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Abstract

Using nearly a trillion dollars of live trading data from a large institutional money manager across 19 developed equity markets over the period 1998 to 2011, we measure the real-world transactions costs and price impact function facing an arbitrageur and apply them to size, value, momentum, and short-term reversal strategies. We find that actual trading costs are less than a tenth as large as, and therefore the potential scale of these strategies is more than an order of magnitude larger than, previous studies suggest. Furthermore, strategies designed to reduce transactions costs can increase net returns and capacity substantially, without incurring significant style drift. Results vary across styles, with value and momentum being more scalable than size, and short-term reversals being the most constrained by trading costs. We conclude that the main anomalies to standard asset pricing models are robust, implementable, and sizeable.

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Empirical asset pricing studies largely focus on the expected gross returns of assets, without taking transaction costs into account. For investors, however, the net of transaction costs returns are the critical input for investment decisions. A large literature documents several strong predictors for the cross-section of average returns, which have been thrust into the efficient markets debate as challenges or anomalies to standard asset pricing models. However, an understanding of the net of transaction costs returns and capacity limits of strategies based on these predictors is crucial in determining whether they are robust, implementable, and sizeable or face significant arbitrage limits that prevent traders from profiting from them.

We explore the cross-section of net of trading cost returns using a unique dataset of live trading data, representing nearly one trillion dollars of live trades from a large institutional money manager from 1998 to 2011 across 19 developed equity markets. The data offer a singular look into the real-time trading costs of an investor who resembles the theoretical "arbitrageur." Indeed, our institutional investor implemented strategies similar to those we examine here. Specifically, we evaluate the robustness of several prominent capital market anomalies—size, value, momentum, and short-term reversals, which dominate the cross-sectional return landscape[1]—to trading costs and assess their implied capacity limits.

The results help shed light on the market efficiency debate surrounding these return predictors. If some predictors do not survive real-world trading costs, or only survive at small dollar investments (low capacity), then limits to arbitrage may simply prevent them from being exploited and disappearing. On the other hand, if returns after trading costs are significantly positive at very large size, then strategies based on these predictors may be implementable and sizeable, either offering profit opportunities to be exploited by an arbitrageur or perhaps representing a risk factor in the economy that a significant fraction of the market is exposed to. We remain agnostic on risk versus non-risk based explanations for these predictors and simply estimate the implementation frictions and real-world costs of each strategy.

Our live trading data contains some unique features not previously studied in the literature. For example, our dataset contains both the actual trade as well as the *intended* trade of our manager. The intent of the trade provides us with a model-implied portfolio weight and theoretical price, where the difference between the executed trade and intended (theoretical) trade can be used to measure "implementation shortfall" in order to capture the opportunity cost of a trade in addition to its execution cost. The unique look into the opportunity cost of a trade allows us to explore the tradeoff

[1] See Fama and French (1996, 2008, and 2012), Asness, Moskowitz, and Pedersen (2012), and Ilmanen (2011) for a description of cross-sectional stock return predictors that appear most robust in the data.

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between price impact and the opportunity cost of not trading at various fund sizes. In addition, we can look separately at buy versus sell initiated trades, trades to speculate versus cover a position, and sell versus short-sell trades, all of which may face different costs, and hence can provide a more accurate measure of the trading costs of long-short portfolios common in the literature. Finally, in addition to estimating these costs for NYSE and NASDAQ stocks, our data also covers 18 other developed equity markets internationally, providing the first look at the trading costs of similar strategies deployed in many different markets simultaneously.

Armed with our trading cost measures from live trade data, we evaluate the robustness of size, value, momentum, and short-term reversal strategies to trading costs and compute their break-even sizes or capacities. Assessing standard long-short strategies commonly used in the literature we find that size, value, and momentum survive transactions costs at fairly substantial sizes, but that short-term reversals do not. Break-even fund sizes for the Fama and French long-short factors of size, value, and momentum are 103, 83, and 52 billion dollars among U.S. securities, respectively, and 156, 190, and 89 billion dollars globally. Short-term reversal strategies, on the other hand, do not survive transactions costs at sizes greater than $9 billion in the U.S. or $13 billion globally. Moreover, a combination of value and momentum has even higher capacity ($100 billion U.S., $177 billion globally) due to netting of trades across negatively correlated positions.

However, since the standard academic portfolios are not designed to address or consider transactions costs in any way, it may be misleading to conclude what the efficacy or capacity of these strategies might be without recognizing what a real-world trader might do to respond to transactions costs. For example, Garleanu and Pedersen (2012) shows theoretically how a strategy's net performance can be significantly improved by taking transaction costs into account when computing optimal portfolios.

We therefore construct portfolios that seek to minimize trading costs or maximize net of trading cost returns, subject to maintaining similar exposure to the style of the strategy. Using a static portfolio optimization, we minimize trading costs subject to a tracking error constraint that seeks to avoid style drift, using our estimated transaction cost model from the live trading data that measures both price impact and the opportunity cost of a trade. We analyze how much after-transaction costs returns can be improved across styles through portfolio optimization and assess the tradeoff between reducing trading costs and introducing tracking error across the equity style strategies. We find that value and momentum offer the most favorable tradeoffs, where after-cost net alphas can be improved significantly without incurring large tracking error or reductions in before-cost gross alphas. Size and especially short-term reversal strategies are more difficult to optimize since minimizing trading costs results in a steeper decline in gross alpha or increase in tracking error. At their maximums, within one percent tracking error, optimized versions of size, value, momentum, and short-term reversal strategies applied globally generate break-even sizes of 1,807, 811, 122 and 17 billion dollars, respectively. These findings indicate that the main anomalies to standard asset pricing models, particularly size, value, and momentum, are robust, implementable, and sizeable.

We view our trading cost estimates as those facing a real arbitrageur engaged in similar strategies, and not those of the average investor. However, the ability to generalize our cost estimates to other strategies studied in the academic literature or perhaps to other institutional money managers depends on how exogenous the trading costs are to the portfolios being traded by our manager. For instance, if the portfolios themselves are a function of trading costs, then the cost estimates we obtain would be most relevant to the specific portfolios run by the manager, but less relevant for portfolios not identical to the manager's. We argue and show that the trading costs we estimate are indeed fairly independent from the portfolios being traded. First, we only examine the live trades of our manager's longer-term strategies, where the portfolio formation process generating the desired trade is separate from the trading process executing it. This is because the set of intended trades is primarily created from longer-term models (such as value and momentum effects, which we study here) and from specific client mandates that often adhere to a benchmark subject to a tracking error constraint of a few percent. This separation of portfolio formation from the trading process is a feature of the longer-term trades we examine and would not apply, for instance, to high frequency trading strategies. Hence, we exclude all high frequency trading from our trade data. The manager then trades to the intended positions in the cheapest possible manner using a proprietary trading algorithm, but importantly, that algorithm does not make any buy or sell decisions. The algorithm merely determines how best to execute the desired trades. The average realized trade horizon for completion is less than one day and no more than three days, indicating relatively low tracking error to the intended trade.

To rule out any concerns over the endogeneity of portfolio weights to (expected) transactions costs, we also estimate our trading costs using only the first trade from new inflows from long-only mandates that specifically adhere to a benchmark. The initial trades to a benchmark from inflows only are exogenous to trading costs since there is no scope for deviation. We find that the trading costs from these exogenous, initial trades are identical to those from all other trades.[2]

[2] These costs, however, are not applicable to high frequency (intra-day) strategies where the trading process and the portfolio weights are determined simultaneously and endogenously. Again, these strategies are excluded from our sample.

Our unique approach has some advantages and disadvantages relative to other studies on trading costs. While explicit trading costs, such as commissions and bid/ask spreads, can be measured relatively easily and have been studied at length,[3] the full costs of trading are typically dominated by the cost of price impact (Kyle (1985), Easley and O'Hara (1987), Glosten and Harris (1988), Hasbrouck (1991a, 1991b), Huberman and Stanzl (2000), Breen, Hodrick, and Korajczyk (2002), Loeb (1983), Keim and Madhavan (1996, 1997), and Knez and Ready (1996)), where transaction costs are a function of the size of a trade and its ability to move the market price of an asset. Previous studies attempting to estimate transactions costs, particularly price impact costs, rely on theoretical models and typically use one of two types of data: daily spread and price information or intra-day transaction level data (containing time-stamped volume, transactions prices, and bid-ask spread information), which are often aggregated over time intervals ranging from five to 30 minutes.[4] However, both the daily and intra-daily data provide estimates of transactions costs for the average trader, which reflects the aggregated volume of a variety of traders with different intentions, objectives, and information. Our unique trade data reflects the trading costs of a single arbitrageur, which may be a better proxy for the marginal investor's trading costs. While a few other studies have looked at proprietary trade data from a large institution or set of institutions (e.g., Keim (1995), Keim and Madhavan (1997), Engle, Ferstenberg, and Russell (2008)), the data used in these studies is much more limited in breadth and depth and covers only a few years of U.S. data. In addition, the data used in all of these studies only reflects *executed* trades and contains no information about the process generating the trades (at least we are not aware of any study that does). Our unique look at the intent of the trade provides a novel source of trading cost information across a variety of markets.

The drawback of our data is that we can only measure costs for one particular money manager, albeit a large manager who invests in many of the anomalies we investigate. In addition, our sample period is shorter than those that use daily or TAQ data, though it is much longer (and covers a broader cross-section of stocks and markets) than the few other papers that use proprietary trade data (e.g., Keim (1995), Keim and Madhavan (1997), and Engle, Ferstenberg, and Russell (2008)). We

[3] Schultz (1983) and Stoll and Whaley (1983) examine the effect of commissions and spreads on size portfolios.

[4] Examples of trading cost measures using daily price and spread data include Roll (1984), Huang and Stoll (1996), Chordia, Roll, and Subrahmanyam (2000), Amihud (2002), Acharya and Pedersen (2005), Pastor and Stambaugh (2003), Watanabe and Watanabe (2006), Fujimoto (2003), Korajczyk and Sadka (2008), Hasbrouck (2009), and Bekaert, Harvey, and Lundblad (2007). Examples using transaction level data from the trade and quote (TAQ) data from the NYSE, Rule 605 data (e.g., Goyenko, Holden, and Trzcinka (2009)), or proprietary broker data (e.g., Engle, Ferstenberg, and Russell (2008)), include Goyenko (2006), Sadka (2006), Holden (2009) and for a comparison of daily versus intra-daily measures, see Lesmond, Ogden, and Trzcinka (1999), Lesmond (2005), Engle, Ferstenberg and Russell (2008), Lehmann (2003), Werner (2003), Hasbrouck (2009), and Goyenko, Holden, and Trzcinka (2009).

also use our trading data to estimate a price impact model using observable stock and market variables that we then project out of sample to estimate trading costs in earlier time periods.

Our conclusions differ substantially from other papers that use trading cost estimates from daily or intra-daily data to study the effects of price impact on the profitability and size of investment strategies. Generally, these papers find a huge effect from transactions costs on the viability of these strategies. For instance, Knez and Ready (1996) find that trading costs swamp the profitability of lead-lag trading strategies between large and small firms. Mitchell and Pulvino (2001) estimate commissions and price-impact costs for a merger arbitrage portfolio and find that trading costs account for 300 basis points per year. Chen, Stanzl, and Watanabe (2002) estimate that only small maximal fund sizes are attainable before costs eliminate profits on size, book-to-market, and momentum strategies. Lesmond, Schill, and Zhou (2003) find that trading costs eliminate the profits to momentum strategies and Korajczyk and Sadka (2004) find that break-even fund sizes for long-only momentum portfolios are in the two to five billion dollar range. We find that our real-world estimates of transaction costs are much smaller—approximately one tenth as large—as those reported in the literature. As a result our estimates of the break-even capacity of size, value, and momentum strategies are more than an order of magnitude larger than those implied by previous studies.

Two key differences between our study and the previous literature are driving these results. First, previous studies compute trading costs for the *average* investor using aggregated daily or transaction-level data, which are about ten times higher than the costs we estimate for a large arbitrageur. Second, these studies examine portfolios that do not consider transactions costs in their design, which we show can significantly reduce trading costs. Both innovations result in trading cost estimates (break-even fund sizes) that are an order of magnitude smaller (larger) than previous studies suggest. We believe our estimates are much closer to the real-world transactions costs facing an arbitrageur, which has import for addressing market efficiency questions with respect to the asset pricing anomalies we study.

The paper proceeds as follows. Section I describes the trade data and equity style strategies we examine and discusses the manager's portfolio formation and trading processes. Section II outlines our methodology for measuring transactions costs and examines realized trading costs across time, markets, and trade types, which we then use to estimate a price impact function based on observable trade, stock, and market characteristics. Section III presents the cross-section of net after-cost returns to the style strategies and computes their break-even capacities. Section IV constructs strategies using trading cost optimization that seek to maximize net after-cost returns subject to a tracking error or style constraint. Section V concludes.

I. Data and Methodology

We detail the equity style portfolios and trading data we examine, including a description of how our institutional manager generates desired trades and executes and manages trading.

A. Equity style portfolios

We examine long-short equity style portfolios commonly used in the literature pertaining to size, value, momentum, and short-term reversals, as well as combinations of these portfolios. We focus on these equity styles because research shows they capture much of the cross-sectional variation in returns (Fama and French [1996, 2008] and Asness, Moskowitz and Pedersen [2012]), and are also, not coincidentally, the focus of attention in the investment management industry.

We examine equity strategies in U.S. and developed international markets. We obtain stock returns and accounting data from the union of the CRSP tapes and the XpressFeed Global database. Our U.S. equity data include all available common stocks on the merged CRSP/Compustat data between July 1926 and December 2011. Our global equity data include all available common stocks on the XpressFeed Global database for 18 developed markets. The international data run from January 1983 to December 2011. We assign individual stocks to the corresponding market based on the location of the primary exchange. For international companies with securities traded in multiple markets, we use the primary trading vehicle identified by XpressFeed. Table A1 in the Appendix presents the summary statistics for the global data, including number of stocks and average market caps for stocks in each country.

Our portfolio construction closely follows Fama and French (1996 and 2012) and Asness and Frazzini (2012). Our global portfolios are country neutral in the sense that we form long-short portfolios within each country and then compute a global factor by weighting each country's long-short portfolio by the country's total (lagged) market capitalization. The market factor, MKT, is the value-weighted return on all available stocks minus the one-month U.S. Treasury bill rate. The size and value factors are constructed using six value-weighted portfolios formed on size and book-to-market. At the end of June of year *t*, stocks are assigned to two size-sorted portfolios based on their market capitalization. For the U.S., the size breakpoint is the median NYSE market equity. For the international sample the size breakpoint is the 80th percentile by country (to roughly match the U.S. size portfolios). Since some countries have a small cross section of stocks in the early years of our sample, we use conditional sorts that first sort on size, then on book-to-price, in order to ensure we have enough securities in each portfolio (the U.S. sorts are always independent). Portfolios are value-weighted and reconstructed every month using the most recent price following the methodology of

Asness and Frazzini (2012), and rebalanced every calendar month to maintain value weights.[5] Following Fama and French (1992) we assume that accounting variables are known with a minimum six-month lag and align book value of the firm at the end of the firm's fiscal year, ending anywhere in calendar year *t-1* to June of calendar year *t*. In order to be included in any of our tests we require a firm to have a non-negative book value and non-missing price at fiscal yearend as well as in June of calendar year *t*.

The size factor, SMB (Small minus Big), is the average return on the three small portfolios minus the average return on the three big portfolios = 1/3 (Small Value + Small Neutral + Small Growth) - 1/3 (Big Value + Big Neutral + Big Growth). The value factor, HML (High minus Low), is the average return of the two value portfolios minus the average return of the two growth portfolios = HML = 1/2 (Small Value + Big Value) - 1/2 (Small Growth + Big Growth).

The momentum and short term reversal long-short portfolios are constructed in a similar manner, where we use six value-weighted portfolios formed on size and prior returns (the cumulative return in local currency from months *t-12* to *t-2* in the case of momentum or the return at *t-1* for short-term reversals). The portfolios are the intersections of two portfolios formed on size and three portfolios formed on prior returns. The momentum factor UMD is constructed as UMD = 1/2 (Small High + Big High) - 1/2(Small Low + Big Low) and the short term reversal factor STR = 1/2 (Small Low + Big Low) - 1/2(Small High + Big High).

All portfolio returns are in $US and excess returns are relative to the one-month U.S. Treasury bill rate. We include delisting returns when available in CRSP. Finally, since some of our variables are computed from closing prices we skip one trading day between portfolio formation and investment, both when reconstituting the breakpoints and when rebalancing stocks in the portfolio. This serves two purposes. First, it ensures that our portfolios are implementable in that they use only information available at portfolio formation. Second, it avoids a mechanical negative autocorrelation in returns induced by bid-ask bounce, which would overstate returns to STR and HML and understate returns to UMD.

While the above portfolios are standard to the literature (see Fama and French (2012) and Ken French's webpage), these portfolios are not designed in any way to deal with transactions costs. As

[5] To obtain shareholders' equity we use Stockholders' Equity (SEQ), but if not available, we use the sum of Common Equity (CEQ) and Preferred Stock (PSTK). If both SEQ and CEQ are unavailable, we will proxy shareholders' equity by Total Assets (TA) minus the sum of Total Liabilities (LT) and Minority Interest (MIB). To obtain book equity, we subtract from shareholders' equity the preferred stock value (PSTKRV, PSTKL, or PSTK depending on availability). Finally, to compute book value per share (B) we divide by common shares outstanding (CSHPRI). If CSHPRI is missing, we compute company-level total shares outstanding by summing issue-level shares (CSHOI) at fiscal yearend for securities with an earnings participation flag in the security pricing file.

such, they may not be proper benchmarks to evaluate the after-cost efficacy or capacity of an anomaly since break-even capacities may depend critically on the design of the portfolios, including simple steps taken to limit trading costs. Therefore, we also consider in Section IV a set of style portfolios specifically designed to optimize for trading costs and maximize after-cost returns and examine their trading costs and break-even sizes.

B. Trade execution data

Our trading costs data are drawn from the internal trade execution database maintained by our institutional manager, who manages global strategies ranging from benchmark-oriented long-only strategies (including mutual funds) to absolute return alternative strategies in a variety of asset classes. The database is compiled by the execution desk and covers all trades executed algorithmically (described below) in any of the firm's funds since inception, excluding trades associated with high frequency (intra-day) models.

The data contains information about orders, execution prices, and quantities. We exclude all non-equity and emerging markets trades, restricting the sample to developed market equity transactions, which includes cash equities and equity swap transactions. For each individual order we collect the trade-order-set identifier, stock identifier, order size, trade horizon, order type, region and country, size and portfolio type, benchmark, ex-ante predicted beta (described below), model-implied price P^{theory}, arrival price P^{start}, execution time, execution price P^{ex} and execution quantity Q^{ex}.

The trade-order-set has a unique identifier that allows the mapping of individual trades to corresponding portfolios. A trade-order-set is simply a basket of trades submitted together. Usually a trade-order-set contains trades from a single fund, but in some cases trades from multiple funds are bundled in the same set. In a given day, the same stock can appear in multiple order sets. Therefore, we use a trade-order-set stock pair as our cross sectional unit of observation for each date. Hereafter, we'll use the term "order" to indicate a trade-order-set stock pair submission date triplet.

