ICSI431/ICSI531 Data Mining
Lecture 4-B
Classification

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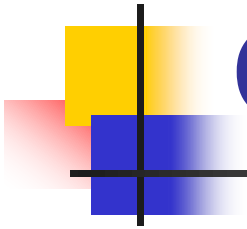
http://www.cs.albany.edu/~fchen/course/2016-ICSI-431-531

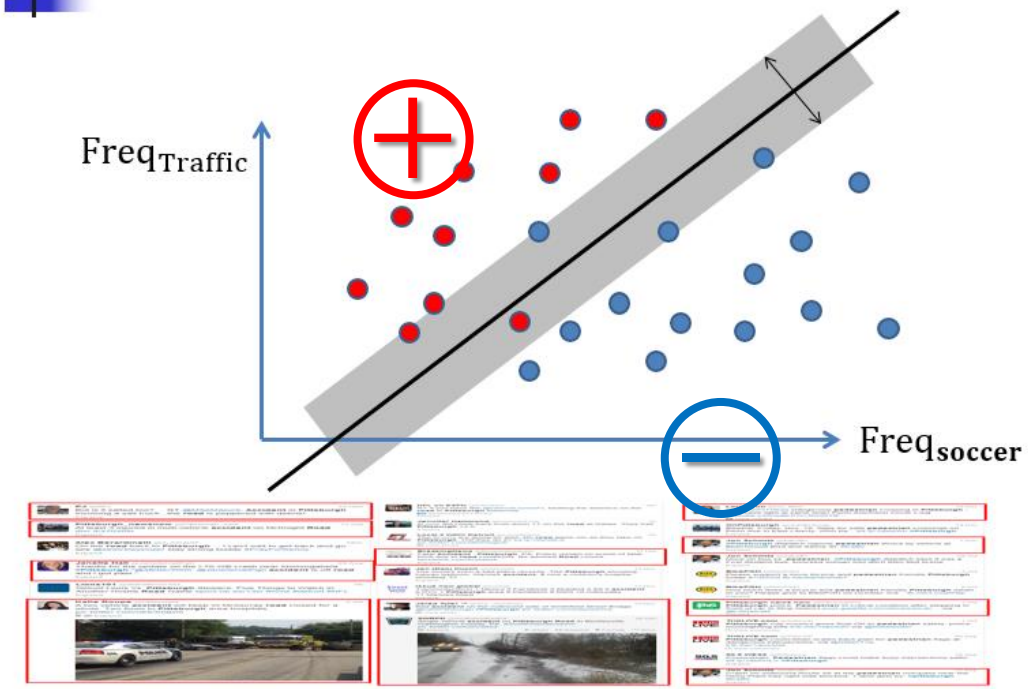


Classification Techniques

- Decision Tree based Methods
- Support Vector Machines
- Logistic Regression
- Rule-based Methods
- Memory based reasoning
- Neural Networks
- Naïve Bayes and Bayesian Belief Networks

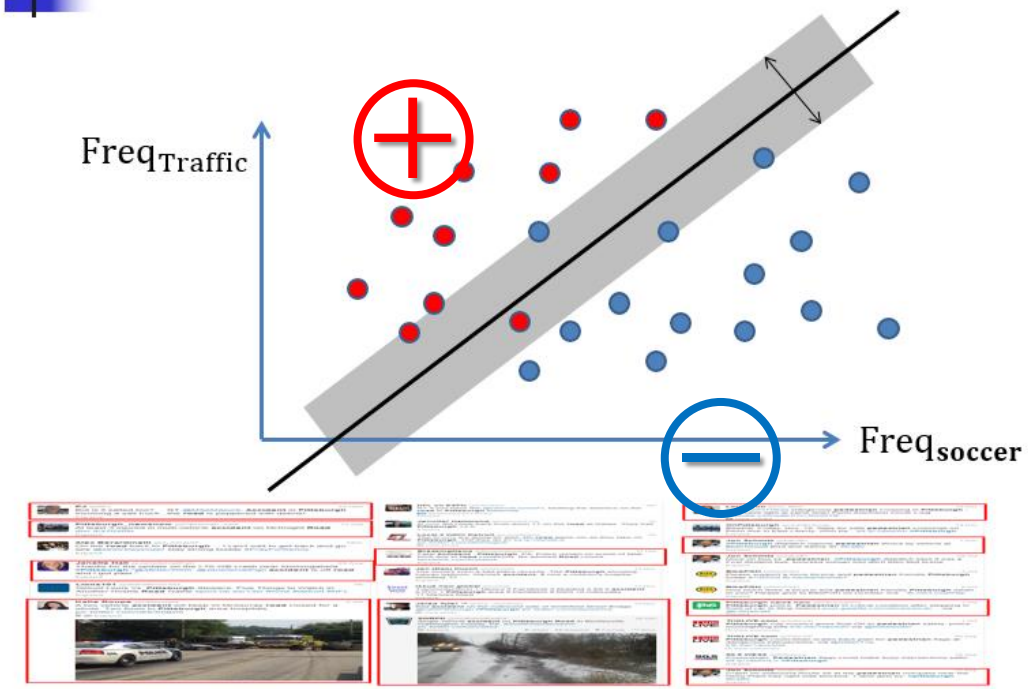
Mathematical Background

Example: Tweet Classification

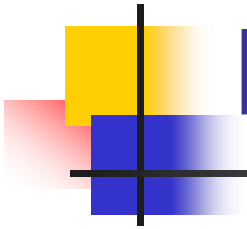


Mathematical Background

Example: Tweet Classification



SVM is about the learning of a good separating line (hyper-plane) that separates objects into different classes!

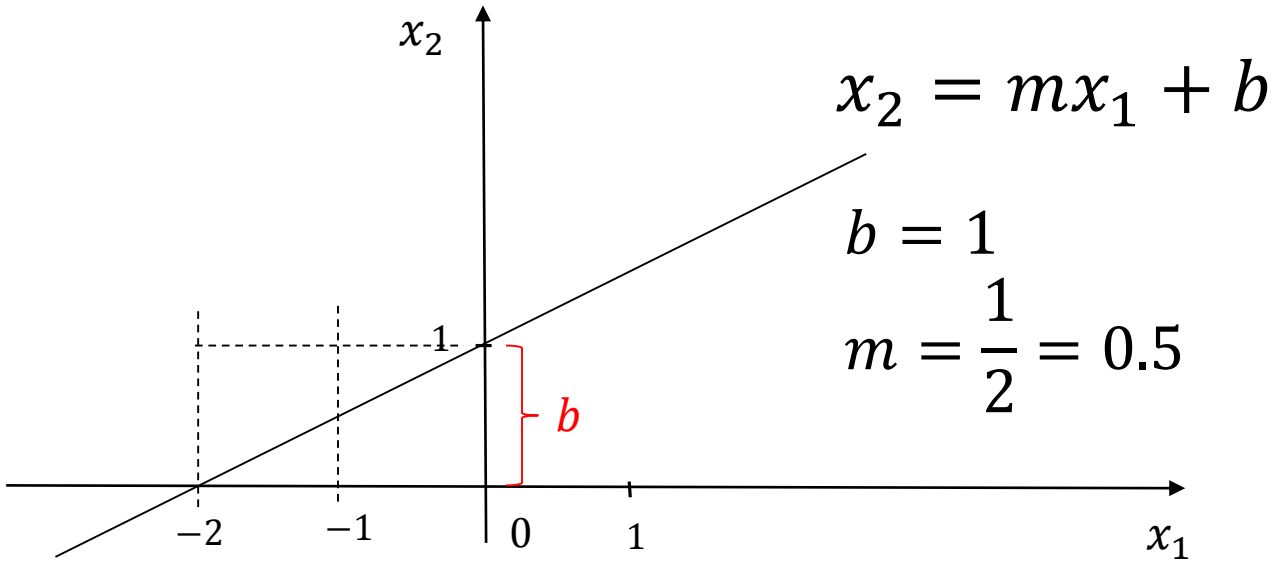


Mathematical Background

- Straight Line
- Perpendicular Line
- Parallel Lines
- Distance Between Parallel Lines
- Distance From a Point to a Line
- Separating Line

Straight Line Equation-1

- In high school: slope + intercept
- Slope-intercept form
  - m is the slope or gradient of the line
  - b is the x_2-intercept of the line

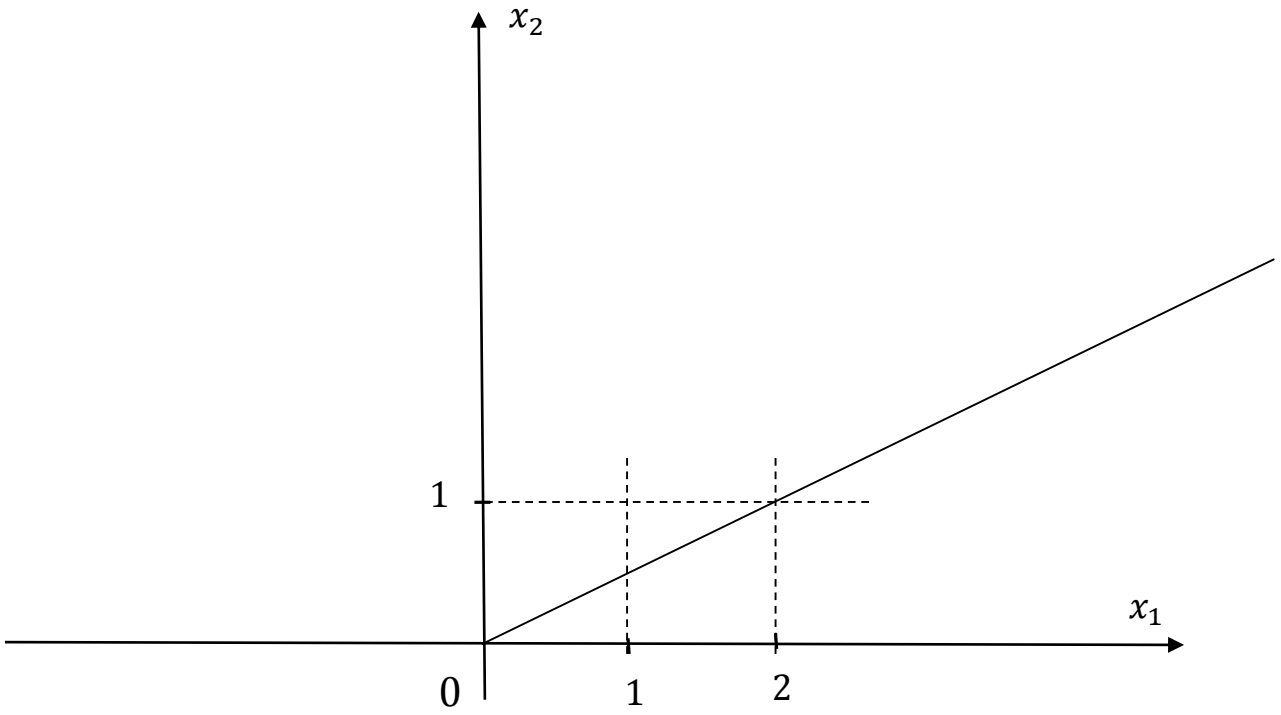


$$x_2 = mx_1 + b$$

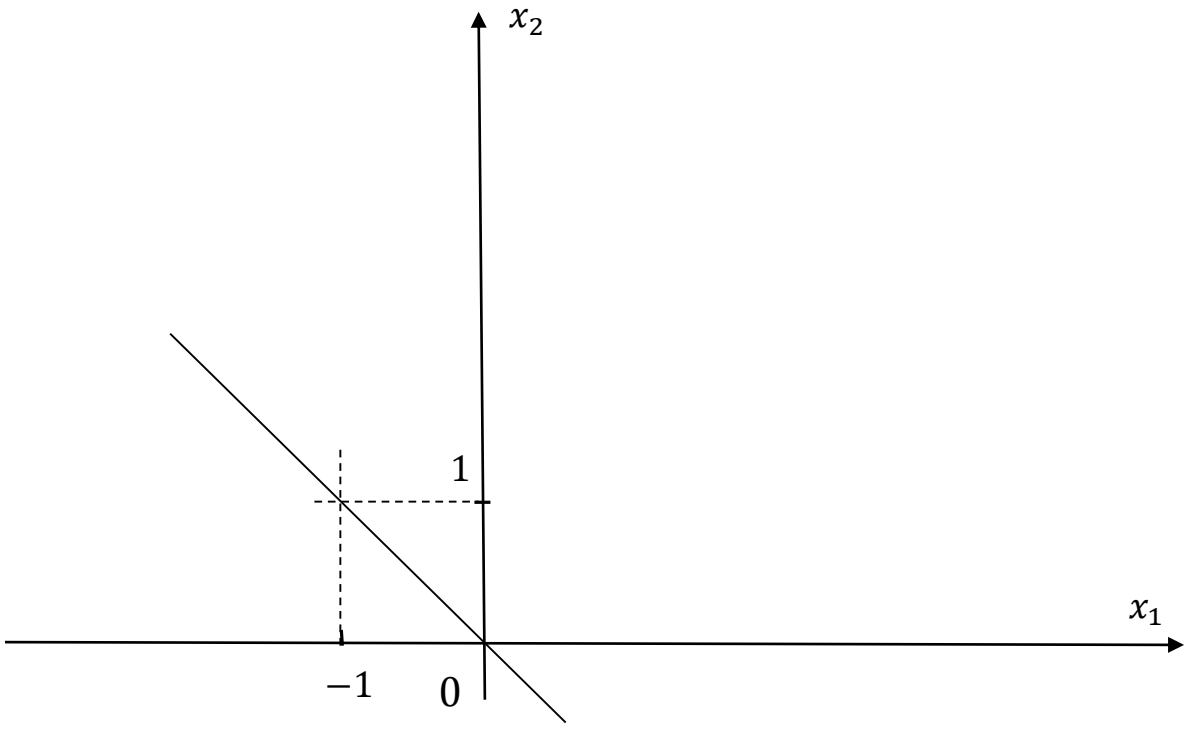
$$b = 1$$

$$m = \frac{1}{2} = 0.5$$

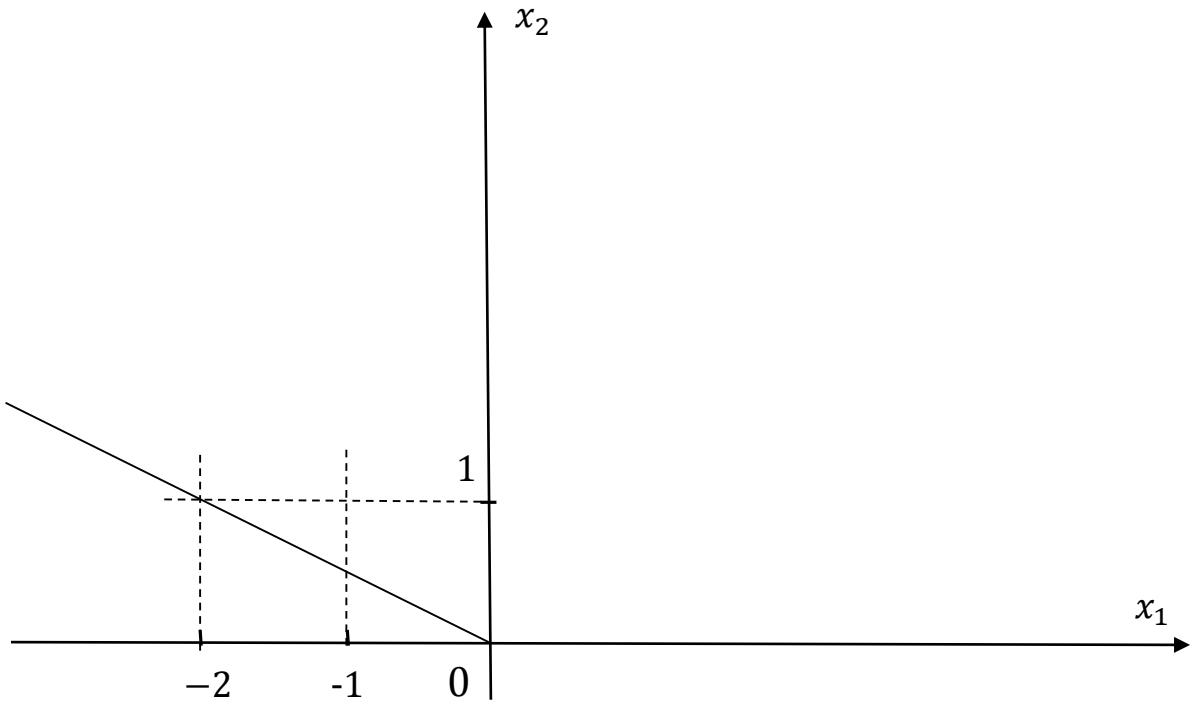
Straight Line Equation-1



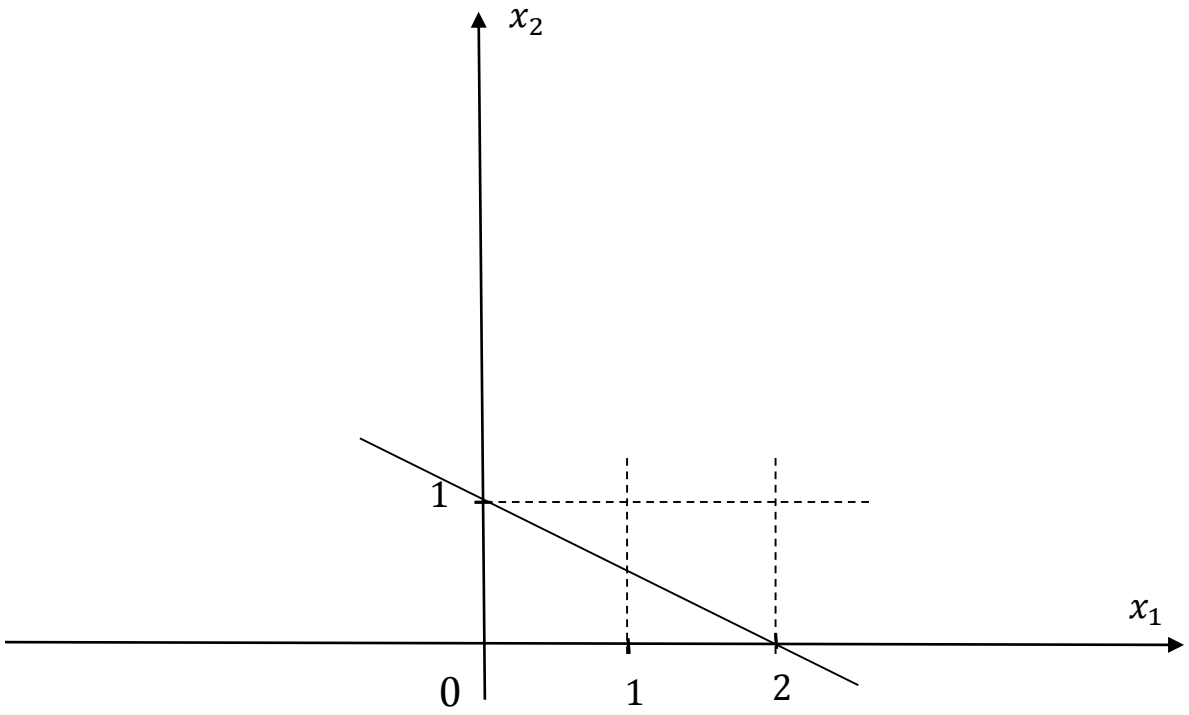
Straight Line Equation-1



Straight Line Equation-1

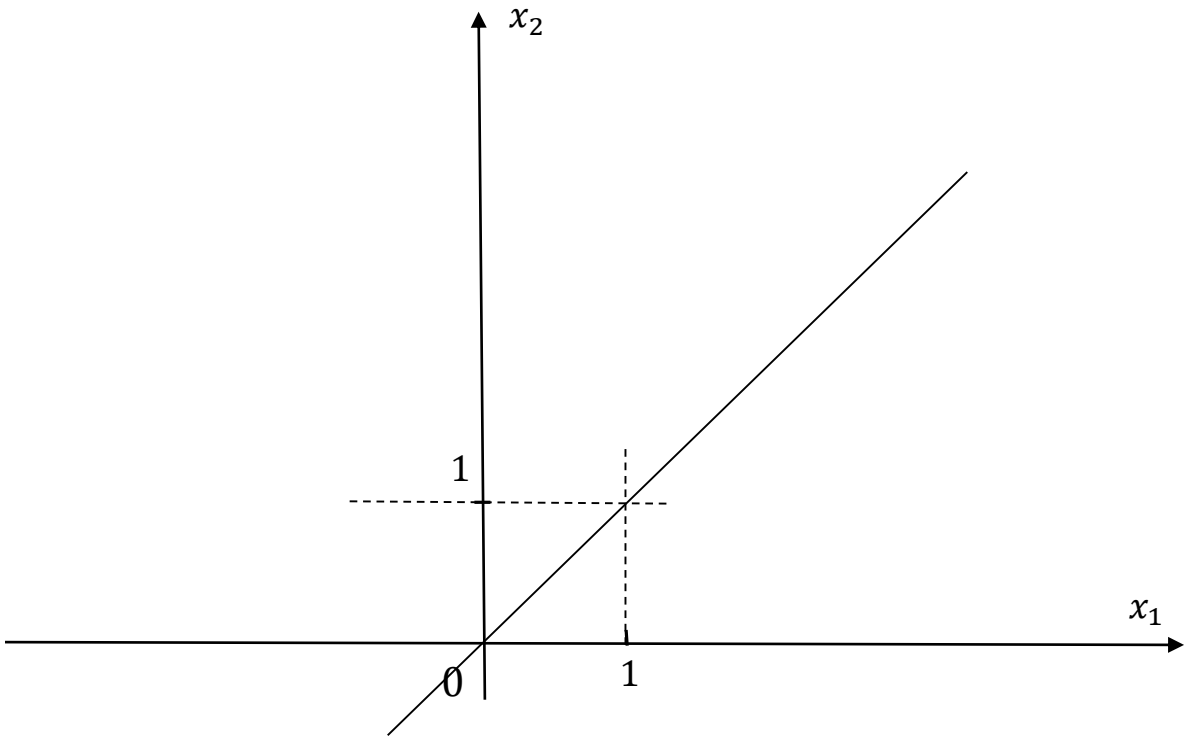


Straight Line Equation-1



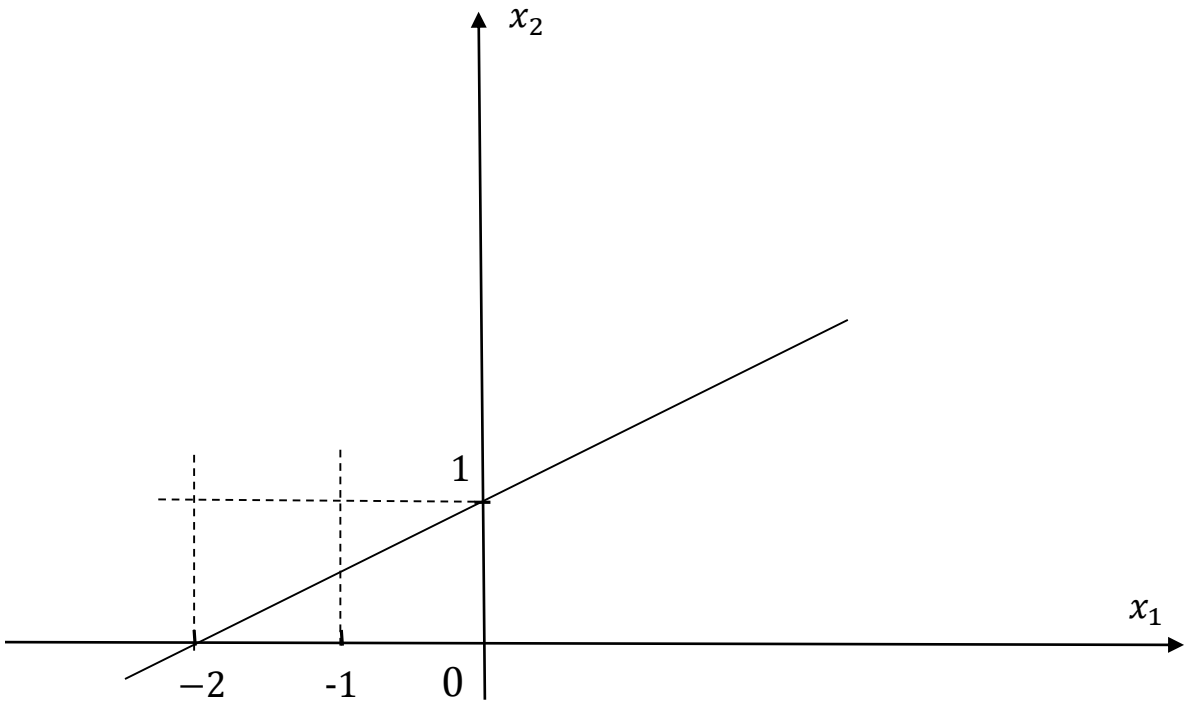
Straight Line Equation-1

- In class exercise 1



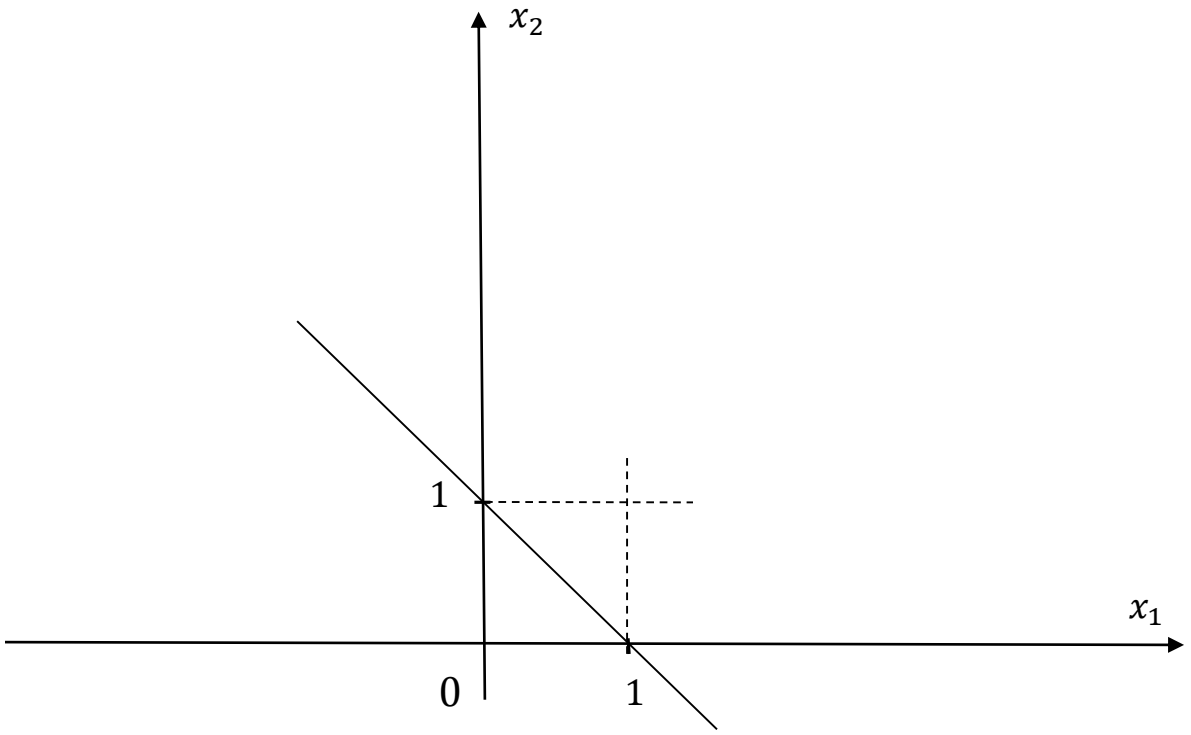
Straight Line Equation-1

- In class exercise 2



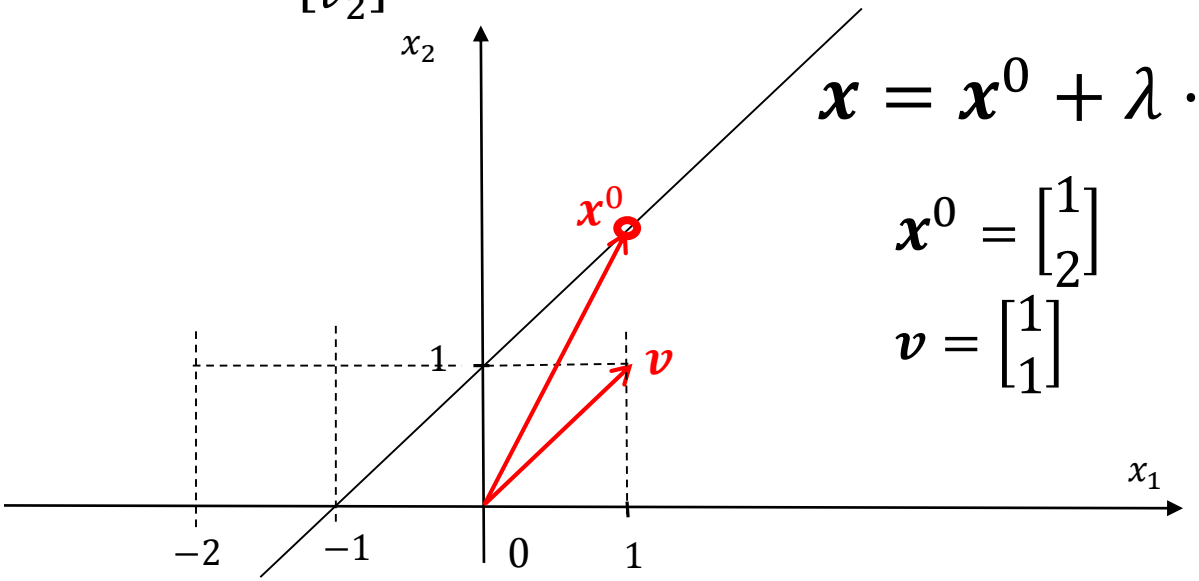
Straight Line Equation-1

- In class exercise 3



Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
  - $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a vector parallel to the line



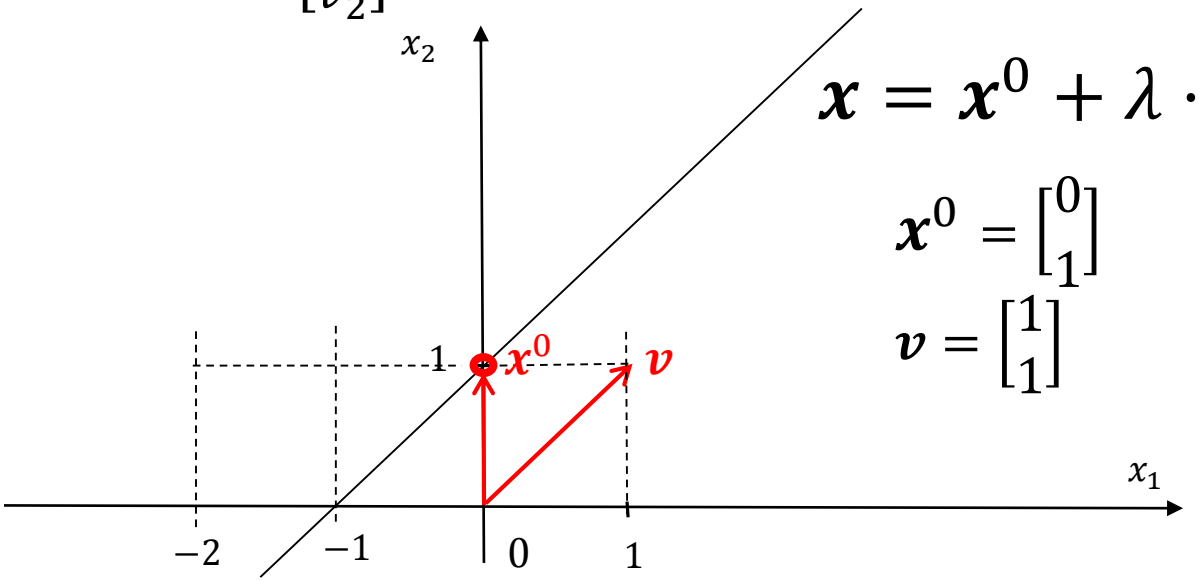
$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
  - $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a vector parallel to the line



