c is the potential difference across the capacitor. At the first instant, $V_c = 20$ kV, so

$$I = \frac{V_c}{R} = \frac{20 \times 10^3 \text{ V}}{7.0 \times 10^6\ \Omega} = 2.9 \text{ mA}$$

The potential across the capacitor, as well as the charge on it, will decrease to 0.37 of its original value in one time constant. The required time is

$$RC = (7.0 \times 10^6\ \Omega)(5.0 \times 10^{-6} \text{ F}) = 35 \text{ s}$$

23. The proton carries a positive charge. It will therefore move from regions of high potential the regions of low potential if left free to do so. Its change in potential energy as it moves through a potential different V is Vq. In our case, $V = -5$ kV.

Therefore, change in $PE_E = Vq$

$$= (-5 \times 10^3 \text{ V})(1.6 \times 10^{-19} \text{ C}) = -8 \times 10^{-16} \text{ J}$$

24. The ray enters the prism without deviation, since it strikes side AB normally. It then make an incidence angle of 45.0° with normal to side AC. The critical angle of the prism must be smaller than 45.0°, if the ray is to be totally reflected at side AC and thus turned through 90°.

$$\text{Minimum } n_i = \frac{1}{\sin 45.0°} = 1.41$$

25. First, notice that $r_1 > 0$ and $r_2 > 0$ because both surfaces have their centres of curvature to the right. Consequently,

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= (1.54 - 1)\left(\frac{1}{20} - \frac{1}{40}\right) = \frac{0.54}{40} \text{ cm}$$

or $\quad f = +74$ cm

Since, f turns out to be positive, the lens is converging.

26. From the grating equation $m\lambda = a \sin \theta_2$

$$\lambda = \frac{a \sin \theta_2}{2} = \frac{\left(\frac{1}{4000} \text{ cm}\right)(0.559)}{2}$$

$$= 6.99 \times 10^{-5} \text{cm} = 699 \text{ nm}$$

27. We are interested in the following transitions

$n = 2 \rightarrow n = 1$:

$\Delta E_{(2,1)} = -3.4 - (-13.6) = 10.2$ eV

$n = 3 \rightarrow n = 1$:

$\Delta E_{(3,1)} = -1.5 - (-13.6) = 12.1$ eV

$n = 4 \rightarrow n = 1$:

$\Delta E_{(4,1)} = -0.85 - (-13.6) = 12.8$ eV

To find the corresponding wavelengths, we can use $\Delta E = hf = \frac{hc}{\lambda}$.

For example, for the $n = 2$ to $n = 1$ transition,

$$\lambda = \frac{hc}{\Delta E_{(2,1)}}$$

$$= \frac{(6.63 \times 10^{-34} \text{ J-s})\ (2.998 \times 10^8 \text{ m/s})}{(10.2 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}$$

$$= 122 \text{ nm}$$

The other lines are found in the same way to be 102 nm and 96.9 nm. These are the first three lines of the Lyman series.

28. X-OR ⇒ Exclusive OR

NOR ⇒ NOT + OR

Symbols 2 and 3 represent XOR and NOR gates respectively.

29. Modulation index m_a is given by

$$m_a = \sqrt{2\left[\left(\frac{I_{tot}}{I_{carrier}}\right)^2 - 1\right]}$$

where, $\quad I_{tot} = 9$ A, $I_{carrier} = 8$ A

$$\therefore \quad m_a = \sqrt{2\left[\left(\frac{9}{8}\right)^2 - 1\right]}$$

$$m_a = \sqrt{2\left(\frac{81-64}{8}\right)}$$

$$= 72.9\% \text{ (approx)}$$

Hence, option (b) is true.

30. $M_f = \frac{\delta}{f_m} = \frac{10}{2} = 5$

For $\quad M_f = 5$,

Highest J coefficient is J_8.

Thus 8th pair sidebands is the farthest from the carrier to be included here. Thus, bandwidth

$$D = 2 \times 8 \times 2 = 32 \text{ kHz}$$

Practice Set 2

Instructions

For instructions refer to Practice Set 1.

1. A car and a truck are moving with same kinetic energy. To stop them brakes exerting equal force are applied simultaneously. Then, which of the following statement is correct?

(a) The truck stops earlier as it covers less distance
(b) The car stops earlier as it covers less distance
(c) The car and the truck stop simultaneously covering equal distances
(d) The car stops earlier though it covers the same distance as the truck before coming to stop

2. A DC voltage with appreciable ripple expressed as $V = V_1 + V_2 \cos \omega t$ is applied to a resistor R. The amount of heat generated per second is

(a) $\frac{V_1^2 + V_2^2}{2R}$
(b) $\frac{2V_1^2 + V_2^2}{2R}$
(c) $\frac{V_1^2 + 2V_2^2}{2R}$
(d) depends on whether V_1 is greater than or less than V_2

3. The work function for the surface of aluminium is 4.2 eV. How much potential difference will be required to just stop the emission of maximum energy electrons emitted by light of 2000 Å?

(a) 1.51 V (b) 1.99 V
(c) 2.99 V (d) None of these

4. A charge of 20μC is situated at the origin of an xy-coordinate system. The potential difference between two points $(5a, 0)$ and $(-3a, 4a)$ will be

(a) a (b) $2a$ (c) zero (d) $\frac{a}{\sqrt{2}}$

5. A car accelerates from rest at a constant rate α for some time after which it decelerates at a constant speed β to come to rest. If the total time elapsed is t, the maximum velocity acquired and total distance travelled by the car are given by

(a) $\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)t, \frac{1}{2}\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)t^2$
(b) $\left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)t, \frac{1}{2}\left(\frac{\alpha^2 - \beta^2}{\alpha\beta}\right)t^2$
(c) $\left(\frac{\alpha + \beta}{\alpha\beta}\right)t, \frac{1}{2}\left(\frac{\alpha + \beta}{\alpha\beta}\right)t^2$
(d) $\left(\frac{\alpha\beta}{\alpha + \beta}\right)t, \frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2$

6. A body slides down a plane inclined at an angle θ to the horizontal. The coefficient of friction μ down the plane varies in direct proportion to the distance moved down the plane. The body will move down the plane with a

(a) constant acceleration $g \sin\theta$
(b) constant acceleration $(g \sin\theta - \mu g \cos\theta)$
(c) constant acceleration $(\mu g \cos\theta - g \sin\theta)$
(d) variable acceleration that first decreases, then becomes negative

7. Unit of LCR is

(a) ohm × sec (b) ohm s^{-1}
(c) ohm × sec^2 (d) No unit

8. Two factories are sounding their sirens at 800 Hz each. A man goes from one factory to the other at a speed of $2\ ms^{-1}$. The speed of sound is $320\ ms^{-1}$. Therefore, the number of beats heard by the person in one second will be

(a) 2 (b) 4
(c) 8 (d) 10

9. A container open to atmosphere contains air (assumed to be an ideal gas) at temperature 27°C. The temperature is now raised to 227°C. The ratio of number of atoms in the container now and at the beginning is

(a) $\frac{3}{5}$ (b) $\frac{5}{3}$ (c) $\frac{3}{4}$ (d) $\frac{4}{5}$

➡ **Directions** *Q. no. 10 and 11 are based on following paragraph.*
A block of mass 2 kg is suspended by a spring of force constant k = 10 N/m. An another spring of same value of force constant is 1 m below it. Initially both the springs were unstretched. Mass is released from rest.

10. What will be the maximum extension in the upper spring?

(a) 3.26 m (b) 1.67 m
(c) 2.82 m (d) None of these

11. In the above problem, equilibrium position of the block is at ... m from where it was released.

(a) 1.0 (b) 2.0
(c) 1.2 (d) 1.5

12. A ray of light is incident normally on the face *PQ* of a glass prism ($\mu = 1.52$) as shown in the figure

(i) Assuming that the prism is immersed in air, find the largest value for angle ϕ, so that the ray is totally reflected at face *PR*.

(ii) Find ϕ if the prism is immersed in water ($\mu = 1.33$). Choose the correct answer from the following code.

(a) (i) 28.9° (ii) 48.9°
(b) (i) 48.9° (ii) 28.9°
(c) (i) 41.1° (ii) 61.1°
(d) (i) 36° (ii) 30°

13. Four charges *Q* each are located at four vertices of a regular tetrahedron of side *L*. The potential energy of the system is

(a) $\frac{4Q^2}{\pi\varepsilon_0 L^2}$ (b) $\frac{6Q^2}{4\pi\varepsilon_0 L}$ (c) $\frac{Q^2}{\pi\varepsilon_0 L}$ (d) $\frac{8Q^2}{4\pi\varepsilon_0 L}$

14. The moment of inertia of a body about a given axis is $1.2\ kg\text{-}m^2$. Initially, the body is at rest. In order to produce a rotational kinetic energy of 1500 J, an angular acceleration of $25\ rad\text{-}s^{-2}$ must be applied about that axis for a duration of

(a) 4 s (b) 2 s (c) 8 s (d) 10 s

15. Light passes from a denser medium 1 to a rarer medium 2. When the angle of incidence is θ, the reflected and refracted rays are mutually perpendicular. The critical angle will be

(a) $\sin^{-1}(\cot\theta)$ (b) $\sin^{-1}(\tan\theta)$
(c) $\sin^{-1}(\cos\theta)$ (d) $\sin^{-1}(\sec\theta)$

16. A person looking through telescope *T* just sees the point *A* on the rim at the bottom of a cylindrical vessel when the vessel is empty. When the vessel is completely filled with a liquid ($\mu = 1.5$), he observes a mark *B* at the centre of the bottom without moving the telescope or the vessel. The diameter of the base of the vessel is 10 cm. The height of the vessel is

(a) 10 cm (b) 15 cm (c) 4.85 cm (d) 8.45 cm

17. A particle A has charge $+q$ and particle B has charge $+4q$ with each of them having same mass m. When allowed to fall from rest through the same electric potential difference, the ratio of their speeds $\frac{v_A}{v_B}$ will become

(a) 2:1 (b) 1:2 (c) 1:4 (d) 4:1

18. A radio transmitter operates at 880 kHz and its power is 10 kW. The number of photons emitted per second is

(a) 1.72×10^{31} (b) 1.72×10^{32}

(c) 3.44×10^{31} (d) None of these

19. The whistle of a railway engine is heard in winter at much longer distances. This is due to

(a) decrease in the velocity of sound in winter
(b) decrease in the density of air with respect to height from the surface of the earth
(c) cold air absorbs much small energy from sound waves
(d) increase in the density of air with respect to height from the surface of the earth

20. The position vector of an electron is $\mathbf{r} = 5\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$. To an observer moving along x direction with speed $0.6\,c$, the magnitude of position vector is

(a) $5\sqrt{2}$ (b) $\sqrt{34}$ (c) $\sqrt{41}$ (d) 5

21. Magnus effect is related to

(a) magnetic effect of current
(b) production of eddy currents
(c) Bernoulli's effect
(d) None of the above

22. The displacement of a particle is given by

$$y = 2\sin(\omega t) + 2\sin\left(\omega t + \frac{\pi}{3}\right)$$

Then, the incorrect statement is

(a) The amplitude of motion is $\sqrt{12}$
(b) The angular frequency is ω
(c) The velocity at $t = 0$ is 3ω
(d) The initial phase of motion is $\pi/3$

23. In Young's double slit experiment, constructive interference is produced at a certain point P. The intensities of light at P due to the individual sources are 4 and 9 units. The resultant intensity at point P will be

(a) 13 units (b) 25 units
(c) $\sqrt{97}$ units (d) 5 units

24. Substances for which permeability μ is slightly greater than 1, are called

(a) Diamagnetic (b) Paramagnetic
(c) Ferromagnetic (d) Non-magnetic

25. An isolated sphere S of radius R carries an electric charge. S is momentarily connected to a distant uncharged sphere T which has a radius r. The ratio of surface charge density of S to that of T is

(a) $\frac{R}{r}$ (b) $\frac{r}{R}$ (c) $\frac{r}{2R}$ (d) $\frac{(r+R)}{R}$

26. In post office box, the graph of galvanometer deflection *versus* resistance R (pulled out of resistance box) for the ratio 100:1 is given as shown (due to unsuitable values of R, galvanometer shows deflection). The two consecutive values of R are shown in the figure.

The value of unknown resistance would be

(a) 3.2 Ω (b) 3.24 Ω (c) 3.206 Ω (d) 3.237 Ω

27. The forward bias characteristics of two diodes D_1 and D_2 are shown, the knee voltages for D_1 and D_2 are respectively (approx)

(a) 0.4 V and 0.7 V (b) 0.6 V and 0.9 V
(c) 0.6 V and 0.8 V (d) 0.4 V and 0.9 V

➡ **Directions** *Q. no. 28 and 29 are Assertion-Reason type questions. Each of these question contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below*

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(b) Statement I is true, Statement II is false
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true Statement II is a correct explanation of Statement I

28. **Statement I** An empty test tube dipped into water in a beaker appears silvery, when viewed from a suitable direction.

Statement II Due to refraction of light, the substance in water appears silvery.

29. **Statement I** The electric field due to a dipole on its axial line at a distance r is E. Then, electric field due to the same dipole on the equatorial line and at the same distance will be $\frac{E}{2}$.

