The volatility puzzle of the beta anomaly*

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Abstract

This paper shows that leading theories of the beta anomaly fail to explain the anomaly's conditional performance. Abnormal returns and Sharpe ratios of betting-against-beta (BAB) factors rise following months with below-median realized volatility, even controlling for mispricing, limits to arbitrage, lottery preferences, analyst disagreement, and sentiment. Moreover, the leverage constraints theory counterfactually predicts that market and BAB Sharpe ratios increase with volatility. We further show that institutional investors shift their demand from high- to low-beta stocks as volatility increases, and the resulting price impact is sufficient to explain the difference in abnormal BAB returns between high- and low-volatility states.

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"We keep regurgitating the data to find yet one more variation of the size, value, or momentum anomaly, when the mother of all inefficiencies may be standing right in front of us—the risk anomaly."

—Robin Greenwood quoted in Ang (2014), page 332.

1. Introduction

Understanding the relationship between risk and return is perhaps the most central pursuit of asset pricing. One of the oldest and most well known facts about this relationship is that stocks with low market betas earn positive abnormal returns relative to the CAPM of Sharpe (1964) and Lintner (1965), with the opposite result holding for high-beta stocks. This anomaly dates to at least Friend and Blume (1970) and Black, Jensen, and Scholes (1972) but it remains robust in the fifty years of data after these seminal studies and its cause is still actively debated, lacking a single universally accepted explanation (see, e.g., Frazzini and Pedersen, 2014; Liu, Stambaugh, and Yuan, 2018; Asness, Frazzini, Gormsen, and Pedersen, 2020). In addition to academic interest, "defensive equity" strategies have experienced "massive capital inflows" in the words of Novy-Marx and Velikov (2022) and provide a foundation for a multitude of exchange-traded funds and other investment products.

In this paper, we take a novel approach to evaluating leading theories of the beta anomaly based on their ability to explain the conditional returns of "betting-against-beta" (BAB) factors that buy low-beta stocks and short high-beta stocks. Overwhelming evidence exists that risk and risk premia are time-varying, and the true cause of an anomaly must match this time variation (see, e.g., Cochrane, 2011). The literature also posits several theories of the beta anomaly that, suspiciously, are all able to explain the unconditional returns of BAB factors even though they are based on very different economic mechanisms. As noted by Nagel and Singleton (2011), exploiting conditional variation in anomaly performance provides more statistical power to reject false explanations that "pass" unconditional asset pricing tests. We use lagged volatility of the BAB factor as conditioning information following

Moreira and Muir (2017) and Cederburg, O'Doherty, Wang, and Yan (2020), who show that volatility timing beta factors results in significant performance improvements because their Sharpe ratios rise following low realizations of volatility. We also show that the loadings of BAB returns on several asset pricing factors fall with volatility so that the returns of BAB are most anomalous when risk is low. The main results of this paper show that none of the leading theories of the beta anomaly completely explain this strong performance in low-volatility periods.

We consider six leading explanations for the low-risk anomaly: leverage constraints, risk factors missing from the CAPM, limits to arbitrage, lottery preferences, sentiment, and analyst disagreement. The leverage constraints theory, proposed by Black (1972) and expanded by Frazzini and Pedersen (2014), posits that leverage-constrained investors with relatively low risk aversion bid up the prices of high-beta stocks because of their high expected returns, rendering their CAPM alphas negative. Consistent with this theory, Adrian, Etula, and Muir (2014), Frazzini and Pedersen (2014), Boguth and Simutin (2018), Jylhä (2018), and Lu and Qin (2021) show that proxies for leverage constraints forecast returns on BAB factors. However, prior studies ignore the predictions this theory makes about the relationship between volatility and subsequent BAB performance. We compare several calibrations of Frazzini and Pedersen's equilibrium model that generate different levels of BAB volatility. They show that the Sharpe ratios of both BAB and the market portfolio should increase with these factors' respective volatilities, the opposite of the results found in the data. These counterfactual predictions arise from a key assumption in the model that all investors are risk averse and therefore demand a positive risk-return tradeoff (see, e.g., Merton, 1973).

Turning to multifactor explanations, Novy-Marx and Velikov (2022) show that the Fama and French (2018) six-factor model (FF6) prices BAB because low-beta stocks move like those with robust profitability, low asset growth, and high momentum. We confirm this result, but also show that the FF6 fails to explain the conditional performance of BAB. When lagged volatility is "low" (below its median value), the BAB Sharpe ratio more than doubles compared to its whole-sample estimate. At the same time, the factor loadings that explain BAB returns unconditionally shrink significantly, or even flip signs, resulting in significantly positive alpha that is one percentage point higher per month than in high-volatility states. Thus, BAB returns appear most anomalous precisely when they require bearing the least risk to earn. We further show that this pattern extends to the CAPM and Fama and French (1993) three-factor model, as well as more economically motivated models with factors based on the "unfiltered" consumption of Kroencke (2017) and the intermediary leverage measure of Adrian et al. (2014), which prices BAB unconditionally. In all models considered, a one standard deviation *decrease* in volatility predicts an *increase* in abnormal return of 0.7 percentage points per month.

We next investigate whether the remaining theories of the beta anomaly rectify the failure of the FF6 in low-volatility states. Liu et al. (2018) posit that the beta anomaly could arise from the strong cross-sectional correlation between idiosyncratic volatility (IVOL) and beta. This argument expands on that of Stambaugh, Yu, and Yuan (2015), who argue IVOL is a proxy for arbitrage risk and interacts with limitations on short selling to magnify overpricing. Using the mispricing measure of Stambaugh, Yu, and Yuan (2012), Liu et al. (2018) show that the cross-sectional relationship between beta and abnormal returns disappears when excluding "overpriced" high-IVOL stocks or when controlling for IVOL or mispricing. Motivated by these results, we construct three new BAB factors: one that excludes overpriced high-IVOL stocks, one that buys and sells stocks with similar levels of IVOL (but different levels of beta), and one that buys and sells stocks with similar levels of mispricing. Consistent with Liu et al. (2018), all three factors earn insignificant unconditional alpha, however, this alpha becomes positive when volatility is low and negative when volatility is high, with a significant difference between the two states of 0.74 to 1.08 percentage points per month. The Sharpe ratios of these factors also increase by 0.5 to 0.6 going from high- to low-volatility months. Thus, limits to arbitrage evidently fail to explain the conditional performance of the beta anomaly.

Bali, Cakici, and Whitelaw (2011) and Bali, Brown, Murray, and Tang (2017) argue that investor demand for stocks with lottery like payoffs could bid up the prices of high-risk stocks at the expense of subsequently earning negative abnormal returns. They measure a stock's lottery demand in a given month using the average of the highest five daily returns in that month, and the latter study shows it subsumes the cross-sectional relationship between beta and returns. Hypothetically, lottery demand could explain the time-varying performance of BAB factors as long as this demand is high when BAB volatility is low. However, similar to our limits-to-arbitrage tests, we form a BAB factor that matches the lottery demand of stocks bought and sold, and show it earns significantly higher Sharpe ratios and alphas in low-volatility states than high-volatility ones. Thus, lottery demand cannot explain the conditional performance of the beta anomaly.

Next we consider two explanations of the beta anomaly that predict time-series variation in BAB returns. Antoniou, Doukas, and Subrahmanyam (2016) posits that high-beta stocks are especially exposed to sentiment and find that the anomaly is significant only in high-sentiment months. Similarly, Hong and Sraer (2016) argue that beta amplifies disagreement about the economy, which, coupled with short-sale constraints, renders high-beta stocks overpriced when disagreement is high. Consistent with this argument, they show that the beta anomaly is significant only following relatively high levels of analyst earnings forecast dispersion. However, we find that the negative relationship between BAB alphas and lagged volatility holds regardless of the level of analyst disagreement or sentiment.

For the abnormal returns of BAB to be negatively related to lagged volatility, it must be the case that high-beta stocks become less "overvalued" relative to low-beta stocks as the volatility of BAB increases. We next investigate whether trading by institutional investors can explain this dynamic relationship by estimating a demand system following Koijen and Yogo (2019). On average, all institutional investors have a significant preference for high-beta stocks. However, going from low- to high-volatility months, this preference falls to essentially zero and becomes statistically insignificant, while household investors consistently prefer low-beta stocks regardless of volatility. As a result, a counterfactual experiment shows that low-beta stocks appreciate relative to high-beta stocks by 3.5% going from low- to high-volatility months. This figure is within 0.1 percentage points of the appreciation required to fully eliminate the conditional alpha of BAB based on an estimate derived from the Campbell and Shiller (1988) decomposition following Han, Roussanov, and Ruan (2022). These findings are consistent with a nascent literature arguing that widely used performance evaluation contracts incentivize institutional managers to overweight high-beta stocks, even if their alphas are negative, because, on average, their high returns exceed those of unit-beta benchmarks like the S&P 500 (e.g., Baker, Bradley, and Wurgler, 2011 and Christoffersen and Simutin, 2017). The same incentives also penalize tracking error volatility and so, as BAB volatility increases, managers have the incentive to retreat from their positions in high-beta stocks toward their benchmarks, leading them to sell high-beta stocks and buy low-beta ones.[1]

Overall, the results in this paper challenge current explanations from the extensive literature on the beta anomaly. Our study is most closely related to Asness et al. (2020), who examine the compatibility of this anomaly with essentially the same leading explanations as we do. Their key innovation is creating betting-against-risk factors that separate the cross-sectional effects of the systematic risk in beta from those of idiosyncratic volatility. In contrast, our key innovation focuses on the time series, which leads us to very different conclusions. They find that multiple theories contribute to explaining the unconditional beta anomaly while we find that none of these theories are consistent with the conditional performance of BAB. More broadly, our results show that five decades worth of attempts to explain a single anomaly fail to produce a single theory that can explain this anomaly's returns conditional on volatility. This fact highlights the importance of not ignoring conditioning information in asset pricing, which is still widely done in practice.

[1]Several investments professionals we discussed these results with had observed similar behavior, namely managers retreating towards benchmarks when risk of positions increases. One common stated reason is that managers are especially concerned about outflows or termination from poor performance when risk increases and investors are paying relatively close attention.

This paper proceeds as follows. Section 2 specifies our data sources and the construction of our BAB portfolios. Section 3 presents motivational evidence on the performance of volatility-managed BAB factors. Section 4 evaluates leading theories of the beta anomaly. Section 5 reports institutional demand results. Section 6 concludes.

2. Data and variable construction

This section describes our data sources and the construction of our betting-against-beta (BAB) factor.

2.1. Data sources

We obtain returns on U.S. common stocks from CRSP, accounting data from COMPUSTAT, institutional common stock holdings from the Thomson Reuters Institutional Holdings Database, and analyst forecasts from I/B/E/S. We obtain return data for the risk-free rate and the six-factor asset pricing model of Fama and French (2018), FF6, from the website of Kenneth French. FF6 includes the market (MKT) and value (HML) factors of Fama and French (1993), as well as factors based on size (SMB), robust (high)-minus-weak (low) profitability (RMW), conservative (low)-minus-aggressive (high) real investment (CMA), and momentum (MOM) following Carhart (1997). We use the stock-level mispricing measure of Stambaugh et al. (2012), Stambaugh et al. (2015), and Stambaugh and Yuan (2017) obtained from the website of Robert Stambaugh. They construct their measure each month as the average of the stock's rankings with respect to 11 variables associated with prominent anomalies relative to the Fama and French (1993) three-factor model (FF3) such that a higher level of mispricing negatively predicts subsequent abnormal returns. Finally, we obtain the monthly sentiment index of Baker and Wurgler (2007) orthogonalized to economic conditions comes from the website of Jeffrey Wurgler, the "unfiltered" consumption growth time series of Kroencke (2017) from Tim Kroencke's website, and yields on BAA- and AAArated bonds from the St. Louis Fed's website. Due to data availability, our sample period is from July 1965 to December 2016.[2]

2.2. Betting-against-beta factor

We estimate betas following Liu, Stambaugh, and Yuan (2018). Each stock-month, (i, t), we estimate CAPM regressions using the prior 60 monthly returns (36 month minimum), applying a Dimson (1979) correction for nonsynchronous trading:

$$r_{it} - r_{ft} = a_i + \beta_{i,0} MKT_t + \beta_{i,1} MKT_{t-1} + \epsilon_t, \tag{1}$$

where MKT is the return on the market in excess of the risk-free rate, r_{ft}. We then shrink the resulting estimates, $\hat{\beta}_{i,t}^{ts} := \hat{\beta}_{i,0} + \hat{\beta}_{i,1}$, toward one using the procedure of Vasicek (1973) to mitigate estimation error:

$$\hat{\beta}_{i,t} := w_i \hat{\beta}_{i,t}^{ts} + (1 - w_i) \times 1, \tag{2}$$

where:

$$w_i = \frac{1/\hat{\sigma}^2(\hat{\beta}_{i,t}^{ts})}{1/\hat{\sigma}^2(\hat{\beta}_{i,t}^{ts}) + 1/\hat{\sigma}^2(\beta)}. \tag{3}$$

The $\hat{\sigma}(\hat{\beta}_{i,t}^{ts})$ is the standard error of $\hat{\beta}_{i,t}^{ts}$ and $\hat{\sigma}^2(\beta) := \hat{\sigma}_{cs}^2(\hat{\beta}_{i,t}^{ts}) - \overline{\hat{\sigma}^2(\hat{\beta}^{ts})}$ is a measure of the cross-sectional variance of true betas, where $\hat{\sigma}_{cs}^2(\hat{\beta}_{i,t}^{ts})$ is the cross-sectional variance of $\hat{\beta}_{i,t}^{ts}$ and $\overline{\hat{\sigma}^2(\hat{\beta}^{ts})}$ is the cross-sectional mean of $\hat{\sigma}^2(\hat{\beta}_{i,t}^{ts})$.

At the beginning of each month, we sort all common stocks in CRSP into value-weighted

[2] Links to the websites cited in this section are: Kenneth French: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`; Robert Stambaugh: `http://finance.wharton.upenn.edu/~stambaug/`; Tim Kroencke: `https://sites.google.com/site/kroencketim/`; and the St. Louis Fed: `https://fred.stlouisfed.org/series/BAA`. The 11 anomalies used in the mispricing measure are: failure probability (e.g., Campbell, Hilscher, and Szilagyi, 2008), O-Score (Ohlson, 1980), net stock issuance (Loughran and Ritter, 1995), composite equity issuance (Daniel and Titman, 2006), accruals (Sloan, 1996), net operating assets (Hirshleifer, Hou, Teoh, and Zhang, 2004), momentum (Levy, 1967; Jegadeesh and Titman, 1993; and Fama and French, 1996), gross profitability (Novy-Marx, 2013), asset growth (Titman, Wei, and Xie, 2004*b*; Lyandres, Sun, and Zhang, 2007; Xing, 2007; and Cooper, Gulen, and Schill, 2008), return on assets (Fama and French, 2006), and abnormal capital investment (Titman, Wei, and Xie, 2004*a*).

decile portfolios based on their $\hat{\beta}_{i,t}$, denoting the return on the lowest-beta (highest-beta) portfolio as $r_{L,t}$ ($r_{H,t}$). Following Novy-Marx and Velikov (2022), we define our BAB factor to target beta neutrality as follows:

$$BAB_t = r_{L,t} - r_{H,t} - (\beta_{L,t-1} - \beta_{H,t-1}) * MKT_t, \tag{4}$$

where $\beta_{L,t-1}$ ($\beta_{H,t-1}$) denotes the value-weighted average of the $\hat{\beta}_{i,t}$ across stocks in the low-beta (high-beta) portfolio. Without hedging the market exposure with $(\beta_{L,t-1} - \beta_{H,t-1}) * MKT_t$, BAB would have insignificant mean returns but significantly positive alpha. The beta neutrality, which need not be perfect, transforms this alpha into positive mean returns, which is necessary for Sharpe ratio comparisons that are central to our study.[3] Frazzini and Pedersen (2014) propose a widely cited alternative construction of a BAB factor that targets beta neutrality by scaling each leg of the factor by the inverse their beginning-of-month betas. However, this scaling results in factors that are "net-long," placing more weight on r_L than r_H, which Liu et al. (2018) and Han (2022) show results in abnormal returns that are unrelated to the beta anomaly. Novy-Marx and Velikov (2022) further argue that the Frazzini and Pedersen (2014) net-long scaling puts extreme weight on small-cap stocks because it uses the long and short legs of BAB to hedge each other's beta, whereas the "direct hedging" that we use hedges with the value-weighted market factor. Using value-weighted portfolios also mitigates another concern raised by Novy-Marx and Velikov (2022), specifically that the rank-weighting scheme of Frazzini and Pedersen (2014) puts extreme weights in micro-cap stocks.

