Distributed Source Location in Wireless Sensor Networks

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Introduction

What?
Why?
How?
Why only this?

Sub-gradient methods

Nowak's decentralized algo.(ICASSP'04)
Improvements
Sequential Bayesian analysis

Conclusions/ Future work

Who leads the race?
Oversimplified assumptions?
What else can be done?

Introduction

What?

Given the measurements of a source/ target
by a set of sensors,
obtain/ estimate its location/ position

Why?

Tracking
Surveillance

How?

- Triangulation
    - **Lateration** (from distances)
    - Angulation (from angles/ bearings)
- Scene analysis
    - Image processing
- Proximity analysis
    - Physical contact….

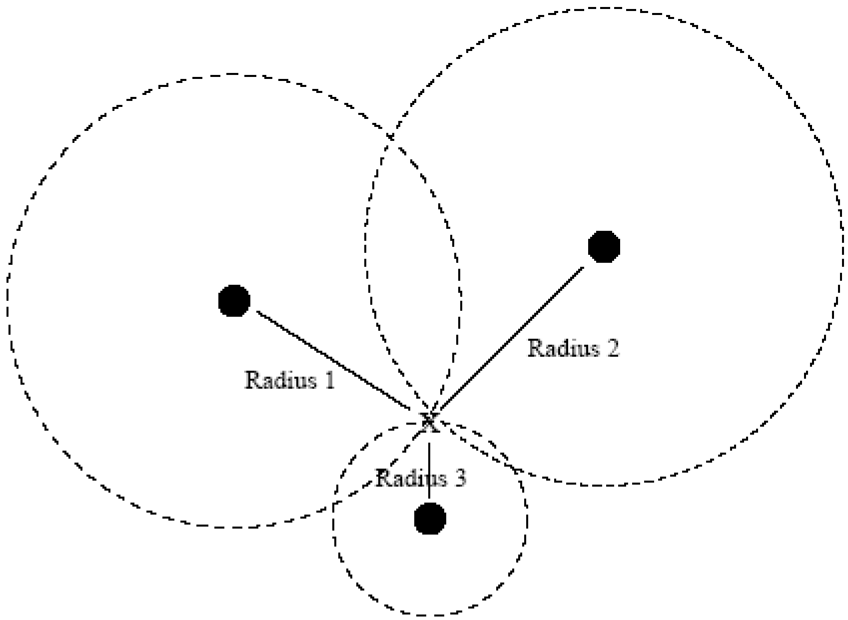
- Centralized
- **Distributed**

Introduction

Why only this?

Lateration

Distributed



Sub-gradient methods

Model :

$$s_j(i) = \frac{A}{((\theta_1 - x_j)^2 + (\theta_2 - y_j)^2)^{\beta/2}} + e_j(i)$$

$s_j(i)$ is the i^{th} measurement at the j^{th} sensor

A is the acoustic energy

$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$ and $\begin{bmatrix} x_j \\ y_j \end{bmatrix}$ are the locations of the source and the j^{th} sensor.

$e_j(i)$ are i.i.d. samples of zero-mean Gaussian noise process with variance σ^2

β is a parameter dependent on the transmission medium

Assumptions:

$\beta = 2$

σ^2

A are known

$\begin{bmatrix} x_j \\ y_j \end{bmatrix}$

Sub-gradient methods

Objective function to be minimized under least-squared cost is:

centralized

$$f = \frac{1}{MN}\sum_{j=1}^{M}\sum_{i=1}^{N}\left[s_j(i) - \frac{A}{(\theta_1 - x_j)^2 + (\theta_2 - y_j)^2}\right]^2$$

M is the number of sensors
N is the number measurements in one cycle
(a sensor contributes only once in a cycle)

Sub-gradient method (Nowak et al.'s algo.)

$$\hat{\theta}_{j+1}^k = \hat{\theta}_j^k - \alpha \nabla f_{j+1}(\hat{\theta}_j^k)$$

distributed

$$\hat{\theta}_j^k = \begin{bmatrix} \hat{\theta}_1 \\ \hat{\theta}_2 \end{bmatrix} \text{ at the } j^{th} \text{ sensor during the } k^{th} \text{ cycle,}$$

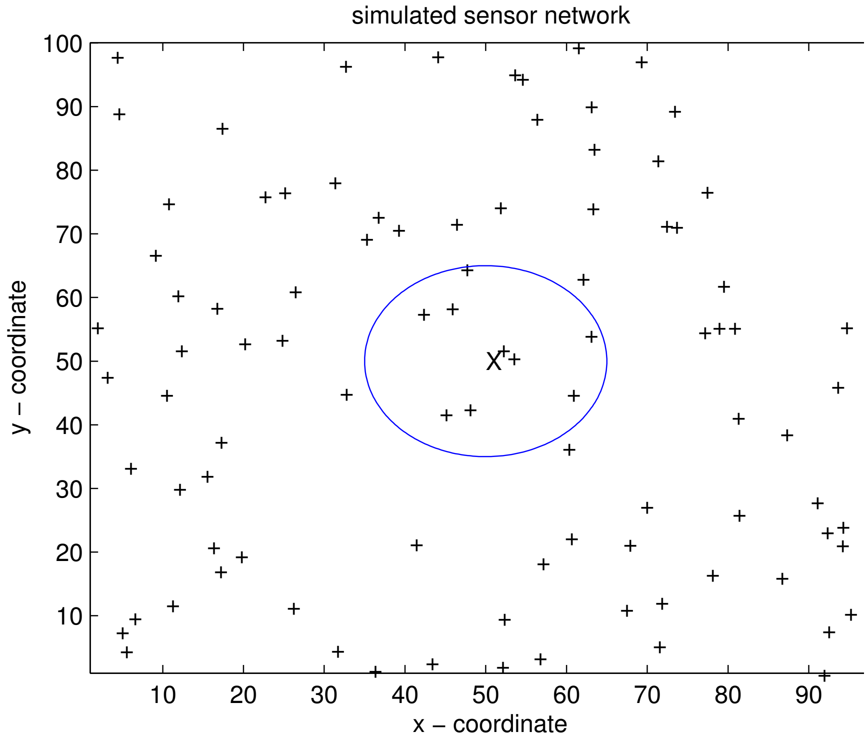
$$f_j = \frac{1}{N}\sum_{i=1}^{N}\left[s_j(i) - \frac{A}{(\theta_1 - x_j)^2 + (\theta_2 - y_j)^2}\right]^2,$$

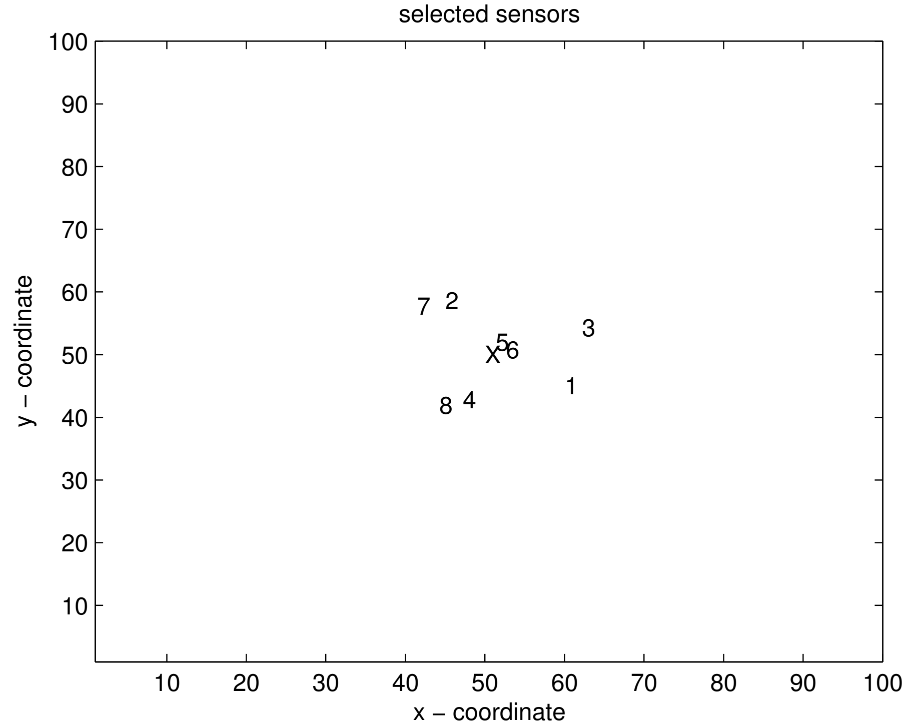
$$\nabla f_j = \frac{4A\sum_{i=1}^{N}\left[s_j(i) - \frac{A}{(\theta_1 - x_j)^2 + (\theta_2 - y_j)^2}\right]}{N((\theta_1 - x_j)^2 + (\theta_2 - y_j)^2)}\begin{bmatrix} \theta_1 - x_j \\ \theta_2 - y_j \end{bmatrix};$$

α is the step size in the steepest decent direction.