The order size is the desired number of shares to trade when orders are submitted. This, of course, can be different from the sum of the actual execution quantities, such as when the trading algorithm does not fully execute orders. The trade horizon is the target trade duration (in days) at the time of order submission. Similar to the trade horizon, the actual total trade duration is endogenous and depends on realized fill rates and may be different from the target duration. The order type indicator identifies the transaction as "buy-long", "buy-to-cover", "sell-long", and "sell-short". The size and portfolio indicators classify each portfolio as "large" or "small" in terms of market capitalization (based on their relevant benchmark, such as Russell 1000 or 2000 in U.S. equities, for example) and "long/short" or "long-only" based on the portfolio's mandate.[6] The benchmark identifies the relevant benchmark, while the ex-ante predicted beta is a forecasted beta for each stock to the benchmark at the time of order submission, which is based on a regression using the past year of daily stock returns on the corresponding benchmark. The model price is defined as the stock price at portfolio formation. For portfolios constructed during market hours this is equal to the current price. For portfolios constructed off market hours this is the latest closing price. The arrival price is defined as the current price at the time the first order is submitted to the market, which is recorded by the trading algorithm when orders are submitted. The execution prices are time-stamped and incorporate commissions. Since the trading algorithms break orders into smaller pieces (and typically dynamically submit and cancel a range of limit orders), there are many executions per order.

The final dataset contains 5,310,387 distinct orders covering 9,128 stocks between August 1998 and December 2011 and totaling $721,430,000,000 in trades.[7] Panel A of Table I reports the amount traded (in $billions) for each year from 1998 to 2011 in the U.S. equity market, across the 18 international markets, for large cap and small cap stocks separately, and for long-short and long-only portfolios separately. Not surprisingly, the amounts traded have grown substantially over time, with faster growth in the U.S. from $1.3 billion traded in 1998 to $95.5 billion in 2011. Equities traded in international markets have grown from $1.7 to $48.7 billion. Large cap (as defined by the Russell 1000 or MSCI) comprises the bulk of trades, accounting for $699.6 billion out of the total $721.4 billion traded over the sample period. Nevertheless, nearly $22 billion worth of trades were placed among small cap stocks. Finally, $536 billion, or roughly 75%, of trades occur for long-short strategies similar to those constructed in the literature.

Panel B of Table I reports time-series means, medians, standard deviations, minimums, and maximums of the number of stocks traded, number of countries traded, and number of exchanges traded on per year. A minimum of 389 stocks across eight countries on 11 exchanges are traded each year with a maximum of over 5,000 stocks in 19 markets across 33 exchanges. Panel C of Table I reports year-by-year time series averages in the style of Fama and MacBeth (1973) of the average trade size (in $thousands) and fraction of daily volume traded. The average (median) trade over time is $695,300 ($401,500), but there is substantial heterogeneity in the size of trades. The standard deviation of trade size is over $1 million and ranges from $52,600 to $5,939,100 over our sample

[6] We classify relaxed constraint portfolios, such as 130-30 or 140-40, as long-only. These represent only a small fraction of trades and excluding them or reclassifying them does not alter any of our results.
[7] Throughout our analysis we exclude netting trades (which are trades settled internally among accounts). Including netting trades does not affect any of our results.

period. As a fraction of daily volume, the average (median) trade represents 1.2% (0.5%) with a standard deviation of 2.1% of total daily volume that ranges from 0.1% to 13.1%. This variation allows us to measure the price impact function across a wide range of trade sizes.

Table A2 in the Appendix reports the fraction of firms covered and the fraction of total market cap covered by our trading database for all stocks on CRSP and Xpressfeed Global. Panel A of Table A2 reports the year-by-year coverage, which reaches as high as 60 percent of names and 97 percent of market cap. Panel B reports the coverage by region and year. Finally, Panel C reports the fraction of the style strategies our data covers that were actually traded (fraction of absolute value of portfolio weights). For the U.S., the trade data covers about 65% of the portfolio weights of the factors we study. Internationally, we cover about 36%, with little variation across style portfolios.

C. Portfolio Formation and Trading Process

Understanding how portfolios and trades are created and executed is important for interpreting the trading costs measures we obtain from these data. Trades are executed using proprietary, automated trading algorithms designed and built by the manager. Before describing how these algorithms are designed, we first briefly describe the portfolio generation process.

Importantly, we examine trades where the portfolio generation process is separate from the trading process. Specifically, we only examine the live trades of our manager's longer-term strategies, where the portfolio formation process generating the desired trade is separate from the trading process executing it. This is because the set of intended trades is primarily created from longer-term models (such as value and momentum effects, which we study here) and from specific client mandates that often adhere to a benchmark subject to a tracking error constraint of a few percent. This separation of portfolio formation from the trading process is a feature of the longer-term trades we examine and would not apply, for instance, to high frequency trading strategies. Hence, we exclude all high frequency trading from our trade data. For these non-high frequency trades, a set of desired trades is first obtained for a portfolio from a set of optimal holdings based on a particular style or model, often using historical data. While the models employed are often more complex, a lot of the portfolio generation process is based on style portfolios similar to those from the academic literature and those we investigate in this paper. Once a theoretical portfolio is determined, an optimization process generates portfolio weights subject to various constraints which can include risk constraints and other client mandated constraints (e.g., tracking error to a benchmark, shorting constraints, industry and position limits, etc.). Once an optimal portfolio is generated, it implies a set of trades that moves the current portfolio to the desired portfolio. These trades are then loaded into the algorithms which are responsible for implementing or executing the desired trades, where the algorithms do not make any explicit aggregate buy or sell decisions. Those decisions are made solely by the portfolio generation process. The trading algorithms, however, determine the trade horizon—how long it will take to buy or sell those positions. The average realized trade horizon for completion is less than one day and no more than three days, indicating relatively low tracking error to the intended trade.

The trading algorithms directly and anonymously access market liquidity through electronic exchanges and, in order to minimize market impact, are designed to provide rather than demand liquidity by using a system of limit orders (with prices generally set to buy at the bid or below and sell at the ask or above) that dynamically break up total orders (parent orders) into smaller orders (child orders), with the sizes of child orders and the time in which they are sent being randomized. Given these features of trading, the costs we estimate are not those facing the average investor in the market, but rather those of an arbitrageur engaged in these strategies and hence much closer to the marginal investor in such strategies. We assume other large arbitrageurs are (or could be) trading similarly to our manager. We are treating the trading costs as (mostly) exogenous to the portfolio weights and the set of securities being traded. There are several reasons to justify this assumption. First, as described above the portfolio determination process is separate from the trading process. Second, we throw out high frequency trades where this isn't true. Third, many of the client mandates force the portfolio to trade to a benchmark with low tracking error, leaving little scope for deviation. Fourth, the trading algorithm controls trade execution, not stock selection, and since most trades are completed within a day to three days, the intended trades for the long-term strategies we study are met in short order with only small deviations in timing. Finally, to rule out any remaining concerns over the endogeneity of portfolio weights to transactions costs, we also estimate our trading costs using only the first trade from new inflows from long-only mandates that specifically adhere to a benchmark. The initial trades to a benchmark from inflows only are exogenous to trading costs since there is no scope for deviation. In the next section we show that the trading costs from these exogenous initial, trades are identical to those from all other trades.

II. Realized Trading Costs

We first describe our methodology for estimating trading costs. Then, using the live trading data we present summary statistics of trading costs over time, for different types of stocks, and across markets for various types of trades. Finally, we estimate an econometric model for the price impact function based on observable variables that we can then apply to various portfolios out of sample.

A. Measuring Transactions Costs: Price Impact and Implementation Shortfall

To measure transactions costs, we use an *implementation shortfall* (*IS*) methodology as defined in Perold (1988). Implementation shortfall measures the difference between a theoretical or benchmark price (e.g., a model price) and an actual traded price, scaled by the amount traded. We also define *market impact* (*MI*) in a similar way by looking at the difference between an arrival price (i.e., the price that exists when a trade begins in the market) and an actual traded price, scaled by the amount traded. The difference between these two measures represents pre-trade moves that might occur from the time a model, or theoretical, portfolio is generated and the time trading begins. The following equations illustrate the two costs:

$$\begin{aligned} IS &= ret_{p,theory} - ret_{p,actual} \\ &= ret_{p,theory} - \text{cost}_{execution} - \text{cost}_{opportunity} \end{aligned} \tag{1}$$

where the opportunity cost is the difference in returns before trading costs between the theoretical and actual portfolio and the cost of execution is measured as

$$\text{cost}_{execution} = Q_{+}^{ex}(P^{ex} - P^{theory}) + Q_{-}^{ex}(-P^{ex} + P^{theory}) \tag{2}$$

with Q_{+}^{ex} and Q_{-}^{ex} representing the quantity of shares bought and sold, respectively. Equation (2) takes into account bid-ask spread, market impact, and commissions. Part of the cost of execution is market impact costs, which we measure as

$$MI = Q_{+}^{ex}(P^{ex} - P^{start}) + Q_{-}^{ex}(-P^{ex} + P^{start}) \tag{3}$$

where the cost of execution is generally at least as great as the market impact cost.

Both *IS* and *MI* are effective measures of the costs of trading. Implementation shortfall measures the total amount of slippage a strategy might experience from its theoretical returns. For the strategies we examine, the theoretical or benchmark price is often defined as the closing price at the time the strategy's desired holdings and trades are generated (e.g., prior day's closing price). Market impact is similarly defined as the opening price on the day the strategy begins trading to the new desired holdings (e.g., current day's opening price). Our unique data allow us to calculate implementation shortfall since we have the underlying theoretical model prices our institutional manager was attempting to trade to. This gives us a novel glimpse into the slippage between theoretical and actual portfolio construction and implementation costs. We can therefore measure the elusive opportunity cost of trading, which is absent from previous studies since data on the model generating the portfolios is typically unavailable to the researcher.

In expectation, *IS* and *MI* should be the same, since we assume that the pre-trade moves (or the difference between the two) are randomly distributed with expected mean of zero. In other words, we assume that the strategies are not forming opinions about the overnight moves in the stocks they desire to hold such that there would be an expected difference in the two measures, or in cases where trading begins almost immediately after portfolio formation, the model price and arrival price are effectively the same. Empirically, we will estimate *IS* and *MI* separately and examine the differences.

There are many ways to think about transactions costs and many different objectives when doing so. Our notion of trading costs computes the difference between the results of a theoretical portfolio which has zero transactions costs by definition and the results of a practical portfolio which attempts to track the theoretical portfolio but is subject to actual traded prices. Effectively, our cost estimates measure how much of the theoretical returns to a strategy can actually be achieved in practice.

There are other ways to measure transactions costs, such as to compare actual traded prices over the trading period to other possible traded prices that existed during the same period. For example, one could compare the actual trade price to the average trade weighted price achieved in the market, such as the volume weighted average price (VWAP) or the price that would have been achieved using market orders over the same period. For example, we could define market impact costs relative to the VWAP as,

$$MI^{relative} = Q_{+}^{ex}(P^{ex} - VWAP) + Q_{-}^{ex}(-P^{ex} + VWAP). \tag{4}$$

However, these calculations tell us more about the effectiveness of a trade relative to other traders in the market at the same time, but they do not help us understand whether the theoretical returns of a strategy are achievable or what the breakeven capacity is to being profitable, which is the goal of this paper. For comparison, we show both market impact measures, recognizing that the relative market impact measure in equation (4) will generally be a lot lower than our estimates of trading costs from the theoretical portfolio in equation (3), where the latter is most relevant for thinking about capacity.

B. Estimated Trading Costs

Table II reports the mean, median, and value-weighted mean (weighted by dollar value of trades) of the market impact (*MI*) and implementation shortfall (*IS*) estimates we get from the live trading data, following equations (1)–(3). Specifically, the cross-sectional mean, median, and weighted mean are computed each month and the time-series average of these monthly measures are reported, where each month is weighted by the number of stocks traded in that month. Standard errors of the monthly estimates are reported in the style of Fama and MacBeth (1973) in the bottom half of each panel. Panel A reports results for the full sample period from 1998 to 2011. The first column reports the summary statistics for all trades, where the mean market impact measure is 12.18 basis points and the mean implementation shortfall measure is 13 basis points. The 0.82 basis point difference between the two represents the opportunity cost of trading or difference between the intended model and the actual portfolio at the time of trading. The median *MI* and *IS* costs are quite a bit lower at 7.98 and 10.34 basis points, respectively, suggesting that trading costs are positively skewed by a few much more expensive trades. For the median trade, the opportunity cost is much higher at 2.36 basis points. Weighting trades by their dollar value, the value-weighted means are quite a bit higher at 18.65 basis points for market impact and 20.20 basis points for implementation shortfall, which indicates that the largest trades are the most expensive trades, consistent with non-proportional trading cost models that argue that trading costs increase substantially with trade size. The standard errors for each of these estimates are small and typically close to one basis point, so the difference between the *MI* and *IS* measures are usually within two standard errors of zero.

Permanent vs. temporary price impact. In measuring the price impact of a trade, as defined in equation (3), it is interesting to assess how much of the impact is permanent versus temporary. The permanent component, while still a cost relative to the theoretical price, is driven by outside forces such as market demand, while the temporary component should reflect the trader's demand for immediacy, which once alleviated should result in price reversion. Figure 1 plots the average price movement in the 24 hours following a trade for the average trade from our live trading data. We restrict the sample to U.S. stock trades with a trading horizon of 1 trading day and compute average price impact during trading and the average overnight and intraday return over the subsequent 24 hours.[8] For simplicity, we estimate the average price impact during the day and split it equally across time during trading hours, doing the same for overnight returns and for the next day's returns. Hence, the plots in Figure 1 are linear. As Figure 1 shows, most (around 70%) of the initial price impact in our trade executions data appears to be permanent. Of the average 9 basis point market impact, only about 2.5 basis points are reversed over the next 24 hours. For our break even fund size calculations, we will use the entire price impact (permanent + temporary), but recognize that less than a third of this impact is temporary and represents immediacy.

By exchange. The next three columns of Panel A of Table II report the same statistics for NYSE-AMEX and NASDAQ traded stocks as well as for all stocks traded on international exchanges separately. Whether measured by market impact or implementation shortfall, the average

[8] We use U.S. trades only since coverage of open prices on XpressFeed global tends to be sparse. We use CRSP open prices to decompose daily returns into overnight and trading-day return.

costs of trading on the NYSE appear smaller than on NASDAQ or in international markets, which makes sense since the NYSE contains larger and more liquid firms. However, the median traded costs are more similar across exchanges. Looking at the differences between the *IS* and *MI* measures, the implied opportunity cost of trading appears to be similar across the exchanges.[9]

By size. The fifth and sixth columns of Panel A of Table II report results for large and small cap stocks separately. Clearly, large cap stocks face lower trading costs than small cap stocks. The average large cap stock trade generates 11.21 basis points of market impact costs compared to 21.27 basis points for small cap stocks. For the median and average dollar-weighted trade, the difference in trading costs between small and large cap stocks is similar. The opportunity cost or difference between *IS* and *MI* is similar among large and small cap stocks.

By portfolio type. The last two columns of Panel A of Table II report trading costs for trades made within the context of long-short portfolios and long-only portfolios separately. The trading costs for long only portfolios are larger than those made in long-short portfolios. The average long-only trade faces nearly 16 basis points of market impact costs compared to only 10 basis points for the average long-short trade. However, on a value-weighted basis, long-short portfolios face about the same or slightly larger costs, since long-short portfolios trade more extreme positions.

Robustness. Panel B of Table II repeats the estimates over the more recent sample period from 2003 to 2011. As shown in Table I, trading activity increased substantially starting in 2003, which also coincides (but not coincidentally) with the adoption of an electronic trading algorithm. Since the time-series averages we report in Table II weight each month by the number of stocks traded, which overweights the more recent periods, the results from 2003 to 2011 look very similar to those from the full sample from 1998 to 2011. Figure 2 summarizes the results from Table II for the price impact measure.[10]

Alternative cost measure. For comparison, Table A4 in the Appendix reports market impact costs relative to VWAP following equation (4), for our sample of U.S. trades. As Table A4 indicates, the trading costs when measured versus the VWAP are much lower than those in Table II that measured costs relative to the theoretical price. Instead of the 9.4 basis point average price impact in

[9] Figure A1 in the Appendix plots the mean and median market impact costs separately for each of the 19 equity markets we examine: Australia, Austria, Belgium, Canada, Switzerland, Germany, Denmark, Spain, Finland, France, U.K., Hong Kong, Italy, Japan, Netherlands, Norway, Singapore, Sweden, and the U.S. Singapore, Hong Kong, and Japan have some of the highest price impact costs, along with Australia. The U.S. and most of the European countries have the lowest price impact costs.

[10] Table A3 in the Appendix reports pooled means of the trading cost measures, rather than Fama and MacBeth averages, over the full sample period and the post-2003 sample period. The point estimates of trading costs are very similar.

Table II, the average price impact measured relative to the VWAP is only 3.7 basis points. The latter measure indicates that our manager paid 3.7 basis points on average relative to the VWAP or average traded priced during the same trading day. The former measure indicates, however, that our manager paid 9.4 basis points more relative to the theoretical prices it was trying to trade to. These differences are also roughly consistent with the permanent versus temporary decomposition of trading costs from Figure 1, where the demand for immediacy should show up as both a cost relative to the average price paid by traders as well as be reversed the following trading day.

We believe the theoretical price impact numbers are the most relevant for answering the question we pose in this paper, which is what are the implementation costs and break even sizes of strategies designed to exploit the asset pricing anomalies we investigate in this paper. Our conclusions using the theoretical price impact numbers will therefore be conservative relative to those using alternative price impact measures from average traded prices like the VWAP.

C. Exogenous Trades from Inflows

We argued above that the trading costs we estimate are (mostly) exogenous to the portfolio weights and set of securities being traded. To rule out any remaining concerns over the endogeneity of portfolio weights to transactions costs, we re-estimate our trading costs using only the first trade from new inflows coming from long-only mandates that specifically adhere to a benchmark. Specifically, we restrict the set of trades to new inflows coming from long-only mandates (that typically specify a benchmark and a tracking error constraint of no more than a few percent) and only examine the initial trade from each. These trades provide little scope for deviation or optimization and hence offer an exogenous estimate of trading costs.

Table III reports the mean, median, and value-weighted mean market impact measures for long-only trades coming from inflows only and for all other trades for comparison. The estimates are very similar and show no systematic differences in market impact across the two types of trades. The last column performs a formal statistical test on the differences and cannot reject that the two measures are identical. Cutting the trades by small and large cap stocks yields the same conclusions. Figure 3 summarizes the results, showing no difference in market impact costs from the exogenous initial inflow trades relative to all other trades in our sample. This indicates that our trading cost estimates are fairly independent and exogenous to the portfolios being traded, and hence can be applied more broadly to other portfolios or settings, including the portfolios from the academic literature we examine in the next section.