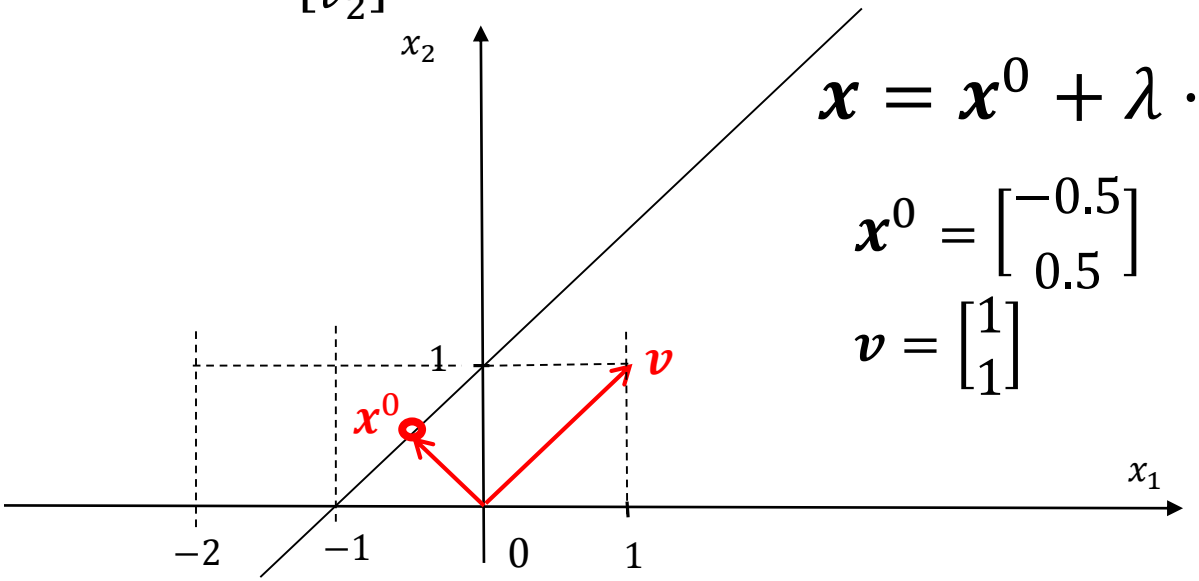
$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
  - $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a vector parallel to the line



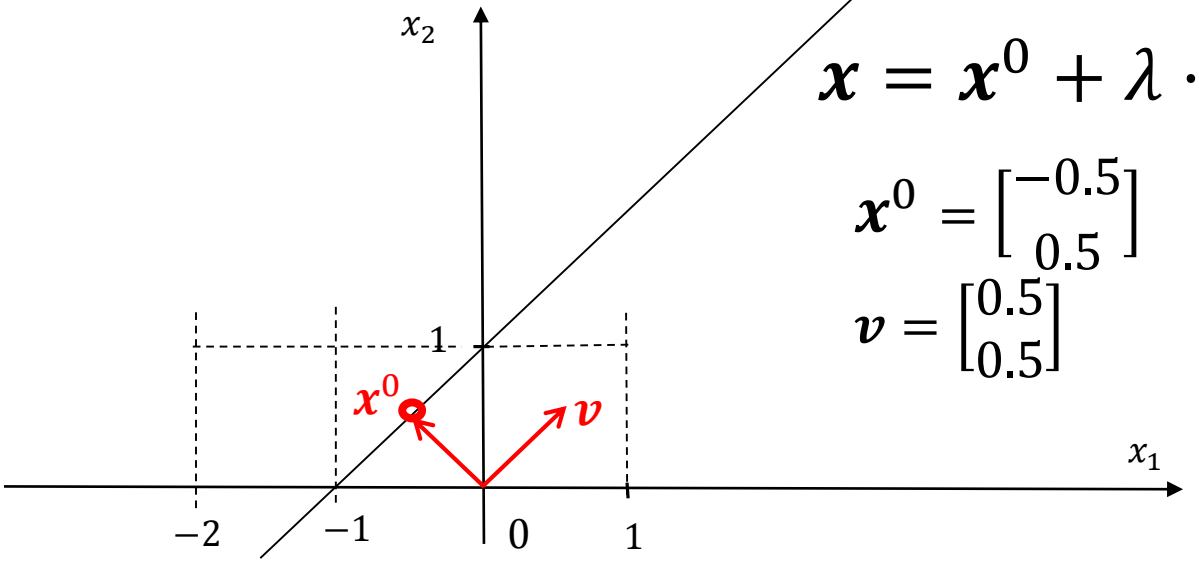
$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
  - $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a vector parallel to the line



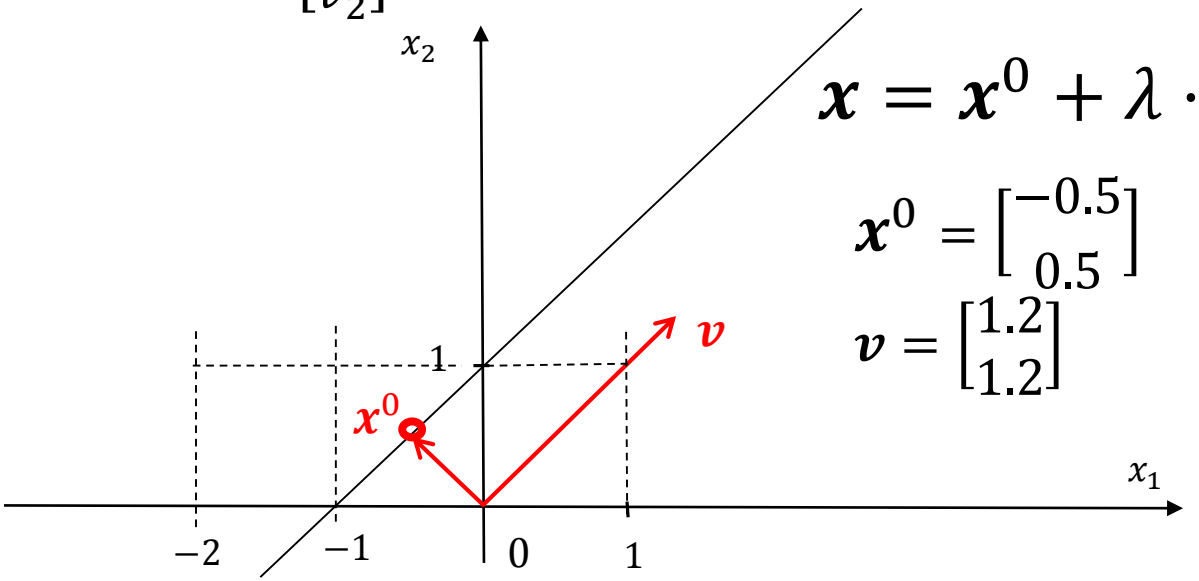
$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
  - $\boldsymbol{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ is a vector parallel to the line



$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} 1.2 \\ 1.2 \end{bmatrix}$$

Straight Line Equation-2

- In college: A parallel vector + A point
- Parametric form
  - $\boldsymbol{x}^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \end{bmatrix}$ is any point on the line
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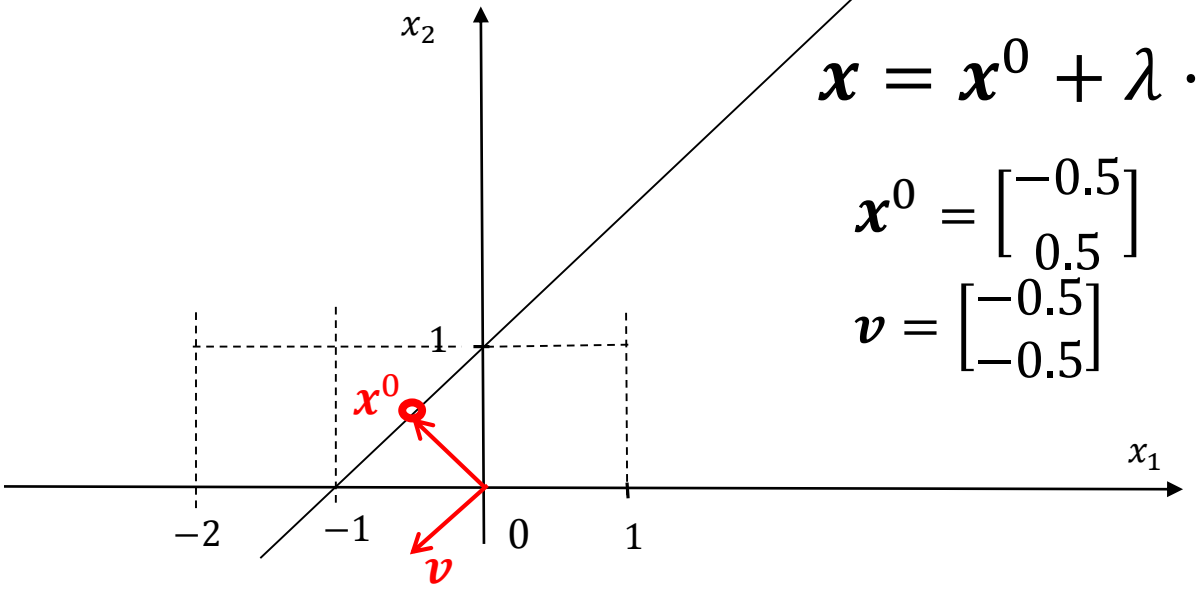
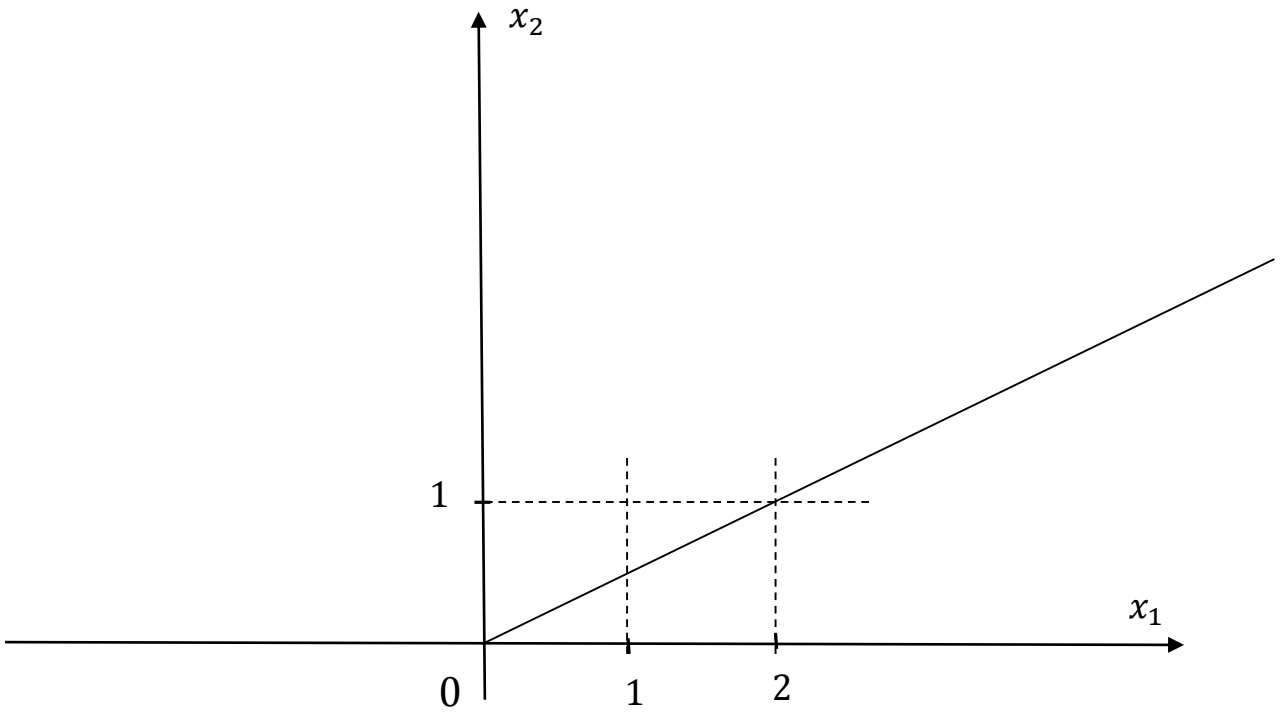


$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \cdot \boldsymbol{v}, \quad \lambda \in \mathbb{R}$$

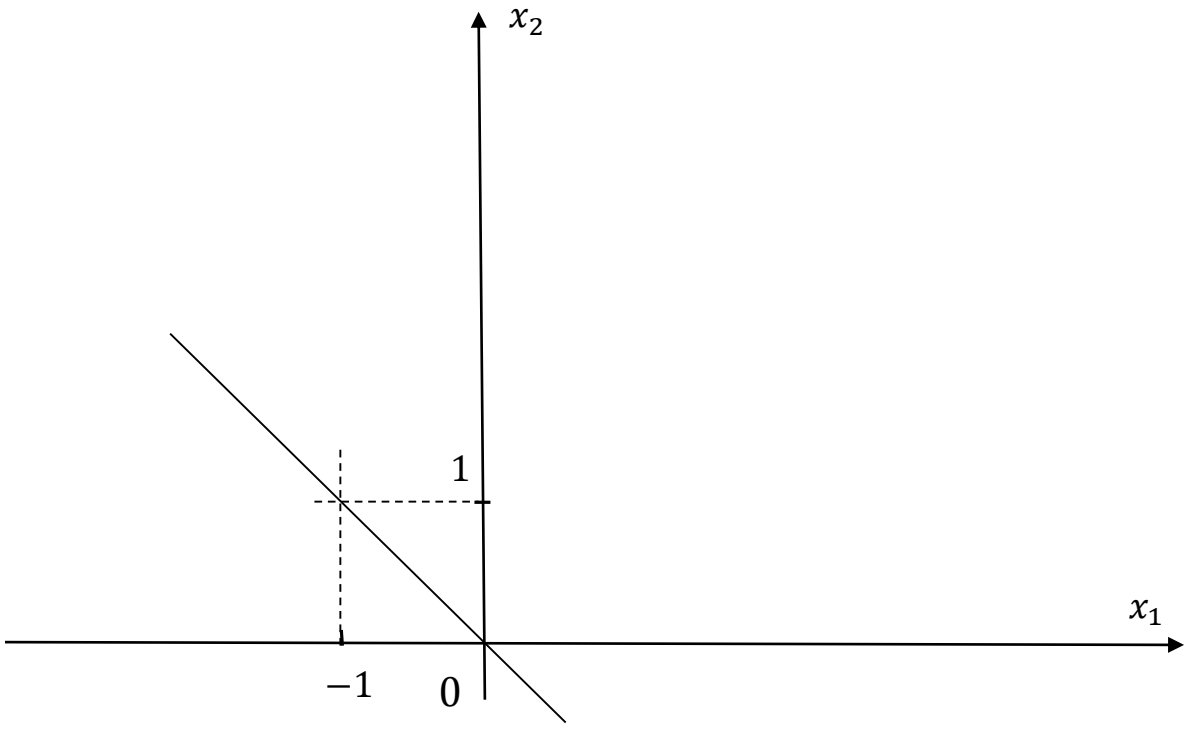
$$\boldsymbol{x}^0 = \begin{bmatrix} -0.5 \\ 0.5 \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} -0.5 \\ -0.5 \end{bmatrix}$$

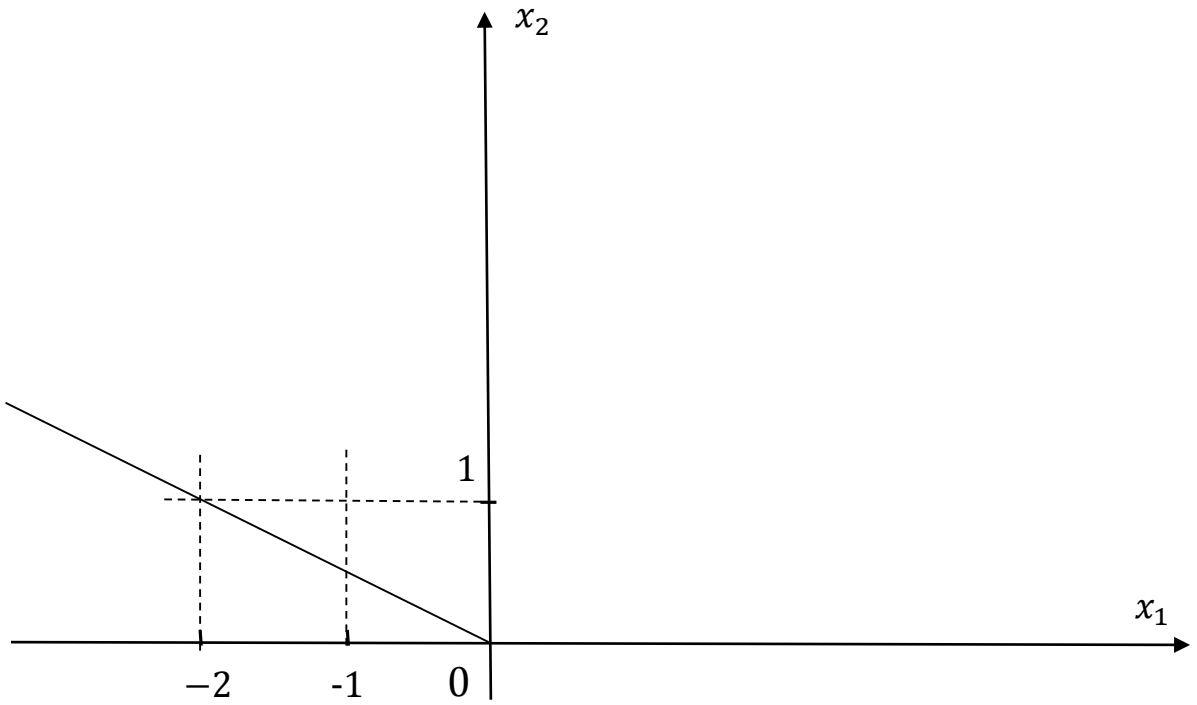
Straight Line Equation-2



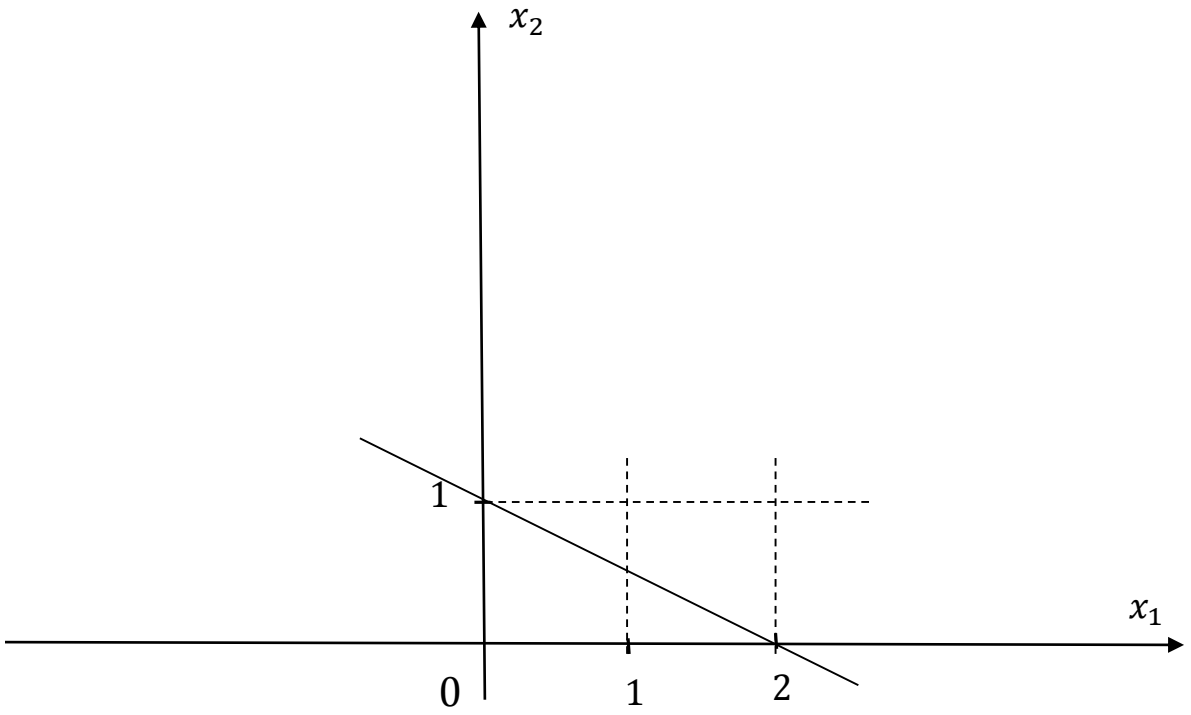
Straight Line Equation-2



Straight Line Equation-2

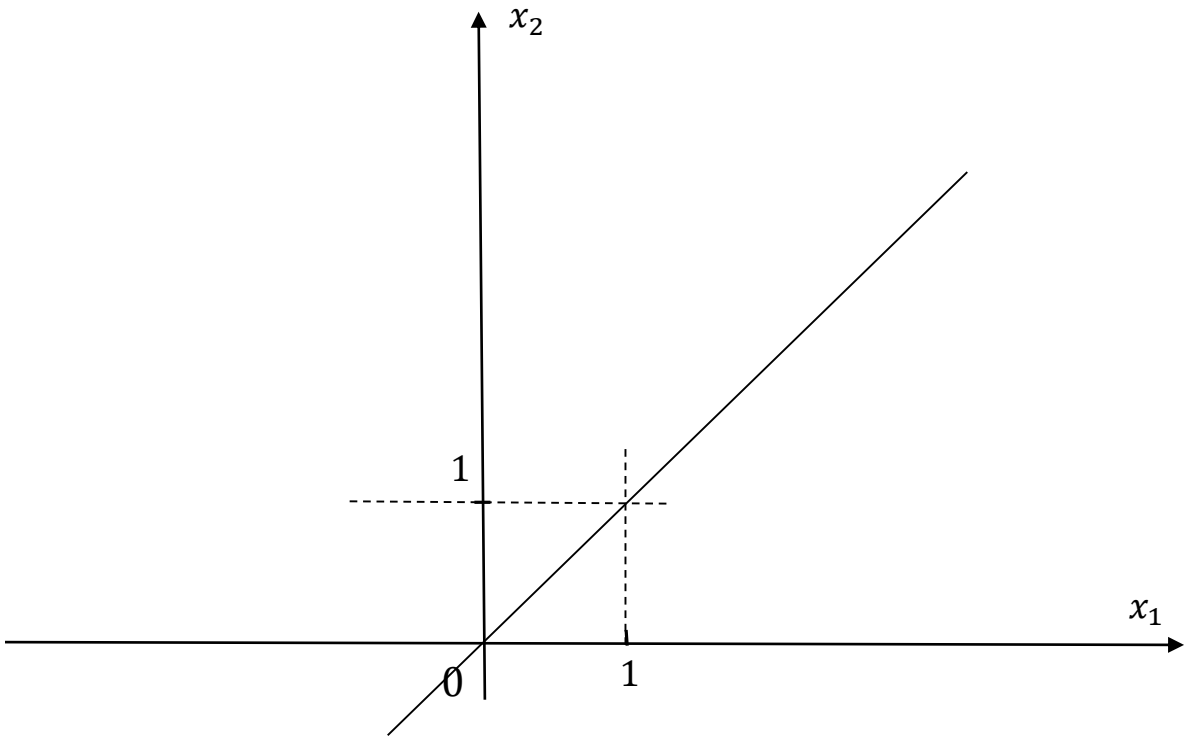


Straight Line Equation-2



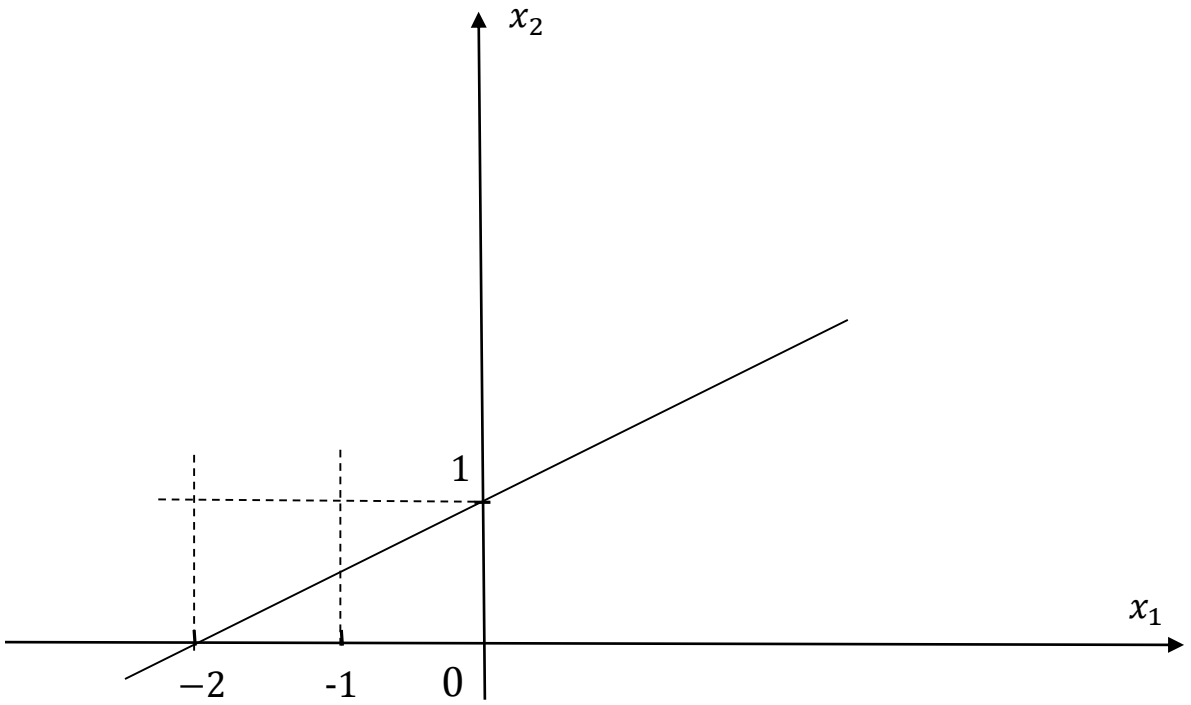
Straight Line Equation-2

- In class exercise 1



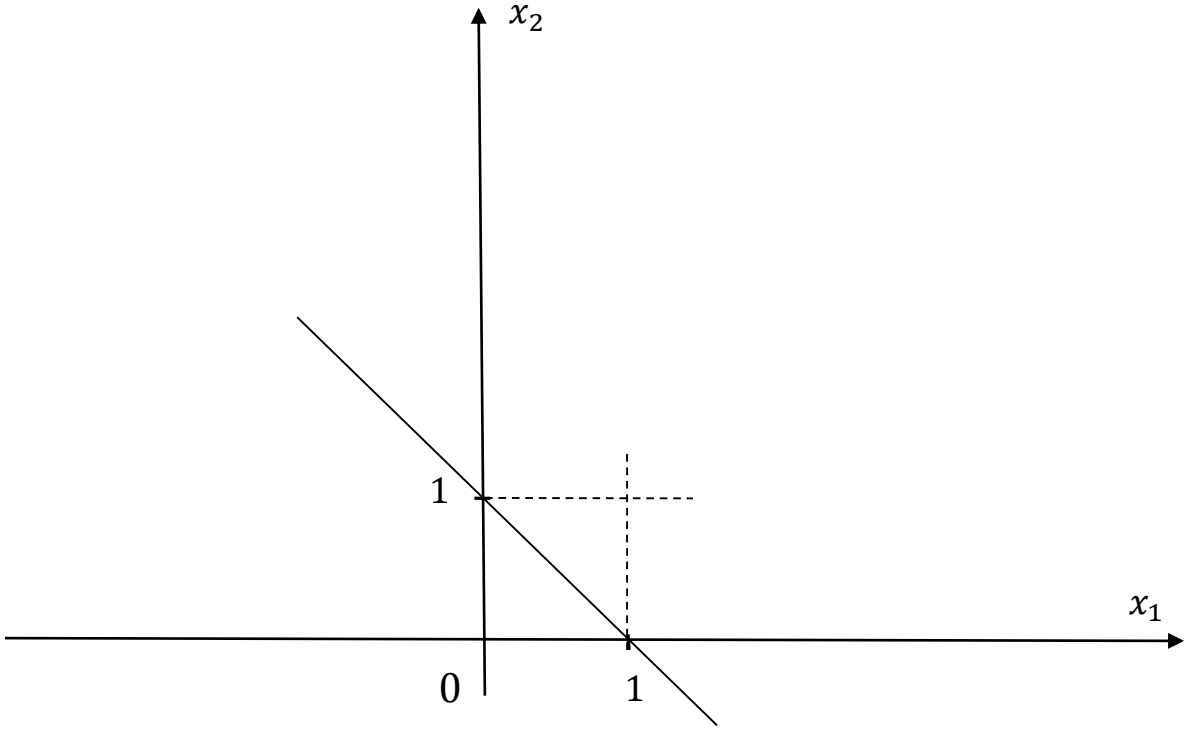
Straight Line Equation-2

- In class exercise 2



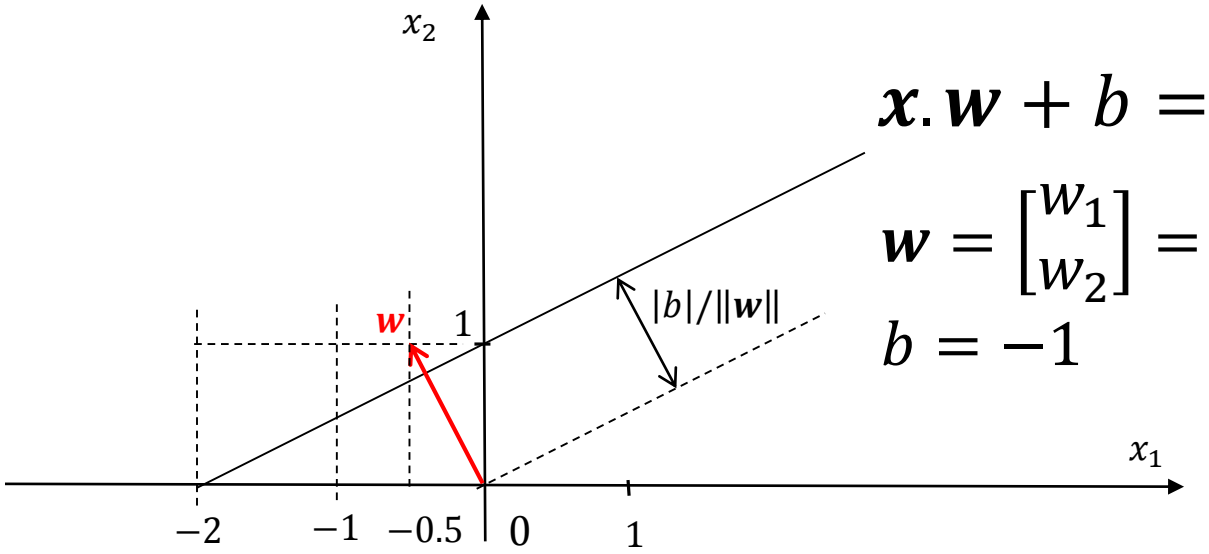
Straight Line Equation-2

- In class exercise 3



Straight Line Equation-3

- In data mining class:
  - A perpendicular vector + A constant
- Vector form
  - $\boldsymbol{w}$ is a vector perpendicular to the line
  - $|b|/\|\boldsymbol{w}\|$ is the distance from the line to the origin



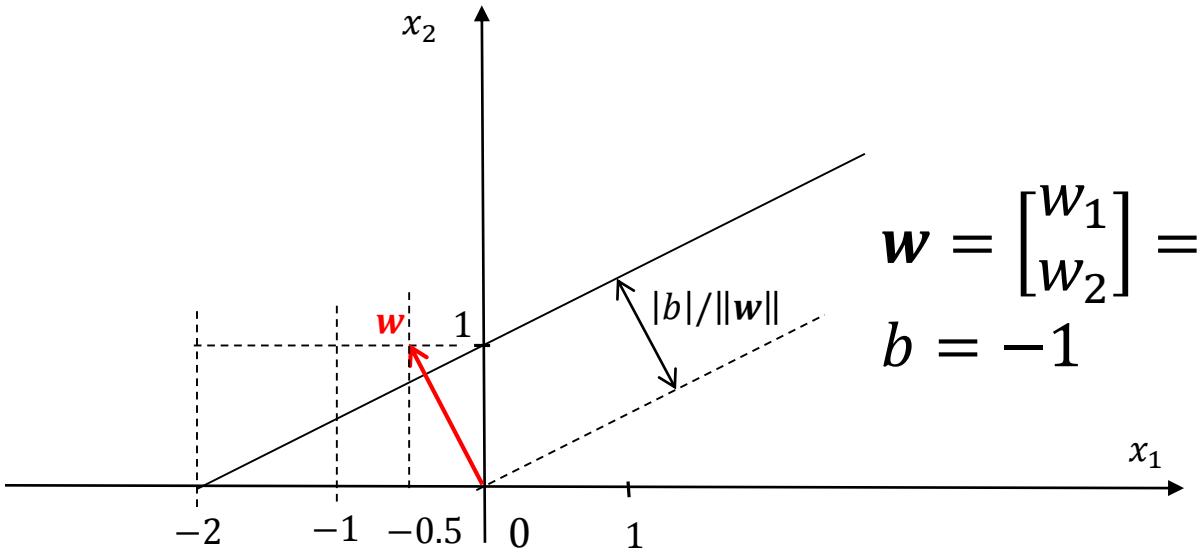
$$\boldsymbol{x}.\boldsymbol{w} + b = 0$$

$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}$$

$$b = -1$$

Estimation of $\boldsymbol{w}$ and b

- Line representation: $\boldsymbol{x}.\boldsymbol{w} + b = 0$
- How to calculate $\boldsymbol{w}$ and b?