[3]The hedge reduces the ex post market beta of BAB to 0.14, as shown in Internet Appendix Table IA.1. Untabulated results show that the strategy $(\beta_{L,t-1} - \beta_{H,t-1}) * MKT_t$ has a CAPM alpha that is essentially zero (-0.01%, $t = -0.19$) and its abnormal return never drives our results.

3. The performance of volatility-managed BAB

Several recent studies that find that volatility-managed versions of numerous asset pricing factors, which increase leverage following low realizations of volatility, produce significant Sharpe ratio gains relative to their unmanaged counterparts.[4] To set the stage for our main results, this section replicates these findings for our specification of BAB and discusses their implications for this factor's time-varying performance.

Following Barroso and Santa-Clara (2015), we define the volatility-managed betting-against-beta strategy as:

$$BAB^{\sigma}_{t+1} = \left(\frac{c}{\hat{\sigma}_t}\right) BAB_{t+1}, \tag{5}$$

where BAB_{t+1} denotes the month-$t+1$ buy-and-hold ("unmanaged") return and $\hat{\sigma}_t$ is the the realized volatility of the last 21 daily returns of the prior month, $BAB_{d,t},\ d = 1, ..., 21$:

$$\hat{\sigma}_t = \sqrt{\sum_{d=1}^{21} BAB^2_{d,t}}. \tag{6}$$

This timing approach is simple to construct, avoids hindsight biases documented by Liu, Tang, and Zhou (2019), and does not produce as extreme weights as the common alternative practice of scaling by realized variance, (e.g., Fleming et al., 2001, 2003; Kirby and Ostdiek, 2012; and Moreira and Muir, 2017). The scalar c is arbitrary and has no impact on statistical inference, but for expositional purposes, we choose it to equate the standard deviation of the returns on the managed and unmanaged factors. Figure 1 presents a monthly time-series of the realized volatility of BAB defined by Eq. (6).

Table 1 reports descriptive statistics for the performance of BAB and BAB$^{\sigma}$. By con-

[4] Barroso and Santa-Clara (2015), Daniel and Moskowitz (2016), Eisdorfer and Misirli (2020), and Barroso and Detzel (2021) demonstrate this results for the market portfolio, momentum, the beta anomaly, and the distress anomaly. Moreira and Muir (2017) and Cederburg et al. (2020) expand them to numerous factors. See also Kirby and Ostdiek (2012), who consider volatility timing strategies for portfolio allocations across stock portfolios. Much of this literature follows from Fleming, Kirby, and Ostdiek (2001, 2003) and Marquering and Verbeek (2004), who demonstrate large utility gains from volatility timing allocations across several asset classes.

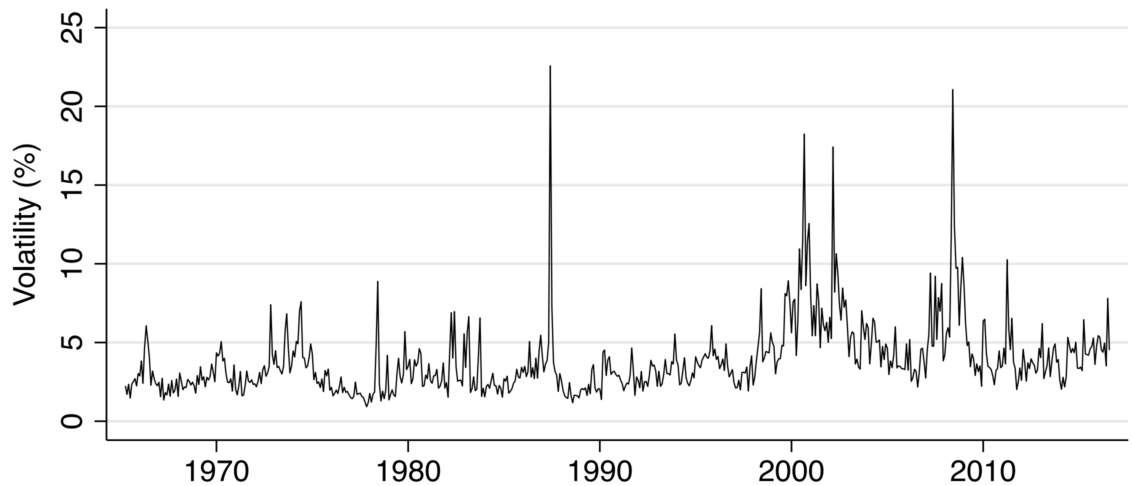

Figure 1: Monthly time series of realized volatility of the betting-against-beta factor

struction, BAB$^{\sigma}$ has the same standard deviation, but 50% higher mean return than the unmanaged BAB (0.76% vs. 0.49%). As a result, BAB$^{\sigma}$ has a higher Sharpe ratio than BAB (0.55 vs. 0.35) with the difference significant at the 1% level. The economic and statistical significance of this Sharpe ratio improvement is noteworthy since Cederburg et al. (2020) find that only eight out of 103 managed factors have significantly higher Sharpe ratios than their unmanaged counterparts, and investors would find it difficult to realize benefits implied by other measures of performance, such as alphas, in real time. Volatility management also reduces higher order risk of BAB$^{\sigma}$, flipping the skew from negative to positive and decreasing the excess kurtosis.

The finding of Table 1 that BAB$^{\sigma}$ earns a higher Sharpe ratio than BAB indicates that the Sharpe ratio of BAB increases when volatility falls; and this fact raises the bar for explanations of the beta anomaly. To see why, suppose that arbitrage opportunities do not exist so that assets are priced by a stochastic discount factor, m_t. Standard arguments (see, e.g., Cochrane, 2005, Ch. 1) show that the maximum Sharpe ratio in the market is

Table 1: Performance of BAB and its volatility-managed counterpart

At the beginning of each month, we sort all stocks into value-weighted decile portfolios based on beta. From these sorts, we form a betting-against-beta factor, BAB, that goes long the lowest-beta portfolio, short the highest-beta portfolio, and hedges market risk based on the ex ante betas of each portfolio (see, Eq. (4) for details). This table presents descriptive statistics of the returns on BAB and BAB$^{\sigma}$, which leverages BAB each month proportionally to its inverse realized volatility estimated over the previous month. The statistics shown are: mean excess return (Mean), standard deviation (SD), skewness (Skew), excess kurtosis (Kurt), and Sharpe ratio (SR). The sample period is July 1965 to December 2016. The parenthesis contains a z-statistic from a Jobson and Korkie (1981) test of the difference in Sharpe ratio between BAB and BAB$^{\sigma}$.

	Mean (%)	SD (%)	Skew	Kurt	SR
BAB	0.49	4.78	−0.80	7.23	0.35
BAB$^{\sigma}$	0.76	4.78	0.26	4.79	0.55
					(2.91)

$\sigma_t(m_{t+1})/E_t(m_{t+1})$, and the conditional Sharpe ratio of BAB is:

$$\mathrm{SR}_t(BAB) := \frac{E_t(BAB_{t+1})}{\sigma_t(BAB_{t+1})} = -\rho_t(m_{t+1}, BAB_{t+1})\frac{\sigma_t(m_{t+1})}{E_t(m_{t+1})}. \tag{7}$$

For this equation to hold, the negative relation between the volatility and Sharpe ratio of BAB necessarily requires that when its volatility *falls*, BAB loads more heavily on priced risk factors or the maximum Sharpe ratio in the economy rises. Especially given the commonality of volatility across factors, these facts are counterintuitive since investors should demand higher compensation for risk when volatility rises (see, e.g., Moreira and Muir, 2017). Alternatively, BAB may earn higher abnormal returns when volatility falls due to mispricing, but this possibility is also counterintuitive since one would expect arbitrage risk to be positively related to volatility.

4. Evaluating explanations of the beta anomaly

In this section, we investigate whether leading theories of the beta anomaly are consistent with the strong performance of BAB conditional on low volatility.

4.1. The leverage constraints theory

Black (1972) and Frazzini and Pedersen (2014) posit that high-beta stocks have embedded leverage that has economic value for investors facing borrowing constraints. Since they cannot borrow, less risk averse investors will instead invest heavily in high-beta stocks in equilibrium, bidding up their prices and lowering their expected returns relative to the CAPM. Sophisticated financial intermediaries able to leverage and assume the opposite side of the trade (e.g. hedge funds "betting against beta") are exposed to funding risk and margin requirements, which limits their capacity to eliminate the anomaly completely. Consistent with this theory, Adrian et al. (2014), Frazzini and Pedersen (2014), Boguth and Simutin (2018), Jylhä (2018), and Lu and Qin (2021) all show that various proxies for the tightness of leverage constraints forecast the returns on BAB factors. However, prior studies generally ignore the implications that the leverage constraints theory has for the predictive relationship between BAB's volatility and its subsequent returns.[5] In this section, we identify this predictive relation in their model by comparing several calibrations with different levels of volatility.

In the Frazzini and Pedersen (2014) model, each agent i has initial wealth W_t^i and forms a portfolio of S securities. The number of shares of each security she wants to own is given by the vector $x_i = (x_i^1, ...x_i^S)$. The agent has a quadratic utility function and maximizes the respective objective function:

$$\max_{x_i} x_i'(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t) - \frac{\gamma^i}{2} x_i' \Omega_t x_i, \tag{8}$$

where P_{t+1} and δ_{t+1} are the price and dividend at time $t+1$ (both are S-by-1 vectors), γ^i is a risk aversion parameter, and Ω is the covariance matrix of $P_{t+1} + \delta_{t+1}$. Agents face the restriction that invested wealth cannot exceed the amount required to cover margin

[5]This is arguably a similar omission as the common practice of motivating cross-sectional asset pricing studies with the ICAPM of Merton (1973) while ignoring the time-series predictions of the model like the positive risk-return tradeoff. Moreira and Muir (2017) also note that this positive tradeoff is a feature of all leading macro-finance asset pricing models.

requirements:

$$m_t^i \sum_s x_s^i P_t^s \leq \; W_t^i, \tag{9}$$

where m_t^i is the margin constraint. An investor that cannot use leverage, for example, has $m_t^i = 1$, meaning that the value of her risky portfolio of assets cannot exceed her wealth. The higher the m_t^i, the tighter becomes the margin requirement. This feature plays a crucial role in the model as investors have different levels of risk aversion and some, the less risk-averse ones, can be constrained by margin requirements, creating a shadow cost of funding constraints.

In equilibrium, for any security, the expected return should then be:

$$E\left(r_{t+1}^s\right) = r^f + \psi_t + \gamma \mathrm{cov}_t(r_{t+1}^s, r_{t+1}^M) P_t' x^*, \tag{10}$$

where r_{t+1}^s, r_{t+1}^M are, respectively, the security and market returns, ψ_t is an endogenous variable reflecting the aggregate shadow cost of leverage (a function of the average Lagrange multiplier of the constraint in Eq. (9)), and x^* is the S-by-1 vector with the total number of shares outstanding in the economy that must all be owned in equilibrium. Applying Eq. (10) to the market itself yields:

$$E\left(r_{t+1}^M\right) = r^f + \psi_t + \gamma \mathrm{var}_t(r_{t+1}^M) P_t' x^*. \tag{11}$$

Solving Eq. (11) for $P_t' x^*$, plugging into Eq. (10), and rearranging terms yields the main equilibrium relationship of the model:

$$E\left(r_{t+1}^s\right) - r^f = \psi_t(1 - \beta_t^s) + \beta_t^s \left(E\left(r_{t+1}^M\right) - r^f\right). \tag{12}$$

In particular, $\psi_t(1 - \beta_t^s)$ is the alpha of stock s. As long as some agents are constrained ($\psi_t > 0$), this alpha is negative for high-beta stocks ($\beta_t^s > 1$) and positive for low-beta stocks.

Eq. (11) highlights a previously neglected feature of the leverage constraints model: a positive market risk-return tradeoff. Because investors are risk averse, the representative risk aversion γ will be positive and the expected return and Sharpe ratio of the market should rise with volatility, counterfactual to the empirical results that the predictive relationship between market volatility and expected return is weak, or even negative, in the data, and the market Sharpe ratio falls with volatility (see, e.g., Moreira and Muir, 2017). We next show the model is also inconsistent with the conditional performance of BAB.

Frazzini and Pedersen (2014) provide the calibration exercise of their model in the Internet Appendix (page B18) for values "chosen to roughly match the empirical volatilities and correlations of the asset returns." They consider an economy with two agents and two assets, which have the same expected payoff of $\frac{1}{2} \times 100/(1+r^f)$. The payoffs have variances of 40 and 205 and a covariance of 84. The risk-free rate is 3.6% and total wealth split by the two agents is 100. The share of wealth of each agent, their risk aversion parameters, and margin requirements are the only exogenous variables needed to fully characterize equilibrium.

The first three columns of Table 2 replicate baseline calibrations of Frazzini and Pedersen (2014) and the remaining columns show results for different levels of volatility. In the standard CAPM scenario, investors are not constrained and so no BAB effect emerges in equilibrium. But a sizable BAB effect appears when the less risk-averse investor is constrained, which occurs in columns (2) and (3). Frazzini and Pedersen (2014) argue that the results in column (3) are the most suitable to explain the observed BAB effect in the data and adopt it as their baseline calibration, so we focus on that scenario.

In columns (4) through (7), we multiply the covariance matrix of payoffs in the original calibration by a volatility scalar ranging from 0.25 to 2. The expected return of the BAB portfolio monotonically increases from 2% in the low volatility scenario to 9% in the high volatility scenario. The increase in expected excess returns is so large that, in spite of higher volatility, the Sharpe ratio of the strategy increases in equilibrium. Both results are the opposite of the empirical results documented in Table 1, where expected returns are either constant or decreasing with volatility and Sharpe ratios fall with volatility.

Table 2: Calibrations of the Frazzini and Pedersen (2014) leverage constraints model

In the model, two agents with different relative risk aversion, shares of wealth, and leverage constraints (m1 and m2) trade a pair of risky assets that together comprise the market. The first column shows a baseline calibration in which leverage constraints do not bind and the CAPM holds. The following two columns replicate scenarios with leverage constraints from Frazzini and Pedersen (2014). Columns (4) to (7) present calibrations that each multiply the baseline covariance matrix of asset payoffs by a "volatility scalar" ranging from 0.25 to 2 (see Section 4.1 for details). The last column shows the combined effects of high volatility with high margin requirements. The first six rows specify the exogenous variables. Remaining rows specify the endogenous outcome variables, which include the annual volatility, expected excess return, Sharpe ratio, and beta of, the low-risk (L) asset, the high-risk asset (H), the market portfolio (MKT), and the betting-against-beta (BAB) factor. The last row contains the shadow cost of leverage constraints for the less risk averse investor (ψ).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Exogenous variables								
Volatility scalar	1	1	1	0.25	0.5	1.5	2	2
Risk aversion 1	1	1	1	1	1	1	1	1
Risk aversion 2	1	10	10	10	10	10	10	10
Wealth share 1	0.5	0.5	0.8	0.8	0.8	0.8	0.8	0.8
m1	1	1	1.2	1.2	1.2	1.2	1.2	1.5
m2	1	1	0	0	0	0	0	0
Endogenous variables								
Vol L	13%	14%	14%	7%	9%	18%	21%	22%
Vol H	30%	33%	33%	15%	22%	42%	51%	55%
Vol MKT	21%	23%	23%	11%	15%	29%	35%	37%
Vol BAB	7%	7%	7%	3%	5%	9%	11%	12%
Excess return L	3%	9%	10%	3%	6%	14%	17%	26%
Excess return H	6%	16%	15%	4%	8%	22%	28%	37%
Excess return MKT	4%	12%	13%	3%	7%	18%	22%	31%
Excess return BAB	0%	4%	6%	2%	3%	8%	9%	16%
SR MKT	0.20	0.55	0.55	0.33	0.44	0.61	0.64	0.84
SR BAB	0.00	0.47	0.79	0.51	0.66	0.81	0.78	1.32
β_L	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
β_H	1.4	1.4	1.4	1.4	1.4	1.4	1.5	1.5
ψ	0%	4%	7%	2%	4%	9%	10%	19%

A second concern is that margin requirements typically increase with volatility. Brokers and central clearing counterparties routinely raise margin requirements on their clients when volatility is high to guard against the elevated likelihood of loss. In column (8), we combine the high volatility scenario in column (7) with higher margin requirements. In this scenario, the BAB Sharpe ratio is even higher than in column (7) since the less risk averse investor becomes even more constrained.