D. Price Impact by Time and Trade Type

Since the estimates for market impact and implementation shortfall are so similar, for brevity we focus on market impact costs in Table IV, which reports trading cost estimates over time and by trade type. Panel A of Table IV reports market impact by year. Trading costs are quite a bit larger in the early time period prior to 2003, and then decline through the sample period, primarily due to market conditions (such as electronic trading, more venues, increased volumes, etc.). The time variation in average costs is also interesting. Costs appear to decline through time but jump up before and during the financial crisis in 2007 and 2008. The patterns in international markets mirror those of the U.S.

Panel B of Table IV reports trading costs for different types of trades: buy long, buy to cover, sell long, and sell short. The first two columns report the percentage of trades, in both dollar terms and numbers, of each trade type. Buying long and selling long account for two-thirds of all trades, with the remaining one third split evenly between short selling and covered buys. Over the full sample, buying long generates about 12.2 basis points of price impact, but buying to cover has 19.5 basis points of price impact, and the 7.3 basis point difference between them is statistically significant (as the bottom of Panel B of Table IV shows). The fact that buying to cover is more expensive than buying long makes sense since covering is potentially a more liquidity-demanding trade. Comparing selling long versus short selling, we see that short selling is slightly more expensive by 3.7 basis points on average, but the difference is not statistically significant.[11] Although a large literature discusses the additional costs associated with short-selling, conditional on actually shorting, we see no marked difference in trading costs between selling a long position versus selling short. If short selling is indeed costlier, it is likely to be from opportunity cost (i.e., not being able to short). The execution costs of short selling versus selling long appear no different, even with uptick rules and other barriers to shorting. Results for international exchanges are similar: buying to cover is more expensive than buying long, but short selling versus selling long faces roughly the same price impact. (In fact, in the U.S. selling short is actually 2.67 basis points cheaper than selling long on average, though this difference is not statistically significant.)

E. Trading Cost Model: What Determines Trading Costs?

To better understand what affects trading costs, we investigate what observable market, stock, and trade characteristics are related to realized trading costs. Through this exercise, we seek to better

[11] Shorting costs and shorting revenues from lending fees are not included in these costs, which just capture price impact. The majority (around 99%) of short positions are in stocks that are not hard to borrow and hence the general collateral rate applies, which is small.

understand what determines price impact costs, but also build a model based on observable characteristics that we apply out of sample in other time periods and for other trading strategies.

When it comes to modeling transactions costs, there are a number of different types of models. Proportional trading costs models, where the cost of trading does not vary with the size of the portfolio traded, do not match theory or data (or practice for that matter). Nonproportional trading costs models, where the cost of trading does vary with the size of the portfolio traded, are more realistic, but the shape of the price impact function is an important ingredient. Often these are modeled as concave or linear functions (Hasbrouck (1991), Hausman, Lo, and MacKinlay (1992), and Keim and Madhavan (1996)), which don't appear to match actual trade data, where the functional form appears to be convex—the price impact of large trades increases considerably.

Figure 4 plots the average and median price impact measures from the live trading data as a function of trade size, measured by fraction of daily trading volume traded. The plot shows the convex relation between market impact and trade size. Fitting a curve to our sample of trade data (superimposed on the graph) statistically supports a convex function of price impact. Hence, for our econometric model of transactions costs we estimate a convex function.

Table V presents our econometric model for trading costs. The explanatory variables used in the model are characterized into three groups. The first set of variables captures how trading costs vary with the size of a trade. These variables include the fraction of daily trading volume traded, which is the dollar size of the trade divided by that stock's average daily trading volume over the past year, and the square root of the fraction of daily volume traded to capture the convexity we see in the data (Figure 4). The second group of variables include firm characteristics that may be related to the cost of trading, such as the size of the firm, which is the log of 1 plus the market value of equity [log(1+ME)] and the idiosyncratic volatility of firm's equity return, which is the standard deviation of the residuals from a regression of one-year daily stock returns of the firm on the corresponding value-weighted market index for that country (expressed as an annualized percentage). Finally, we include a set of market variables to capture variation in trading costs over time that result from different market conditions. We include the "VIX" to capture market volatility, which is the monthly variance of the CRSP-value weighted index computed using daily returns (expressed as an annualized percentage), and also include the variable "Beta*IndexRet*buysell", which is the beta of the stock times the index return for that market times a variable "buysell" equal to 1 for buys and -1 for sells. "Beta" is the stock's predicted beta at time of order submission and "IndexRet" is the market index return over the life of the trade. This variable is used to account for contemporaneous (beta-adjusted) market returns over the trade period and to sign the trades correctly, since our data contains both buys and sells. This variable is only used to parameterize our model when looking at historical transactions. It plays no role in estimating future transactions costs or when applying the model out of sample, since we assume that daily market returns are unforecastable (i.e., E[IndexRet] = 0). Thus, when estimating transaction costs out of sample, we simply drop this term while holding the other parameters fixed.

The model is estimated separately for the full sample, U.S. trades only, and international trades only, using pooled regressions with country fixed effects. The dependent variable is the realized market impact from the live trading data, in basis points. Table V shows the results from the model regressions. The economic intuition is fairly clear. Larger trades lead to higher transaction costs. Smaller and more volatile firms have higher transaction costs. An increase in market volatility is associated with higher transactions costs, which is consistent with the notion that market makers need to be compensated more in more volatile markets. These variables are generally statistically significant, with only one variable (idiosyncratic volatility) not being significant at the 5% level for the full sample. The R-squares of the regressions are about 7%, which is reasonably large for these types of models given the noisiness of trade data. The variable with the largest impact on trading costs is the square root of the fraction of daily volume, consistent with theory and data that costs of trading increase significantly and non-linearly as trades become larger. The coefficient estimates are also very similar across markets, with U.S. and international data giving similar parameterizations.[12]

III. The Cross-Section of After-Trading Cost Returns

Using the parameters from Table V, we estimate the price impact function for the standard portfolios used in the literature to capture the anomalies related to size, value, momentum, and short-term reversals. We first apply the models in sample over the same time period with which the trading costs are measured (from 1998 to 2011). We then extend our model using the same parameters to estimate price impact costs out of sample for earlier time periods going back to 1926 for U.S. data and to 1983 for international data. Using these estimates, we compute the net after-trading-cost returns of the strategies and their break-even capacities.

[12] Our price impact function is, of course, limited by the range of trading size our data provides. Hence, we cannot say what price impact might look like beyond these sizes. However, the range of trading volumes (as shown in Figure 3) used in our transactions cost model are mostly less than 5% of a stock's daily volume (the max is 13%). This is a reasonable range of trading volumes for the types of strategies we investigate in this paper and for the type of trader, a large arbitrageur, we wish to model. Trades done at sizes well beyond these levels tend to reflect idiosyncratic trades, perhaps around an event, and are often informational trades.

A. In Sample Net Returns

We start by computing the trading costs and net returns to our portfolio strategies in sample over the 1998 to 2011 period. We examine the strategies SMB, HML, UMD, STR, an equal-weighted combination of HML and UMD we call ValMom, and an equal-weighted combination (Combo) of all four strategies. We examine ValMom based on the benefits of combining value with momentum shown in Asness, Moskowitz, and Pedersen (2012), Asness and Frazzini (2012), and Daniel and Moskowitz (2012) and examine the combination of all four strategies to see what costs, net returns, and capacity look like for a diversified portfolio of all four strategies.

By examining these strategies over the same period that we have live trading data, we are able to compute actual realized trading costs of these theoretical portfolios. Moreover, since our trading costs are shown to be exogenous to the portfolios, the results should provide a reasonably accurate assessment of what the live trading costs of these strategies would have been historically. Specifically, we restrict the universe of stocks that appear in our trade execution database at portfolio formation, and build long/short portfolios to match those in the literature based on that universe. This ensures that we have trading cost estimates for each stock in the portfolio. Since the manager was trading similar types of strategies during this time, many of the stocks in these portfolios are covered by the execution data. Our data on average covers 75% (58%) of the market cap of U.S. (International) stocks, and a large fraction of the portfolio weights of the strategies we aim to mimic from the literature (see Table A2 in the Appendix).

For each stock we estimate its market impact at each rebalance using the realized market impact of all trades executed in that stock in the prior six months. For example, if the UMD portfolio required buying Microsoft on June 30, 2003, we assume that the market impact of that purchase is equal to the average market impact of Microsoft trades between January 1, 2003, and June 30, 2003. We use six month windows of trading activity to maximize stock coverage and to reduce noise. This exercise assumes that the trade size is equal to the average trade size of our manager over the past six months. In essence, we replicate SMB, HML, UMD, and STR using only those stocks that were traded by the manager in the most recent six months at the actual sizes the manager traded and use the actual prices and actual trading costs borne for those trades.

This calculation produces actual trading costs on a stock-by-stock basis, but assumes that trading costs are the same no matter what portfolios the trades belong to. Given the independence between portfolios and trading costs we showed earlier, this assumption seems valid. On the other hand, if market makers knew the reason behind each trade and adjusted prices depending on the type of portfolio being traded (e.g., SMB versus UMD), then this assumption would be violated. However, given the anonymity of trading through the trading algorithm and the virtual impossibility of knowing which portfolio each trade belongs to, it seems quite reasonable to assume that these costs would remain essentially the same.

U.S. equity 1998 – 2011. Panel A of Table VII reports results for U.S. equity style strategies over the sample of live trading data. The first row reports the average dollar volume of trades used to estimate trading costs at each rebalance. For example, when rebalancing SMB, our trading cost estimate for that month is based on an average of 7.6 billion dollars of trades. The second row reports the "implied fund size" from these trades. For example, given turnover (which we report near the bottom of the table as the sum of long and short trades per dollar) of SMB is 50% per month, our estimates are based on a fund size of 7.6/0.5 = 15.2 billion dollars. In other words, we are carving out a 15.2 billion dollar SMB portfolio from our live trading data that would generate the same dollar volume of trades in SMB stocks as our manager. The next row reports the realized trading cost of the SMB strategy at this size, which is 1.46 percent per year. The corresponding average market impact cost is 24.2 basis points per dollar traded.[13]

Looking across the columns at the other strategies, HML has a slightly higher realized cost of 1.54 percent per year because it has higher turnover of 68%, though it has lower average market impact costs of 19 basis points. UMD has an annual drag of 3.51 percent per year from trading costs, due to its 127% turnover and an average market impact cost of 23 basis points. STR has the largest trading costs of 6.75 percent per year, due to its 305% turnover per month with a market impact cost of 18.4 basis points per trade. A ValMom combination has a realized cost of 2.18 percent per year with turnover of only 79% since value and momentum trades tend to offset each other, resulting in lower turnover which has real transactions costs benefits. Finally, a combination of all four strategies experiences a 2.39 percent per year trading cost from 102% turnover with an average price impact of 19.4 basis points.

To compare the realized trading costs to the profitability of these strategies, the third row of Panel A of Table VI reports the break-even cost of each strategy, which we estimate for each strategy using historical data over the largest sample of returns available. For example, for SMB we compute the average return in U.S. data from July 1926 to December 2011, which is 2.61 percent. We use the longest sample of returns available because estimating mean returns is notoriously difficult and the

[13] Total trading costs depend on turnover and market impact of each stock. We define the *average market impact* as the constant market impact ($\overline{MI}$) that would generate the same level of total trading costs: $Realized\ cost = \overline{MI} \times Turnover$.

short sample from 1998 to 2011 may provide a distorted view of what the true expected return is on these strategies. For instance, we report the actual gross average return over the 1998 to 2011 sample period for SMB to be 8.58 percent, which is four times larger than its historical average of 2.61, which seems like a much more reasonable estimate of SMB's expected return. Given the 1.46 percent trading cost, this implies that a $15.2 billion SMB portfolio would deliver in expectation 1.16 percent per year net of trading costs. This difference between the expected return and expected trading cost, which is the net expected return, is highly statistically significant with a *t*-statistic of 5.20. (We also report the gross and net Sharpe ratios.) This indicates that SMB still remains profitable in expectation after trading costs at a size of $15 billion.

The next row calculates the break-even size of SMB given an expected gross return of 2.61 percent per year and the implied price impact function we estimate from live trade data. We find that the break-even size for SMB would be $103 billion.

We conduct the same exercise for the other long-short equity strategies. For HML, using a historical average return from 1926 to 2011 of 3.88 percent per year, the net return to HML trading on average $4.74 billion is 2.34 percent per year (*t*-statistic of 10.20), and its break-even fund size is almost $83 billion. For UMD, the net expected return at its implied fund size of $3.7 billion is 4.34 percent per year (*t*-statistic of 8.42) and its break-even fund size is $52 billion. For short-term reversals, STR, the expected net returns are only 1.52 percent per year with a marginally significant 2.14 *t*-statistic, as trading costs significantly eat into the return of this strategy at an implied fund size of $1.5 billion. As such, STR's break-even size is only $9.5 billion. The ValMom combination generates a 4.05 percent net expected return (*t*-statistic of 13.11) at a fund size of $8.25 billion and its break-even size is $98 billion, which is larger than both value and momentum independently. Thus, significant gains in trading costs are obtained from combining value with momentum because of offsetting trades. This is another positive feature of combining value and momentum (Asness, Moskowitz, and Pedersen (2012) and Israel and Moskowitz (2012)). Finally, the combination of all four strategies produces a net expected return of 3 percent per year (*t*-statistic of 11.37) for an average trading size of $7.8 billion, and an implied break-even fund size of $51 billion.

The remaining rows of the table report the in-sample gross and net returns and Sharpe ratios of the portfolios over the 1998 to 2011 trading sample period. Over this sample period SMB experiences a very positive realization relative to history, while HML, UMD, and STR experience relatively poor performance by historical standards. This is why using the historical long-term estimates of average returns is a better measure of the strategy's expected returns going forward. Nevertheless, the combination of all four strategies net of transactions costs would have produced

2.42 percent per year over this sample period, which is similar to the 3 percent net expected return we calculated using the longer historical mean estimates.

Comparison with the Literature. Before turning to the international evidence and the out of sample results, it is important to reconcile our results with those in the literature that estimate much higher transactions costs and far lower break-even sizes for similar strategies in U.S. stocks. For instance, Chen, Stanzl, and Watanabe (2002) estimate very small maximal fund sizes before costs eliminate profits on size, book-to-market, and momentum strategies. Lesmond, Schill, and Zhou (2003) find that trading costs eliminate the profits to momentum strategies at small fund sizes and Korajczyk and Sadka (2004) find that break-even fund sizes for long-only momentum portfolios are about $2 to $5 billion. Our estimates of trading costs are many times smaller than these studies and, consequently, our estimated break-even sizes are many times larger. For example, instead of the $2 to $5 billion Korajczyk and Sadka (2004) estimate for momentum, we find a break-even fund size for momentum of over $50 billion.

Why do our results look so different than previous studies? First, previous studies compute trading costs using aggregated daily or TAQ data and using simple microstructure models. The models employed are typically too conservative and overestimate price impact costs. Moreover, use of the TAQ data, which approximates the average trade, includes informed traders, retail traders, liquidity demanders, and those traders facing high price impact costs. The costs facing such investors are an order of magnitude higher than those facing a large institution engaged in exploiting these strategies. As a result, the estimates from these papers may represent the average trading costs and break-even sizes for the average investor seeking a size, value, or momentum trade, but these costs are conservative for a large arbitrageur.

For a more direct comparison, we use the trading cost estimates from Korajczyk and Sadka (2004) using TAQ data to back out the implied break-even size of the long-only momentum portfolio they examine, which is the top 10% of momentum winners. Using their methodology and data, we indeed get a similar break-even fund size of around $2 billion. Applying our trading cost model estimated from our live trading data to the exact same portfolio, we get a break-even fund size of $33.5 billion; an almost 17-fold increase.

For another comparison, we repeat the same exercise for the Russell 1000 and 2000, where we estimate the alphas of each with respect to the market to be about 36 basis points and 2.5% per year, respectively. The break-even fund sizes according to the Korajczyk and Sadka (2004) trading cost estimates are $785 billion for the Russell 1000 and $127 billion for the Russell 2000. Under our trading cost estimates, we get a break-even size of $4,655 billion for the Russell 1000 and $1,114 billion for the Russell 2000, which are both almost an order of magnitude larger. To get a sense of the actual capacity of these indexes, we assume, according to ICI, that about half is held by money managers and half of that is indexed, such that about 25% of the total market cap of each index is held passively. This alone accounts for $3,750 billion in assets for the Russell 1000 and $250 billion for the Russell 2000, which already exceeds the break even sizes estimated by Korajczyk and Sadka (2004) by a lot. In addition, if you add to this the amount being benchmarked to the Russell 1000 and 2000 from active investors, which according to Sensoy (2009) is 13.2% of all active funds for the Russell 1000 and 21.6% for the Russell 2000, multiplied by the roughly three trillion dollars in assets in active U.S. funds, then the total current amounts invested in the Russell 1000 and 2000 are $4,146 and $898 billion, respectively, which are just under our estimated break-even sizes (and you wouldn't expect actual sizes to reach the break even zero profit levels in any case). Hence, our estimated costs seem more in line with the real sizes of aggregate institutional investment, which previous studies grossly underestimate.

Finally, our money manager has also been running long-only momentum indexes in large and small cap U.S. stocks and internationally, as well as momentum mutual funds based off of these indexes, since July 2009. The live realized price impact costs in these funds have been 8, 18.2, and 5.9 basis points in large cap, small cap, and international momentum, respectively, which is in line with, but slightly lower than, our estimates from historical trading data.

Previous models in the literature and aggregate transaction level data, therefore, seem ill-equipped to address whether anomalies survive transactions costs. The marginal investor's costs in these markets are not the average investor's costs, and our data from a large arbitrageur shows that these costs can be very different and are substantially lower. We turn now to international evidence on transactions costs on the same strategies.

International equity 1998 – 2011. Panel B of Table VI reports the results for the international portfolios over the trading sample period. The results are remarkably similar in the sense that the estimated trading costs are very close to those we get for U.S. equities. The market impact costs for each of the strategies are very similar to those we found in the U.S., and the turnover of each portfolio is nearly identical to its U.S. counterpart. The implied fund sizes at which we calculate these costs are higher internationally, indicating our dataset has a robust international sample.

Conducting the same break-even exercise we did for the U.S. strategies, where we estimate the mean returns to the strategies using the longest historical return information we have internationally (which is from 1983 to 2011),[14] net expected returns to SMB are marginally significant at 45 basis points per year, but are a highly significant 5.21, 4.57, and 2.26 percent per year for HML, UMD, and STR, respectively. The break-even fund sizes we estimate are about the same in international markets as they are in the U.S. For SMB, HML, UMD, and STR we calculate break-even fund sizes of $53, $106, $37, and $3.3 billion.

Global equity 1998 – 2011. Finally, Panel C of Table VI reports results for global strategies that combine both the U.S. and international data. The fund sizes used to calculate the trading costs obviously increase as do the break-even fund sizes. Since an arbitrageur seeking to exploit these anomalies would apply these strategies globally, perhaps the most relevant break-even fund sizes are reported here, where SMB can be run at $156 billion, HML at $190 billion, UMD at $89 billion, and STR at $12.8 billion. Moreover, a combination of value and momentum can be run at $177 billion and all four strategies applied globally can reach $89 billion before wiping out returns entirely through trading costs. At these dollar magnitudes, the anomalous returns to size, value, momentum, and short-term reversals appear to be robust to transactions costs and implementable.

