Importance Sampling in PET Collimator Simulations

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***Abstract*—*SimSET* (a Simulation System for Emission Tomography) incorporates importance sampling (IS) to accelerate emission tomography simulations. To this point IS has been used in the object being imaged, but not in the collimator. This is an effective strategy for simulations using geometric or perfect-absorber collimators, but breaks down when more realistic collimators are simulated. We are improving the efficiency of collimator simulation with algorithms that complement the IS algorithms used in the object. Recent improvements to the IS in the object have led to many more photons passing through the collimator without intersecting the septa. However, the photons that do intersect septa have much higher weights, resulting in poor quality factors if they are detected. The collimator IS algorithms force these high-weight photons either to penetrate the septa or to scatter toward the detector and then penetrate the septa. This will reduce the weights of the photons, improve detection efficiency, and raise the quality factor. The new algorithms are currently focused on collimators for position emission tomography (PET), but the underlying techniques are more broadly applicable.**

I. Introduction

The Simulation System for Emission Tomography (SimSET) is a public domain software package that is widely used in the nuclear medicine research community. It provides photon tracking through the tomograph field-of-view (FOV), collimator and detector models, and user-configurable binned output [1].

Recent improvements in our in-FOV IS for PET led to a large improvement in the number of events detected per simulated decay (by a factor of 2-7, depending on the simulation) [2]. However, a drop in the quality factor [3] of the data (by 30-65%) made the overall efficiency gains small.

The new IS achieved the improvements in the number of detected events by greatly increasing the likelihood that tracked photons would pass through the collimator without intersecting the septa. However, in direct consequence, those photons that did intersect the septa had much higher weights. Most of these photons failed to reach the detectors, but the few that did created high-weight events that reduced the quality factor.

We have devised an algorithm that applies importance sampling in the collimator. When a photon intersects septa, it is forced to penetrate the septa, or to scatter towards the detector and then penetrate the septa. These forcing steps increase the number of photons that penetrate the collimator while greatly reducing their weight, which we hope will result in higher detection efficiency and improvements to the quality factor.

II. Algorithm

The collimator receives a list of weighted photons from the object. Each photon is tracked using an analog algorithm (i.e., without IS). Each time the photon crosses a boundary or scatters we create a copy of it and either eliminate the copy using Russian roulette or force the copy to pass through the collimator to the detector. The copy is forced to detection either (1) by forcing it to penetrate through any septa its path intersects or (2) by forcing it to scatter within the current segment of the collimator (e.g., the current septum), forcing the scatter direction to be towards a nearby strip on the detector, and forcing the scattered photon to penetrate the remaining collimator to the detector. Once the tracking of the copy is finished, tracking of the original photon continues normally. The photon will be discarded when it leaves the collimator even if it would intersect the detector. This prevents double-counting of possibilities, which would bias the resulting output. The forced detection accounts for all detectable photon paths. Fig. 1 gives an overview of the algorithm.

A. Forced penetration or forced scatter

The first decision to be made in the algorithm is whether the photon should be forced to penetrate or forced to scatter. We need to sample both possibilities, but can choose the frequency with which we will sample each.

In general the incoming photon will have a much higher weight, $w_{incoming}$, than the desired target weight, w_{target}. Our goal is to have the outgoing weight of the photon, $w_{outgoing}$,

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as close to w_{target} as possible. We can calculate $w_{outgoingPenetration}$ for forced penetration directly. If there are $x_{penetration}$ free-paths before the photon leaves the collimator,

$$w_{outgoingPenetration} = \frac{w_{incoming} * \exp(-x_{penetration})}{f_{penetration}} \tag{1}$$

where $f_{penetration}$ is the probability with which we attempt forced penetration for a given forced detection attempt.

We cannot pre-compute the outgoing weight, $w_{outgoingScatter}$, for the case where we force a scatter. It is dependent on many factors that are determined randomly during the process of forcing the scatter. However, we know that

$$w_{outgoingScatter} < \frac{w_{incoming} * \left(1 - \exp(-x_{septum})\right) * p_{scatter}}{f_{scatter}} \tag{2}$$

where x_{septum} is the number of free-paths before the photon leaves the septum, $p_{scatter}$ is the probability that an interaction in the septum would be a scatter (rather than absorption), and $f_{scatter}$ is the probability with which we attempt forced scatter for a given forced detection attempt. $w_{outgoingScatter}$ will be considerably less than this upper limit, as the photon weight will also be reduced when a scatter angle towards a nearby detector strip is chosen, and also when the photon is forced to penetrate the septum to the detector strip. An estimate of the upper bound of these further factors can be included as extra terms in the upper limit for $w_{outgoingScatter}$.

