ee621
Project Report on
Space-Frequency coded OFDM

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Abstract:
We implement and study a space-frequency coded OFDM system consisting of two transmitters and a single receiver. Simple Alamouti space-time code is used. An M-ary PSK modulation is used to modulate the symbols across an OFDM channel. We will also propose a variation of the scheme which tries to spread additional symbols across time-frequency attempting to increase the rate of transmission without changing the type of modulation employed or increasing the bandwidth. A Rayliegh frequency selective slow fading channel is assumed through-out the analysis. SER performance of the above systems is carried-out with emphasis on the modulation scheme, No. of carriers and bit SNR.

Key words:
Space time/ frequency coding, OFDM, MIMO, frequency-selective channel

Introduction:

High data rate reliable transmission over wireless channels is seemingly possible due to the advent of Space-time codes. Space-time codes rely on transmit diversity and are particularly suitable when the signal undergoes frequency flat fading due to the channel. In the original space-time code scheme [1], Alamouti showed that it is possible to obtain the same diversity as with multiple receivers. Since then, transmit diversity has been pursued with great interest among the research community. However, the fundamental assumption based on the which the scheme works is that that channel is frequency flat, i.e., the coherence bandwidth of the channel is much smaller than bandwidth of the signal which may not be true in wideband communication [2-5]. This assumption may not be generally true in wideband communication systems. For example, high-data rates are made possible with increased resources in terms of bandwidth in WWAN and outdoor wireless WANs. There is need to developing new technologies for providing wideband wireless communications.

OFDM (orthogonal frequency division multiplexing) has matured into a very practicable technique and has been incorporated into the IEEE 802.11a [2]. OFDM splits the channel into sub-channels equal to the no. of carriers under use. Each sub-channel is treated independently and the multiplexed modulated symbols are sent over each carrier. This operation is performed via IFFT at the transmitter side and with FFT at the receiver side. This is another interesting aspect of OFDM.

Thus marrying OFDM with Space-time codes appears vary natural in frequency selective fading scenarios. OFDM splits the channel into near frequency -flat sub-channels and Space-time codes exploit the transmit diversity under these frequency-flat sub-channels. Together they form a promising technological alternative for high data rate broadband communications. This space-frequency coded OFDM system [6]is shown in Fig. 1.

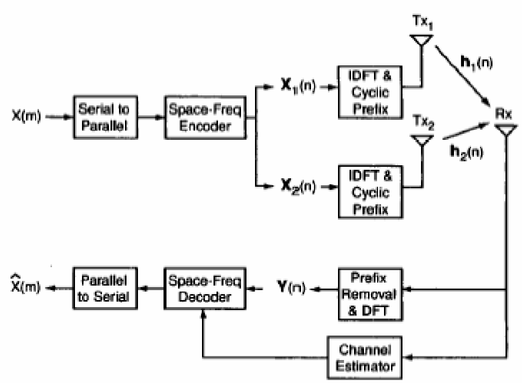


Fig. 1. A space-frequency OFDM system

In this report, we simulate 2 x 1 Alamouti space-time code with OFDM (which is called as space-frequency code, as the space-time codes are transmitted over another carrier rather than another time-slot). A frequency selective Raleigh channel is assumed. Information bits are M-ary PSK modulated, which are then converted into space-frequency codes. These are multiplexed to form OFDM frames for final transmission. We compute symbol error rates at different bit-SNRs for different M in the M-ary PSK. Later on, we will investigate ways to improve the system. In particular, we study the performance of the system in which information about few more symbols is spread over all the carriers by varying both the phase and energy of that particular constellation in a controlled manner. We will also give details about the encoding and decoding mechanisms. Symbol error rates are also compared.