Sub-gradient methods

Set-up:

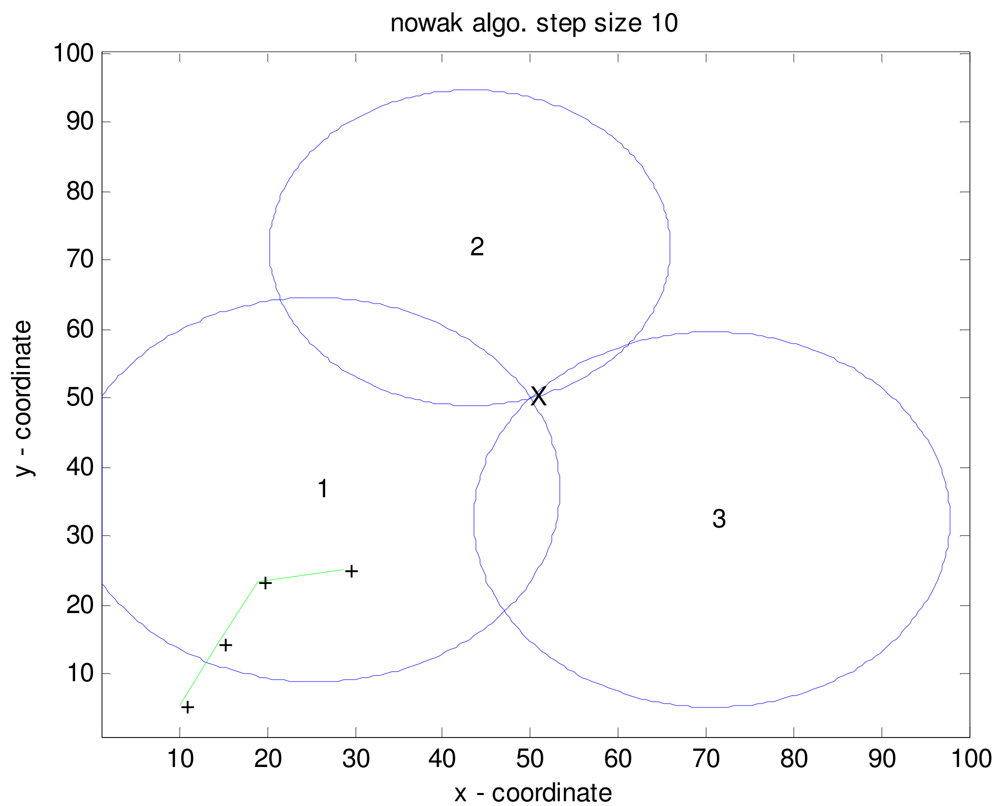




All the sensor within the radius of 15 units and not less than 1 unit from the source collaborate in estimating the location

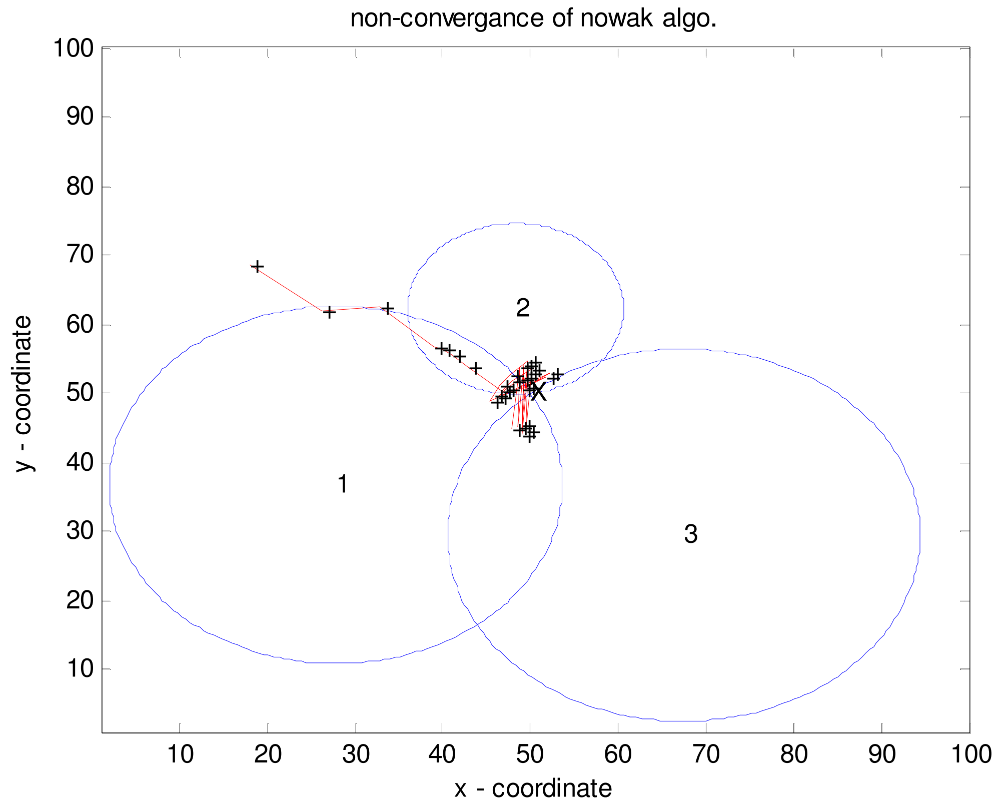
Sub-gradient methods

(Nowak et al.'s algo.)



Sub-gradient methods

(Nowak et al.'s algo.)



Sub-gradient methods

(Modified Nowak et al.'s algo.)

Rewrite the objective function w.r.t the current sensor:

$$f = \frac{1}{MN} \sum_{j=1}^{M} \sum_{i=1}^{N} \left[s_j(i) - \frac{A}{r_j^2} \right]^2$$

Then, solution to the normal eqn. is,

ML estimate

$$r_j^2 = \frac{A}{\bar{s}_j}$$

$\bar{s}_j$ is the sample average of the measurements at the j^{th} sensor in the k^{th} cycle

$$\begin{aligned}
r_j^k &= \sqrt{\frac{A}{\bar{s}_j^k}} \\
\phi_j^k &= \tan^{-1}\left(\frac{\hat{\theta}_{j2}^{k-1} - y_j}{\hat{\theta}_{j1}^{k-1} - x_j}\right) \\
\hat{\theta}_{j1}^k &= y_j + r_j^k \cos(\phi_j^k) \\
\hat{\theta}_{j2}^k &= x_j + r_j^k \sin(\phi_j^k)
\end{aligned}$$

Sub-gradient methods

(Modified Nowak et al.'s algo.)

$$d_j^{k-1} = \left((\hat{\theta}_{j1}^{k-1} - x_j)^2 + (\hat{\theta}_{j2}^{k-1} - y_j)^2\right)^{\frac{1}{2}}$$

$$\begin{bmatrix} \hat{\theta}_{j1}^{k} \\ \hat{\theta}_{j2}^{k} \end{bmatrix} = \begin{bmatrix} \hat{\theta}_{j1}^{k-1} \\ \hat{\theta}_{j2}^{k-1} \end{bmatrix} - \frac{d_j^{k-1} - r_j^{k-1}}{d_j^{k-1}} \begin{bmatrix} \hat{\theta}_{j1}^{k-1} - x_j \\ \hat{\theta}_{j2}^{k-1} - y_j \end{bmatrix}$$