Overall, we conclude that the leverage constraints model cannot explain the variation in BAB returns and Sharpe ratios driven by volatility. Intuitively, it is hard to reconcile the economic rationale of the model with the absence of risk-return tradeoffs for both BAB and the market necessary to generate these factors' volatility-timing benefits. In the model, the constrained investors, who are more risk tolerant and therefore cause the beta anomaly, are not unsophisticated, optimistic, or driven by lottery preferences. Rather, they are rational utility-maximizing investors that choose, in equilibrium, to accept low CAPM alphas in high-beta stocks to benefit from their embedded leverage. As risk averse investors, however, they still require larger returns to hold risky investments when volatility increases.

4.2. Multifactor explanations of the beta anomaly

Novy-Marx and Velikov (2022) show that the FF6 prices BAB due to loadings on profitability, investment, and momentum factors. In this subsection, we examine whether this result continues to hold conditional on the level of BAB volatility and extend this result to other models.

Panel A of Table 3 shows the results from spanning regressions of the returns on BAB on those of the FF6 factors over different sample periods. The first column, "Unc," presents estimates from the regression using the full sample period. These estimates confirm the finding of Novy-Marx and Velikov (2022) that BAB earns insignificant unconditional alpha of -0.18% per month due largely to positive loadings on RMW, CMA, and MOM. The second column, "Low," reports a similar regression following months where realized volatility is below its median value. It shows that the alpha increases significantly and flips signs to a positive 0.51%. In contrast, the third column,"High," reports a similar regression following months where realized variance is above its median value and shows that BAB earns a negative alpha of -0.50% that is marginally significant. The fourth column, "L−H," presents

Table 3: Performance of BAB conditional on volatility

Panel A reports regressions of the returns on BAB (defined in Table 1) on those of the factors from the Fama and French (2018) six-factor model (FF6). The four columns report: a regression over the full sample ("Unc."), one using the subsample of months with below-median lagged volatility ("Low"), one using the subsample period with above-median volatility ("High"), and the corresponding low-minus-high differences ("L−H"). Rows labeled "Alpha" present the intercepts from these regressions. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. Panel B reports the Sharpe ratios (SR) of BAB over the subsamples specified by the column headings. The bottom row of Panel B reports a p-value for the low-minus-high difference in Sharpe ratios based on a GMM estimation with heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	Unc.	Low	High	L−H
Panel A: Fama and French (2018) six-factor model				
Alpha (%)	−0.18	0.51	−0.50	1.01
	(−0.98)	(2.69)	(−1.76)	(2.96)
MKT	0.47	0.39	0.55	−0.16
	(8.85)	(6.80)	(6.87)	(−1.62)
SMB	−0.53	−0.59	−0.52	−0.08
	(−6.86)	(−7.28)	(−4.55)	(−0.55)
HML	0.14	0.16	0.08	0.08
	(1.41)	(1.30)	(0.57)	(0.46)
RMW	0.41	−0.15	0.56	−0.71
	(4.10)	(−0.95)	(4.42)	(−3.49)
CMA	0.64	0.29	0.76	−0.47
	(4.88)	(1.50)	(4.43)	(−1.83)
MOM	0.32	0.10	0.38	−0.27
	(3.95)	(1.63)	(3.92)	(−2.35)
R^2 (%)	34.17	29.35	38.82	
Panel B: Sharpe ratios				
SR	0.35	0.72	0.15	0.57
p-value(SR)	-	-	-	0.043

low-minus-high differences between the second and third columns and shows that the alpha is a full 1.01 percentage points higher in months with low volatility compared to those with high volatility and this difference is significant at the 1% level.

The large increase in abnormal returns of BAB going from high- to low-volatility states reflects the confluence of two patterns. First, Panel B shows that BAB's Sharpe ratio rises nearly five fold (0.15 to 0.72), going from high- to low-volatility states, with the difference significant at the 5% level. Second, Panel A of Table 2 shows that BAB's exposure to every risk factor in the FF6 falls as volatility decreases. As discussed above, Eq. (7) shows that for a risk-based explanation to hold, it must be the case that BAB loads more heavily on risk factors when volatility is low, or for the maximum Sharpe ratio in the economy to increase proportionally with that of BAB. However, since the FF6 factor loadings fall with volatility, for the former possibility to hold, it must be the case that BAB loads very heavily on risk factors that are both missing from the FF6 and economically significant enough to justify Sharpe ratios that exceed 0.74. Even if this were the case, and the correlation between BAB and the stochastic discount factor remained the same, the maximum Sharpe ratio in the economy would also have to double from its unconditional level to that in the half of months with below-median BAB volatility given corresponding increases in BAB Sharpe ratio of 0.35 ("Unc.") to 0.72 ("Low"). But, if the unconditional market Sharpe ratio of 0.5 is deemed a "puzzle," as in Mehra and Prescott (1985), its more than doubling must be even more so.[6] Overall, the evidence in Panels A and B supports explanations of the low-risk anomaly that do not require a risk-factor structure in returns because BAB returns seem highest precisely when they require bearing the least observable risk. In interpreting these results, it is also worth noting that "risk-based" explanations need not be rational. Kozak, Nagel, and Santosh (2018) show that, in the absence of near arbitrage, asset returns can conform to a factor model even if the fundamental source of return predictability is entirely driven by sentiment. Indeed, Stambaugh et al. (2015) and Stambaugh and Yuan (2017) even classify several characteristics (e.g., momentum, profitability, investment) as mispricing that are the basis for three risk factors in the FF6. However, even if the premiums on MOM, RMW, and CMA are entirely driven by sentiment, it is still the case that these factors do not explain the conditional returns on BAB since their correlations with BAB is low precisely when the Sharpe ratio of BAB is highest.

Table 3 clearly shows that the momentum, profitability, and investment factors are critical to the ability of FF6 to price BAB unconditionally. Moreover, the dwindling of BAB's

[6]Sharpe ratios of approximately twice the market are often considered "good deals" that are unlikely to survive arbitrage. See, e.g., Ross (1976), Shanken (1992), and Cochrane and Saá-Requejo (2000). MacKinlay (1995) argues that a Sharpe ratio of 0.6 borders on implausible for anomaly factors.

exposure to these factors as volatility falls also contributes to BAB's abnormal returns in low-volatility states. However, these factors are empirically motivated, lacking a clear link to economic fundamentals. We next investigate whether BAB's volatility predicts its abnormal returns for several models that either exclude these factors or have a more straightforward link to macroeconomic conditions. We do so based on the conditional CAPM of Shanken (1990), who proposes allowing abnormal returns and factor betas to vary with lagged instruments such as the market dividend-price ratio. Specifically, we estimate regressions of the form:

$$BAB_t = \alpha_0 + \boldsymbol{\alpha}_1' \boldsymbol{z}_{t-1} + \sum_{k=1}^{K} (\gamma_{k,0} + \boldsymbol{\gamma}_{k,1}' \boldsymbol{z}_{t-1}) f_{kt} + \nu_t, \tag{13}$$

where $\boldsymbol{z}_{t-1}$ is the vector of lagged instruments and f_{kt} is the return on factor $k = 1, ..., K$. Table 4 presents estimates of the α parameters that characterize the abnormal returns in Eq. (13) and Table IA.1 in the Internet Appendix presents estimated γ parameters that characterize the slopes. Following Cederburg and O'Doherty (2016), $\boldsymbol{z}_{t-1}$ includes realized BAB volatility in addition to the default spread, the market dividend-price ratio, and the ex ante beta of BAB ($\beta_{L,t-1} - \beta_{H,t-1}$ in Eq. (4)). The instruments are demeaned and standardized such that α_0 is the average abnormal return from the model and elements of $\boldsymbol{\alpha}_1$ represent the impact of a one-standard deviation change in the instrument on the conditional abnormal return. Each pair of columns corresponds to a model, with odd-numbered ones presenting estimates from the "unconditional" version of the model (all $\boldsymbol{\alpha}_1$ and $\boldsymbol{\gamma}_1$ restricted to be zero), and even-numbered columns presenting estimates from the unrestricted model given by Eq. (13).[7]

The first six columns of Table 4 present results for the CAPM, Fama and French (1993)

[7] Cederburg and O'Doherty (2016) show that a conditional CAPM with similar instruments prices the beta anomaly, although Liu et al. (2018) show this finding is not robust to the technique used to estimate beta. Asness et al. (2020) also argue that the tests from Cederburg and O'Doherty (2016) suffer from low power to reject the null that the conditional CAPM holds and are ruled out by the simple fact that the BAB factors have a *constant* beta of zero by construction so that time-varying betas cannot explain their returns. Some conditional CAPM studies restrict the $\boldsymbol{\alpha}_1 = 0$. We do not do this because time-variation in abnormal returns associated with volatility is central to our study, but this has no impact on α_0 since the elements of $\boldsymbol{z}_{t-1}$ are demeaned.

Table 4: Performance of BAB relative to conditional factor models

Even numbered columns of this table report α_0 and $\boldsymbol{\alpha}_1$ estimates from regressions of the form:

$$BAB_t = \alpha_0 + \boldsymbol{\alpha}_1' \boldsymbol{z}_{t-1} + \sum_{k=1}^{K} (\gamma_{k,0} + \boldsymbol{\gamma}_{k,1}' \boldsymbol{z}_{t-1}) f_{kt} + \nu_t,$$

where f_k is the return on factor $k = 1, .., K$. Odd numbered columns report estimates from restricted versions of these regressions in which $\boldsymbol{\alpha}_1 = \mathbf{0}$ and $\boldsymbol{\gamma}_{k,1} = \mathbf{0}$, $k = 1, 2, ..., K$. The vector of instruments, $\boldsymbol{z}_{t-1}$, includes: the log realized volatility on BAB (VOL), the difference between BAA and AAA bond yields (DEF), the log dividend-price ratio on the CRSP value-weighted index (DP), and the difference in market betas between the long and short legs of BAB ($Beta$) before hedging the market exposure. The instruments are demeaned and standardized so that the $\boldsymbol{\alpha}_1$ coefficients are the impact, in percentage points, of a one standard deviation shock to the instrument. In Columns (1) and (2), the only factor is MKT; in Columns (3) and (4), the factors are MKT, SMB, and HML; in Columns (5) and (6), the factors are all six in the FF6 model; in Columns (7) and (8), the only factor is the mimicking portfolio for growth in the unfiltered consumption series of Kroencke (2017); and in Columns (9) and (10), the only factor is the mimicking portfolio for intermediary leverage of Adrian et al. (2014) (see Section 4.2 for details). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	CAPM		FF3		FF6		Consumption		Leverage	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
α_0	0.42	0.39	0.38	0.44	−0.18	0.31	0.42	0.40	−0.01	0.19
	(2.17)	(2.14)	(2.20)	(2.82)	(−0.98)	(1.92)	(2.25)	(2.08)	(−0.05)	(1.04)
α_1^{VOL}		−0.70		−0.70		−0.66		−0.70		−0.69
		(−2.59)		(−3.24)		(−3.29)		(−2.19)		(−2.17)
α_1^{DEF}		0.07		0.38		0.55		0.26		0.34
		(0.20)		(1.49)		(2.64)		(0.66)		(1.10)
α_1^{DP}		−0.38		−0.35		−0.32		−0.44		−0.48
		(−1.33)		(−1.41)		(−1.16)		(−1.23)		(−1.26)
α_1^{Beta}		−0.11		−0.12		0.13		−0.02		−0.08
		(−0.63)		(−0.70)		(0.68)		(−0.08)		(−0.35)

three-factor model (FF3), and FF6. Consistent with Liu et al. (2018) and Table 3, Columns (1), (3), and (5) of Table 4 show that, unlike the FF6, the CAPM and FF3 fail to price BAB unconditionally, earning average alphas of 0.4% that are significant at the 5% level. However, columns (2), (4), and (6) show that a one-standard deviation decrease in BAB volatility predicts an increase in abnormal returns of 0.7 percentage points, regardless of model.

The last four columns of Table 4 present estimates from models with factors based on macroeconomic conditions. Columns (7) and (8) use a single-factor model similar to the consumption CAPM of Breeden, Gibbons, and Litzenberger (1989), but based on a mimicking portfolio for the "unfiltered" annual consumption growth series of Kroencke (2017).

This portfolio is formed by projecting the consumption growth series onto the annual excess returns of the base assets used in the FF6 model and then sampling the resulting portfolio at the monthly frequency (see the Internet Appendix for details). Columns (9) and (10) use a single-factor model based on the mimicking portfolio of intermediary leverage from Adrian et al. (2014) based on Federal Reserve Flow of Funds data. They form this factor by projecting the quarterly time series of intermediary leverage growth onto the returns of Fama and French (1993) and Carhart (1997) size, value, and momentum portfolios.

Columns (7) and (8) show that the consumption CAPM fails to price BAB, earning average alpha of 0.4% that is significant at the 5% level. In contrast, consistent with Adrian et al. (2014) and the leverage constraints theory of the beta anomaly, column (9) shows that the intermediary leverage factor prices BAB unconditionally, reducing the abnormal return to −0.01% per month ($t = -0.05$). However, columns (8) and (10) show that BAB volatility negatively predicts subsequent BAB abnormal returns relative to both the consumption and leverage models with essentially the same economic magnitude as the empirically motivated Fama and French (2018) models.

In summary, Tables 3 and 4 show that, while the FF6 and an intermediary leverage factor can price BAB unconditionally, the returns on BAB become more anomalous when its volatility falls, regardless of model considered. Moreover, the economic significance is similar across models with a one-standard deviation *decrease* in volatility predicting a 0.7 percentage point *increase* in monthly abnormal returns with this point estimate significant at the 1% or 5% level, depending on specification.

4.3. Limits to arbitrage

Shleifer and Vishny (1997) and Pontiff (2006) argue that bearing IVOL risk is an important holding cost of arbitrage, and, consistent with this argument, many studies show that anomaly returns increase in the cross section with high levels of IVOL (see, e.g., Ali, Hwang, and Trombley, 2003; Mashruwala, Rajgopal, and Shevlin, 2006; Doukas, Kim, and Pantzalis, 2010; Li and Zhang, 2010; and McLean, 2010). Stambaugh et al. (2012) observe that a disproportionate amount of capital exploits underpricing relative to overpricing and show, presumably as a result of this "arbitrage asymmetry," that overpricing is generally more prevalent than underpricing, and even more so in times of optimistic sentiment. Expanding on this work, Stambaugh et al. (2015) posit that arbitrage risk and asymmetry combine to explain the IVOL puzzle. IVOL deters arbitrage in both underpriced and overpriced stocks, but the effect in overpriced stocks is larger due to the asymmetry. Hence, the effect in overpriced stocks dominates when sorting stocks based on IVOL alone. Consistent with this argument, they show that IVOL is positively related to FF3 alpha in underpriced stocks, but the average relation across all stocks is negative. Liu et al. (2018) extend these results and show that the unconditional beta anomaly results from the strong positive cross-sectional correlation between beta and IVOL. Specifically, they show that BAB abnormal returns exist only in overpriced stocks and disappear controlling for IVOL or when dropping "overpriced" high-IVOL stocks from the construction of BAB. In this subsection, we investigate whether the conditional performance of the beta anomaly is consistent with the anomaly's limits-to-arbitrage explanation.