Statement II Electric field due to dipole varies inversely as the square of distance.

30. The output current of a 60% modulated AM generator is 1.5 A. To what value will the current rise, if the generator is additionally modulated by another audio wave of modulation index 0.7?
(a) 0.64 A
(b) 1.64 A
(c) 2.34 A
(d) 5.32 A

Answers

1. (d)	**2.** (b)	**3.** (b)	**4.** (c)	**5.** (d)	**6.** (d)	**7.** (c)	**8.** (d)	**9.** (a)	**10.** (c)
11. (d)	**12.** (b)	**13.** (b)	**14.** (b)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (a)	**19.** (b)	**20.** (c)
21. (c)	**22.** (d)	**23.** (b)	**24.** (b)	**25.** (b)	**26.** (b)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (b)

Solutions

1. Momentum, $p = \sqrt{2mE}$

As energy E is the same, car will have lesser momentum than the truck. Final momentum is zero for both. Hence, the change in momentum is less for the car.

Now, change in momentum = Impulse = $F \times t$

Since, the force F is the same, it is clear that the car stops earlier.

Again, $E = F \times s$

where, s is the distance travelled before coming to rest. This is the same for both the car and the truck as E and F are same.

2. Heat generated, $Q = \frac{V^2}{R} = \frac{(V_1 + V_2 \cos \omega t)^2}{R}$

$$= \frac{V_1^2 + V_2^2 \cos^2 \omega t + 2V_1V_2 \cos \omega t}{R}$$

Over a cycle,

$$(\cos^2 \omega t)_{\text{average}} = \frac{1}{2}\left[\because \cos^2 \omega t = \frac{1-\cos 2\omega t}{2}\right]$$

$$(\cos \omega t)_{\text{average}} = 0$$

$$(\cos 2\omega t)_{\text{average}} = 0$$

and $(\cos \omega t) = 0$

$$\Rightarrow \quad Q = \frac{V_1^2 + \frac{1}{2}V_2^2}{R}$$

$$\Rightarrow \quad Q = \frac{2V_1^2 + V_2^2}{2R}$$

3. From equation of photoelectric effect, we have

$$E_k = h\nu - W = \frac{hc}{\lambda} - W \qquad \left[\text{as } \nu = \frac{c}{\lambda}\right] \ldots \text{(i)}$$

$$W = 4.2 \text{ eV} = 4.2 \times 1.6 \times 10^{-19} = 6.7 \times 10^{-19} \text{ J}$$

From Eq. (i), we get

$$E_k = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{2000 \times 10^{-10}} - 6.72 \times 10^{-19}$$

$$\Rightarrow \quad E_k = 9.9 \times 10^{-19} - 6.72 \times 10^{-19}$$

$$\Rightarrow \quad E_k = 3.18 \times 10^{-19} \text{ J}$$

Hence, the stopping potential

$$V_0 = \frac{E_k}{e} = \frac{3.18 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.99 \text{ V}$$

4. The distance of point $(5a, 0)$ from origin

$$r_1 = 5a$$

$$\Rightarrow \quad V_1 = \frac{20\,K}{5a}$$

The distance of point $(-3a, 4a)$ from origin

$$r_2 = \sqrt{9a^2 + 16a^2}$$

$$= 5a$$

$$\Rightarrow \quad V_2 = \frac{20K}{5\,a}$$

Hence, $\quad V_1 - V_2 = 0$

5. Let the car accelerate for time t_1, travelling distance S_1 and acquiring maximum velocity v.

Then, $S_1 = \frac{1}{2}\alpha t_1^2$ [from the equation, $S = ut + \frac{1}{2}at^2$]

and $\quad v = \alpha t_1 \quad$ [from the equation, $v = u + at$]

$$\Rightarrow \quad S_1 = \frac{1}{2}\alpha \times \frac{v^2}{\alpha^2}$$

$$= \frac{v^2}{2\alpha} \quad \ldots\text{(i)}$$

After this car decelerates for time t_2 to come to rest.

Hence, $\quad S_2 = vt_2 - \frac{1}{2}\beta t_2^2$

[from the equation, $S = ut - \frac{1}{2}at^2$]

and $\quad 0 = v - \beta t_2$

[from the equation, $u = v - at$]

$$\Rightarrow \quad S_2 = \frac{v^2}{\beta} - \frac{v^2}{2\beta} = \frac{v^2}{2\beta} \quad \ldots\text{(ii)}$$

Now, $\quad t_1 + t_2 = t$

$$\Rightarrow \quad \frac{v}{\alpha} + \frac{v}{\beta} = t$$

$$\Rightarrow \quad v = \left(\frac{\alpha\beta}{\alpha + \beta}\right)t$$

Also, total distance travelled,

$$S = S_1 + S_2$$

$$= \frac{v^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{1}{2}\left(\frac{\alpha\beta t}{\alpha + \beta}\right)^2 \left(\frac{\alpha + \beta}{\alpha\beta}\right)$$

$$= \frac{1}{2}\left(\frac{\alpha\beta}{\alpha + \beta}\right)t^2$$

6. The force on the body moving down the plane

$$F = mg \sin\theta - \mu N$$

$$\Rightarrow \quad F = mg \sin\theta - \mu\, mg \cos\theta$$

Acceleration down the plane

$$a = (g \sin\theta - \mu g \cos\theta)$$

As μ varies in direct proportion to the distance moved down the plane, it is clear that first the acceleration decreases, becomes zero and then reverses.

7. $\frac{L}{R}$ and RC are time constants. Each has unit second.

So, $\quad LCR = \left(\frac{L}{R}\right)(RC)(R)$

$$= \text{sec} \times \text{sec} \times \text{ohm} = \text{ohm} \times \text{sec}^2$$

8. Apparent frequency is given by $n_1 = \frac{v - v_L}{v - v_s}n$

where, n is frequency of source.

$$n_1 = \frac{320 - 2}{320 - 0} \times 800 = 795 \text{ Hz}$$

[minimum frequency]

$$n_2 = \frac{320+2}{320-0} \times 800 = 805 \text{ Hz}$$

[maximum frequency]

So, number of beats heard per second is

$$n_b = n_2 - n_1 = 805 - 795 = 10 \text{ beats/s}$$

9. $pV = nkT$

$\Rightarrow \quad n_1 kT_2 = n_2 kT_2$

$\Rightarrow \quad \frac{n_2}{n_1} = \frac{T_1}{T_2} = \frac{273+27}{273+227} = \frac{300}{500} = \frac{3}{5}$

10. Let lower spring compresses maximum by x metre. From conservation of mechanical energy, decreases in potential energy of block is equal to increases in elastic potential energy of both the springs.

$$mgh = \frac{1}{2}kx^2 + \frac{1}{2}k(x+1)^2$$

$$(2)(10)(x+1) = \frac{1}{2} \times 10 \times x^2 + \frac{1}{2} \times 10 \times (x+1)^2$$

or $\quad 20x + 20 = 5x^2 + 5x^2 + 5 + 10x$

$\therefore \quad 10x^2 - 10x - 15 = 0$

or $\quad 2x^2 - 2x - 3 = 0$

$\therefore \quad x = \frac{2 \pm \sqrt{4+24}}{4}$

or $\quad x = 1.82$ m

$\therefore$ Maximum extension of upper spring

$= 1 + x = 2.82$ m

11. At equilibrium position, net force on the block should be zero. So, let it be at distance y from where it was released.

Then, $\quad mg = ky + k(y-1)$

or $\quad 20 = 10y + 10(y-1) \quad$ or $\quad 20y = 30$

$\therefore \quad y = 1.5$ m

12. If the ray is to be totally reflected, then α must be equal to or greater than the critical angle. Since, $\alpha + \phi = 90°$, ϕ is a maximum when α is a minimum allowed value. Hence, α should be equal to the critical angle.

Now, $\quad \sin\alpha = \frac{1}{\mu} \quad [\because \alpha = C]$

$\Rightarrow \quad \sin\alpha = \frac{1}{1.52} \quad \left[\because \sin C = \frac{1}{\mu}\right]$

$\Rightarrow \quad \sin\alpha = 0.6579 \quad \Rightarrow \quad \alpha = 41.1°$

Hence, $\quad \phi = 90° - \alpha = 90° - 41.1° = 48.9°$

In this case, $\quad \sin\alpha = \frac{1}{{}^w\mu_g} = {}^g\mu_w$

$= \frac{{}^a\mu_w}{{}^a\mu_g} = \frac{1.33}{1.52} = 0.8750$

$\alpha = 61.1°$

Hence, $\quad \phi = 90° - \alpha = 90° - 61.1° = 28.9°$

13. The potential energy of the system

= Total work done in bringing all four charges

$(Q_0 = Q_1 = Q_2 = Q_3 = Q)$ in their position

$$U = 0 + \frac{Q_0 Q_1}{4\pi\varepsilon_0 L} + \frac{(Q_0 + Q_1)Q_2}{4\pi\varepsilon_0 L} + \frac{(Q_0 + Q_1 + Q_2)Q_3}{4\pi\varepsilon_0 L}$$

$$U = 0 + \frac{Q^2}{4\pi\varepsilon_0 L} + \frac{2Q^2}{4\pi\varepsilon_0 L} + \frac{3Q^2}{4\pi\varepsilon_0 L}$$

$$U = \frac{6Q^2}{4\pi\varepsilon_0 L}$$

14. $E = \frac{1}{2} I\omega^2$

$\Rightarrow \quad 1500 = \frac{1}{2} \times 1.2\,\omega^2 \quad \Rightarrow \quad \omega^2 = \frac{3000}{1.2} = 2500$

$\Rightarrow \quad \omega = 50 \text{ rad-s}^{-1}$

$\Rightarrow \quad t = \frac{\omega}{\alpha} \quad [\because \omega = 0 + \alpha t]$

$\Rightarrow \quad t = \frac{50}{25} = 2$ s

15. ${}_2\mu_1 = \frac{\sin(90° - \theta)}{\sin\theta} = \cot\theta$

Also, $\quad {}_2\mu_1 = \dfrac{1}{\sin C}$

$\Rightarrow \quad \sin C = \dfrac{1}{{}_2\mu_1} = \dfrac{1}{\cot\theta} = \tan\theta$

$\Rightarrow \quad C = \sin^{-1}(\tan\theta)$

16. Suppose the position of the telescope is at T when the point A is visible on the rim as shown in the figure. Now, when the vessel is filled with a liquid, point B is observed at the same arrangement due to refraction of the ray at point D.

In ΔBCA, $\sin i = \dfrac{BC}{BD}$

$$= \frac{BC}{\sqrt{(BC)^2 + (CD)^2}} = \frac{5}{\sqrt{25 + h^2}}$$

Again, $\quad \angle EDT = \angle ADC = \angle r$

$\therefore \quad \sin r = \dfrac{AC}{AD}$

$$= \frac{AC}{\sqrt{(AC)^2 + (CD)^2}} = \frac{10}{\sqrt{100 + h^2}}$$

Now, applying Snell's law,

$$\frac{\sin i}{\sin r} = {}_w\mu_a = \frac{1}{{}_a\mu_w} \qquad [\because \mu_a = 1]$$

$\Rightarrow \quad \dfrac{5}{\sqrt{25 + h^2}} \times \dfrac{\sqrt{100 + h^2}}{10} = \dfrac{1}{1.5}$

or $\quad \dfrac{100 + h^2}{25 + h^2} = \dfrac{16}{9}$

or $\quad h = 8.45$ cm

17. Acceleration $= \dfrac{\text{Force}}{\text{Mass}}$

$$a = \frac{F}{m} = \frac{qE}{m}$$

$\therefore \quad a_A = \dfrac{1}{4} a_B$

Now, $\quad v^2 = u^2 + 2ax$

Since, x is the same for both the particles.

$\Rightarrow \quad v_A^2 = 0 + 2a_A x$ and $v_B^2 = 0 + 2a_B x$

$\Rightarrow \quad \dfrac{v_A^2}{v_B^2} = \dfrac{a_A}{a_B} = \dfrac{1}{4} \Rightarrow \dfrac{v_A}{v_B} = \dfrac{1}{2}$

18. Number of photons emitted per second

$$n = \frac{P}{h\nu} \qquad \left[\because n = \frac{P}{E}\right]$$

$$= \frac{10 \times 1000}{6.6 \times 10^{-34} \times 880 \times 10^3} = 1.72 \times 10^{31}$$

19. In winter, the density of air near the ground is more, producing total internal reflection. Hence, the sound is heard loud and to a greater distance.

20. Only x-component of radius vector contracts

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}} = 5\sqrt{1 - (0.6)^2}$$

$$= 5 \times 0.8 = 4$$

$$\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$$

[$\because$ only x-component will be altered]

$$|r| = \sqrt{(4)^2 + (4)^2 + (-3)^2} = \sqrt{41}$$

21.

According to Bernoulli's theorem, pressure difference on a ball thrown spinning exerts a force on the ball due to which a swing is produced in the ball. This effect is known as Magnus effect.