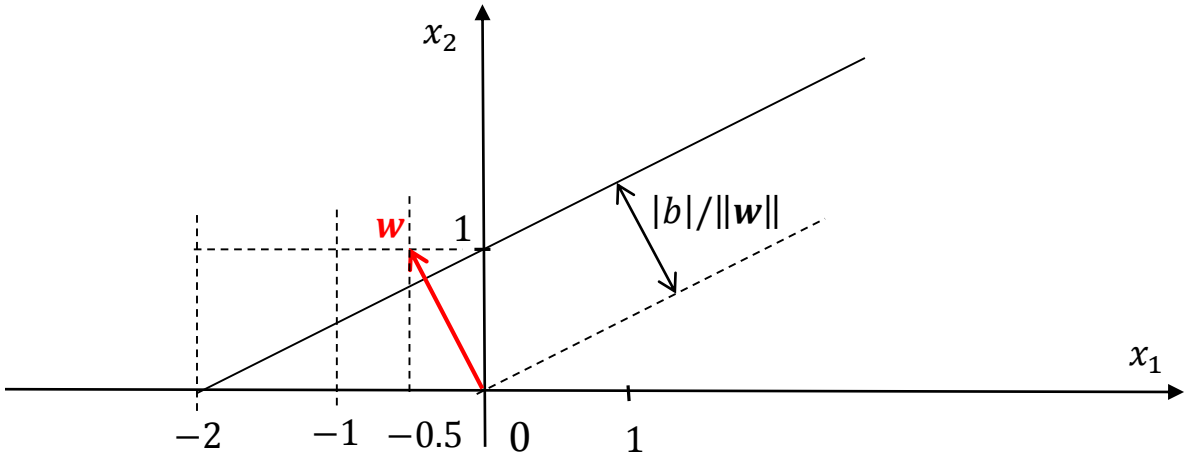


$$\boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}$$

$$b = -1$$

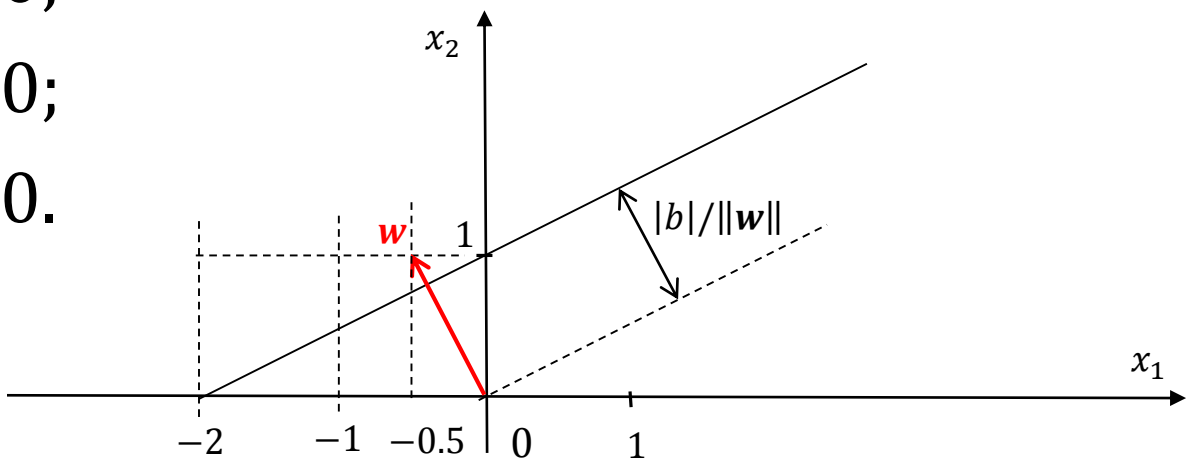
Calculating $\boldsymbol{w}$ and b: Method 1

- Step 1: Identify a perpendicular vector $\boldsymbol{w}$
  - $\boldsymbol{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \boldsymbol{w}.\boldsymbol{v} = 0 \rightarrow \frac{w_1}{w_2} = \frac{-v_2}{v_1} = -\frac{1}{2} \rightarrow \boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}$
- Step 2: Given $\boldsymbol{w}$ and any point $\boldsymbol{x}^0$ within the line, estimate b:
  - $\boldsymbol{x}^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \boldsymbol{w}.\boldsymbol{x}^0 + b = 0 \rightarrow b = -\boldsymbol{w}.\boldsymbol{x} = -\begin{bmatrix} -0.5 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} = -1$



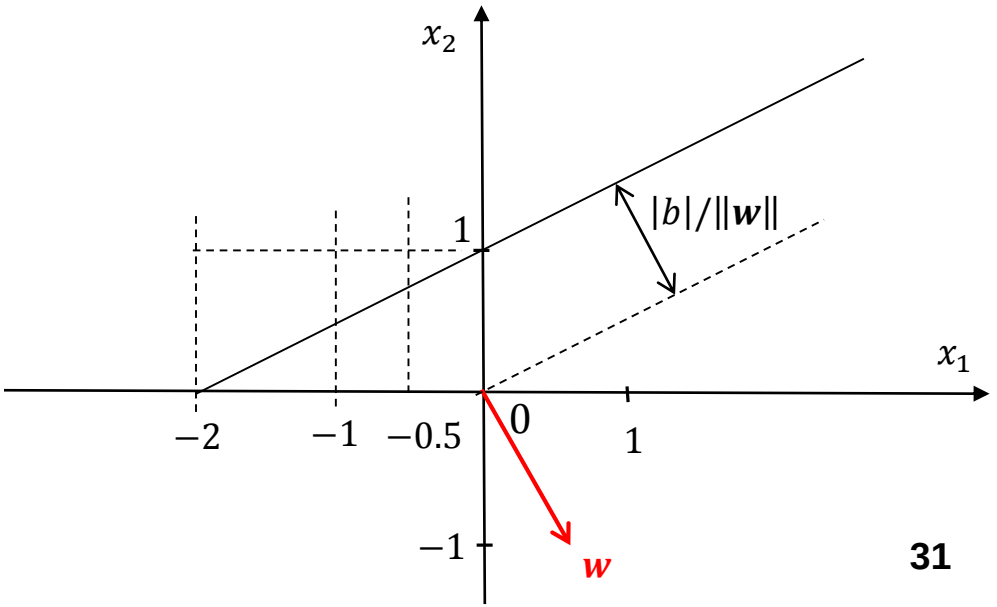
Calculating $\boldsymbol{w}$ and b: Method 2

- Step 1: Find any three points within the line
  - $\boldsymbol{x}^1 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \boldsymbol{x}^2 = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \boldsymbol{x}^3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$
- Step 2: Identify $\boldsymbol{w}$ and b by solving the system of three linear equations:
  - $\boldsymbol{w}.\boldsymbol{x}^1 + b = 0;$
  - $\boldsymbol{w}.\boldsymbol{x}^2 + b = 0;$
  - $\boldsymbol{w}.\boldsymbol{x}^3 + b = 0.$

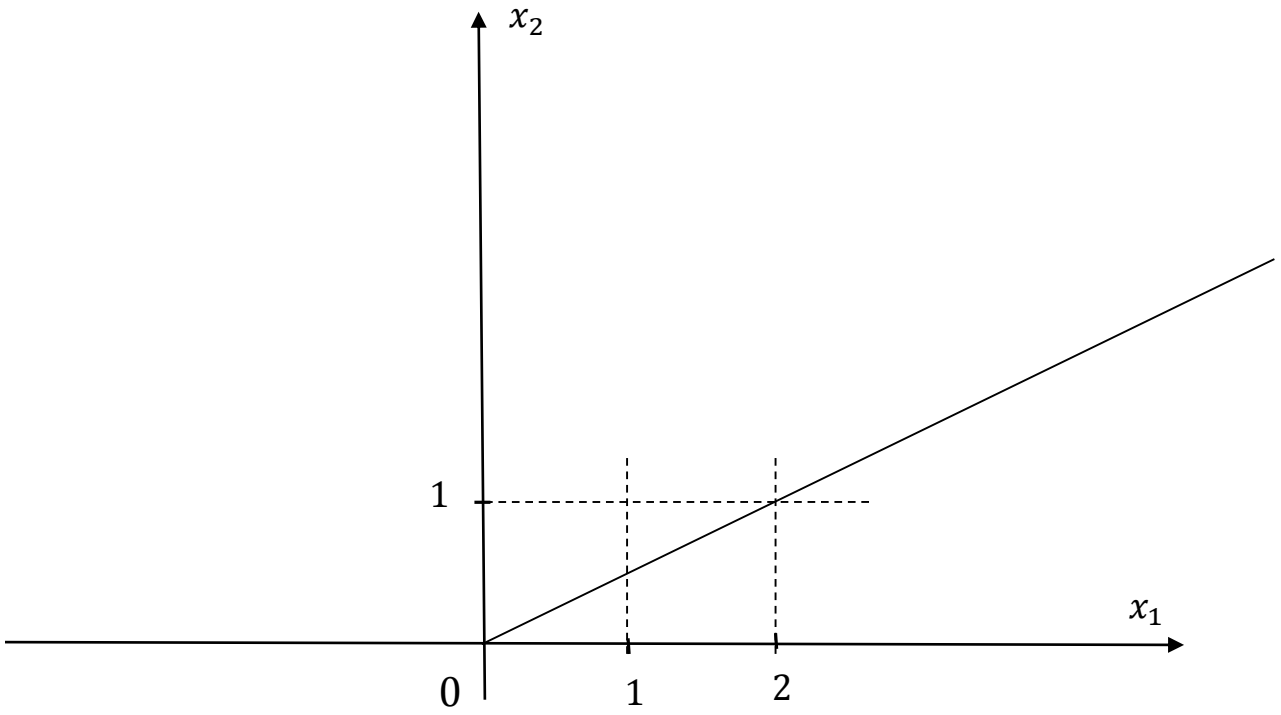


Calculating $\boldsymbol{w}$ and b: Method 3

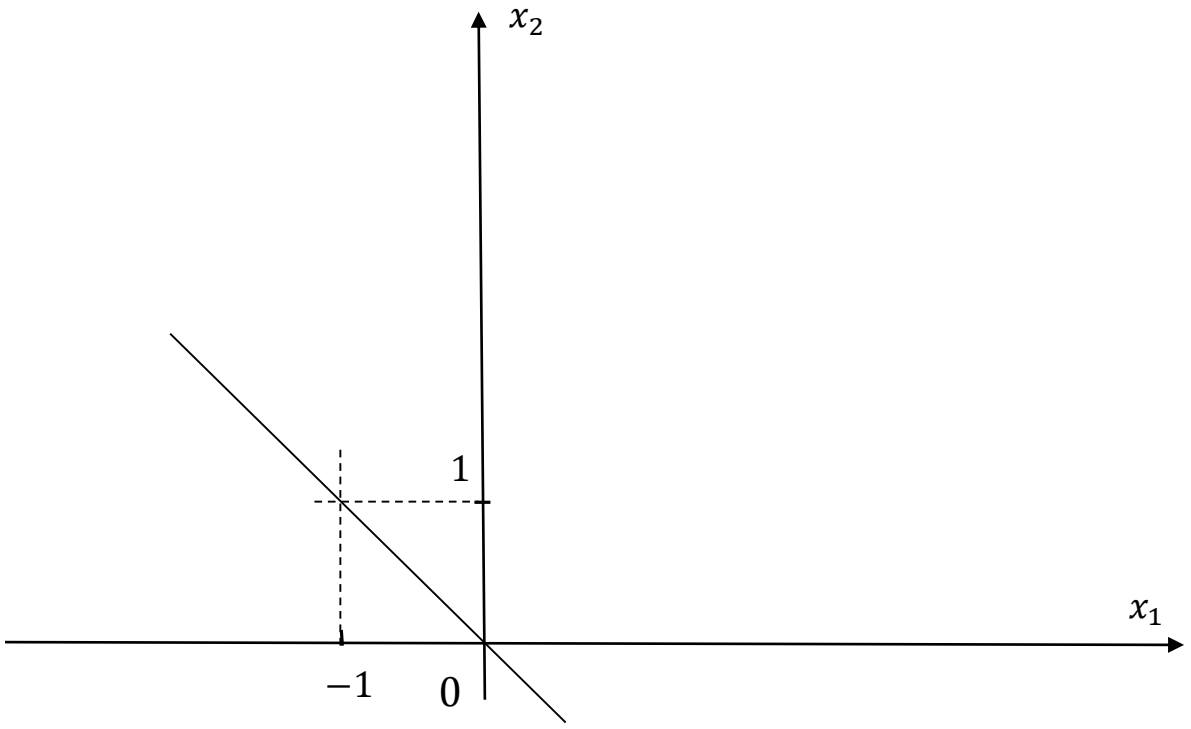
- Step 1: Find the representation based on slope and intercept
  - $x_2 = m \cdot x_1 + b, m = \frac{1}{2}, b = 1$
- Step 2: Rearrange x_1 and x_2 to the vector form $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$
  - $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \cdot \begin{bmatrix} m \\ -1 \end{bmatrix} + b = 0$
  - $\boldsymbol{w} = \begin{bmatrix} m \\ -1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ -1 \end{bmatrix}$
  - $b = 1$



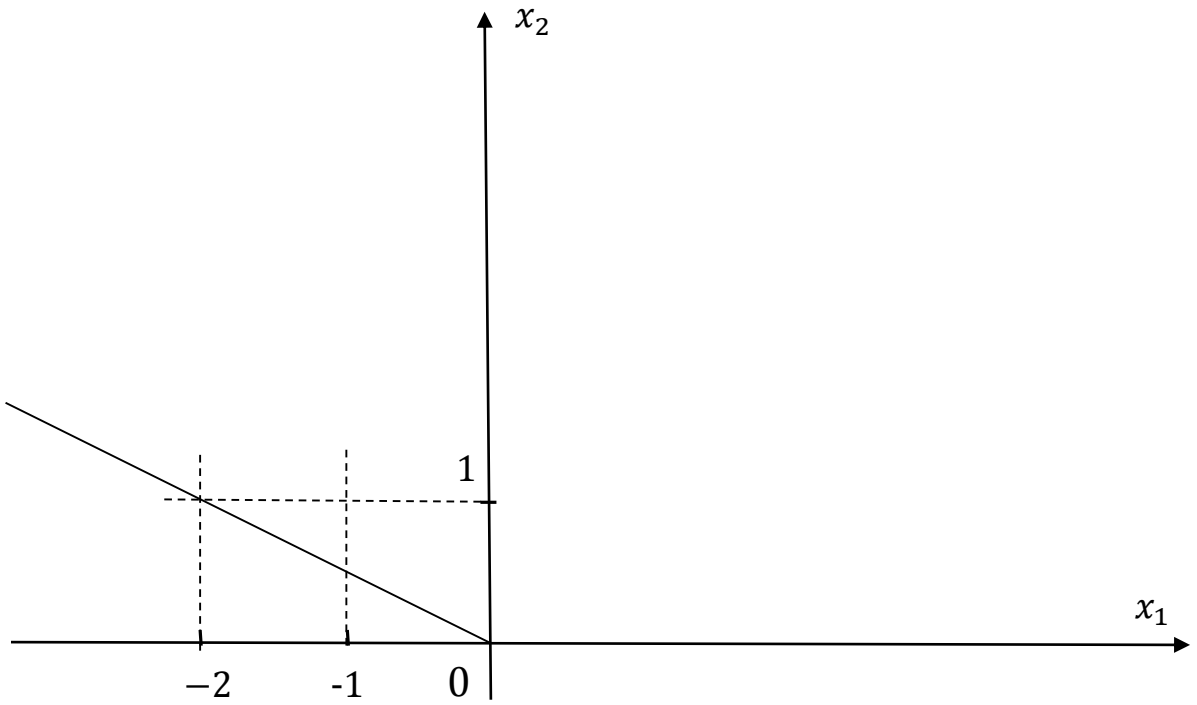
Straight Line Equation-3



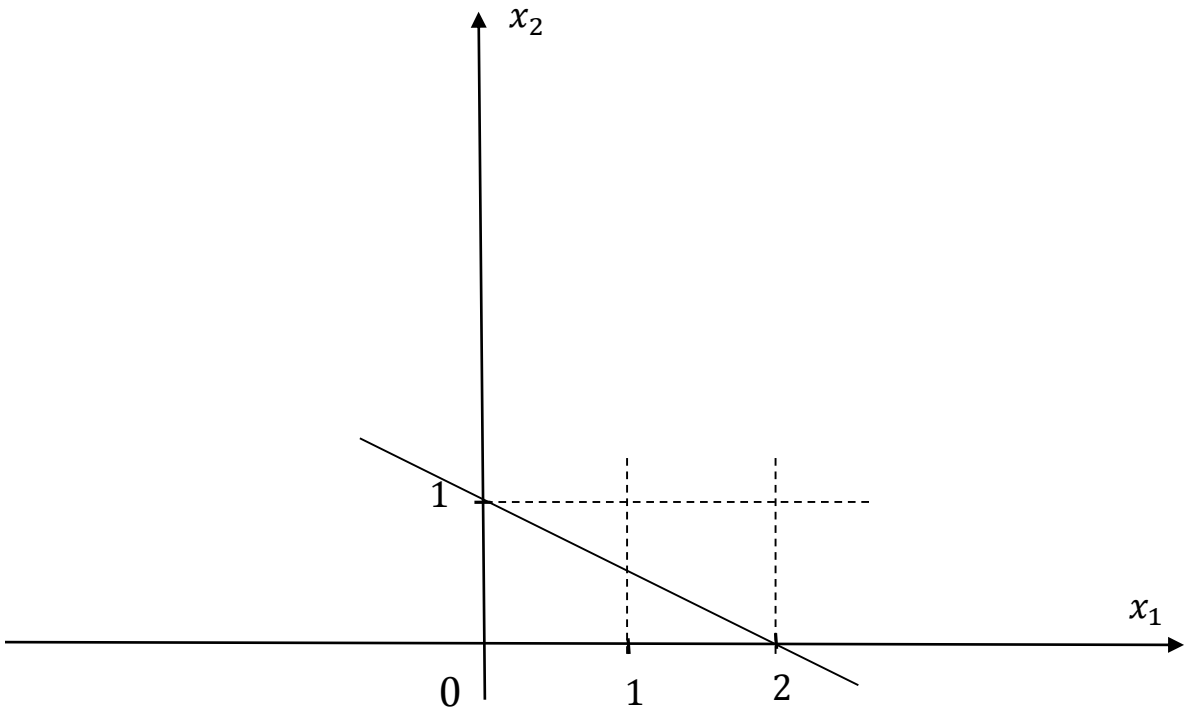
Straight Line Equation-3



Straight Line Equation-3

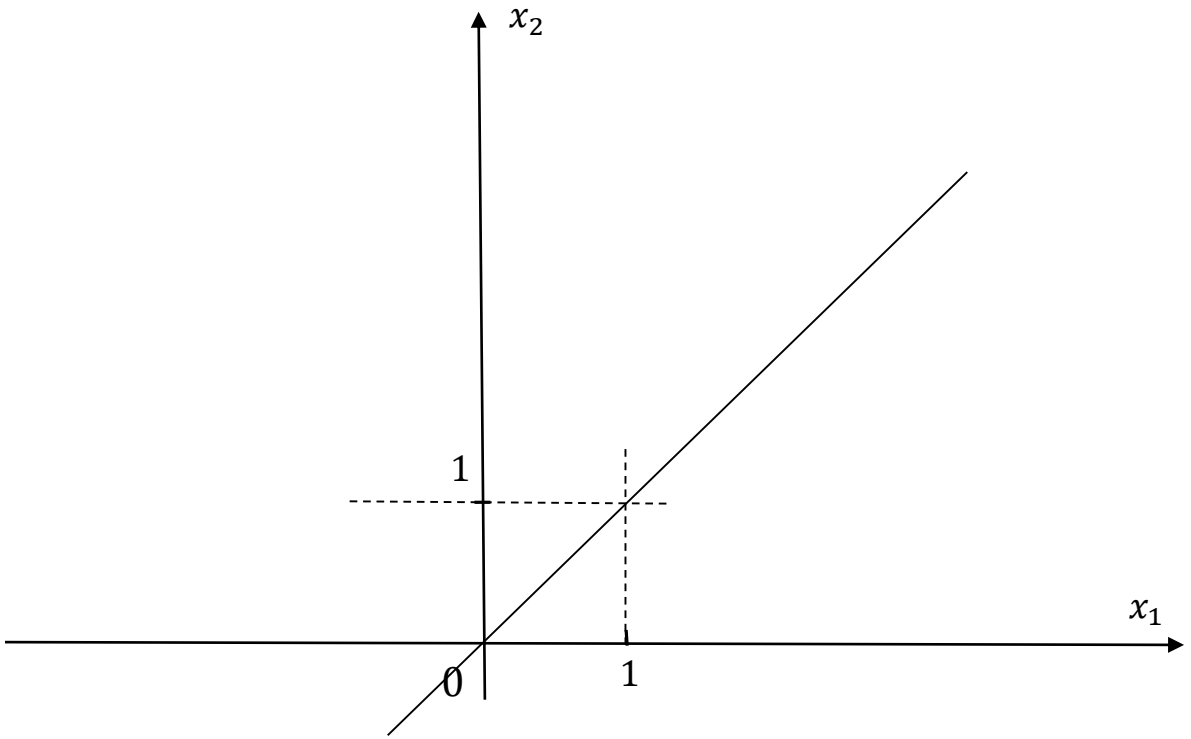


Straight Line Equation-3



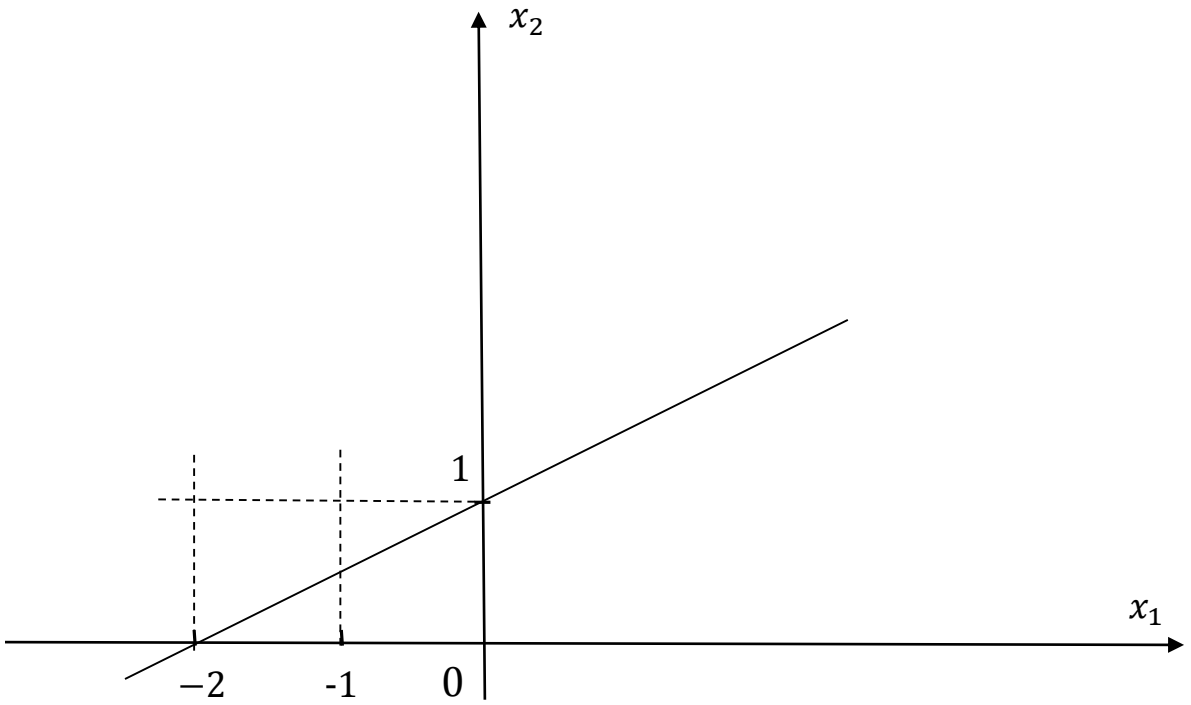
Straight Line Equation-3

- In class exercise 1



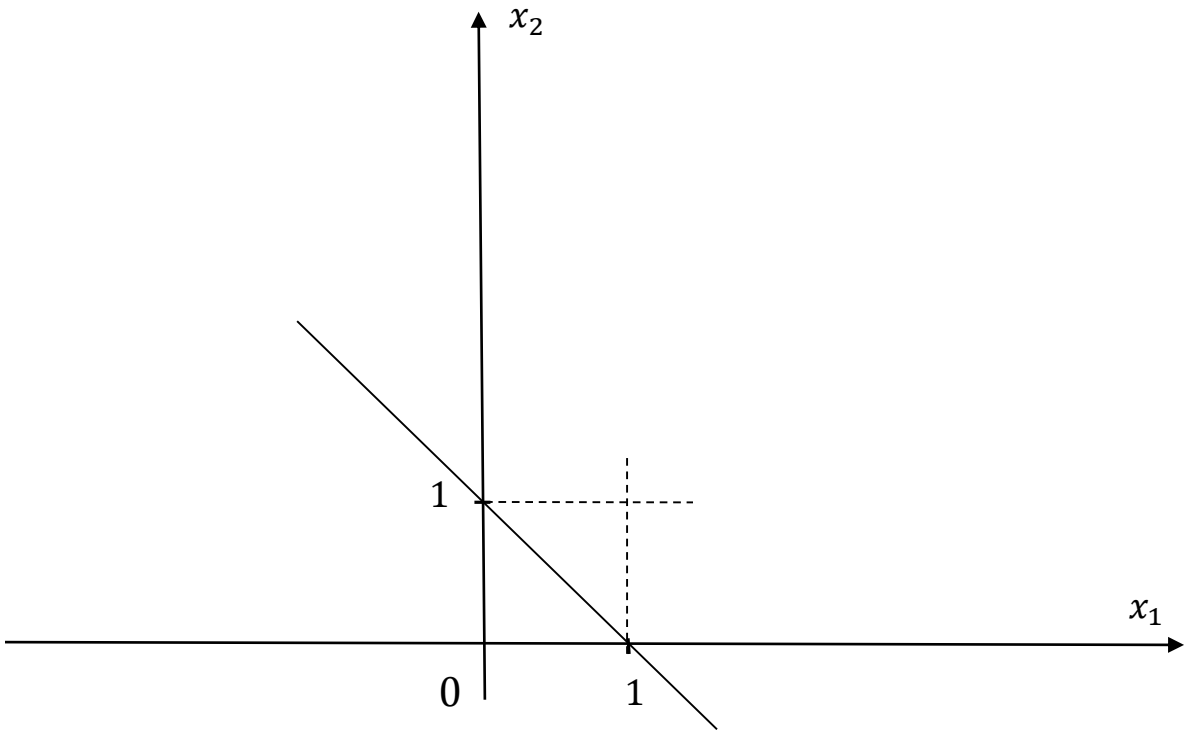
Straight Line Equation-3

- In class exercise 2



Straight Line Equation-3

- In class exercise 3

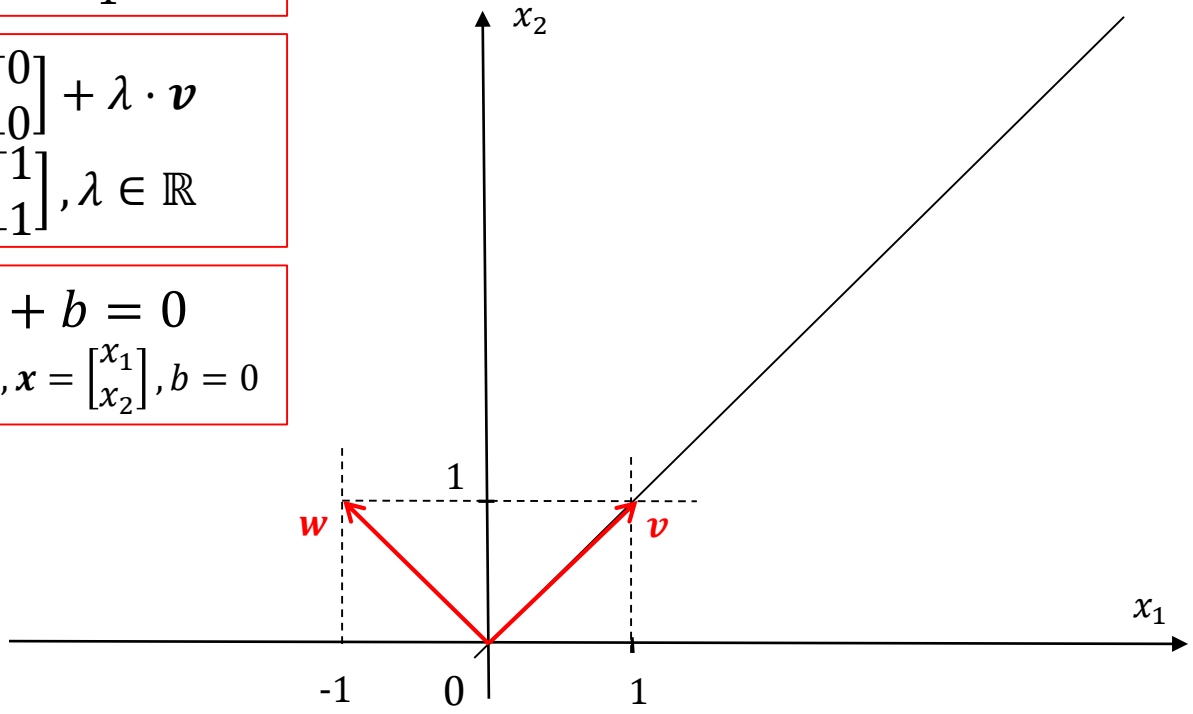


Straight Line Equation

$$x_2 = 1 \cdot x_1 + 0$$

$$\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$
$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$
$$\boldsymbol{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 0$$