B. Out of Sample Net Returns

Since returns to our strategies are available over a much longer sample period in both U.S. and international markets, we also assess the net expected returns of our strategies out of sample over these longer time periods. In order to do so, we use our trading cost model estimated in Table V and apply the coefficients to observable variables in the out of sample time periods. We assume that the relation between the observable variables and trading costs from our model are the same over time. This assumes, for instance, that the relation between trade size and market impact is constant through time, or the relation between VIX and market impact costs is the same through time. To be clear, the model allows for time variation in costs as VIX or other observable variables change, but it assumes that the relation between those variables and costs remains the same through time. While this assumption may not be true, it provides a rough and reasonable estimate of what trading costs might have looked like in earlier time periods if our model were accurate and predictive.

This exercise produces market impact estimates for the entire cross section of stocks from 1926 to 2011. To compute these estimates we project the coefficients from our trading cost model on observable variables out of sample. We use firm size, market and firm volatility (we require one-year of daily return and volume data) and fix the average trade size (as a fraction of daily volume) equal to

[14] Alternatively, we could use the U.S. historical estimates for the expected returns to these strategies since they cover a much longer period and there is no a priori reason why the returns should be different internationally.

the median fraction of volume traded in our execution data. For stocks covered by our execution data, we take a conservative approach and take the maximum between the realized market impact (averaged over the past six months) and the predicted impact from the regression model. Table A5 in the Appendix reports the predicted price impact costs for U.S. stocks from 1926 to 2011 and for international stocks from 1983 to 2011 by 20-year subsamples and for large and small stocks separately.

U.S. equity 1926 – 2011. Panel A of Table VII applies the out of sample estimates of trading costs to the U.S. equity portfolios over the entire 1926 to 2011 historical sample. The results are fairly consistent with those from the shorter and more recent sample period, but provide a more robust assessment of the long-term expected net returns to these strategies. Focusing on the gross and net expected return numbers, SMB and STR, despite having significant gross expected returns, do not have significant net expected returns after trading costs. Especially for STR, trading costs wipe out most of its alpha. Hence, even for a large arbitrageur facing low trading costs, the viability of a short-term reversal strategy is not very compelling. HML delivers a marginally significant net expected return of 2.92 percent after trading costs (*t*-statistic of 2.00) that weakly survives, but UMD offers a 5.37 percent net of trading cost return (*t*-statistic of 2.99) that is robust to transactions costs. A combination of value and momentum is even more powerful, generating a 4.87 percent net expected return (*t*-statistic of 6.84) after trading costs. Finally, a combination of all four strategies produces a net expected return of 5.39 percent (*t*-statistic of 9.99).

The break-even fund sizes implied by these estimated trading costs are large: $354 billion for SMB, $190 billion for HML, $66 billion for UMD, except for STR, which is only $9.4 billion. These numbers are similar to those obtained over the shorter sample period of actual trading costs. A value and momentum combination has a $130 billion break-even capacity and a portfolio of all four strategies has a capacity of $54 billion, limited by the inclusion of the short-term reversal strategy.

International equity 1982 – 2011. Panel B of Table VII reports similar findings for the international portfolios, where SMB and STR do not survive trading costs and HML and UMD do, where a combination of value and momentum is even more powerful and produces the highest net expected returns after trading costs. Again, the break-even fund sizes are very consistent with what we find in U.S. stocks and for what we found for the more recent time period of actual trading data. Significant capacity sizes are obtained for size, value, and momentum, but short-term reversals can only be run profitably at a very small size of $3.3 billion.

Global equity 1926 – 2011. Panel C of Table VI reports the results for the global strategies that combine the U.S. and international data. Here, all the strategies produce positive net expected returns after trading costs that are significant at the 10% level, where value and momentum, and their combination, produce the strongest and most reliable after-cost returns. Break-even fund sizes globally for SMB and HML are in the hundreds of billions. UMD is also above $100 billion in capacity and a value and momentum combination has a $236 billion break-even size. Only STR is severely limited in scale globally, with only $8.3 billion capacity potential after trading costs.

Finally, as a robustness check, Appendix Table A6 inflates costs by factors of two or three and then calculates breakeven costs. Even with these much more conservative cost estimates, size, value, and momentum continue to produce significant positive excess returns after transactions costs.

IV. Optimizing Portfolios for Trading Costs

To more fully address the trading cost efficiency of the equity style strategies, in this section we attempt to maximize the after-trading costs returns and capacity of each style. We design "trading costs managed" versions of our portfolios that try to maximize after-trading costs returns of each strategy, subject to a tracking error constraint. Portfolios analyzed in the literature are not designed to optimize or pay attention to transactions costs in any way and hence may be quite trading-cost inefficient. In order to answer how cost-efficient various investment styles are, it seems crucial to evaluate how trading costs can be optimized within a style. Comparing the after-trading costs returns of the original versions of the style portfolios to those optimized for trading costs provides a sense of how large the improvements are in trading costs across styles.

A. Baseline Optimization

The objective is to maximize after-trading cost returns subject to maintaining the style of the original portfolio. We place a one percent (annualized) constraint on the amount of tracking error or style drift we allow the optimized portfolio to have. We want to optimize for trading costs but not at the expense of producing a portfolio that is too dissimilar from the equity style itself.[15] We focus on the trading cost consequences of trading weighed against the opportunity cost of not trading by simply minimizing the total amount of expected trading costs, while imposing a cost for tracking error or style drift and a penalty for trading amounts in excess of 5% of a stock's average daily trading volume. This constraint ensures that we do not dominate a market for very small stocks and is also consistent with our estimated trading cost model in that no trades are designed to exceed this size and only very rarely do some trades actually exceed this threshold. For simplicity, we use the

[15] For example, we could buy and hold a portfolio and never trade for the entire sample period in order to minimize trading costs, but this portfolio would not look anything like its intended style.

CAPM as a risk model so the inputs of the covariance matrix are a stock's beta, its idiosyncratic volatility, and market volatility.[16] The optimization problem is

$$\begin{aligned}
&\min_{\boldsymbol{w}} Total\ Trading\ Cost\ (\boldsymbol{w}) \\
&\text{Subject to:} \\
&\textit{Tracking Error Constraint}: \sqrt{(\boldsymbol{w}-\boldsymbol{B})\boldsymbol{\Omega}(\boldsymbol{w}-\boldsymbol{B})} \leq 1\% \qquad (5) \\
&\textit{\$1 long and \$1 short}: \quad \boldsymbol{w}'\boldsymbol{i} = 0 \text{ and } |\boldsymbol{w}|'\boldsymbol{i} = 2 \\
&\textit{Trading Constraint: Fraction of daily volume <=5\%}
\end{aligned}$$

where $\boldsymbol{w}$ is the vector of chosen portfolio weights, $\boldsymbol{B}$ is the vector of original weights for SMB, HML, UMD, STR, ValMom or Combo, $\boldsymbol{i}$ is a vector of 1s and $\boldsymbol{\Omega}$ is the CAPM-implied covariance matrix estimated using daily data over the prior 12 months. The trading cost function is based on column (4) of Table V.

Since we are interested in studying implementable portfolios, and to ease computational burden and because volume information and trading cost estimates are more noisy in the earlier time periods, we restrict our sample to the period 1980 to 2011 and use stocks with relatively active trading markets, mimicking the universe of liquid stocks traded by large institutional managers. We focus on the top 2,000 stocks in the U.S. based on their combined rank of size and daily volume, and do the same for the top 2,000 stocks outside of the U.S. across all other markets. We refer to this sample of 4,000 stocks as our "tradable" universe. These additional restrictions allow the optimization to run in reasonable time and are also consistent with the trading cost model we estimate in Table V.

In order to run the optimizations and estimate trading costs, we must input a portfolio size. Panel A of Table VIII reports results where we start with a fund size of \$100 million in net asset value (NAV) and Panel B starts with a \$200 million NAV. The second row of Panel A reports the ending NAV for each strategy, which ranges from only \$112.8 million for STR to \$988.2 million for HML, which is a function of both the gross returns on the strategy over the sample period as well as the trading costs of each strategy. The third and fourth rows report the gross returns and *t*-statistics of the non-optimized portfolios using the tradable sample from 1980 to 2011. The gross returns for SMB and HML are pretty consistent with those over the full sample from 1926 to 2011, but the returns to UMD and STR over this shorter period are much lower than they are over the full period (compared to Table VII). The next two rows of Panel A of Table VIII report the gross returns and *t*-

[16] Using the Fama and French (1993) three factor model augmented with a fourth momentum factor does not have a material impact on the results.

statistics for the optimized portfolios we obtain from equation (5). The returns are very close to those for the non-optimized portfolios, suggesting that little style drift is imposed from the optimization when we require less than 1% of tracking error.

Computing the trading costs of both the non-optimized and optimized versions of the trading strategies, we see that the optimized portfolios significantly reduce trading costs. Rows 9 and 10 report the total trading costs for the non-optimized and optimized versions of the strategies. The non-optimized versions correspond to those portfolios typically studied in the literature. For example, the original or non-optimized SMB generates 99 basis points of total trading costs per year, but the optimized version of SMB we construct generates only 21 basis points of trading costs per year over the same time period. Since the gross returns of the two portfolios are similar (actually, the optimized version added 29 basis points of gross returns), the net result is that the optimized-for-trading-costs version of SMB outperforms the original SMB portfolio by 128 basis points per year after trading costs.

Repeating the same exercise for the other strategies, we see even larger improvements in after-trading-cost returns. HML's trading costs are 2.28 percent per year, but an optimized version of HML that adjusts portfolio weights to explicitly account for the cost of trading generates only 57 basis points of costs. Again, since the gross returns to the non-optimized versus optimized versions of HML are similar, this results in significant improvement in HML's net returns after trading costs. For UMD, the results are similar. The non-optimized version of UMD faces 4.78% trading cost drag per year, but an optimized version can reduce those costs to 2.14% per year, leaving significant after-trading-cost returns to momentum. Even STR, which faces a 10.82 percent trading cost per year, can be reduced to 6.17 percent, but at these levels trading costs more than wipe out the average profits to STR, even the optimized version of STR. In other words, at $100 million in staring assets, a short-term reversal strategy would not have survived trading costs even when optimized to minimize those trading costs.

The next set of rows decompose the reduction in trading costs for each strategy. Rows 11 and 12 report the turnovers of the non-optimized versus optimized versions of the strategies. In all cases, turnover is reduced considerably. Hence, one of the main factors in reducing trading costs across all styles is to reduce trading, which isn't surprising. More interestingly, however, are the results in rows 13 and 14 which report the average market impact cost of trades for the non-optimized and optimized versions. As Panel A of Table VIII shows, the average market impact of trades also declines significantly for the optimized portfolios. Hence, not only do the optimized portfolios trade less, but they also trade stocks with the least predicted price impact in order to reduce transactions costs. On both dimensions the optimization is successful.

An arbitrageur facing trading costs who wishes to implement these strategies would both manage the turnover of the strategies and be strategic in which stocks he/she traded in order to minimize price impact costs. Moreover, as the optimization shows, significant turnover and price impact reduction can be achieved without incurring substantial tracking error or degradation in gross returns. The tracking errors are small,[17] and the betas with respect to the original non-optimized portfolios are very close to one. The net effect of managing these two variables is a significant improvement in after-trading cost net returns, without incurring significant style drift.

Panel B of Table VIII reports results for strategies that begin with twice the amount of assets ($200 million). The main results are similar. SMB, HML, and UMD all experience significant improvements in after-trading cost returns, with the largest improvements for HML and UMD. Ending NAV's for SMB, HML, and UMD reach 1.7, 1.8, and 1.3 billion, respectively. However, STR, even after trading cost optimization, faces too high a cost to remain profitable, as its NAV deteriorates from 200 to 150 million over the sample period and its net returns remain significantly negative after trading costs. In this sense, SMB, HML, and UMD seem to easily survive trading costs and their after-trading-cost net performances can be improved substantially further through portfolio optimization, but STR appears too constrained by trading costs.

Finally, the ValMom combination and composite combination portfolio of all four strategies shows significant net returns and significant improvements following optimization. The largest improvements are for the ValMom combination, which generates significant excess net returns after trading costs both before and after optimization, and whose trading costs decline the most after optimization, going from 4.57 (5.54) before optimization to 1.11 (1.44) percent per year after optimization starting at $100 ($200) million in NAV. The source of this large reduction in trading costs comes both from turnover, which is reduced by almost 2/3, and from price impact per trade, which is cut almost in half. These reductions are larger than those for either HML or UMD themselves, indicating that there are significant trading cost synergies from combining value and momentum that allow trading costs to be reduced even further than a simple averaging of the two. Value and momentum trades within the same portfolio provide useful interaction benefits that help further reduce trading costs, which is another virtue of combining value and momentum in the same

[17] The ex-ante tracking error constraint is 1% per year, which is based on forecasted volatility and covariances, but the realized tracking error is closer to 2% for the strategies. Still, the optimized versions bear little tracking error to their original portfolios.

portfolio (Asness (1997), Asness, Moskowitz, and Pedersen (2012), Daniel and Moskowitz (2012), and Israel and Moskowitz (2012)). Panels C and D of Table VIII report similar results for the international and global portfolios, which highlight substantial reductions in trading costs from portfolio optimization, especially for value and momentum.

B. The Tracking Error Frontier of After-Trading Cost Returns

The previous results in Table VIII pertain to optimized portfolios with a tracking error constraint of 100 basis points. In this subsection we examine how the results change across styles as we vary the tracking error constraint. This analysis is informative about the tradeoff between trading cost management and tracking error across styles.

Table IX reports the total trading costs for the portfolios at ex ante tracking errors ranging from zero (the non-optimized/original version) to 200 basis points per year. Panel A reports the U.S. results, and Panels B and C the international and global results. For each style strategy, trading costs decrease monotonically as tracking error is allowed to increase, indicating that the optimization is doing what it is supposed to. The most dramatic reductions occur for HML and UMD. HML's 2.88 annual trading cost in the U.S. can be reduced to 30 basis points through portfolio optimization allowing as much as 2% tracking error. UMD's 5.83 annual cost is reduced to 111 basis points at 2% tracking error. But, the largest improvements in trading costs occur for the value and momentum combination, ValMom, where annual costs of 5.54 percent per year are reduced to 31 basis points per year. The combination of all four strategies can similarly be vastly improved on an after-cost basis through portfolio optimization.

Below the trading cost estimates are the net Sharpe ratios of the strategies after trading costs across tracking errors. The Sharpe ratios reflect significant improvements in after-cost returns to these strategies, with the biggest benefits accruing to value and momentum and especially a combination of value and momentum. This evidence indicates that trading costs can be significantly reduced without incurring large return consequences or tracking error costs. The tradeoff between tracking error and trading costs appears most favorable for value and momentum.

Figures 5 and 6 illustrate and summarize the results in Table IX. Figure 5 plots the total trading costs of each strategy across tracking error. As the graph illustrates, HML and UMD benefit the most from portfolio optimization as their trading costs decline the fastest as tracking error increases. However, the steepest decline in trading costs appears for the value and momentum combination. Figure 6 shows that the reduction in trading costs does not impose equally high and offsetting return consequences. The net Sharpe ratios of the strategies improve significantly as tracking error increases, especially for value and momentum. SMB's after-cost Sharpe ratios improve only slightly and while STR's net Sharpe ratios also improve, they still produce significantly negative performance after trading costs. Hence, STR does not appear to survive transactions costs even when optimized to try to minimize those costs. Finally, the combination of value and momentum benefits the most from portfolio optimization, with the steepest improvement in net Sharpe ratio among all strategies as tracking error increases.[18]

C. Break-Even Capacity After Optimization

How much do break-even sizes of these strategies improve from portfolio optimization? In order to compare to our previous break-even sizes, we first compare the same universe of tradable stocks and consider the same sample period from 1980 to 2011. Our previous break-even sizes pertained to the much longer 1926 to 2011 time period and covered the entire universe of listed equities. To facilitate comparisons, we use the results from our trading-cost optimized portfolios and apply them to the average returns to the strategies over the longest historical time period (1926 to 2011) to calculate break even sizes.

The last two rows of Table IX report the break-even sizes for the optimized portfolios at zero and 100 basis points of (ex ante) tracking error. The zero tracking error break even sizes are identical to those in Table VII and represent the non-optimized break even fund sizes. Comparing those numbers to break even sizes for the optimized portfolios at 100 basis points of tracking error, we see marked increases in fund sizes for all strategies, except STR. SMB goes from $354 billion to $1.58 trillion in capacity after optimization. HML's capacity more than doubles from $190 to $486 billion, and UMD's break-even capacity increases by 50 percent from $66 billion to just under $100 billion. STR's break-even size, however, only improves from $9.4 to $13.2 billion after trading cost optimization. A combination of value and momentum doubles capacity to nearly $250 billion.

Panels B and C repeat these calculations for the international and global samples and find similar improvements from optimization. On a global scale, we estimate that through portfolio optimization, SMB, HML, and UMD have $1.8 trillion, $811 billion, and $122 billion in capacity, while STR has only $17 billion. A global value and momentum combination strategy has $415 billion in capacity. These break-even sizes are orders of magnitude larger than those previously estimated in the literature. We conclude that the returns to size, value, and momentum appear to

[18] It's worth noting that the significant improvement in trading costs through portfolio optimization is another indication that the trading costs we estimate from the live trade data are largely independent from the portfolios being traded. If the portfolio weights and trading costs has already been simultaneously optimized, and given the manager was running similar strategies, we would have found no further improvement from optimization.

survive transactions costs at very high asset size, and hence appear to be robust, implementable, and sizeable. Returns to short-term reversal strategies, however, do not.

V. Conclusion

We examine the trading costs, net of cost returns, and break even fund sizes of equity strategies designed to capture several of the main asset pricing anomalies documented in the literature. Using a unique dataset of live trades from a large institutional investor, who engages in many similar strategies, we approximate the trading costs of a large arbitrageur. We find that our trading cost estimates are many times smaller and our fund sizes are more than an order of magnitude larger than those claimed in the literature. These results are driven by two key innovations in our study. First, we use actual trading costs from a real-world arbitrageur to estimate price impact rather than aggregated trade and quote level data used in other studies. The latter represents the average trader's cost, while the former is likely closer to the marginal investor's cost, and we find costs that are five to six times smaller than those estimated in the literature. Second, we use portfolio optimization techniques to design strategies that account for trading costs, as a real-world investor facing transactions costs would do, that further decreases realized trading costs. The combination of these two effects provides orders of magnitude larger break-even sizes for strategies designed to capture several of the key asset pricing anomalies in the literature.

The results and tradeoffs between trading costs and tracking error vary across the different anomalies/styles. Value and momentum benefit the most from trading cost optimization and face the most favorable tradeoffs. Short-term reversals, however, do not survive trading costs at reasonable size and do not offer much improvement from portfolio optimization. Our results indicate that strategies based on size, value, and momentum can be deployed at very high asset size and still survive trading costs, while short-term reversals cannot. Hence, the return premia associated with size, value, and momentum appear to be robust, sizeable, and implementable.