4.3.1. Idiosyncratic volatility and beta

The first implication of the limits-to-arbitrage theory we test is that the beta anomaly is an artifact of the IVOL effect, and thus, BAB factors that control for IVOL by buying and selling stocks with similar levels of IVOL (but different levels of beta) should have zero abnormal returns. The same result should also obtain for BAB factors formed after dropping overpriced high-IVOL stocks. To form a factor that controls for IVOL, we use the standard double-sorting approach following Ang, Hodrick, Xing, and Zhang (2006), Stambaugh et al. (2015), Liu et al. (2018), and Asness et al. (2020). Each month, we sort stocks into quintiles based on their IVOL, and then, within each quintile, we sort stocks into value-weighted decile portfolios based on their beta. We form BAB factors within each IVOL quintile that

Table 5: Abnormal returns of portfolios based on idiosyncratic volatility and beta

This table presents FF6 alphas of 50 value-weighted portfolios formed at the beginning of each month by sorting stocks into quintiles based on IVOL, and then within each quintile, into decile portfolios based on beta. In each panel, the last column shows low-minus-high differences in alphas between the low- and high-beta portfolios; and the last row shows high-minus-low differences in alphas between the high- and low-IVOL portfolios. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is from July 1965 to December 2016.

IVOL	Beta decile										Low−
quintile	Low	2	3	4	5	6	7	8	9	High	High
Low	−0.08	−0.03	−0.16	0.08	−0.10	0.02	−0.27	−0.24	−0.18	0.04	−0.13
	(−0.75)	(−0.35)	(−1.57)	(0.84)	(−1.02)	(0.21)	(−2.83)	(−2.55)	(−1.66)	(0.33)	(−0.71)
2	−0.04	−0.12	−0.08	−0.13	−0.08	0.01	−0.12	−0.13	0.10	0.08	−0.12
	(−0.30)	(−1.03)	(−0.73)	(−1.14)	(−0.78)	(0.08)	(−1.13)	(−1.20)	(0.72)	(0.68)	(−0.65)
3	−0.17	−0.24	0.08	−0.24	−0.04	0.44	0.08	0.10	0.04	0.23	−0.41
	(−1.13)	(−1.63)	(0.56)	(−1.55)	(−0.22)	(3.17)	(0.62)	(0.62)	(0.21)	(1.54)	(−1.77)
4	0.01	−0.05	−0.09	0.10	0.20	−0.02	0.32	−0.01	0.03	0.12	−0.11
	(0.05)	(−0.29)	(−0.56)	(0.59)	(1.14)	(−0.12)	(1.60)	(−0.06)	(0.19)	(0.66)	(−0.43)
High	−0.87	−0.24	−0.49	−0.63	−0.11	−0.23	−0.69	−0.72	0.01	−0.36	−0.51
	(−3.85)	(−0.97)	(−1.98)	(−2.62)	(−0.33)	(−0.97)	(−2.66)	(−2.65)	(0.03)	(−1.41)	(−1.47)
High−	−0.78	−0.20	−0.33	−0.71	−0.01	−0.25	−0.42	−0.48	0.19	−0.40	−0.38
Low	(−3.11)	(−0.74)	(−1.26)	(−2.73)	(−0.02)	(−0.93)	(−1.42)	(−1.62)	(0.55)	(−1.50)	(−1.06)

go long the low-beta portfolio, short the high-beta portfolio as in Eq. (4). The BAB factor that controls for IVOL is given by the simple average of the corresponding factors across the five IVOL quintiles.

Table 5 reports the unconditional FF6 alphas of the 50 IVOL-beta portfolios before hedging market risk. The results are partially consistent with Liu et al. (2018) who report FF3 alphas. The bottom row shows that, in at least two of beta groups, the high-minus-low-IVOL quintile spread in abnormal returns is significant at the 5% level, and nine out of ten spreads have the "right" negative sign. However, the last column shows that none of the low-minus-high-beta decile spreads in abnormal returns are significant at the 5% level and none have the "right" positive sign. Overall, this evidence is consistent with the beta anomaly arising unconditionally from the IVOL anomaly.

Next, we investigate whether the IVOL anomaly subsumes the beta anomaly, conditional on the state of realized volatility. The first four columns of Panel A of Table 6 presents FF6 alphas, similar to those in Table 3, for the BAB factor that controls for IVOL. Consistent with Table 5, the columns labeled "Unc." show that over the full sample, BAB factor earns

Table 6: Conditional performance of BAB controlling for idiosyncratic volatility

Using the 50 IVOL-beta portfolios from Table 5, we make a BAB factor within each IVOL quintile that goes long the quintile's low-beta portfolio, short the high-beta portfolio, and hedges market risk as in Eq. (4). The BAB factor that controls for IVOL is defined as the simple average of the five factors across IVOL quintiles. The first four columns of Panel A present spanning regressions of the returns on this factor on those of the FF6 factors. The four columns report: an unconditional regression over the full sample ("Unc."), one using the subsample of months with below-median lagged volatility ("Low"), one using the subsample period with above-median volatility ("High"), and the corresponding low-minus-high differences ("L−H"). The last four columns of Panel A present similar estimates as the first four columns for a BAB factor constructed similarly as the baseline factor in Table 1, but after excluding stocks that are in the top quintiles of both the Stambaugh et al. (2012) mispricing score and IVOL. Panel B reports the Sharpe ratios (SR) of the BAB factors over the subsamples specified by the column headings. The bottom row of Panel B reports a p-value for the low-minus-high differences in Sharpe ratios based on GMM estimation with heteroskedasticity-robust standard errors. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	Controlling for IVOL				No high-IVOL overpriced stocks			
	Unc.	Low	High	L−H	Unc.	Low	High	L−H
Panel A: Fama and French (2018) six-factor model								
Alpha (%)	−0.25	0.34	−0.40	0.74	−0.20	0.53	−0.55	1.08
	(−1.41)	(1.79)	(−1.49)	(2.25)	(−1.10)	(2.76)	(−1.94)	(3.16)
MKT	0.49	0.37	0.60	−0.23	0.47	0.39	0.55	−0.16
	(8.93)	(6.05)	(7.84)	(−2.34)	(8.95)	(6.75)	(6.97)	(−1.68)
SMB	−0.45	−0.49	−0.45	−0.05	−0.52	−0.58	−0.51	−0.07
	(−5.62)	(−6.57)	(−3.88)	(−0.32)	(−6.81)	(−7.15)	(−4.55)	(−0.51)
HML	0.21	0.19	0.15	0.03	0.16	0.13	0.11	0.02
	(1.78)	(1.38)	(1.02)	(0.16)	(1.57)	(1.09)	(0.83)	(0.12)
RMW	0.44	−0.13	0.62	−0.75	0.38	−0.15	0.51	−0.66
	(3.11)	(−0.77)	(3.69)	(−3.16)	(3.83)	(−0.96)	(4.04)	(−3.27)
CMA	0.51	0.27	0.56	−0.29	0.63	0.30	0.74	−0.44
	(3.50)	(1.48)	(2.84)	(−1.09)	(4.76)	(1.53)	(4.33)	(−1.72)
MOM	0.32	0.11	0.39	−0.29	0.31	0.10	0.37	−0.27
	(4.29)	(1.80)	(4.52)	(−2.75)	(3.93)	(1.56)	(3.91)	(−2.34)
R^2	32.80	25.67	39.40		33.00	28.36	37.22	
Panel B: Sharpe ratios								
SR	0.34	0.64	0.17	0.47	0.33	0.73	0.10	0.63
p-value(SR)	-	-	-	0.098	-	-	-	0.026

insignificant alpha. However, when volatility is low, these alphas become positive, and negative when volatility is high, with a spread of 0.74 percentage points between the two states that is significant at the 5% level. Panel B shows that the corresponding Sharpe ratio of BAB controlling for IVOL nearly quadruples going from high- to low-volatility states (0.17 to 0.64). The right four columns of Table 6 present results for BAB factors that drop what

Liu et al. (2018) call overpriced high-IVOL stocks, that is, those that are in the top quintiles of both IVOL and mispricing. These columns of Panel A and B show results that are at least as strong as those in Table 3 that ignore mispricing and IVOL. The spread in alphas between low- and high-volatility states is 1.08 percentage points per month and significant at the 1% level. Similarly, the Sharpe ratios increase sevenfold from high- to low-volatility states (from 0.10 to 0.73). Overall, the results in Table 6 show that the limits to arbitrage captured by IVOL do not explain the strength of the beta anomaly conditional on low volatility.

4.3.2. Beta, idiosyncratic volatility, and mispricing

To provide further evidence on the arbitrage explanation of the conditional beta anomaly, we next examine the performance of a BAB factor that controls for mispricing. We use the same double-sorting approach to construct mispricing-beta portfolios that we use for the IVOL-beta portfolios in Table 5. Specifically, each month, we sort stocks into quintiles based on the Stambaugh et al. (2012, 2015) mispricing measure, and then, within each quintile, we sort stocks into value-weighted decile portfolios based on their beta. We form a BAB factor within each mispricing quintile that goes long the low-beta portfolio, short the high-beta portfolio, and hedge market risk as in Eq. (4). The BAB factor that controls for the cross-sectional effects of mispricing is defined as the simple average of the BAB factors across the five mispricing quintiles.

Table 7 presents FF6 alphas for the 50 mispricing-beta portfolios before beta neutralization along with low-minus-high differences between the extreme portfolios along both dimensions. The last row show that seven out of ten high-minus-low mispricing alphas are significantly negative at the 10% level. However, the last column shows that none of the low-minus-high-beta spreads are significantly positive. Thus Table 7 shows, in terms of unconditional FF6 alphas, a clear mispricing effect, but no beta anomaly.

Table 8 reports performance statistics for the BAB factor that controls for mispricing conditional on volatility. This table shows similar results as Tables 3 and 6. The BAB that

Table 7: Abnormal returns of portfolios sorted on mispricing and beta

This table presents FF6 alphas of 50 value-weighted portfolios formed at the beginning of each month by sorting stocks into quintiles based on the mispricing measure of Stambaugh et al. (2015), and then within each quintile, into decile portfolios based on beta, as specified by the panel heading. The last column shows low-minus-high differences in alphas between the low- and high-beta portfolios; and the last row shows high-minus-low differences in alphas between the high- and low-mispricing portfolios. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is from July 1965 to December 2016.

Mispricing	Beta decile										Low−
quintile	Low	2	3	4	5	6	7	8	9	High	High
Low	−0.06	−0.11	−0.08	0.12	−0.15	0.17	0.14	0.19	0.17	0.39	−0.45
	(−0.54)	(−1.03)	(−0.76)	(0.98)	(−1.19)	(1.38)	(1.10)	(1.39)	(1.05)	(2.74)	(−2.27)
2	−0.08	−0.17	−0.16	−0.01	0.03	−0.05	0.12	−0.19	−0.10	0.15	−0.22
	(−0.53)	(−1.26)	(−1.37)	(−0.08)	(0.26)	(−0.43)	(0.85)	(−1.57)	(−0.78)	(0.92)	(−1.05)
3	−0.16	−0.28	−0.22	−0.14	−0.21	−0.01	0.06	−0.16	0.35	0.32	−0.48
	(−1.02)	(−2.19)	(−1.75)	(−1.11)	(−1.70)	(−0.10)	(0.42)	(−1.08)	(2.45)	(2.14)	(−2.09)
4	−0.04	−0.22	−0.30	−0.23	−0.52	−0.43	−0.29	−0.31	−0.05	0.08	−0.12
	(−0.25)	(−1.47)	(−2.70)	(−1.77)	(−3.96)	(−3.09)	(−1.88)	(−2.24)	(−0.34)	(0.47)	(−0.48)
High	−0.35	−0.35	−0.19	−0.39	−0.41	−0.44	−0.28	−0.35	−0.45	−0.31	−0.04
	(−2.11)	(−2.08)	(−1.34)	(−2.46)	(−2.29)	(−3.01)	(−1.72)	(−2.02)	(−2.85)	(−1.59)	(−0.13)
High−	−0.29	−0.24	−0.11	−0.51	−0.27	−0.61	−0.42	−0.54	−0.62	−0.70	0.41
Low	(−1.91)	(−1.37)	(−0.63)	(−2.45)	(−1.20)	(−3.43)	(−1.95)	(−2.44)	(−2.96)	(−3.05)	(1.47)

controls for the mispricing measure is priced unconditionally by the FF6, but its alpha and Sharpe ratio increase significantly going into low-volatility states, indicating that mispricing does not subsume the conditional version of the beta anomaly. Overall, the evidence in Tables 5 through 8 is not consistent with the arbitrage asymmetry explanation of the beta anomaly.

4.4. Lottery preferences

The lottery preferences theory of the beta anomaly posits that some investors demand high-risk stocks, even if they have poor average performance, because they offer a possibility of very high returns. Consistent with this explanation, several studies find evidence that lottery demand drives down expected returns of risky stocks and plays at least a partial role explaining both the beta and IVOL anomalies (e.g., Bali, Cakici, and Whitelaw, 2011; Conrad, Kapadia, and Xing, 2014; and Bali et al., 2017). Bali et al. (2011) and Bali et al. (2017) measure lottery demand with a variable called MAX, which is defined for each stock and month as the average of the stock's highest five daily returns over that month. They

Table 8: Conditional performance of BAB controlling for mispricing

Using the 50 mispricing-beta portfolios from Table 7, we form a BAB factor within each mispricing quintile that goes long the quintile's low-beta portfolio, short the high-beta portfolio, and hedges market risk as in Eq. (4). The BAB factor that controls for mispricing is defined as the simple average of the five factors across the corresponding quintiles. Panel A reports spanning regressions of the returns on this BAB factor on those of the factors from the FF6. The four columns report: an unconditional regression over the full sample ("Unc."), one using the subsample of months with below-median lagged volatility ("Low"), one using the subsample period with above-median volatility ("High"), and the corresponding low-minus-high differences ("L−H"). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. Panel B reports the Sharpe ratios (SR) of the BAB factor over the subsamples specified by the column headings. The bottom row of Panel B reports a p-value for the low-minus-high difference in Sharpe ratios based on GMM estimation with heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	BAB			
	Unc.	Low	High	L−H
Panel A: Fama and French (2018) six-factor model				
Alpha (%)	−0.28	0.44	−0.59	1.03
	(−1.50)	(2.33)	(−2.04)	(2.98)
MKT	0.46	0.39	0.53	−0.14
	(8.43)	(7.01)	(6.37)	(−1.38)
SMB	−0.49	−0.58	−0.47	−0.11
	(−6.20)	(−7.13)	(−3.98)	(−0.75)
HML	0.23	0.19	0.22	−0.03
	(2.17)	(1.61)	(1.47)	(−0.15)
RMW	0.37	−0.11	0.45	−0.56
	(3.71)	(−0.68)	(3.65)	(−2.81)
CMA	0.55	0.15	0.68	−0.53
	(4.06)	(0.82)	(3.74)	(−2.07)
MOM	0.24	0.05	0.30	−0.25
	(3.39)	(0.78)	(3.51)	(−2.46)
R^2	30.55	27.09	33.92	
Panel B: Sharpe ratios				
SR	0.25	0.63	0.03	0.60
p-value(SR)	-	-	-	0.032

find that MAX subsumes the low-risk anomaly. Asness et al. (2020) argue that stocks can have a high MAX either because their returns are volatile or because they are right-skewed. They measure lottery demand with a variable called SMAX, which scales a given stock's MAX by the stock's standard deviation of daily returns in the month MAX is measured. As a result, SMAX captures skewness on a stock while neutralizing the mechanical effect of volatility.

For the conditional performance of BAB to be consistent with the lottery theory, lottery demand must be lower in high-volatility periods when risky stocks appear less "overvalued" relative to low-risk stocks. But lower demand of lotteries in high-vol states is not consistent with extant facts on lottery preferences. Kumar (2009), and cites therein, all find that lottery demand for a given investor increases during economic downturns when volatility is relatively high. Kumar further argues: "When volatility is high, investors might believe that the extreme return events observed in the past are more likely to be realized again." However, an opposite effect can be at work as high-volatility periods are also "bad times" when investors commonly classified as "sentiment traders" tend to exit the market and these traders are hard to distinguish empirically from traders characterized by lottery preferences (e.g., Antoniou, Doukas, and Subrahmanyam, 2013; Antoniou et al., 2016). Thus, increases in volatility can increase demand of risky stocks by lottery-motivated traders that stay in the market, while simultaneously decreasing the number of such traders in the market, rendering the net effect an empirical question.

