5.3.4. K-Means Clustering

The K-means clustering algorithm, also known as generalized Lloyds algorithm, is an iterative method for classifying the realizations (Vaseghi, 1996). The random vector process $\boldsymbol{y}$ is partitioned into M clusters or regions $\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_M$, and each cluster $\boldsymbol{y}_i$, is represented by centroid $\boldsymbol{c}_i$. The centroid computation and classification of samples are performed iteratively. Each iteration consists of two steps: (a) Partitioning the realizations of the random process into M regions or clusters and (b) Computing the centroid of each of the clusters. The centroids obtained can be considered as initial mean vector estimates given by:

$$\hat{\mu}_{i_0} = c_i = \frac{1}{N_i} \sum_{n=1}^{N_i} y_i(n) \qquad \text{for } i = 1,2,\ldots,M \ . \tag{5.42}$$

Instead of using an arbitrary covariance matrix as initial estimate, we may compute the initial covariance matrix of the i^{th} component from the samples in that cluster as:

$$\hat{\Sigma}_{i_0} = \frac{1}{N_i} \sum_{n=0}^{N_i} (y_i(n) - \hat{\mu}_{i_0})(y_i(n) - \hat{\mu}_{i_0})^T, \quad \text{for } i = 1,2,\ldots,M \ . \tag{5.43}$$

For the comparison purpose we initialize the parameters for each segment $\boldsymbol{y}_i$, using a stance of the EM algorithm. From then onwards, Eqn. (5.39) can be implemented recursively. Once the spectrogram is modeled appropriately by mixture modeling, our objective is to synthesize signals corresponding to the components given by the mixture model. In the next section, we present a novel algorithm to directly synthesize a signal from a component given by the mixture model.

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5.4 MAPPING OF COMPONENTS

The estimated components have to be utilized in synthesizing the signal. Coates *et al* (Coates *et al*, 1998) have used these components to separate multicompoennt signals and the isolated components are mapped into a time domain signal by using time-frequency projection filters (Hlawatsch, 1994). It is equivalent to saying that the pass region required by the filters is specified in terms of the mixture model with a choice to select the pass region in an orderly manner. Hence, the whole effort has been just confined to select the pass region. In the present work, we try to use the information provided by the mixture model alone to synthesize the signal rather than using the time-frequency projection filters, which require eigen value decomposition. With the belief that a bivariate pdf and a joint distribution of time and frequency have strong correlation, we make the analogy between a bivariate pdf and time-frequency distribution, since the general concepts of average, standard deviation, moments, etc. apply to any distributions irrespective of the physical quantities they represent (Cohen, 1995). Now at our disposal we have a bivariate pdf and TFD, and we need to make transition from the former to the latter. Since our pdf is a normal pdf, it can be sufficiently represented by standard deviation, mean and covariance. Henceforth, we must have a TFD that can also be represented by these quantities. For a pdf, the mean vector specifies where the distribution is located (or centered around) and the standard deviation specifies the spread. When we talk about TFD, the mean vector has to represent the time-frequency centering of the distribution and the standard deviation vector has to represent the spread in time and spread in frequency. Modulating a signal with a window to obtain different spreads and modulating with a complex exponential to obtain the desired frequency shift,

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together with a shift in time parameter, essentially capture all the information given by the pdf parameters. However, the additional parameter involved in pdf is the covariance. This can be considered as a shearing operator and multiplying the signal with a chirp signal can take care of this parameter. Now we resort to comparing the normal pdf and spectrogram of the signal having the above parameters.

Let $s(t)$ be a signal of the form given by

$$s(t) = \left(\frac{\alpha}{\pi}\right)^{\frac{1}{4}} e^{-\frac{\alpha(t-t_c)^2}{2}} e^{j\beta(t-t_c)^2/2 + j\omega_c(t-t_c)}, \tag{5.44}$$

with the parameters as explained in Eqn. (5.2).