System Description:

No. of carriers: N_c

Total Bandwidth: W Hz

Input symbols (M-ary coded) in a frame: $X_{0,} X_1, ..., X_{N_c}$

Modulation: M-ary PSK

No. of Tx: 2

No. of Rx: 1

Space time code: Alamouti 2 x 2 code

Channel Model [3][4][5]:

No. of taps: L = 6

$$h_i(n) = \sum_{l=1}^{L} a_l(n)\delta(t - l/W)$$

where a_l are zero mean complex Gaussian random variables with variance 1/L

$$H_i(k) = \sum_{j=1}^{N_c} h_i(j)\exp(-j2\pi kn/N_c)$$

Space-frequency code at Tx -1 (total N_c symbols) :

$$\mathbf{X}_1 = [X_{0,} - X_1^*, ..., X_{N_c-2} - X_{N_c-1}^*]^T$$

Space-frequency code at Tx -2 (total N_c symbols):

$$\mathbf{X}_2 = [X_1, X_0^*, ..., X_{N_c-1} X_{N_c-2}^*]^T$$

Received symbols at the receiver:

$$\mathbf{Y}_e = \mathbf{\Lambda}_{1e}\mathbf{X}_{1e} + \mathbf{\Lambda}_{1e}\mathbf{X}_{2e} + \mathbf{N}_e$$

$$\mathbf{Y}_o = \mathbf{\Lambda}_{1o}\mathbf{X}_{1o} + \mathbf{\Lambda}_{2o}\mathbf{X}_{2o} + \mathbf{N}_o$$

where

$$\begin{aligned} \mathbf{X}_{ie} &= \mathbf{X}_i(2k) \\ \mathbf{X}_{io} &= \mathbf{X}_i(2k+1), k = 0,1..Nc/2 \end{aligned}$$ and $\mathbf{N}_e, \mathbf{Y}_e, \mathbf{\Lambda}$ defined likewise.

$\mathbf{\Lambda}$ is a diagonal matrix with $H_i(k)$ as its diagonal elements

The estimated (decoded) symbols (assuming that two adjacent sub-channels have approximately same frequency response) after stripping the cyclic prefix and performing FFT operation on the received symbols, are [6],

$$\hat{\mathbf{X}}_e = (|\mathbf{\Lambda}_{1e}|^2 + |\mathbf{\Lambda}_{2e}|^2)\mathbf{X}_e + \mathbf{\Lambda}^*{}_{1e}\mathbf{N}_e + \mathbf{\Lambda}_{1e}\mathbf{N}^*{}_o$$

$$\hat{\mathbf{X}}_e = (|\mathbf{\Lambda}_o|^2 + |\mathbf{\Lambda}_{2o}|^2)\mathbf{X}_o + \mathbf{\Lambda}^*{}_{2e}\mathbf{N}_e - \mathbf{\Lambda}_{1o}\mathbf{N}^*{}_o$$

This equation enables us to study the performance of the scheme completely in the constellation domain.

Performance analysis:

We studied the performance of the above scheme with different M-ary groupings and different number of carriers. We varied the SNR from 0 to 20dB. M is varied at from 2 to 4. As we can see from the graph in Fig. 2, the SER is decaying with almost unity slope for all the schemes except the case with Nc=16. The reason might be that, there bandwidth is too small to assume that the channels are frequency flat and also the assumption that adjacent channels have approximately same frequency response may also be violated. The effect of employing carriers from 256 to 512 did not seem very significant as the capacity almost linearly adds up and the sub-channels would tend to be frequency flat. However, this constant SER is at the expense of bandwidth. The effect of varying M from 2 to 4 did shift the SER curve by about 2dB which is expected in M-ary modulation. To achieve the same SER, more power is required so as to push the constellation further away from the origin.

Over-loaded OFDM:

We tried to spread a message symbol onto all the carriers. An explanation is sought to suggest a way of spreading the message symbols. In the OFDM case, message symbols are modulated on to separate carriers which are orthogonal. Suppose if we use time-varying signals to modulate these message symbols, we can overload the time-frequency plane i.e., besides all the carriers, we can modulate some additional symbols using these time-varying signals. In the context of M-ary FSK, this affects the constellation in two ways:

The phase as well as radius of the constellation is varied. In the regular M-ary OFDM, the constellation is cylinder. The radius is equal to the symbol energy and each section of the cylinder corresponds to a carrier. When chirp signals are used to modulate additional symbols, then, the phase of the message symbol in a particular is crrupted by the phase of the chirp signal and phase of the message symbols this chirp signal is carrying. The radius is affected by the addition of instantaneous energy of the chirp signal. The modified constellation can be represented as:

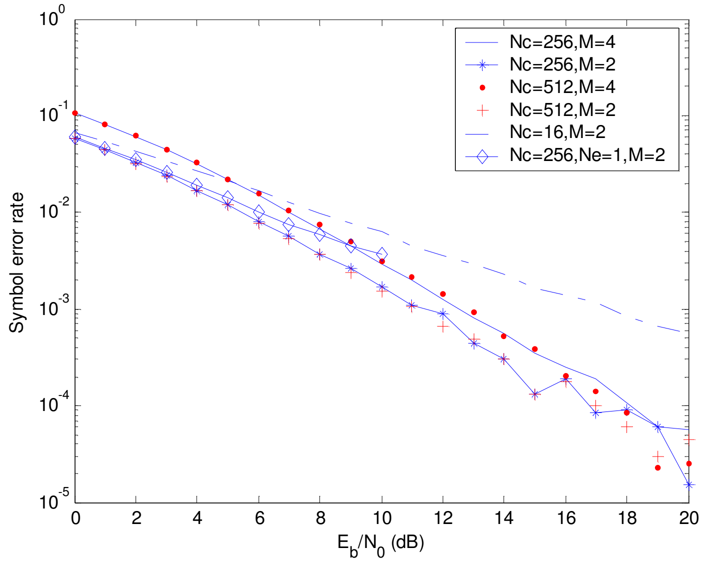
$$\begin{bmatrix} S'_o \\ : \\ S'_{Nc} \end{bmatrix} = \begin{bmatrix} 1\ 0 \ldots 0\ \mathrm{W}_1^0 \ldots \mathrm{W}_{\mathrm{Ne}} \\ : \\ 0\ \ldots\ 1\ \mathrm{W}_1^{\mathrm{Nc}} \ldots \mathrm{W}_{\mathrm{ne}}^{\mathrm{Nc}} \end{bmatrix} \begin{bmatrix} S_o \\ : \\ S_{Nc} \\ S_{Nc+1} \\ : \\ S_{Nc+Ne} \end{bmatrix}$$

S_i : mapped symbol $\mathrm{i} \in [0, \mathrm{Nc} - 1]$

W_i^k : weight of ith symbol over ith carrier

$$\sum_{i=0}^{\mathrm{Nc-1}} \mathrm{W}_i^k = 1$$

These modified symbols are now used to form the space-frequency codes. The decoding is performed by inverting the weight matrix to obtain an approximate Nc+Ne constellation vectors which is subjected to M-ary PSK detection. In the simulation studies, we have used BPSK modulation with Nc=256 and Ne=1. The choice of the weight matrix affects the performance matrix severely. It is of much interest to study how to do design the weight matrix for a given modulation and given number of carriers. Our initial results were not promising and a careful study to construct such pseudo orthogonal weight matrix is of great interest.



Conclusions:

For large number of carriers for a given band-width, the SER rate is almost the same. This is because, the sub-channels are frequency flat, each sub-channel can be used to its full capacity. The assumption that adjacent sub-channels are identical is also valid. As we increase the M, the SER shifts by 2dB. This is expected because, to achieve the same SER, more power is required to push the constellation further away from the origin to offer more resolution or detection capability. The intial weight matrix we have chosen was not performing as expected. We expect that there should be no degradation in the SER because, the Ne symbols are spread over all the carriers and all the carriers suffer independent fading. So we expect frequency diversity to offer some advantage here. This lets us think to design new weight pseudo orthogonal matrices.