22. $y = 2\sin\omega t + 2\sin\left(\omega t + \dfrac{\pi}{3}\right)$

$$= 2\sin\omega t + 2\left[\sin\omega t\cos\frac{\pi}{3} + \cos\omega t\sin\frac{\pi}{3}\right]$$

$$= 2\sin\omega t + 2\left[\sin\omega t\,\frac{1}{2} + \cos\omega t\,\frac{\sqrt{3}}{2}\right]$$

$$= 3\sin\omega t + \sqrt{3}\cos\omega t$$

$$= 3 \times \frac{2}{\sqrt{3}} \sin \omega t \cdot \frac{\sqrt{3}}{2} + 2\sqrt{3} \cos \omega t \cdot \frac{1}{2}$$

$$= \sqrt{12}\left[\sin \omega t \cos \frac{\pi}{6} + \cos \omega t \sin \frac{\pi}{6}\right]$$

$$y = \sqrt{12} \sin\left(\omega t + \frac{\pi}{6}\right)$$

Comparing it with standard equation

$y = a \sin(\omega t + \phi)$, we get

Amplitude, $a = \sqrt{12}$

Angular velocity $= \omega$

Velocity, $v = \frac{dy}{dt}$

$\because$ $v = \sqrt{12} \cos\left(\omega t + \frac{\pi}{6}\right) \cdot \omega$

At $t = 0$, $v = \sqrt{12} \cos \frac{\pi}{6} \cdot \omega$

$$= \sqrt{12} \times \frac{\sqrt{3}}{2} \times \omega = \frac{2\sqrt{3} \times \sqrt{3}\,\omega}{2} = 3\omega$$

Initial phase, $\phi = \frac{\pi}{6}$

It is not equal to $\frac{\pi}{3}$.

23. Amplitudes of the sources are 2 and 3. The resultant amplitude at point P is $2 + 3 = 5$.
So, resultant intensity at point P will be

$$(5)^2 = 25 \text{ units}$$

24. We know that when a paramagnetic substance is placed in a magnetic field, the magnetic flux density is slightly greater than that in the free space. Thus, the relative permeability for them is slightly greater than 1.

24. S and T will have same potential. Thus,

$$\frac{Q_1}{4\pi\varepsilon_0 R} = \frac{Q_2}{4\pi\varepsilon_0 r} \Rightarrow \frac{Q_1}{Q_2} = \frac{R}{r}$$

Again, if σ_1 and σ_2 are the surface densities of spheres S and T, then

$$\frac{Q_1}{Q_2} = \frac{4\pi R^2 \sigma_1}{4\pi r^2 \sigma_2} = \frac{R}{r} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{r}{R}$$

26. For post office box (Wheatstone bridge), under balanced conditions

$\frac{P}{Q} = \frac{R}{S}$, where S is unknown resistance.

$S = \frac{Q}{P} \times R$ $\left[\because \frac{P}{Q} = 100, \text{ given}\right]$

From the given graph, the galvanometer shows zero deflection at

$R = 323.75 = 324\ \Omega$ [upto 3 significant digits]

So, $S = 3.24\ \Omega$ $\left[\because S = \frac{R}{100}\right]$

27. The forward voltage when current in circuit starts increasing rapidly is the knee voltage. As it is clear from the graph, current starts increasing rapidly when voltage is about 0.4 and 0.7 V.

28. Rays of light travelling from water to air at the surface of test tube strikes at an angle of incidence greater than the critical angle for water-air interface. Therefore, they suffer total internal reflection and reach the eye. Thus, the surface of test-tube appears silvery.

29. We know that for an electric dipole,

$$E_{\text{axial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{2p}{r^3}\right) \quad \ldots\text{(i)}$$

and $$E_{\text{equatorial}} = \frac{1}{4\pi\varepsilon_0}\left(\frac{p}{r^3}\right) \quad \ldots\text{(ii)}$$

From Eqs. (i) and (ii), we get

$$\frac{E_{\text{axial}}}{2} = E_{\text{equatorial}}$$

Hence, $E_{\text{equatorial}} = \frac{E}{2}$

Reason is false as electric field due to dipole varies inversely as cube of distance *i.e.*, $E \propto \frac{1}{r^3}$.

30. $I = I_c \sqrt{\frac{1 + m_1^2}{2}}$

$$I_c = \frac{I}{\sqrt{1 + \frac{m_1^2}{2}}} = \frac{1.5}{\sqrt{1 + \frac{(0.6)^2}{2}}} = \frac{1.5}{\sqrt{1.18}} \left[\because m_1 = \frac{60}{100}\right]$$

$$I_{\text{net}} = I_c \sqrt{1 + \frac{m_1^2}{2} + \frac{m_2^2}{2}}$$

$$= \frac{1.5}{\sqrt{1.18}} \sqrt{1 + 0.18 + \frac{0.49}{2}} = 1.64 \text{ A}$$

Hence, option (b) is true.

Practice Set 3

Instructions

For instructions refer to Practice Set 1.

1. The length of the string of a simple pendulum is measured with a meter scale to be 92.0 cm, the radius of the bob plus the hook is measured with the help of vernier calliper to be 2.17 cm. Mark out the correct statement.

(a) Least count of meter scale is 0.1 cm
(b) Least count of vernier calliper is 0.01 cm
(c) Effective length of simple pendulum is 94.2 cm
(d) All of the above

2. Two bodies A and B of equal mass are suspended from two separate massless springs of spring constant k_1 and k_2, respectively. If the bodies oscillate vertically such that their maximum velocities are equal, the ratio of the amplitude of A to that of B is

(a) $\frac{k_1}{k_2}$ (b) $\sqrt{\frac{k_1}{k_2}}$ (c) $\frac{k_2}{k_1}$ (d) $\sqrt{\frac{k_2}{k_1}}$

3. As the object moves from infinity to focus, then which is true, about the image formed by single concave mirror?

(a) Always real and speed of image continuously increases
(b) Always real and speed is initially smaller and finally larger than object speed
(c) Initially real and moving with speed smaller than object speed but later on image becomes virtual and moving with speed of object
(d) Always virtual and speed is less than object speed

4. When a glass capillary tube of radius 0.015 cm is dipped in water, the water rises to a height of 15 cm within it. Assuming contact angle between water and glass to be 0°, the surface tension of water is [$\rho_{water} = 1000$ kg/m^3, $g = 9.81$ m/s^2]

(a) 0.11 N/m (b) 0.7 N/m
(c) 0.072 N/m (d) None of these

5. Two bodies of different masses have been released from the top of tower. One is thrown in horizontal direction while other is dropped, then which will reach the ground first?

(a) The body which has been thrown horizontally
(b) The body which has been dropped
(c) Both will reach the ground simultaneously
(d) Depends on the velocity with which the first body has been projected horizontally

6. A cylinder of radius R made of a material of thermal conductivity K_1 is surrounded by a cylindrical shell of inner radius R and outer radius $2R$ made of a material of thermal conductivity K_2. The two ends of the combined system are maintained at two different temperatures. There is no loss of heat across the cylindrical surface and the system is in steady state. Find the effective thermal conductivity of the system.

(a) $K_1 + K_2$ (b) $\frac{K_1 K_2}{(K_1 + K_2)}$
(c) $\frac{(3K_1 + K_2)}{4}$ (d) $\frac{(K_1 + 3K_2)}{4}$

➡ **Directions** **Q. Nos.** *7 to 9 are based on following paragraph.*

A solid sphere of mass M and radius R is surrounded by a spherical shell of same mass M and radius 2R as shown in figure. A small particle of mass m is released from rest from a height h(< < R) above the shell. There is a hole in the shell.

7. In what time will it enter the hole at A?

(a) $2\sqrt{\frac{hR^2}{GM}}$ (b) $\sqrt{\frac{2hR^2}{GM}}$

(c) $\sqrt{\frac{hR^2}{GM}}$ (d) None of these

8. What time will it take to move from A to B?

(a) $= \frac{R^2}{\sqrt{GMh}}$ (b) $> \frac{R^2}{\sqrt{GMh}}$

(c) $< \frac{R^2}{\sqrt{GMh}}$ (d) None of these

9. With what approximate speed will it collide at B?

(a) $\sqrt{\frac{2GM}{R}}$ (b) $\sqrt{\frac{GM}{2R}}$ (c) $\sqrt{\frac{3GM}{2R}}$ (d) $\sqrt{\frac{GM}{R}}$

10. An AC source producing emf $e = e_0[\cos(100\pi s^{-1})t + \cos(500\pi s^{-1})t]$ is connected in series with a capacitor and resistor. The steady state current in the circuit is found to be

$$i = i_1 \cos[(100\pi s^{-1})t + \phi_1] + i_2 \cos[(500\pi s^{-1})t + \phi_2].$$

(a) $i_1 > i_2$

(b) $i_1 = i_2$

(c) $i_1 < i_2$

(d) The information is insufficient to find the relation between i_1 and i_2

11. A block of wood has a mass of 25 g. When a 5 g metal piece with a volume of 2 cm^3 is attached to the bottom of the block, the wood barely floats in water. What is the volume V of the wood?

(a) 28 cm^3 (b) 35 cm^3

(c) 48 cm^3 (d) 12 cm^3

12. A stone is swinging in a horizontal circle 0.8 m in diameter, at 30 rev/min. A distant light causes a shadow of the stone to be formed on a nearby wall. What is the amplitude of the motion of the shadow? What is the frequency?

(a) 0.4 m, 1.5 Hz (b) 0.4 m, 0.5 Hz

(c) 0.8 m, 0.5 Hz (d) 0.2 m, 0.5 Hz

13. A 20 g bullet pierces through plate of mass $m_1 = 1$ kg and then comes to rest inside a second plate of mass $m_2 = 2.98$ kg. It is found that the two plates, initially at rest, now move with equal velocities. The percentage loss in the initial velocity of bullet when it is between m_1 and m_2 (neglect any loss of material of the bodies, due to action of bullet) will be

(a) 20% (b) 25% (c) 30% (d) 45%

14. Given, $\text{force} = \frac{\alpha}{\text{density} + \beta^3}$

What are the dimensions of α, β?

(a) $[ML^2T^{-2}]$, $[ML^{-1/3}]$

(b) $[M^2 L^4T^{-2}]$, $[M^{1/3} L^{-1}]$

(c) $[M^2 L^{-2}T^{-2}]$, $[M^{1/3} L^{-1}]$

(d) $[M^2 L^{-2}T^{-2}]$, $[ML^{-3}]$

15. If electric potential due to some charge distribution is given by $V = 3/r^2$, when r is radial distance, then find electric field at (1, 1, 1).

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{2(\mathbf{i}+\mathbf{j}+\mathbf{k})}{3}$

(c) $\frac{2}{8(\mathbf{i}+\mathbf{j}+\mathbf{k})}$ (d) $\frac{3}{2(\mathbf{i}+\mathbf{j}+\mathbf{k})}$

16. In the diagram, a plot between δ (deviation) *versus* i (angle of incidence) for a triangular prism is given. From the observed plot, some conclusions can be drawn. Mark out the correct conclusions.

(a) The range of deviation for which two angles of incidence are possible for same deviation is $\delta_0 - \delta_m$

(b) The curve is unsymmetrical about i_0

(c) For a given δ, i is unique

(d) Both (a) and (b) are correct

17. Mark the correct option

(a) When the branch of the circuit containing battery is open, then potential difference across the terminals of battery is same as emf

(b) When two terminals of a battery are shorted, then potential difference across the terminals of battery is zero

(c) If external resistance of the circuit changes, then potential difference across the terminals of the battery changes but emf remains same

(d) All of the above

18. Two clocks are being tested against a standard clock located in a national laboratory. At 12:00:00 noon by the standard clock, the readings of the two clocks are given as follows.

	Clock I	**Clock II**
Monday	12 : 00 : 05	10 : 15 : 06
Tuesday	12 : 01 : 15	10 : 14 : 59
Wednesday	11 : 59 : 08	10 : 15 : 18
Thursday	12 : 01 : 50	10 : 15 : 07
Friday	11 : 59 : 15	10 : 14 : 53
Saturday	12 : 01 : 30	10 : 15 : 24
Sunday	12 : 01 : 19	10 : 15 : 11

If you are doing an experiment that requires precision time interval measurements, which of the two clocks will you prefer?

(a) Clock I

(b) Clock II

(c) Both have same precision time interval measurements

(d) None of the above

19. The emf and internal resistance of the battery shown in figure are 4.3 V and 1Ω, respectively. The external resistance R is 50 Ω. The resistance of the voltmeter and ammeter are 200 Ω and 2 Ω, respectively. Find the readings of the two meters.

(a) 0.1 A, 2 V (b) 0.1 A, 4 V

(c) 0. 4 A, 1 V (d) 0.4 A, 4 V

20. All the accelerations shown in figure are with respect to ground, find acceleration of B.

(a) $3\ \text{m/s}^2$ upward

(b) $5\ \text{m/s}^2$ upward

(c) $3\ \text{m/s}^2$ downward

(d) None of the above

21. Three dielectric slabs of thickness $d/4$, $d/7$ and $d/2$ having dielectric constants 2, 8/7 and 4 respectively are inserted between the plates of a parallel plate capacitor having plate separation d and plate area A. The remaining space is filled with a conducting medium. Find the capacitance of the system.

