EE 524

Project Report on

Wavelet Packets and Signal Adapted Filterbanks

Submitted by

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Principal Component Filter Banks, Compaction Filters and Wavelet Packets: A Comparative Study

Objective:

Investigate the methods for designing signal adapted bases from filter banks, in particular, principal component filter banks, FIR compaction filters and wavelet packets. Study the performance of these methods in context of discrete multitone modulation and/or subband coding.

Signal-adapted filter-banks, when viewed as a form of designing best-basis, find many applications in communication, source coding, feature detection etc.. There exist many of ways of designing these filter banks which arise mainly due to various properties that can be attributed these filters like, finite length, number of channels, smoothness, regularity, compactness, existence in all practical cases etc.. Quite naturally, choosing a design mechanism is not a simple task. In this project, we will study the design methods for adapting the filter banks and carry-out an objective analysis of performance in the context of DMT modulation and/or subband coding.

In particular, we will study, principal component filter banks (PCFBs), FIR compaction filters (FIR-CFs))and wavelet packets (WPs) and the interplay among them. PCFBs are optimal in the sense of coding gain but may not exist always. On the other hand, FIR compaction filters are particularly suitable for implementation but are sub-optimal. Again, based the number of channels, filter length, the possibilities vary and we try to understand their implication in each setting. As an alternative to designing filter banks for optimal energy compaction, one can consider wavelet packets. A fully-grown wavelet tree is nothing but a uniform filter bank, the band-width of each band proportional to the depth of the tree. Pruning this fully-blown tree can lead to non-uniform filter-bank or can lead to any structure as driven by the design criteria. Thus, WPs can be considered as alternative to PCFBs and FIR-CFs.

Recently, PCFBs have been applied in DMT and it is very well known that WPs are used in subband coding. Hencce, we can take any of the application and carry-out an application specific performance (this is not yet determined, in what sense we compare?) study.

Introduction:

Signal Adapted Filter-Banks find numerous applications in signal representation, data compression, noise suppression, progressive transmission. These filters are designed based on the input power spectral density subject to certain constraints on the number of channels, order of the filter lengths, bandwidth of the filters etc.. There exist numerous procedures to derive and design filters. Some of them include Principal Component filter banks, Compaction filters [1-3].

Based on the design criteria and input spectral density, the nature these filters vary. For example, it turns that, an optimal transform coder is an optimal subband coder with constant polyphase matrix.

Due to the presence of many constraints in the design stage, one should be very careful in designing appropriate filters. For example, if the number of channels is more than two and filter length is constrained, then PCFBs may not exist. These difficulties pose significant challenge to the designer.

In this report, we will compare the alternative to these signal adapted filter banks and try to implement a class of filter-banks derived from wavelet packets.

Filter-bank:

A filterbank, essentially decomposes the input spectrum into certain number of sub-bands where the subbands may or may not overlap. The advantage of dividing the spectrum into smaller bands is that most of the energy might be present in only few of the subbands while the remaining bands may not have any energy in them. This helps us in processing only the subband corresponding to the maximum energy. Applications of such principle can be found in data compression, progressive transmission.

A subband coder is optimal it follows the following two properties (a necessary and sufficient condition for optimality)

The coding gain is maximized if:

a) Total decorrelation (the subbands are decorrelated one from the other)
b) The subbands majorize the spectrum

Where the coding gain is defined as:

$$G_{SBC}(M) = \frac{\sum_{i=0}^{M-1} \sigma_i^2 / M}{\left(\prod_{i=0}^{M-1} \sigma_i^2\right)^{1/M}} = \frac{\sigma_x^2}{\left(\prod_{i=0}^{M-1} \sigma_i^2\right)^{1/M}}.$$

σ_i^2, is the variance of the i^{th} subband 'σ_x^2' is the variance of the input.

Principal component filter-banks (PCFBS)

Definition: Let C be a class of FBs (like transform coder class, class of filter of infinite order etc..), then for a given input spectral density, PCFB is the FB that if its the variance vector majorizes all vectors in the subbands belonging to the class C.

PCFBs are optimal in the sense that, they maximize the coding gain (G_{SBC}).