Nowak's algo
(adaptive step-size)

$$\begin{bmatrix} \hat{\theta}_{j1}^{k} \\ \hat{\theta}_{j2}^{k} \end{bmatrix} = \begin{bmatrix} \hat{\theta}_{j1}^{k-1} \\ \hat{\theta}_{j2}^{k-1} \end{bmatrix} - \frac{d_j^{k-1} - r_j^{k-1}}{d_j^{k-1}} \begin{bmatrix} \hat{\theta}_{j1}^{k-1} - x_j \\ \hat{\theta}_{j2}^{k-1} - y_j \end{bmatrix}$$

Only quantity being computed
At the current sensor in the current cycle

$$r_j^k = \sqrt{\frac{A}{\bar{s}_j^k}}$$

Improve the estimate by using moving average

$$\bar{s}_j^k = \frac{\bar{s}_j + (k-1)\bar{s}_j^{k-1}}{k}$$

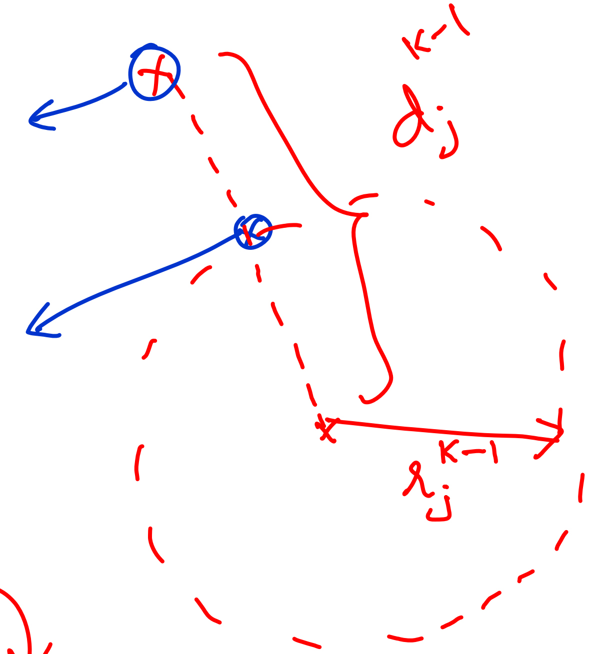
Sub-gradient methods

(Modified Nowak et al.'s algo.)

$$\begin{bmatrix} \hat{\theta}_{j1}^{k} \\ \hat{\theta}_{j2}^{k} \end{bmatrix} = \begin{bmatrix} \hat{\theta}_{j1}^{k-1} \\ \hat{\theta}_{j2}^{k-1} \end{bmatrix} - \underbrace{\frac{d_j^{k-1} - r_j^{k-1}}{d_j^{k-1}}}_{\text{magnitude}} \underbrace{\begin{bmatrix} \hat{\theta}_{j1}^{k-1} - x_j \\ \hat{\theta}_{j2}^{k-1} - y_j \end{bmatrix}}_{\text{direction vector}}$$

Step-size can be imaginary when the data is noisy

$$r_j^k = \begin{cases} 0 & \text{if } \bar{s}_j^k \le th \\ \sqrt{\frac{A}{\bar{s}_j^k}} & \text{otherwise} \end{cases}$$

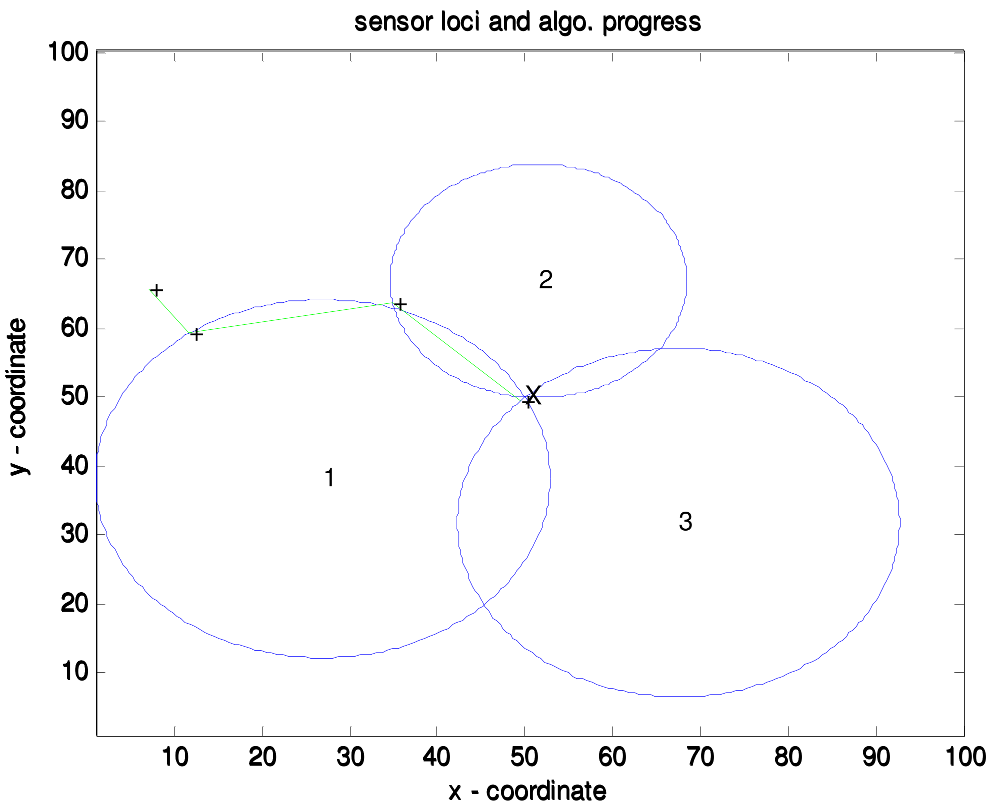


- **forward-only**
- **not greedy**

ML estimate $\leftarrow r_j^{k-1} e^{j\phi}$ $\rightarrow$ least-squared estimate (also ML)

Sub-gradient methods

(Modified Nowak et al.'s algo.)



Sequential Bayesian analysis

Posterior density at J-th sensor during kth cycle becomes Prior for the (j+1)-th sensor in the same cycle.

$$\Pi^k_{j+1}(\theta_1,\theta_2) = P^k_j(\theta_1,\theta_2)$$

Normal **prior**

$$\Pi(\theta_1,\theta_2) = \frac{exp(-\frac{(\theta_2-\mu_2)^2}{2\sigma_2^2})}{(2\pi\sigma_2^2)^{0.5}} \frac{exp(-\frac{(\theta_1-\mu_1-\rho\frac{\sigma_1}{\sigma_2}(\theta_2-\mu_2))^2}{2\sigma_1^2(1-\rho^2)})}{(2\pi\sigma_1^2(1-\rho^2))^{0.5}}$$

$$\begin{aligned} \mu_1 &= E[\theta_1] \\ \mu_2 &= E[\theta_2] \\ \sigma_1^2 &= E[(\theta_1-\mu_1)^2] \\ \sigma_2^2 &= E[(\theta_2-\mu_2)^2] \\ \rho &= \frac{E[(\theta_1-\mu_1)(\theta_2-\mu_2)]}{\sigma_1\sigma_2}. \end{aligned}$$

Likelihood function of the data

$$L(\theta_1,\theta_2) = \frac{1}{(2\pi\sigma^2)^{0.5}} exp(-\frac{\sum_{i=1}^{N}(s_j^i - \frac{A}{(\theta_1-x_j)^2+(\theta_2-y_j)^2})^2}{2\sigma^2})$$

Posterior density

$$P(\theta_1,\theta_2) = \frac{L(\theta_1,\theta_2)\,\Pi(\theta_1,\theta_2)}{\int\int L(\theta_1,\theta_2)\,\Pi(\theta_1,\theta_2)\,d\theta_1\,d\theta_2}$$