Spectrogram of $s(t)$ is given by

$$SP(t,\omega) = \frac{P(t-t_c)}{\sqrt{2\pi\sigma^2_{\omega/t}}} exp\left[-\frac{(\omega - \omega_c - \langle\omega\rangle_t)^2}{2\sigma^2_{\omega/t}}\right] \quad \text{where} \tag{5.45}$$

$$P(t) = \sqrt{\frac{a\alpha}{\pi(\alpha+a)}}\ exp\left[-\frac{a\alpha}{\alpha+a}\ t^2\right],$$

$$\sigma^2_{\omega/t} = \frac{1}{2}(\alpha+a) + \frac{1}{2}\frac{\beta^2}{(\alpha+a)} \qquad \text{and}$$

$$\langle\omega\rangle_t = \frac{a}{(\alpha+a)}\beta(t-t_c)\ .$$

A bivariate Gaussian pdf is given by

$$f(y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\rho^2)}} exp\left(-\frac{(y_1-\mu_1)^2\sigma_2^2 + (y_2-\mu_2)^2\sigma_1^2 - 2\sigma_1\sigma_2(y_1-\mu_1)(y_2-\mu_2)}{2\sigma_1^2\sigma_2^2(1-\rho^2)}\right), \tag{5.46}$$

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where $[\,\mu_1, \mu_2\,]^T$ form the mean vector μ as $\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix}$, and σ_1, σ_2, ρ form the elements of the covariance matrix Σ as $\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}$.

After rewriting Eqn. (5.46) and by comparing the like terms of Eqns. (5.45) and (5.46), we obtain the following relationships:

$$\frac{a}{(a+\alpha)}\beta = \rho\frac{\sigma_2}{\sigma_1},$$

$$\begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} = \begin{bmatrix} t_c \\ \omega_c \end{bmatrix},$$

$$\frac{a}{(a+\alpha)}\alpha = \frac{1}{2\sigma_1^2} \quad \text{and}$$

$$\sigma_2^2(1-\rho^2) = \frac{1}{2}(\alpha+a) + \frac{1}{2}\frac{\beta^2}{(\alpha+a)}. \tag{5.47}$$

The following mapping rules satisfy the above equation simultaneously. They are:

$$\alpha = \frac{a\left(\frac{\sigma_2}{\sigma_1}\right)^2(1-\rho^2)}{a^2 + \left(\frac{\rho\sigma_2}{\sigma_1}\right)^2} \quad \text{and}$$

$$\beta = \frac{\rho(\mathrm{a}+\alpha)}{\mathrm{a}}\left(\frac{\sigma_2}{\sigma_1}\right). \tag{5.48}$$

And the weight of the mixture can be used to measure the weight of the chirplet, i.e., $\lambda_i = c_i$. Hence, λ, μ *and* Σ can totally estimate the parameters c, t_c, ω_c, α *and* β which are used to decompose/reconstruct a signal in terms of the chiprlets as given in Eqn. (5.1).

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5.5. ANALYSIS OF THE MAPPING RULES

The argument we made earlier that the Gaussain mixture model parameters sufficiently characterize the chirplets will be reinforced with the following analysis. However, we are least interested in the mean vector information, as it only determines the location of the chirplets. We take up the following cases:

Case 1: $\frac{\sigma_2}{\sigma_1} = 1$

The specific case of interest is at $\rho = 0$, where the pdf will be totally unskewed and has equal spreads in either direction. The equivalent representation in the t-f plane is that it forms the oblique cell, i.e., it is the only point where the Fourier transform and the signal will have equal duration. It is to be observed that the mapping rule depends only on the ratio $\frac{\sigma_2}{\sigma_1}$ but not individually on σ_1 and σ_2; and it is a many-to-one transformation. For example, the histogram of the pdf $\frac{\sigma_2}{\sigma_1} = 1$ with larger individual variances has larger spread than the one with lower individual variances, whereas in the time-frequency plane it cannot be as exemplified as in Figs. 5.4 and 5.5. The covariance matrices and the mean vectors are also shown in figures. Any σ_1 and σ_2 that satisfy $\frac{\sigma_2}{\sigma_1} = 1$ are mapped to the same signal. The variation of ρ in the positive direction causes skewing of the pdf and hence the effect of ρ now is a steady increase in β and duration of the window as shown in Fig. 5.6. Finally, when $\rho = 1$, the signal is a chirp of infinite duration with $\beta = 1$ which can be inferred from the mapping rules.