PCFBs have strong connection with transforms coders (in the case of FBs belonging to the class C), to Wiener filters (when the objective is to estimate the signal in the presence of noise) and are also the filters which minimize the transmit power in the discrete multi-tone modulation. In particular, when the filter order is less than the number of channels, KLT maximizes the coding gain which in this case is the PCFB.

Despite their many desirable properties, their existence is not always guaranteed. At least in the case of DFT filtebanks and cosine-modulated filter banks PCFBs cease to exist. However, in the two-channel case, FBs with unconstrained order and in the transorm coder case, they always exist. In other situations, one has to analyze case-by-case. This makes them less interesting despite their many desirable attributes.

In the case when we don't have PCFBS, we design compaction filters,. An FIR compaction filter is said to be the one that compacts most of the energy into only one band. Then, remaining N-1 filters are completed by considering the complement of the power spectral density. or . A compaction filter is necessarliry an Nyquist(M) filter, i.e.,

$$\left.|H_i(e^{j\omega})|^2\right|_{\downarrow M} = 1$$

Let $G(e^{j\omega}) = |H(e^{j\omega})|^2$., and compaction gain is defined as:

$$G_{comp}(M,N) = \frac{\sigma_y^2}{\sigma_x^2} = \frac{\int_{-\pi}^{\pi} G(e^{j\omega}) S_{xx}(e^{j\omega}) \frac{d\omega}{2\pi}}{\int_{-\pi}^{\pi} S_{xx}(e^{j\omega}) \frac{d\omega}{2\pi}}$$

The method of designing compaction filters is in designing a filter which maximizes the compaction gain subject to the constraints,

$$g(Mn) = \delta(n) \text{ (Nyquist}(M) \text{ condition)} \quad \text{and} \quad G(e^{j\omega}) \geq 0 \text{ (nonnegativity)}$$

Several methods were proposed to designing such filters in [4]. In the present context, we motivate ourselves to consider the following :

We want to design filters that are easily computable and share the elegance of PCFBs. That is we want to retain the existence of FIR compaction filters and retain and yet try to maximize the coding gain. We try to solve this by resorting to wavelet packets.

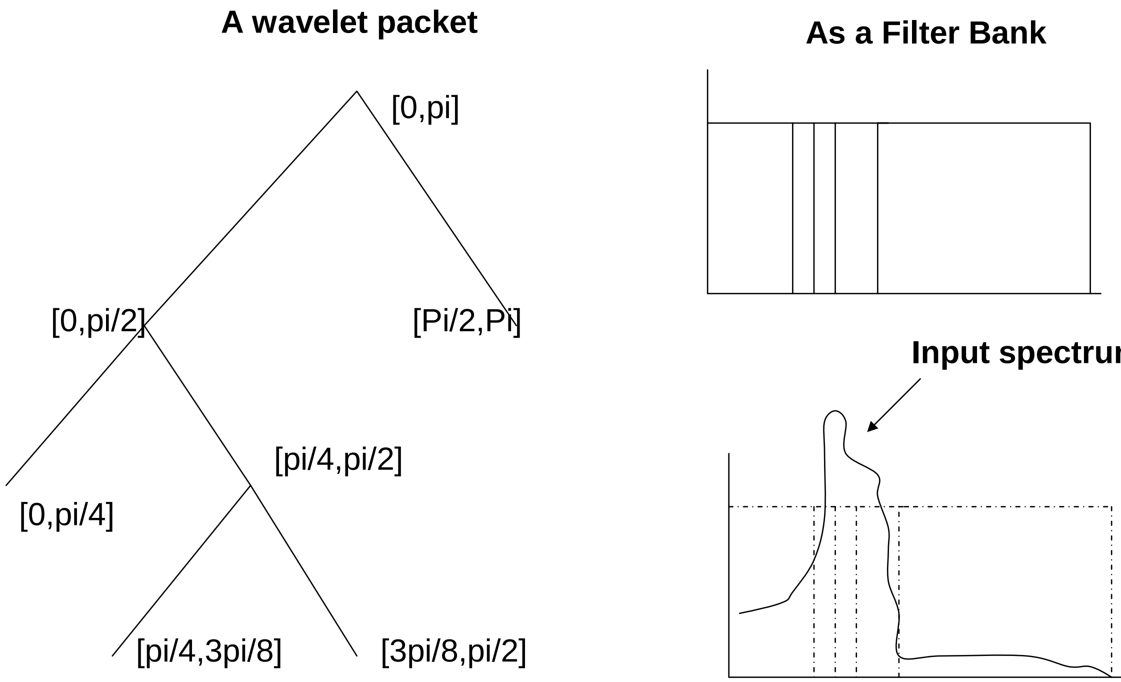
Wavelet Packets:

A Wavlet packet is a complete grown wavelet tree. That is in dyadic wavelet transform, we only further filter the approximation coefficnets and leave the detailed coefficients unattended. If think of the first analysis filter in as an approximate low-pass filter, then we repeatedly filter this low-pass filtered signal. Thus, after M levels of decomposition, we would have one approximate coefficient band and M-1 detailed coefficnet band. We keep splitting the low-pass band repeatedly. In wavelet packets, we can process any of these low-pass or high-pass bands. If we selectively perform this operation of splitting, we can obtain a non-uniform filter-bank. A fully-grown wavelet packet upto M levels, would have M bands each of width 2Pi/M. This is the central idea behind using WPs as signal-adapted FBs.

Wavelet Packets as signal-adapted FBs

We achieve this by pruning the WP tree based on a certain criteria. It is suggested that certain entropy based techniques can be used to prune the tree. This problem was considered in the context of best-basis selection. Indeed, WP offer a wide variety of bases over which the signal can be projected. Thus, it is very suitable for processing large class of signals. As we increase the number of bands or in other words if we increase the depth of the decomposition, we search for even larger number of bases that can best approximate the signal under consideration.

In the figure below are shown a wavelet packet and its equivalent subband decomposition:



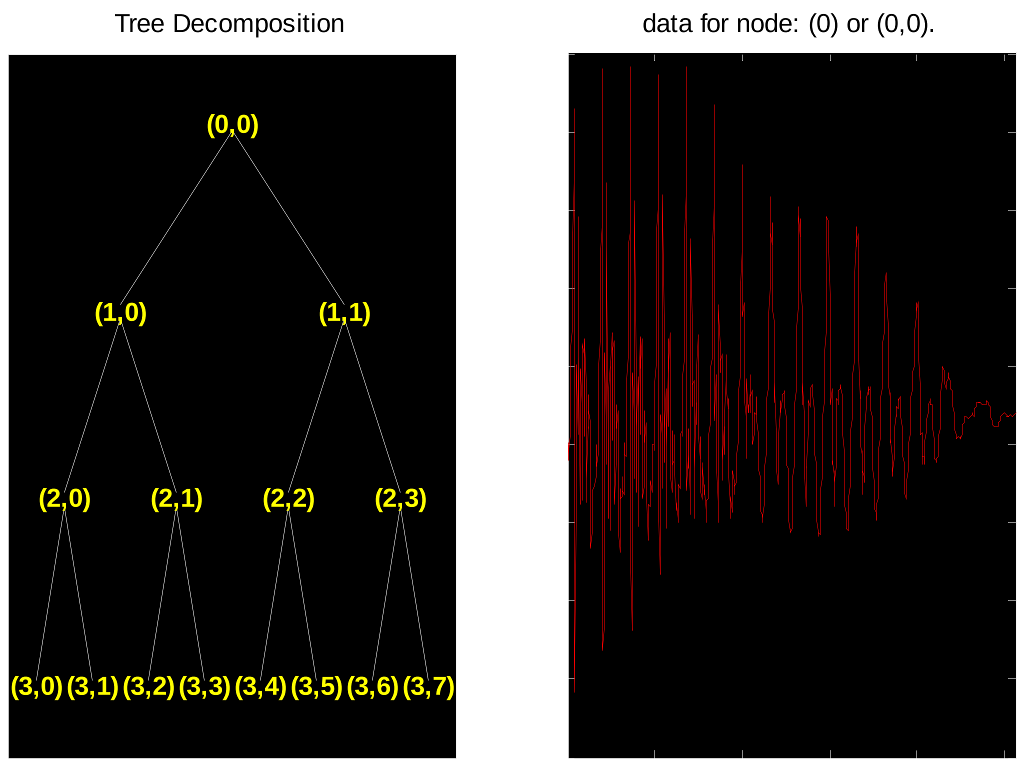
The definition of best tree (i.e., the best way to prune the tree) very much depends on the objective function being used. In the context of best basis selection, [5], have used entropy as the criteria. Those nodes are pruned which minimize the cost function specified in terms of entropy. For example, when “pruning” corresponds to merging, i.e., we first grow the the wavelet tree to its fullest depth possible, then we try to merge those two nodes which result in minimum increase in the entropy. We repeat this process until a stopping criteria is met. This stopping criteria can be given computational complexity, number of bands needed. On the other hand it is also possible to prune the tree by merging. In this case, we start with a initial tree and try to that node which results in maximum reduction in entropy. Like the earlier case, here also the stopping criteria can be desired number of subbands or desired SNR. The entropy function used in this operation is:

$$-\sum x^2 ln(x^2)$$

And this function is an additive function. However, as mentioned earler, we want to prune the WP tree such that coding gain is maximized. For simplicity in implementation, we start with just two bands in the tree and try to split the bands which result in maximum increase in coding gain!

Results:

Data: A speech signal resampled at 8kHz was considered. Total of 1024 samples were taken. A WP of depth 4 was created using “db1” wavelet. The pruned tree using the “entropy” criteria is shown below:

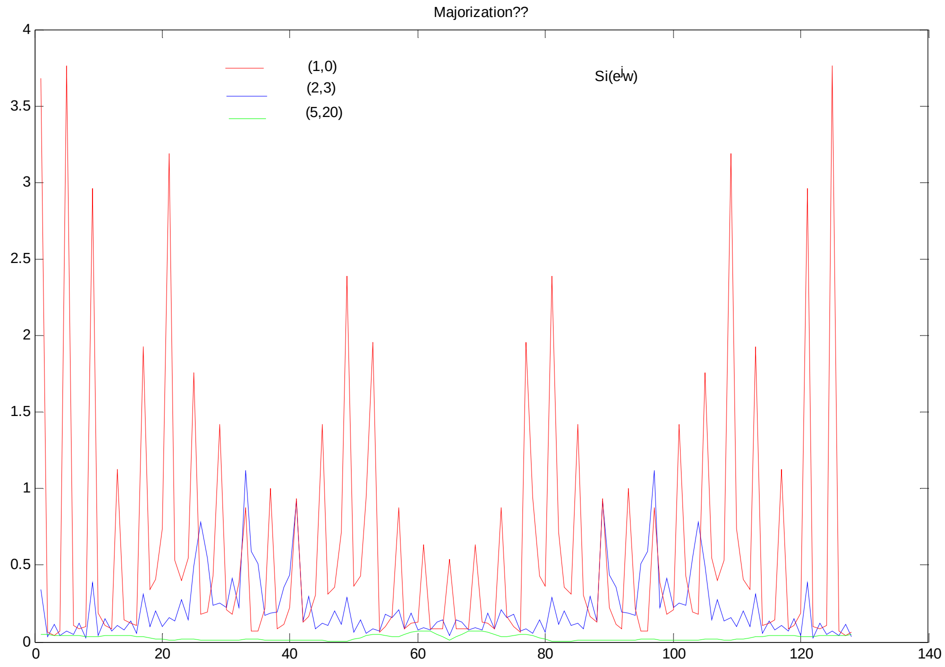


Wavelet Packet Decomposition

Pruned tree using “coding gain” is shown below:

Decomposition using "coding gain"

The PSD of the different subbands are shown in the figure below:

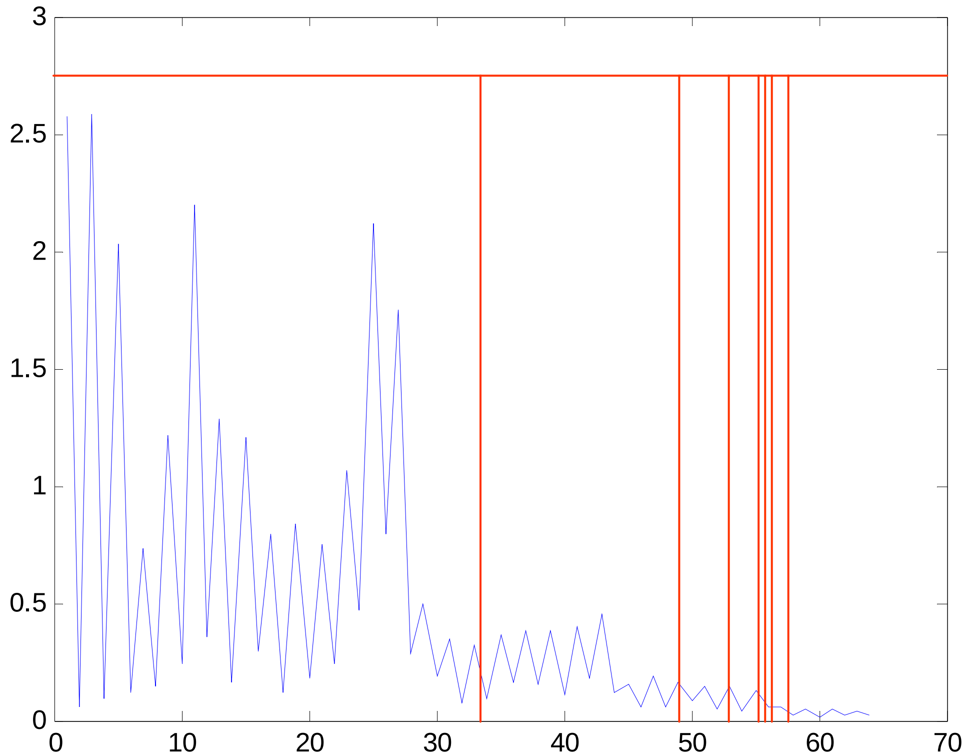


We can observe from the figure that, roughly, the subband having higher variance has PSD greater than the others. i.e.,

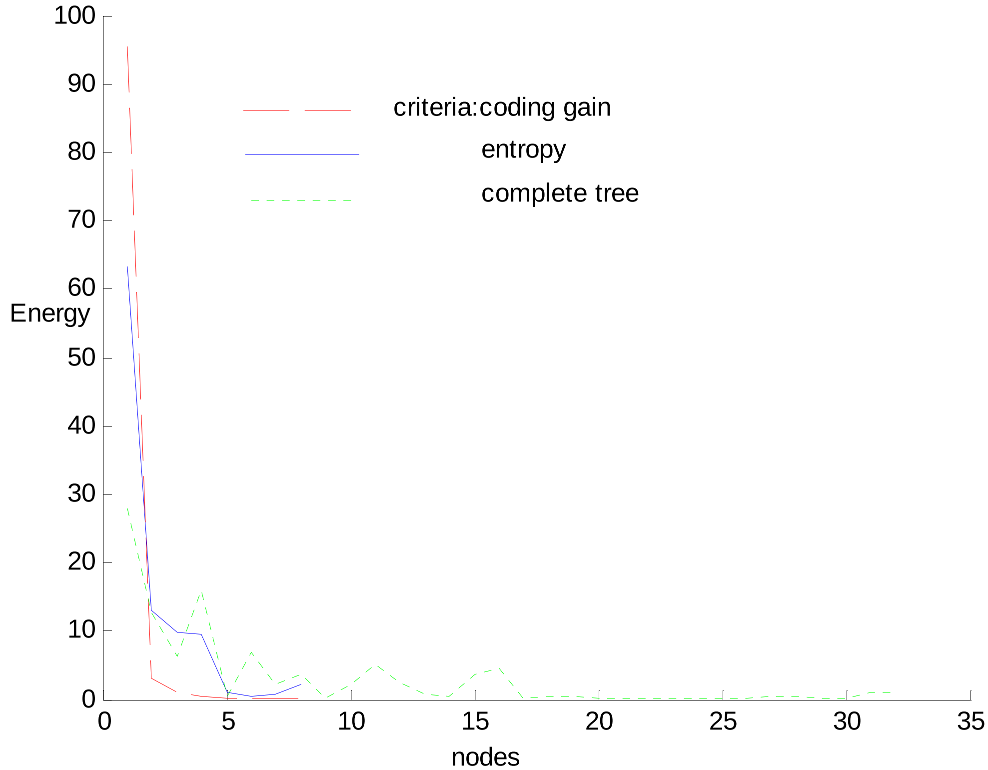
$$S_0(e^{j\omega}) \geq S_1(e^{j\omega}) \geq \cdots \geq S_{M-1}(e^{j\omega}).$$