(a) $\frac{8\varepsilon_0 A}{d}$ (b) $\frac{8\varepsilon_0 A}{3d}$

(c) $\frac{26\varepsilon_0 A}{35d}$ (d) $\frac{12\varepsilon_0 A}{35d}$

22. The electron emitted from an electron gun are having 2 eV of kinetic energy, then they are accelerated by applying a potential of V volts and are allowed to fall on atomic hydrogen gas. It has been observed that red light of wavelength 656.3 nm has been emitted by the hydrogen. Assume that total energy of electron has been given, then V has the value given by

(a) 2 V (b) 0.11 V

(c) –2 V (d) – 0.11 V

23. A body dropped from a height H reaches the ground with a speed of $1.2\sqrt{gH}$. Calculate the work done by air-friction.

(a) 2.8 mgH (b) –1.3 mgH

(c) 1.3 mgH (d) –0.28 mgH

24. The ratio of the acceleration due to gravity at the bottom of a deep mine and that on the surface of the earth is 978/980. Find the depth of the mine, if the density of the earth is uniform throughout and the radius of the earth is 6300 km.

(a) 12.86 km (b) 13.0 km

(c) 25.38 km (d) 90.9 km

25. A sky wave with a frequency 55 MHz is incident on *D*-region of the earth's atmosphere at 45°. The angle of refraction is (electron density for *D*-regions is 400 electron/cm^3)

(a) 60° (b) 45° (c) 30° (d) 15°

➡ **Directions** **Q. Nos.** 26 *and* 27 *are Assertion-Reason type questions. Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below*

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(b) Statement I is true, Statement II is false
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

26. **Statement I** The rocket works on the principle of conservation of linear momentum.

Statement II Whenever there is the change in momentum of one body, the same change occurs in the momentum of the second body of the same system but in opposite direction.

27. **Statement I** If the half-life of a radioactive substance is 40 days, then 25% substance decays in 20 days.

Statement II $N = N_0\left(\frac{1}{2}\right)^n$

where, $n = \frac{\text{time elapsed}}{\text{half-life period}}$

28. The voltage gain of an amplifier without feedback is 100. If a negative feedback is introduced with a feedback fraction $\beta = 0.1$, then the gain of the feedback amplifier is

(a) 9.09 (b) 10
(c) 100.1 (d) 90.0

29. In frequency modulated wave,

(a) frequency varies with time
(b) amplitude varies with time
(c) both frequency and amplitude vary with time
(d) both frequency and amplitude are constant

30. The output current of a 60% modulated AM generator is 1.5 A. If the generator is additionally modulated by another radio wave of modulate index 0.7, the new modulation index is (in percentage)

(a) 20% (b) 92%
(c) 80% (d) 55%

Answers

1. (c)	**2.** (d)	**3.** (b)	**4.** (a)	**5.** (c)	**6.** (d)	**7.** (a)	**8.** (c)	**9.** (d)	**10.** (c)
11. (a)	**12.** (b)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (d)	**18.** (b)	**19.** (b)	**20.** (d)
21. (b)	**22.** (d)	**23.** (d)	**24.** (a)	**25.** (b)	**26.** (d)	**27.** (c)	**28.** (a)	**29.** (a)	**30.** (b)

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Solutions

1. Effective length of the pendulum is (92.0 + 2.17) cm = 94.2 cm after rounding off to 3 significant digits.

2. Since, maximum velocity is same, so maximum KE *i.e.,* TE would be same. Also, maximum PE would also be same. Let A_1 and A_2 be the amplitudes of two bodies A and B, respectively.

Then, $$\frac{k_1A_1^2}{2} = \frac{k_2A_2^2}{2} \Rightarrow \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

3. As the object moves from infinity to centre of curvature, the image formed by a concave mirror would be real and is moving from focus to centre of curvature, but as the object crosses centre of curvature and moves towards focus the image is still real but moves from centre of curvature towards infinity and when the object is at focus, the real image would be formed at infinity.

So, image speed is smaller in beginning when the object is moving from infinity to centre of curvature and increases thereafter.

4. When liquid rises to height h, then

$$2\pi r \times T\cos\theta = \pi r^2 h\rho g$$

$$\Rightarrow \quad T = \frac{rh\rho g}{2}$$

Given, $r = 0.015\text{ cm} = 0.015\times10^{-2}$ m

$h = 15\text{ cm} = 15\times10^{-2}$ m, $\rho = 1000$ kg/m^3

$$\therefore \quad T = \frac{0.015\times10^{-2}\times15\times10^{-2}\times1000\times9.8}{2}$$

$T = 0.11$ N/m

5. Since, vertical displacement is same as well as initial velocity in vertical downward direction is zero for both the bodies.

$$\because \quad h = ut + \frac{1}{2}gt^2$$

So, $$h = \frac{gt_1^2}{2} \quad \text{[for horizontal throwing]}$$

$$h = \frac{gt_2^2}{2} \quad \text{[for dropping]}$$

$$\therefore \quad t_1 = t_2$$

6. This can be considered as a parallel combination of two, one the inner cylinder and the other surrounding cylinder.

$$A_1 = \pi R^2, \quad A_2 = \pi(4R^2 - R^2) = 3\pi R^2$$

$$L_1 = L_2 = L$$

Heat is flowing only along the length of tube.

$$\therefore H_{eq} = H_1 + H_2$$

$$= \frac{K_1A_1(T_1 - T_2)}{L} + \left[\frac{K_2 \times A_2(T_1 - T_2)}{L}\right]$$

$$\Rightarrow \quad K_{eq} \times 4 = K_1 + 3K_2$$

$$\Rightarrow \quad K_{eq} = \frac{(K_1 + 3K_2)}{4}$$

7. Acceleration due to gravity near the surface of shell can be assumed to be uniform ($h << R$).

$$g = \frac{G(2M)}{(2R)^2} = \frac{GM}{2R^2}$$

From $$h = \frac{1}{2}gt^2,$$

$$t = \sqrt{\frac{2h}{g}} = 2\sqrt{\frac{hR^2}{GM}}$$

8. Speed of the particle

$$u_A = \sqrt{2gh} = \sqrt{2\times\frac{GM}{2R^2}h}$$

$$= \frac{\sqrt{GMh}}{R}$$

From A to B, field due to shell is zero but field due to sphere is non-zero.

Hence, $$t_{AB} < \frac{R}{u_A} < \frac{R^2}{\sqrt{GMh}}$$

9. Potential between A and B due to shell is constant. From energy conservation, we can write

$$K_A + U_A = K_B + U_B$$

or $$K_B = U_A - U_B = m(V_A - V_B) \qquad [K_A \approx 0]$$

or $$\frac{1}{2} mv_B^2 = m(V_A - V_B)$$

$$\therefore \qquad v_B = \sqrt{2(V_A - V_B)}$$

$$= \sqrt{2\left[-\frac{GM}{2R} + \frac{GM}{R}\right]} = \sqrt{\frac{GM}{R}}$$

10. $$i_1 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_1 C}\right)^2}} = \frac{e_0}{Z_1}, \text{ where } \omega_1 = 100\,\pi$$

$$i_2 = \frac{e_0}{\sqrt{R^2 + \left(\frac{1}{\omega_2 C}\right)^2}} = \frac{e_0}{Z_2}, \text{ where } \omega_2 = 500\,\pi$$

So, $Z_1 > Z_2$, therefore $i_1 < i_2$

11. Let volume of wood be V cm^3, then total volume of displaced water is $(V + 2)$ cm^3, then for translational equilibrium,

$$(V + 2)\rho g = (25g + 5g)$$

where, all the quantities are in CGS unit and ρ is the density of water.

$$\Rightarrow \qquad (V + 2) \times 1 = 30 \quad \therefore V = 28 \text{ cm}^3$$

12. The amplitude is the radius of the circle

$$R = \frac{0.8}{2}\text{m} = 0.4 \text{ m}$$

The frequency of the shadow is the same as that of the circular motion, so

$$\omega = 30 \text{ rev/min} = 0.5 \text{ rev/s} = \pi \text{ rad/s}$$

and $$v = \frac{\omega}{2\pi} = \frac{\pi}{2\pi} = 0.5 \text{ Hz}$$

13. The situation is shown in figure.

Conserve the momentum for the two cases and then solve the equation.

Firstly, take 1st sheet and bullet as the system,

$$mu = mu_1 + m_1 v$$

Now, take 2nd sheet and bullet as the system.

$$mu_1 = (m_1 + m_2)v$$

Solving this equation, we get the value of

$$\frac{(u - u_1)}{u} \times 100\% = 25\%$$

14. Given, force $= \dfrac{\alpha}{\text{density} + \beta^3}$

Two quantities can only be added, if their dimensions are same. Therefore, dimension of β^3 is same as that of density.

$$\beta^3 = \text{density} = [ML^{-3}]$$

$$\therefore \qquad \beta = [M\,L^{-3}]^{1/3} = [M^{1/3}L^{-1}]$$

Two quantities with same dimensions can only be equated.

Also, $$\alpha = \text{force} \times \text{density}$$

$$= [MLT^{-2}]\,[ML^{-3}]$$

$$= [M^2L^{-2}T^{-2}]$$

15. $$V = \frac{3}{r^2}$$

$$\mathbf{E} = -\left(\frac{dV}{dr}\right)\mathbf{r}$$

$$= -\frac{\partial}{\partial r}\left[\frac{3}{r^2}\right]\frac{\mathbf{r}}{r}$$

$$= -\frac{3 \times (-2)}{r^3}\frac{\mathbf{r}}{r} = \frac{6}{r^4}\mathbf{r}$$

$$= \frac{2}{3}(\mathbf{i} + \mathbf{j} + \mathbf{k})$$

16. The range of deviation for which two angles of incidence are possible for same deviation is $\delta_0 - \delta_m$ and the given curve is symmetrical about i_0.

17. In the option (a), the circuit is open, as a result no current flows through it, so the potential difference across internal resistor is zero and hence terminal potential difference is same as emf of battery.

For option (b), the terminal potential difference gets zero as battery is shorted although emf remains same.

In option (c), discharging of battery takes place, current has been withdrawn from battery, whose value is depending on value of resistance, which is turn changes the value of terminal potential difference but emf remains same.

18. A simple look at the data shows that average reading of clock I is much closer to the standard time that average reading of clock II. Therefore, zero error in clock II is much larger than the zero error in clock I. But zero error can always be corrected by applying necessary correction.

Now, over the seven days, range of variation of clock I is 12 : 01 : 50 – 11 : 59 : 08 = 162 s and range of variation of clock II is 10 : 15 : 24 – 10 : 14 : 53 = 31 s. Therefore, precision of clock II is much better than the precision of clock I. We will prefer clock II.

19. In the given circuit, 200 Ω and 50 Ω are in parallel, hence equivalent resistance is

$$\frac{1}{R} = \frac{1}{200} + \frac{1}{50} = \frac{4+1}{200} = \frac{5}{200}$$

$\Rightarrow \quad R = 40\,\Omega$

The equivalent circuit is as follows in this circuit, then 40 Ω, 2 Ω and 1 Ω resistors are connected in series, hence equivalent resistance is

$$R_{eq} = 40 + 2 + 1 = 43\,\Omega$$

From Ohm's law, $V = IR$

$$\Rightarrow \quad I = \frac{V}{R} = \frac{4.3}{43} = 0.1\text{ A}$$

Hence, voltmeter reading = 4.3 – 0.1 × 3 = 4 V

20. Consider downward direction as positive

$$\mathbf{a}_{AP} = -\mathbf{a}_{BP}$$
$$\mathbf{a}_{AG} = \mathbf{a}_{AP} + \mathbf{a}_{PG}$$
$$3 = \mathbf{a}_{AP} - 2 \Rightarrow \mathbf{a}_{AP} = 5\text{ m/s}^2$$

$$\therefore \quad \mathbf{a}_{BG} = \mathbf{a}_{BP} + \mathbf{a}_{PG} = -5 - 2 = -7\text{ m/s}^2$$

21. Here, three slabs are in series.

$$C_{eq} = \frac{\varepsilon_0 A}{\left(\frac{d/4}{2} + \frac{d/7}{8/7} + \frac{d/2}{4}\right)} = \frac{\varepsilon_0 A}{\left(\frac{d}{8} + \frac{d}{8} + \frac{d}{8}\right)} = \frac{8\varepsilon_0 A}{3d}$$

22. Total energy of the electron

$$2 + eV = \frac{1242}{656.3}$$

$\Rightarrow \quad eV = 1.89 - 2$

$\Rightarrow \quad = -0.11\text{ eV}$

$\therefore \quad V = -0.11\text{ V}$

23. *The forces acting on the body are*
(i) force of gravity, (ii) air-friction

According to work-energy theorem, total work done on the body = Gain in KE

$$W = \frac{1}{2}mv^2 = \frac{1}{2}m(1.2\sqrt{gH})^2 = 0.72\,mgH$$

As work done by gravity, $W_1 = mgH$

∴ Work done by friction,

$$W_2 = W - W_1 = 0.72\,mgH - 1\,mgH = -0.28\,mgH$$

24. The ratio of the acceleration due to gravity at bottom and by deep mine is given by.

$$\frac{g'}{g} = \frac{978}{980} = 1 - \frac{d}{R} \quad \left[\because \text{at depth, } g' = g\left(1 - \frac{d}{R_e}\right)\right]$$

or $\quad \frac{d}{R} = 1 - \frac{978}{980} = \frac{2}{980}$

or $\quad d = \frac{2R}{980} = \frac{2 \times 6300}{980} = 12.86\text{ km}$

25. $$n_{eff} = n_0\sqrt{1 - \left(\frac{80.5N}{v^2}\right)}$$

$$= 1\sqrt{1 - \frac{80.5 \times (400 \times 10^6)}{(55 \times 10^6)^2}} = 1$$

Also, $\quad n_{eff} = \frac{\sin i}{\sin r} \Rightarrow \sin i = \sin r$

$\Rightarrow \quad r = i = 45°$

26. Since, the rocket fuel is undergoing combustion, the gases produced in this process leave the body of the rocket with large velocity and produce upthrust to the rocket.