We choose $f_{penetration}$ and $f_{scatter}$ to make $w_{outgoingPenetration}$ and the upper limit for $w_{outgoingScatter}$ as close as possible to w_{target}, with the restriction that

$$f_{penetration} + f_{scatter} \le 1. \tag{3}$$

To determine whether a photon copy will undergo forced penetration or forced scatter we generate a random number, R_1, between 0 and 1. If

$$R_1 < f_{penetration} \tag{4}$$

we do forced penetration. Otherwise, if

$$R_1 < f_{penetration} + f_{scatter} \tag{5}$$

we do forced penetration. Finally, if

$$R_1 \geq f_{penetration} + f_{scatter} \tag{6}$$

we discard the photon copy.

B. Forced penetration

When the photon copy undergoes forced penetration, it is projected in its current direction to the back face of the collimator. Its weight is adjusted using (1).

C. Forced scatter

Forced scatter consists of several steps: choosing a scatter location within the septum; choosing a scatter direction towards the target detector strip; and forcing the photon to penetrate to the back face of the collimator.

To set the scatter location, a number of free-paths is randomly sampled from the truncated exponential distribution with maximum x_{septum}. The photon is then projected this many free-paths into the septum, and its weight is adjusted by the likelihood it would have interacted before leaving the septum, as well as for $f_{scatter}$:

$$w_{atScatter} = w_{incoming} * \frac{1 - \exp(-x_{septum})}{f_{scatter}}. \tag{7}$$

The scatter direction is chosen so that the photon will hit somewhere within a strip of the detector that extends to either side of the septum and includes all the detector face up to the next septum in both directions (Fig. 2). A very crude approximation to the probability density function giving the probability of scattering into directions that hit this strip is created, $\mathrm{P}_{forcedScatter}(z)$, where *z* is the location on the detector strip. A scatter direction is randomly chosen from a cumulative distribution based on $\mathrm{P}_{forcedScatter}$. The photon weight is changed to reflect the fact that we didn't allow for photoabsorption and to account for the difference between $\mathrm{P}_{forcedScatter}$ and $\mathrm{P}_{scatter}$, the true probability density function for scatter angles (including both Compton and coherent scatter):

$$w_{afterScatter} = w_{atScatter} * p_{scatter} * \frac{\mathrm{P}_{scatter}(z)}{\mathrm{P}_{forcedScatter}(z)}. \tag{8}$$

Finally, the photon is projected to the chosen point and its weight is adjusted to reflect the probability of penetration:

$$w_{outgoingScatter} = w_{afterScatter} * \exp(-x_{scatter}) \tag{9}$$

where $x_{scatter}$ is the number of free-paths between the scatter point and the chosen point on the detector.

III. Discussion

We have not yet completed our implementation of the algorithms described. In particular, there are still bugs in the code that forces scatter in the collimator. Thus we cannot yet determine how great an efficiency improvement these algorithms will achieve. We are able to look at the effect of the forced penetration algorithm, and have found that it increases efficiency of true-event simulations by 0-10%. The number of detections went up by a factor of 1.5 to 5. However, this improvement was cancelled out by a lower quality factor and longer run times. This efficiency improvement is disappointingly small, and we hope that we can improve on this figure once we have the entire collimator importance sampling package working. In particular we hope to improve the quality factor. We expect that a Russian roulette step will be needed after forced scatter: the upper bound given for $w_{outgoingScatter}$ in (2) and used in determining $f_{scatter}$ is so generous that $w_{outgoingScatter}$ is usually extremely small. We are investigating better approximations for this upper bound.

SimSET is available on our web site (http://depts.washington.edu/~simset/html/simset_home.html). It is distributed as user-modifiable source code.

IV. References

[1] T. Lewellen, R. Harrison, and S. Vannnoy, "The SimSET Program, in Monte Carlo Calculations," *Monte Carlo Calculations in Nuclear Medicine*, M. Ljungberg, S. Strand, and M. King, Editors. 1998, Institute of Physics Publishing: Philadelphia. p. 77-92.

[2] R. Harrison. (2002, November). Presented at IEEE Medical Imaging Conference. [CD]. Available at http://www.ieee.org/products/onlinepubs/pop/pop.html.