Sequential Bayesian analysis

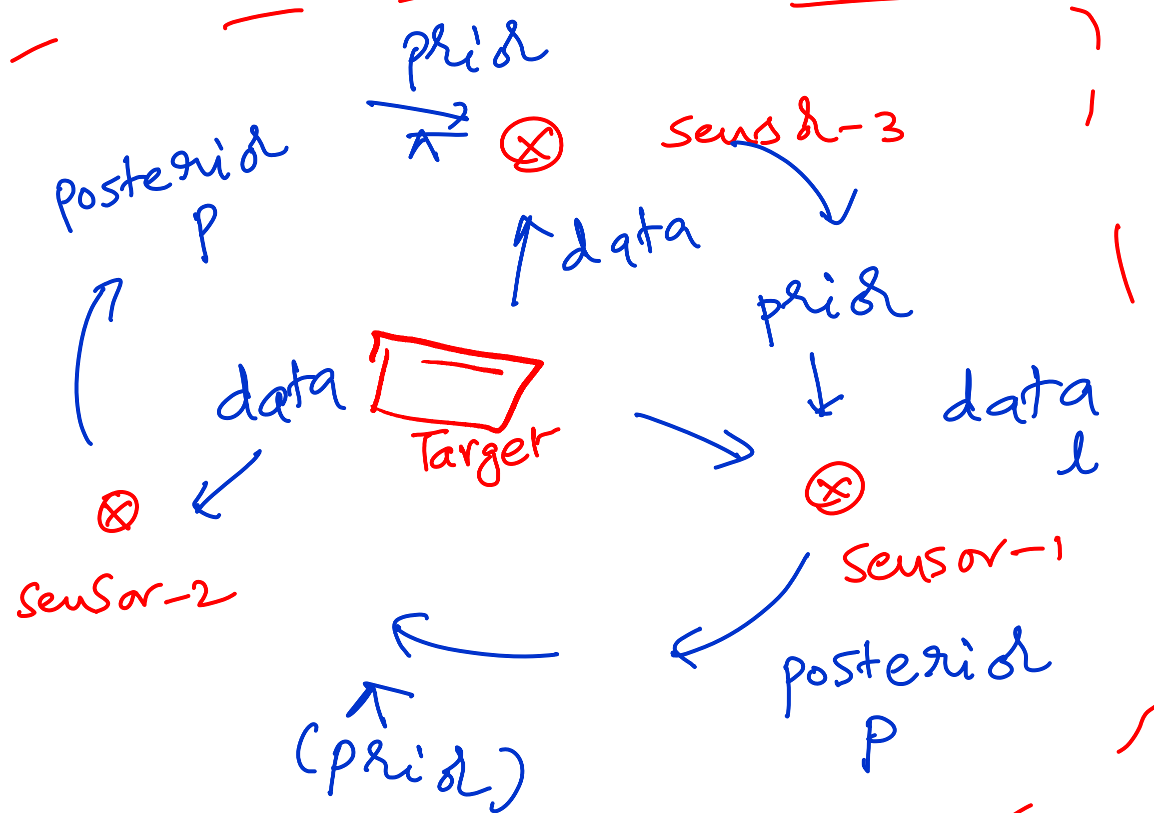
Bayasian analysis

- incorporate subjective knowledge into the problem
- can make inference even when data is insufficient
- suitable for online processing (sequential processing)

posterior (for inference) $\propto$ likelihood (data) $\times$ prior (prior knowledge)

Sequential Bayesian analysis

Bayasian analysis in wireless sensor networks



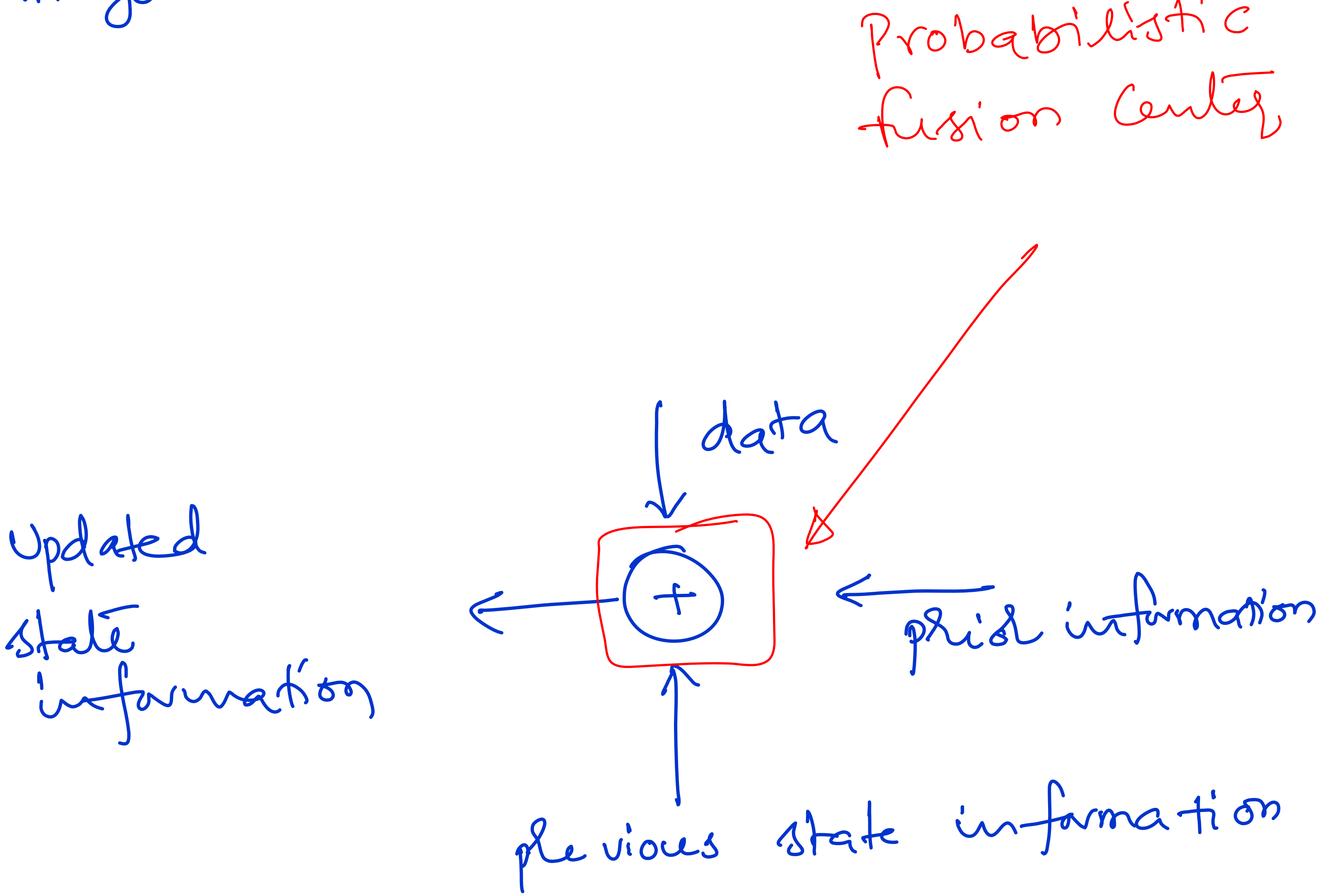
ABCD

geometry of the target locus
(one possible prior)

cyclical processing

Sequential Bayesian analysis

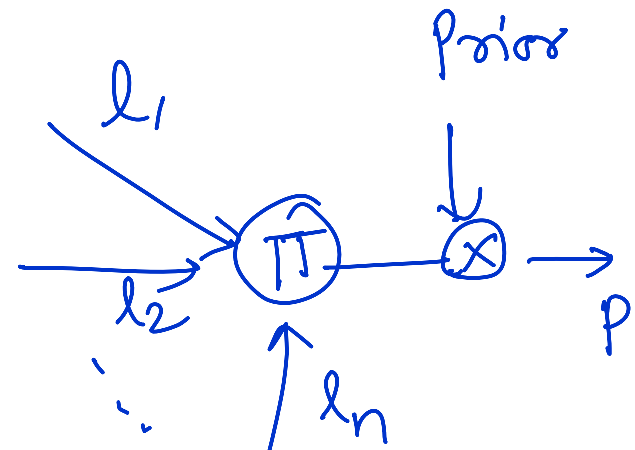
in general



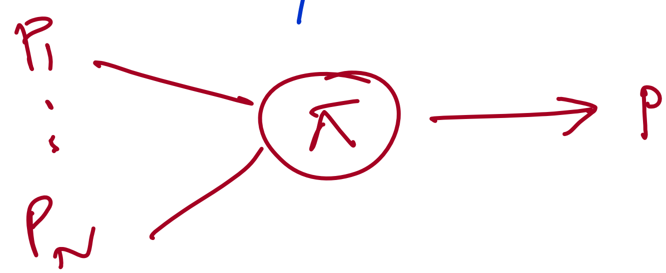
Sequential Bayesian analysis

Types of fusion:

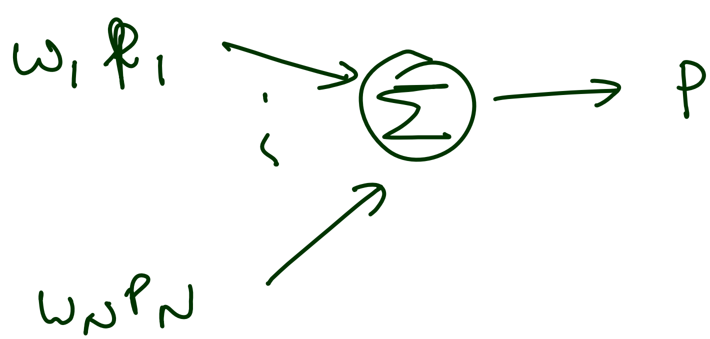
- Independent likelihood pool



- Independent opinion pool



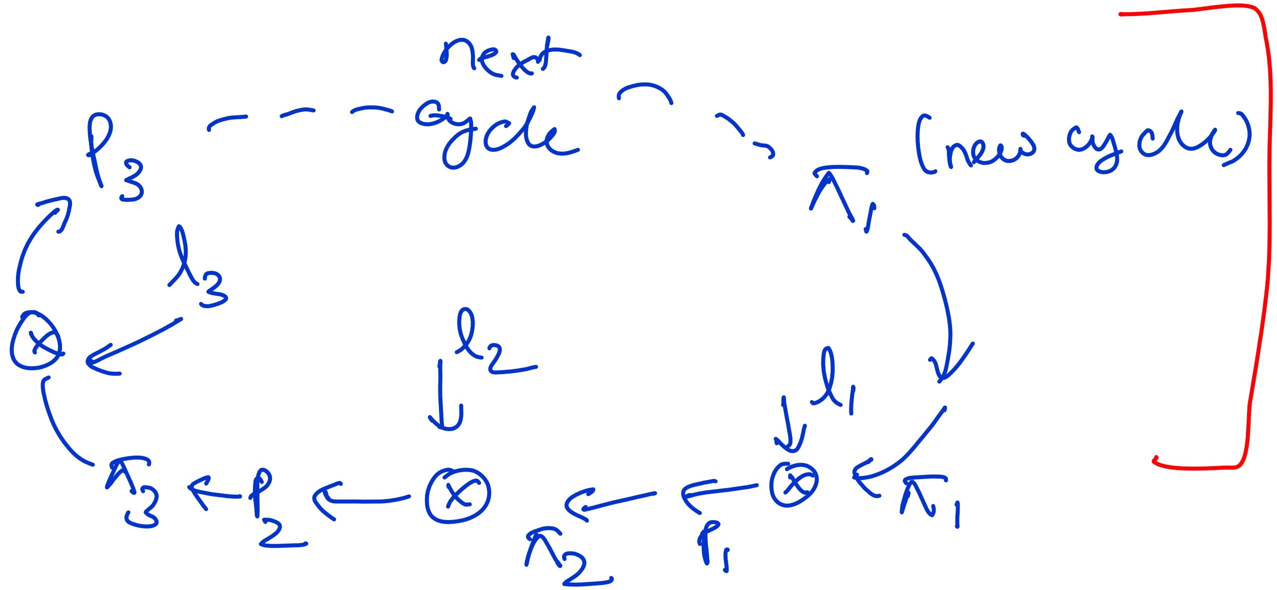
- Linear opinion pool



Sequential Bayesian analysis

Independent likelihood pool:

- Suitable if all the sensors have the same prior
- and can independently estimate the state information