Let us assume that the fuel be undergoing combustion at the constant rate, then rate of change of momentum of the rocket will be constant. Since, more and more fuel will be burnt, the mass of rocket will go on decreasing, so it will lead to increase the velocity of the rocket more and more rapidly.

27. Here, $N = N_0 \left(\frac{1}{2}\right)^{t/T}$

or $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{t/T}$ ...(i)

where, T is the half-life period and $\frac{N}{N_0}$ is fraction of atoms left after time t. Here, $T = 40$ days and $\frac{N}{N_0} = \frac{25}{100} = \frac{1}{4}$.

Putting the values of T and $\frac{N}{N_0}$ in Eq. (i), we get

$$\frac{1}{4} = \left(\frac{1}{2}\right)^{t/40} \text{ or } \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{t/40}$$

or $\frac{t}{40} = 2$

or $t = 80$ days

28. The gain of feedback amplifier

$$A_f = \frac{A}{1+\beta A} = \frac{100}{1+0.1\times 100} = \frac{100}{11} = 9.09$$

29. In frequency modulation, frequency of a carrier signal varies in accordance with the modulating signal. The amplitude of the carrier wave is fixed while its frequency is changing.

When the instantaneous amplitude of the modulating voltage is large, the instantaneous carrier frequency is higher. It is lower when the modulating voltage is small. Hence, frequency varies with time.

30. Output current by generator is

$$I = I_c\sqrt{1+\frac{m_1^2}{2}}$$

$$I_c = \frac{I}{\sqrt{1+\frac{m_1^2}{2}}}$$

$$= \frac{1.5}{\sqrt{1+\frac{(0.6)^2}{2}}} = \frac{1.5}{\sqrt{1.18}}$$

$$I_{net} = I_c = \sqrt{1+\frac{m_1^2}{2}+\frac{m_2^2}{2}}$$

$$= \frac{1.5}{\sqrt{1.18}}\sqrt{1+0.18+\frac{0.49}{2}} = 1.64 \text{ A}$$

$$m_a = \sqrt{m_1^2+m_2^2}$$

$$= \sqrt{(0.6)^2+(0.7)^2}$$

$$= \sqrt{36+0.49}$$

$$= \sqrt{0.85} = 0.92 \text{ or } 92\%$$

Practice Set 4

Instructions

For instructions refer to Practice Set 1.

1. A particle of mass 10 kg starts from point A, with an initial velocity of 3 m/s towards negative x-axis, it has been acted by a force of 10 N towards positive x-axis. Find the distance travelled by particle in 4 s.

(a) 4 m (b) 0.5 m (c) 4.5 m (d) 5 m

2. Find the charge on 5 μF capacitor.

(a) 5.45 μC (b) 16.37 μC
(c) 10.9 μC (d) 18 μC

3. A uniform slender rod 1m long is initially standing vertically on a smooth horizontal surface. It is struck by a sharp horizontal blow at the top end, with the blow directed at right angles to the rod axis. As a result, the rod acquires an angular velocity of 3.00 rad/s. What is the translational velocity of the centre of mass of the rod after the blow?

(a) 0.25 m/s (b) 0.64 m/s
(c) 0.5 m/s (d) 1.2 m/s

4. Mark the incorrect option.

(a) During the discharging of battery, the battery supplies electrical energy at the rate of $VI = EI - I^2r$. Here, EI represents the rate at which non-electrical energy is converted to electrical energy

(b) During the charging of battery, the battery consumes energy at the rate of $VI = EI + I^2r$. Here, EI represents the rate at which electrical energy is converted to non-electrical energy

(c) In both the above cases, the term I^2r represents the rate of dissipation of energy in the internal resistance of battery

(d) None of the above

5. Why does a glass sometimes break, if we quickly pour boiling water into it ?

(a) Hot water expands, pushing the glass out

(b) The hot water cools when it touches the glass, shrinking and pulling the glass in

(c) The glass becomes hot and expands causing the molecules to break

(d) The inside of the glass expands faster than the outside of the glass, causing the glass to break

6. Find the equivalent capacitance between A and B.

(a) $\frac{2}{3}$ μF (b) 1 μF (c) $\frac{4}{3}$ μF (d) 6 μF

7. A 0.5 kg ball is dropped from rest at a point 1.20 m above the floor. The ball rebounds straight upward to a height of 0.7 m. What is the magnitude and the direction of the impulse of the net force applied to the ball during the collision with the floor?

(a) 4.28 N-s, upward (b) 4.28 N-s, downward
(c) 8.56 N-s, upward (d) 8.56 N-s, downward

8. A body is moving with uniform speed v in a horizontal circle in anti-clockwise direction as shown in figure. The motion starts from point A, find the change in velocity in second quarter of revolution.

(a) $\sqrt{2}\,v$ N-W (b) $\sqrt{2}\,v$ N-E
(c) $\sqrt{2}\,v$ S-W (d) $\sqrt{2}\,v$ S-E

9. N moles of an ideal diatomic gas are in a cylinder at temperature T. If we supply some heat to it, then $N/3$ moles of gas dissociates into atoms while temperature remains constant. Heat supplied to the gas is

(a) $\frac{NRT}{6}$ (b) $\frac{5NRT}{2}$ (c) $5.6\,NRT$ (d) $\frac{8NRT}{3}$

10. Electrons having KE 15 eV is collided with hydrogen atom and 80.6% of it is used to excite the electrons from its ground state. Find the number of emitted wavelengths.

(a) 1 (b) 2 (c) 3 (d) 4

➡ **Directions** **Q. Nos.** 11 *and* 12 *are based on following paragraph.*

Work function of metal A is equal to the ionisation energy of hydrogen atom in first excited state. Work function of metal B is equal to the ionisation energy of He^+ ion in second orbit. Photons of same energy E are incident on both A and B. Maximum kinetic energy of photoelectrons emitted from A is twice that of photoelectrons emitted from B.

11. Value of E (in eV) is

(a) 20.8 (b) 32.2 (c) 24.6 (d) 23.8

12. The difference in maximum kinetic energy of photoelectrons from A and from B

(a) increases with increase in E
(b) decreases with increase in E
(c) first increases then decreases with increase in E
(d) remains constant

13. A girl is seated on the top of a hemispherical mound of ice. She is given a very small push and starts sliding down the ice. She leaves the ice at a point whose height is

(a) $\frac{2R}{5}$ (b) $\frac{R}{3}$ (c) $\frac{R}{5}$ (d) $\frac{2R}{3}$

14. A person of mass m is standing on a structure made up of pulley, strings and platform as shown in figure. Find the force exerted by the person on the rope, so that the system (person + structure) remains in equilibrium.

(a) mg
(b) $(m + M' + M)\,g$
(c) $\frac{(M + M' + m)\,g}{2}$
(d) None of the above

15. Find the distance between the object and its doubly magnified real image by a concave mirror of focal length f.

(a) $2f$ (b) $\frac{3f}{2}$
(c) $\frac{5f}{2}$ (d) $3f$

16. A point source of light is taken away from the experimental set up of photoelectric effect, then which is the most appropriate statement?

(a) Saturation photocurrent remains same, while stopping potential increases
(b) Saturation photocurrent and stopping potential both decrease
(c) Saturation photocurrent decreases while stopping potential remains same
(d) Saturation photocurrent decreases and stopping potential increases

17. The pitch of a screw gauge is 0.5 mm and there are 50 divisions on circular scale. When there is nothing between the two ends (studs) of screw gauge, 45th division of circular scale is coinciding with screw gauge and in this situation, zero of main scale is not visible. When a wire is placed between the studs, the linear scale reads 2 divisions and 20th division of circular scale coincides with reference line. For this situation, mark the correct statement(s).

(a) Least count of the instrument is 0.01 mm
(b) Zero correction for the instrument is + 0.45 mm
(c) Thickness of wire is 1.65 mm
(d) All of the above

18. A man in an empty swimming pool has a telescope focussed at 4'O clock sun. When the swimming pool is filled with water, the man (now inside the water with his telescope undisturbed) observes the setting sun. Find the refractive index of water, if sun rises and sets at 6'O clock.

(a) $\frac{4}{3}$ (b) $\frac{2}{\sqrt{3}}$ (c) $\frac{8}{5}$ (d) $\frac{2}{5}$

19. A 25 kg bear slides from rest, 12 m down a pine tree, moving with a speed of 5.6 m/s just before hitting the ground. What is the average frictional force that acts on the sliding bear?

(a) 210 N (b) 180 N
(c) 110 N (d) 170 N

➡ **Directions Question Nos** 20 *to* 22 *are Assertion-Reason type questions. Each of these questions contains two statements : Statement* I *(Assertion) and Statement* II *(Reason). Each of these questions also has four alternative choices, only one of which is correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below*

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(b) Statement I is true, Statement II is false
(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

20. **Statement I** A car battery is of 12 V. Eight dry cells of 1.5 V connected in series can give 12 V. Still such cells are not used in starting a car.

Statement II It is easier to start a car engine on a warm day than on a rainy day.

21. **Statement I** The minimum inductance that can be obtained by combining three inductors of 2H, 3H and 6H is 1H.

Statement II Minimum inductance is obtained when the inductors are connected in series.

22. **Statement I** A small body of mass 0.1 kg is undergoing SHM of amplitude 1.0 m and period 0.2 s. The maximum value of the force acting on it is 98.7 N.

Statement I Maximum force acting on it is given by $F = m\omega^2 r$.

23. A Carnot air conditioner takes energy from the thermal energy of a room at 70°F and transfers it to the outdoors, which is at 96°F. For each joule of electric energy required to operate the air conditioner, how many joules are removed from the room?

(a) 15 J (b) 10 J (c) 20 J (d) 10.5 J

24. A UHF television loop antenna has a diameter of 11 cm. The magnetic field of a TV signal is normal to the plane of the loop and at one instant of time, its magnitude is changing at the rate 0.16 T/s. The magnetic field is uniform. What emf is induced in the antenna?

(a) 1.52×10^{-3} V
(b) 0.35×10^{-2} V
(c) 0.54×10^{-3} V
(d) 1.39×10^{-2} V

25. What direct current will produce the same amount of thermal energy, in a particular resistor, as an alternating current that has a maximum value of 2.60 A?

(a) 2.35 A (b) 1.84 A
(c) 1.20 A (d) 1.29 A

26. A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What is the smallest magnetic field (not necessarily vertical) that would cause the rod to slide?

(a) 0.10 T (b) 0.20 T (c) 0.50 T (d) 0.30 T

27. A zener diode is operating in its normal region *i.e.,* the breakdown region for which the circuit diagram is as shown in the figure.

Here, take $V_Z = 7\,\text{V}$ and $R = 10\,\text{k}\Omega$. For potential difference equal to 8 V across *AB*, what is the current through microammeter?