References:

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Appendix: MATLAB code

Main program

```
% OFDM parameters
% to be varied
Nc = 256; % no of carriers
Ne = 0; % no of over-loaded bits (rate = log2(M)*(Ne+Nc)/Nc );

% Ne = 0 becomes regular OFDM
% fixed
Ntx = 2; %no of tx
Nrx = 1; %no of rcvrs
L = 6; % no of taps of the channel FIR filter
W = get_weights(Nc,Ne); % returns Nc x Ne weight matrix
code_weight = [eye(Nc) W];
inv_code_weight = pinv(code_weight);
if(Ne==0)
   code_weight = eye(Nc);%hadamard(Nc);
end
%H = get_H(Nc); % channel gain matrix (from FIR filer);
% simulation settings
% to be varied
EbN0dB = [0:10];  % bit SNR
M = 2; % M-ary PSK
k = log2(M);
% PSK detection boundary
for p = 1:M-1
   det_bound_left(p) =  ((p-1)*2*pi/M)+(pi/M);
   det_bound_right(p) =  det_bound_left(p)+(2*pi/M);
end
Lframe = Nc+Ne; % length of the freme (# symbols in a frame)
N = (Lframe)*floor(4*1e5/(Lframe));  % number of bits to simulate
Nframe = N/(Lframe); % No. of frames
[H1e,H1o,H2e,H2o] = get_H(L,Nc,Nframe); % Nc x Nframe complex GWN with vari 1/L
per dimension
%HC = get_HC(M,Nc,Ne,code_weight); % HC is Nc x # M^(Nc+Ne)
% fixed
C = exp(j*2*pi*(1:M)/M);  % the complex constellation
%CC = exp(-j*2*pi*(1:M)/M);
d = randint(N,1,M)+1;  % the digital symbols
s = C(d);  % constellation mapping
%Cs = CC(d); % conjugate symbols, needed for alamouti code
s=s(:);
% form them into symbol streams for OFDM frames (1symbol per carrier)
symFrames = zeros(Lframe,Nframe);
sym = zeros(Nc,1);
%Csym = zeros(Nc,1);
for p = 1:Nframe
   symFrames(:,p) = s((p-1)*Lframe+1:p*Lframe);
end
symFrames = code_weight*symFrames;
eveSym = symFrames(1:2:end,:);
oddSym = symFrames(2:2:end,:);
clear h_temp car_l car_r al_code temp1 temp2 sym symFrames
% map them to the alamouti code const
for i=1:length(EbN0dB)
   fER = 0;
    fprintf('Eb/N0=%2d dB, ', EbN0dB(i));
    Eb = 1; % we can fix Eb;
    %Es = log2(M)*Eb*Lframe/Nc; %log2M for M-ary, Lframe/Nc o-ofdm
```

```
   Es = log2(M)*Eb; %log2M for M-ary, Lframe/Nc o-ofdm
    N0=Eb/10^(EbN0dB(i)/10);
   n = sqrt(N0/2)*(randn(Nc, Nframe)+ j*randn(Nc, Nframe));
   neve = n(1:2:Nc,:);nodd = n(2:2:Nc,:);

   dhat = zeros(Lframe*Nframe,1);
   for p = 1:Nframe % detect frame-by-frame
      dh1e=diag(H1e(:,p));
      dh1o=diag(H1o(:,p));
      dh2e=diag(H2e(:,p));
      dh2o=diag(H2o(:,p));
      SymEst(1:2:Nc)                                                          =
((abs(dh1e).^2+abs(dh2e).^2)*sqrt(Es)*eveSym(:,p))+(conj(dh1e)*neve(:,p))+(dh2o*c
onj(nodd(:,p)));
      SymEst(2:2:Nc)                                                          =
((abs(dh1o).^2+abs(dh2o).^2)*sqrt(Es)*oddSym(:,p))+(conj(dh2e)*neve(:,p))-
(dh1o*conj(nodd(:,p)));
      SymEst=SymEst(:);
      dhatFrame = det_Frame(M,Nc,Ne,inv_code_weight,SymEst); % Last Ne symbols are
corrected.
      %dhatFrame                                                               =
det_symFrame(M,Nc,Ne,code_weight,SymEst,Es,dh1e,dh1o,dh2e,dh2o);    %    Last    Ne
symbols are corrected.
      dhat((p-1)*Lframe+1:p*Lframe)=dhatFrame;
      %mod(floor((angle(SymEst)+pi)/(2*pi/M)+0.5)+3, M)+1;
      if(sum(abs((dhatFrame-d((p-1)*Lframe+1:p*Lframe)))))
         fER = fER+1;
      end
   end
   %ph = mod(atan2(imag(rx),real(rx))+(2*pi),2*pi); % phase is now 0 - 2*pi
   %rx = M*ones(size(ph));
   % perform detection for each symbol on the matrix rather than on each
   % symol (MATALB is slow in for-looping)
   %for q = 1:M-1
   %    rx( find( (ph>=det_bound_left(q))&(ph < det_bound_right(q) ) ) ) = q;
   %end

    %r = s*sqrt(Es) + sqrt(N0/2)*(randn(1, K)+ j*randn(1, K));  % received
    % dhat = mod(floor((angle(r)+pi)/(2*pi/M)+0.5)+3, M)+1;
   SER(i) = length(find(d-dhat))/N;
   FER(i) = fER/Nframe;

   % the theoretical bounds
    fprintf('SER=%.2e, FER=%.2e\n', SER(i),FER(i));
   %fprintf('SER=%.2e\n',SER(i));
end
clear temp rx r t_m t_i fER C d s;
save sf_ofdm_stfc_nc256_M2.mat
%cd u:\soma\ee621
% plot the result
% semilogy(EbN0dB, SER, 'b');
% legend('Monte-Carlo simulation');
% xlabel('E_b/N_0 (dB)');
% ylabel('Symbol error rate');
% title(sprintf('Symbol error rate of %d-ary PSK', M));
% grid on;
```