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Table I
Trade Execution Data, 1998 – 2011. Summary Statistics.

This table shows summary statistics of the trade execution database. Panel A reports the total amount traded by year. Amounts are in billion USD. "U.S." are trades executed in the United States. "INT" are trades executed outside of the United States. "LC" are trades in large cap stocks, "SC" are trades in small cap stocks. The distinction between large cap and small cap is based on the portfolio's benchmark. "L/S" are trades executed in long-short accounts and "LO" are trades executed in long-only accounts. Relaxed constraints portfolios (130-30 and 140-40) are classified as "LO". Panel B reports times series means of summary statistics as of December of each year (14 annual observations). Panel C reports Fama-MacBeth averages. Each calendar month, we compute value-weighted means of each variables. Weighs are equal to the total amount traded and Panel C reports the time series means of the cross sectional coefficients. "Fraction of daily average trading volume" is equal to the trade's dollar size divided by the stock's average 1-year dollar volume. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011.

Panel A: Amount Traded (Billion USD)		By Region		By Size		By Portfolio type	
Year	Total	U.S.	INT	Large Cap	Small Cap	Long short	Long only
1998*	2.96	1.29	1.67	2.96		2.96	
1999	5.29	1.99	3.30	5.29		5.29	
2000	1.99	0.76	1.23	1.99		1.86	0.13
2001	1.08	0.55	0.53	1.08		1.00	0.08
2002	4.21	0.71	3.50	4.21	0.00	1.40	2.81
2003	5.43	2.69	2.75	5.43	0.00	4.17	1.26
2004	10.00	2.95	7.05	9.99	0.01	6.38	3.62
2005	16.16	8.06	8.10	15.75	0.41	11.45	4.71
2006	67.01	34.79	32.22	64.23	2.78	44.69	22.31
2007	127.76	68.89	58.88	123.79	3.97	95.52	32.25
2008	107.82	56.06	51.76	103.91	3.90	69.21	38.60
2009	110.74	75.77	34.96	107.88	2.86	85.44	25.30
2010	116.86	78.44	38.42	113.52	3.34	91.87	24.99
2011	144.12	95.46	48.67	139.54	4.58	114.80	29.32
Total	721.43	428.41	293.02	699.57	21.85	536.04	185.39

Panel B: Annual Time Series	Mean	Median	Std	Min	Max
Number of stocks per year	3,147.3	3,271.0	1,676.9	389.0	5,132.0
Number of countries per year	16.4	17.5	3.8	8.0	19.0
Number of exchange venues per year	24.1	24.5	6.9	11.0	33.0
Number of trade baskets per year	789.4	244.0	978.9	9.0	2,833.0
Number of orders per year (1,000s)	109.1	23.5	140.5	0.6	393.6
Number of trade executions per year (1,000,000s)	2.4	2.9	1.4	0.1	4.0
Panel C: Fama MacBeth averages					
Average trade size (1,000$)	695.3	401.5	1,034.1	52.6	5,939.1
Fraction of daily average trading volume (%)	1.2	0.5	2.1	0.1	13.1
Trade horizon (days)	2.1	1.5	2.2	0.0	8.8

* From 19980831

Table II
Trade Execution Data, 1998 – 2011. Realized Trading Costs.

This table shows average Market Impact (MI) and Implementation Shortfall (IS). Each calendar month, we compute average, median and weighted average cost ("vw_mean") of all trades during the month. When computing weighted average cost, trades are weighted by their dollar amount. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. "U.S." indicates trades executed in the United States. "INT" indicates trades executed outside of the United States. "LC" indicates trades in large cap stocks, "SC" indicates trades in small cap stocks. The distinction between large cap and small cap is based on the portfolio's benchmark. "L/S" indicates trades executed in long-short accounts, "LO" indicates trades executed in long-only accounts. Relaxed constraints portfolios (130-30 and 140-40) are classified as "LO". Market Impact and Implementation Shortfall are in basis points and standard errors are reported in the bottom panel.

Panel A:	All sample	By Region			By Size		By Portfolio type	
Full sample: 1998 - 2011		U.S. Nyse-Amex	U.S. Nasdaq	INT	Large Cap	Small Cap	Long short	Long only
MI mean	12.18	9.44	11.93	14.09	11.21	21.27	10.13	15.98
MI median	7.98	6.60	6.51	9.88	7.47	14.95	6.99	10.45
MI vw mean	18.65	15.51	18.30	19.11	18.07	27.25	18.24	17.97
IS mean	13.00	10.04	11.92	15.33	12.02	22.19	11.67	15.45
IS median	10.34	8.28	8.31	12.70	9.67	18.35	9.40	12.23
IS vw-mean	20.20	17.79	21.35	20.30	19.58	30.62	19.93	19.04
Standard errors								
MI mean	1.01	1.07	2.20	1.00	1.06	1.71	1.25	1.11
MI median	0.56	0.80	1.04	0.72	0.59	1.50	0.69	0.93
MI vw mean	1.23	1.50	2.78	1.22	1.26	2.39	1.52	1.27
IS mean	1.32	1.36	2.61	1.36	1.38	2.05	1.63	1.35
IS median	0.82	1.08	1.55	1.06	0.85	1.99	0.97	1.11
IS vw-mean	1.53	2.02	3.29	1.39	1.57	2.67	1.80	1.54

Panel B:	All sample	By Region			By Size		By Portfolio type	
Recent sample: 2003 - 2011		U.S. Nyse-Amex	U.S. Nasdaq	INT	Large Cap	Small Cap	Long short	Long only
MI mean	12.13	9.57	12.44	13.73	11.11	21.56	10.02	16.00
MI median	7.79	6.40	6.45	9.62	7.24	15.04	6.71	10.43
MI vw mean	18.46	15.65	18.73	18.76	17.84	27.68	18.04	18.16
IS mean	12.98	10.14	12.63	14.99	11.96	22.48	11.63	15.47
IS median	10.16	8.09	8.32	12.48	9.45	18.48	9.14	12.26
IS vw-mean	20.00	17.90	21.65	19.97	19.35	31.07	19.73	19.24
Standard errors								
MI mean	0.85	0.92	1.28	1.06	0.89	1.64	0.93	1.12
MI median	0.57	0.65	0.70	0.78	0.60	1.45	0.66	0.96
MI vw mean	1.26	1.49	2.06	1.33	1.27	2.39	1.52	1.23
IS mean	1.25	1.30	1.59	1.54	1.31	2.00	1.46	1.40
IS median	0.88	0.96	1.00	1.23	0.92	1.96	1.01	1.15
IS vw-mean	1.64	2.15	2.65	1.56	1.67	2.67	1.85	1.55

Table III
Trade Execution Data, 1998 – 2011. Realized Trading Costs – Inflows

This table shows average Market Impact (MI). Each calendar month, we compute average, median and weighted average cost ("vw_mean") of all trades during the month. When computing weighted average cost, trades are weighted by their dollar amount. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions executed in long-only accounts (cash equities and equity swaps) in our data between August 1998 and December 2011. The distinction between large cap and small cap is based on the portfolio's benchmark. "Inflows" are defined as the first trade for a given account. Market Impact is in basis points.

Long - Only Trades - 1998 - 2011		Only Inflows	All other Trades	difference	*t*-statistic
MI mean	All Trades	14.68	14.10	0.58	0.07
MI median	All Trades	9.62	8.10	1.52	0.19
MI vw mean	All Trades	11.36	11.75	-0.38	-0.05
MI mean	Large Cap	10.33	10.64	-0.31	-0.02
MI median	Large Cap	4.58	6.02	-1.44	-0.12
MI vw mean	Large Cap	3.19	10.04	-6.85	-0.63
MI mean	Small Cap	19.80	21.31	-1.52	-0.27
MI median	Small Cap	16.17	13.49	2.68	0.57
MI vw mean	Small Cap	21.20	27.19	-5.99	-0.93

Table IV
Realized Trading Costs by Time period and Trade Type, 1998 – 2001

This table shows average Market Impact (MI) and Implementation Shortfall (IS). Each calendar month, we compute average, median and weighted average cost ("vw_mean") of all trades executed during the month. When computing weighted average cost trades are weighted by their dollar amount. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. "U.S." indicates trades executed in the United States. "INT" indicates trades executed outside of the United States. "LC" indicates trades in large cap stocks, "SC" indicates trades in small cap stocks. The distinction between large cap and small cap is based on the portfolio's benchmark. "L/S" indicates trades executed in long-short accounts, "LO" indicate trades executed in long-only accounts. Relaxed constraints portfolios (130-30 and 140-40) are classified as "LO". Market Impact and Implementation Shortfall are in basis points and standard errors are reported in the bottom panel.

Panel A:	U.S.			International		
Market Impact by year	MI mean	MI median	MI vw mean	MI mean	MI median	MI vw mean
1998*	164.40	139.95	152.25	103.06	90.09	119.70
1999	49.12	21.16	61.97	83.62	39.56	124.59
2000	44.68	24.21	63.72	39.74	25.99	61.97
2001	-76.16	-4.78	3.90	43.97	35.31	60.52
2002	8.06	8.65	38.62	14.88	15.01	28.25
2003	10.74	7.58	31.87	9.38	7.01	19.75
2004	9.70	5.24	17.80	17.41	14.92	15.81
2005	20.84	12.75	20.93	9.39	7.10	7.29
2006	9.29	5.15	11.40	8.08	5.17	26.02
2007	15.12	9.32	35.35	15.80	8.87	33.11
2008	14.36	7.64	34.94	23.74	14.96	34.64
2009	8.25	3.10	11.96	14.34	8.18	31.22
2010	6.04	3.57	6.92	7.89	5.61	15.20
2011	7.36	4.89	9.47	7.79	5.91	12.86

Panel B: Market Impact by Trade Type		% of sample		All sample	By Region		By Size	
		Dollars	Trades		U.S.	INT	Large Cap	Small Cap
MI (VW-mean)	Buy Long	0.35	0.32	12.22	14.05	10.43	10.87	32.72
	Buy Cover	0.15	0.17	19.53	20.12	18.92	19.20	27.71
	Sell Long	0.32	0.34	19.74	12.97	23.36	19.25	28.12
	Sell Short	0.18	0.18	23.47	10.31	29.99	23.16	29.64
Differences	Buy Cover - Buy Long			7.31	6.07	8.49	8.33	-5.01
	Sell Short - Sell Cover			3.73	-2.67	6.62	3.91	1.52
t-statistics	Buy Cover - Buy Long			1.82	1.07	1.89	1.95	-0.26
	Sell Short - Sell Cover			1.10	-0.50	1.37	1.08	0.07

Table V
Regression Results

This table shows results from pooled regressions. The left-hand side is a trade's Market Impact (MI), in basis point. The explanatory variables include the contemporaneous market returns, firm size, volatility and trade size (all measured at order submission). "Beta*IndexRet*buysell" is the contemporaneous (beta-adjusted) market returns. Beta is the stock's predicted beta at time of order submission. "indexRet" is the corresponding index return over the life of the trade. "BuySell" is a dummy equal to 1 for buy orders and -1 for sell orders. "Size" is the equal to the log of 1 plus the market value of equity log(1+ME) . ME is in Billion USD. "Fraction of daily volume" is equal to the trade's dollar size divided by the stock's average 1-year dollar volume (in %). "Idiosyncratic Volatility" is the volatility of the residuals of a regression of 1-year daily stock returns on the corresponding value-weighted benchmark (annualized , in %), "Market Volatility" is the monthly variance of the CRSP-value weighted index , computed using daily returns (annualized, in %). Country fixed effects are included when indicated, t-statistics are shown below the coefficient estimates and 5% statistical significance is indicated in bold. Standard errors are clustered by calendar month.

	All sample				U.S.				International			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Beta*IndexRet*buysell	**0.20**	**0.20**	**0.20**	**0.20**	**0.20**	**0.20**	**0.20**	**0.20**	**0.21**	**0.21**	**0.21**	**0.21**
	(15.92)	(15.93)	(15.93)	(15.90)	(8.08)	(8.08)	(8.07)	(8.08)	(20.98)	(20.98)	(20.99)	(21.11)
Time trend	**-0.11**	-0.06	-0.07	-0.09	-0.05	0.00	-0.01	-0.02	**-0.16**	**-0.12**	**-0.14**	**-0.15**
	-(2.16)	-(1.31)	-(1.40)	-(1.77)	-(0.59)	-(0.03)	-(0.07)	-(0.19)	-(5.65)	-(4.32)	-(4.75)	-(5.20)
Log of ME (Billion USD)	**-4.54**	**-3.74**	**-3.20**	**-2.18**	**-3.89**	**-3.21**	**-2.77**	**-2.05**	**-5.55**	**-4.60**	**-3.96**	**-2.46**
	-(9.00)	-(7.51)	-(6.63)	-(7.39)	-(5.95)	-(4.97)	-(4.70)	-(3.70)	-(11.14)	-(9.13)	-(7.52)	-(8.79)
Fraction of daily volume		**1.68**	**1.06**	**1.01**		**1.81**	**1.34**	**1.30**		**1.57**	**0.77**	**0.71**
		(8.59)	(3.97)	(3.79)		(4.32)	(2.57)	(2.45)		(11.63)	(3.89)	(3.76)
Sqrt(Fraction of daily volume)			**5.61**	**7.39**			4.43	6.03			**6.85**	**8.77**
			(2.24)	(2.88)			(1.01)	(1.27)			(4.20)	(5.69)
Idiosyncratic Volatility				0.10				0.07				**0.19**
				(1.78)				(0.87)				(4.64)
Vix				**0.36**				**0.25**				**0.43**
				(6.62)				(3.97)				(5.13)
Obs (1,000s)	1,561	1,561	1,561	1,561	790	790	790	790	771	771	771	771
R2	0.070	0.071	0.071	0.072	0.066	0.066	0.067	0.067	0.075	0.076	0.076	0.077
Country FE	Yes	Yes	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes

Table VI
Returns Results – Trade Execution Data Sample

This table reports portfolio returns gross and net of trading costs. We report returns of value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. All the portfolios are the intersections of two portfolios formed on size and three portfolios formed on book to market or prior returns. At the end of each calendar month stocks are assigned to two size-sorted portfolios based on their market capitalization. The size breakpoint for the US sample is the median NYSE market equity. The size breakpoint for the international sample is the 80th percentile by country. We measure book to market and lagged book divided by current price and update monthly (Asness and Frazzini (2012)). We use one year return (in local currency) skipping the most recent month for momentum (UMD) and (minus) the local currency return in the most recent month for short term reversal (STR). Stocks are further ranked in 3 groups (low, neutral, high) based on NYSE-based breakpoints (or breakpoints based on top 20% of market capitalization by country for the international sample). The size portfolio SMB is the average return on the three small portfolios minus the average return on the three big portfolios. The value portfolio HML is the average return on the two value portfolios minus the average return on the two growth portfolios. UMD and STR and constructed in the same manner. All portfolios are value-weighted, refreshed every month and rebalanced every calendar month to maintain value weights. This table includes all available stocks in our Trade Execution Data at portfolio formation. The sample period runs from 1998 to 2011. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. For each security, the predicted market impact MI is equal the average market impact over the prior 6-months. Returns and costs are annualized, MI is in basis points, t-statistics are reported below the coefficient estimates and 5% statistical significant is indicated in bold.

Panel A: Realized TC sample 1998 - 2011 - U.S.	SMB	HML	UMD	STR	ValMom	Combo
Average USD traded at each rebalance (Billion)	7.59	4.74	4.70	4.62	6.50	7.79
Implied fund size (Billion, USD)	15.11	7.02	3.71	1.51	8.25	7.61
Realized cost	1.46	1.54	3.51	6.75	2.18	2.39
Break-even cost*	2.61	3.88	7.85	8.27	6.23	5.39
Realized minus breakeven	**-1.16**	**-2.34**	**-4.34**	**-1.52**	**-4.05**	**-3.00**
t-statistics	-(5.20)	-(10.20)	-(8.42)	-(2.14)	-(13.11)	-(11.37)
Break-even fund size (billions)	$103.19	$82.95	$52.18	$9.51	$98.74	$51.21
Return (Gross)	**8.58**	3.44	2.56	4.65	3.00	**4.81**
	(2.87)	(0.74)	(0.43)	(1.10)	(1.23)	(2.76)
Return (Net)	**7.12**	1.90	-0.95	-2.10	0.82	2.42
	(2.39)	(0.40)	-(0.16)	-(0.49)	(0.33)	(1.38)
Turnover (monthly)	0.50	0.68	1.27	3.05	0.79	1.02
MI (bps)	24.20	19.01	23.09	18.42	23.05	19.44
Sharpe ratio (gross)	0.78	0.20	0.12	0.30	0.34	0.75
Sharpe ratio (net)	0.65	0.11	-0.04	-0.14	0.09	0.37
Obs	162	162	162	162	162	162

*Average returns estimated from 1926 to 2011.

Table VI
Returns Results – Trade Execution Data Sample (Continued)

Panel B: Realized TC sample 1998 - 2011 - International

	SMB	HML	UMD	STR	ValMom	Combo
Average USD traded at each rebalance (Billion)	27.46	16.21	18.37	18.24	24.59	30.19
Implied fund size (Billion, USD)	51.04	21.47	13.70	6.00	27.17	28.24
Realized cost	1.26	1.58	2.77	5.61	1.88	2.06
Break-even cost*	1.71	6.78	7.34	3.35	7.12	4.79
Realized minus breakeven	-0.45	**-5.21**	**-4.57**	**2.26**	**-5.24**	**-2.73**
t-statistics	-(1.89)	-(18.68)	-(12.04)	(3.42)	-(18.95)	-(10.56)
Break-even fund size (billions)	$53.21	$106.74	$37.14	$3.28	$78.17	$38.00
Return (Gross)	3.05	5.60	1.58	3.16	**3.59**	**3.35**
	(1.41)	(1.71)	(0.34)	(0.86)	(2.09)	(2.32)
Return (Net)	1.80	4.03	-1.19	-2.45	1.71	1.29
	(0.83)	(1.22)	-(0.26)	-(0.66)	(0.98)	(0.87)
Turnover (monthly)	0.54	0.76	1.34	3.04	0.91	1.07
MI (bps)	19.45	17.38	17.19	15.37	17.31	16.07
Sharpe ratio (gross)	0.38	0.46	0.09	0.23	0.57	0.63
Sharpe ratio (net)	0.23	0.33	-0.07	-0.18	0.27	0.24
Obs	162	162	162	162	162	162

*Average returns estimated from 1983 to 2011.