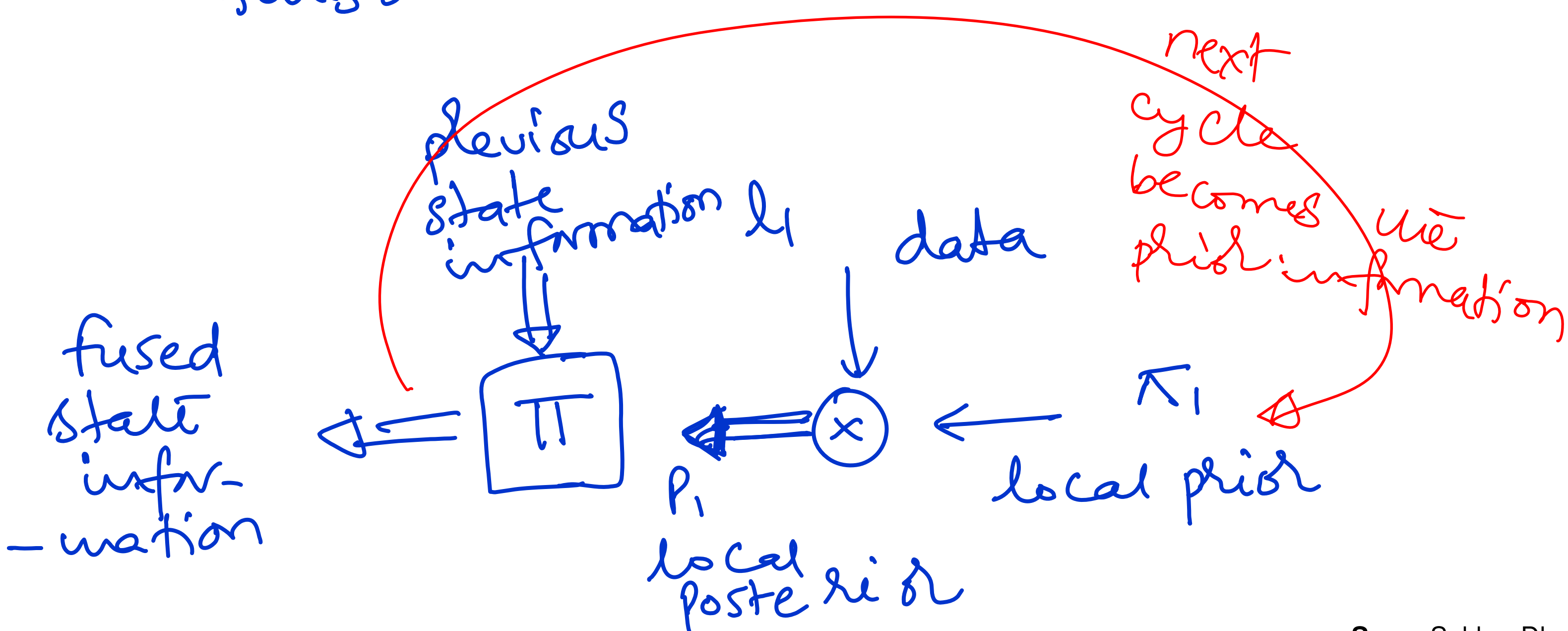


a seq. version

Sequential Bayesian analysis

Independent opinion pool

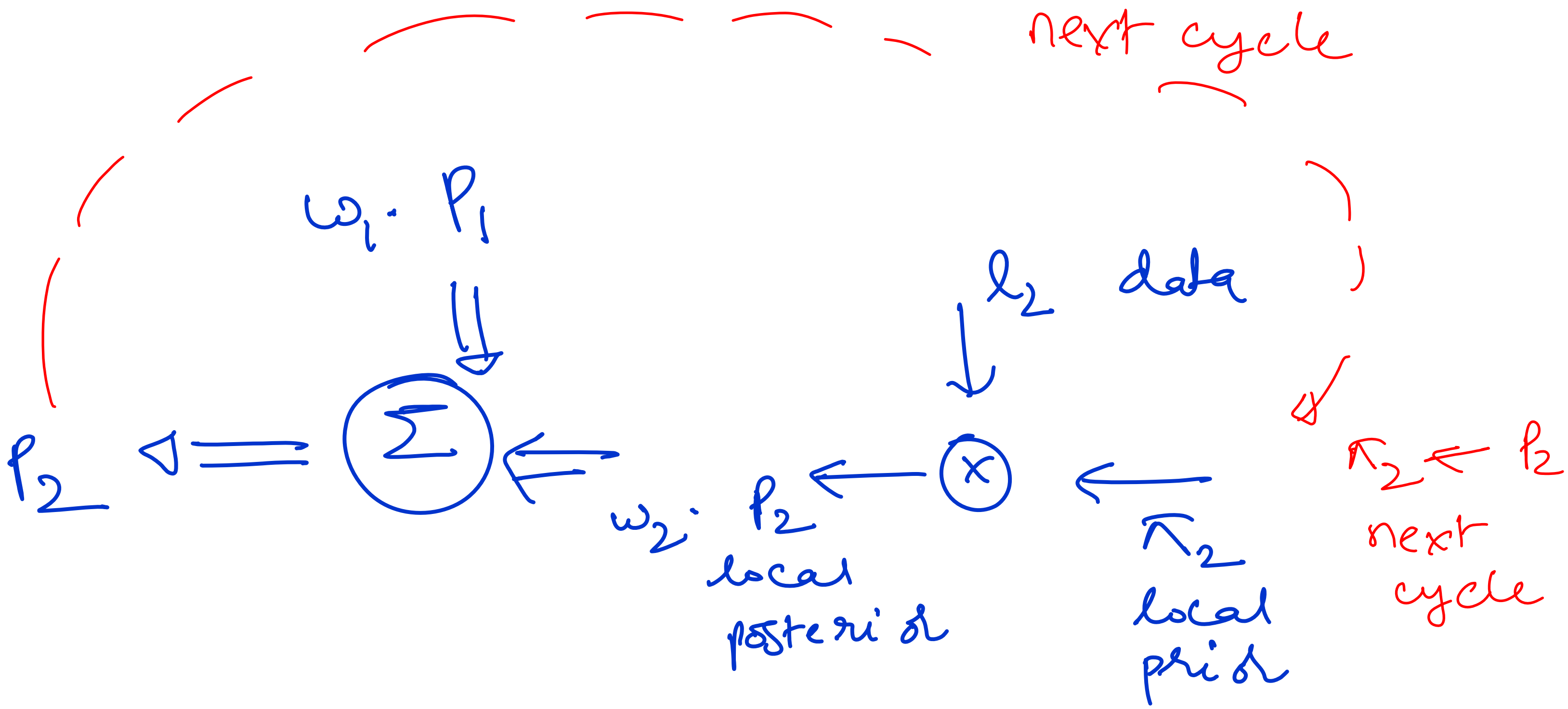
- Suitable when each sensor has different prior information
- and can estimate the state information reliably by each sensor



Sequential Bayesian analysis

Linear opinion pool.

- sensor estimates have strong dependency (possibly linear)



Sequential Bayesian analysis

- prior is normal $\rightarrow N(\mu, \Sigma)$
- posterior is approximated as normal
- likelihood $= c\, N\left(\frac{A}{d^2}, \sigma^2\right)$

posterior calculations are based on

- Monte-carlo integrations &
- Gauss-Hermite numerical approximations

Sequential Bayesian analysis

Efficient calculation of Posterior density:

(Using Gauss Hermite numerical approximation)

$$E_{post}(g(\theta_1,\theta_2)) = \int\int g(\theta_1,\theta_2)\, L(\theta_1,\theta_2)\, \Pi(\theta_1,\theta_2)\, d\theta_1\, d\theta_2$$

Upon expanding, we get,

$$= \int \frac{exp(-\frac{(\theta_2-\mu_2)^2}{2\sigma_2^2})}{(2\pi\sigma_2^2)^{0.5}} \int g(\theta_1,\theta_2) \frac{exp(-\frac{(\theta_1-\mu_1-\rho\frac{\sigma_1}{\sigma_2}(\theta_2-\mu_2))^2}{2\sigma_1^2(1-\rho^2)})}{(2\pi\sigma_1^2(1-\rho^2))^{0.5}}\, d\theta_1\, d\theta_2$$

15 x 15 grid
Is chosen

A numerical approximation to the above equation is sought using the Hermite polynomials [Ref2] which leads us to the following:

$$E_{post}[g(\theta_1,\theta_2)] \approx \sum_{P=1}^{N1} m_{1,p} \sum_{q=1}^{N2} m_{2,q}\, g(z_{1,p}, z_{2,q}) L(z_{1,p}, z_{2,q})$$

$$\begin{aligned}
\mu_1' &= \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(\theta_2 - \mu_2) \\
\sigma_1' &= \sigma_1(1-\rho^2)^{0.5} \\
m_{1,p} &= w_{1,p}\, exp(t_{1,p})\sqrt{2}\sigma_2 \\
z_{1,p} &= \mu_2 + \sqrt{2}\,\sigma_2\, t_{1,p} \\
m_{2,q} &= w_{2,q}\, exp(t_{2,q})\sqrt{2}\,\sigma_1' \\
z_{2,q} &= \mu_1' + \sqrt{2}\,\sigma_1'\, t_{2,q}
\end{aligned}$$

$$\begin{aligned}
\hat{\theta_1} &= M_{11} \\
\hat{\theta_2} &= M_{21} \\
\hat{\sigma_1^2} &= M_{12} - \hat{\theta_1}^2 \\
\hat{\sigma_2^2} &= M_{22} - \hat{\theta_2}^2 \\
\hat{\rho} &= \frac{M_{co} - \hat{\theta}_1\hat{\theta}_2}{\hat{\sigma}_1\hat{\sigma}_2}
\end{aligned}$$

$$\begin{aligned}
M_{11} &= E_{post}[\theta_1] \\
M_{21} &= E_{post}[\theta_2] \\
M_{12} &= E_{post}[\theta_1^2] \\
M_{22} &= E_{post}[\theta_2^2] \\
M_{co} &= E_{post}[\theta_1\theta_2]
\end{aligned}$$

Sequential Bayesian analysis

Some approximations:

$$p_{new} \propto \underbrace{p_j^K(\theta)}_{N_{sly}} \cdot \underbrace{p_{j-1}^K(\theta)}_{N(\mu_{j-1}^K,\ \Sigma_{j-1}^K)}$$

[Indep. opinion pool]

$$p_{new} \approx \sim N\left[\mu_{new},\ \Sigma_{new}\right]$$

$$\mu_{new} = \Sigma_{new}\left(\Sigma_j^K \mu_j^K + \Sigma_{j-1}^K \mu_{j-1}^K\right)$$

$$\Sigma_{new} = \left({\Sigma_{j-1}^K}^{-1} + {\Sigma_j^K}^{-1}\right)^{-1}$$

(mean: a weighted average of the local state & previous state)

Sequential Bayesian analysis

Some approximations:

$$p_{new} \propto \underbrace{w_1 p_1}_{\mathcal{N}(\mu_1, \Sigma_1)} + \underbrace{w_2 p_2}_{\mathcal{N}(\mu_2, \Sigma_2)} \quad \left\{ \text{linear opinion pool} \right\}$$

. $p_{new} \approx \mathcal{N}(\mu, \Sigma)$ $\left\{ \text{normal approx is sought} \right\}$

when

$$\Sigma_{new} = w_1 (\Sigma_1 + \mu_1 \mu_1^T) + w_2 (\Sigma_2 + \mu_2 \mu_2^T)$$

$$- \mu_{new} \mu_{new}^T$$

$$\mu_{new}^T = w_1 \mu_1 + w_2 \mu_2$$

Sequential Bayesian analysis

Some approximations:

$$P_{new} \propto w_1 \underbrace{P_1}_{\mathcal{N}(\mu_1, \Sigma_1)} + w_2 \underbrace{P_2}_{\mathcal{N}(\mu_2, \Sigma_2)}$$