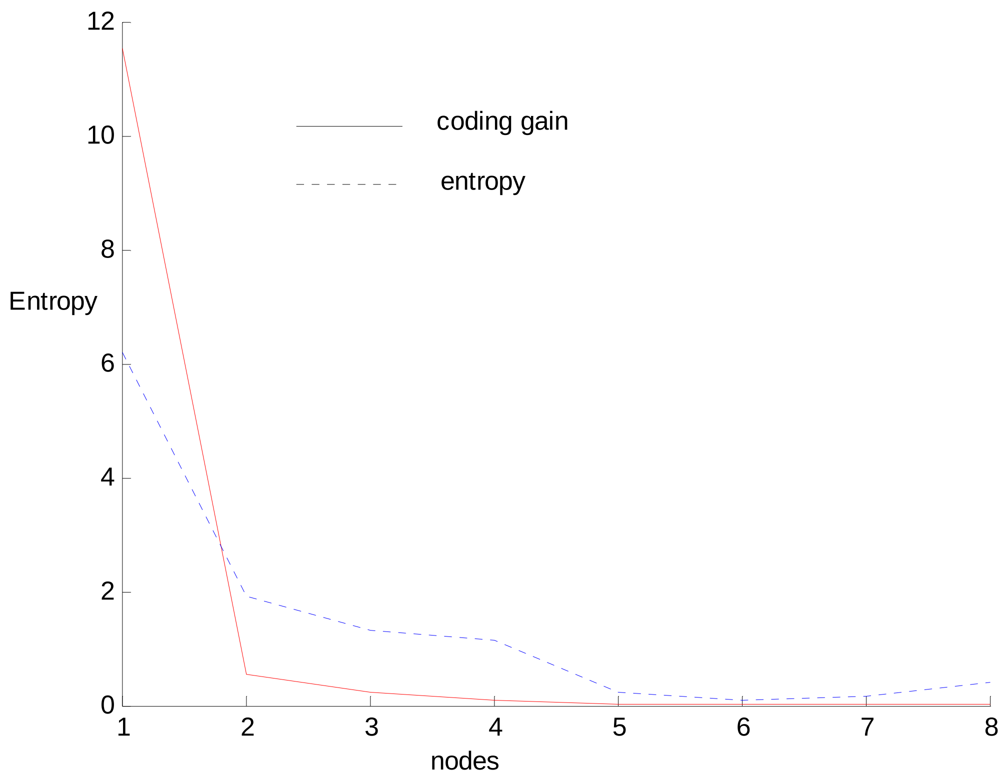
which is nothing but the majorization theorem.

In the frequency domain, the filter-bank splits the bands as shown in the figure below:



Energy and entropy of the subnands in both case are shown in the following figures, respectively.


100
90
80
70
60
50
40
30
20
10
0
Energy
criteria:coding gain
entropy
complete tree
0
5
10
15
20
25
30
35
nodes


12
10
8
6
4
2
0
Entropy
coding gain
entropy
1
2
3
4
5
6
7
8
nodes

As expected, the energy of the subband (corresponding to higher energy within the tree) has more energy compared to the subband in tree pruned based on entropy. Also, the entropy of the former is less than the latter's.

Conclusions:

The results are not very interesting -> rather they produce obvious bands

The criteria for pruning the tree play an important role in determining the structure

Wavelet packet are practically feasible…we need alternate criteria for speeding-up the pruning

Entropy used in the current pruning is additive but not directly related to Information!