(a) 1000μA (b) 1 mA (c) 10μA (d) 100μA

28. The output of a 2 input NAND gate is fed to a NOT gate. The truth table for the output of the combination for all possible inputs of *A* and *B* is

(a)	*A*	*B*	*Y*		(b)	*A*	*B*	Y
	0	0	0			0	0	0
	1	0	0			1	0	1
	0	1	0			0	1	1
	1	1	1			1	1	1

(c)	*A*	*B*	*Y*		(d)	*A*	*B*	*Y*
	0	0	1			0	0	1
	1	0	1			1	0	0
	0	1	1			0	1	1
	1	1	0			1	1	0

29. A signal wave of frequency 12 kHz is modulated with a carrier wave of frequency 2.51 MHz. The upper and lower side band frequencies are respectively

(a) 2512 kHz and 2508 kHz
(b) 2522 kHz and 2488 kHz
(c) 2502 kHz and 2498 kHz
(d) 2522 kHz and 2498 kHz

30. An oscillator is producing FM waves of frequency 2 kHz with a variation of 10 kHz. The modulation index is

(a) 0.20 (b) 5.0 (c) 0.67 (d) 1.5

Answers

1. (d)	**2.** (b)	**3.** (c)	**4.** (d)	**5.** (d)	**6.** (b)	**7.** (a)	**8.** (d)	**9.** (a)	**10.** (c)
11. (d)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (c)	**17.** (d)	**18.** (b)	**19.** (a)	**20.** (a)
21. (b)	**22.** (d)	**23.** (c)	**24.** (a)	**25.** (b)	**26.** (a)	**27.** (d)	**28.** (a)	**29.** (d)	**30.** (b)

Solutions

1. This is a question based on the concept that when acceleration and initial velocity are in opposite direction, then particle comes to rest momentarily at some instant and after that direction of velocity becomes same as that of acceleration. Upto this instant, distance and displacement are equal.

$$a = \frac{F}{m} = \frac{10}{10} = 1\,\text{m/s}^2$$

Let velocity becomes zero after t second, then applying

$$v = u + at$$

$$\Rightarrow \quad 0 = 3 - 1 \times t$$

$$\Rightarrow \quad t = 3\text{ s}$$

Distance travelled in 3 s = Displacement in 3 s

$$\Rightarrow \quad s_1 = ut + \frac{1}{2}at^2$$

$$= 3 \times 3 - \frac{1}{2} \times 1 \times 3^2$$

$$= \frac{9}{2}\text{ m} = 4.5\text{ m}$$

[Since, the above equation is a vector equation and gives us displacement and not distance]

Displacement of particle in 4 s,

$$s = ut + \frac{1}{2}at^2$$

$$= 3 \times 4 - \frac{1}{2} \times 1 \times 4^2$$

$$= 12 - 8 = 4\text{ m}$$

From the diagram, distance travelled in 4 s

$$= AB + BC$$

$$= 4.5 + 0.5 = 5\text{ m}$$

2. We will solve this question by using point potential theory.Consider point O at zero potential and then mark the potentials of various junctions as shown in figure. Apply KCL at junction having potential V_1.

$$5 \times (V_1 - 0) + 2 \times (V_1 - 6) + 4 \times (V_1 - 6) = 0$$

$$\Rightarrow \quad (5 + 2 + 4)\,V_1 = 12 + 24$$

$$\Rightarrow \quad V_1 = \frac{36}{11}\text{ V}$$

So, charge on $5\,\mu\text{F} = \frac{36}{11} \times 5\,\mu\text{C} = 16.37\,\mu\text{C}$

3. Let the magnitude of the impulse delivered by the blow be represented by p. From the translational and rotational forms of Newton's second law,

$$v_c = \frac{p}{M} \quad \text{and} \quad I_c\omega_c = \frac{pl}{2}$$

Here, v_c is the rightward velocity of the centre of mass after the blow and ω_c is the (positive clockwise) angular velocity of the rod after the blow about the point C.

Using the fact that $I_c = \frac{Ml^2}{12}$, we find that

$$\omega_c l = \frac{6p}{M}$$

Therefore, $$v_c = \frac{(\omega_c l)}{6}$$

Since, $\omega_c = 3.00$ rad/s and $l = 1.00$ m, we find

$$v_c = 0.5\text{ m/s}$$

4. During discharging, battery supplies electric power to external circuit, while during charging the charger supplies electrical energy to the battery. In both the cases, some part of the energy is dissipated as heat in internal resistance of the battery.

5. When hot water is poured into the glass, the inner portion of the glass comes into the contact of hot water immediately while of outer portion after sometimes as heat is conduced through glass. As a result, the inside of the glass expands faster than the outside of the glass and hence causes the glass to break.

6. Equivalent circuit can be drawn as shown in figure.

$$\left[\because C_p = C_1 + C_2 \text{ and } \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}\right]$$

The middle capacitor is shorted and hence no charge flows through it.

So, the equivalent circuit can be redrawn as

7. The velocity of ball just before collision

$$= \sqrt{2g \times 1.2}$$
$$= \sqrt{2.4\,g} \text{ m/s}$$

in downward direction.

The velocity of ball just after collision

$$= \sqrt{2g \times 0.7}$$
$$= \sqrt{1.4\,g} \text{ m/s in upward direction.}$$

Impulse $= \mathbf{p}_f - \mathbf{p}_i$

$$= m\,[\sqrt{1.4\,g} - (-\sqrt{2.4\,g})]$$
$$= 4.28 \text{ N-s}$$

in the direction of final momentum, *i.e.,* upward.

8. The diagram is clearly showing the change in velocity vector

$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$$

9. Heat supplied = Change in internal energy

$$\Delta U = U_f - U_i$$

$$U_f = \left(N - \frac{N}{3}\right) \times \frac{5RT}{2} + \frac{N}{3} \times 2 \times \frac{3RT}{2}$$

$$= \frac{5NRT}{3} + NRT = \frac{8}{3}NRT$$

and $\quad U_i = 5 \times \dfrac{NRT}{2} \quad \therefore \quad \Delta Q = \dfrac{NRT}{6}$

10. Change in kinetic energy of electron

$$\frac{80.6}{100} \times 15 = \Delta E = 12.09 = |E_3 - E_1|$$

So, electron can make the hydrogen atom to go into 2nd excited state, *i.e,* $n = 3$ energy state. From there 3 transitions are possible, *i.e,* from 3 to 2, 3 to 1 and 2 to 1.

11. We know that, $E_n = \dfrac{-13.6}{n^2}$ in nth orbit.

W_A = Ionisation energy of electron in 2nd orbit of hydrogen atom = 3.4 eV.

W_B = Ionisation energy of electron in 2nd orbit of He^+ ion = 13.6 eV

Now, given that $K_A = 2K_B$

or $\quad (E - W_A) = 2(E - W_B)$

$\therefore \quad E = 2W_B - W_A = 23.8$ eV

12. $(E - W_A) - (E - W_B) = W_B - W_A$

$$= 10.2 \text{ eV} = \text{constant}$$

13. The free body diagram for the girl is shown in the figure.

Net inward force = $mg \cos\theta - N$

Also, $mg \cos\theta - N = \dfrac{mv^2}{R}$ ...(i)

where, v is the speed of the girl.

At the point where girl leaves

$$N = 0$$

So, $g \cos\theta = \dfrac{v^2}{R}$ ...(ii)

Conservation of energy gives

$$0 = \frac{1}{2}mv^2 - mgR\,(1 - \cos\theta)$$

or $v^2 = 2gR\,(1 - \cos\theta)$ ...(iii)

From Eqs. (ii) and (iii), we have

$$g \cos\theta = 2g\,(1 - \cos\theta)$$

$$\Rightarrow \quad \cos\theta = \frac{2}{3}$$

Thus, the required height = $R \cos\theta = \dfrac{2R}{3}$

14. The net force on the system can be calculated directly after drawing the free body diagram.

$T = (M + M' + m)g$

[∵ upward force = downward force]

So, the required force, $F = \dfrac{T}{2} = \dfrac{(M + M' + m)g}{2}$

15. $m = -\left(\dfrac{v}{u}\right) \Rightarrow -2 = \dfrac{v}{u}$

[–ve magnification because image is real]

$\Rightarrow \quad v = 2u$

From mirror formula, $\left(\dfrac{1}{v}\right) + \left(\dfrac{1}{u}\right) = \left(\dfrac{1}{f}\right)$

$$\Rightarrow \quad \left(\frac{1}{2u}\right) + \left(\frac{1}{u}\right) = \left(\frac{1}{f}\right)$$

$$\Rightarrow \quad \frac{3}{2u} = \frac{1}{f} \Rightarrow u = \frac{3f}{2}$$

16. As the source of light is taken away, the intensity of light at the location of experimental setup decreases.

∴ Photocurrent ∝ Intensity

So, photocurrent decreases.

Now, $V_0 = \dfrac{hc}{\lambda_0 e} - \dfrac{\phi}{e}$

which is independent of intensity.

17. Least count ,

$$\text{LC} = \frac{\text{Pitch}}{\text{Divisions on circular scale}}$$

$$= \frac{0.5}{50} = 0.01\,\text{mm}$$

Zero error is negative in nature and is

$$e = -45 \times \text{LC} = -0.45\,\text{mm}$$

Zero correction $= -$ Zero error $= 0.45\,\text{mm}$

Reading of the screw gauge,

$$d = 2 \times 0.5 + 20 \times 0.01 + \text{Zero correction } (0.45\,\text{mm})$$

$$= 1.65\,\text{mm}$$

18. When no water is there in the swimming pool, the telescope is along the line PQ. When water has been filled into the pool, the person is able to see the setting sun, *i.e,* refracted ray will be along QP while incident ray is along RP, as shown in figure. That is from the object (sun) to the telescope.

Let μ be the refractive index of water, then

$$\frac{\sin 90°}{\sin 60°} = \frac{\mu}{1}$$

$$\Rightarrow \qquad \mu = \frac{1}{\sin 60°} = \frac{2}{\sqrt{3}}$$

19. The change in the mechanical and thermal energy must sum to zero. Since, the change in thermal energy is $\Delta E_{th} = fL$, where f is the magnitude of the average frictional force.

$$f = -\frac{\Delta K + \Delta U}{L}$$

$$= -\frac{(K_f - K_i) + (U_f + U_i)}{L}$$

$$= -\frac{\left(\frac{1}{2}mv^2 - 0\right) + (-mgL - 0)}{L}$$

$$= -\frac{\frac{1}{2} \times 25 \times (5.6)^2 - 25 \times 9.8 \times 12}{12}$$

$$= -\frac{3.92 \times 10^5 - 2.94 \times 10^3}{12}$$

$$= 210\,\text{N}$$

20. To start a car, a very high current is required. A car battery has very low internal resistance, so that it can provide the required high current. When eight dry cells are joined in series, the internal resistance of the combination becomes very high. Due to this high internal resistance, small current will be drawn from it.

Hence, such cells cannot be used to start a car. On a warm day, the internal resistance of car battery decreases and so large current can be drawn from the battery. But on a chilly day, reverse process occurs.

21. We know that minimum inductance can be obtained when the inductors are connected in parallel. Hence,

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{6}{6}$$

$$\Rightarrow \qquad L_p = 1\,\text{H}$$

22. The maximum force is given by

$$F = m\omega^2 r = m \times \left(\frac{2\pi}{T}\right)^2 \times r = 0.1 \times \left(\frac{2\pi}{0.2}\right)^2 \times 1.0$$

$$= 0.1 \times (10\pi)^2 = 98.7\ \text{N}$$

23. $K = \dfrac{T_C}{T_H - T_C}$ or $K = \dfrac{294}{309 - 294} = 19.6$

The coefficient of performance is the energy Q_C drawn from the cold reservoir as heat divided by the work done

$$K = \frac{|Q_C|}{|W|}$$

Thus, $\quad |Q_C| = K|W| = (19.6)(1.0) = 20\ \text{J}$

24. The magnetic field is normal to the plane of the loop and is uniform over the loop. Thus, at any instant, the magnetic flux through the loop is given by $\phi_B = AB = \pi r^2 B$, where $A\ (= \pi r^2)$ is the area of the loop.

According to Faraday's law, the magnitude of the emf in the loop is

$$e = \frac{d\phi_B}{dt} = \pi r^2 \frac{dB}{dt}$$

$$= \pi\ (0.055\ \text{m})^2 (0.16\ \text{T/s})$$

$$= 1.52 \times 10^{-3}\ \text{V}$$

25. The average rate with which thermal energy is generated in resistance R when the current is alternating is given by

$$P_{av} = I_{rms}^2 R$$

or

$$P_{av} = \frac{I^2 R}{2}$$

The rate of thermal energy generation in the same resistor when the current is direct is given by $P = i^2 R$, where i is the current. Set the two rates equal to each other and solve for i. You should get

$$i = \frac{I}{\sqrt{2}}, = \frac{2.60\ \text{A}}{\sqrt{2}} = 1.84\ \text{A}$$

26. Suppose the magnetic field makes the angle θ with the vertical. The diagram to the right shows the view from the end of the sliding rod. The forces are also shown. Notice that the magnetic force makes the angle θ with the horizontal. When the rod is on the verge of sliding, the net force acting on it is zero and the magnitude of the frictional force is given by $f = \mu_s N$, where μ_s is the coefficient of static friction.

The magnetic field is perpendicular to the wire, so the magnitude of the magnetic force is given by $F_B = ILB$, where I is the current in the rod and L is the length of the rod.

The vertical component of Newton's second law yields

$$N + ILB\sin\theta - mg = 0 \quad \ldots(i)$$

and the horizontal component yields

$$ILB\cos\theta - \mu_s N = 0 \quad \ldots(ii)$$

Solve the Eq. (ii) and substitute the resulting expression into the Eq. (i), then solve for B. You should get

$$B = \frac{\mu_s mg}{IL(\cos\theta + \mu_s \sin\theta)}$$

The minimum value of B occurs when $\cos\theta + \mu_s \sin\theta$ is a maximum. Set the derivative of $\cos\theta + \mu_s \sin\theta$ equal to zero and solve for θ. You should get $\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.60) = 31°$. Now, evaluate the expression for the minimum value of B.

$$B_{min} = \frac{0.60\,(1.0)(9.8)}{(50)(1.0)(\cos 31° + 0.60\sin 31°)}$$

$$= 0.10\text{ T}$$

The magnetic field makes an angle of 31° with the vertical.