{ linear opinion pool }

- $P_{new} \approx \mathcal{N}(\mu_{new}, \Sigma_{new})$ { normal apprx is sought }

when

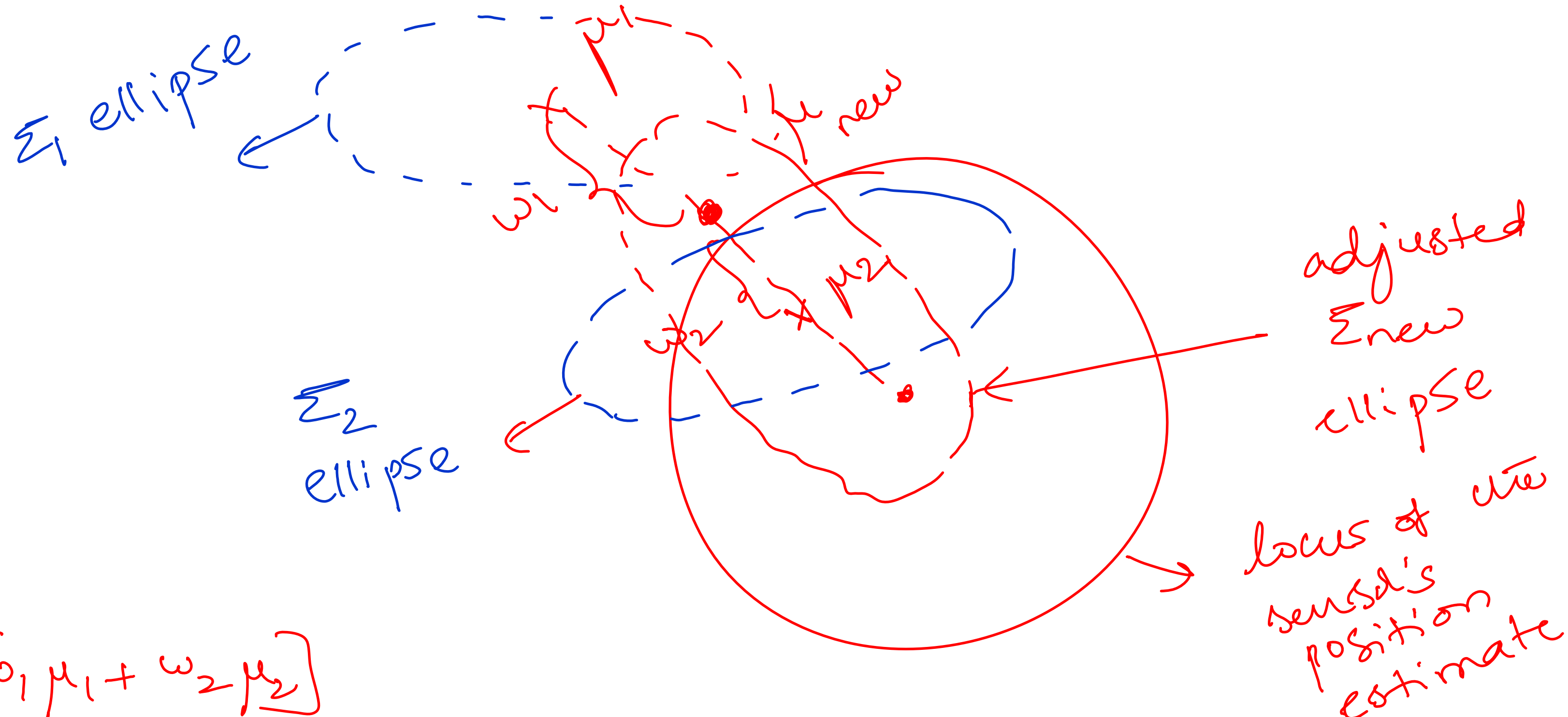
if $\Sigma_1 \to 0$ (absolutely certain about it)

$$\mu_{new} = w_1 \mu_1 + w_2 \mu_2$$

$$\Sigma_{new} = w_2 \Sigma_2 + w_1 w_2 (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

Sequential Bayesian analysis

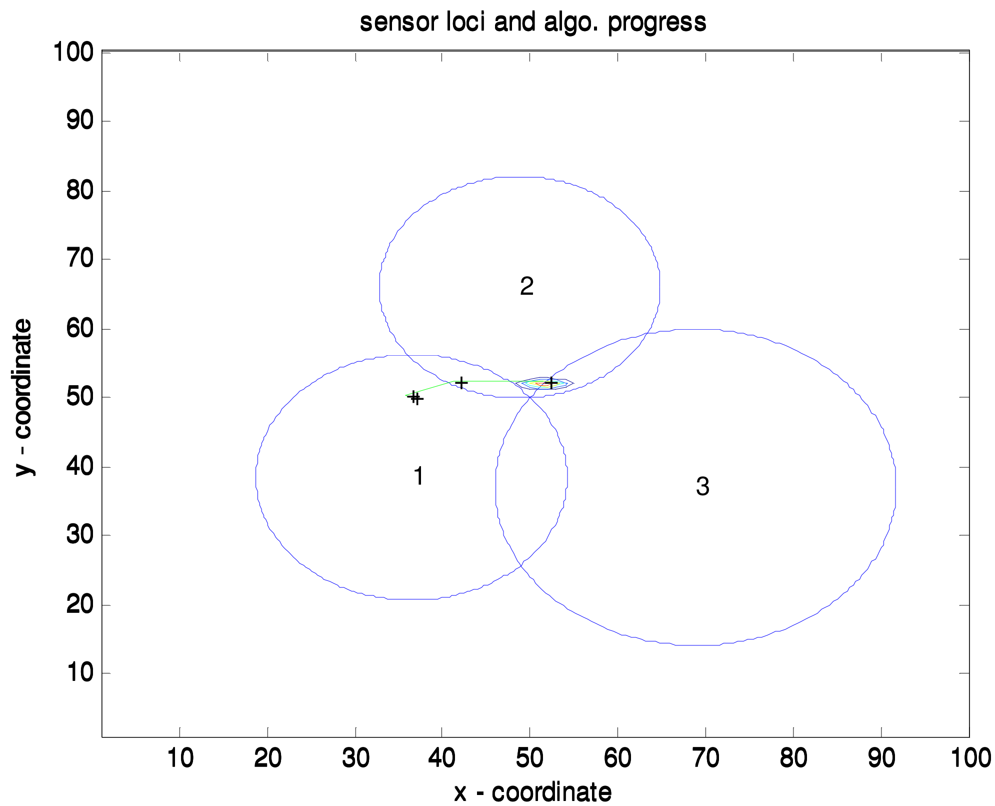
Some approximations: geometrical interpretation