Panel C: Realized TC sample 1998 - 2011 - Global

	SMB	HML	UMD	STR	ValMom	Combo
Average USD traded at each rebalance (Billion)	35.05	20.95	23.07	22.86	31.09	37.98
Implied fund size (Billion, USD)	67.40	29.18	17.75	7.51	36.75	36.27
Realized cost	1.34	1.56	3.19	6.19	2.07	2.23
Break-even cost*	2.61	4.88	7.88	8.33	6.75	5.63
Realized minus breakeven	**-1.27**	**-3.32**	**-4.70**	**-2.14**	**-4.68**	**-3.40**
t-statistics	-(6.48)	-(14.82)	-(11.73)	-(3.38)	-(18.10)	-(14.12)
Break-even fund size (billions)	$156.40	$189.70	$89.32	$12.79	$176.91	$89.20
Return (Gross)	**6.49**	4.39	3.03	4.17	**3.71**	**4.52**
	(3.12)	(1.20)	(0.60)	(1.18)	(2.00)	(3.33)
Return (Net)	**5.15**	2.83	-0.16	-2.02	1.64	2.29
	(2.48)	(0.77)	-(0.03)	-(0.57)	(0.87)	(1.66)
Turnover (monthly)	0.52	0.72	1.30	3.05	0.85	1.05
MI (bps)	21.55	18.11	20.42	16.93	20.34	17.73
Sharpe ratio (gross)	0.85	0.33	0.16	0.32	0.54	0.91
Sharpe ratio (net)	0.67	0.21	-0.01	-0.16	0.24	0.45
Obs	162	162	162	162	162	162

*Average returns estimated from 1926 to 2011.

Table VII
Returns Results – Full sample 1926 – 2011

This table reports portfolio returns gross and net of trading costs. We report returns of value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. All the portfolios are the intersections of two portfolios formed on size and three portfolios formed on book to market or prior returns. At the end of each calendar month stocks are assigned to two size-sorted portfolios based on their market capitalization. The size breakpoint for the US sample is the median NYSE market equity. The size breakpoint for the international sample is the 80th percentile by country. We measure book to market and lagged book divided by current price and update monthly (Asness and Frazzini (2012)). We use one year return (in local currency) skipping the most recent month for momentum (UMD) and (minus) the local currency return in the most recent month for short term reversal (STR). Stocks are further ranked in 3 groups (low, neutral, high) based on NYSE-based breakpoints (or breakpoints based on top 20% of market capitalization by country for the international sample). The size portfolio SMB is the average return on the three small portfolios minus the average return on the three big portfolios. The value portfolio HML is the average return on the two value portfolios minus the average return on the two growth portfolios. UMD and STR and constructed in the same manner. All portfolios are value-weighted, refreshed every month and rebalanced every calendar month to maintain value weights. This table includes all available stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1926 to 2011. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. For each security, the predicted market impact MI is equal the maximum of the average market impact over the prior 6-months (if available) and the predicted market impact. To compute predicted market impact for each stock we use coefficients from column (4) Table IV. We assume that market returns are unpredictable (Beta*IndexRet*buysell = 0) and set the "fraction to trading volume" equal to the average fraction to trading volume in our trade execution database. Returns and costs are annualized, MI is in basis points, t-statistics are reported below the coefficient estimates and 5% statistical significant is indicated in bold.

Panel A: All Stocks 1926 - 2011 - U.S.	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.55	0.97	2.49	6.25	1.36	1.89
Break-even cost	2.61	3.88	7.85	8.27	6.23	5.39
Realized minus breakeven	**-2.07**	**-2.92**	**-5.37**	**-2.02**	**-4.87**	**-3.51**
t-statistics	-(104.98)	-(90.97)	-(74.27)	-(16.61)	-(142.46)	-(81.19)
Break-even fund size (billions)	$354.27	$189.56	$65.92	$9.45	$129.47	$54.36
Return (Gross)	**2.61**	**3.88**	**7.85**	**8.27**	**6.23**	**5.39**
	(2.13)	(2.65)	(4.41)	(6.37)	(8.79)	(9.99)
Return (Net)	2.07	**2.92**	**5.37**	2.02	**4.87**	**3.51**
	(1.68)	(2.00)	(2.99)	(1.56)	(6.84)	(6.47)
Turnover (monthly)	0.27	0.45	1.13	3.06	0.69	0.99
MI (bps)	16.92	17.71	18.32	16.99	16.46	15.93
Sharpe ratio (gross)	0.27	0.34	0.48	0.69	1.13	1.28
Sharpe ratio (net)	0.22	0.26	0.32	0.17	0.88	0.83
Obs	729	729	1,023	1,034	729	729

Table VII
Returns Results – Full sample 1926 – 2011 (Continued)

Panel B: All Stocks 1982 - 2011 - International	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.94	1.48	2.70	6.58	1.86	2.37
Break-even cost	1.71	6.78	7.34	3.35	7.12	4.79
Realized minus breakeven	**-0.77**	**-5.31**	**-4.64**	**3.23**	**-5.26**	**-2.42**
t-statistics	-(13.69)	-(63.21)	-(45.53)	(14.30)	-(74.58)	-(30.16)
Break-even fund size (billions)	$113.45	$189.85	$49.57	$3.30	$106.42	$40.22
Return (Gross)	1.71	**6.78**	**7.34**	3.35	**7.12**	**4.79**
	(1.12)	(3.87)	(3.01)	(1.76)	(7.17)	(6.25)
Return (Net)	0.77	**5.31**	4.64	-3.23	**5.26**	**2.42**
	(0.50)	(3.04)	(1.89)	-(1.66)	(5.29)	(3.13)
Turnover (monthly)	0.37	0.57	1.16	3.03	0.78	1.04
MI (bps)	21.27	21.74	19.45	18.11	19.88	18.97
Sharpe ratio (gross)	0.21	0.72	0.56	0.32	1.34	1.17
Sharpe ratio (net)	0.09	0.57	0.35	-0.30	0.99	0.58
Obs	345	345	349	360	345	345

Panel C: All Stocks 1926 - 2011 - Global	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.59	0.99	2.45	6.11	1.37	1.85
Break-even cost	2.61	4.88	7.88	8.33	6.75	5.63
Realized minus breakeven	**-2.02**	**-3.88**	**-5.43**	**-2.22**	**-5.38**	**-3.78**
t-statistics	-(87.14)	-(107.88)	-(76.71)	-(18.75)	-(153.80)	-(90.36)
Break-even fund size (billions)	$467.72	$379.41	$115.50	$12.75	$235.89	$94.58
Return (Gross)	**2.61**	**4.88**	**7.88**	**8.33**	**6.75**	**5.63**
	(2.50)	(3.86)	(4.67)	(6.80)	(10.66)	(11.73)
Return (Net)	2.02	**3.88**	**5.43**	2.22	**5.38**	**3.78**
	(1.93)	(3.08)	(3.19)	(1.81)	(8.47)	(7.85)
Turnover (monthly)	0.29	0.48	1.14	3.07	0.71	1.00
MI (bps)	16.92	17.39	17.88	16.57	16.06	15.37
Sharpe ratio (gross)	0.32	0.50	0.51	0.73	1.37	1.51
Sharpe ratio (net)	0.25	0.40	0.35	0.20	1.09	1.01
Obs	729	729	1,023	1,029	729	729

Table VIII
Returns Results – Optimized Portfolios, Tradable Sample, 1980 – 2011

This table reports portfolio returns gross and net of trading costs. We report returns of value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. This table includes all available tradable stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1980 to 2011. To select the tradable universe, at the end of each calendar month we rank all stocks with available market capitalization, 1-year average daily volume, market betas and idiosyncratic volatility based on their market capitalization and on their 1-year average daily volume. We sum the rank and select the top 2,000 U.S. securities and the top 2,000 International securities based on the combined rank. Portfolio are optimized for trading costs by minimizing the total dollar trading costs subject to a 1% tracking error constraint relative the unconstrained portfolio and a maximum trade constraint equal to 5% of the stock's average daily volume. For each security, the predicted market impact MI is computed using coefficients from column (4) Table IV. Returns and costs are annualized, MI is in basis points, t-statistics are reported below the coefficient estimates and 5% statistical significant is indicated in bold.

Panel A: Tradable Stocks 1980- 2011 - U.S. - Starting NAV 100M	SMB	HML	UMD	STR	ValMom	Combo
Starting Nav (Million, USD)	100.00	100.00	100.00	100.00	100.00	100.00
Ending Nav (Million, USD)	856.86	988.18	738.30	112.77	1,322.47	902.08
Non-optimized Excess Return (Gross)	1.99	3.96	4.95	**4.23**	**4.46**	**3.78**
	(1.31)	(1.68)	(1.75)	(1.98)	(3.89)	(4.81)
Optimized Exess Return (Gross)	2.28	3.56	4.61	2.12	**4.48**	**4.29**
	(1.56)	(1.60)	(1.71)	(1.07)	(3.55)	(4.31)
Optimized Excess Return (Net)	2.07	2.99	2.47	**-4.05**	**3.37**	**2.06**
	(1.42)	(1.35)	(0.91)	-(2.07)	(2.66)	(2.10)
Total trading costs (non-optimized)	0.99	2.28	4.78	10.82	4.57	7.52
Total trading costs	0.21	0.57	2.14	6.17	1.11	2.22
Turnover (non-optimized)	0.27	0.47	1.09	3.00	0.97	1.70
Turnover	0.10	0.22	0.71	1.95	0.39	0.71
MI (non-optimized, bps)	30.76	40.79	36.37	30.01	39.35	36.96
MI (bps)	17.67	21.72	25.23	26.36	23.95	26.17
Sharpe ratio (gross, non-optimized)	0.23	0.30	0.31	0.35	0.69	0.85
Sharpe ratio (gross)	0.28	0.28	0.30	0.19	0.63	0.76
Sharpe ratio (net)	0.25	0.24	0.16	-0.37	0.47	0.37
Beta to non-optimized	0.94	0.93	0.95	0.90	1.06	1.16
Tracking error to non-optimized (%)	1.77	2.05	1.92	2.44	2.03	2.15
Portfolio volatility	8.21	12.52	15.24	11.05	7.14	5.53
Obs	382	382	382	382	382	382

Table VIII
Returns Results – Optimized Portfolios, Tradable Sample, 1980 – 2011 (Continued)

Panel B: Tradable Stocks 1980- 2011 - U.S. - Starting NAV 200M						
	SMB	HML	UMD	STR	ValMom	Combo
Starting Nav (Million, USD)	200.00	200.00	200.00	200.00	200.00	200.00
Ending Nav (Million, USD)	1,710.52	1,854.77	1,275.90	152.16	2,433.47	1,475.60
Non-optimized Excess Return (Gross)	1.99	3.96	4.96	**4.23**	**4.46**	**3.79**
	(1.31)	(1.68)	(1.75)	(1.98)	(3.89)	(4.81)
Optimized Exess Return (Gross)	2.32	3.56	4.57	2.03	**4.56**	**4.18**
	(1.58)	(1.60)	(1.70)	(1.01)	(3.58)	(4.30)
Optimized Excess Return (Net)	2.08	2.81	2.00	**-5.27**	**3.12**	1.43
	(1.42)	(1.26)	(0.75)	-(2.68)	(2.45)	(1.50)
Total trading costs (non-optimized)	1.24	2.88	5.83	12.31	5.54	8.85
Total trading costs	0.25	0.76	2.56	7.30	1.44	2.75
Turnover (non-optimized)	0.27	0.47	1.09	3.01	0.97	1.70
Turnover	0.11	0.25	0.68	1.97	0.41	0.72
MI (non-optimized, bps)	38.53	51.51	44.34	34.12	47.81	43.46
MI (bps)	19.39	25.57	31.28	30.84	28.95	31.94
Sharpe ratio (gross, non-optimized)	0.23	0.30	0.31	0.35	0.69	0.85
Sharpe ratio (gross)	0.28	0.28	0.30	0.18	0.64	0.76
Sharpe ratio (net)	0.25	0.22	0.13	-0.48	0.43	0.27
Beta to non-optimized	0.95	0.94	0.94	0.90	1.07	1.13
Tracking error to non-optimized (%)	1.71	2.00	2.05	2.64	2.10	2.08
Portfolio volatility	8.29	12.56	15.15	11.09	7.20	5.39
Obs	382	382	382	382	382	382

Table VIII
Returns Results – Optimized Portfolios, Tradable Sample, 1980 – 2011 (Continued)

Panel C: Tradable Stocks 1980- 2011 - International - Starting NAV 200M

	SMB	HML	UMD	STR	ValMom	Combo
Starting Nav (Million, USD)	200.00	200.00	200.00	200.00	200.00	200.00
Ending Nav (Million, USD)	384.18	856.43	332.70	195.68	641.65	467.84
Non-optimized Excess Return (Gross)	0.84	**5.85**	3.73	4.14	**4.79**	**3.44**
	(0.63)	(2.34)	(1.11)	(1.33)	(4.08)	(3.37)
Optimized Exess Return (Gross)	0.90	**6.10**	2.77	2.91	**4.24**	**3.09**
	(0.69)	(2.58)	(0.86)	(0.97)	(3.31)	(2.62)
Optimized Excess Return (Net)	0.57	**5.18**	0.67	-2.40	**3.26**	1.57
	(0.43)	(2.19)	(0.21)	-(0.81)	(2.54)	(1.34)
Total trading costs (non-optimized)	1.40	2.97	5.14	12.25	4.56	7.21
Total trading costs	0.34	0.92	2.10	5.31	0.98	1.52
Turnover (non-optimized)	0.37	0.57	1.19	3.01	1.03	1.67
Turnover	0.16	0.34	0.78	2.14	0.39	0.58
MI (non-optimized, bps)	31.47	43.21	36.01	33.86	36.86	35.90
MI (bps)	17.96	22.80	22.33	20.69	20.83	21.98
Sharpe ratio (gross, non-optimized)	0.14	0.54	0.25	0.30	0.93	0.77
Sharpe ratio (gross)	0.16	0.59	0.20	0.22	0.76	0.60
Sharpe ratio (net)	0.10	0.50	0.05	-0.19	0.58	0.31
Beta to non-optimized	0.96	0.94	0.96	0.94	1.05	1.09
Tracking error to non-optimized (%)	1.31	1.45	1.37	2.07	1.58	1.67
Portfolio volatility	5.74	10.32	14.09	12.88	5.62	5.12
Obs	229	229	229	229	229	229

Panel D: Tradable Stocks 1980- 2011 - Global - Starting NAV 200M

	SMB	HML	UMD	STR	ValMom	Combo
Starting Nav (Million, USD)	200.00	200.00	200.00	200.00	200.00	200.00
Ending Nav (Million, USD)	1,615.25	2,888.74	1,170.16	174.14	2,412.09	1,373.86
Non-optimized Excess Return (Gross)	1.43	**4.77**	4.76	**4.08**	**4.76**	**3.76**
	(1.19)	(2.38)	(1.91)	(2.16)	(5.11)	(5.38)
Optimized Exess Return (Gross)	2.04	**4.68**	3.57	1.57	**3.99**	**3.07**
	(1.77)	(2.50)	(1.53)	(0.92)	(3.83)	(3.88)
Optimized Excess Return (Net)	1.77	**3.97**	1.43	**-5.02**	**3.01**	1.16
	(1.54)	(2.12)	(0.61)	-(3.01)	(2.88)	(1.49)
Total trading costs (non-optimized)	1.58	3.51	6.48	13.40	6.16	9.50
Total trading costs	0.27	0.71	2.14	6.58	0.98	1.91
Turnover (non-optimized)	0.30	0.50	1.12	3.01	0.99	1.70
Turnover	0.11	0.23	0.59	1.70	0.30	0.47
MI (non-optimized, bps)	43.66	58.07	48.07	37.07	51.73	46.68
MI (bps)	20.30	26.00	30.35	32.19	27.57	34.21
Sharpe ratio (gross, non-optimized)	0.21	0.42	0.34	0.38	0.91	0.95
Sharpe ratio (gross)	0.31	0.44	0.27	0.16	0.68	0.69
Sharpe ratio (net)	0.27	0.38	0.11	-0.53	0.51	0.26
Beta to non-optimized	0.92	0.92	0.92	0.86	1.06	0.98
Tracking error to non-optimized (%)	2.03	2.14	2.23	2.83	2.01	2.06
Portfolio volatility	6.50	10.56	13.16	9.40	5.90	4.39
Obs	382	382	382	382	382	382

Table IX
Tracking Error Frontier, Tradable Sample, 1980 – 2011

This table reports portfolio net Sharpe ratios and total trading costs. We report results for value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. This table includes all available tradable stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1980 to 2011. To select the tradable universe, at the end of each calendar month we rank all stocks with available market capitalization, 1-year average daily volume, market betas and idiosyncratic volatility based on their market capitalization and on their 1-year average daily volume. We sum the rank and select the top 2,000 U.S. securities and the top 2,000 International securities based on the combined rank. Portfolio are optimized for trading costs by minimizing the total dollar trading costs subject to a tracking error constraint relative the unconstrained portfolio and a maximum trade constraint equal to 5% of the stock's average daily volume. The starting NAV is 200 Million USD, corresponding to Panel B, Table VII. For each security, the predicted market impact MI is computed using coefficients from column (4) Table IV. Sharpe ratios and costs are annualized.

Panel A: Tradable Stocks 1980- 2011 - U.S. - Starting NAV 200M

	Ex Ante TE (bps)	SMB	HML	UMD	STR	ValMom	Combo
Total trading costs	0	1.24	2.88	5.83	12.31	5.54	8.85
	50	0.47	1.38	3.81	9.79	3.46	5.30
	75	0.32	1.02	3.13	8.59	2.25	4.41
	100	0.25	0.76	2.56	7.30	1.44	2.75
	200	0.16	0.30	1.11	3.40	0.31	0.39
Sharpe ratio (net)	0	0.09	0.08	-0.05	-0.67	-0.02	-0.44
	50	0.17	0.17	0.07	-0.55	0.26	0.08
	75	0.20	0.20	0.11	-0.51	0.38	0.11
	100	0.25	0.22	0.13	-0.48	0.43	0.27
	200	0.25	0.20	0.11	-0.38	0.17	0.26
Break-even size (USD billion)	0	354.27	189.56	65.92	9.45	129.47	54.36
	100	1,584.36	486.44	94.44	13.17	248.51	64.80

Table IX
Tracking Error Frontier, Tradable Sample, 1980 – 2011 (continued)

Panel A: Tradable Stocks 1980- 2011 - International - Starting NAV 200M	Ex Ante TE (bps)	SMB	HML	UMD	STR	ValMom	Combo
Total trading costs	0	1.40	2.97	5.14	12.25	4.56	7.21
	50	0.70	1.63	3.42	9.56	2.45	4.33
	75	0.50	1.25	2.84	8.31	1.60	2.94
	100	0.36	1.00	2.33	5.70	1.08	1.62
	200	0.23	0.39	0.86	3.29	0.33	0.34
Sharpe ratio (net)	0	-0.09	0.26	-0.09	-0.81	0.14	-0.46
	50	0.03	0.37	0.00	-0.67	0.45	-0.01
	75	0.06	0.42	0.03	-0.63	0.55	0.12
	100	0.09	0.49	0.03	-0.24	0.55	0.27
	200	0.26	0.63	-0.05	-0.40	0.56	0.50
Break-even size (USD billion)	0	36.72	107.70	15.99	2.75	27.33	7.40
	100	223.49	325.42	27.25	3.84	167.08	56.39

Panel A: Tradable Stocks 1980- 2011 - Global - Starting NAV 200M	Ex Ante TE (bps)	SMB	HML	UMD	STR	ValMom	Combo
Total trading costs	0	1.58	3.51	6.48	13.40	6.16	9.50
	50	0.57	1.59	4.01	10.54	3.26	5.30
	75	0.40	1.12	3.15	8.93	1.96	3.72
	100	0.30	0.81	2.44	7.29	1.12	2.13
	200	0.17	0.29	0.88	2.97	0.31	0.33
Sharpe ratio (net)	0	-0.02	0.11	-0.12	-0.87	-0.08	-0.71
	50	0.12	0.25	0.04	-0.72	0.37	-0.04
	75	0.19	0.31	0.08	-0.65	0.47	0.12
	100	0.26	0.36	0.10	-0.60	0.50	0.22
	200	0.22	0.33	0.02	-0.48	0.17	0.23
Break-even size (USD billion)	0	390.99	297.26	81.91	12.20	156.80	61.76
	100	1,807.84	811.85	121.68	17.00	415.60	121.19

Figure 1
Event-Time Average Market Impact, 1998 – 2011, U.S. only

This table shows average Market Impact (MI). We restrict the sample to each calendar month, we compute the average cost of all trade baskets executed during the month. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of baskets executed during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. Market Impact is in basis points.