27. Write KVL equation, $-V_{BA} + IR + V_Z = 0$

[as diode is operating in breakdown region]

$\Rightarrow \quad IR = V_{BA} - V_Z = 8 - 7$

$\Rightarrow \quad I = \dfrac{1\text{V}}{10\text{ k}\Omega} = 100\,\mu\text{A}$

28. If the output of two inputs NAND gate is used as the input of NOT gate as shown below, we get back AND gate.

Here, the Boolean expression for output Y' is

$$Y' = \overline{A \cdot B} \text{ and } Y = \overline{\overline{A \cdot B}} = A \cdot B$$

The truth table of this combination will be as shown below, which is a truth table of AND gate.

A	B	Y'	Y
0	0	1	0
1	0	1	0
0	1	1	0
1	1	0	1

29. Upper side band (USB) frequency $= f_c + f_m$

Given, $f_c = 2.51$ MHz $= 2.51 \times 10^6$ Hz $= 2510$ kHz

$f_m = 12$ kHz

$\therefore$ Upper side band (USB) frequency

$= (2510 + 12)$ kHz $= 2522$ kHz

Lower side band (LSB) frequency

$= f_c - f_m = 2510 - 12 = 2498$ kHz

30. Modulation index is given by $\beta = \dfrac{\Delta f}{f_m}$

where, Δf is frequency deviation, f_m is the highest modulation frequency.

Given, $\Delta f = 10$ kHz $= 10 \times 10^3$ Hz

$f_m = 2$ kHz $= 2 \times 10^3$ Hz

$\therefore \quad \beta = \dfrac{10 \times 10^3}{2 \times 10^3} = 5$

Practice Set 5

Instructions

For instructions refer to Practice Set 1.

1. A carpet is to be installed in a room whose length is measured to be 12.71 m (four significant figures) and whose width is measured to be 3.46 m (three significant figures). Find the area of the room.

(a) 43.97 m^2 (b) 43.9766 m^2
(c) 43.98 m^2 (d) 44.0 m^2

2. If the charge of 10μC and –2μC are given to two plates of a capacitor which are connected across a battery of 12 V, find the capacitance of the capacitor.

(a) 0.33μF (b) 0.5μF (c) 0.41μF (d) 0.66 μF

3. A passenger at the rear of a train travelling at 15 m/s relative to earth throws a baseball with a speed of 15 m/s in the direction opposite the motion of the train. What is the velocity of the baseball relative to earth?

(a) Zero (b) 15 m/s
(c) 30 m/s (d) –15 m/s

4. A sled and its rider together weigh 800 N. They move down on a frictionless hill through a vertical distance of 10.0 m. Use conservation of mechanical energy to find the speed of the sled at the bottom of the hill, assuming the rider pushes off with an initial speed of 5.00 m/s. Neglect air resistance.

(a) 21.5 m/s (b) 14.9 m/s
(c) 4.9 m/s (d) 20.3 m/s

5. A body when projected vertically up covers a total distance *s*, during its time of flight. If we neglect gravity, then how much distance the particle will travel during the same time? Will it fall back?

(a) *s*, Yes (b) *s*, No
(c) 2 *s*, Yes (d) 2 *s*, No

6. Satellite dishes do not have to change directions in order to stay focussed on a signal from a satellite. This means that the satellite always has to be found at the same location with respect to the surface of earth. For this to occur, the satellite must be at a height such that its revolution period is the same as that of earth, 24 h. At what height must a satellite be to achieve this?

(a) $\left(\frac{T^2}{4\pi^2} GM_e\right)^{1/3}$ (b) $\left(\frac{T^2}{4\pi^2} GM_e\right)$
(c) $\left(\frac{T^2}{4\pi^2} GM_e\right)^{1/2}$ (d) $\left(\frac{T}{4\pi^2} GM_e\right)^{1/3}$

7. Which of the following curves may represent the speed of the electron in a hydrogen atom as a function of the principal quantum number *n*?

(a) *A* (b) *B* (c) *C* (d) *D*

8. Water with a mass of 2.0 kg is held at constant volume in a container while 10.0 kJ of energy is slowly added by a flame. The container is not well insulated and as a result 2.0 kJ of energy leaks out to the surroundings. What is the temperature increase of the water?

(a) 0.28°C (b) 27°C
(c) 0.96°C (d) 1.27°C

➡ **Directions** *Q. no. 9 to 11 are based on following paragraph.*

We have two radioactive nuclei A and B. Both convert into a stable nucleus C. Nucleus A converts into C after emitting two α-particles and three β-particles. Nucleus B converts into C after emitting one α-particle and five β-particles. At time t = 0, nuclei of A are $4N_0$ and that of B are N_0. Half life of A (into the conversion of C) is 1 min and that of B is 2 min. Initially number of nuclei of C are zero.

9. If atomic numbers and mass numbers of A and B are Z_1, Z_2, A_1 and A_2, respectively. Then,

(a) $Z_1 - Z_2 = 6$
(b) $A_1 - A_2 = 4$
(c) Both (a) and (b) are correct
(d) Both (a) and (b) are wrong

10. What are number of nuclei of C, when number of nuclei of A and B are equal?

(a) $2N_0$ (b) $3N_0$ (c) $\frac{9N_0}{2}$ (d) $\frac{5N_0}{2}$

11. At what time rate of disintegrations of A and B are equal?

(a) 4 min (b) 6 min (c) 8 min (d) 2 min

12. When two particles q and $-q$ are placed at a distance r, then Coulomb's force of attraction between them has magnitude F. If another charge particle having charge $+3q$ is placed near to $-q$, then the force exerted by $-q$ on q will be

(a) $4F$, repulsive (b) $2F$, attractive
(c) F, attractive (d) $2F$, repulsive

13. Find the equivalent thermal resistance of combination of rods as shown in figure. Every rod has same length l and cross-sectional area A. Thermal conductivities are mentioned in figure.

(a) $\frac{l}{4KA}$ (b) $\frac{7l}{4KA}$ (c) $\frac{7l}{12KA}$ (d) $\frac{l}{12KA}$

14. If the blood vessels in a human being acted as simple pipes (which they do not in actual), what would be the difference in blood pressure between the blood in a 1.80 m tall man's feet and in his head when he is standing? Assume the specific gravity of blood to be 1.06.

(a) 16.23 kPa (b) 18.7 kPa
(c) 17.3 kPa (d) 15.9 kPa

15. In J J Thomson experiment, the potential difference of 320 V is accelerating the electron. The electron beam is entering a region having uniform magnetic field 6×10^{-5} T acting perpendicular to it. Find the value of electric field in this region so that the electron does not experience any deflection. (Take $m_e = 9.1 \times 10^{-31}$ kg)

(a) 6.4 V/m (b) 64 V/m
(c) 640 V/m (d) 6.4×10^4 V/m

16. A wire of length 100 cm is connected to a cell of emf 2V and negligible internal resistance. The resistance of the wire is 3Ω. The additional resistance required to produce a potential difference of 1 mV/cm is

(a) 47 Ω (b) 57 Ω (c) 60 Ω (d) 55 Ω

17. A 20 g bullet is fired horizontally with a speed of 600 m/s into a 7 kg block sitting on a table top; the bullet b lodges in the block B. If the coefficient of kinetic friction between the block and the table top is 0.4, what is the distance the block will slide?

(a) 0.5 m (b) 1.2 m (c) 0.37 m (d) 0.85 m

➡ **Directions** *Q. no. 18 is Assertion-Reason type questions. Each of these questions contains two statements : Statement I (Assertion) and Statement II (Reason). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice in the codes (a), (b), (c) and (d) in the given below*

(a) Statement I is true, Statement II is true; Statement II is not a correct explanation of Statement I
(b) Statement I is true, Statement II is false

(c) Statement I is false, Statement II is true
(d) Statement I is true, Statement II is true; Statement II is a correct explanation of Statement I

18. **Statement I** When the range of projectile is maximum the time of flight is the largest.

Statement II Range is maximum when angle of projection is 45°.

19. A uniform rope, of mass m per unit length, hangs vertically from a support so that the lower end just touches the table top. If it is released, then at the time a length y of the rope has fallen, the force on the table is equivalent to the weight of the length ky of the rope. Find the value of k.

(a) 1 (b) 2
(c) 3 (d) 3.5

20. The two blocks shown in figure have equal masses and $\mu_s = \mu_k = 0.3$ for both blocks. Wedge W is fixed and block A is given initial speed of 1 m/s down the plane. How far will it move before coming to rest if inclines and strings are quite long ? [Take g = 10 m/s^2]

(a) 0.45 m (b) 0.9 m
(c) + 4.8 m (d) Question is irrevalant

21. As shown in figure, a uniform solid sphere rolls on a horizontal surface at 20 m/s. It then rolls up the incline shown. What will be the value of h, where the ball stops?

(a) 28.6 m (b) 8.6 m
(c) 6 m (d) 18.6 m

22. A 1.6 kg block on a horizontal surface is attached to a spring with a spring constant of 1.0×10^3 N/m. The spring is compressed to a distance of 2.0 cm and the block is released from rest. Calculate the speed of the block as it passes through the equilibrium position, $x = 0$, if the surface is frictionless.

(a) 0.75 m/s (b) 0.50 m/s
(c) 0.25 m/s (d) 2.25 m/s

23. A small conducting circular loop is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is

(a) clockwise
(b) anti-clockwise
(c) zero
(d) clockwise or anti-clockwise depending on whether the current is increased or decreased

24. An inductor coil joined to a 6 V battery draws a steady current of 12 A. This coil is connected to a capacitor and an AC source of rms voltage 6 V in series. If the current in the circuit is in phase with the emf, the rms current will be

(a) 16.9 A (b) 12 A (c) 8 A (d) 9.87 A

25. Two wires, each having a weight per unit length of 1.0×10^{-4} N/m, are strung parallel to one another above earth's surface, one directly above the other. The wires are aligned in a North-South direction so that earth's magnetic field will not affect them. When their distance of separation is 0.10 m, what must be the current in each in order for the lower wire to levitate the upper wire? Assume that the wires carry the same currents, travelling in opposite directions.

(a) 2.7 A (b) 0.1 A (c) 3.5 A (d) 7.1 A

26. Two converging lenses are placed 20.0 cm apart, as shown in figure. If the first lens has a focal length of 10.0 cm and the second has a focal length of 20.0 cm, locate the final image formed of an object 30.0 cm infront of the first lens.

(a) 6.67 cm left (b) 6.67 cm right
(c) 15.0 cm left (d) 15.0 cm right

27. Calculate the minimum thickness of a soap-bubble film ($n = 1.33$) that will result in constructive interference in the reflected light, if the film is illuminated by light with a wavelength in free space of 602 nm.

(a) 98 nm (b) 113 nm (c) 125 nm (d) 25 nm

28. A ground receiver station is receiving a signal at 100 MHz, transmitted from a ground transmitter at a height of 300 m located at a distance of 100 km. Then,

(a) signal is coming *via* space wave
(b) signal is coming *via* sky wave
(c) signal is coming *via* satellite transponder
(d) None of the above

29. The output waveform (Y) of AND gate for the following inputs A and B given below is

30. A load resistor of 2 kΩ is connected in the collector branch of an amplifier using CE mode. $\beta = 50$ and input resistance of the transistor 500 Ω. If the input current is changed by 50 μA, then the power gain is

(a) 10^2 (b) 10^3
(c) 10^4 (d) 10^6

Answers

1. (d)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (d)	**6.** (a)	**7.** (b)	**8.** (c)	**9.** (b)	**10.** (c)
11. (b)	**12.** (c)	**13.** (c)	**14.** (b)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (c)	**19.** (c)	**20.** (a)
21. (a)	**22.** (b)	**23.** (c)	**24.** (b)	**25.** (d)	**26.** (a)	**27.** (b)	**28.** (c)	**29.** (b)	**30.** (c)

Solutions

1. If we multiply 12.71 m by 3.46 m, we will get an answer of 43.9766 m^2. We have only three significant figures in our least accurate measurement, so we should express our final answer as 44.0 m^2.

Note that in the answer given, we used a general rule for rounding off numbers, which states that the last digit retained is to be increased by 1, if the first digit dropped was equal to 5 or greater.

2. Charge of capacitor is the charge on facing surfaces of the plates of capacitor

$$Q = \left(\frac{q_1 - q_2}{2}\right) = \frac{[10 - (-2)]}{2} = 6\,\mu C$$

Potential difference across the capacitor = 12 V

So, $$C = \frac{Q}{V} = \left[\frac{(6 \times 10^{-6})}{12}\right] F = 0.5\,\mu F$$

3. We first write our knowns and unknowns with appropriate subscripts.

$v_{te} = +15$ m/s [velocity of the train relative to earth]

$v_{bt} = -15$ m/s

[velocity of the baseball relative to the train]

We need v_{bc}, the velocity of the baseball relative to earth.

$$v_{bc} = v_{bt} + v_{tc} = +15 - 15 = 0$$

4. The initial energy of the sled-rider-earth system includes kinetic energy because of the initial speed.

$$\therefore \quad \frac{1}{2} mv_i^2 + mgy_i = \frac{1}{2} mv_f^2 + mgy_f$$

or $$\frac{1}{2} v_i^2 + gy_i = \frac{1}{2} v_f^2 + gy_f$$

If we set the origin of our coordinates at the bottom of the incline, the initial and final y coordinates of the sled are $y_i = 10.0$ m and $y_f = 0$.

