EIGEN VALUES AND EIGEN VECTORS

1. Create a 3x3 matrix with rank 2, find eigenvectors. Show with Matlab, that eigenvectors correspond to non-zero eigenvalues lie in column space.

```
% Generate a random 3x3 A matrix with rank 2
A = randi(9,3,2)*randi(9,2,3)
A = 3 \times 3
  105
       102
              52
              28
   54
        60
   21
        30
              12
% Generate eigen vectors
[eigen vectors, eigen values] = eig(A)
eigen vectors = 3 \times 3
   0.8622 0.6631
                   -0.3123
   0.4653
          -0.3015
                   -0.1562
   0.2003
          -0.6852
                   0.9370
eigen values = 3 \times 3
 172.1197
                          0
                0
       0
            4.8803
                          0
        0
                     0.0000
% check ranks
rank(A)
ans = 2
check =[A eigen_vectors(:,1)];
rank(check)
ans = 2
check2=[A eigen_vectors(:,3)];
rank(check2)
ans = 3
check3=[A eigen_vectors(:,2)];
rank(check3)
ans = 2
fprintf("Since the rank of non zero eigen value's corresponding \n eigen vector
didn't change, the eigen vectors \n with non zero eigen values lie in the
columnspace of A ");
Since the rank of non zero eigen value's corresponding
eigen vector didn't change, the eigen vectors
```

2. Construct a symmetric 3 by 3 matrix - rank 1 matrix

with non zero eigen values lie in the columnspace of A

```
% Generate a random matrix
  A=randi(5,3,1)*randi(5,1,3);
 % Make it symmetric since AA' or A'A matrices are symmetric
  A=A*A'
  A = 3 \times 3
     45
          135
                 45
    135
          405
                135
     45
          135
                 45
  rank(A)
  ans = 1
3. Construct a random 3 by 3 matrix A of rank 3 and show that AAT is symmetric.
  % Generate a random matrix
  A=randi(5,3,3)*randi(5,3,3)
  A = 3 \times 3
     23
           34
                 30
     16
           24
                 16
      10
           18
                 23
  A=A*A';
  Α
  A = 3 \times 3
         2585
                     1664
                                 1532
         1664
                                  960
                     1088
         1532
                                  953
                      960
  Α'
  ans = 3 \times 3
         2585
                                 1532
                     1664
                     1088
                                  960
         1664
         1532
                      960
                                  953
4. Construct a random 3 by 3 matrix A of rank 3 and show that the eigen values of A and AT are the same
 % Generate random matrix
 A=randi(5,3,3)*randi(5,3,3);
```

```
% Compute Eigen values
eig(A)
```

ans = 3×1 98.8025 0.5323 -4.3348

```
eig(A')
```

```
ans = 3 \times 1
```

98.8025

0.5323

-4.3348

5. Construct a 3x3 matrix with all elements non-zero and one eigen value as zero

```
% Generate a non zero 3x3 matrix with rank 2
A = randi([1,5],3,2) * randi([2,4],2,3)
```

 $A = 3 \times 3$

17 16 20

22 16 24

24 22 28

% Compute the eigen values
eig(A)

ans = 3×1

63.5794

-0.0000

-2.5794