function[dhatFrame]=det_Frame(M,Nc,Ne,inv_code_weight,SymEst)

```
framEst = inv_code_weight*SymEst;
```

```
Lframe = Nc+Ne;
dhatFrame =  zeros(Lframe,1);
detSym = M*ones(size(framEst));
ph = mod(atan2(imag(framEst),real(framEst))+(2*pi),2*pi); % phase is now 0 - 2*pi
for p = 1:M-1
   det_bound_left(p) =  ((p-1)*2*pi/M)+(pi/M); % detection regions
   det_bound_right(p) =  det_bound_left(p)+(2*pi/M); % detection regions
   detSym( find( (ph>=det_bound_left(p))&(ph < det_bound_right(p) ) ) ) = p;
end
dhatFrame(1:Lframe)=detSym;
```

function [H1e,H1o,H2e,H2o] = get_H(L,Nc,Nframe)

```
% 12-tap urban wireless channel model
% taps are obtained by random taps FIR filter
% two channels are assumed
%rand('state',2);
N0 = 1/L;
tap = sqrt(N0/2)*(randn(L, Nframe)+ j*randn(L, Nframe));
%tap = rand(L,Nframe);
H = fft(tap,Nc); % H is Nc x Nframe size channel matrix;
%H=ones(Nc,Nframe)/2;
H1e = H(1:2:Nc,:);
H1o = H(2:2:Nc,:);

tap = sqrt(N0/2)*(randn(L, Nframe)+ j*randn(L, Nframe));
%tap = rand(L,Nframe);
H = fft(tap,Nc); % H is Nc x Nframe size channel matrix
%H=ones(Nc,Nframe)/2;
H2e = H(1:2:Nc,:);
H2o = H(2:2:Nc,:); % Nc/2 x Nframes
clear H1 H2 tap;
```

function W = get_weights(Nc,Ne)

```
W = zeros(Nc,Ne);
w = linspace(0,0.5,Ne+1);
for p = 2:Ne+1
   %sp = tfrwv(amgauss(256,128,32).*fmlin(256,0,w(p)),1:256,Nc);
   %x=sum(sp');
   %x=x/sum(x);
   x = fft(fmlin(256,0,w(p)),2*Nc);
   x=x(1:Nc);
   x=x/sum(abs(x));
   x=x(:);
   W(:,p-1)=x;
end
```