Thus, we get

$$\frac{1}{2} v_i^2 + gy_i = \frac{1}{2} v_f^2 + 0$$

$$v_f^2 = v_i^2 + 2gy_i$$

$$= (5.00)^2 + 2\,(9.80)(10.0)$$

$$v_f = 14.9 \text{ m/s}$$

5. Let particle be projected with speed u, so total time of flight

$$T = \left(\frac{2u}{g}\right)$$

and $s = 2 \times$ Maximum height

$$= 2 \times \frac{u^2}{2g} = \frac{u^2}{g}$$

If there is no gravity, then

$$s' = u \times T = \frac{2u^2}{g} = 2s$$

If gravity is not there, it will never fall back.

6. The force that produces the centripetal acceleration of the satellite is the gravitational force, so

$$G\frac{M_e m}{r^2} = \frac{mv^2}{r} \quad \ldots(i)$$

where, M_e is earth's mass and r is the satellite's distance from the centre of earth.

Also, we find the speed of the satellite to be

$$v = \frac{d}{T} = \frac{2\pi r}{T} \quad \ldots(ii)$$

where, T is the orbital period of the satellite.

Solving Eqs. (i) and (ii) simultaneously for r yields

$$r = \left(\frac{T^2}{4\pi^2} GM_e\right)^{1/3}$$

7. Velocity in nth orbit, $v_n = \dfrac{2.2 \times 10^6}{n}$ ms^{-1}

where, $v_1 = 2.2 \times 10^6$ m/s

For hydrogen, which implies velocity is inversely proportional to n. Hence, the plot is a rectangular hyperbola.

8. Recall that an isovolumetric process is one that takes place at constant volume. In such a process, the work done is equal to zero because there is no change in volume. Thus, the first law of thermodynamics gives

$$\Delta U = Q \quad [\because Q = \Delta U + W]$$

This indicates that the net energy Q added to the water goes into increasing the internal energy of the water. The net energy added to the water is

$Q = 10.0 - 2.0 = 8.0$ kJ

Because $Q = mc\Delta T$, the temperature increase of the water is

$$\Delta T = \frac{Q}{mc} = \frac{8.0 \times 10^3}{(2.0)(4.186 \times 10^3)} = 0.96°C$$

9. $Z_1 - 2 \times 2 + 3 \times 1 = Z_2 - 2 \times 1 + 5 \times 1 = Z_c$

$\therefore \quad Z_1 - Z_2 = 4$

$A_1 - 4 \times 2 = A_2 - 1 \times 4 = A_c$

$\therefore \quad A_1 - A_2 = 4$

10. At A, $4N_0 \xrightarrow{1\text{ min}} 2N_0 \xrightarrow{1\text{ min}}$

$$N_0 \xrightarrow{1\text{ min}} \frac{N_0}{2} \xrightarrow{1\text{ min}} \frac{N_0}{4}$$

At B, $N_0 \xrightarrow{2\text{ min}} \dfrac{N_0}{2} \xrightarrow{2\text{ min}} \dfrac{N_0}{4}$

After 4 min, $N_A = N_B = \dfrac{N_0}{4}$

$$\therefore \quad N_C = (4N_0 + N_0) - \left(\frac{N_0}{4} + \frac{N_0}{4}\right) = \frac{9N_0}{2}$$

$[\because \text{initial} - \text{final} = N_C]$

11. Given, $R_A = R_B \quad \therefore \quad \lambda_A N_A = \lambda_B N_B$

or $$\left(\frac{\ln 2}{T_A}\right)(4N_0 e^{-\lambda_A t}) = (N_0)\left(\frac{\ln 2}{T_B}\right) e^{-\lambda_B t}$$

$$e^{(\lambda_A - \lambda_B)t} = 8$$

$$\therefore \quad (\lambda_A - \lambda_B)t = \ln 8 = 3(\ln 2)$$

$$\therefore \quad \left(\frac{\ln 2}{1} - \frac{\ln 2}{2}\right)t = 3\ln(2) \quad \text{or} \quad t = 6\,\text{min}$$

12. From Coulomb's law, since charge of opposite polarity is placed, the force exerted will be attractive, also force between two charge particles is independent of presence of any other charge from the principle of superposition. Hence, option (c) is true.

13. $R_1 = \frac{l}{KA}, \quad R_2 = \frac{l}{2KA}, \quad R_3 = \frac{l}{4KA}$

$$R_{eq} = \left(\frac{R_1 R_2}{R_1 + R_2} + R_3\right) = \frac{7l}{12\,KA}$$

$$\left[\because \frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} \text{ and } R_S = R_1 + R_2 \right.$$
as rods 1 and 2 are in parallel
and equivalent in series with 3 $\Big]$

14. Specific gravity $= \frac{\text{Density of substance } (\rho)}{\text{Density of water } (\rho_w)}$

$[\because \rho = \rho_w \times 1000]$

$$\Delta p = \rho g h = 1060 \times (9.8)(1.8) = 18.7 \text{ kPa}$$

15. Electric potential energy = Kinetic energy

i.e., $qV = \frac{mv^2}{2}$

$$\Rightarrow \quad v = \sqrt{\frac{2qV}{m}} \quad \text{and} \quad qvB = qE \quad \Rightarrow \quad E = vB$$

$$E = \sqrt{\frac{2qV}{m}} \times B$$

After substituting these values, we get

$$E = 640 \text{ V/m}$$

16. Let a resistance R be connected in series with the battery and wire.

Voltage drop across wire

$$= 1 \times 10^{-3} \times 100 = 0.1 \text{ V}$$

Let current in circuit be I.

$$\therefore \quad 0.1 = I \times 3 = 3 \times \frac{2}{R+3} \quad \Rightarrow R = 57\ \Omega$$

17. By conservation of momentum, the momentum of the block bullet system just after the interaction is $p = mv$.

where, m is the mass of bullet and v is its velocity before striking the block.

Hence, the kinetic energy of the system just after the lodging of bullet into the block is

$$K = \frac{p^2}{2(M+m)}$$

$$= \frac{(mv)^2}{2(M+m)}$$

The friction force does work

$$W_f = -f_s = -\mu_k (m+M) g s$$

In stopping the block, where s is the distance traversed by block-bullet system on the tabletop.

From work-energy theorem,

$$\Delta K = W_f$$

$$\Rightarrow \quad 0 - K = -\mu_k (m+M) g s$$

$$\therefore \quad s = 0.37 \text{ m}$$

18. The horizontal range,

$$R = \frac{u^2 \sin 2\theta}{g}$$

Time of flight, $T = \frac{2u \sin\theta}{g}$

Range is maximum when $\theta = 45°$.

So, $\sin 2\theta = \sin 90° = 1$

$$R_{max} = \frac{u^2}{g}$$

Time of flight is maximum when $\theta = 90°$.

So, $\sin\theta = \sin 90° = 1$

$$\therefore \quad T_{max} = \frac{2u}{g}$$

19. The descending part of the rope is in free fall. It has speed $v = \sqrt{2gy}$ at the instant all its points have descended a distance y. The length of the rope which lands on the table during an interval dt following this instant is vdt.

The increment of momentum imparted to the table by this length in coming to rest is $m(vdt)v$. Thus, the rate at which momentum is transferred to the table is

$$\frac{dp}{dt} = mv^2 = (2\,my)\,g$$

and this is the force arising from stopping the downward fall of the rope. Since, a length of rope y of the weight $(my)g$, already lies on the tabletop, the total force on the tabletop is
$(2my)g + (my)g = (3my)\,g$ or the weight of a length $3y$ of rope.

So, $k = 3$

20. Find acceleration

$$2ma = mg \sin 53° - \mu mg \cos 53° - mg \sin 37° - \mu mg \cos 37°$$

Then, use $v^2 = u^2 + 2as$, $v = 0$, $u = 1$ m/s.

On solving, we get $s = 0.45$ m

21. The rotational and translational kinetic energy of the ball at the bottom will be changed to gravitational potential energy, when the sphere stops. We therefore write

$$\left(\frac{Mv^2}{2} + \frac{I\omega^2}{2}\right)_{\text{start}} = (Mgh)_{\text{end}}$$

For a solid sphere, $I = \dfrac{2\,Mr^2}{5}$

Also, $\omega = \dfrac{v}{r}$. Then, above equation becomes

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}Mr^2\right)\left(\frac{v}{r}\right)^2 = Mgh$$

or $$\frac{1}{2}v^2 + \frac{1}{5}v^2 = (9.8)\,h$$

Using $v = 20$ m/s, gives $h = 28.6$ m.

Note The answer does not depend upon the mass of the ball or the angle of the incline.

22. The initial elastic potential energy of the compressed spring is

$$\text{PE}_s = \frac{1}{2}kx_i^2$$

Because the block is always at the same height above earth's surface, the gravitational potential energy of the system remains constant. Hence, the initial potential energy stored in the spring is converted to kinetic energy at $x = 0$ *i.e.,*

$$\frac{1}{2}kx_i^2 = \frac{1}{2}mv_f^2$$

Solving for v_f gives

$$v_f = \sqrt{\frac{k}{m}}\,x_i = \sqrt{\frac{1.0 \times 10^3}{1.6}}\,(2.0 \times 10^{-2})$$
$$= 0.50 \text{ m/s}$$

23. The angle between magnetic field and area vector is 90°, so the flux associated with coil is zero. Although magnetic field is changing but flux is remaining constant equal to zero, so emf induced and hence current in the loop is equal to zero.

24. Resistance of coil $= \dfrac{V_{DC}}{I_{DC}} = \dfrac{6}{12} = 0.5\ \Omega$

In an AC circuit, the current is in phase with emf. This means that the net reactance of the circuit is zero. The impedance is equal to the resistance, *i.e.,* $Z = 0.5\ \Omega$.

$$\text{RMS current} = \frac{\text{RMS voltage}}{Z} = \frac{6}{0.5} = 12 \text{ A}$$

25. If the upper wire is to float, it must be in equilibrium under the action of two forces : the force of gravity and magnetic repulsion. The weight per unit length here 1.0×10^{-4} N/m must be equal and opposite the magnetic force per unit length. Because the currents are the same, we have

$$\frac{F_1}{l} = \frac{mg}{l} = \frac{\mu_0 I^2}{2\pi d}$$

$$\Rightarrow \quad 1.0 \times 10^{-4} = \frac{(4\pi \times 10^{-7})\,(I^2)}{(2\pi)(0.10)}$$

We solve for the current to find $I = 7.1$ A.

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26. First we make ray diagrams roughly to scale to see, where the image from the first lens falls and how it acts as the object for the second lens. The location of the image formed by the first lens is found *via* the thin-lens equation

$$\frac{1}{30.0} + \frac{1}{v_1} = \frac{1}{10.0} \quad \Rightarrow \quad v_1 = +\,15.0 \text{ cm}$$

The image formed by this lens becomes the object for the second lens. Thus, the object distance for the second lens is 20.0 cm – 15.0 cm = 5.00 cm. We again apply the thin-lens equation to find the location of the final image (see figure).

$$\frac{1}{5.00} + \frac{1}{v_2} = \frac{1}{20.0} \quad \Rightarrow \quad v_2 = -\,6.67 \text{ cm}$$

Thus, the final image is 6.67 cm to the left of the second lens.

27. For constructive interference,

$2nt = \frac{\lambda}{2}$, we have

$$t = \frac{\lambda}{4n} = \frac{602}{(4)(1.33)} = 113 \text{ nm}$$

28. Maximum distance covered by space wave communication

$$= \sqrt{2Rh} = \sqrt{2 \times 6.4 \times 10^6 \times 300} = 62 \text{ km}$$

Since, receiver-transmitter distance is 100 km, this is ruled out for signal frequency.

Further f_c for ionospheric propagation is

$$f_c = 9\,(N_{max})^{1/2} = 9 \times (10^{12})^{1/2} = 9 \text{ MHz}$$

So, the signal of 100 MHz ($7f_c$) comes *via* the satellite mode.

29. For $t \le t_1$; $A = 0, B = 0$; hence $Y = 0$

For t_1 to t_2; $A = 1, B = 0$; hence $Y = 0$

For t_2 to t_3; $A = 1, B = 1$; hence $Y = 1$

For t_3 to t_4; $A = 0, B = 1$; hence $Y = 0$

For t_4 to t_5; $A = 0, B = 0$; hence $Y = 0$

For t_5 to t_6; $A = 1, B = 0$; hence $Y = 0$

For $t > t_6$; $A = 0, B = 1$; hence $Y = 0$

Based on the above, the output waveform for AND gate can be drawn as given below

30. $\Delta V_0 = \beta\, \Delta I_b (R_L)$ $\left[\because \beta = \frac{\Delta V_0}{\Delta V_i}\right]$

Given, $\beta = 50, \Delta I = 50\,\mu\text{A} = 50 \times 10^{-6}$ A

$R_L = 2\text{ k}\Omega = 2000\,\Omega$

$\therefore \quad \Delta V_0 = 50\,(50 \times 10^{-6}) \times 2000$

$\Delta V_0 = 5$ V

$$\therefore \quad A_p = \beta^2 \cdot \frac{R_L}{r_i} = (50)^2 \times \frac{2000}{500} = 10^4$$

Hence, option (c) is true.