MATLAB code:

```
%load sig.mat %loads data x (size 128);
load x.mat;x=x(1025:1025+1024);
M = log2(length(x)); % max. of levels of decomposition
N = 8;
%x=ones(128,1);
t=wpdec(x,1,'db1');
cN = 2;% current no of nodes
node = zeros(cN,3);
code_gain = ones(cN,1);
for p = 1:cN
    node(p,1) = var(wpcoef(t,[1,p-1]));
    node(p,2)=1;
    node(p,3)=p-1;
end

code_gain_a(2)=mean(node(:,1));
code_gain_b(2)=prod(node(:,1))^(1/cN); % this is coding gain at
Mth level with N bins
code_gain(2) = code_gain_a(2)/code_gain_b(2);

% try_splitting here...N greater than 2
dN = cN;
for q = 2:N-1%

    gain_table = zeros(dN,1);% inefficient usage of previously
computed gains
    for p = 1:dN
        % check wether the node can be split for this db1 wavelet
        if(node(p,1)==M)
```

```
            % stop splitting this node. it can not be split
anymore
            gain_table(p) = -1; % set this to a value that can be
detected easily;
        else
            mt = wpsplt(t,[node(p,2),node(p,3)]);
            tx_a = wpcoef(mt,[node(p,2)+1,2*node(p,3)+0]);
            tx_b = wpcoef(mt,[node(p,2)+1,2*node(p,3)+1]);
            var_a = var(tx_a);
            var_b = var(tx_b);
            gain_table_a(p)= ((code_gain_a(q)*dN)-
node(p,1)+var_a+var_b)/(dN+1);
            gain_table_b(p)=
(code_gain_b(q))^(dN/(dN+1))*(var_a*var_b/node(p,1))^(1/(dN+1))
            gain_table(p) = gain_table_a(p)/gain_table_b(p);
        end
    end
    %pick-up the maximum
    [val,ind]=max(gain_table);
    if(val<0)%
        q = q-1;
        % this should not go infinite loop as long as N<2^M;
    else
        % split the node with index "ind";
        dN = dN+1;
        new_node = zeros(dN,3);
        new_node(1:ind-1,:)=node(1:ind-1,:);
        new_node(ind+2:dN,:)=node(ind+1:dN-1,:);
        % repeat the step..update the table;
        p = ind;
        t = wpsplt(t,[node(p,2),node(p,3)]);
        tx_a = wpcoef(t,[node(p,2)+1,2*node(p,3)+0]);
        tx_b = wpcoef(t,[node(p,2)+1,2*node(p,3)+1]);
        new_node(p,1) = var(tx_a);
        new_node(p,2) = node(p,2)+1;
        new_node(p,3) = 2*node(p,3)+0;
        new_node(p+1,1) = var(tx_b);
        new_node(p+1,2) = node(p,2)+1;
        new_node(p+1,3) = 2*node(p,3)+1;
        code_gain_a(q+1)= ((code_gain_a(q)*dN)-
node(p,1)+new_node(p,1)+new_node(p+1,1))/(dN+1);
        code_gain_b(q+1)=
(code_gain_b(q))^(dN/(dN+1))*(new_node(p,1)*new_node(p+1,1)/node(
p,1))^(1/(dN+1));
        code_gain(q+1) = code_gain_a(q)/code_gain_b(q);
        clear node;
        node = new_node;
        clear new_node;
    end
 end
```

References:

(1) Sony Akkaakaran, P.P. Vaidyanathan, “Principal component filter banks- Existence issues and applications to modulated filter banks”, IEEE IISCAS –May 2000, pp. I-523-526

(2) P.P. Vaidyanathan, “Theory of optimal subband coders”, IEEE Trans. on Sig. Proc., Vol. 46, No. 6, June, 1998

(3) P.P. Vaidyanathan, “The best basis problem, compaction filters and PCFB design problems”, IEEE ICAS, Orlando, Florida, June, 1999.

(4) A. Kirac, P.P. Vaidyanathan, “Efficient methods of optimal FIR compaction filters for M-chennel FIR subband coders”, Proc. Oof 30th ASILMORE conf. on Signals, Sysmtes and Computers, 1996

(5) Ronald R. Coifman, M. V. Wickerhauser, “Entropy-based Algoritms for best basis selection”, IEEE Trans. Oon. Info. Th., Vol. 38, No. 2, March, 1992, pp. 713-718