We test whether lottery demand can explain the conditional performance of BAB using similar methods as in Tables 6 and 8. Specifically, we sort stocks each month into lottery-demand quintiles based on SMAX, and then, within each quintile, sort stocks into value-weighted beta portfolios. Within each SMAX quintile, we form a BAB factor that goes long the low-risk portfolio and shorts the high-risk portfolio while hedging market risk as in Eq. (4). Averaging the BAB factors across SMAX quintiles generates a factor based on variation in risk that controls for the cross-sectional effects of lottery-demand.

Table 9 reports the FF6 alphas of the 50 SMAX-beta portfolios before market hedging and high-minus-low differences between the extreme portfolios along both dimensions. The last row shows that 6 out of 10 high-minus-low-SMAX decile alphas are significantly negative at the 10% level. In contrast, the last column shows that none of the low-minus-high-beta spreads in alphas are significantly positive. Thus, the FF6 fails to price the lottery anomaly unconditionally, but the beta anomaly does not hold relative to the FF6 in any lottery group.

Table 10 reports performance statistics for the BAB factors that control for SMAX condi-

Table 9: Abnormal returns of portfolios sorted on lottery demand and beta

This table presents FF6 alphas of 50 value-weighted portfolios formed at the beginning of each month by sorting stocks into quintiles based on the lottery demand, and then within each quintile, into decile portfolios based on beta. In each panel, the last column shows low-minus-high differences in alphas between the low- and high-beta portfolios; and the last row shows high-minus-low differences in alphas between the high- and low-lottery portfolios. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is from July 1965 to December 2016.

SMAX	Beta decile										Low−
quintile	Low	2	3	4	5	6	7	8	9	High	High
Low	0.25	0.17	0.21	0.19	0.13	0.34	0.31	0.20	0.60	0.62	−0.36
	(1.83)	(1.26)	(1.66)	(1.42)	(0.86)	(2.29)	(1.85)	(1.06)	(3.81)	(3.29)	(−1.60)
2	−0.07	−0.26	0.15	0.00	0.01	−0.02	0.14	0.13	0.09	0.46	−0.52
	(−0.49)	(−1.90)	(1.36)	(0.00)	(0.07)	(−0.14)	(1.00)	(0.83)	(0.61)	(2.51)	(−2.14)
3	−0.04	−0.01	0.05	−0.15	−0.31	−0.17	0.09	−0.02	0.09	0.11	−0.15
	(−0.30)	(−0.08)	(0.42)	(−1.25)	(−2.47)	(−1.27)	(0.71)	(−0.09)	(0.54)	(0.71)	(−0.68)
4	−0.22	−0.31	−0.19	−0.26	−0.18	−0.12	−0.07	0.04	0.05	0.10	−0.32
	(−1.68)	(−2.78)	(−1.95)	(−2.16)	(−1.50)	(−1.00)	(−0.53)	(0.32)	(0.37)	(0.63)	(−1.50)
High	−0.33	−0.13	−0.16	−0.17	−0.12	−0.07	−0.32	−0.11	−0.13	−0.24	−0.09
	(−2.12)	(−0.82)	(−1.19)	(−1.24)	(−0.84)	(−0.40)	(−1.77)	(−0.67)	(−0.56)	(−1.23)	(−0.36)
High−	−0.58	−0.30	−0.37	−0.37	−0.25	−0.41	−0.63	−0.30	−0.73	−0.86	0.28
Low	(−3.51)	(−1.60)	(−1.95)	(−1.70)	(−1.16)	(−1.60)	(−2.54)	(−1.14)	(−2.50)	(−3.25)	(0.95)

tional on volatility. Column "Unc." of Panel A shows that the FF6 yields a negative alpha on BAB that is marginally significant, while the alpha rises to a positive and significant 0.43% when volatility is low, and falls to a negative and significant −0.68%, a spread between high- and low-volatility states of 1.12 percentage points ($t = 3.46$).[8] Similarly, the Sharpe ratio of BAB rises significantly from high- to low-volatility months by 0.65. Overall, Table 10 shows that lottery demand can not explain BAB returns in different volatility states.

4.5. Sentiment, analyst disagreement, and the beta anomaly

The results above consider explanations of the beta anomaly based on beta's cross-sectional correlation with stock-level characteristics such as mispricing or IVOL. Next, we consider explanations whose evidence is based on time-series predictability.

Antoniou et al. (2016) and Liu et al. (2018) show that the beta anomaly is nearly absent in pessimistic periods, consistent with mispricing of beta being caused by unsophisticated investors drawn to high-beta stocks when they feel optimistic. Barroso and Detzel (2021)

[8]This spread is 0.96 percentage points ($t = 2.83$) using MAX in lieu of SMAX.

Table 10: Conditional performance of BAB controlling for lottery demand

Using the 50 lottery demand-beta portfolios from Table 9, we make a BAB factor within each mispricing quintile that go long the quintile's low-beta portfolio, short the high-beta portfolio, and hedges market risk as in Eq. (4). The BAB factor that controls for lottery demand is defined as the simple average of the five factors across quintiles. Panel A reports spanning regressions of the returns on this BAB factor on those of the FF6. The four columns report: an unconditional regression over the full sample ("Unc."), one using the subsample of months with below-median lagged volatility ("Low"), one using the subsample period with above-median volatility ("High"), and the corresponding low-minus-high differences ("L−H"). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. Panel B reports the Sharpe ratios (SR) of the BAB factors over the subsamples specified by the column headings. The bottom row of Panel B reports a p-value for the low-minus-high difference in Sharpe ratios based on GMM estimation with heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	Unc.	Low	High	L−H
Panel A: Fama and French (2018) six-factor model				
Alpha (%)	−0.30	0.43	−0.68	1.12
	(−1.66)	(2.33)	(−2.58)	(3.46)
MKT	0.47	0.37	0.55	−0.18
	(8.79)	(6.66)	(6.87)	(−1.84)
SMB	−0.57	−0.64	−0.53	−0.11
	(−7.69)	(−7.78)	(−5.01)	(−0.84)
HML	0.12	−0.02	0.12	−0.14
	(1.12)	(−0.17)	(0.84)	(−0.72)
RMW	0.35	−0.09	0.50	−0.59
	(3.93)	(−0.55)	(4.37)	(−2.89)
CMA	0.60	0.52	0.62	−0.10
	(4.35)	(2.93)	(3.47)	(−0.40)
MOM	0.30	0.10	0.37	−0.26
	(4.96)	(1.55)	(5.28)	(−2.74)
R^2	35.87	30.29	41.09	
Panel B: Sharpe ratios				
SR	0.24	0.63	−0.02	0.65
p-value(SR)	-	-	-	0.020

document that volatility timing the market portfolio is only profitable when sentiment is high, and significantly diminishes performance during pessimistic periods, reflecting the results of Yu and Yuan (2011) that the time-series risk-return tradeoff of the market is negative in optimistic periods, but positive otherwise. Motivated by these findings, we investigate whether sentiment explains the abnormal returns on BAB when volatility is low. To do so, following Liu et al. (2018), Panel A of Table 11 presents FF6 alphas of BAB in months divided into four regimes according to whether prior-month Baker and Wurgler (2007) sentiment and BAB realized volatility are "high" or "low" relative to their median value. This panel shows that BAB earns alphas that are 0.68 to 1.32 percentage points higher in low-volatility states than in high-volatility states depending on sentiment. The average alpha spread between low- and high-volatility states is 1.00 percentage point across the two sentiment regimes and is significant at the 1% level. If anything, the negative relationship between volatility of BAB and its subsequent alpha is relatively strong when sentiment is low, with a high-minus-low-sentiment difference in the low-minus-high-volatility alpha spreads of -0.64 percentage points. However, this estimated difference is statistically insignificant ($t = -0.92$).

Hong and Sraer (2016) argue that, compared to low-beta stocks, high-beta stocks are more sensitive to aggregate disagreement about the growth prospects of firms, experience greater divergence of opinion about their payoffs, and are thus more overpriced due to short-sales constraints. They propose a measure of aggregate disagreement, which is the beta-weighted average of the cross-sectional standard deviation of long-term earnings growth forecasts from IBES. Table 11 investigates whether disagreement subsumes the volatility-timing benefits of BAB by estimating alphas over four regimes similar to Panel A, but using disagreement in lieu of volatility. Consistent with Hong and Sraer (2016), the average low-minus-high-volatility spread in BAB alpha is 0.58 percentage points higher in high-disagreement regimes than in low-disagreement regimes, although this spread is insignificant. The spread between low- and high-volatility regimes ranges from 0.84 to 1.21 percentage points across disagreement regimes, averaging a significant 1.02 percentage points ($t = 2.48$). In particular, disagreement does not subsume the effect of volatility in predicting abnormal returns on BAB.

4.6. Market volatility vs. BAB volatility

Realized variance of returns is highly correlated across factors (Moreira and Muir, 2017; Eisdorfer and Misirli, 2020; and DeMiguel, Martín-Utrera, and Uppal (2022)). Table 12 investigates whether the conditional performance of BAB documented in this section is driven by its own volatility or that of the market. Similar to the sentiment results from the previous table, market volatility clearly does not subsume BAB volatility. The average

Table 11: Abnormal returns of BAB in periods of high and low volatility, sentiment, and analyst disagreement

We assign each month to one of four regimes ($j = 1, ..., 4$) according to whether prior-month investor sentiment and realized volatility of BAB (defined in Table 1) are above or below their median values. Panel A reports alphas estimated in each of these regimes from a regression of the form:

$$BAB_t = \sum_{j=1}^{4} D_{j,t} \left(\alpha_j + \beta_j MKT_t + s_j SMB_t + h_j HML_t + m_j MOM_t + r_j RMW_t + c_j CMA_t\right) + \nu_t, \quad (14)$$

where $D_{j,t}$ equals one if month t is in regime j, and zero otherwise. Panel B reports similar estimates as Panel A, but using the time series of beta-weighted analyst disagreement about long-term earnings growth forecasts in lieu of sentiment. Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

Panel A: Sentiment

	BAB volatility			
Sentiment	Low	High	Low−High	Average
Low	0.44	−0.87	1.32	−0.21
	(1.58)	(−1.93)	(2.48)	(−0.81)
High	0.56	−0.12	0.68	0.22
	(1.98)	(−0.36)	(1.53)	(0.97)
High−Low	0.11	0.75	−0.64	0.43
	(0.28)	(1.32)	(−0.92)	(1.24)
Average	0.50	−0.50	1.00	
	(2.51)	(−1.76)	(2.88)	

Panel B: Disagreement

	BAB volatility			
Disagreement	Low	High	Low−High	Average
Low	0.21	−0.64	0.84	−0.22
	(0.65)	(−1.31)	(1.45)	(−0.74)
High	0.97	−0.23	1.21	0.37
	(2.28)	(−0.57)	(2.04)	(1.25)
High−Low	0.76	0.40	0.36	0.58
	(1.44)	(0.64)	(0.44)	(1.41)
Average	0.59	−0.44	1.02	
	(2.22)	(−1.37)	(2.48)	

alpha spread between low- and high-BAB volatility states is 0.81 percentage points and significant, while the average spread between high- and low-market volatility states is an insignificant 0.21 percentage points. Thus, it appears to be the specific risk of BAB that drives its performance in low-volatility states, not the risk of the market.

Table 12: Abnormal returns of BAB in periods of high- and low- own-factor and market volatility

This table assigns months to one of four regimes based according to whether prior-month realized market volatility and realized BAB volatility are above or below their median value and presents FF6 alphas of BAB (defined in Table 1) in each of the four regimes. Regression slopes and intercepts are estimated separately in each regime as in Eq. (14). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

Market	BAB volatility			
volatility	Low	High	Low−High	Average
Low	0.47	−0.02	0.48	0.22
	(2.14)	(−0.04)	(0.98)	(0.91)
High	0.59	−0.56	1.14	0.01
	(1.50)	(−1.68)	(2.24)	(0.05)
Low−High	−0.12	0.54	−0.66	0.21
	(−0.27)	(0.98)	(−0.93)	(0.60)
Average	0.53	−0.29	0.81	
	(2.36)	(−1.05)	(2.29)	

5. Institutional ownership and betting-against-beta

The results above show that the conditional performance of BAB is a puzzle relative to leading explanations of the beta anomaly. For this performance to exist, it must be the case that high-beta stocks become less "overvalued" relative to low-beta stocks when the volatility of these factors increases. In this section, we use the demand system framework of Koijen and Yogo (2019) to investigate whether changes in institutional demand for high- and low-beta stocks can explain these changes in valuations sufficiently to produce the spread in BAB abnormal returns we observe between high- and low-volatility states.

Baker et al. (2011) show theoretically that managers judged relative to a unit-beta benchmark like the S&P 500 will overweight high-beta stocks, even if they have negative alpha, because their returns exceed those of the benchmark on average. Consistent with this theory, Christoffersen and Simutin (2017) show that, in the cross-section, mutual fund managers facing the greatest pressure to beat benchmarks tilt their portfolios toward high-beta stocks and Boguth and Simutin (2018) show that active mutual funds have betas that are greater than one on average. While standard performance evaluation incentives reward beating the benchmark on average, they also penalize excessive tracking-error volatility. This tradeoff gives managers an incentive to overweight high-beta stocks unconditionally; but, when these stocks' contribution to tracking-error volatility increases, managers have the incentive to shift their portfolio weights towards those of the benchmark. When BAB volatility increases, the potential contribution of high- and low-beta stocks to tracking error volatility relative to unit-beta indexes should mechanically increase as well because both quantities are driven by the market variance and the residual variance of extreme-beta stocks.[9] Taken together, while managers prefer to unconditionally overweight high-beta stocks, an increase in BAB volatility encourages them to shift their portfolio weights towards those of the benchmark, which increases demand for low-beta stocks and decreases demand for high-beta stocks.

5.1. Demand system

The market has N stocks, $n = 1, ..., N$, and an outside asset, $n = 0$, traded by I investors consisting of $I - 1$ institutions required to file form 13f (those with at least \$100 million of assets under management), denoted $i = 1, ..., I - 1$, and "households," denoted $i = I$, who hold all stocks not held by 13f institutions. Lower case variables denote natural logarithms of uppercase variables, e.g., $me_t(n) = \log(ME_t(n))$; and bold font denotes vectors. Asset n has a vector of observable characteristics $\boldsymbol{x}_t(n) \in \mathbb{R}^K$ besides market value of equity, $ME_t(n)$, and the beta defined in Eq. (2), which we denote in this section as $\beta_t(n)$. Investor $i's$ assets under management are denoted A_{it} and their "investment universe" of assets they are allowed to hold is denoted as $\mathcal{N}_{it} \subset \{1, ..., N\}$.