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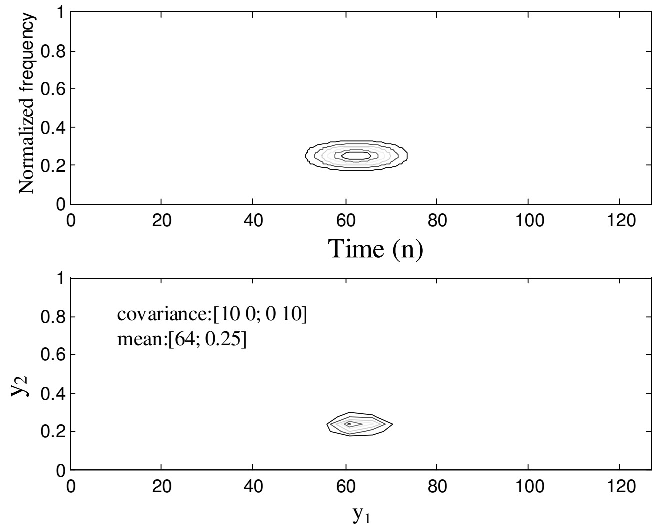

Fig. 5.4. Spectrogram of the synthesized signal using mapping rules and the histogram of the mixture density

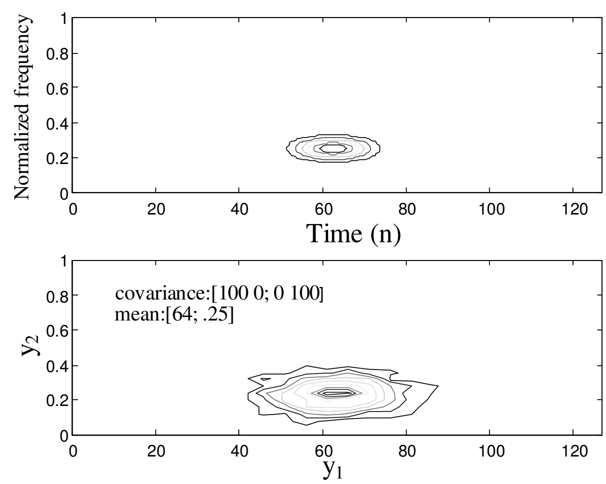

Fig. 5.5. Spectrogram of the synthesized signal using mapping rules and the histogram of the mixture density with lager variance

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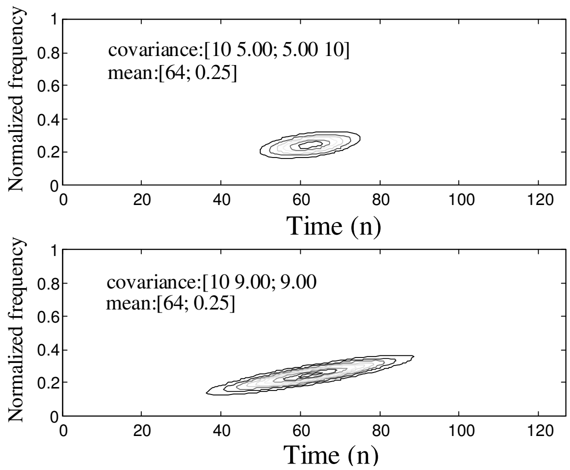

Fig. 5.6. The effect of ρ on the window duration of the chirplets synthesized from mapping rules with different ρ in each case

Case 2: $\rho = 0$

Now from Eqn. (5.48), we obtain $\alpha = \dfrac{2\left(\dfrac{\sigma_2}{\sigma_1}\right)^2}{a}$ and $\beta = 0$. It can be observed that any increase in $\dfrac{\sigma_2}{\sigma_1}$ causes an increase in α and hence shortens the window. An impulse can be modeled theoretically at $\dfrac{\sigma_2}{\sigma_1} \approx \infty$. Impulses at different time instants synthesized from the mapping rules are shown in Fig. 5.7. An impulse located at different time can be modeled by the same covariance matrix but with a different mean vector. When $\dfrac{\sigma_2}{\sigma_1}$ decreases, it has an effect of increasing the window duration and can be considered scaling of the window as depicted in Fig. 5.8.