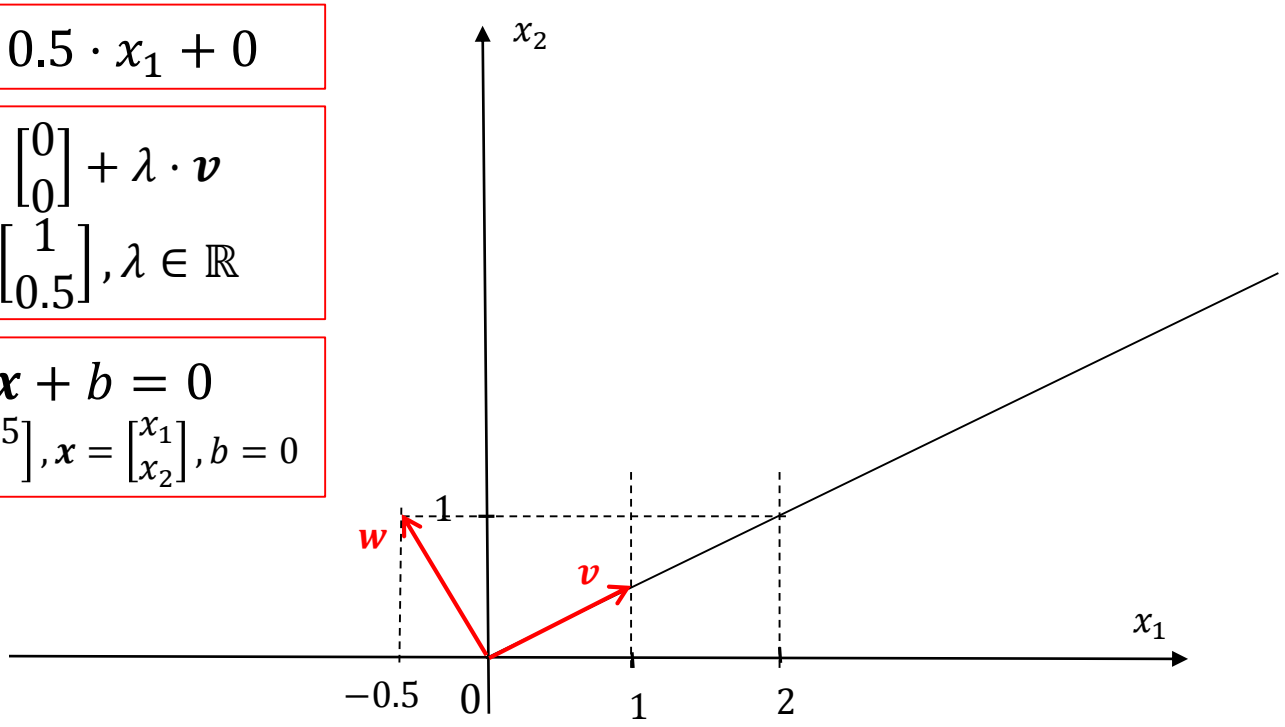


Straight Line Equation

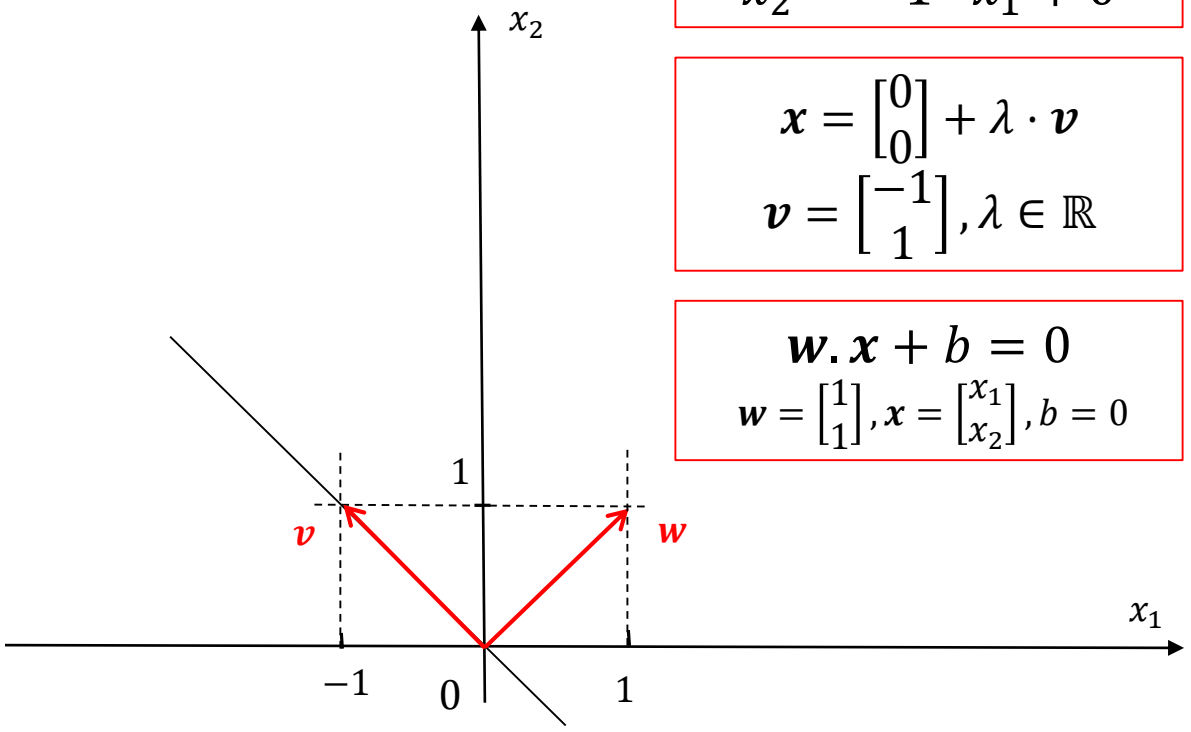
$$x_2 = 0.5 \cdot x_1 + 0$$

$$\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$
$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$
$$\boldsymbol{w} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 0$$



Straight Line Equation



$$x_2 = -1 \cdot x_1 + 0$$

$$\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$

$$\boldsymbol{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 0$$

Straight Line Equation

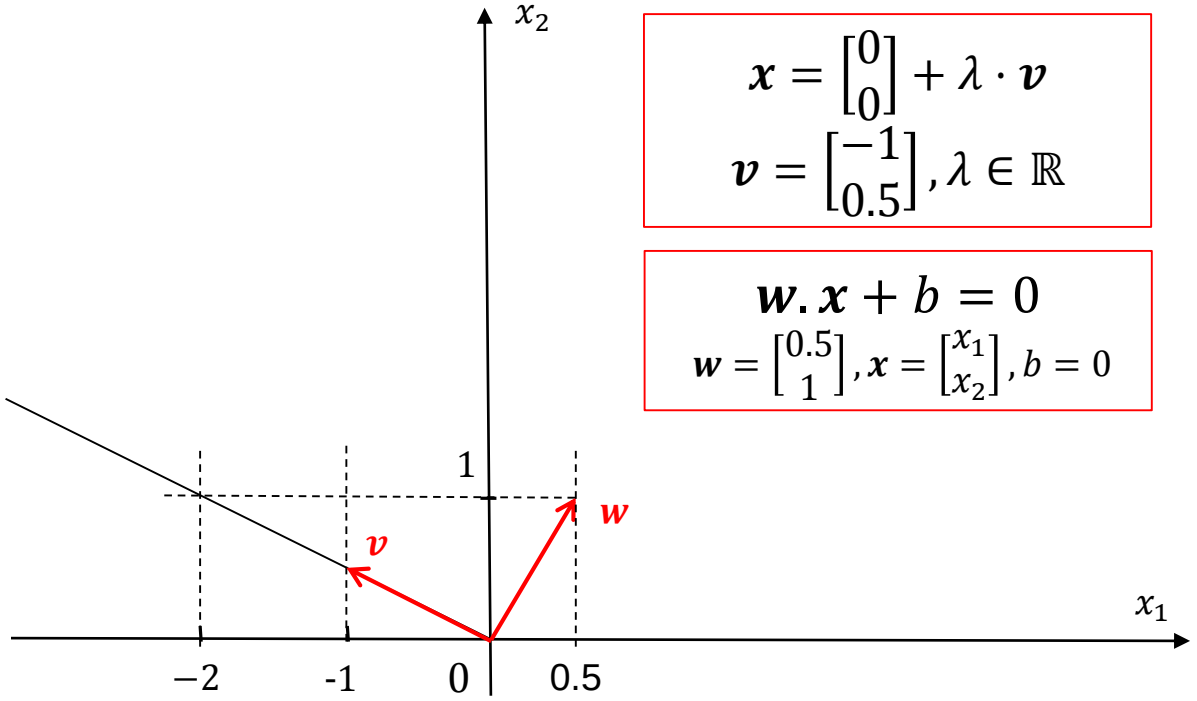
$$x_2 = -0.5 \cdot x_1 + 0$$

$$\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$

$$\boldsymbol{v} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$

$$\boldsymbol{w} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 0$$

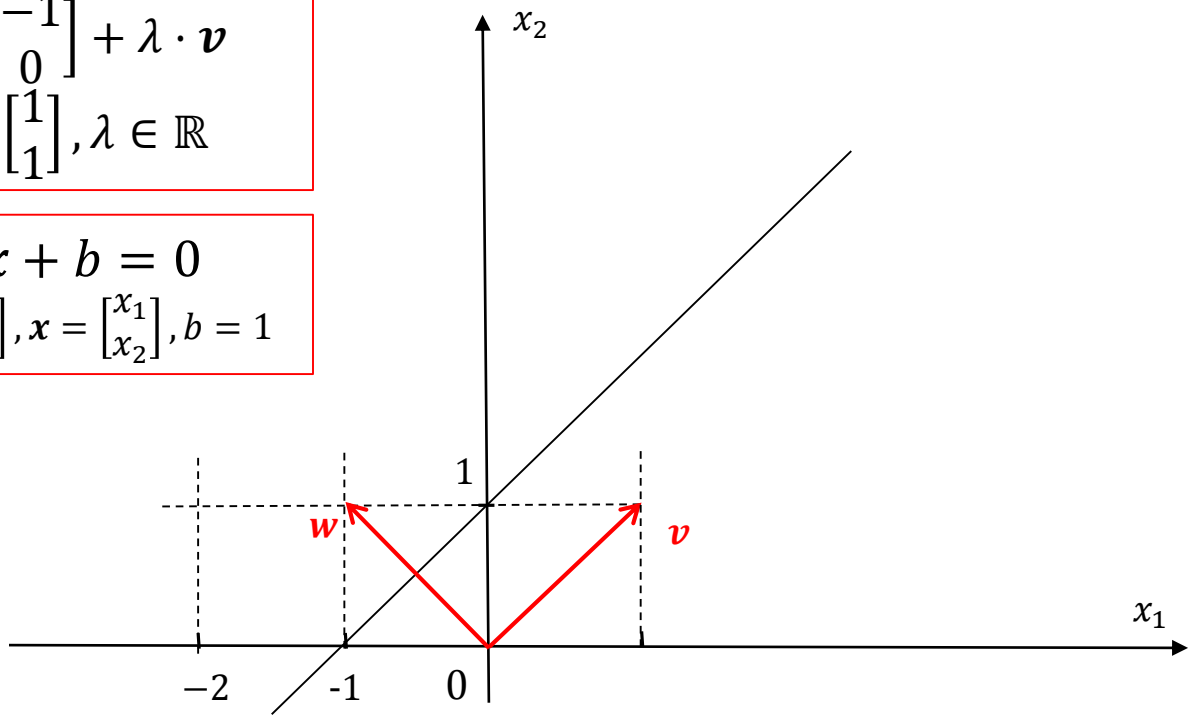


Straight Line Equation

$$x_2 = 1 \cdot x_1 + 1$$

$$\boldsymbol{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$
$$\boldsymbol{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$
$$\boldsymbol{w} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 1$$

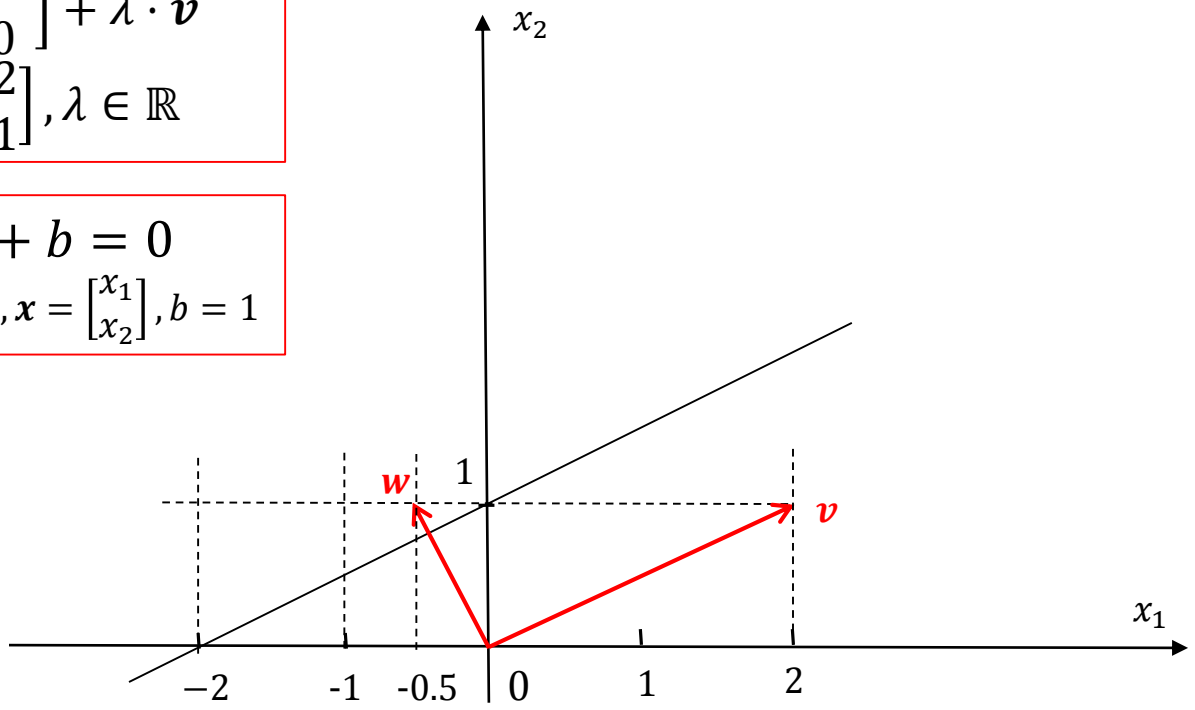


Straight Line Equation

$$x_2 = 0.5 \cdot x_1 + 1$$

$$\boldsymbol{x} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$
$$\boldsymbol{v} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$
$$\boldsymbol{w} = \begin{bmatrix} -0.5 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = 1$$



Straight Line Equation

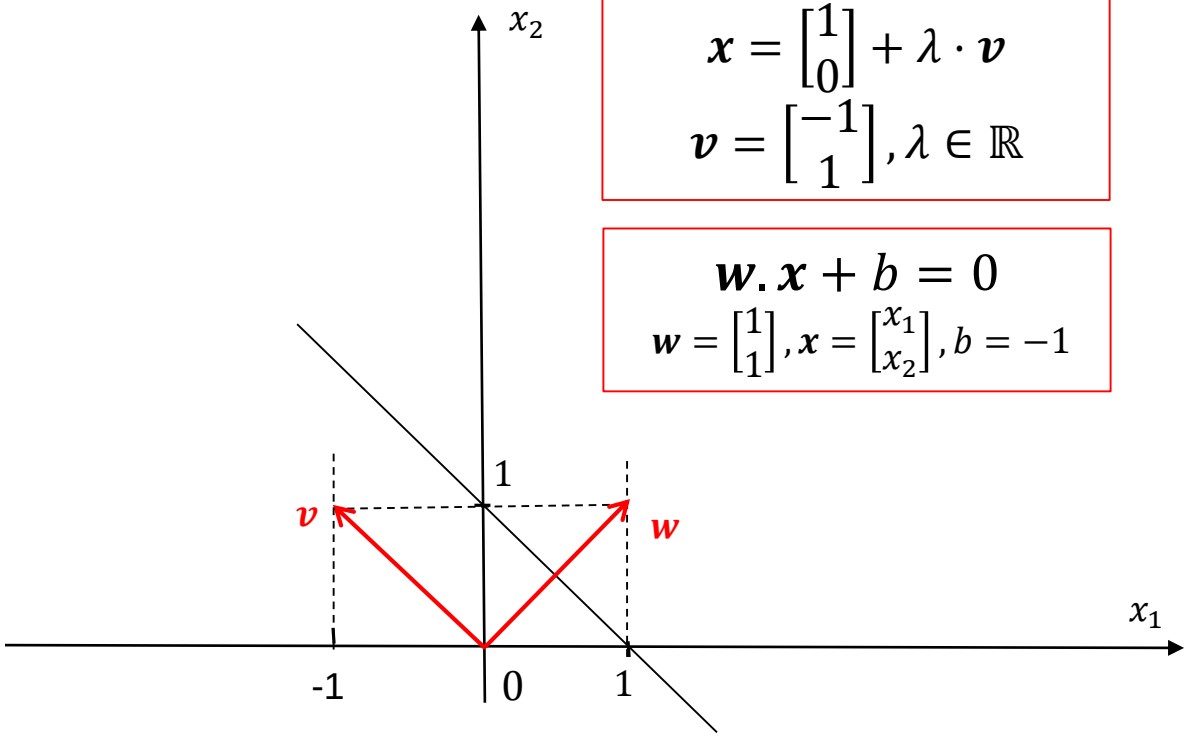
$$x_2 = -1 \cdot x_1 + 1$$

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$

$$\boldsymbol{v} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$

$$\boldsymbol{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = -1$$



Straight Line Equation

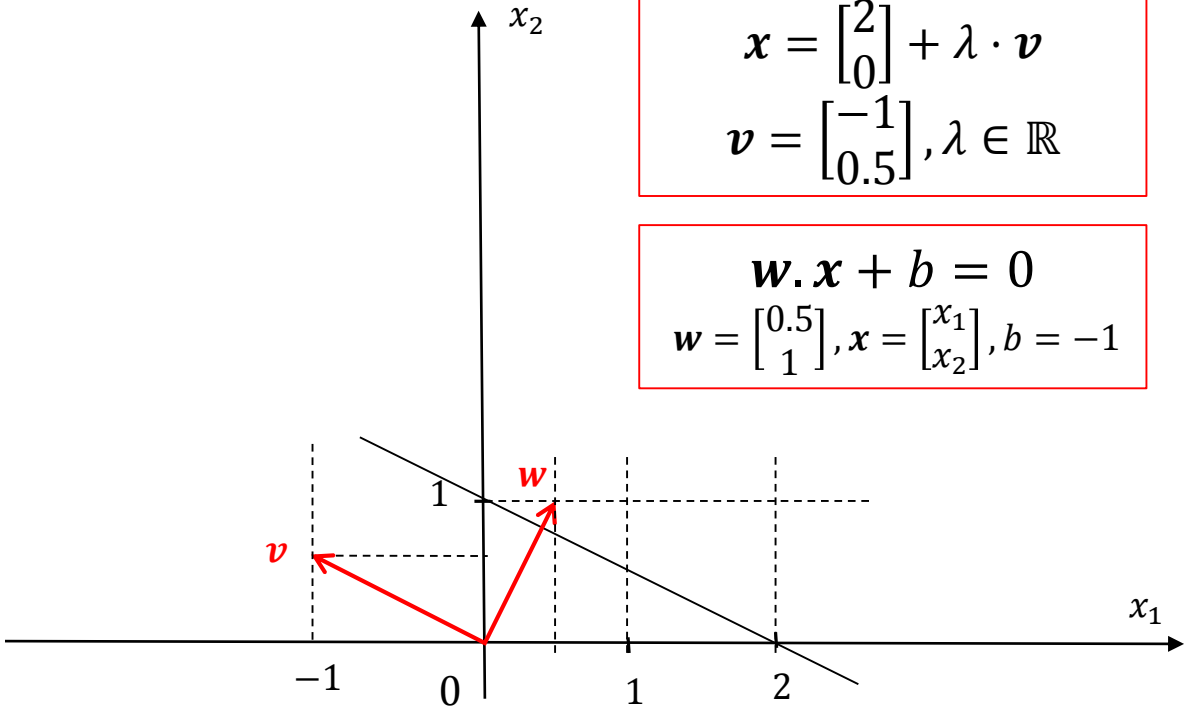
$$x_2 = -0.5 \cdot x_1 + 1$$

$$\boldsymbol{x} = \begin{bmatrix} 2 \\ 0 \end{bmatrix} + \lambda \cdot \boldsymbol{v}$$

$$\boldsymbol{v} = \begin{bmatrix} -1 \\ 0.5 \end{bmatrix}, \lambda \in \mathbb{R}$$

$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$

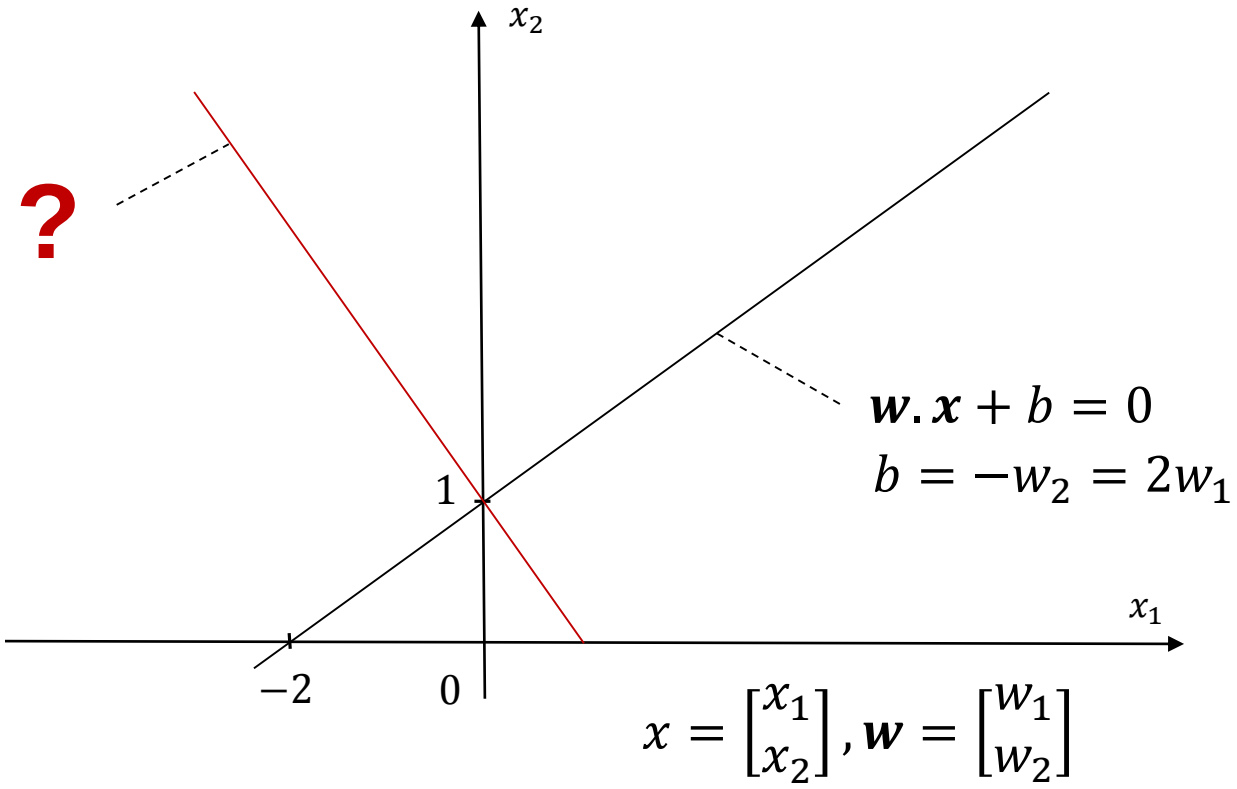
$$\boldsymbol{w} = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, b = -1$$



Mathematical Background

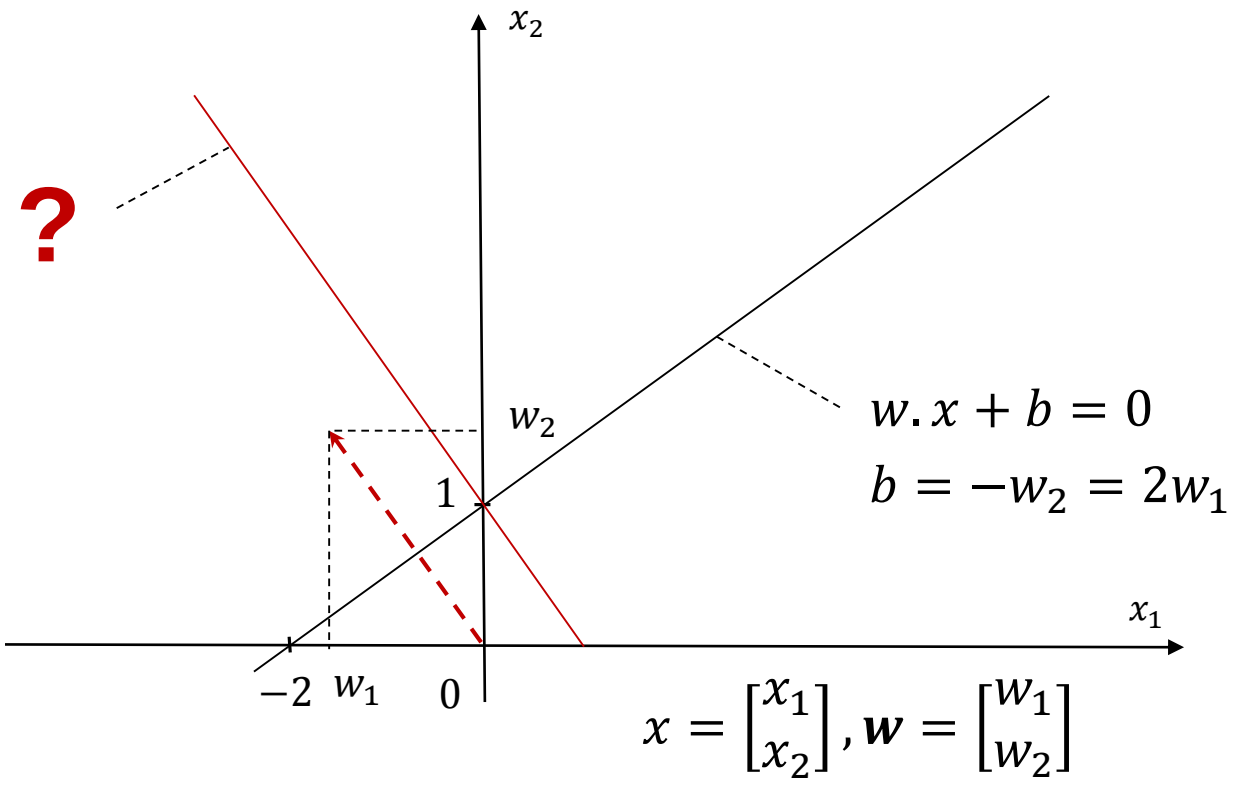
- Straight Line
- Perpendicular Line
- Parallel Line
- Distance Between Parallel Line
- Distance From a Point to a Line
- Separating Line

Perpendicular Line Equation



Question: What is the equation of the perpendicular line?

Perpendicular Line Equation

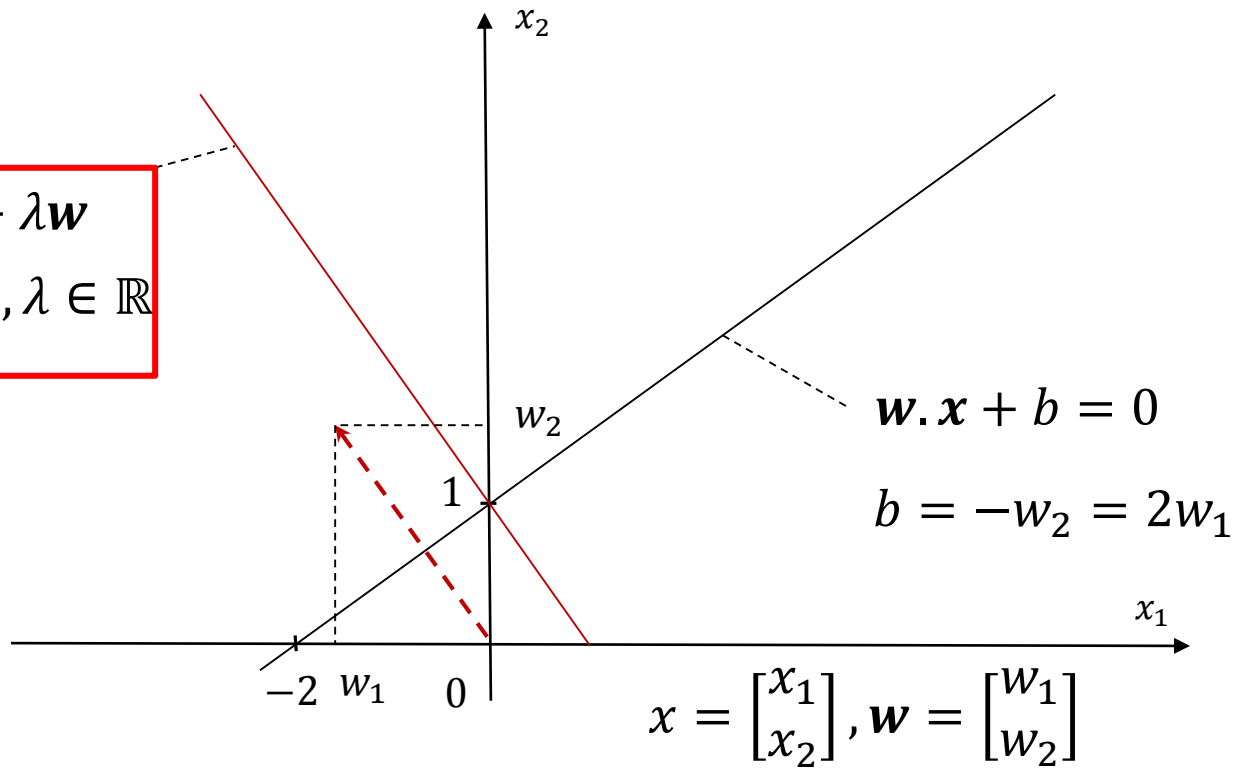


Question: What is the equation of the perpendicular line?

Perpendicular Line Equation

$$\boldsymbol{x} = \boldsymbol{x}^0 + \lambda \boldsymbol{w}$$

$$\boldsymbol{x}^0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \lambda \in \mathbb{R}$$



$$\boldsymbol{w}.\boldsymbol{x} + b = 0$$

$$b = -w_2 = 2w_1$$

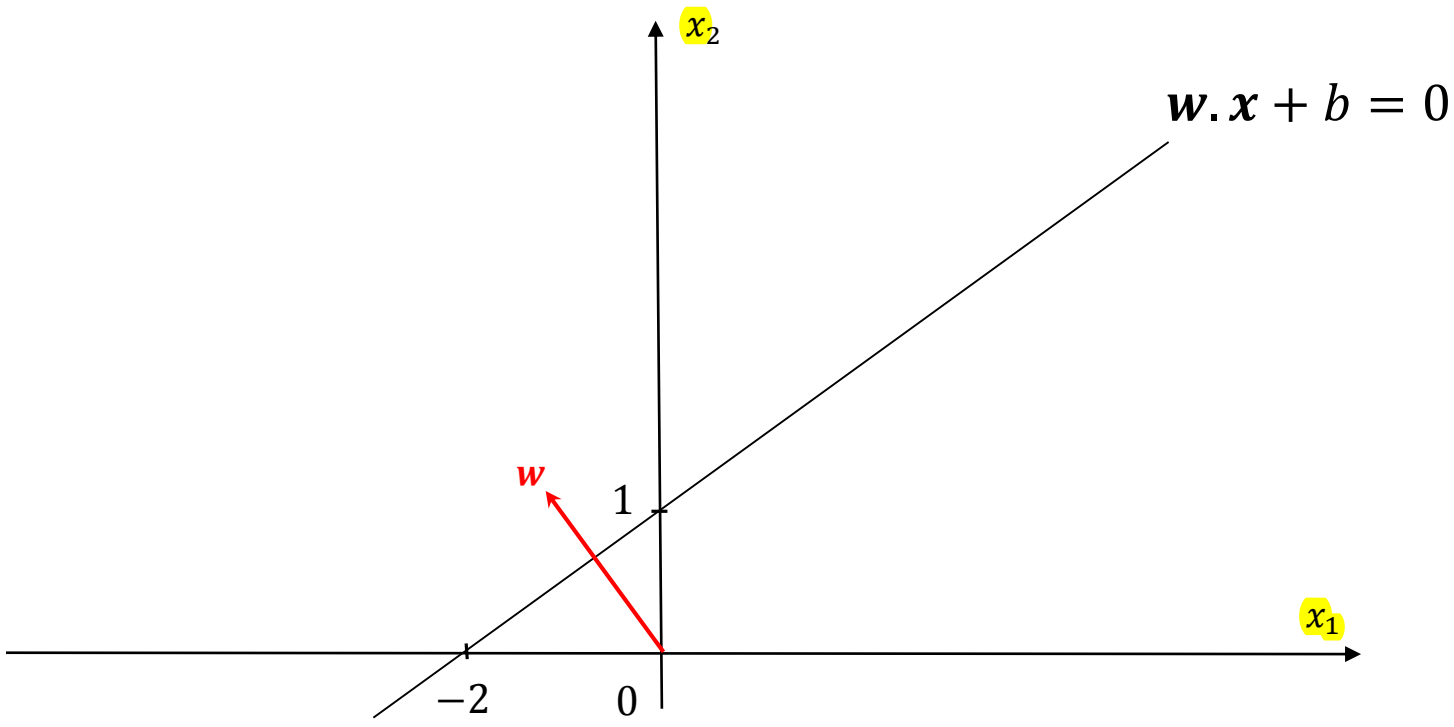
$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \boldsymbol{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$

Question: What is the equation of the perpendicular line?