The weight of asset n in investor i's portfolio is denoted $w_{it}(n)$ and is modeled by:

$$w_{it}(n) = \frac{\delta_{i,t}(n)}{1 + \sum_{m=1}^{N} \delta_{i,t}(m)}, \tag{15}$$

[9] To see this, note that the tracking error volatility of a fund return, r_i, relative to the market benchmark, r_{MKT}, is given by: $\sqrt{\sigma^2(r_i - r_{MKT})} = \sqrt{(1-\beta_i)^2\sigma^2(r_{MKT}) + \sigma^2(\epsilon_i)}$.

where $\delta_{it}(n)$ is the weight normalized by investor i's weight in the outside asset:

$$\delta_{it}(n) := \frac{w_{it}(n)}{w_{it}(0)} = \exp\left(\theta_{it}^{me} me_t(n) + \theta_{it}^{\beta}\beta_t(n) + \sum_{k=1}^{K}\theta_{it}^{x_k} x_{kt}(n) + \theta_{0it}\right)\epsilon_{it}(n). \tag{16}$$

That weights sum to one across positions requires:

$$w_{it}(0) = \frac{1}{1 + \sum_{m=1}^{N}\delta_{it}(m)}. \tag{17}$$

Eq. (16) maps the demand for a given stock to its characteristics and latent demand, $\epsilon_{it}(n)$, in a way that varies across institutions and time. Weights and characteristics are observable, so we can estimate Eq. (16) via nonlinear GMM for each institution-quarter using the moment condition:

$$E(\epsilon_{it}(n)|\widehat{me}_t(n), \boldsymbol{x}_t(n), \beta_t(n)) = 1. \tag{18}$$

Since $me_t(n)$ is endogenous, we use the instrumental variable of Koijen and Yogo (2019), defined as:

$$\widehat{me}_{it}(n) := \log\left(\sum_{j\neq i} A_{jt}\frac{1_{jt}(n)}{1 + \sum_{m\in\mathcal{N}_{it}} 1_{jt}(m)},\right), \tag{19}$$

where $1_{jt}(n)$ equals one if $n \in \mathcal{N}_{jt}$ and zero otherwise. This instrument depends only on the investment universe of other investors and the wealth distribution, which are exogenous by assumption. It can be interpreted as the counterfactual market equity, at the market clearing price, if other investors were to hold an equal-weighted portfolio within their investment universe.[10] The characteristics in $\boldsymbol{x}_t(n)$ are: log book equity, profitability, investment, and dividends to book equity, all defined the same as Koijen and Yogo (2019).[11] The choice of book equity, profitability, and investment is explicitly motivated by the Fama-French five-factor model, which is helpful given that we are trying to explain abnormal returns relative

[10] Koijen and Yogo (2019) also show the demand system results are robust to using other natural modifications to this instrument.

[11] To estimate the demand system, we use the replication code from Ralph Koijen's website, `https://koijen.net/index.html`, modified only to use our definition of beta from Section 2.

to factors based on these characteristics.

5.2. *Institutional demand for high- and low-beta stocks*

For each institution-quarter since 1980q1 (when 13f data become available), we estimate the demand system, Eqs. (16) and (18), via nonlinear GMM following Koijen and Yogo (2019). The "Unc." row of Table 13 presents time-series means of quarterly assets-under-management-weighted averages, $\bar{\theta}_t^{\beta}$, of the parameter θ_{it}^{β}, by institution type. This row shows that, on average, all institutional investors have a significantly positive preference for beta, especially those with high benchmarking motivations (mutual funds, pension funds, and hedge funds, which are split between the investment advisors and "other"). In contrast, households have a negative preference for beta on average and the "Low vol" and "High vol" rows show that household demand remains negative in both volatility states.[12] In contrast, all institutional investor types significantly drop their demand for high-beta stocks as volatility goes from low to high. For "all" institutions, this drop is essentially to zero (0.01, $t = 0.37$), and in the case of investment advisors (which includes hedge funds), it even becomes negative (-0.05, $t = -2.80$). Taken together, this evidence is consistent with institutional demand (controlling for size, value, profitability, and investment) driving an "overvaluation" of high-beta stocks in states with low BAB volatility compared to those with high volatility.

[12] We sample BAB volatility monthly, whereas the institutional holdings data report snapshots at the end of each calendar quarter, that is the end of the months: March, June, September, and December. As "lagged" BAB volatility, we use the value of $\hat{\sigma}_t$ defined in Eq. (6) at the end of February, May, August, and November, respectively.

Table 13: Demand system coefficients by institution type

For each institution-quarter, (i, t), we estimate the following demand system for portfolio weights via GMM:

$$E\left(\epsilon_{it}(n) | \widehat{me}_t(n), \beta_{it}(n), \boldsymbol{x}_t(n)\right) = 1, \tag{20}$$

$$\frac{w_{it}(n)}{w_{it}(0)} = \exp\left(\theta_{it}^{me} me_t(n) + \theta_{it}^{\beta}\beta_t(n) + \sum_{k=1}^{K}\theta_{i,t}^{x_k} x_{kt}(n) + \theta_{0,i,t}\right)\epsilon_{it}(n), \tag{21}$$

where $n = 1, ..., N_t$ indexes stocks that exist at time t, and $w_{it}(n)/w_{it}(0)$ is the portfolio weight in stock n normalized by the weight in the outside asset. This table reports time-series means of quarterly assets under management-weighted averages, $\bar{\theta}_t^{\beta}$, of the parameter θ_{it}^{β}, over: the full sample (row "Unc."), the subsample with below-median realized volatility ("Low vol"), and the subsample with above-median volatility ("High vol"), as well as low-minus-high difference "Low−High"). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is 1980q1 to 2016q4.

	Households	Banks	Insurance Companies	Investment Advisors	Mutual Funds	Pension Funds	Other	All Institutions
Unc.	−0.10	0.06	0.08	0.06	0.10	0.15	0.19	0.09
	(−6.48)	(4.16)	(3.47)	(3.01)	(7.17)	(7.02)	(4.64)	(5.87)
Low vol	−0.10	0.13	0.15	0.20	0.20	0.30	0.30	0.19
	(−4.30)	(4.80)	(3.56)	(5.67)	(6.91)	(7.08)	(3.98)	(6.43)
High vol	−0.11	−0.00	0.01	−0.06	0.03	0.04	0.10	0.01
	(−4.85)	(−0.20)	(0.67)	(−2.80)	(2.15)	(1.77)	(2.28)	(0.37)
Low−High	0.01	0.13	0.14	0.26	0.17	0.26	0.20	0.18
	(0.40)	(4.46)	(3.01)	(6.31)	(5.40)	(5.50)	(2.33)	(5.66)

5.3. Market clearing and counterfactual experiments

Assuming that shares outstanding are exogenous in the short run, we can relate the demand system to prices via market clearing:

$$ME_t(n) = \sum_{i=1}^{I} A_{it} w_{it}(n). \tag{22}$$

Our primary strategy to quantify the impact of change in institutional demand on BAB abnormal returns is to perform a counterfactual experiment in which we shut off institutional demand for beta by restricting $\theta_{it}^{\beta}(n) = 0$ in Eq. (16), and then comparing the resulting valuation impact implied by market clearing across high- and low-volatility states. Following Han et al. (2022), we then use the Campbell and Shiller (1988) decomposition to map how these changes in valuations across low- and high-volatility states compare to those required to produce the BAB alphas we see in the data.

Applying the decomposition to an arbitrary asset (suppressing index n) with return r_t:

$$r_t \approx \kappa_0 + \kappa_1 pd_t - pd_{t-1} + dg_t, \tag{23}$$

where pd_t denotes the log price-dividend ratio, dg_t denotes log dividend growth, and $\kappa_1 = 0.96$. Solving forward for pd_t and taking expectations yields:

$$pd_t = \frac{\kappa_0}{1-\kappa_1} + E_t\left(\sum_{h=0}^{\infty} \kappa_1^j (dg_{t+1+h} - r_{t+1+h})\right). \tag{24}$$

Next, we assume that r_t follows the factor structure:

$$r_t = \alpha_t + \boldsymbol{\beta}'_{t-1}\boldsymbol{f}_t + \nu_t, \tag{25}$$

and the abnormal return decays according to an autoregressive process:

$$\alpha_t = \phi\alpha_{t-1} + \eta_t, \tag{26}$$

with $\phi < 1$. If we further assume that dividend-growth and the factor structure of the asset are the same in the counterfactual as they are in reality, the percentage price impact of asset n in this experiment relative to the baseline case is given by (see Appendix A.1 for details):

$$\%\Delta ME_{nt}^{CF} := \frac{ME_t(n)^{CF} - ME_t(n)}{ME_t(n)} = \exp\left(\frac{\alpha_t - \alpha_t^{CF}}{1-\phi\kappa_1}\right) - 1, \tag{27}$$

where $ME_t^{CF}(n)$ denotes the market capitalization of asset n in the counterfactual. Using L and H to denote the low- and high-beta portfolios, respectively, we can define the price impact of the BAB portfolio as $\%\Delta ME_{L-H,t}^{CF} = \%\Delta ME_{Lt}^{CF} - \%\Delta ME_{Ht}^{CF}$. Using superscripts LV and HV to denote low- and high-volatility states, respectively, we then compare the price impact of the BAB portfolio across low- and high-volatility states:

$$\begin{aligned}
\%\Delta ME_{L-H}^{CF,LV-HV} &:= \%\Delta ME_{L-H}^{CF,LV} - \%\Delta ME_{L-H,t}^{CF,HV} \\
&= \exp\left(\frac{\alpha_{Lt}^{LV} - \alpha_{Lt}^{CF,LV}}{1-\phi\kappa_1}\right) - \exp\left(\frac{\alpha_{Ht}^{LV} - \alpha_{Ht}^{CF,LV}}{1-\phi\kappa_1}\right) \\
&- \left[\exp\left(\frac{\alpha_{Lt}^{HV} - \alpha_{Lt}^{CF,HV}}{1-\phi\kappa_1}\right) - \exp\left(\frac{\alpha_{Ht}^{HV} - \alpha_{Ht}^{CF,HV}}{1-\phi\kappa_1}\right)\right].
\end{aligned} \tag{28}$$

What price impact would be required to eliminate the spread in FF6 BAB alphas of 1.01 percentage points between low- and high-volatility states reported in Table 3? Consider a counterfactual universe in which $\alpha^{CF} = 0$. In the real world, untabulated regressions show the low-beta portfolio in BAB earns an FF6 alpha of 0.18% in the low-volatility state, and −0.16% in the high-volatility state. The high-beta portfolio earns alpha of −0.33% and 0.34% in these states, respectively. Departing from Han et al. (2022), based on our results in Tables 3 and 4, we further assume that $\alpha_t = c \cdot \hat{\sigma}_t$, for some $c \in (-1, 0)$, where $\hat{\sigma}_t$ is the log realized volatility of BAB. This assumption implies that the AR(1) coefficient, ϕ, in Eq. (26), is equal to the AR(1) coefficient of $\hat{\sigma}_t$, which we estimate to be 0.75. Under these assumptions and Eq. (28), the price impact required to render $\alpha^{CF} = 0$ is:

$$\begin{aligned}
\%\Delta ME_{L-H,min}^{CF,LV-HV} &:= \exp\left(\frac{0.18\%}{1-0.75\cdot 0.96}\right) - \exp\left(\frac{-0.16\%}{1-0.75\cdot 0.96}\right) \\
&\quad - \left[\exp\left(\frac{-0.33\%}{1-0.75\cdot 0.96}\right) - \exp\left(\frac{0.34\%}{1-0.75\cdot 0.96}\right)\right] \\
&\quad = 3.61\%.
\end{aligned} \tag{29}$$

Using the FF3 model, ignoring volatility, and only considering the high-beta portfolio, Han et al. (2022) estimate $\phi = 0.5$, which lowers the figure in Eq. (29) to 1.9%.

Table 14 contains the repricing statistics. On average (row "Unc."), nullifying the institutional demand parameter for beta causes low-beta stocks to appreciate by 12.7% more than high-beta stocks. This is consistent with high-beta stocks being "overvalued" relative

Table 14: Counterfactual repricing statistics

Based on the estimated demand system defined in Table 13, each quarter, we compute the percentage price impact of the low- and high-beta portfolios ("$\%\Delta ME_L$" and "$\%\Delta ME_H$," respectively) in BAB from the counterfactual experiment in which the institutional preference for beta is set to zero ($\theta^{\beta}_{it} = 0$ in Eq. (16)) and stocks are repriced by market clearing. This table reports time-series means of these price impact statistics (in units of %) over the whole sample ("Unc."), months with below-median BAB volatility ("Low vol"), and months with above-median volatility ("High vol"). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors.

	$\%\Delta ME_L$	$\%\Delta ME_H$	$\%\Delta ME_{L-H}$
Unc.	19.61	6.88	12.74
	(35.47)	(14.06)	(18.69)
Low vol	20.33	5.63	14.70
	(22.74)	(10.48)	(18.13)
High vol	19.04	7.88	11.15
	(27.78)	(9.95)	(10.69)
Low−High	1.29	−2.25	3.54
	(1.14)	(−2.36)	(2.68)

to low-beta stocks on average, but this interpretation is confounded by the fact that beta is correlated with other characteristics in the demand system. For example, low-beta stocks have high profitability, which is a desirable characteristic to institutions. In contrast, comparing the change in relative valuation of high- and low-beta stocks between low- and high-volatility regimes better isolates the incremental impact of how beta is valued in these different regimes controlling for the other characteristics. Indeed, we see that in high-volatility states, nullifying preference for beta leads to a smaller counterfactual impact on relative valuations than in low-volatility states, consistent with higher relative "overvaluation" of high-beta stocks when volatility is low. The spread in BAB valuation changes between the two states is 3.54%. This figure is within 0.1 percentage point of the 3.61% from Eq. (29) required to produce the difference in BAB alphas between low- and high-volatility states (and almost double the 1.9% implied by the Han et al., 2022 estimate of ϕ).

5.4. Partial derivative of price with respect to beta

The market clearing relationship captured by Eq. (22) defines the price of a stock as an implicit function of its characteristics and latent demand. Given our estimated demand system, we can compute the partial derivative of this function with respect to beta and estimate the value impact of a change in an asset's beta. This derivative is informative about the asset pricing implications of beta in a way that is independent of the choice of counterfactual experiment. It also addresses a different question than our counterfactual experiment, which answers how a given asset's value would change as *demand preferences* for beta vary with volatility. The derivative addresses the question of what happens to a given asset's value if demand preferences remain the same but its *beta* varies with volatility.

Table 15 presents the time-series means of the partial derivative of the log market capitalization of three value-weighted portfolios with respect to their ex ante betas (see, Appendix A for details). Standard calculus arguments show that these derivatives are the first order approximation to the percent price impact of a unit increase in beta.[13] Conveniently for interpretation, the post-ranking spread in betas between the long- and short-legs of BAB is approximately one.

The column labeled "Market" shows that, on average, the (value-weighted) typical stock would decrease in value by 11.3% with a unit increase in beta. In months with high BAB volatility, this decrease grows in magnitude to 14.5%, but in low-volatility months, it falls to 7.4%, with a spread between the two states of 7.1, consistent with beta increasing valuations more when volatility is low.

The column labeled "High beta" shows that, when volatility is high, a unit-beta *decrease* in beta would *increase* the valuation of high-beta stocks by 9.5%. Conversely, when volatility is low, the same *decrease* in beta would *decrease* the valuation of high-beta stocks by 7.3%, so high-beta stocks are valued more highly when volatility is low, and valued less when

[13]The percentage change is approximately the log level change, $\%\Delta ME_t(n) \approx \Delta me_t(n)$, and, for a given change in beta, $\Delta\beta_t(n)$, the first-order approximation is: $\Delta me_t(n) \approx \frac{\partial me_t(n)}{\partial \beta_t(n)} \Delta\beta_t(n)$.

Table 15: Derivatives of price with respect to beta

Based on the demand system estimation specified in Table 13, each quarter, we compute the partial derivative of the log market capitalization of three value-weighted portfolios with respect to their ex ante betas (see, Eq. (41) for details). The column headings specify the portfolios, which are the low- and high-beta portfolios, and the value-weighted market. This table reports the time-series means of these derivatives over the whole sample, ("Unc.,") low-BAB volatility months ("Low vol"), and high-volatility months ("High vol"). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors.

	Market	High beta	Low beta	Low−High
Unc.	-11.30	-1.98	-15.14	-12.34
	(-4.31)	(-0.81)	(-5.56)	(-9.82)
High Vol	-14.46	-9.54	-16.88	-7.34
	(-5.09)	(-3.64)	(-5.66)	(-10.65)
Low Vol	-7.36	7.26	-12.97	-18.51
	(-1.55)	(1.57)	(-2.65)	(-6.78)
Low-High	7.10	16.80	3.91	-11.17
	(1.28)	(3.16)	(0.68)	(-3.97)

volatility is high, consistent with our results above. The difference between the high- and low-volatility states is 16.8, which is significant at the 1% level. Further consistent with our results thus far, changing low-beta-stocks to high beta stocks by increasing their beta by one would decrease their value by 3.9% more in low-volatility states, when high-beta stocks are in high demand, than in high-volatility states when they are not.

Overall, the evidence in this section shows that institutional demand drives up the prices of high-beta stocks relative to low-beta ones when volatility is low, and by enough to generate the conditional abnormal returns of BAB.