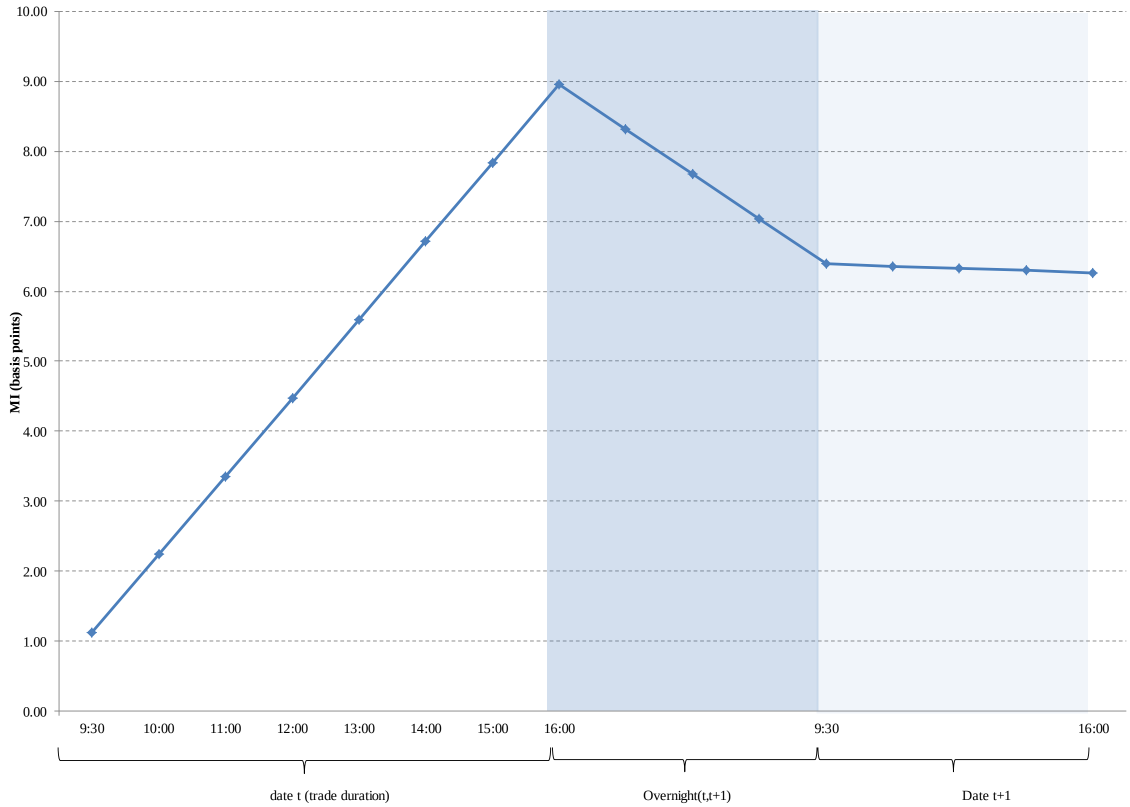


Figure 2
Average Market Impact, 1998 – 2011.

This table shows average Market Impact (MI). Each calendar month, we compute the average cost of all trade baskets executed during the month. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of baskets executed during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. Market Impact is in basis points.

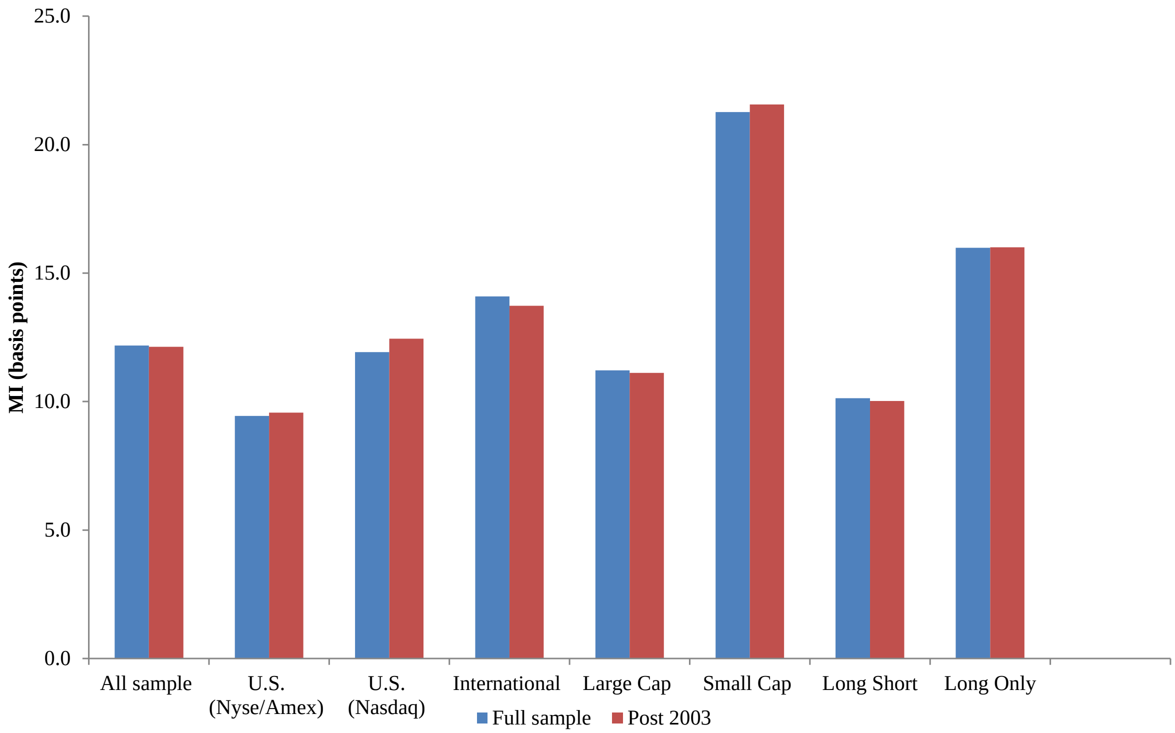


Figure 3
Trade Execution Data, 1998 – 2011. Realized Trading Costs – Inflows

This figure shows average Market Impact (MI). Each calendar month, we compute average cost of all trades during the month. This figure plots time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions executed in long-only accounts (cash equities and equity swaps) in our data between August 1998 and December 2011. The distinction between large cap and small cap is based on the portfolio's benchmark. "Inflows" are defined as the first trade for a given account. Market Impact is in basis points.

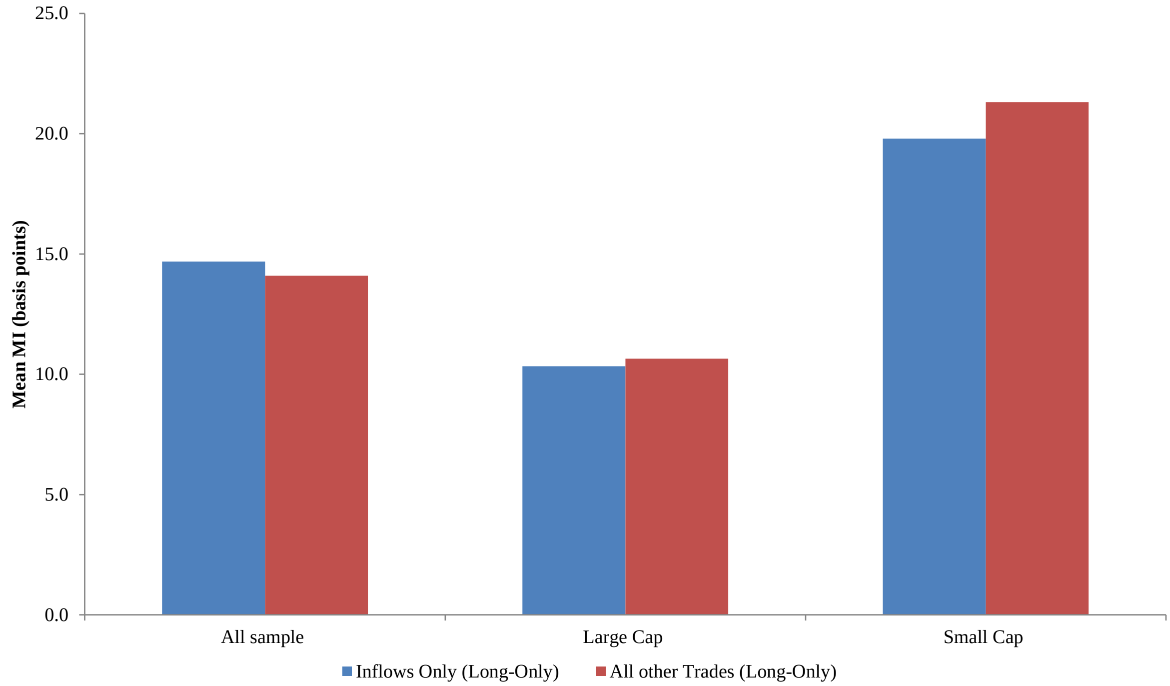


Figure 4
Market Impact by Fraction of Trading Volume

This table shows average Market Impact (MI). We sort all trades in our datasets in 30 bins based on their fraction of daily volume and compute average and median market impact for each bucket. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. Market Impact is in basis points.

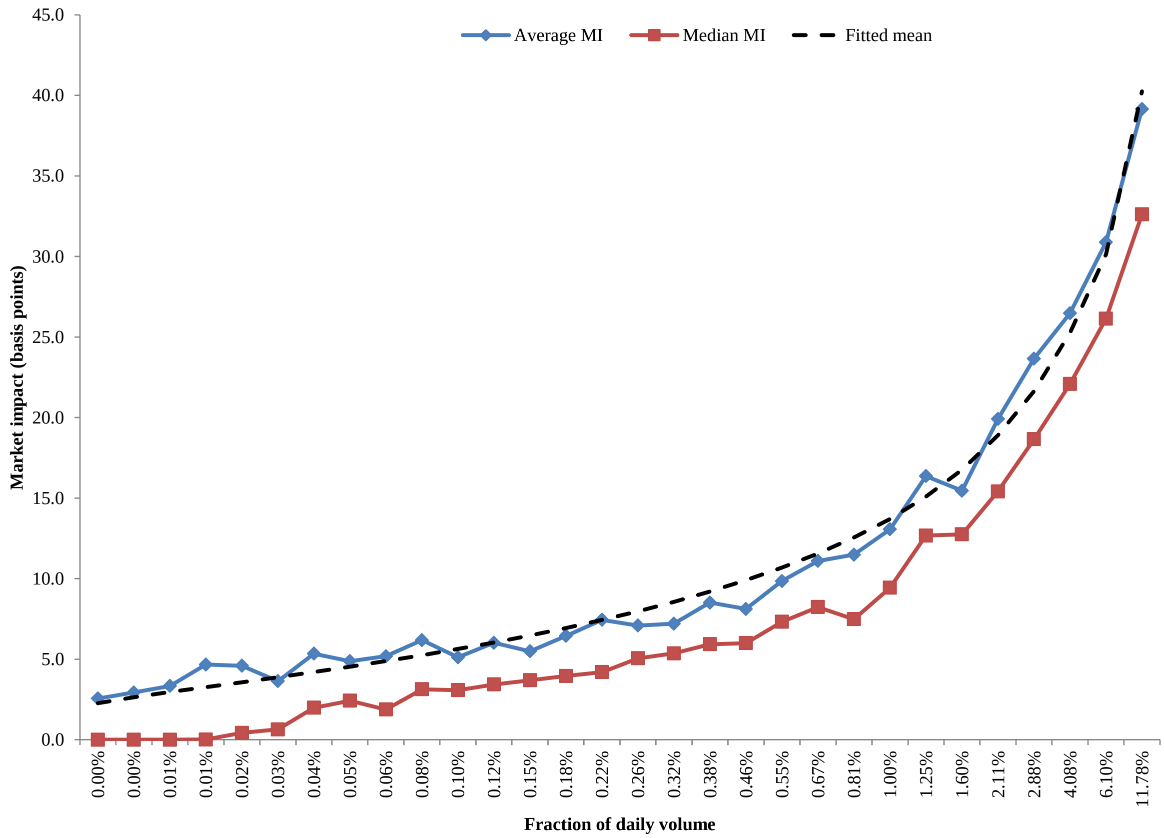
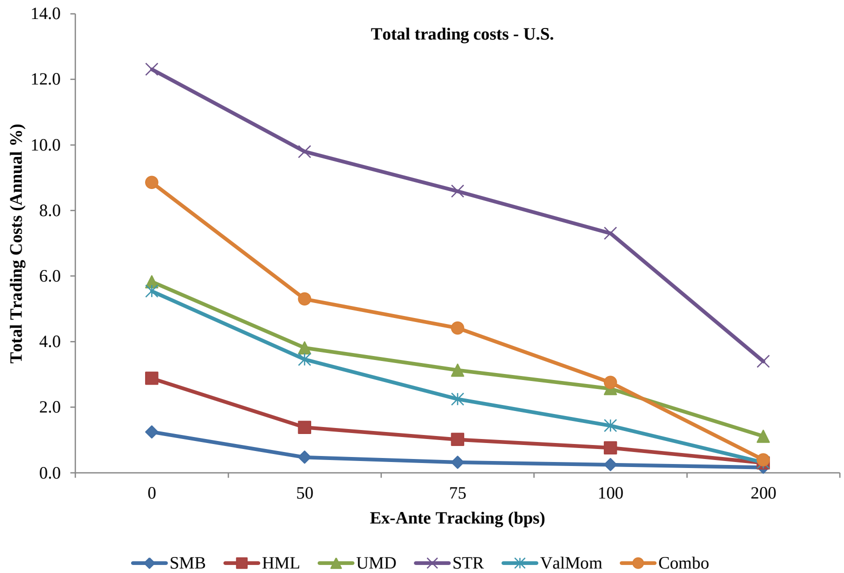


Figure 5
Tracking Error Frontier, Tradable Sample, 1980 – 2011. Total Trading Costs

This figure plot reports portfolio total trading costs. We report results for value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. This table includes all available *tradable* stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1980 to 2011. To select the tradable universe, at the end of each calendar month we rank all stocks with available market capitalization, 1-year average daily volume, market betas and idiosyncratic volatility based on their market capitalization and on their 1-year average daily volume. We sum the rank and select the top 2,000 U.S. securities and the top 2,000 International securities based on the combined rank. Portfolio are optimized for trading costs by minimizing the total dollar trading costs subject to a tracking error constraint relative the unconstrained portfolio and a maximum trade constraint equal to 5% of the stock's average daily volume. For each security, the predicted market impact MI is computed using coefficients from column (4) Table IV. Costs are annualized.



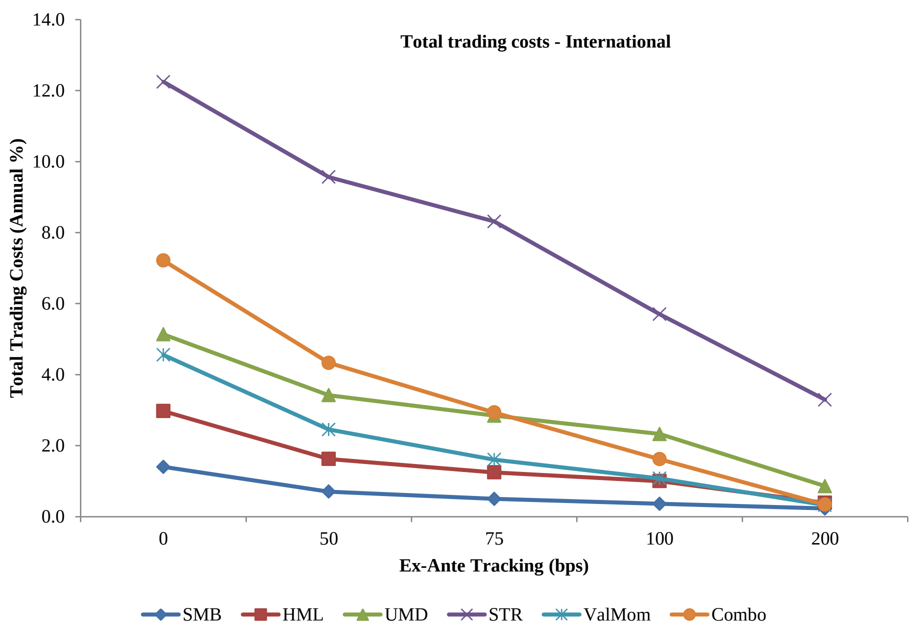
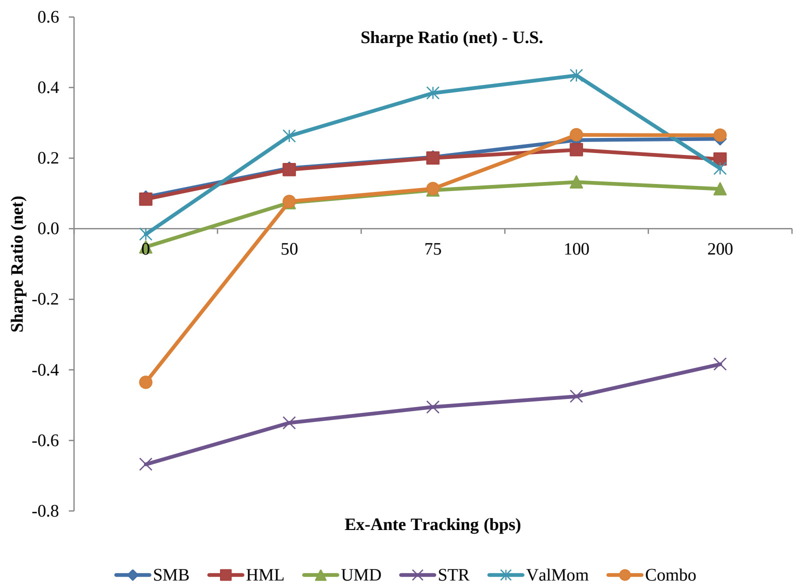
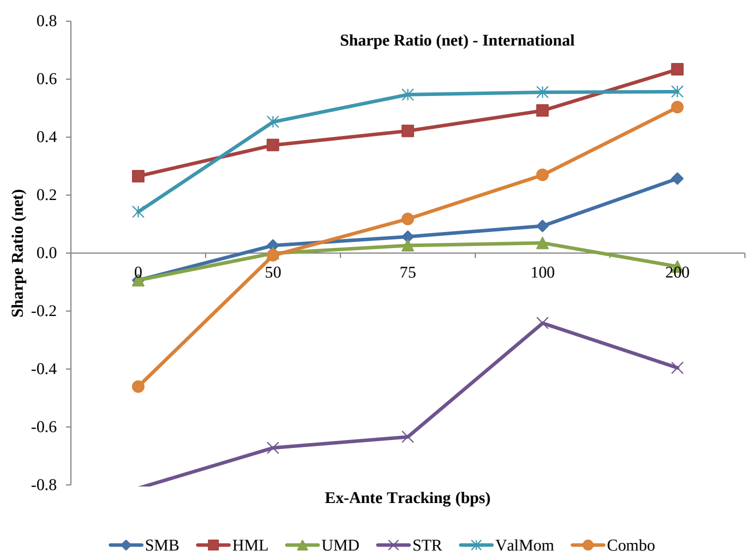


Figure 6
Tracking Error Frontier, Tradable Sample, 1980 – 2011. Sharpe Ratios

This figure plot reports portfolio net Sharpe ratios. We report results for value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. This table includes all available *tradable* stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1980 to 2011. To select the tradable universe, at the end of each calendar month we rank all stocks with available market capitalization, 1-year average daily volume, market betas and idiosyncratic volatility based on their market capitalization and on their 1-year average daily volume. We sum the rank and select the top 2,000 U.S. securities and the top 2,000 International securities based on the combined rank. Portfolio are optimized for trading costs by minimizing the total dollar trading costs subject to a tracking error constraint relative the unconstrained portfolio and a maximum trade constraint equal to 5% of the stock's average daily volume. For each security, the predicted market impact MI is computed using coefficients from column (4) Table IV. Sharpe ratios are annualized.