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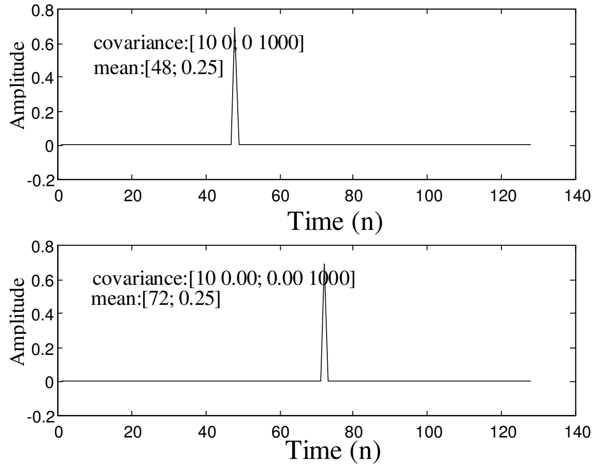

Fig. 5.7. Impulses located at different time instants obtained from the mixture model having different mean vectors

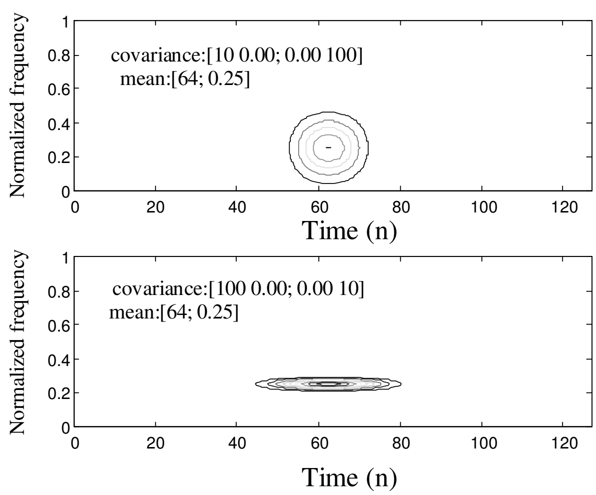

Fig. 5.8. Comparison of the spectrograms of two signals at different scales synthesized from mixture model

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And finally, analytic part of sinusoid of arbitrary frequency can be modeled at $\frac{\sigma_2}{\sigma_1} \approx 0$ with μ_2 determining the frequency of the sinusoid. Two truncated sinusoids of different frequencies derived from the mapping rules are shown in Fig. 5.9.

Case 3: $\rho = 1$

In this particular case, the pdf will be having the maximum skewness and from the mapping rules we obtain an infinite duration window. With increase in $\frac{\sigma_2}{\sigma_1}$, we observe an increase in β by which chirps of infinite duration can be modeled.

Case 4: $\frac{\sigma_2}{\sigma_1} \neq 1$ *and* $0 < \rho < 1$

As we have been mentioning, we can rotate the t-f plane or tile the t-f plane in an arbitrary fashion using chirplets and this can be obtained by coordinated shearing and scaling (Baraniuk, 1996b). Together with $\frac{\sigma_2}{\sigma_1}$, ρ causes the effect of rotation as demonstrated in Fig 5.10. Hence, we can analyze the signal in the space shift in time, shift in frequency, scale, shear in time, shear in frequency and rotation. To test the algorithm in a multicomponent scenario, we have considered a sinusoidal frequency modulated signal, WVD of which is shown in Fig. 5.11. Five components were used in the mixture density with four segments in the incremental EM algorithm. After ten iterations of incremental EM, mapping rules are used to synthesize from the mixture density parameters. The WVD of the synthesized signal is shown in Fig. 5.12. The synthesized signals corresponding to individual components in the mixture can be added according to the weights given by λ. It is the mixture modeling that resolves the

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nonlinearity inherently associated with the spectrogram which necessitates over-estimating the number of components in the mixture.