Mathematical Background

- Straight Line
- Perpendicular Line
- Parallel Line
- Distance Between Parallel Line
- Distance From a Point to a Line
- Separating Line

Parallel Line Equation

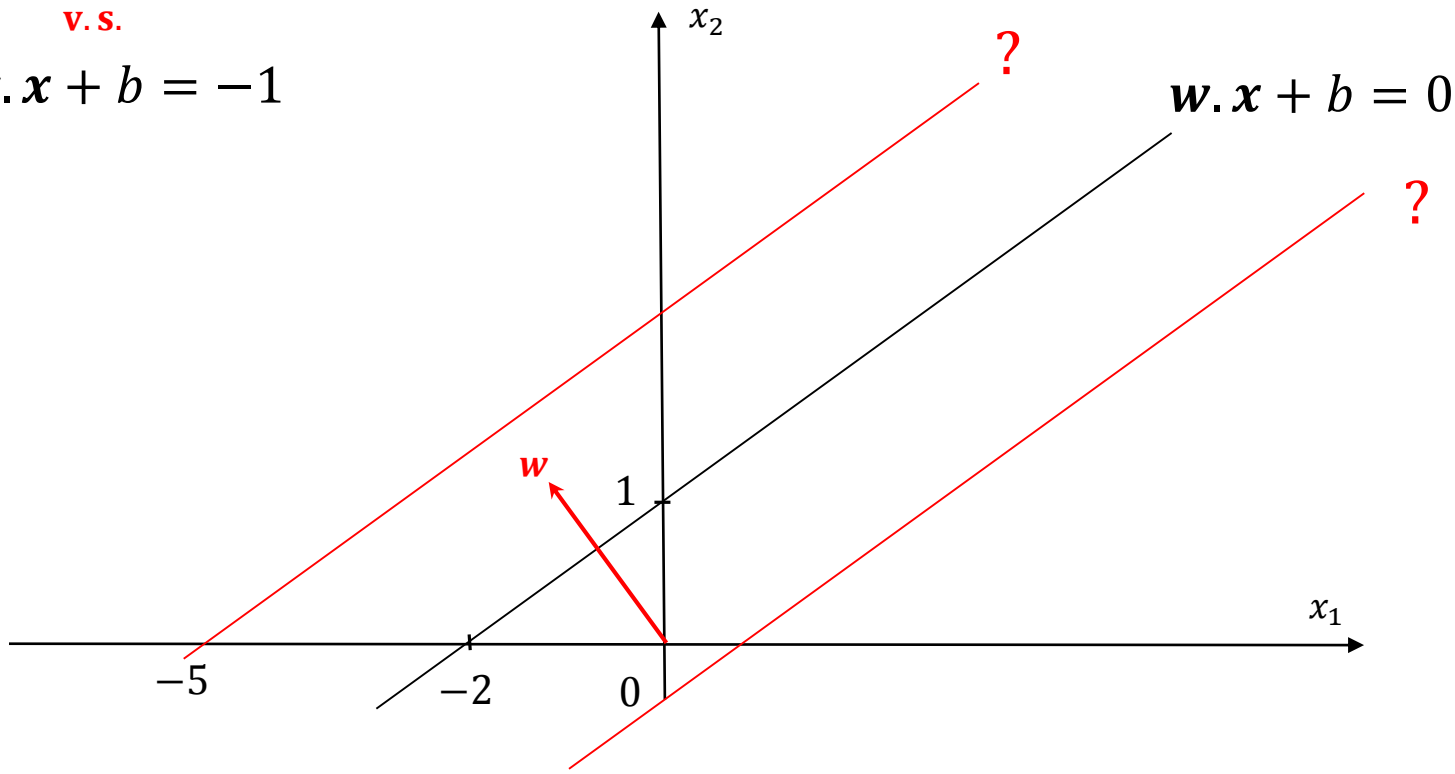


Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 1$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -1$

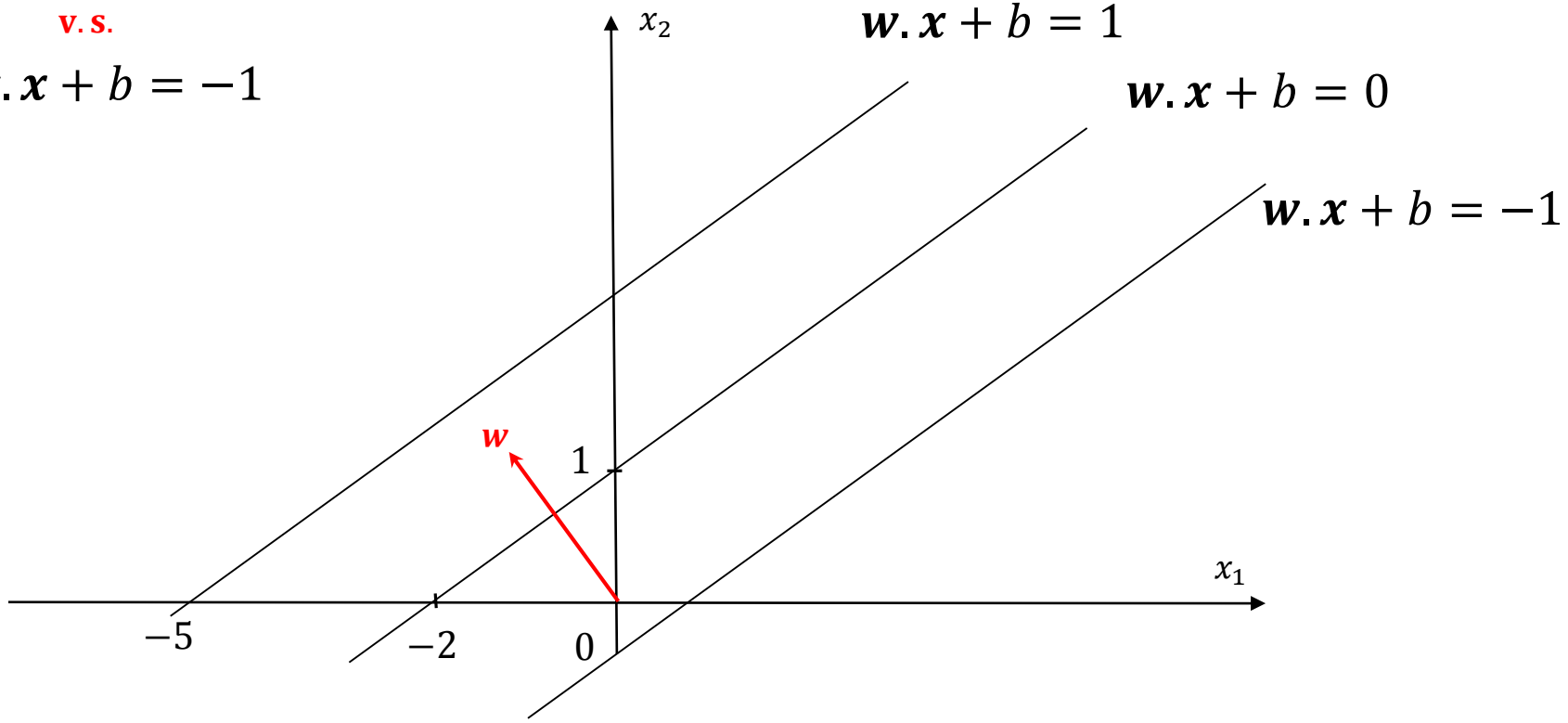


Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 1$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -1$

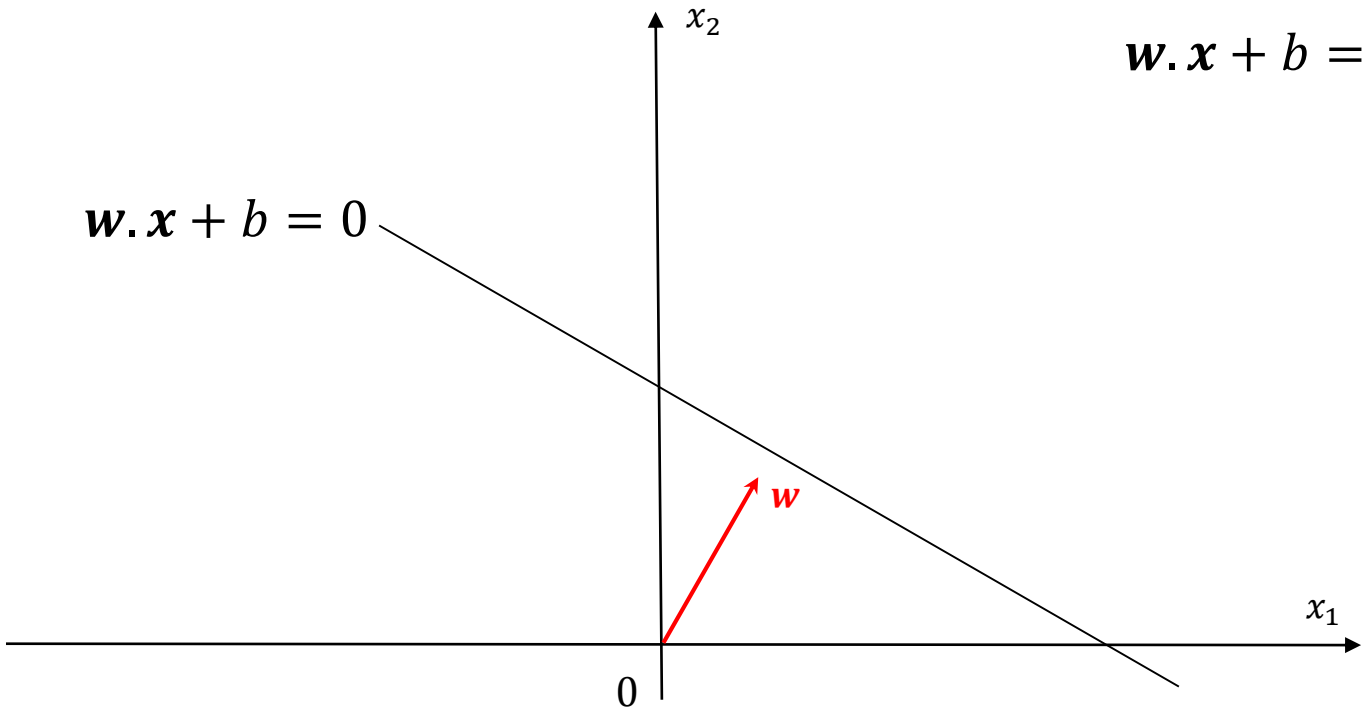


Parallel Line Equation

$$\boldsymbol{w}.\boldsymbol{x} + b = 1$$

v. s.

$$\boldsymbol{w}.\boldsymbol{x} + b = -1$$

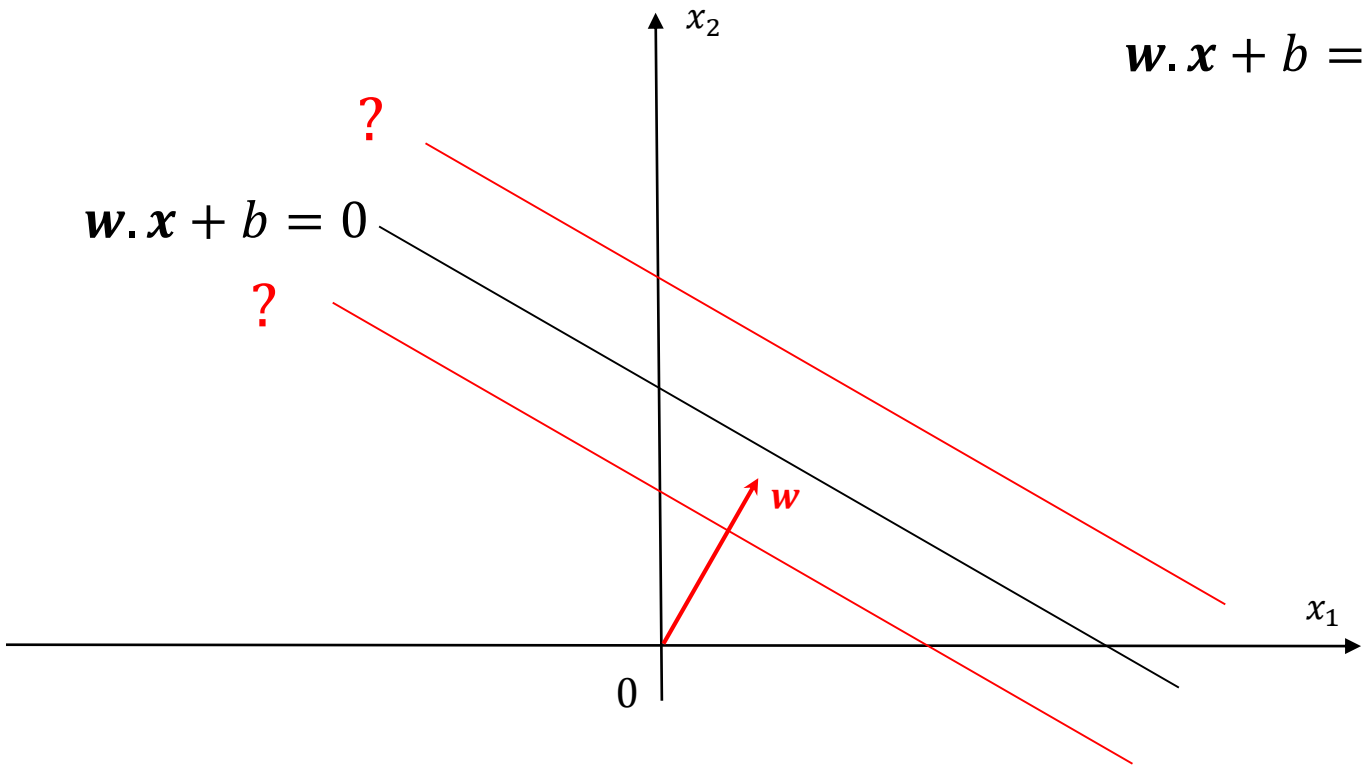


Parallel Line Equation

$$\boldsymbol{w}.\boldsymbol{x} + b = 1$$

v. s.

$$\boldsymbol{w}.\boldsymbol{x} + b = -1$$

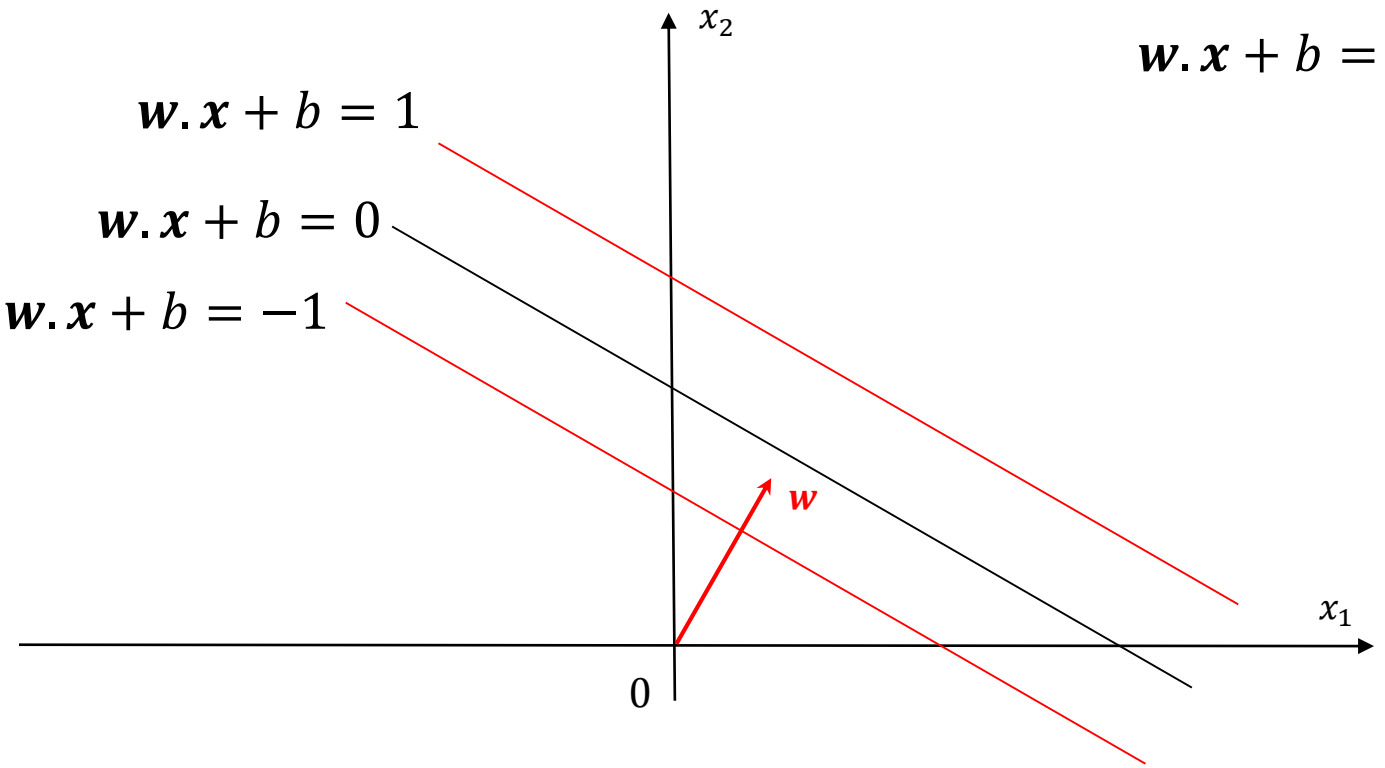


Parallel Line Equation

$$\boldsymbol{w}.\boldsymbol{x} + b = 1$$

v. s.

$$\boldsymbol{w}.\boldsymbol{x} + b = -1$$

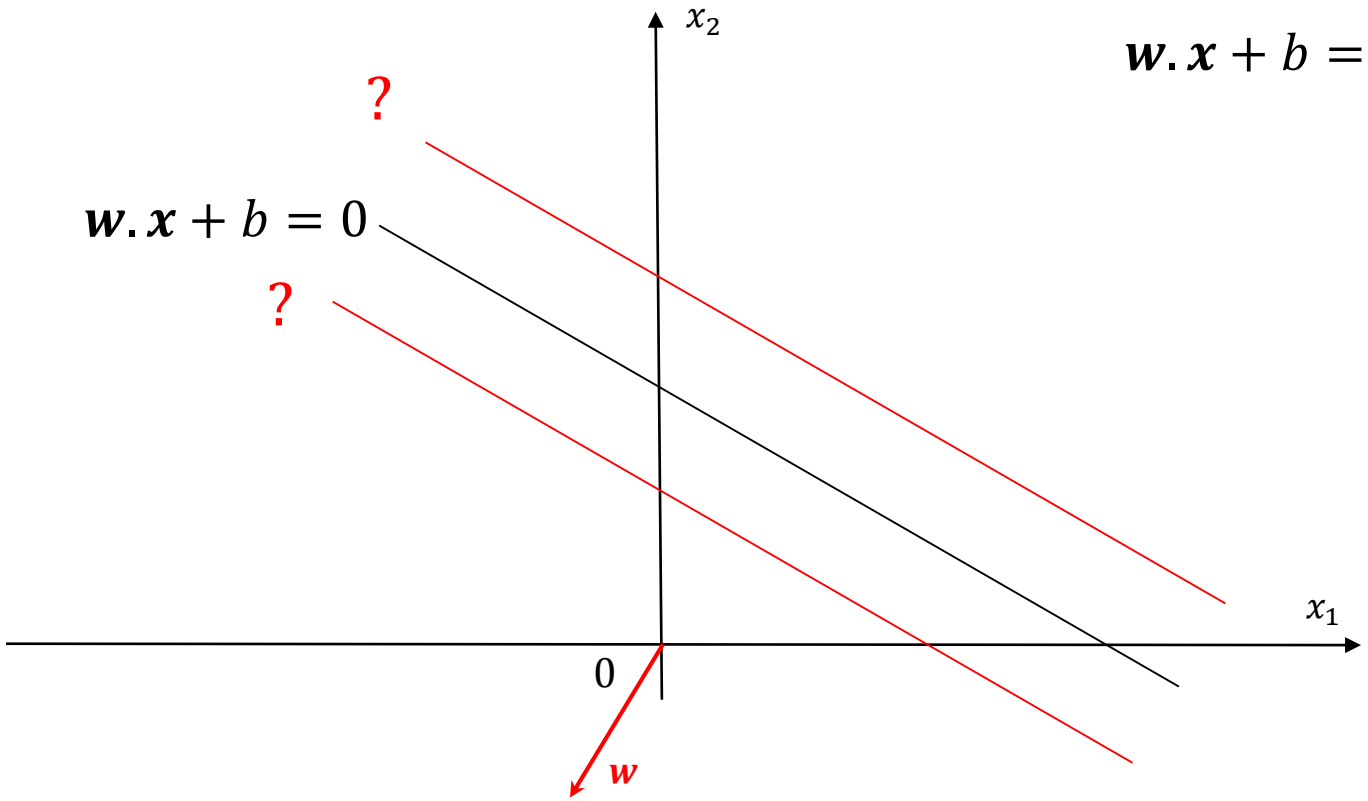


Parallel Line Equation

$$\boldsymbol{w}.\boldsymbol{x} + b = 1$$

v. s.

$$\boldsymbol{w}.\boldsymbol{x} + b = -1$$

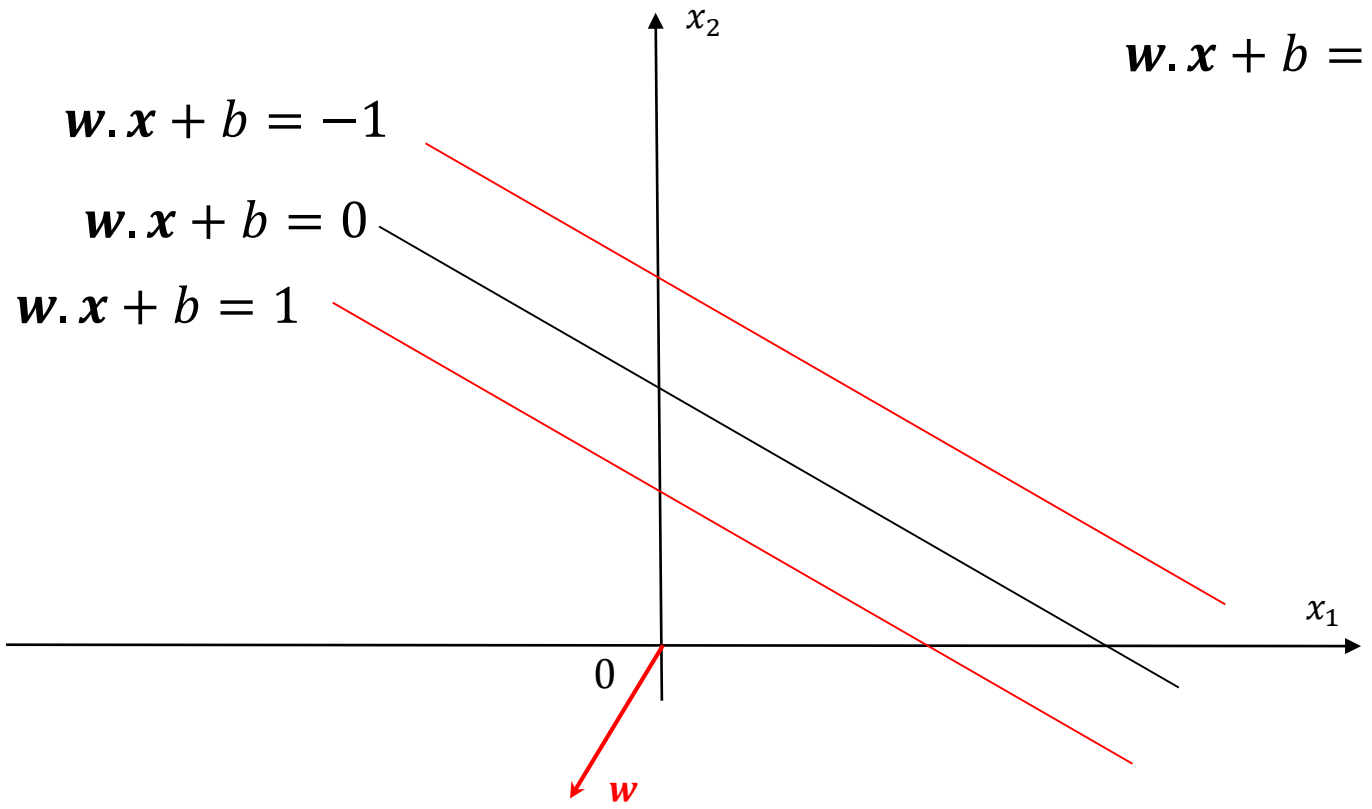


Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 1$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -1$

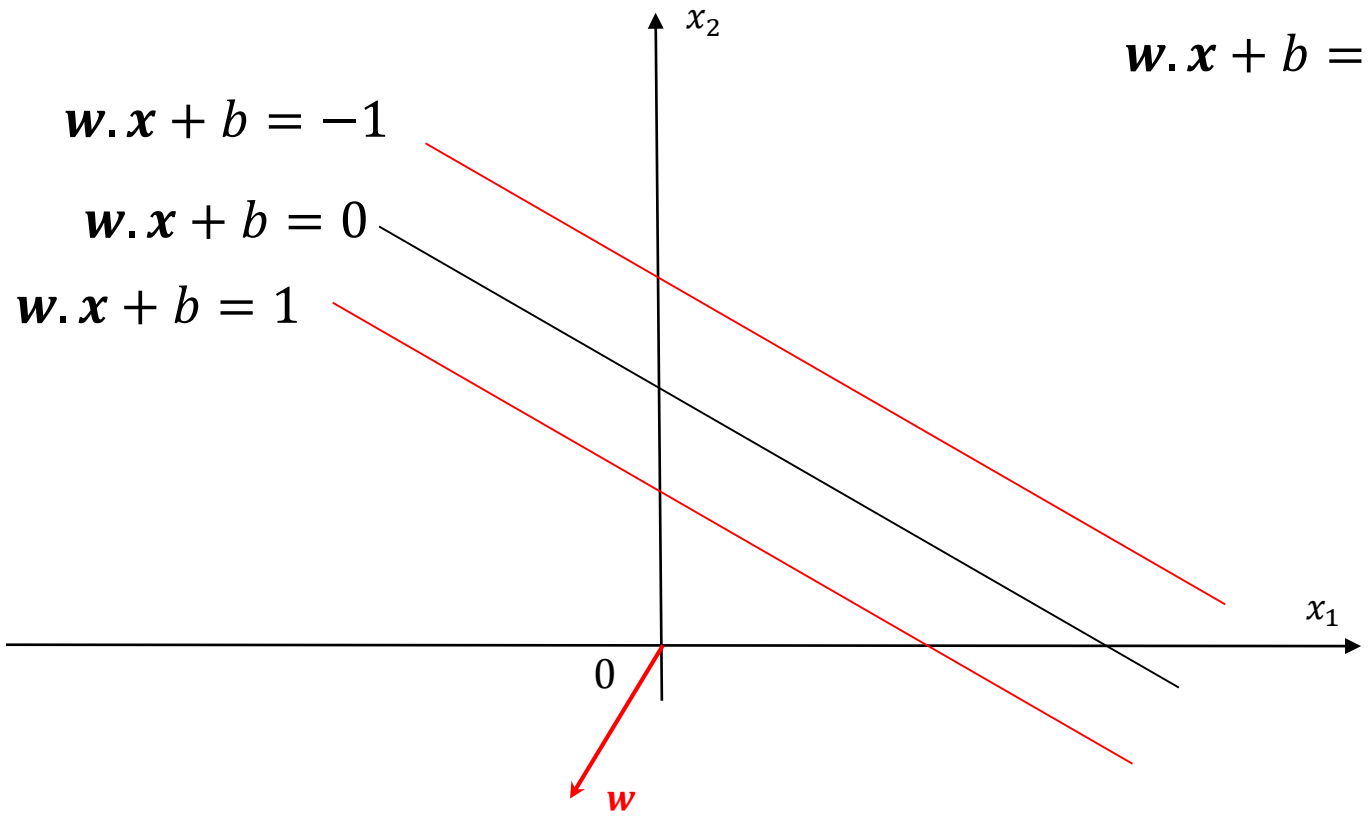


Parallel Line Equation

$$\boldsymbol{w}.\boldsymbol{x} + b = 1$$

v. s.

$$\boldsymbol{w}.\boldsymbol{x} + b = -1$$

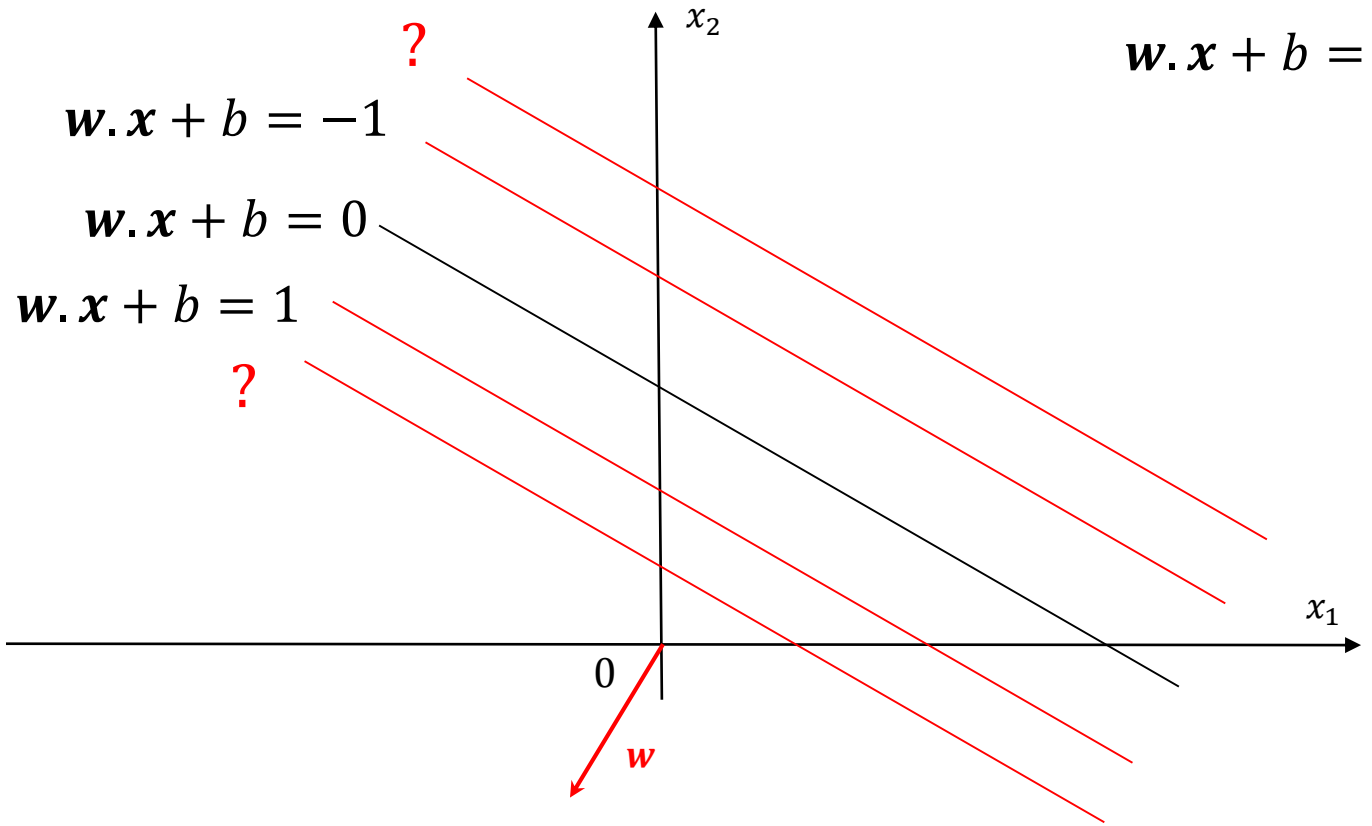


Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 2$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -2$