6. Conclusion

For fifty years, the finance literature has struggled to explain the puzzling weak cross-sectional relationship between beta and return, which challenges foundational principals of asset pricing. This literature posits a wide range of explanations for the anomaly. Perhaps the earliest of them, the leverage constraints theory argues that high-beta stocks appear overvalued because they give relatively risk-tolerant investors the chance to earn high returns without borrowing. More recently, some argue that investors value the lottery-like payoffs of high-risk stocks or are naively optimistic about their prospects. Others argue that limits to arbitrage allow this anomaly to persist or that it can be explained by missing risk factors.

These explanations all have empirical backing, however, prior studies do not rigorously test them in a conditional setting. The abnormal returns and Sharpe ratios of betting-against-beta factors based on the anomaly rise dramatically when the ex ante volatility of these factors is low. This leads us to inquire which explanations can simultaneously match the cross-sectional risk-return patterns and this element of time-series predictability. The resulting exercises show that this dual achievement is more demanding than what the current state of theory can offer.

We show that the leverage constraints theory counterfactually predicts a positive relationship between volatility and subsequent Sharpe ratios of beta factors, intuitively because it depends critically on the assumption of risk averse investors who naturally demand a positive risk-return tradeoff in both the cross-section and time-series. Empirically, we find that multifactor asset pricing models fail to explain the conditional returns of betting-against-beta factors. The loadings that explain their returns unconditionally generally shrink when realized variance falls such that they earn most anomalous returns precisely when they have the lowest risk. This fact remains true for beta factors that control for the cross-sectional effects of lottery preferences and limits to arbitrage. Overall, we bring a new time-series dimension to test leading theories of the beta anomaly and find these theories generally fail to explain this anomaly's conditional performance.

The results in this paper highlight serious problems associated with ignoring conditioning information in asset pricing research. Five decades worth of explanations for the beta anomaly uniformly fail to explain its performance conditional on volatility. While many studies estimate conditional asset pricing models, these studies are generally the exception, and not the norm, with unconditional tests still dominating the literature.

Towards an explanation of our findings, we show that institutional investors have strong demand for high-beta stocks when volatility is high, but this demand diminishes as risk falls. We further show quantitatively that the value impact of this shifting demand is sufficient to fully explain the difference in abnormal returns on the betting-against-beta strategy between high- and low-volatility regimes. Incentives from widely used benchmark-based performance-evaluation contracts can explain these trading patterns. They motivate institutions to robustly demand high-beta stocks when volatility is low to beat unit-beta benchmarks like the S&P500 on average; but they also punish tracking error, which encourages a retreat from high- to low-beta stocks when volatility increases.

A. Derivations of formulas

A.1. Repricing statistics

Here we derive the price impact that a counterfactual experiment would have to yield in order to produce the difference in BAB abnormal returns between low- and high-volatility states. We follow Han et al. (2022), who make a similar statistic for a high-beta portfolio defined similarly to ours. Consider the Campbell and Shiller (1988) decomposition, applied to an arbitrary asset n:

$$r_{nt} = \kappa_0 + k_1 pd_{nt} - pd_{n,t-1} + dg_{nt}, \tag{30}$$

with $k_1 \approx 0.96$. It follows from Eq. (30) that:

$$pd_{nt} = \frac{\kappa_0}{1-\kappa_1} + E_t\left(\sum_{h=0}^{\infty} \kappa_1^j (dg_{n,t+1+h} - r_{n,t+1+h})\right) \tag{31}$$

Next, assume that r_{nt} follows the factor structure:

$$r_{nt} = \alpha_{nt} + \boldsymbol{\beta}'_{n,t-1}\boldsymbol{f}_t + \epsilon_{nt}, \tag{32}$$

and α has the following dynamics:

$$\alpha_{nt} = \phi \alpha_{n,t-1} + \eta_{nt}, \tag{33}$$

with $\phi < 1$. Combining the above yields:

$$pd_{nt} = \frac{k_0}{1-k_1} + \overline{dg - \beta f}^{\infty}_{nt} - \frac{\alpha_{nt}}{1-\phi\kappa_1}, \tag{34}$$

where $\overline{dg - \beta f}^{\infty}_{nt} := E_t\left(\sum_{h=0}^{\infty} \kappa_1^j (dg_{n,t+1+j} - \boldsymbol{\beta}'_{n,t+h}\boldsymbol{f}_{t+1+h})\right)$. Consider a counterfactual experiment and let the percentage price impact of an asset n in this experiment relative to the "real-world" case be denoted $\%\Delta ME^{CF}_{nt}$. Assuming the path of factor loadings and expected dividend-growth do not change in a counterfactual experiment, we can relate counterfactual alphas to counterfactual price-dividend ratios as follows:

$$\begin{aligned} \%\Delta ME^{CF}_{nt} = \frac{ME_t(n)^{CF} - ME_t(n)}{ME_t(n)} &= \left(\frac{PD^{CF}_{n,t} - PD_{n,t}}{PD_{n,t}}\right) \\ &= \exp\left(\frac{\alpha_t - \alpha^{CF}_t}{1-\phi\kappa_1}\right) - 1. \end{aligned} \tag{35}$$

In our counterfactual experiment, we consider the impact of setting institutional preferences for beta to zero, that is, $\theta^{\beta}_{it} = 0$. We estimate the relative valuation impact, $\%\Delta ME^{CF}_{L-H,t} = \%\Delta ME^{CF}_{Lt} - \%\Delta ME_{Lt} - \left(\%\Delta ME^{CF}_{Ht} - \%\Delta ME_{Ht}\right)$, across low- and high-beta portfolios. We then compare this difference across low- and high-volatility states:

$$\begin{aligned} &\%\Delta ME^{CF,LV-HV}_{L-H} := \\ &\qquad \exp\left(\frac{\alpha^{LV}_{Lt} - \alpha^{CF,LV}_{Lt}}{1-\phi\kappa_1}\right) - \exp\left(\frac{\alpha^{LV}_{Ht} - \alpha^{CF,LV}_{Ht}}{1-\phi\kappa_1}\right) \\ &\qquad - \left[\exp\left(\frac{\alpha^{HV}_{Lt} - \alpha^{CF,HV}_{Lt}}{1-\phi\kappa_1}\right) - \exp\left(\frac{\alpha^{HV}_{Ht} - \alpha^{CF,HV}_{Ht}}{1-\phi\kappa_1}\right)\right]. \end{aligned} \tag{36}$$

A.2. Partial derivative of log market cap with respect to beta

Taking logs yields of both sides of Eq. (22) yields:

$$me_t(n) = \log\left(\sum_{i=1}^{I} A_{it} w_{it}(n)\right). \tag{37}$$

Differentiating yields:[14]

$$\frac{\partial me_t(n)}{\partial \beta_t(n)} = ME_t^{-1}(n) \sum_{i=1}^{I} A_{it} \left(\frac{\partial w_{it}(n)}{\partial me_t(n)} \frac{\partial me_t(n)}{\partial \beta_t(n)} + \frac{\partial w_{it}(n)}{\partial \beta_t(n)}\right), \tag{38}$$

and rearranging leads to:

$$\left[ME_t(n) - \sum_{i=1}^{I} A_{it} \frac{\partial w_{it}(n)}{\partial me_t(n)}\right] \frac{\partial me_t(n)}{\partial \beta_t(n)} = \sum_{i=1}^{I} A_{it} \frac{\partial w_{it}(n)}{\partial \beta_t(n)}, \tag{39}$$

and

$$\begin{aligned} \frac{\partial me_t(n)}{\partial \beta_t(n)} &= \left[ME_t(n) - \sum_{i=1}^{I} A_{it} \frac{\partial w_{it}(n)}{\partial me_t(n)}\right]^{-1} \sum_{i=1}^{I} A_{it} \frac{\partial w_{it}(n)}{\partial \beta_t(n)} \\ &= \left[ME_t(n) - \sum_{i=1}^{I} A_{it} \theta_{it}^{me}(n) w_{it}(n)(1 - w_{it}(n))\right]^{-1} \sum_{i=1}^{I} A_{it} \theta_{it}^{\beta}(n) w_{it}(n)(1 - w_{it}(n)). \end{aligned} \tag{40}$$

Consider a value-weighted portfolio, P, and let $\bar{\beta}_t^P$ be the value-weighted average of $\beta_t(n)$ $n \in P$. The beta of each stock in the the portfolio can be decomposed as: $\beta_t(n) = \bar{\beta}_t^P + \epsilon_t^P(n)$, where $\left(\sum_{m \in P} ME_t(m)\right)^{-1} \sum_{n \in P} ME_t(n) \epsilon_t(n) = 0$. With this decomposition, we can derive a derivative of the log portfolio market value of equity, $me_t^P = \log(ME_t^P)$,

[14] Following Koijen and Yogo (2019), we assume that $\frac{\partial me_t(n)}{\partial me_t(m)} = 0$ and $\frac{\partial w_{it}(n)}{\partial x_{kt}(m)} = 0$ for $m \neq n$.

with respect to the portfolio beta as follows:

$$
\begin{aligned}
\frac{\partial me_t^P(n)}{\partial \bar{\beta}_t^P} &= (ME_t^P)^{-1} \frac{\partial}{\partial \bar{\beta}_t^P} \sum_{n \in P} ME_t(n) \\
&= (ME_t^P)^{-1} \frac{\partial}{\partial \bar{\beta}_t^P} \sum_{n \in P} \exp(me_t(n)) \\
&= (ME_t^P)^{-1} \sum_{n \in P} ME_t(n) \frac{\partial me_t(n)}{\partial \bar{\beta}_t^P} \\
&= (ME_t^P)^{-1} \sum_{n \in P} ME_t(n) \frac{\partial me_t(n)}{\partial \beta_t(n)},
\end{aligned} \tag{41}
$$

which is the value-weighted average of the $\frac{\partial me_t(n)}{\partial \beta_t(n)}$ for each stock in the portfolio.

References

Adrian, T., Etula, E. and Muir, T. (2014), Financial intermediaries and the cross-section of asset returns, *Journal of Finance* 69, 2557–2596.

Ali, A., Hwang, L.-S. and Trombley, M. A. (2003), Arbitrage risk and the book-to-market anomaly, *Journal of Financial Economics* 69, 355–373.

Ang, A. (2014), *Asset management: A systematic approach to factor investing*, Oxford University Press.

Ang, A., Hodrick, R. J., Xing, Y. and Zhang, X. (2006), The cross-section of volatility and expected returns, *Journal of Finance* 61, 259–299.

Antoniou, C., Doukas, J. A. and Subrahmanyam, A. (2013), Cognitive dissonance, sentiment, and momentum, *Journal of Financial and Quantitative Analysis* 48, 245–275.

Antoniou, C., Doukas, J. A. and Subrahmanyam, A. (2016), Investor sentiment, beta, and the cost of equity capital, *Management Science* 62, 347–367.

Asness, C., Frazzini, A., Gormsen, N. J. and Pedersen, L. H. (2020), Betting against correlation: Testing theories of the low-risk effect, *Journal of Financial Economics* 135, 629–652.

Baker, M., Bradley, B. and Wurgler, J. (2011), Benchmarks as limits to arbitrage: Understanding the low-volatility anomaly, *Financial Analysts Journal* 67, 40–54.

Baker, M. and Wurgler, J. (2007), Investor sentiment in the stock market, *Journal of Economic Perspectives* 21, 129–152.

Bali, T. G., Brown, S. J., Murray, S. and Tang, Y. (2017), A lottery-demand-based explanation of the beta anomaly, *Journal of Financial and Quantitative Analysis* 52, 2369–2397.

Bali, T. G., Cakici, N. and Whitelaw, R. F. (2011), Maxing out: Stocks as lotteries and the cross-section of expected returns, *Journal of Financial Economics* 99, 427–446.

Barroso, P. and Detzel, A. (2021), Do limits to arbitrage explain the benefits of volatility-managed portfolios?, *Journal of Financial Economics* 140, 744–767.

Barroso, P. and Santa-Clara, P. (2015), Momentum has its moments, *Journal of Financial Economics* 116, 111–120.

Black, F. (1972), Capital market equilibrium with restricted borrowing, *Journal of Business* 45, 444–455.

Black, F., Jensen, M. C. and Scholes, M. S. (1972), The capital asset pricing model: Some empirical tests, in M. Jensen, ed., Studies in the Theory of Capital Markets, Praeger Publishers Inc.

Boguth, O. and Simutin, M. (2018), Leverage constraints and asset prices: Insights from mutual fund risk taking, *Journal of Financial Economics* 127, 325–341.

Breeden, D. T., Gibbons, M. R. and Litzenberger, R. H. (1989), Empirical tests of the consumption-oriented capm, *Journal of Finance* 44, 231–262.

Campbell, J. and Shiller, R. (1988), The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Campbell, J. Y., Hilscher, J. and Szilagyi, J. (2008), In search of distress risk, *Journal of Finance* 63, 2899–2939.

Carhart, M. M. (1997), On persistence in mutual fund performance, *Journal of Finance* 52, 57–82.

Cederburg, S. and O'Doherty, M. S. (2016), Does it pay to bet against beta? On the conditional performance of the beta anomaly, *Journal of Finance* 71, 737–774.

Cederburg, S., O'Doherty, M. S., Wang, F. and Yan, X. S. (2020), On the performance of volatility-managed portfolios, *Journal of Financial Economics* 138, 95–117.

Christoffersen, S. E. K. and Simutin, M. (2017), On the demand for high-beta stocks: Evidence from mutual funds, *Review of Financial Studies* 30, 2596–2620.

Cochrane, J. H. (2005), *Asset Pricing*, Princeton University Press, Princeton, N.J.

Cochrane, J. H. (2011), Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.

Cochrane, J. and Saá-Requejo, J. (2000), Beyond arbitrage: Good-deal asset price bounds in incomplete markets, *Journal of Political Economy* 108, 79–119.

Conrad, J., Kapadia, N. and Xing, Y. (2014), Death and jackpot: Why do individual investors hold overpriced stocks?, *Journal of Financial Economics* 113, 455–475.

Cooper, M. J., Gulen, H. and Schill, M. J. (2008), Asset growth and the cross-section of stock returns, *Journal of Finance* 63, 1609–1651.

Daniel, K. D. and Moskowitz, T. J. (2016), Momentum crashes, *Journal of Financial Economics* 122, 221–247.

Daniel, K. and Titman, S. (2006), Market reactions to tangible and intangible information, *Journal of Finance* 61, 1605–1643.

DeMiguel, V., Martín-Utrera, A. and Uppal, R. (2022), A multifactor perspective on volatility-managed portfolios, London Business School working paper.

Dimson, E. (1979), Risk measurement when shares are subject to infrequent trading, *Journal of Financial Economics* 7, 197–226.

Doukas, J. A., Kim, C. F. and Pantzalis, C. (2010), Arbitrage risk and stock mispricing, *Journal of Financial and Quantitative Analysis* 907–934.

Eisdorfer, A. and Misirli, E. U. (2020), Distressed stocks in distressed times, *Management Science* 66, 2452–2473.

Fama, E. F. and French, K. R. (1993), Common risk factors in the returns on stocks and bonds, *Journal of Financial Economics* 33, 3–56.

Fama, E. F. and French, K. R. (1996), Multifactor explanations of asset pricing anomalies, *Journal of Finance* 51, 55–84.

Fama, E. F. and French, K. R. (2006), Profitability, investments and average returns, *Journal of Financial Economics* 82, 491–518.

Fama, E. F. and French, K. R. (2018), Choosing factors, *Journal of Financial Economics* 128, 234–252.

Fleming, J., Kirby, C. and Ostdiek, B. (2001), The economic value of volatility timing, *Journal of Finance* 56, 329–352.

Fleming, J., Kirby, C. and Ostdiek, B. (2003), The economic value of volatility timing using "realized" volatility, *Journal of Financial Economics* 67, 473–509.

Frazzini, A. and Pedersen, L. H. (2014), Betting against beta, *Journal of Financial Economics* 111, 1–25.

Friend, I. and Blume, M. (1970), Measurement of portfolio performance under uncertainty, *American Economic Review* 60, 561–575.

Han, X. (2022), Understanding the performance of components in betting against beta, *Critical Finance Review* 11, 1–36.