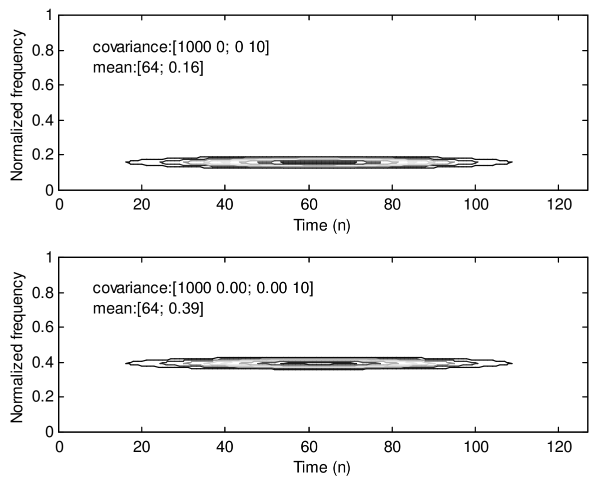

Fig. 5.9 Modeling of sinusoids at different frequencies obtained from the mixture model with different mean vectors

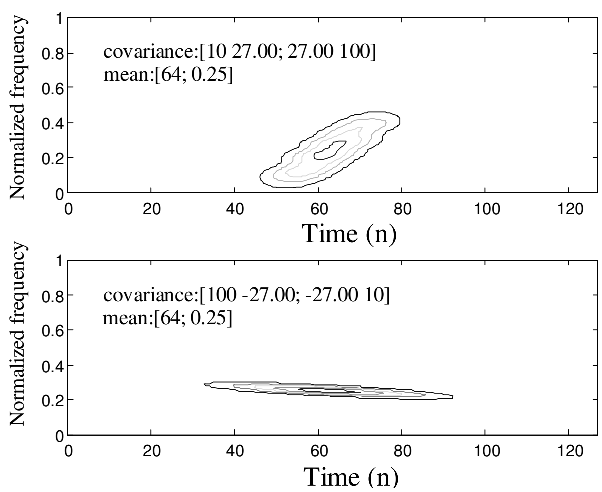

Fig. 5.10. The effect of rotation obtained by a simultaneous shearing and scaling

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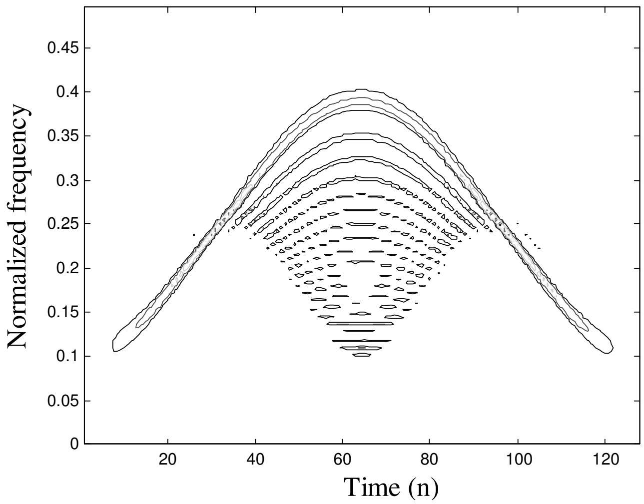

Fig. 5.11. Spectrogram of a sinusoidal frequency modulated signal

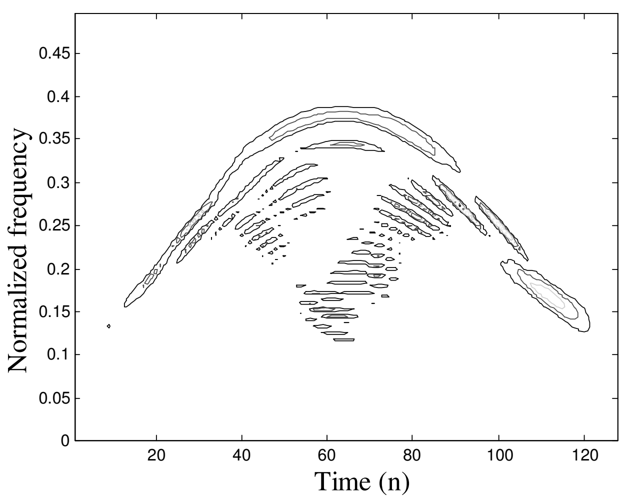

Fig. 5.12. Spectrogram of the signal used in Fig.5.11 synthesized after ten passes of the EM algorithm

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However, if the chirplets are time-frequency disjoint, then the number of components in the mixture is equal to the number of components in the signal, otherwise over estimation is required. The real part of the synthesized signal after thirty iterations of incremental EM algorithm, together with the real part of the signal, subjected to analysis is shown in Fig. 5.13.