[3] D. Haynor, R. Harrison, T. and Lewellen, " The use of importance sampling techniques to improve the efficiency of photon tracking in emission tomography simulations," *Med Phys*, 1991, pp. 290-300.

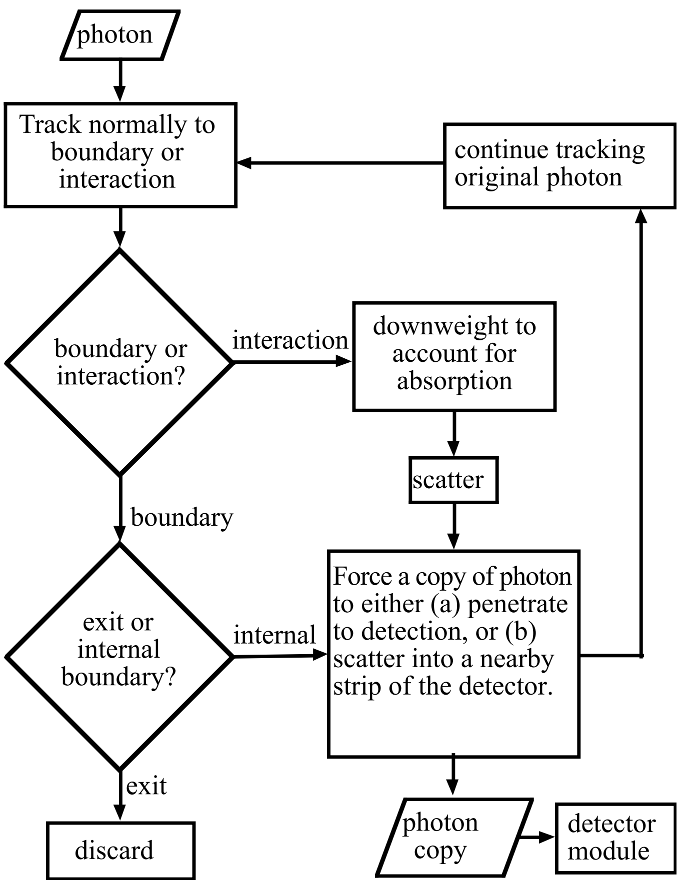

Fig. 1. Flow chart showing how a photon is tracked through the collimator.

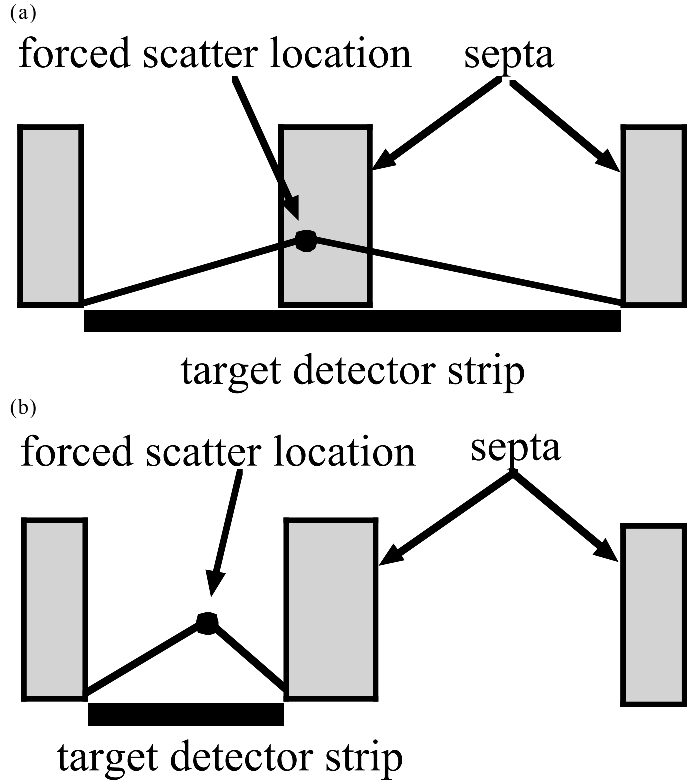

Fig. 2. (a) When forcing scatter in a septum, the photon will be forced to leave the collimator between the septa to either side of the scatter septum. (b) When forcing scatter in a gap between septa, the photon will be forced to leave the collimator between the septa to either side of the gap.