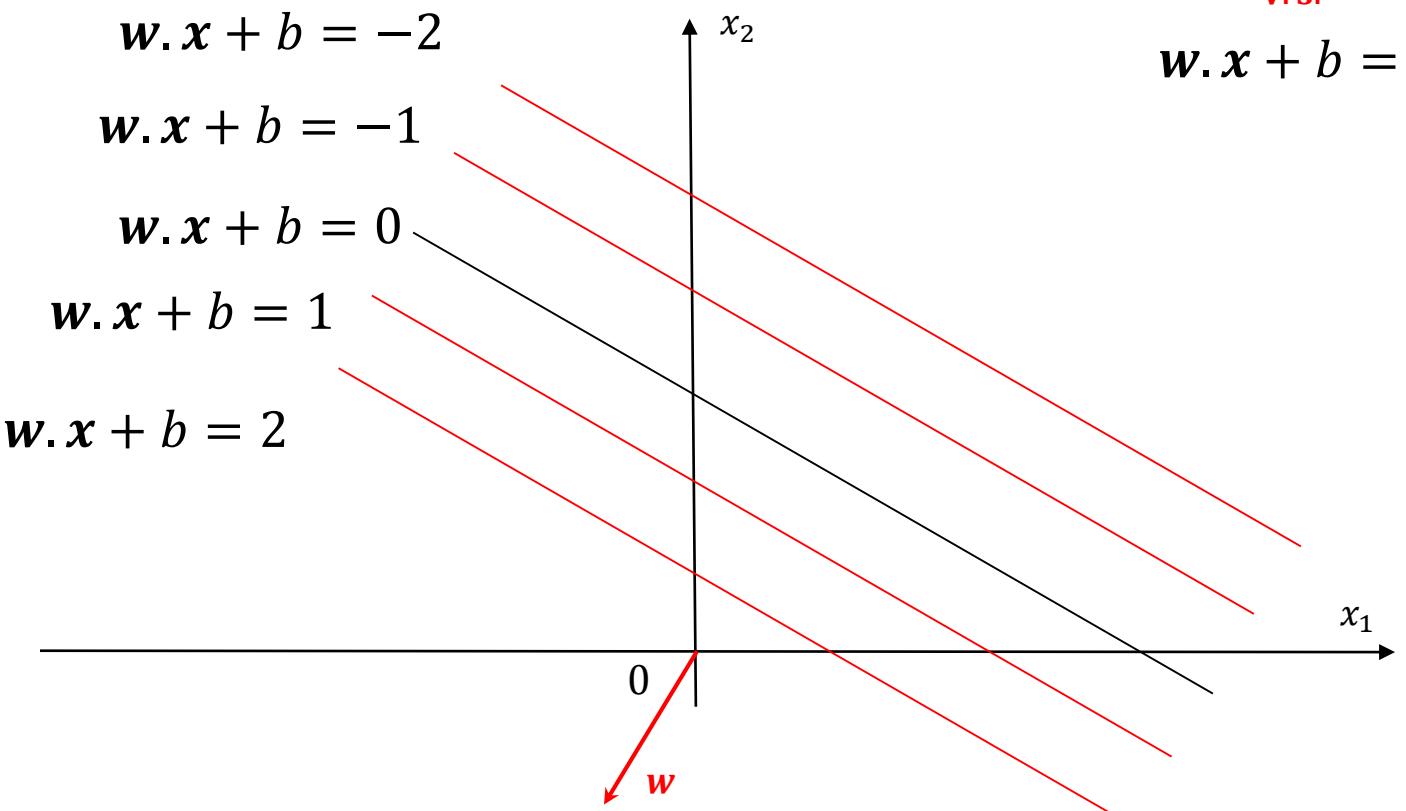


Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 2$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -2$

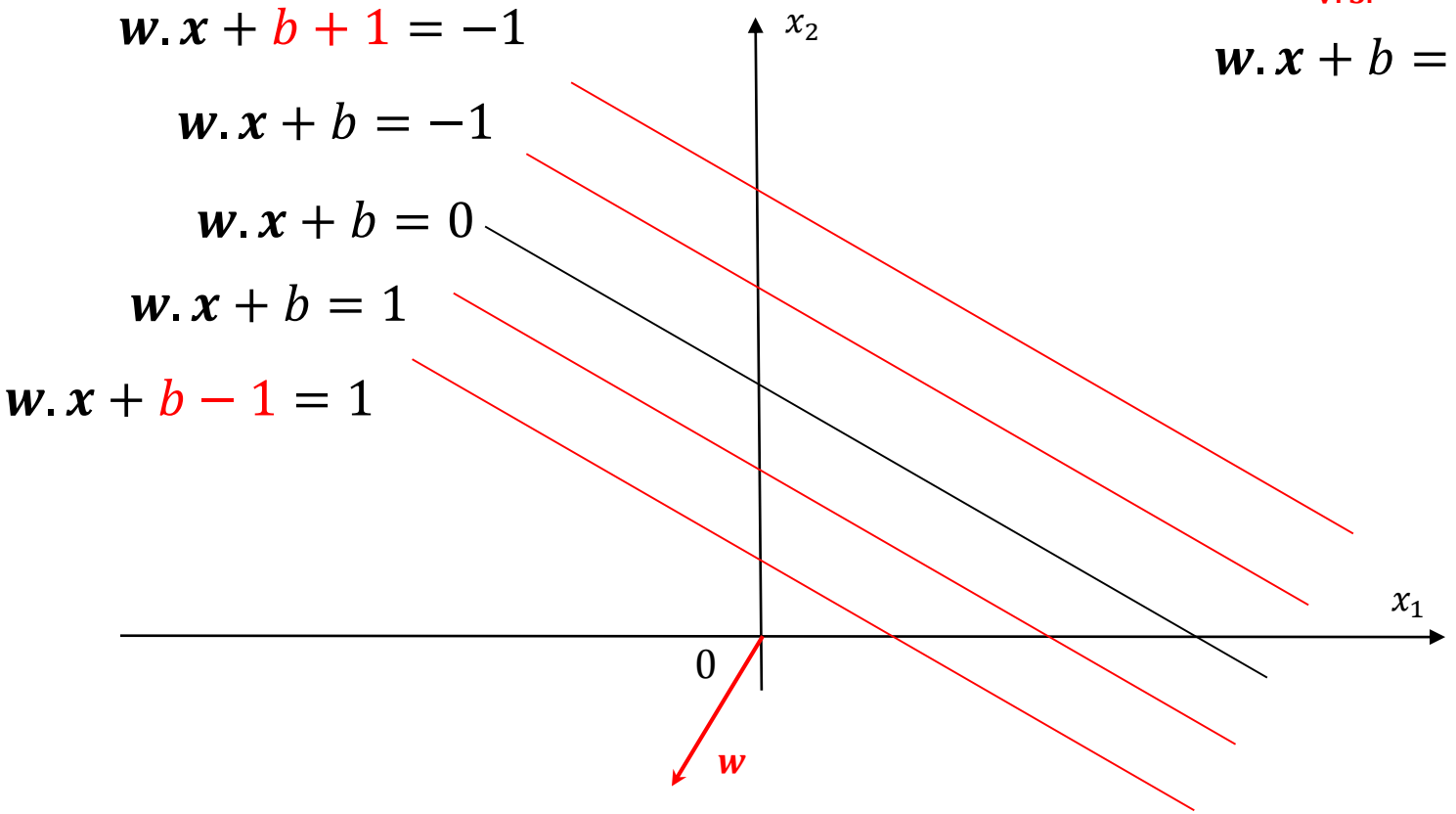


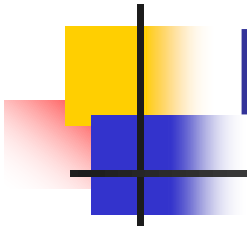
Parallel Line Equation

$\boldsymbol{w}.\boldsymbol{x} + b = 1$

v. s.

$\boldsymbol{w}.\boldsymbol{x} + b = -1$

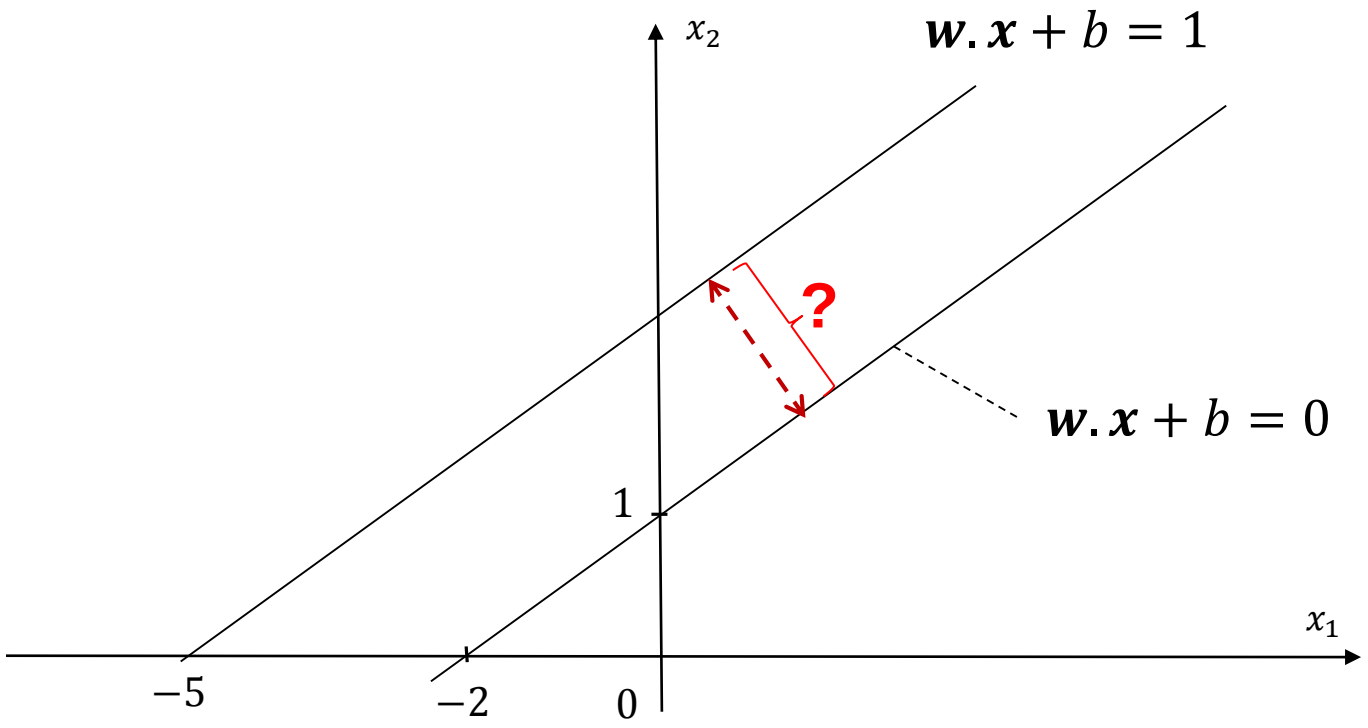




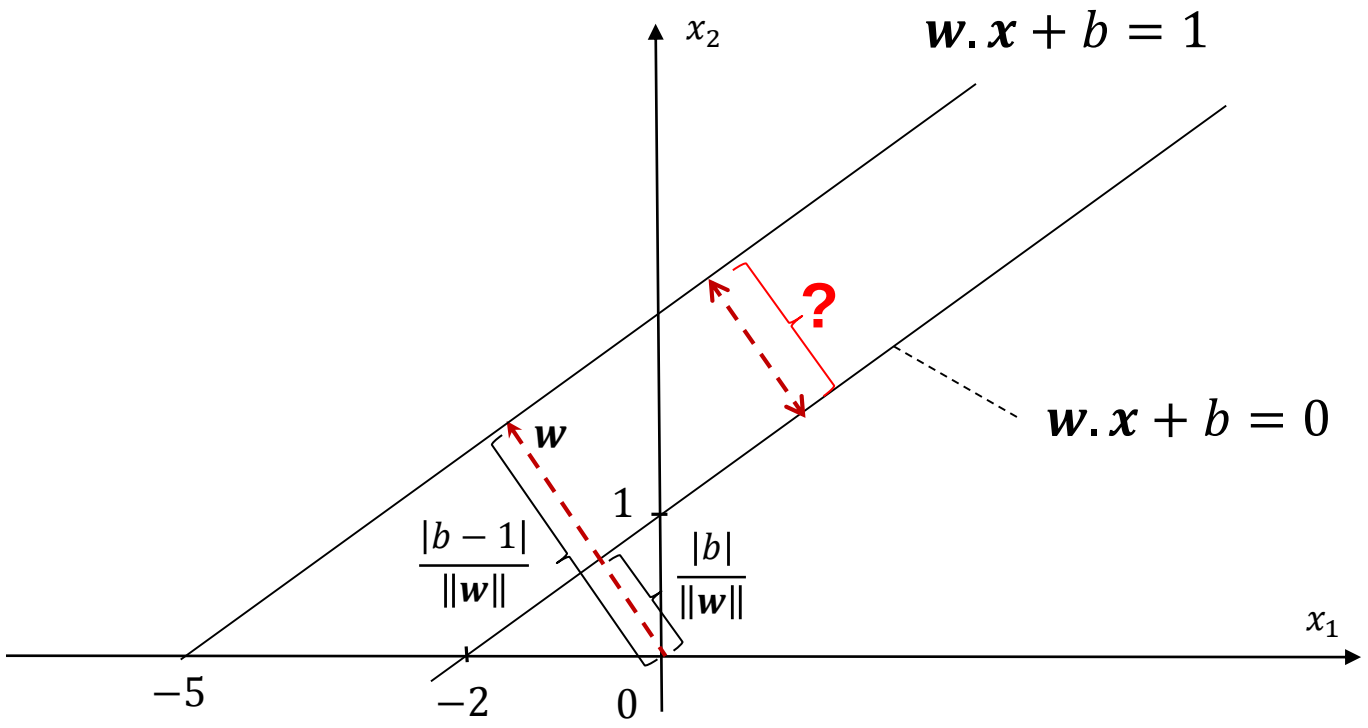
Mathematical Background

- Straight Line
- Perpendicular Line
- Parallel Line
- Distance Between Parallel Line
- Distance From a Point to a Line
- Separating Line

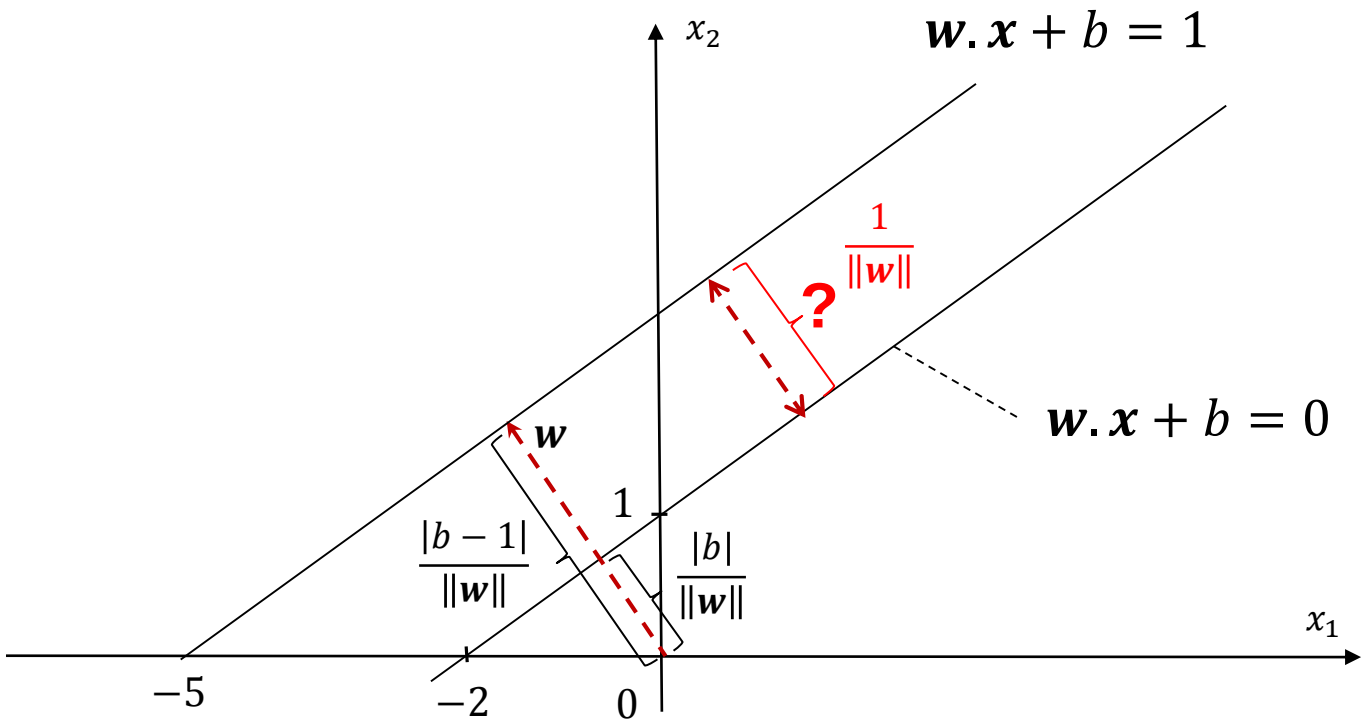
Parallel Line Equation



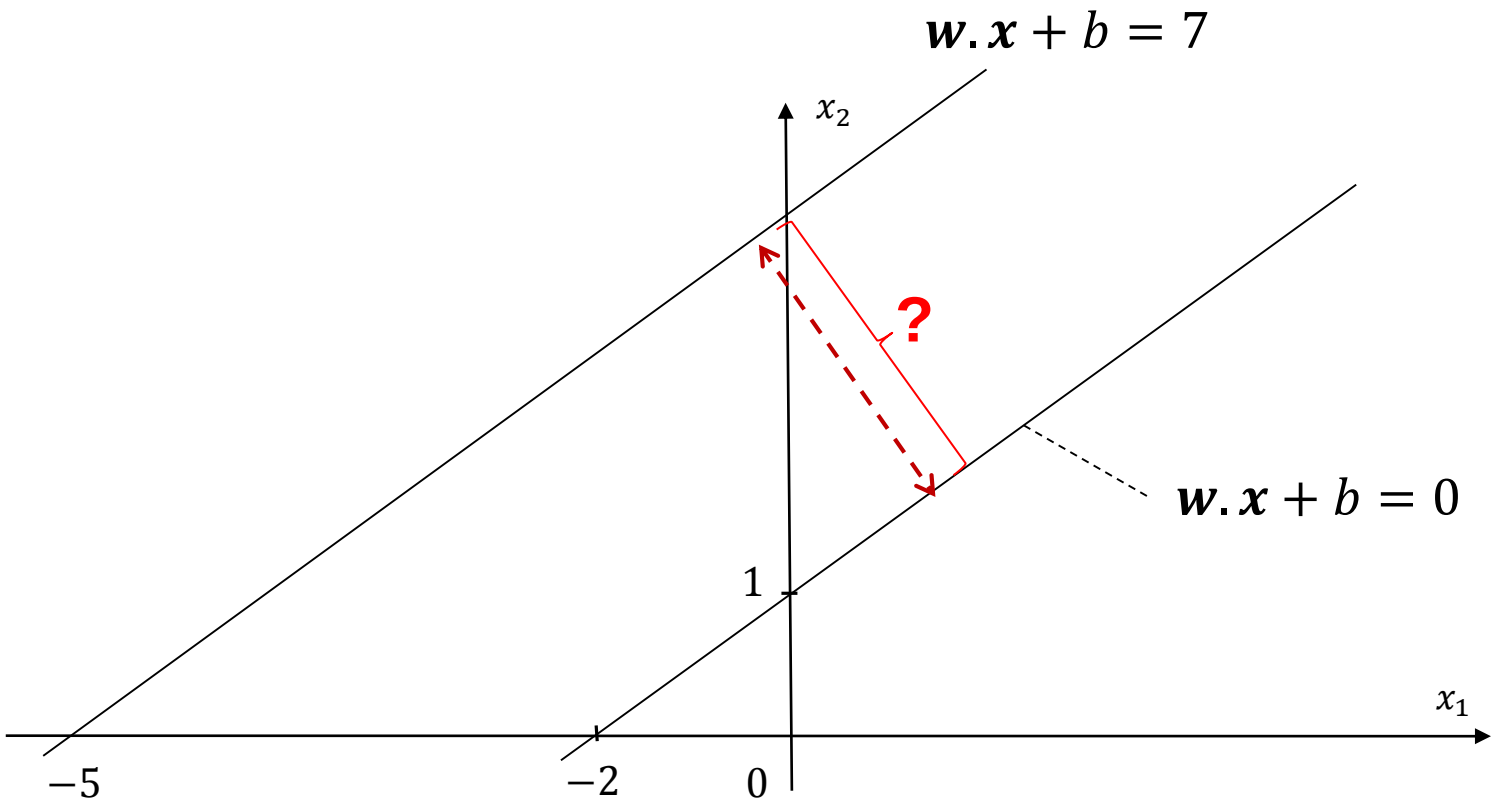
Parallel Line Equation

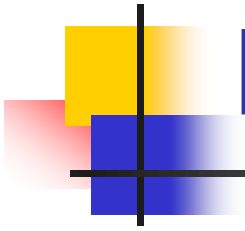


Parallel Line Equation



Parallel Line Equation

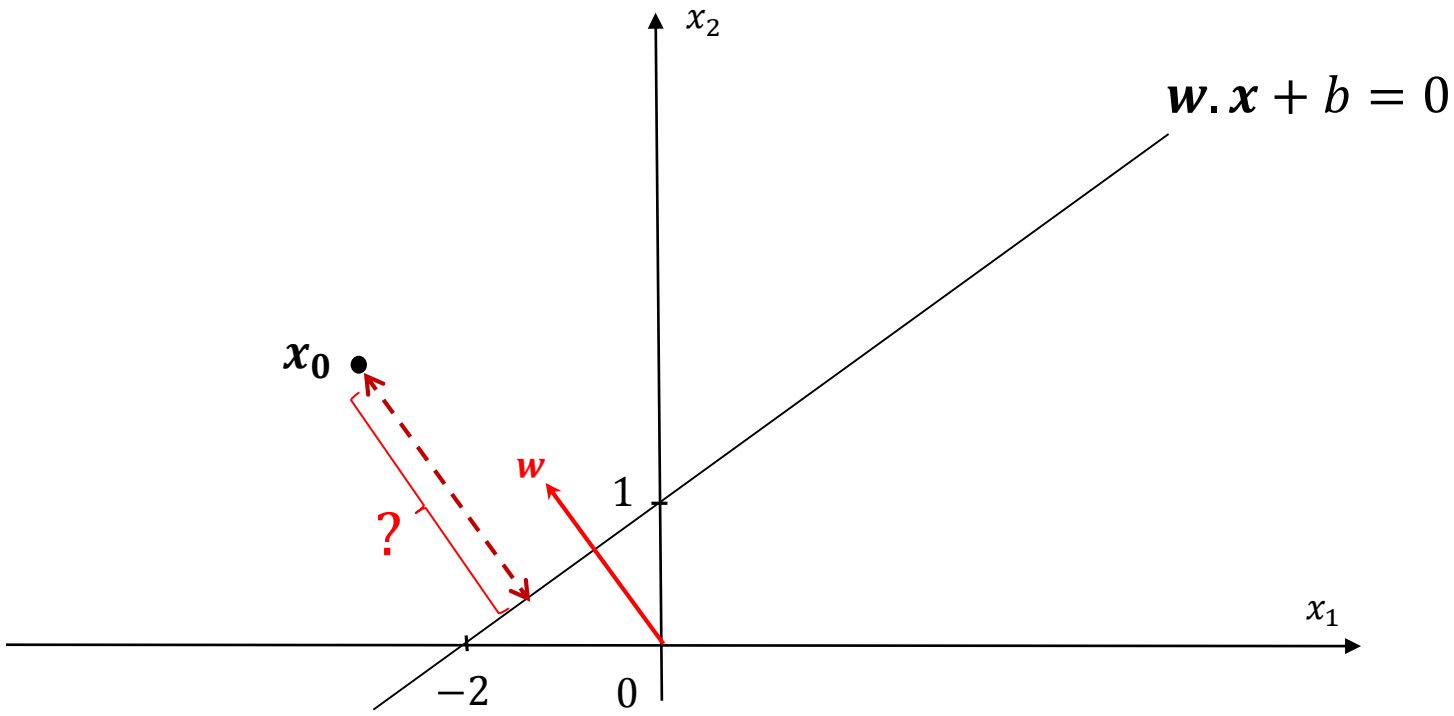




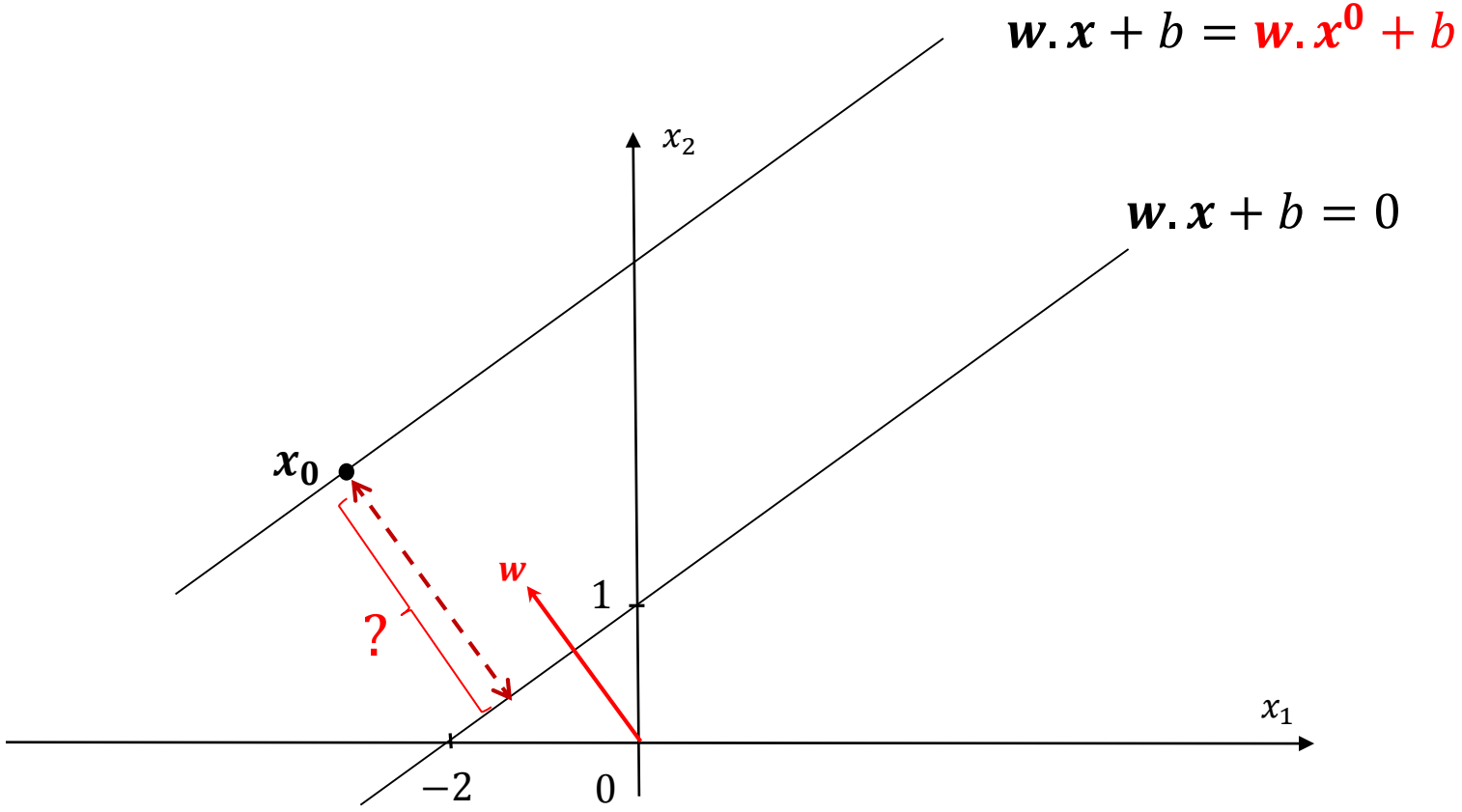
Mathematical Background

- Straight Line
- Perpendicular Line
- Parallel Line
- Distance Between Parallel Line
- Distance From a Point to a Line
- Separating Line