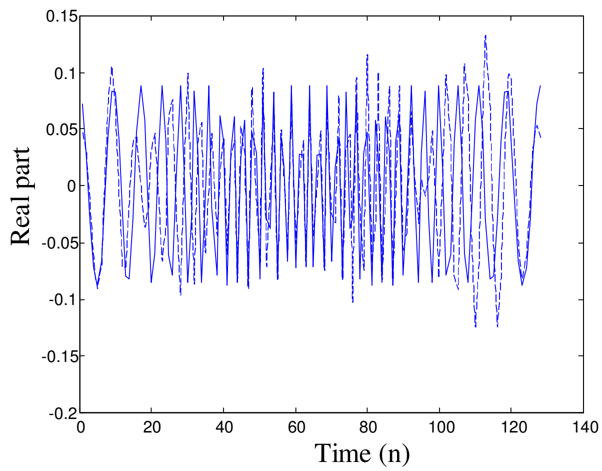

Fig. 5.13. Real part of the signal (solid) and the synthesized signal (dashed) after thirty iterations of incremental EM algorithm

In general, we choose 'a', the analysis window parameter used in computing the spectrogram, in such a way that the length of analysis window will be one fourth of the signal length. Because of this window effect, we cannot obtain the exact envelope in many situations. To demonstrate this point, we have considered a rectangular windowed sinusoid. We first used a single component and estimated the signal using the above algorithm. The estimated and the true envelope of the signal are shown in Fig. 5.14(a). It is theoretically possible to obtain any abrupt edges by assuming it as a weighed

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combination of impulses. Because of the smearing of the signal by the window, the envelope is not preserved, since spectrogram does not satisfy the marginals property. We have considered five components in the mixture to model the spectrogram. It can be observed from Fig. 5.14(b) that even the five component model tries to follow the time marginal. True estimation of the desired signal is not always possible. We cannot consider any other distribution for modeling the t-f plane that satisfies the marginals because they inherently violate the positivity condition required for viewing it as a pdf.

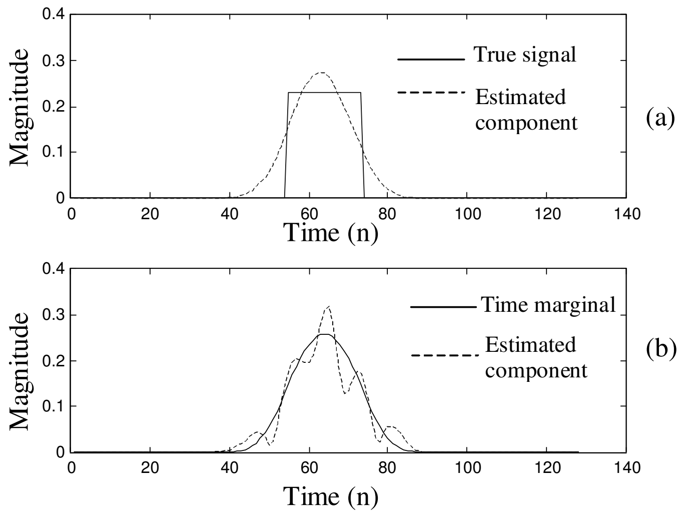

Fig. 5.14. (a) Magnitude of the rectangular windowed signal and the envelope of the estimated using a single component mixture density and (b) Time marginal obtained from spectrogram and the signal estimated from a five component mixture density

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5.6. CONCLUSIONS

We have developed an algorithm to decompose the signal specified as a spectrogram in terms of chirplets. The modeling of the spectrogram is accomplished by mixture modeling. The parameters of the mixture density are estimated using incremental EM algorithm that speeds up the convergence rate to standard EM algorithm. K-Means clustering has been used prior to mixture modeling to speed up the convergence by passing good initial estimates. We have devised a set of mapping rules that synthesize a signal from the mixture-modeled spectrogram of the signal under analysis. We analyzed the rules with an insight into the *chirplet decomposition*, wherein we observed that the orientation of the chirplet is solely determined by the covariance matrix and the location of the chirplet is determined by the mean vector. The mapping rules does not require any projection filters as they use the information provided by mixture modeling, thus considering it not only as a tool to resolve multicomponent signals but also to synthesize them.

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