

Topics: Confidence Intervals

1. For each of the following statements, indicate whether it is True/False. If false, explain why.

I. The sample size of the survey should at least be a fixed percentage of the population size in order to produce representative results.

Ans: True: The representation of the survey results should have a sample size. The sample size must be a fixed percentage of the total population size of the survey.

II. The sampling frame is a list of every item that appears in a survey sample, including those that did not respond to questions.

Ans: False: The sampling frame refers to a list of an item which responds to the question and not the ones which do not respond to the questions.

III. Larger surveys convey a more accurate impression of the population than smaller surveys.

Soln: True: The larger conveys a more accurate impression of the population as larger surveys involve large sample size which reduces the chances of error.

2. PC Magazine asked all of its readers to participate in a survey of their satisfaction with different brands of electronics. In the 2004 survey, which was included in an issue of the magazine that year, more than 9000 readers rated the products on a scale from 1 to 10. The magazine reported that the average rating assigned by 225 readers to a Kodak compact digital camera was 7.5. For this product, identify the following:

A. The population

Soln: Readers of the magazine = 9000

B. The parameter of interest

Soln: Rating of the camera (7.5)

C. The sampling frame

Soln: Sampling frame: All readers of the issue where the survey was included.

D. The sample size

Soln: 225

E. The sampling design

Soln: The sampling design means on what basis the selection of samples from a survey is filtered that is **Kodak compact digital camera.**

F. Any potential sources of bias or other problems with the survey or sample

Soln: It is possible that only those who were particularly pleased or only who are displeased with the product participated in the survey which can makes the results unreliable.

3. For each of the following statements, indicate whether it is True/False. If false, explain why.

I. If the 95% confidence interval for the average purchase of customers at a department store is \$50 to \$110, then \$100 is a plausible value for the population mean at this level of confidence.

Soln: True.

Confidence interval identifies the collection of values for the population parameter that are consistent with the observed sample

II. If the 95% confidence interval for the number of moviegoers who purchase concessions is 30% to 45%, this means that fewer than half of all moviegoers purchase concessions.

Soln: False.

We have evidence in that direction but we cannot confirm 100% based on this data. We have to consider the values out of this range(i.e. more than 95% confidence interval).

III. The 95% Confidence-Interval for μ only applies if the sample data are nearly normally distributed.

Soln: False.

We should have a moderately large sample (usually at least larger than 30 for many cases), the central limit theorem implies that the sampling distribution is normal regardless of the data itself.

4. What are the chances that $\bar{X} > \mu$?

- A. $\frac{1}{4}$
- B. $\frac{1}{2}$
- C. $\frac{3}{4}$
- D. 1

Soln: B.

This is pure assumption. There is a 50% chance that the sample mean (\bar{X}) is greater than the population mean (μ).

5. In January 2005, a company that monitors Internet traffic (WebSideStory) reported that its sampling revealed that the Mozilla Firefox browser launched in 2004 had grabbed a 4.6% share of the market.

I. If the sample were based on 2,000 users, could Microsoft conclude that Mozilla has a less than 5% share of the market?

Soln:

$$\begin{aligned} \hat{x} &= 0.046, n = 2000, z_{.95} = 1.96, q = 0.954 \\ \text{95\% confidence interval for the proportion of web users using Mozilla is} \\ \hat{x} \pm Z \sqrt{\frac{\hat{x}q}{n}} &= 0.046 \pm 1.96 \sqrt{\frac{0.046 \cdot 0.954}{2000}} = 0.046 \pm 0.00918 = 0.0368 - \\ &0.0551. \end{aligned}$$

II. WebSideStory claims that its sample includes all the daily Internet users. If that's the case, then can Microsoft conclude that Mozilla has a less than 5% share of the market?

Soln:

In this case, we have data on the entire population and the sample value accurately reflects the population number. Thus we can conclude that share is less than 5%.

6. A book publisher monitors the size of shipments of its textbooks to university bookstores. For a sample of texts used at various schools, the 95% confidence interval for the size of the shipment was 250 ± 45 books. Which, if any, of the following interpretations of this interval are correct?

A. All shipments are between 205 and 295 books.

Ans: Incorrect.

The interval of (205,295) is for 95% confidence not for 100%.

B. 95% of shipments are between 205 and 295 books.

Ans: Incorrect.

The interval doesn't describe individual shipments.

C. The procedure that produced this interval generates ranges that hold the population mean for 95% of samples.

Ans: Correct.

95% of intervals created in this way contain the true population mean.

D. If we get another sample, then we can be 95% sure that the mean of this second sample is between 205 and 295.

Ans: Incorrect.

The interval doesn't describe the mean of another sample.

E. We can be 95% confident that the range 160 to 340 holds the population mean.

Ans: Incorrect.

The interval doesn't correspond to a 95% confidence level.

7. Which is shorter: a 95% z -interval or a 95% t -interval for μ if we know that $\sigma = s$?

- A. The z -interval is shorter
- B. The t -interval is shorter
- C. Both are equal
- D. We cannot say

Soln:

A. the Z - interval is shorter

Questions 8 and 9 are based on the following: To prepare a report on the economy, analysts need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

8. How many randomly selected employers (minimum number) must we contact in order to guarantee a margin of error of no more than 4% (at 95% confidence)?

- A. 600
- B. 400
- C. 550
- D. 1000

Soln:

here, n =number of employers, Assume $\hat{P}=0.5, \hat{q}=0.5$ Margin of Error=0.04
For 95% confidence interval, the critical value $Z=1.96$

$$ME = Z * \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$0.04 = 1.96 * \sqrt{\frac{0.5*0.5}{n}}$$

$$n = \frac{1.96^2 * 0.5 * 0.5}{0.04^2} = \frac{0.9604}{0.0016} = 600 = A$$

9. Suppose we want the above margin of error to be based on a 98% confidence level. What sample size (minimum) must we now use?

- A. 1000
- B. 757
- C. 848
- D. 543

Soln:

$$Z = 2.326$$

$$0.04 = 2.326 * \sqrt{\frac{0.5 * 0.5}{n}}$$

$$n = \frac{2.326^2 * 0.5 * 0.5}{0.04^2} = \frac{1.3525}{0.0016} = 845.35 = C$$