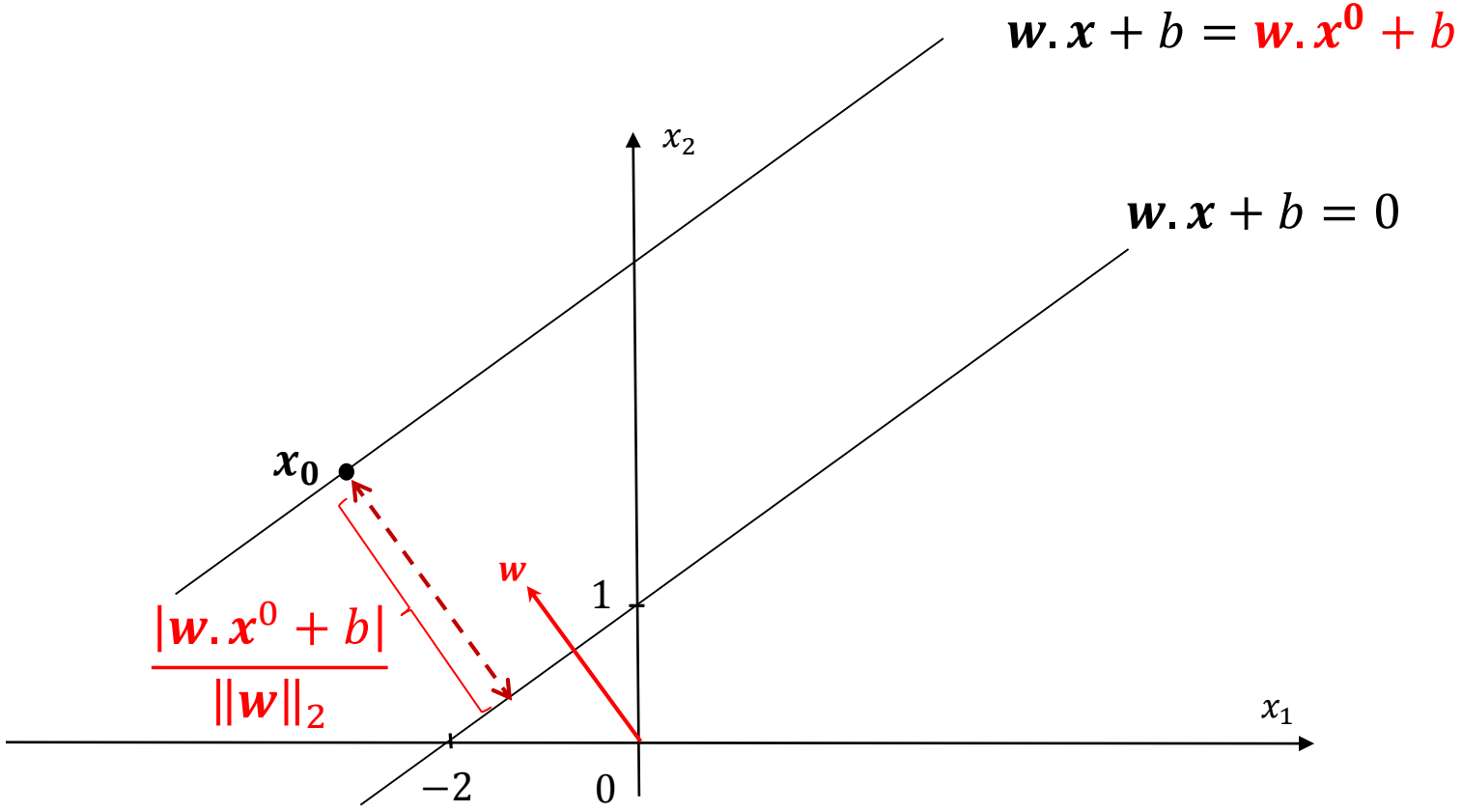
Distance from a Point to a Line

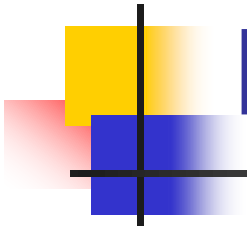


Distance from a Point to a Line



Distance from a Point to a Line

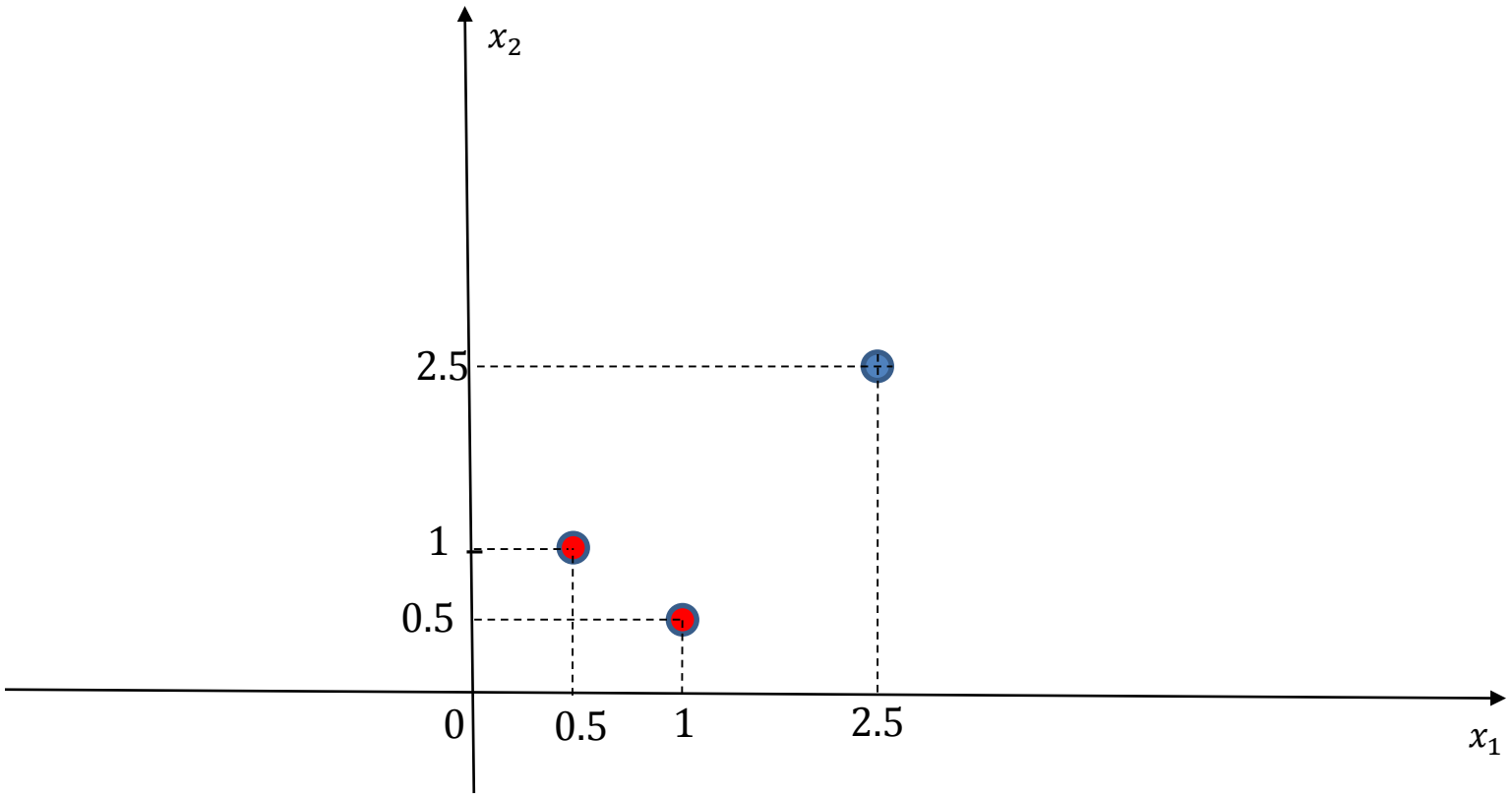




Mathematical Background

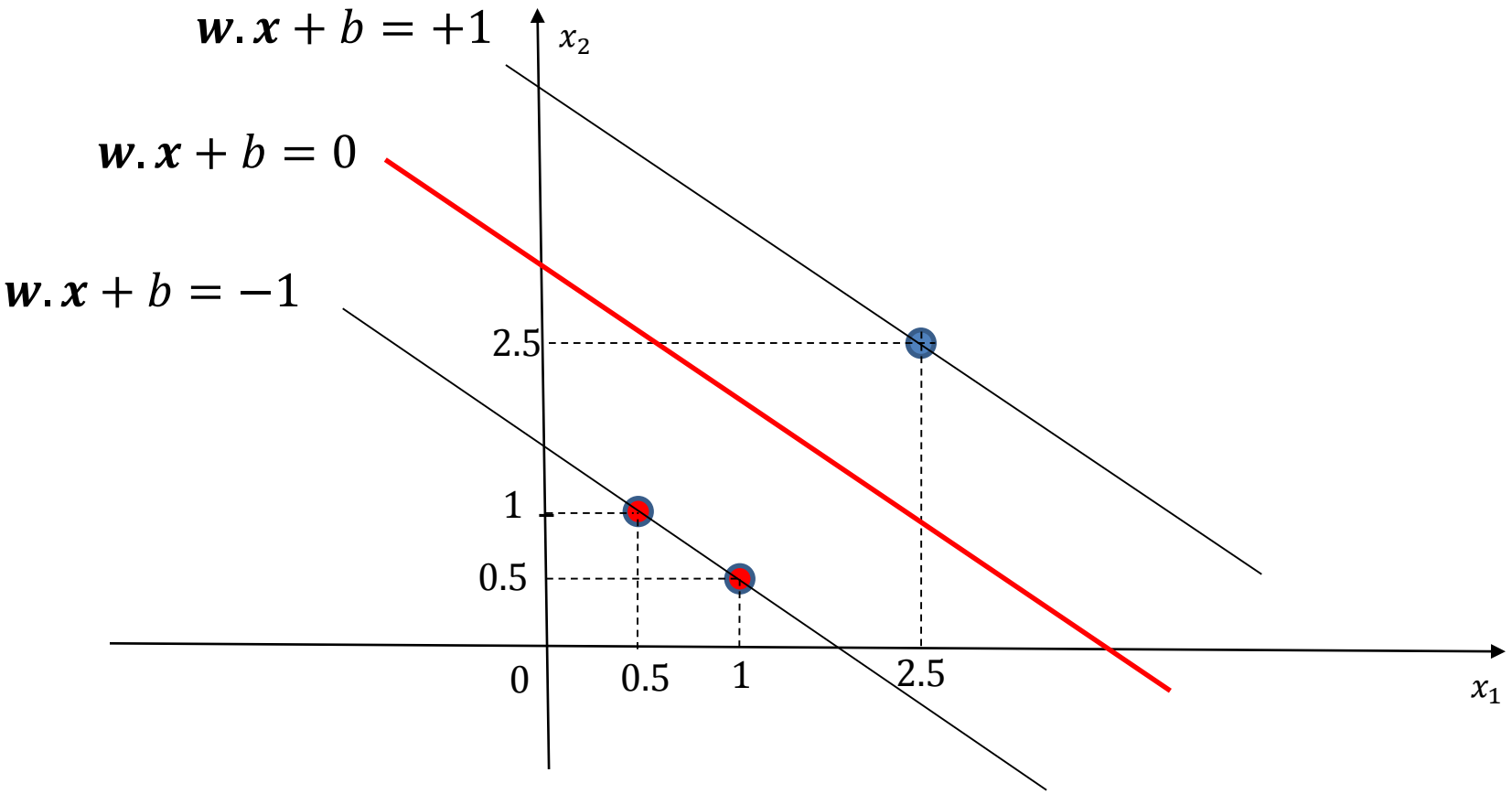
- Straight Line
- Perpendicular Line
- Parallel Line
- Distance Between Parallel Line
- Distance From a Point to a Line
- Separating Line

Separating Line



Question: Where will be the best separating line located?

Separating Line



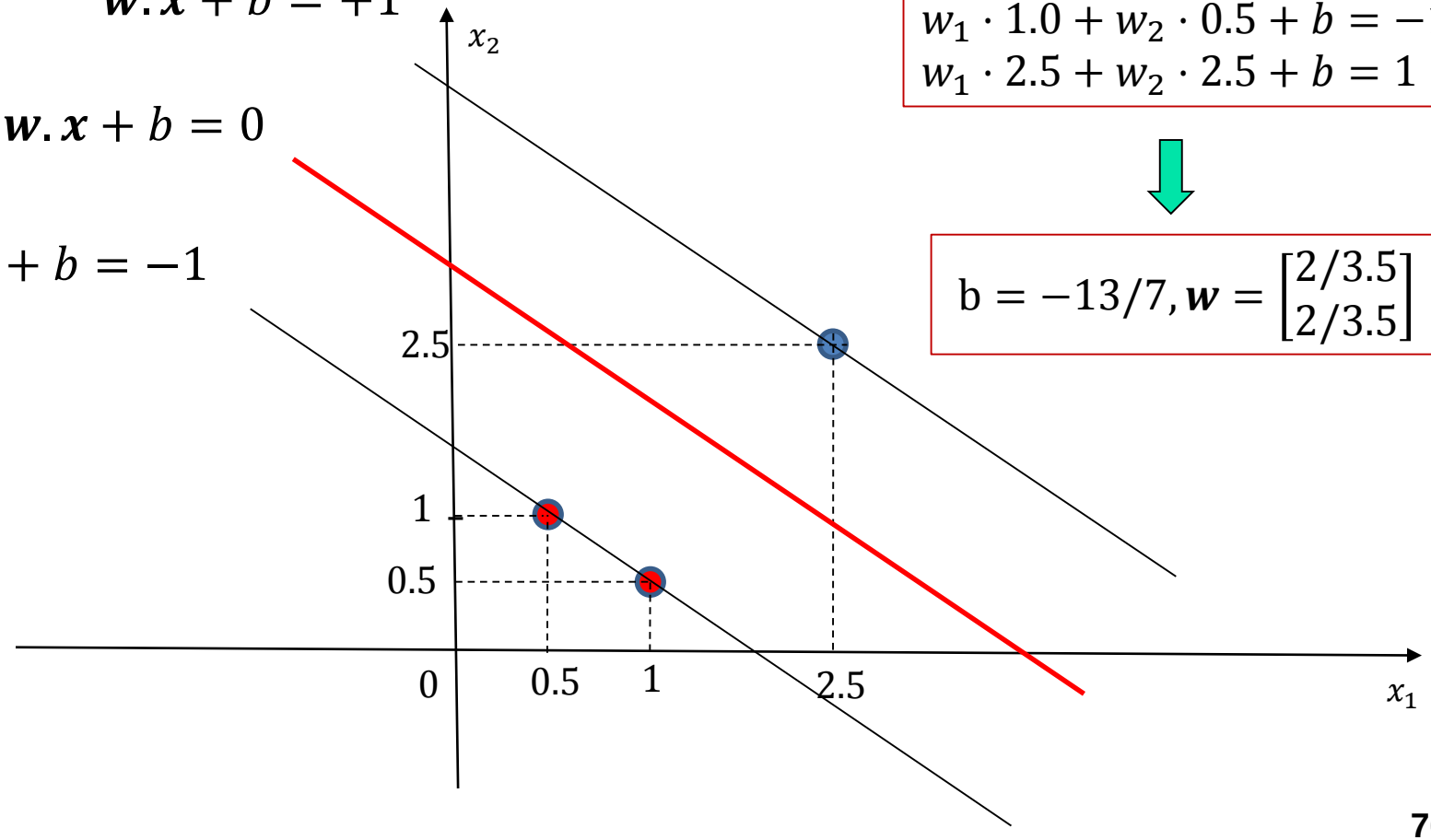
Question: What are the values of w and b?

Separating Line

$\boldsymbol{w}.\boldsymbol{x} + b = +1$

$\boldsymbol{w}.\boldsymbol{x} + b = 0$

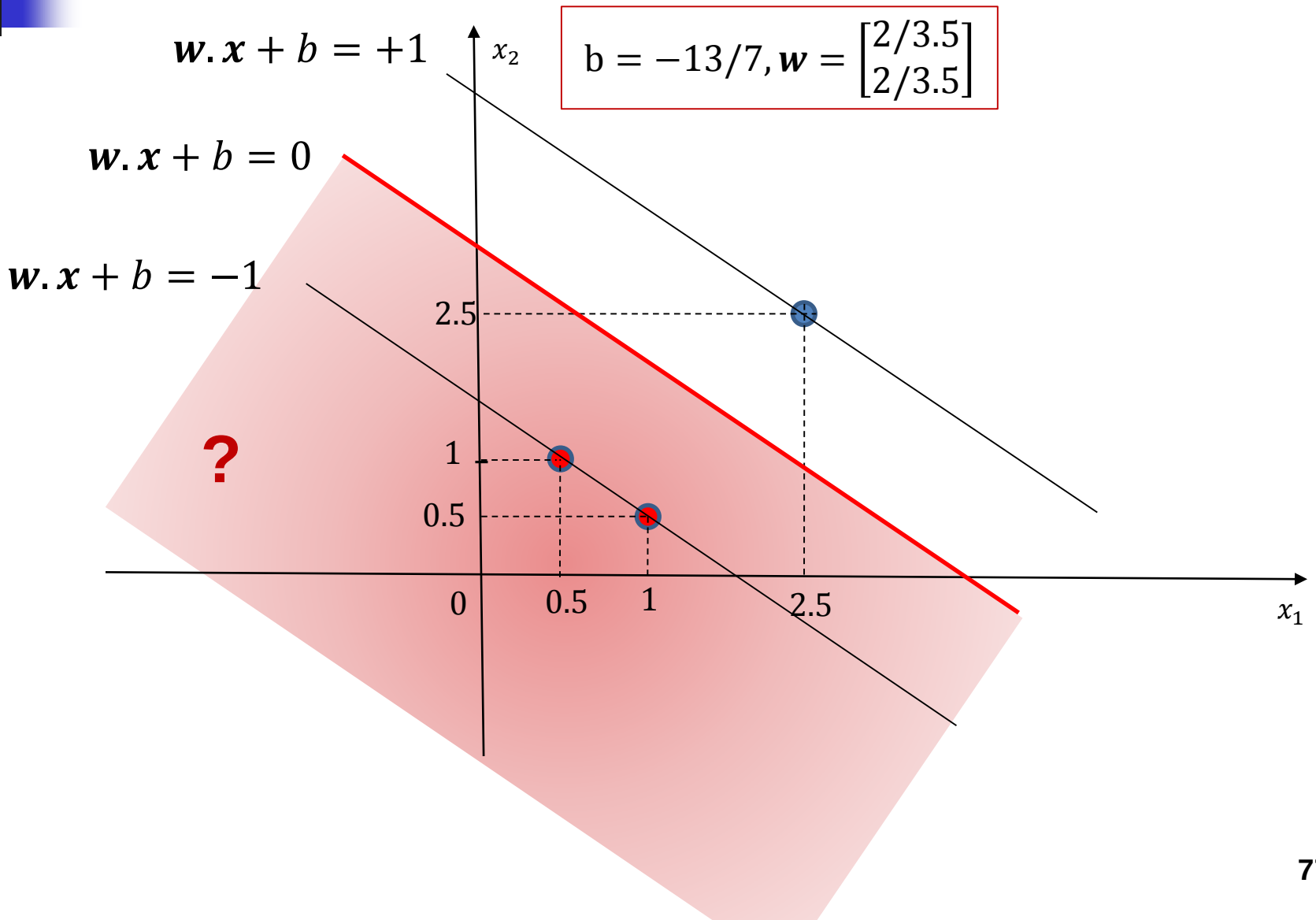
$\boldsymbol{w}.\boldsymbol{x} + b = -1$



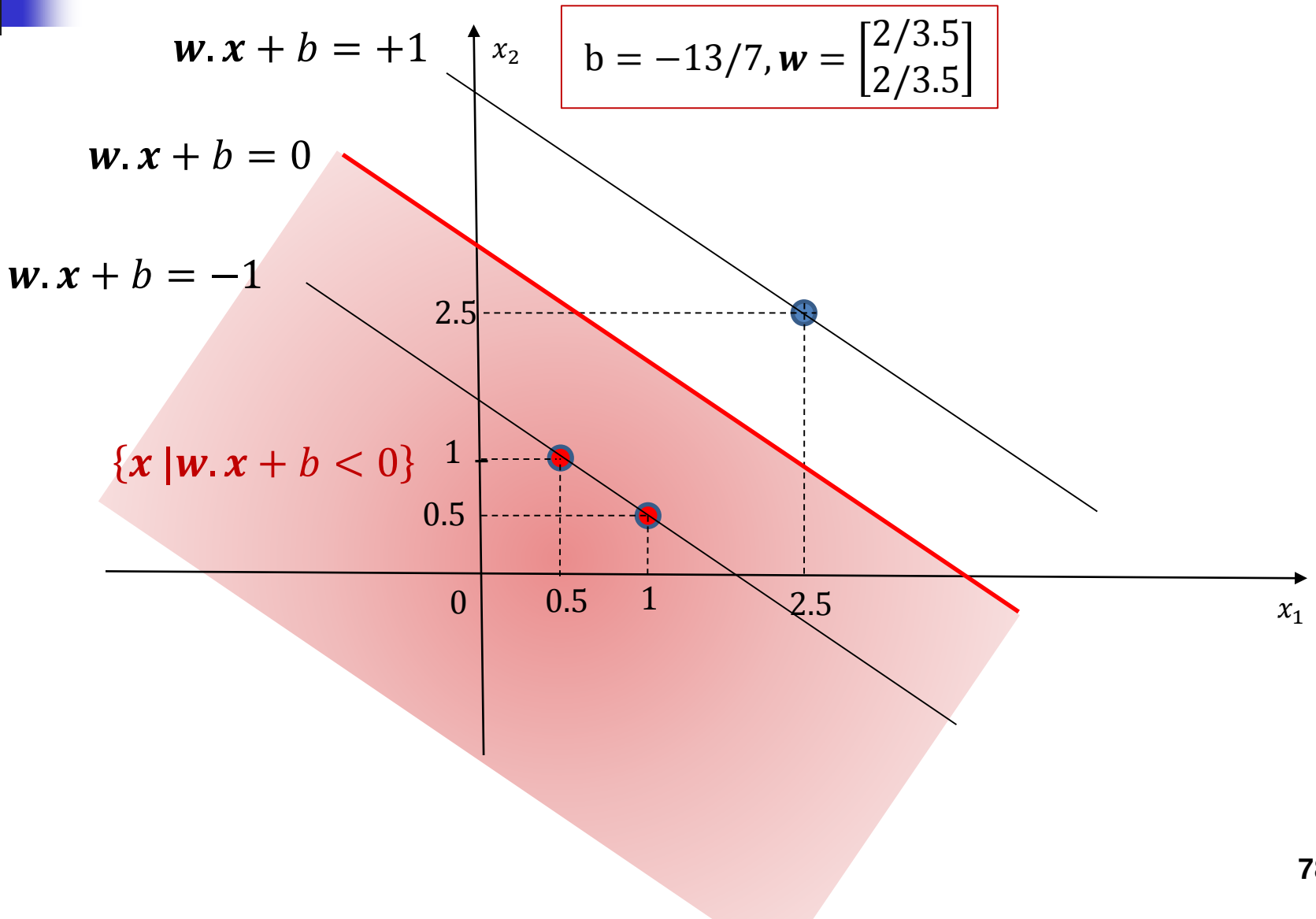
$$w_1 \cdot 0.5 + w_2 \cdot 1.0 + b = -1$$
$$w_1 \cdot 1.0 + w_2 \cdot 0.5 + b = -1$$
$$w_1 \cdot 2.5 + w_2 \cdot 2.5 + b = 1$$

$$\mathrm{b} = -13/7, \boldsymbol{w} = \begin{bmatrix} 2/3.5 \\ 2/3.5 \end{bmatrix}$$

Separating Line



Separating Line



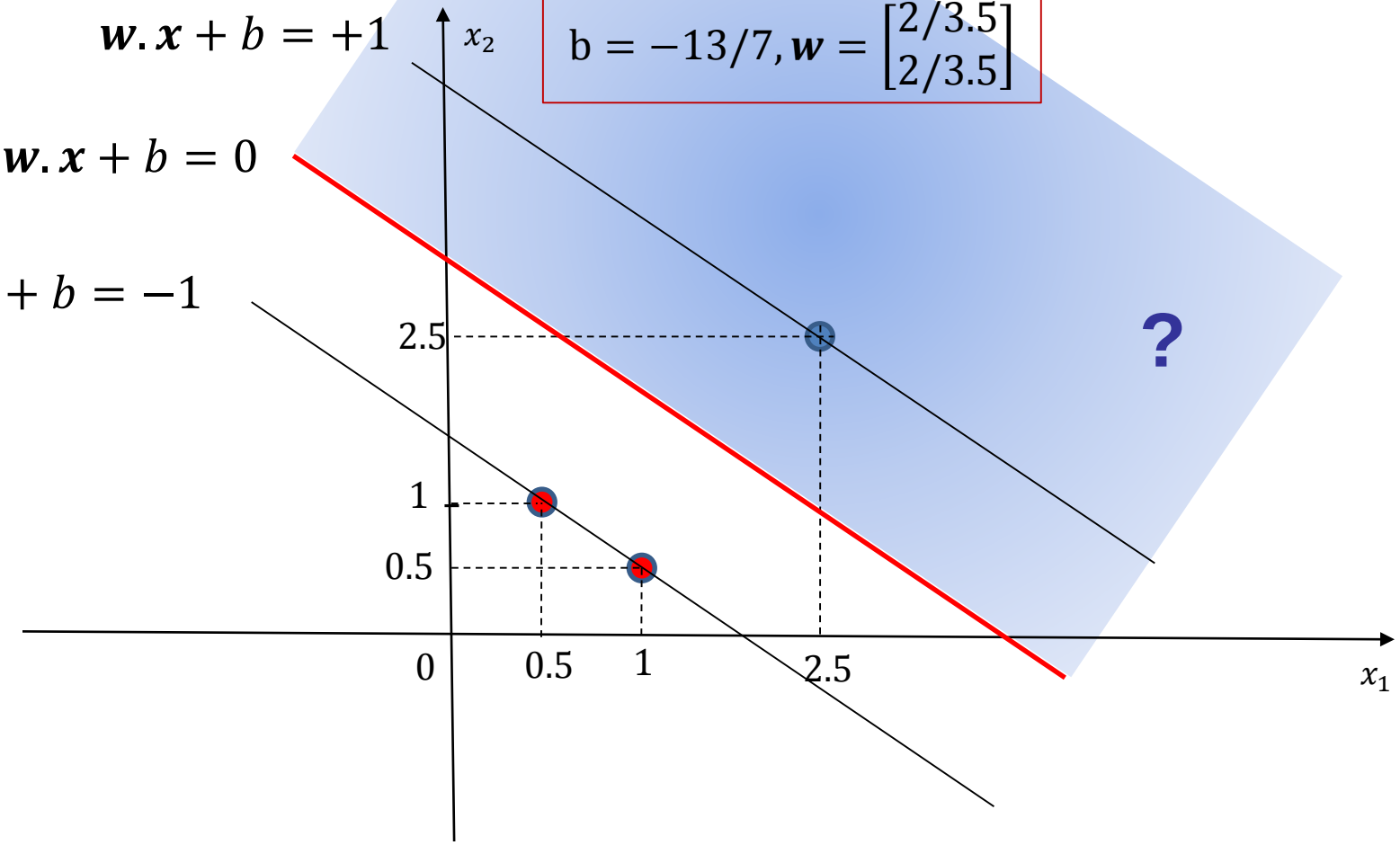
Separating Line

$\boldsymbol{w}.\boldsymbol{x} + b = +1$

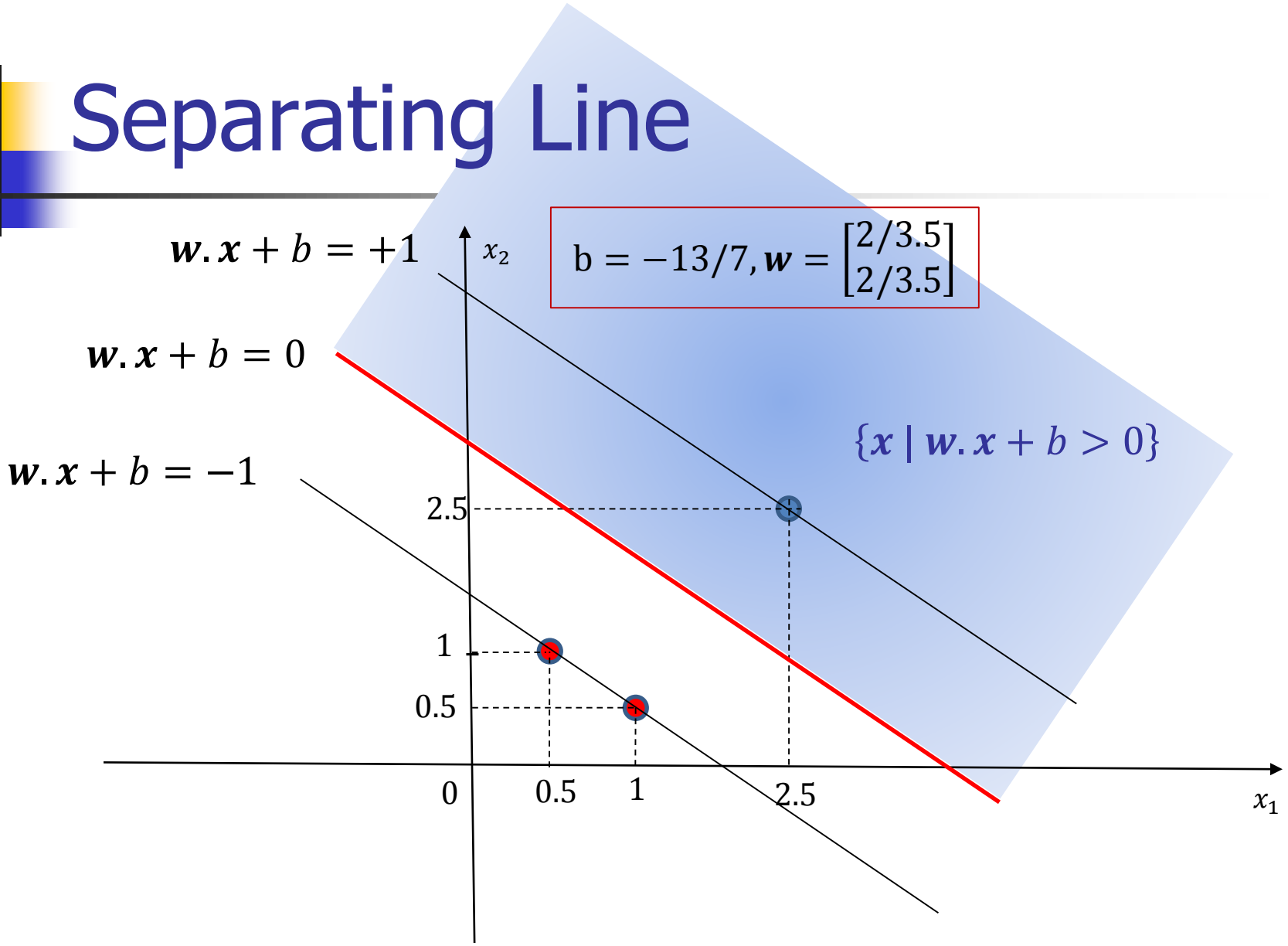
$\boldsymbol{w}.\boldsymbol{x} + b = 0$

$\boldsymbol{w}.\boldsymbol{x} + b = -1$

$\mathrm{b} = -13/7, \boldsymbol{w} = \begin{bmatrix} 2/3.5 \\ 2/3.5 \end{bmatrix}$