Han, X., Roussanov, N. and Ruan, H. (2022), Mutual fund risk shifting and risk anomalies.

Hirshleifer, D., Hou, K., Teoh, S. H. and Zhang, Y. (2004), Do investors overvalue firms with bloated balance sheets?, *Journal of Accounting and Economics* 38, 297–331.

Hong, H. and Sraer, D. A. (2016), Speculative betas, *Journal of Finance* 71, 2095–2144.

Jegadeesh, N. and Titman, S. (1993), Returns to buying winners and selling losers: Implications for stock market efficiency, *Journal of Finance* 48, 65–91.

Jobson, J. D. and Korkie, B. M. (1981), Performance hypothesis testing with the Sharpe and Treynor measures, *Journal of Finance* 889–908.

Jylhä, P. (2018), Margin requirements and the security market line, *Journal of Finance* 73, 1281–1321.

Kirby, C. and Ostdiek, B. (2012), It's all in the timing: Simple active portfolio strategies that outperform naive diversification, *Journal of Financial and Quantitative Analysis* 47, 437–467.

Koijen, R. S. J. and Yogo, M. (2019), A demand system approach to asset pricing, *Journal of Political Economy* 127, 1475–1515.

Kozak, S., Nagel, S. and Santosh, S. (2018), Interpreting factor models, *Journal of Finance* 73, 1183–1223.

Kroencke, T. A. (2017), Asset pricing without garbage, *The Journal of Finance* 72, 47–98.

Kumar, A. (2009), Who gambles in the stock market?, *Journal of Finance* 64, 1889–1933.

Levy, R. A. (1967), Relative strength as a criterion for investment selection, *Journal of Finance* 22, 595–610.

Li, D. and Zhang, L. (2010), Does q-theory with investment frictions explain anomalies in the cross section of returns?, *Journal of Financial Economics* 98, 297–314.

Lintner, J. (1965), The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets, *Review of Economics and Statistics* 13–37.

Liu, F., Tang, X. and Zhou, G. (2019), Volatility-managed portfolio: Does it really work?, *Journal of Portfolio Management* 46, 38–51.

Liu, J., Stambaugh, R. F. and Yuan, Y. (2018), Absolving beta of volatility's effects, *Journal of Financial Economics* 128, 1–15.

Loughran, T. and Ritter, J. R. (1995), The new issues puzzle, *Journal of Finance* 50, 23–51.

Lu, Z. and Qin, Z. (2021), Leveraged funds and the shadow cost of leverage constraints, *Journal of Finance* 76, 1295–1338.

Lyandres, E., Sun, L. and Zhang, L. (2007), The new issues puzzle: Testing the investment-based explanation, *Review of Financial Studies* 21, 2825–2855.

MacKinlay, A. (1995), Multifactor models do not explain deviations from the capm, *Journal of Financial Economics* 38, 3–28.

Marquering, W. and Verbeek, M. (2004), The economic value of predicting stock index returns and volatility, *Journal of Financial and Quantitative Analysis* 39, 407–429.

Mashruwala, C., Rajgopal, S. and Shevlin, T. (2006), Why is the accrual anomaly not arbitraged away? The role of idiosyncratic risk and transaction costs, *Journal of Accounting and Economics* 42, 3–33.

McLean, R. D. (2010), Idiosyncratic risk, long-term reversal, and momentum, *Journal of Financial and Quantitative Analysis* 45, 883–906.

Mehra, R. and Prescott, E. C. (1985), The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Merton, R. C. (1973), An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.

Moreira, A. and Muir, T. (2017), Volatility-managed portfolios, *Journal of Finance* 72, 1611–1644.

Nagel, S. and Singleton, K. J. (2011), Estimation and evaluation of conditional asset pricing models, *Journal of Finance* 66, 873–909.

Novy-Marx, R. (2013), The other side of value: The gross profitability premium, *Journal of Financial Economics* 108, 1–28.

Novy-Marx, R. and Velikov, M. (2022), Betting against betting against beta, *Journal of Financial Economics* 143, 80–106.

Ohlson, J. A. (1980), Financial ratios and the probabilistic prediction of bankruptcy, *Journal of Accounting Research* 18, 109–131.

Pontiff, J. (2006), Costly arbitrage and the myth of idiosyncratic risk, *Journal of Accounting and Economics* 42, 35–52.

Ross, S. A. (1976), The arbitrage theory of capital asset pricing, *Journal of Economic Theory* 13, 341–360.

Shanken, J. (1990), Intertemporal asset pricing: An empirical investigation, *Journal of Econometrics* 45, 99–120.

Shanken, J. (1992), The current state of arbitrage pricing theory, *Journal of Finance* 47, 1569–1574.

Sharpe, W. F. (1964), Capital asset prices: A theory of market equilibrium under conditions of risk, *Journal of Finance* 19, 425–442.

Shleifer, A. and Vishny, R. W. (1997), The limits of arbitrage, *Journal of Finance* 52, 35–55.

Sloan, R. G. (1996), Do stock prices fully reflect information in accruals and cash flows about future earnings?, *Accounting Review* 71, 289–315.

Stambaugh, R. F., Yu, J. and Yuan, Y. (2012), The short of it: Investor sentiment and anomalies, *Journal of Financial Economics* 104, 288–302.

Stambaugh, R. F., Yu, J. and Yuan, Y. (2015), Arbitrage asymmetry and the idiosyncratic volatility puzzle, *Journal of Finance* 70, 1903–1948.

Stambaugh, R. F. and Yuan, Y. (2017), Mispricing factors, *Review of Financial Studies* 30, 1270–1315.

Titman, S., Wei, K. C. J. and Xie, F. (2004*a*), Capital investments and stock returns, *The Journal of Financial and Quantitative Analysis* 39, 677–700.

Titman, S., Wei, K. J. and Xie, F. (2004*b*), Capital investments and stock returns, *Journal of Financial and Quantitative Analysis* 39, 677–700.

Vasicek, O. A. (1973), A note on using cross-sectional information in bayesian estimation of security betas, *Journal of Finance* 28, 1233–1239.

White, H. (1980), A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48, 817–838.

Xing, Y. (2007), Interpreting the value effect through the q-theory: An empirical investigation, *Review of Financial Studies* 21, 1767–1795.

Yu, J. and Yuan, Y. (2011), Investor sentiment and the mean–variance relation, *Journal of Financial Economics* 100, 367–381.

Internet Appendix for: The volatility puzzle of the low-risk anomaly

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Abstract

This appendix describes the construction of the consumption mimicking portfolio used in Section 4.2 of the main body of the paper and reports intercept and slope parameters from the conditional factor model regressions presented in Table 4.

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Table IA.1 shows conditional factor model regressions similar to those in Table 4 that include factor loadings. Following Breeden, Gibbons, and Litzenberger (1989), we form a mimicking portfolio for Consumption growth using the "unfiltered" Consumption growth time series of Kroencke (2017), who recognizes that NIPA Consumption contains a filter that helps ensure the *level* of Consumption is accurate, but comes at the expense of reduced accuracy in measuring *changes* in Consumption, which are relevant for asset pricing tests. He finds that the unfiltered Consumption growth betas line up with average returns on portfolios formed on book-to-market and investment. The use of a mimicking portfolio is necessary since the Consumption growth series is measured annually, but monthly frequency sampling is essential for our study. We form the portfolio as the fitted values from a regression of the Consumption growth time series on the annual returns of the 30 "2x3" base assets used to make the FF6 factors (e.g., the six size-book-to-market portfolios for HML). If these factors span BAB, and BAB's returns arise as compensation for consumption risk, then this factor should also price BAB (Breeden et al., 1989, pp. 239–242).

Table IA.1 shows that, as discussed for Table 4, only the FF6 and intermediary leverage models price BAB unconditionally, but volatility predicts BAB abnormal returns by roughly the same amount in all models, with a one standard deviation *decrease* in BAB volatility predicting an *increase* in monthly alpha of 0.70 percentage points per month. Table 3 shows that BAB loadings on profitability and momentum fall significantly as volatility rises, with the former effect particularly large. Similarly, Table IA.1 shows that BAB volatility is significantly and positively related to the subsequent exposure to the momentum factor, even controlling for the other instruments. In contrast, however, the RMW loading coefficient on volatility is insignificant ($t = 0.35$), whereas that of the dividend-price ratio and lagged beta are both significant at the 1% level. Thus, it seems that the RMW exposure of BAB falls when market valuations seem "cheap" and the spread in betas across stocks is

Table IA.1: Factor model regressions with scaled variables

Even numbered columns of this table presents estimates from regressions of the form:

$$BAB_t = \alpha_0 + \boldsymbol{\alpha}_1' \boldsymbol{z}_{t-1} + \sum_{k=1}^{K} (\gamma_{k,0} + \boldsymbol{\gamma}_{k,1}' \boldsymbol{z}_{t-1}) f_{kt} + \nu_t,$$

where f_k is the return on factor $k = 1, .., K$. Odd numbered columns present restricted versions of these regressions in which $\alpha_1 = 0$ and $\gamma_{k,1} = 0$, $k = 1, 2, ..., K$. The vector of instruments, $\boldsymbol{z}_{t-1}$, includes: the log realized volatility on BAB (VOL), the difference between BAA and AAA bond yields (DEF), the log dividend-price ratio on the CRSP value-weighted index (DP), and the difference in market betas between the long and short legs of BAB ($Beta$) before hedging the market exposure. The instruments are demeaned and standardized so that the $\boldsymbol{\alpha}_1$ coefficients are the impact, in percentage points, of a one standard deviation shock to the instrument. In Columns (1) and (2), the only factor is MKT; in Columns (3) and (4), the factors are MKT, SMB, and HML; in Columns (5) and (6), the factors are all six in the FF6 model; in Columns (7) and (8), the only factor is the mimicking portfolio for growth in the unfiltered consumption series of Kroencke (2017); and in Columns (9) and (10), the only factor is the mimicking portfolio for intermediary leverage of Adrian, Etula, and Muir (2014) (see Section 4.2 for details). Parentheses below point estimates contain t-statistics based on White (1980) heteroskedasticity-robust standard errors. The sample period is July 1965 to December 2016.

	CAPM		FF3		FF6			Consumption			Leverage	
	(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)		(9)	(10)
α_0	0.42	0.39	0.38	0.44	−0.18	0.31	α_0	0.42	0.40	α_0	−0.01	0.19
	(2.17)	(2.14)	(2.20)	(2.82)	(−0.98)	(1.92)		(2.25)	(2.08)		(−0.05)	(1.04)
α_1^{VOL}		−0.70		−0.70		−0.66	α_1^{VOL}		−0.70	α_1^{VOL}		−0.69
		(−2.59)		(−3.24)		(−3.29)			(−2.19)			(−2.17)
α_1^{DEF}		0.07		0.38		0.55	α_1^{DEF}		0.26	α_1^{DEF}		0.34
		(0.20)		(1.49)		(2.64)			(0.66)			(1.10)
α_1^{DP}		−0.38		−0.35		−0.32	α_1^{DP}		−0.44	α_1^{DP}		−0.48
		(−1.33)		(−1.41)		(−1.16)			(−1.23)			(−1.26)
α_1^{Beta}		−0.11		−0.12		0.13	α_1^{Beta}		−0.02	α_1^{Beta}		−0.08
		(−0.63)		(−0.70)		(0.68)			(−0.08)			(−0.35)
$\gamma_{MKT,0}$	0.14	0.18	0.31	0.38	0.47	0.53	$\gamma_{Cons,0}$	0.25	0.28	$\gamma_{LMP,0}$	0.52	0.41
	(2.41)	(3.97)	(5.46)	(8.82)	(8.85)	(11.39)		(4.68)	(6.59)		(6.53)	(6.38)
$\gamma_{MKT,1}^{VOL}$		−0.01		0.02		0.10	$\gamma_{Cons,1}^{VOL}$		0.71	$\gamma_{LMP,1}^{VOL}$		10.78
		(−0.20)		(0.45)		(1.97)			(0.12)			(1.01)
$\gamma_{MKT,1}^{DEF}$		−0.16		−0.23		−0.17	$\gamma_{Cons,1}^{DEF}$		−8.13	$\gamma_{LMP,1}^{DEF}$		12.16
		(−1.82)		(−3.82)		(−3.29)			(−1.12)			(1.32)
$\gamma_{MKT,1}^{DP}$		0.26		0.28		0.18	$\gamma_{Cons,1}^{DP}$		5.58	$\gamma_{LMP,1}^{DP}$		0.20
		(4.13)		(4.55)		(2.76)			(0.81)			(0.01)
$\gamma_{MKT,1}^{Beta}$		−0.26		−0.27		−0.28	$\gamma_{Cons,1}^{Beta}$		−17.59	$\gamma_{LMP,1}^{Beta}$		−16.06
		(−5.68)		(−5.86)		(−5.41)			(−4.00)			(−1.79)
$\gamma_{SMB,0}$			−0.62	−0.70	−0.53	−0.64						
			(−7.50)	(−11.10)	(−6.86)	(−10.35)						
$\gamma_{SMB,1}^{VOL}$				−0.07		0.01						
				(−0.79)		(0.20)						
$\gamma_{SMB,1}^{DEF}$				−0.02		0.02						
				(−0.16)		(0.28)						
$\gamma_{SMB,1}^{DP}$				−0.15		−0.14						
				(−1.48)		(−1.60)						
$\gamma_{SMB,1}^{Beta}$				0.13		0.10						
				(1.75)		(1.34)						

Continued on next page

especially high, with the confluence of these phenomena coinciding with low BAB volatility.

Table IA.1: Continued

	CAPM		FF3		FF6			Consumption			Leverage	
	(1)	(2)	(3)	(4)	(5)	(6)		(7)	(8)		(9)	(10)
$\gamma_{HML,0}$			0.32	0.25	0.14	0.04						
			(3.45)	(3.51)	(1.41)	(0.44)						
$\gamma^{VOL}_{HML,1}$				0.15		0.10						
				(1.95)		(1.24)						
$\gamma^{DEF}_{HML,1}$				0.06		0.18						
				(0.54)		(2.22)						
$\gamma^{DP}_{HML,1}$				−0.01		0.11						
				(−0.14)		(0.81)						
$\gamma^{Beta}_{HML,1}$				0.11		0.14						
				(1.34)		(1.28)						
$\gamma_{MOM,0}$					0.32	0.17						
					(3.95)	(3.39)						
$\gamma^{VOL}_{MOM,1}$						0.18						
						(3.01)						
$\gamma^{DEF}_{MOM,1}$						0.07						
						(1.39)						
$\gamma^{DP}_{MOM,1}$						0.11						
						(1.41)						
$\gamma^{Beta}_{MOM,1}$						0.01						
						(0.15)						
$\gamma_{RMW,0}$					0.41	0.12						
					(4.10)	(1.12)						
$\gamma^{VOL}_{RMW,1}$						0.03						
						(0.35)						
$\gamma^{DEF}_{RMW,1}$						−0.04						
						(−0.40)						
$\gamma^{DP}_{RMW,1}$						−0.31						
						(−2.97)						
$\gamma^{Beta}_{RMW,1}$						−0.39						
						(−3.44)						
$\gamma_{CMA,0}$					0.64	0.38						
					(4.88)	(3.09)						
$\gamma^{VOL}_{CMA,1}$						−0.00						
						(−0.04)						
$\gamma^{DEF}_{CMA,1}$						−0.19						
						(−1.29)						
$\gamma^{DP}_{CMA,1}$						−0.06						
						(−0.38)						
$\gamma^{Beta}_{CMA,1}$						−0.08						
						(−0.53)						
R^2	0.02	0.12	0.20	0.33	0.34	0.50	R^2	0.05	0.08	R^2	0.12	0.16

References

Adrian, T., Etula, E. and Muir, T. (2014), Financial intermediaries and the cross-section of asset returns, *Journal of Finance* 69, 2557–2596.

Breeden, D. T., Gibbons, M. R. and Litzenberger, R. H. (1989), Empirical tests of the consumption-oriented capm, *Journal of Finance* 44, 231–262.

Kroencke, T. A. (2017), Asset pricing without garbage, *The Journal of Finance* 72, 47–98.

White, H. (1980), A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity, *Econometrica* 48, 817–838.