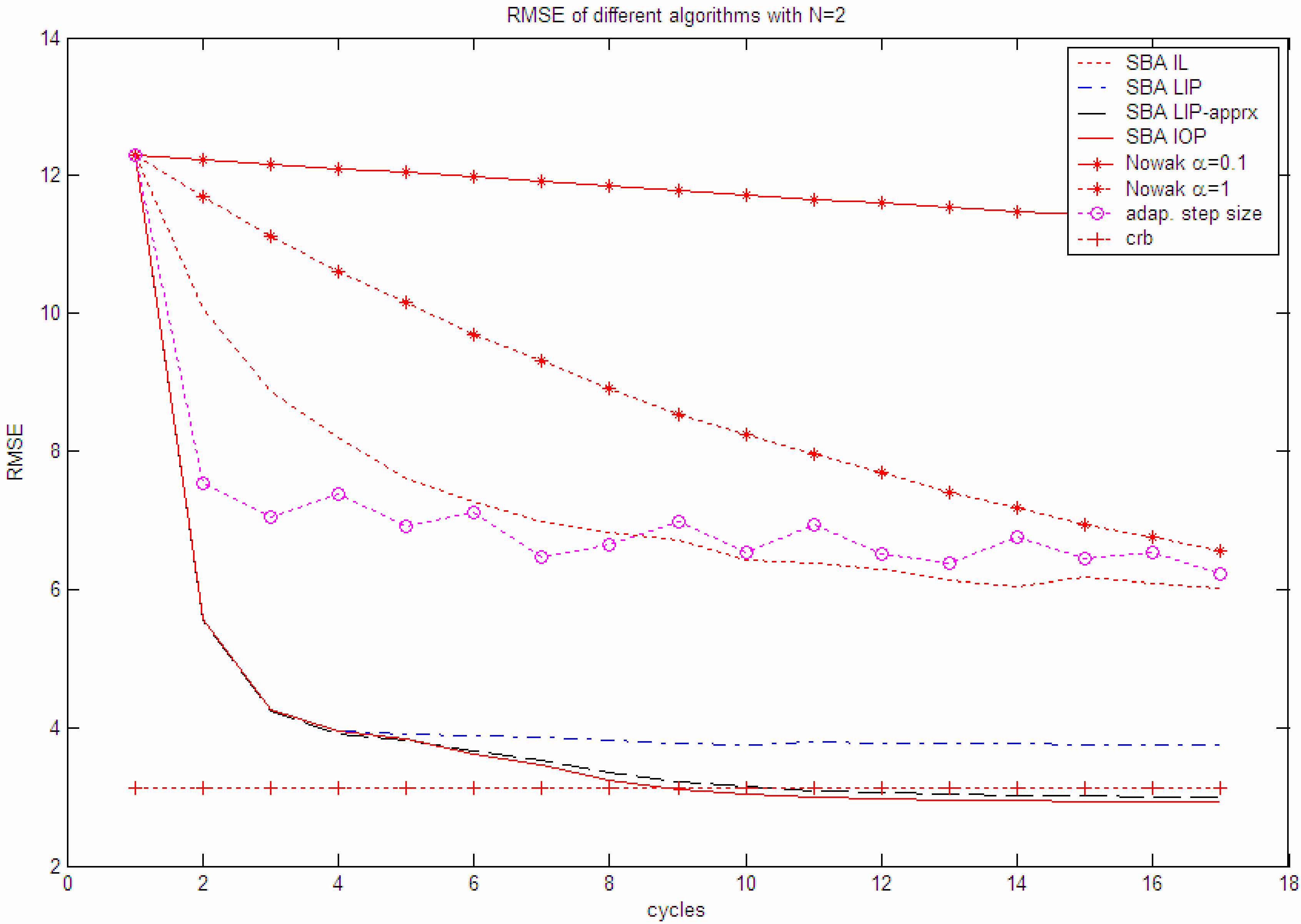
$$\underbrace{\left[\omega_1\mu_1 + \omega_2\mu_2\right]}$$

Covariance matrix is steered in the direction of the error

Sequential Bayesian analysis



Results


RMSE of different algorithms with N=2
RMSE
cycles
SBA IL
SBA LIP
SBA LIP-apprx
SBA IOP
Nowak α=0.1
Nowak α=1
adap. step size
crb
14
12
10
8
6
4
2
0
2
4
6
8
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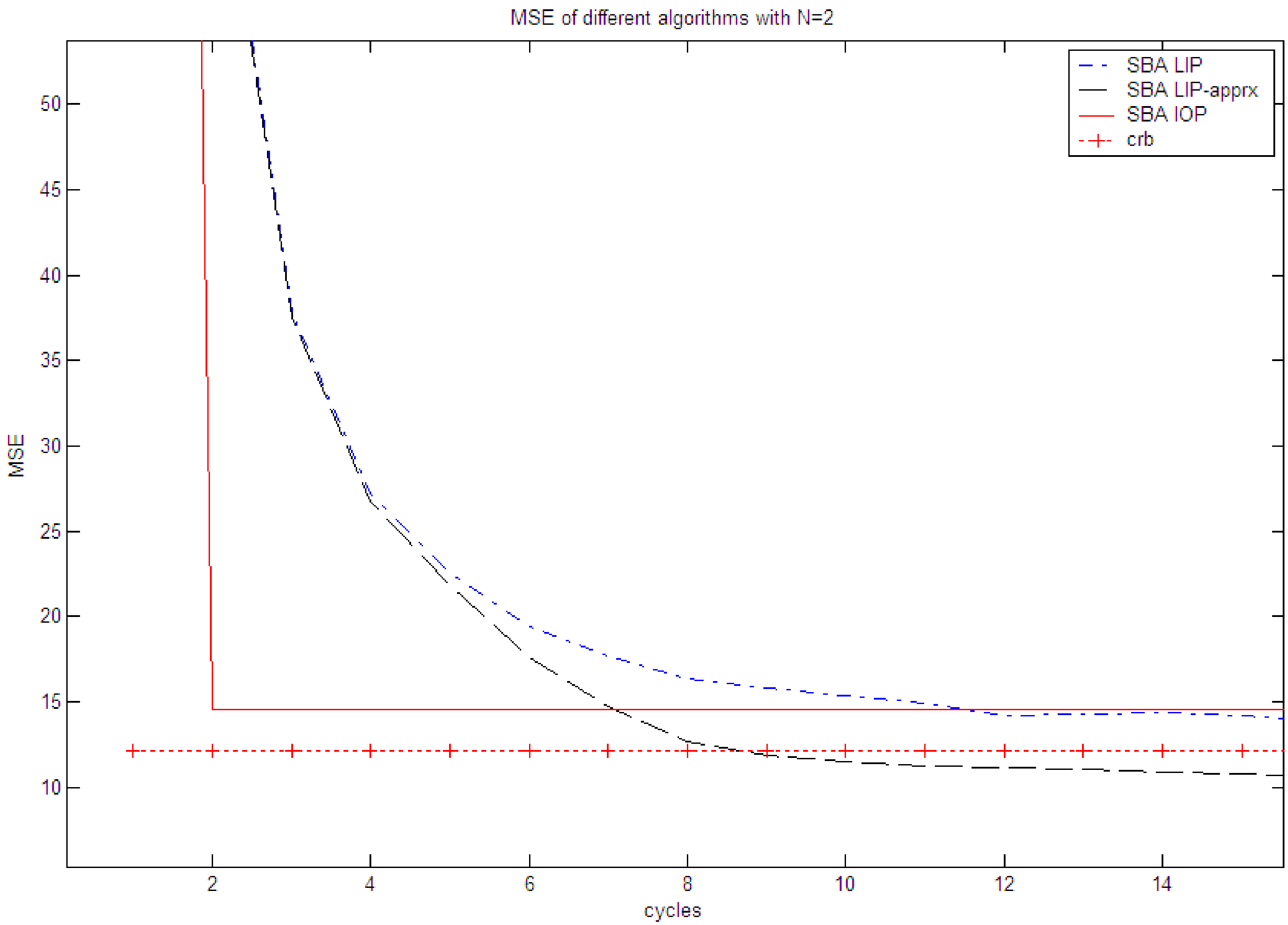
12
14
16
18

Results


MSE of different algorithms with N=2
SBA LIP
SBA LIP-apprx
SBA IOP
crb
MSE
cycles
50
45
40
35
30
25
20
15
10
2
4
6
8
10
12
14

Conclusions/ Future work

Who leads the race?

Computational/ Communication cost
Modified sub-gradient/ Baye's

Oversimplified assumptions?

Too many assumptions
Noise variance unknown
Source energy r.v
Multiple sources
Reverberation (indoor applications)

What else can be done?

Moving source
Bounds on convergence
Advanced Baye's filters
De-simplifying the assumptions to be more realistic

References

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Naylor and **Smith**, “Applications of a method for efficient computation of Posterior Distributions,” Applied Statistics, Vol.31, No.3 (1982), pp. 214-225

Jeffrey Hightower and **Gaetano Borriello**, "Location Sensing Techniques," UW CSE 01-07-01, University of Washington, Department of Computer Science and Engineering, Seattle, WA, July 2001

Thank you!