Trading Costs of Asset Pricing Anomalies

Appendix: Additional Empirical Results

ANDREA FRAZZINI, RONEN ISRAEL, AND TOBIAS J. MOSKOWITZ

This Appendix contains additional analysis and results.

- Table A1 reports additional summary statistics of our merged CRSP/XpressFeed global data.

- Table A2 reports coverage of our Trade Execution Data.

- Table A3 reports average market impact of our Trade Execution data based on pooled averages.

- Table A4 reports summary statistics of our predicted market impact data.

- Table A5 reports average market impact of all U.S. trades in our Trade Execution data, relative to the weighted average price. For each stock we compute the value-weighted average price (VWAP) during the trading interval (one day or multiple days) and compute market impact relative to the VWAP price. VWAPs are compute from TAQ data by averaging all reported trades with a time stamp between 9.30am and 4.00pm.

- Table A6 reports portfolio trading costs. We report difference between realized and breakeven trading costs based on the full sample of stocks (1926 – 2011).

- Table A7 reports average market impact for inflows and for all other trades in our execution database. We restrict the sample to trades of long-only accounts and report average trading costs of inflows (defined as the first trade for a given account) and all other trades.

- Figure A1 reports average and median market impact by country.

Table A1
Summary Statistics CRSP-XpressFeed Global data

This table shows summary statistics as of June of each year. The sample includes all commons stocks on the CRSP/XpressFeed data between 1950 and 2011 and all common stocks on the XpressFeed Global data between 1983 and 2011. "Number of stocks – mean" is the average number of stocks per year. "Mean ME" is the average firm's market value of equity, in billion USD. Means are pooled averages (firm-year) as of June of each year.

Country	Number of stocks - total	Number of stocks - mean	Mean ME (firm , billions USD)	Average weight in international portfolio	Average weight in global portfolio	Start Year	End Year
Australia	3,044	854	0.57	0.019	0.032	1985	2011
Austria	211	77	0.75	0.002	0.004	1985	2011
Belgium	425	133	1.79	0.010	0.017	1985	2011
Canada	5,704	595	0.89	0.023	0.039	1964	2011
Switzerland	566	203	3.03	0.025	0.044	1985	2011
Germany	2,159	696	2.48	0.066	0.112	1985	2011
Denmark	413	140	0.82	0.005	0.008	1985	2011
Spain	376	132	2.99	0.015	0.026	1985	2011
Finland	293	107	1.39	0.006	0.010	1985	2011
France	1,815	568	2.11	0.050	0.084	1985	2011
United Kingdom	6,114	1,819	1.22	0.098	0.168	1984	2011
Hong Kong	1,777	641	1.20	0.029	0.048	1985	2011
Italy	609	221	2.12	0.020	0.034	1985	2011
Japan	5,002	2,882	1.20	0.169	0.284	1985	2011
Netherlands	413	166	3.31	0.024	0.041	1985	2011
Norway	661	154	0.76	0.004	0.007	1985	2011
New Zealand	318	95	0.85	0.003	0.005	1985	2011
Singapore	1,052	360	0.63	0.010	0.016	1985	2011
Sweden	1,049	254	1.30	0.013	0.023	1985	2011
United States	23,503	3,179	0.98	0.409		1926	2011

Table A2
Trade Execution Data, 1998 – 2011. Coverage

This table shows summary statistics of the trade execution database. Panel A reports the fraction of firms with non-missing market impact data as a fractious of the universe of common stocks in the CRSP-Xpressfeed Global data. Panel B reports coverage by regions.

Panel A: Coverage	Firm fraction coverage			Market Cap fraction coverage		
	U.S.	International	Global	U.S.	International	Global
1998	0.03	0.02	0.03	0.21	0.26	0.23
1999	0.07	0.05	0.06	0.43	0.45	0.44
2000	0.09	0.06	0.07	0.46	0.53	0.50
2001	0.14	0.05	0.08	0.51	0.53	0.52
2002	0.29	0.07	0.13	0.56	0.57	0.56
2003	0.24	0.08	0.12	0.83	0.60	0.70
2004	0.33	0.09	0.15	0.84	0.60	0.70
2005	0.45	0.09	0.19	0.93	0.60	0.73
2006	0.42	0.11	0.19	0.94	0.67	0.77
2007	0.45	0.16	0.23	0.94	0.70	0.78
2008	0.50	0.17	0.24	0.97	0.69	0.79
2009	0.56	0.16	0.25	0.97	0.68	0.78
2010	0.60	0.14	0.24	0.96	0.67	0.77
2011	0.59	0.15	0.24	0.96	0.65	0.77
Mean	0.34	0.10	0.16	0.75	0.58	0.64
Median	0.38	0.09	0.17	0.88	0.60	0.72

Panel B: Market Cap fraction coverage by Region							
	Pacific	Australia	Canada	Japan	Europe	UK	U.S.
1998					0.17	0.46	0.21
1999					0.37	0.62	0.43
2000	0.36	0.43		0.61	0.46	0.66	0.46
2001	0.41	0.57		0.58	0.47	0.66	0.51
2002	0.39	0.70	0.62	0.66	0.48	0.71	0.56
2003	0.49	0.74	0.64	0.73	0.51	0.69	0.83
2004	0.46	0.68	0.67	0.75	0.51	0.70	0.84
2005	0.47	0.76	0.71	0.75	0.48	0.74	0.93
2006	0.71	0.77	0.77	0.80	0.57	0.75	0.94
2007	0.75	0.80	0.85	0.86	0.60	0.79	0.94
2008	0.81	0.82	0.87	0.86	0.57	0.76	0.97
2009	0.67	0.85	0.87	0.88	0.56	0.78	0.97
2010	0.64	0.82	0.85	0.85	0.54	0.77	0.96
2011	0.45	0.82	0.90	0.88	0.53	0.78	0.96
Mean	0.55	0.73	0.77	0.77	0.49	0.71	0.75
Median	0.48	0.76	0.81	0.78	0.51	0.72	0.88

Table A2
Trade Execution Data, 1998 – 2011. Coverage (Continued)

Panel C: Fraction of Style Strategies Actually Traded

	USA	International	Global
SMB	0.64	0.37	0.47
HML	0.65	0.36	0.47
UMD	0.65	0.36	0.47
STR	0.63	0.34	0.45
ValMom	0.65	0.36	0.47
Combo	0.64	0.36	0.47

Table A3
Trade Execution Data, 1998 – 2011. Market Impact – Pooled Means

This table shows average Market Impact (MI) and Implementation Shortfall (IS). We compute average, median and weighted average cost ("vw_mean") of all trades in the database. When computing weighted average cost, trades are weighted by their dollar amount. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. "U.S." indicates trades executed in the United States. "INT" indicates trades executed outside of the United States. "LC" indicates trades in large cap stocks, "SC" indicates trades in small cap stocks. The distinction between large cap and small cap is based on the portfolio's benchmark. "L/S" indicates trades executed in long-short accounts, "LO" indicates trades executed in long-only accounts. Relaxed constraints portfolios (130-30 and 140-40) are classified as "LO". Market Impact and Implementation Shortfall are in basis points and standard errors are reported in the bottom panel.

Pooled means	All sample	By Region			By Size		By Portfolio type	
		U.S. Nyse-Amex	U.S. Nasdaq	INT	Large Cap	Small Cap	Long short	Long only
Full sample: 1998 - 2011								
MI mean	10.74	8.51	10.69	12.21	9.89	19.89	9.20	13.68
MI median	6.15	4.47	6.47	7.22	5.69	12.07	5.38	7.78
MI vw mean	21.79	19.35	27.61	21.92	21.53	27.85	25.31	15.81
IS mean	11.42	9.00	10.70	13.24	10.57	20.65	10.72	12.77
IS median	7.79	5.80	7.61	9.10	7.20	14.92	7.05	9.30
IS vw mean	24.60	23.10	31.94	23.96	24.39	29.47	29.46	16.33
Recent sample: 2003 - 2011								
MI mean	10.68	8.60	11.07	11.93	9.80	20.13	9.11	13.68
MI median	6.05	4.44	6.43	7.06	5.55	12.23	5.22	7.77
MI vw mean	20.12	18.80	23.69	20.11	19.78	27.86	22.75	15.81
IS mean	11.39	9.07	11.23	12.97	10.51	20.89	10.67	12.76
IS median	7.66	5.73	7.58	8.90	7.03	15.20	6.84	9.30
IS vw mean	22.83	22.30	27.52	22.21	22.54	29.48	26.80	16.34

Table A4

Trade Execution Data, 1998 – 2011. Realized Trading Costs Relative to VWAP – U.S. Only

This table shows average Market Impact (MI) relative to the stock's value-weighted price (VWAP) during the execution interval. Each calendar month, we compute average, median and weighted average cost ("vw_mean") of all trades during the month. When computing weighted average cost, trades are weighted by their dollar amount. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. "U.S." indicates trades executed in the United States. "LC" indicates trades in large cap stocks, "SC" indicates trades in small cap stocks. The distinction between large cap and small cap is based on the portfolio's benchmark. "L/S" indicates trades executed in long-short accounts, "LO" indicates trades executed in long-only accounts. Relaxed constraints portfolios (130-30 and 140-40) are classified as "LO". Market Impact and Implementation Shortfall are in basis points and standard errors are reported in the bottom panel.

Full sample: 1998 - 2011	All sample	By Exchange		By Size		By Portfolio type	
	U.S.	U.S. Nyse-Amex	U.S. Nasdaq	U.S. Large Cap	U.S. Small Cap	U.S. Long short	U.S. Long only
MI mean	3.73	3.26	4.64	2.70	8.47	3.06	5.56
MI median	2.85	2.85	2.85	2.21	6.08	2.55	3.78
MI vw mean	4.09	4.07	4.01	3.46	8.13	3.40	5.20
Standard errors							
MI mean	0.66	0.46	1.08	0.73	0.76	0.77	0.56
MI median	0.19	0.18	0.23	0.20	0.46	0.22	0.30
MI vw mean	0.74	0.52	1.32	0.82	0.48	0.88	0.35

Table A5
Predicted Market Impact, Summary Statistics, 1926 – 2011

This table shows summary statistics of the predicted market impact used in Table V and VI. At the end of each calendar month, for all common stocks on the combined CRSP-Xpressfeed database we use coefficients from column (4) Table IV to compute predicted impact. We assume that market returns are unpredictable (Beta*IndexRet*buysell = 0) and set the "fraction to trading volume" equal to the average fraction to trading volume in our trade execution database. This table uses all available common stocks on the combined CRSP-Xpressfeed global database. The top panel reports average market impact, the bottom panel reports the number of stocks. "Large Cap" is defines as stocks above the NYSE median market capitalization (U.S.) or in the top 20th percentiles of market capitalization by country (International)

Panel A: Summary Statistics		All sample	By Region		By Size	
			U.S.	INT	Large Cap	Small Cap
MI mean	1926 - 1940	32.62	32.65		20.53	44.76
	1941 - 1960	15.10	15.10		10.71	19.47
	1961 - 1980	16.46	16.49		10.32	18.30
	1981 - 2000	23.99	29.38	20.42	11.20	28.26
	2001 - 2011	30.36	27.57	34.85	14.75	37.53
	All sample	25.61	25.12	28.96	12.79	30.76
Number of stocks	1926 - 1940	1,016	1,016		723	730
	1941 - 1960	1,327	1,327		882	953
	1961 - 1980	8,093	7,975		1,795	7,384
	1981 - 2000	36,717	17,631	16,498	8,228	32,093
	2001 - 2011	35,407	8,692	23,557	8,649	28,954
	All sample	55,221	23,363	28,073	13,545	48,793

Table A6
Returns Results – Full sample 1926 – 2011

This table reports portfolio trading costs. We compute returns of value (HML), Size (SMB), momentum (UMD), reversal (STR), a 50-50 portfolio of UMD and HML (VALMOM) and a composite portfolio (COMBO) that puts equal weights in SMB, HML, UMD and STR. This table includes all available stocks in the combined CRSP Xpressfeed global data. The sample period runs from 1926 to 2011. Country portfolios are aggregated into International and Global portfolios using the country's total market capitalization as of the prior month. For each security, the predicted market impact MI is equal the maximum of the average market impact over the prior 6-months (if available) and the predicted market impact. To compute predicted market impact for each stock we use coefficients from column (4) Table IV. We assume that market returns are unpredictable (Beta*IndexRet*buysell = 0) and set the "fraction to trading volume" equal to the average fraction to trading volume in our trade execution database. Returns and costs are annualized, MI is in basis points, t-statistics are reported below the coefficient estimates and 5% statistical significant is indicated in bold.

Panel A: All Stocks 1926 - 2011 - U.S.

	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.55	0.97	2.49	6.25	1.36	1.89
Break-even cost	2.61	3.88	7.85	8.27	6.23	5.39
Realized minus breakeven	**-2.07**	**-2.92**	**-5.37**	**-2.02**	**-4.87**	**-3.51**
t statistics	-(104.98)	-(90.97)	-(74.27)	-(16.61)	-(142.46)	-(81.19)
Realized x 2 minus breakeven	**-1.52**	**-1.95**	**-2.87**	**4.23**	**-3.52**	**-1.62**
t statistics	-(77.20)	-(60.89)	-(39.69)	(34.78)	-(102.86)	-(37.53)
Realized x3 minus breakeven	**-0.97**	**-0.99**	**-0.38**	**10.48**	**-2.16**	**0.27**
t statistics	-(49.47)	-(30.82)	-(5.26)	(86.17)	-(63.25)	(6.13)

Panel B: All Stocks 1982 - 2011 - International

	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.94	1.48	2.70	6.58	1.86	2.37
Break-even cost	1.71	6.78	7.34	3.35	7.12	4.79
Realized minus breakeven	-0.77	-5.31	-4.64	3.23	-5.26	-2.42
t statistics	-13.69	-63.21	-45.53	14.30	-74.58	-30.16
Realized x 2 minus breakeven	**0.17**	**-3.83**	**-1.94**	**9.80**	**-3.40**	-0.05
t statistics	(2.97)	-(45.62)	-(19.00)	(43.46)	-(48.18)	-(0.67)
Realized x3 minus breakeven	**1.10**	**-2.35**	**0.77**	**16.38**	**-1.54**	**2.32**
t statistics	(19.64)	-(28.03)	(7.53)	(72.62)	-(21.78)	(28.82)

Panel C: All Stocks 1926 - 2011 - Global

	SMB	HML	UMD	STR	ValMom	Combo
Realized cost	0.59	0.99	2.45	6.11	1.37	1.85
Break-even cost	2.61	4.88	7.88	8.33	6.75	5.63
Realized minus breakeven	-2.02	-3.88	-5.43	-2.22	-5.38	-3.78
t statistics	-87.14	-107.88	-76.71	-18.75	-153.80	-90.36
Realized x 2 minus breakeven	**-1.43**	**-2.89**	**-2.98**	**3.89**	**-4.01**	**-1.94**
t statistics	-(61.70)	-(80.25)	-(42.10)	(32.85)	-(114.64)	-(46.26)
Realized x3 minus breakeven	**-0.84**	**-1.90**	**-0.53**	**10.00**	**-2.64**	**-0.09**
t statistics	-(36.26)	-(52.64)	-(7.49)	(84.46)	-(75.47)	-(2.15)

Table A7
Trade Execution Data, 1998 – 2011. Realized Trading Costs – Inflows

This table shows average Market Impact (MI). Each calendar month, we compute average, median and weighted average cost (“vw_mean”) of all trades during the month. When computing weighted average cost, trades are weighted by their dollar amount. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of stocks traded during the month. This table includes all available developed market equity transactions executed in long-only accounts (cash equities and equity swaps) in our data between August 1998 and December 2011. The distinction between large cap and small cap is based on the portfolio’s benchmark. “Inflows” are defined as the first trade for a given account. Market Impact is in basis points.

Long - Only Trades - 1998 - 2011		Only Inflows	All other Trades	diff	t-stistics
MI mean	All Trades	14.68	14.10	0.58	0.07
MI median	All Trades	9.62	8.10	1.52	0.19
MI vw mean	All Trades	11.36	11.75	-0.38	-0.05
MI mean	Large Cap	10.33	10.64	-0.31	-0.02
MI median	Large Cap	4.58	6.02	-1.44	-0.12
MI vw mean	Large Cap	3.19	10.04	-6.85	-0.63
MI mean	Small Cap	19.80	21.31	-1.52	-0.27
MI median	Small Cap	16.17	13.49	2.68	0.57
MI vw mean	Small Cap	21.20	27.19	-5.99	-0.93

Figure A1
Average Market Impact by Country, 1998 – 2011.

This table shows average Market Impact (MI). Each calendar month, we compute the average cost of all trade baskets executed during the month. This table reports time-series averages of the cross sectional estimates. When computing time series averages, we weight each monthly observation by the number of baskets executed during the month. This table includes all available developed market equity transactions (cash equities and equity swaps) in our data between August 1998 and December 2011. Market Impact is in basis points.