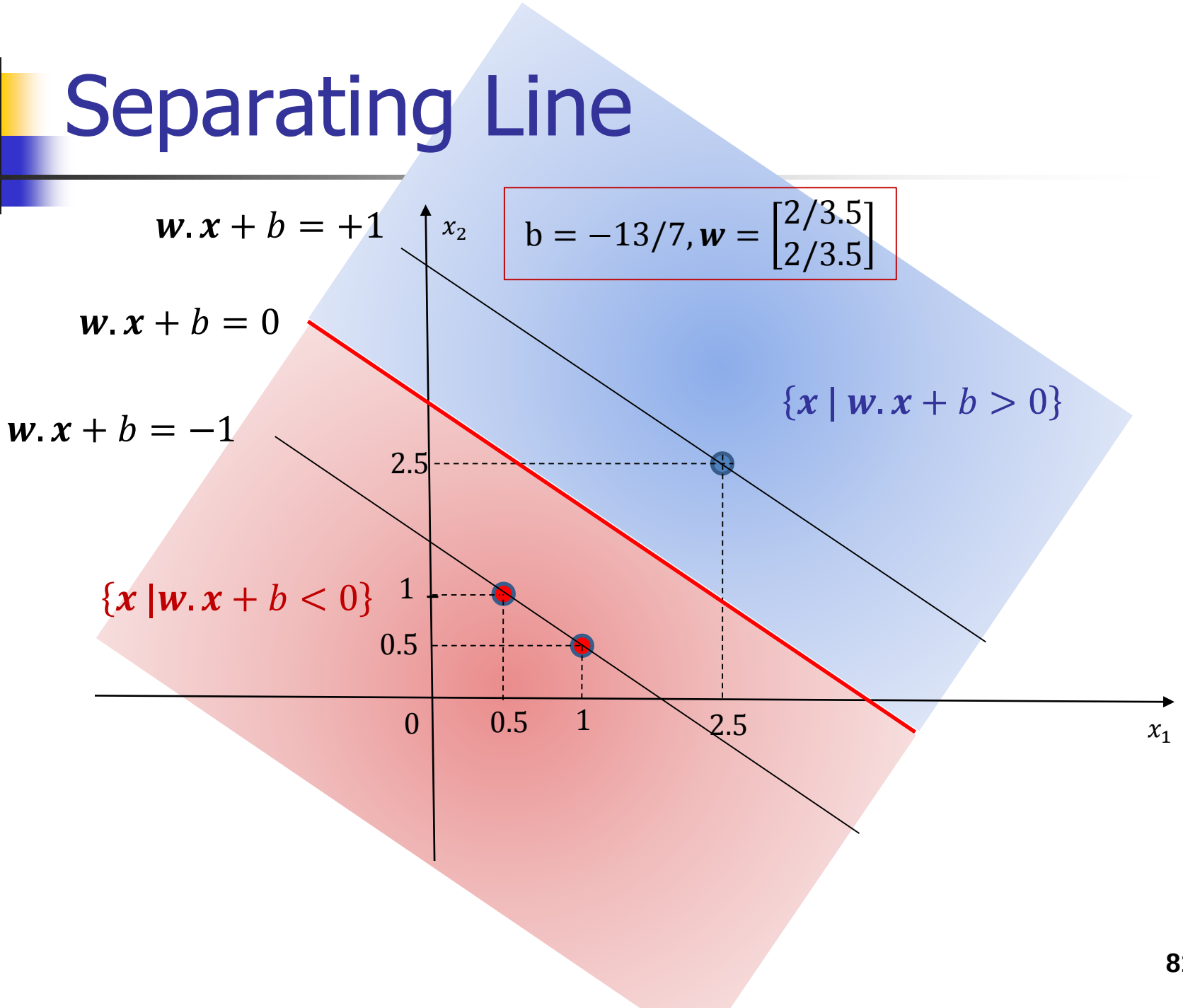


Separating Line

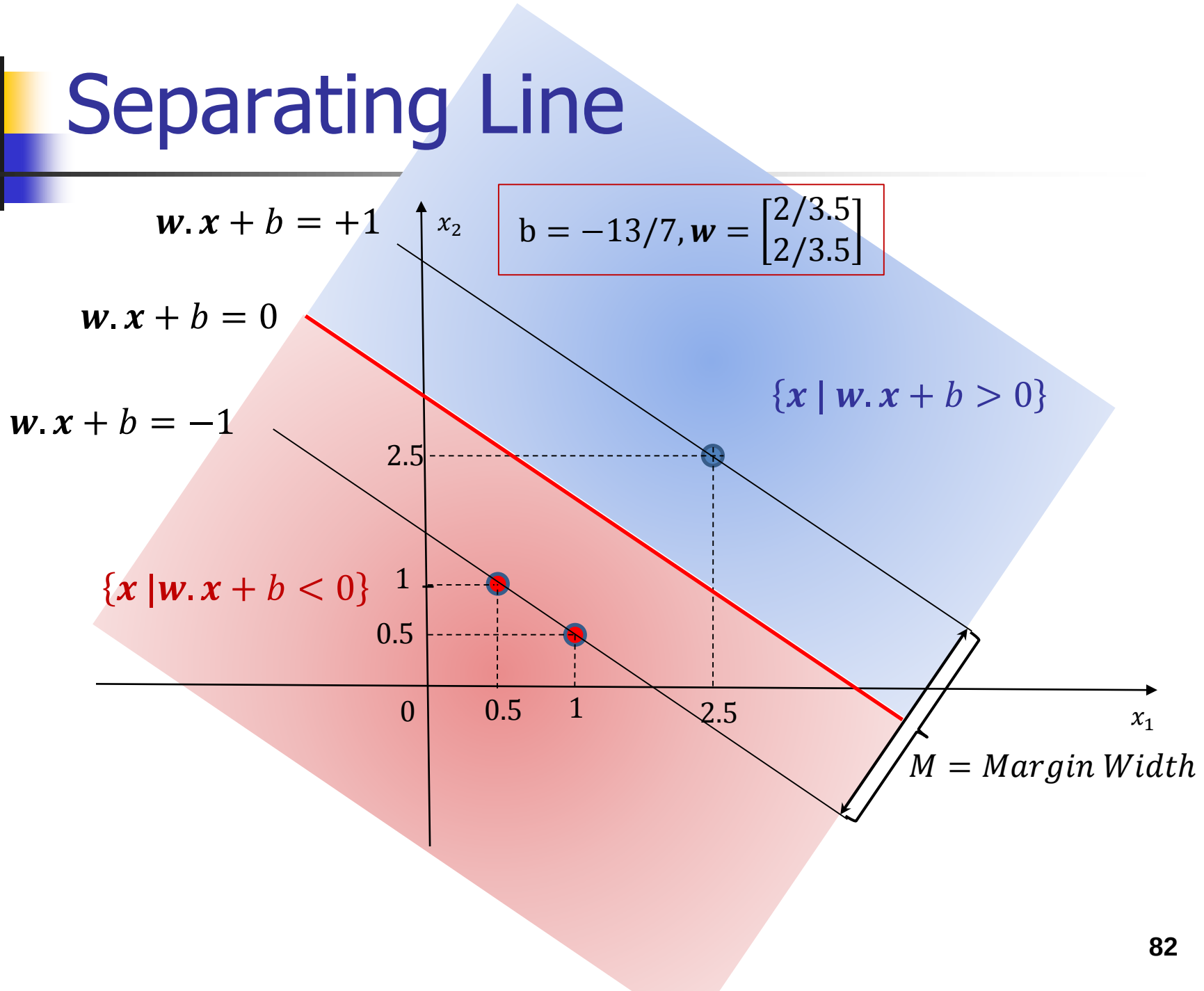


$$b = -13/7, w = \begin{bmatrix} 2/3.5 \\ 2/3.5 \end{bmatrix}$$

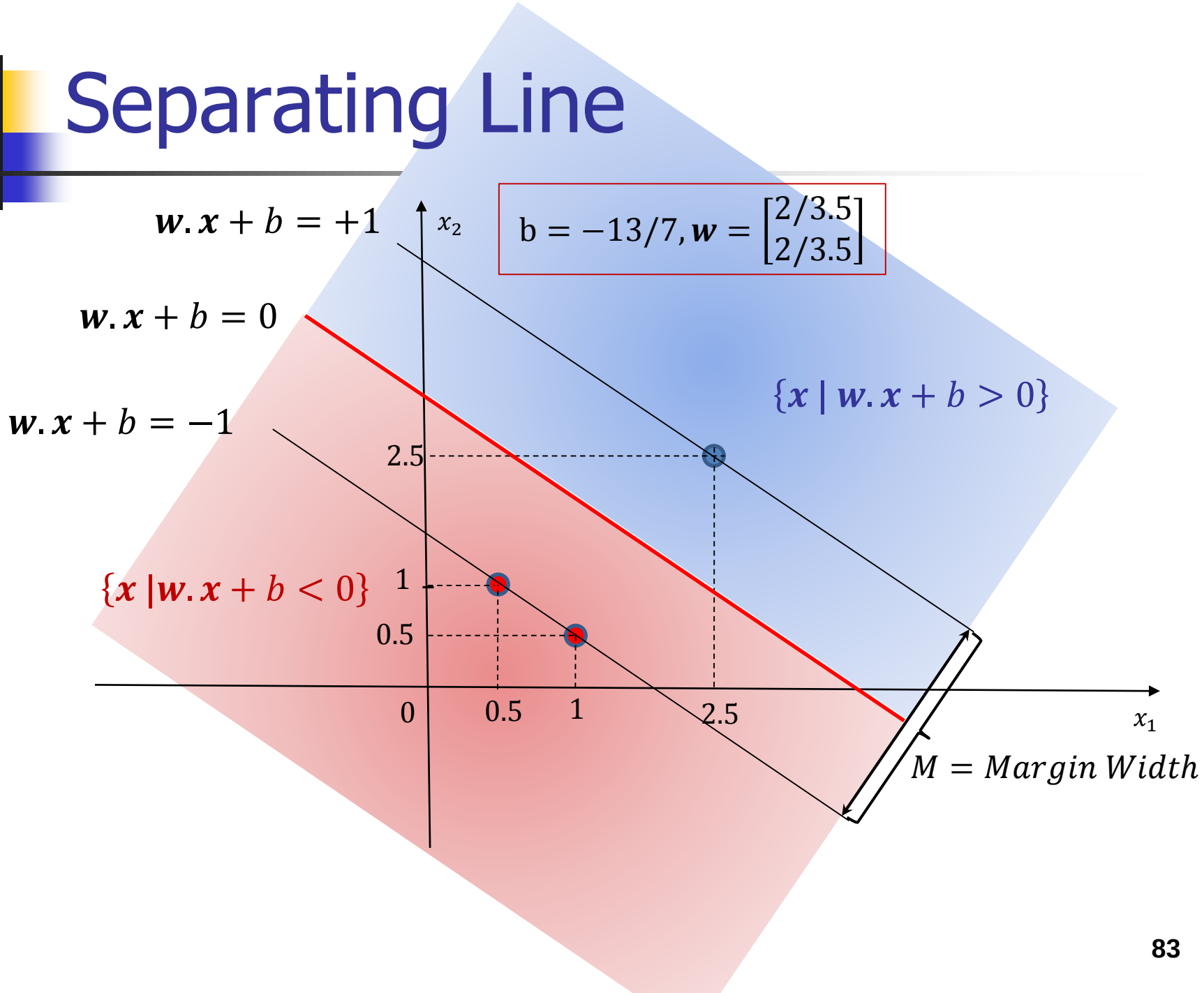
Separating Line



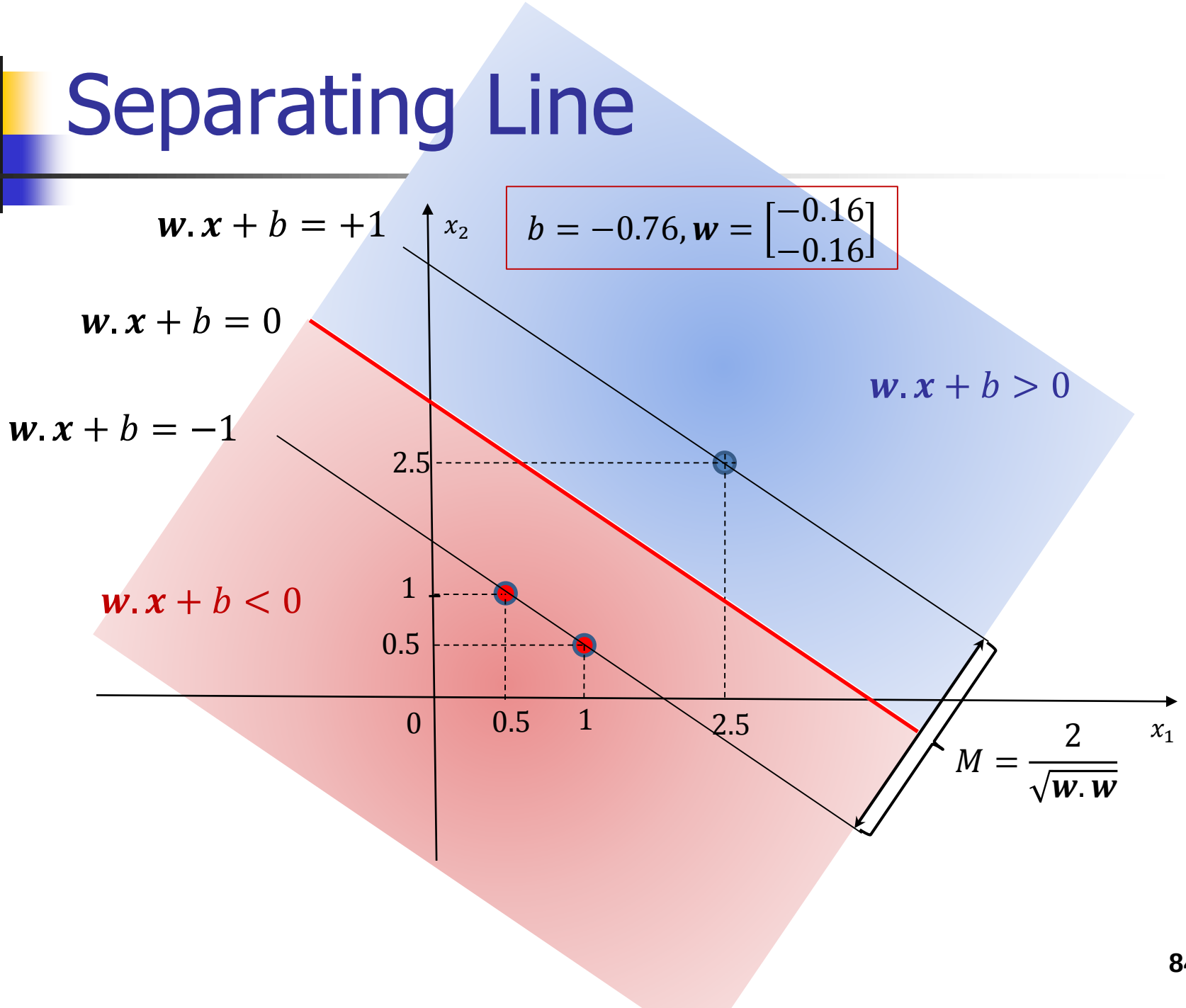
Separating Line



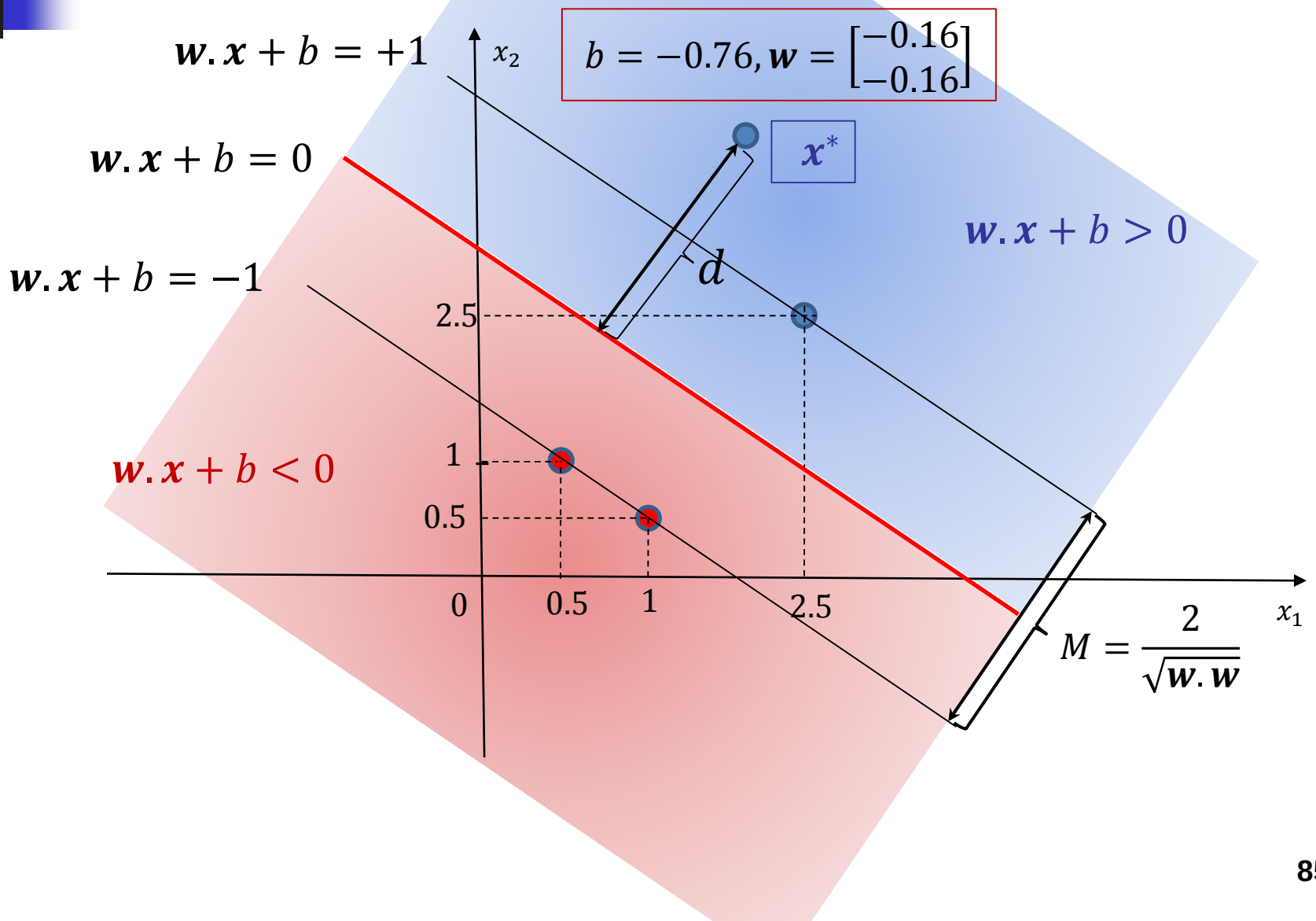
Separating Line



Separating Line

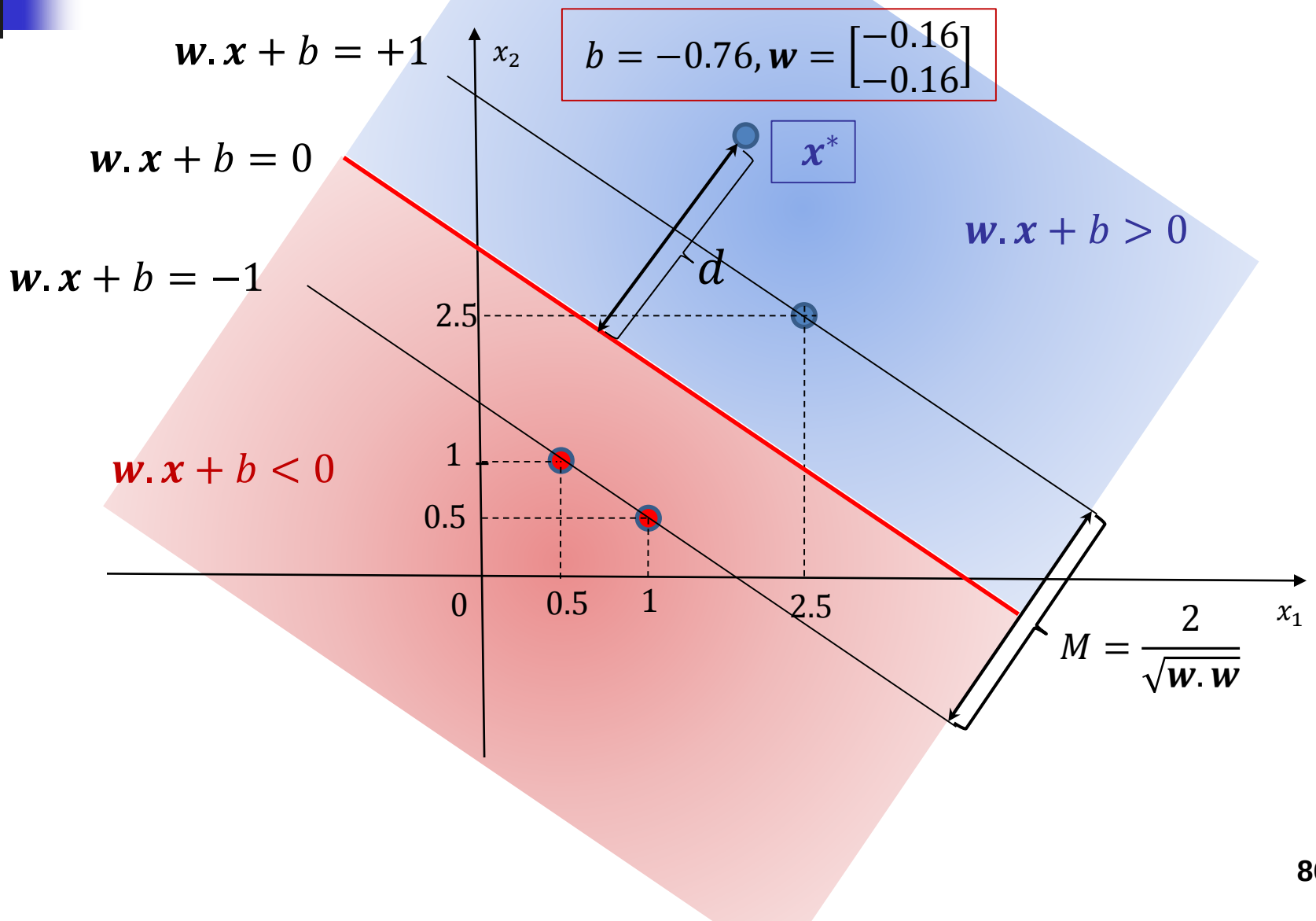


Separating Line



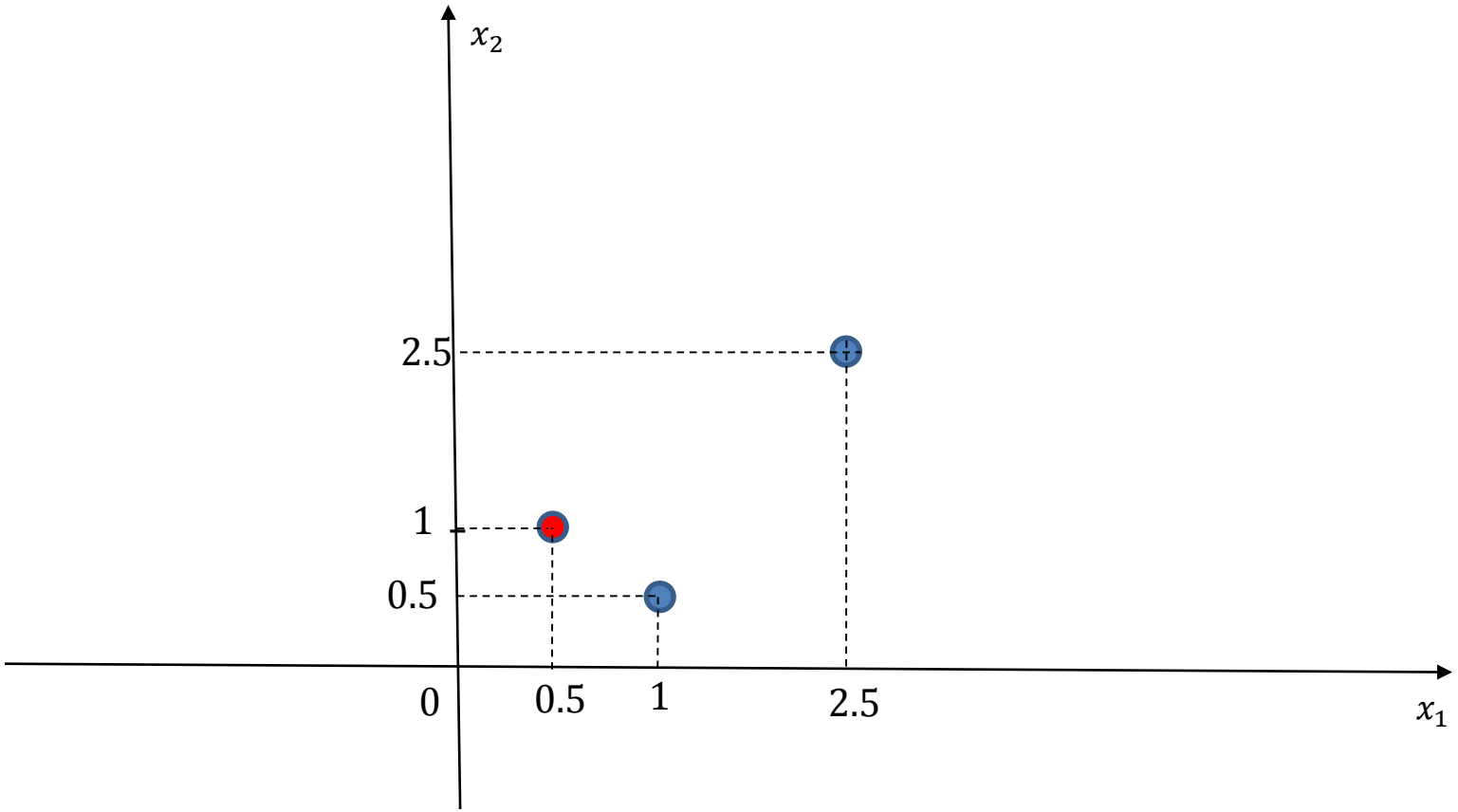
Separating Line

$$d = \frac{|\boldsymbol{w}.\boldsymbol{x}^* + b|}{\sqrt{\boldsymbol{w}.\boldsymbol{w}}}$$



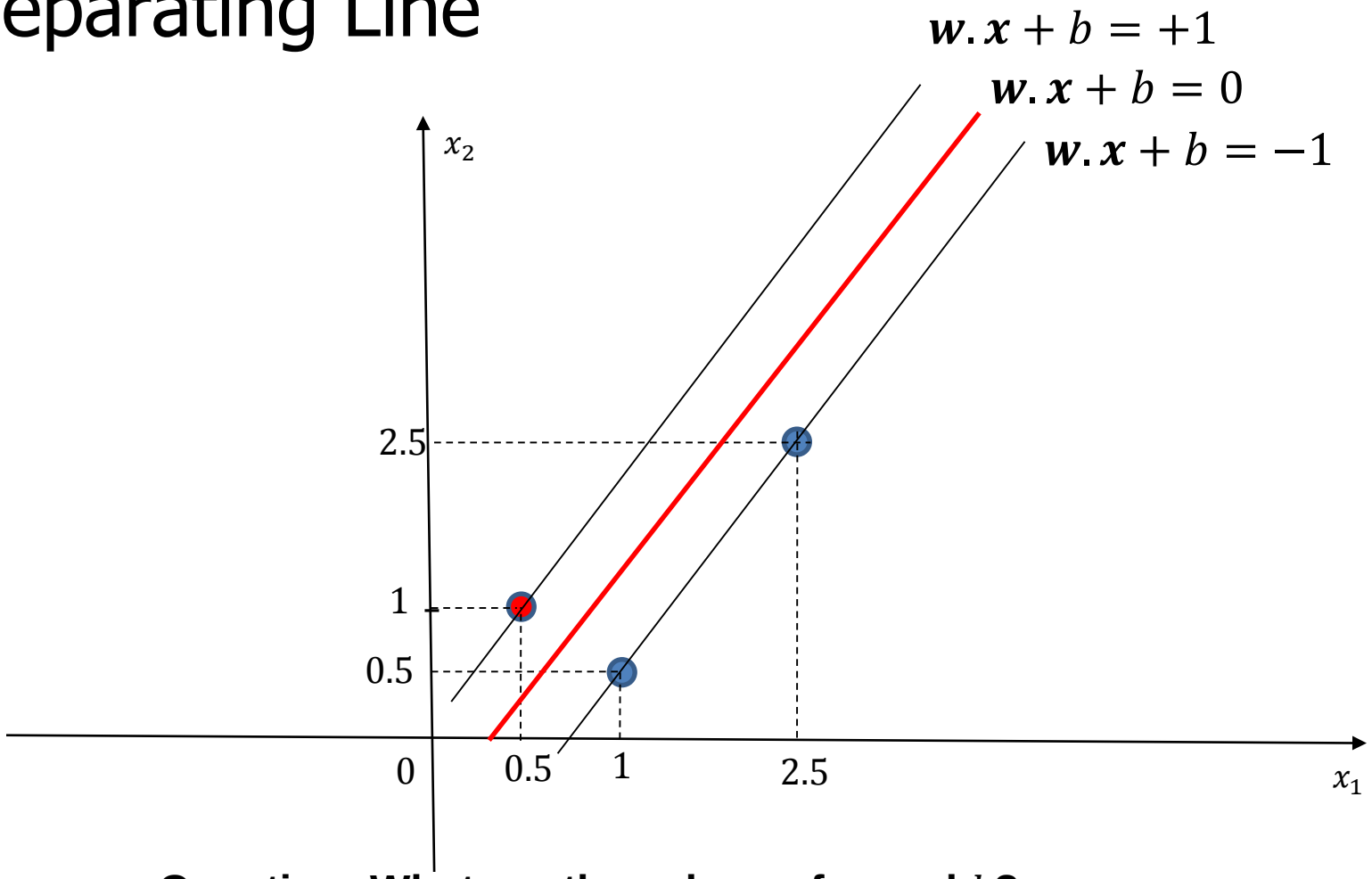
Separating Line

- Separating Line



Separating Line

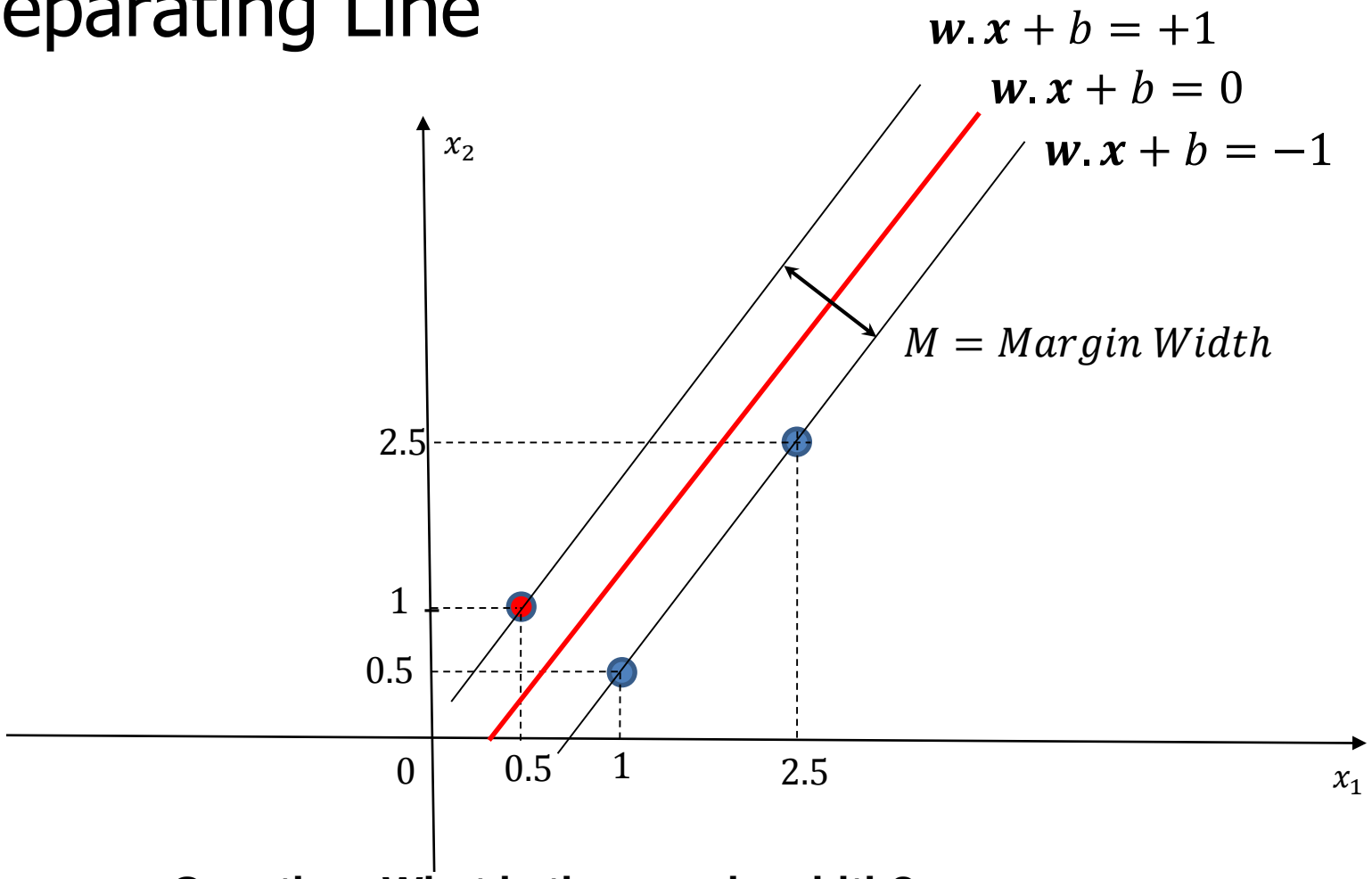
- Separating Line



Question: What are the values of w and b?

Separating Line

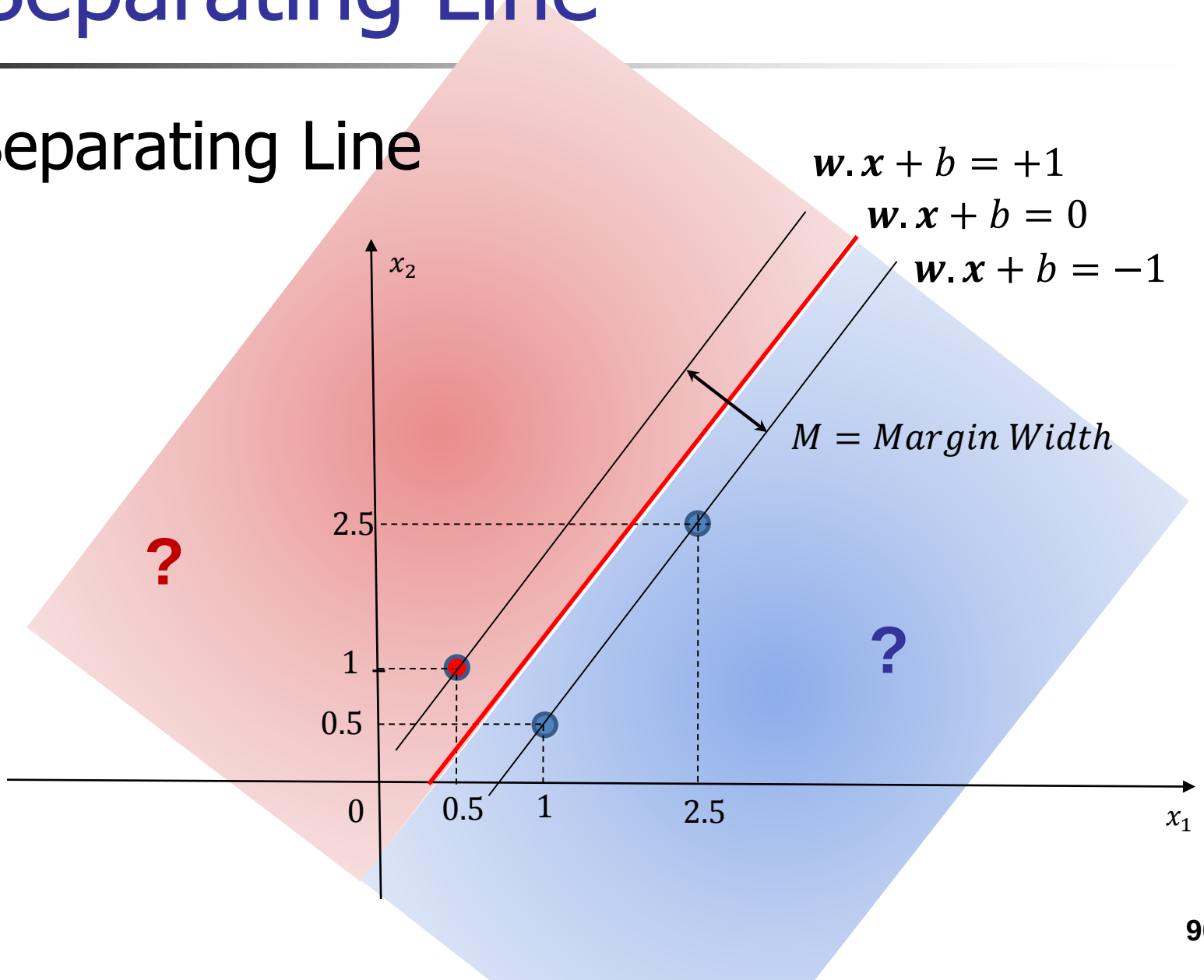
- Separating Line



Question: What is the margin width?

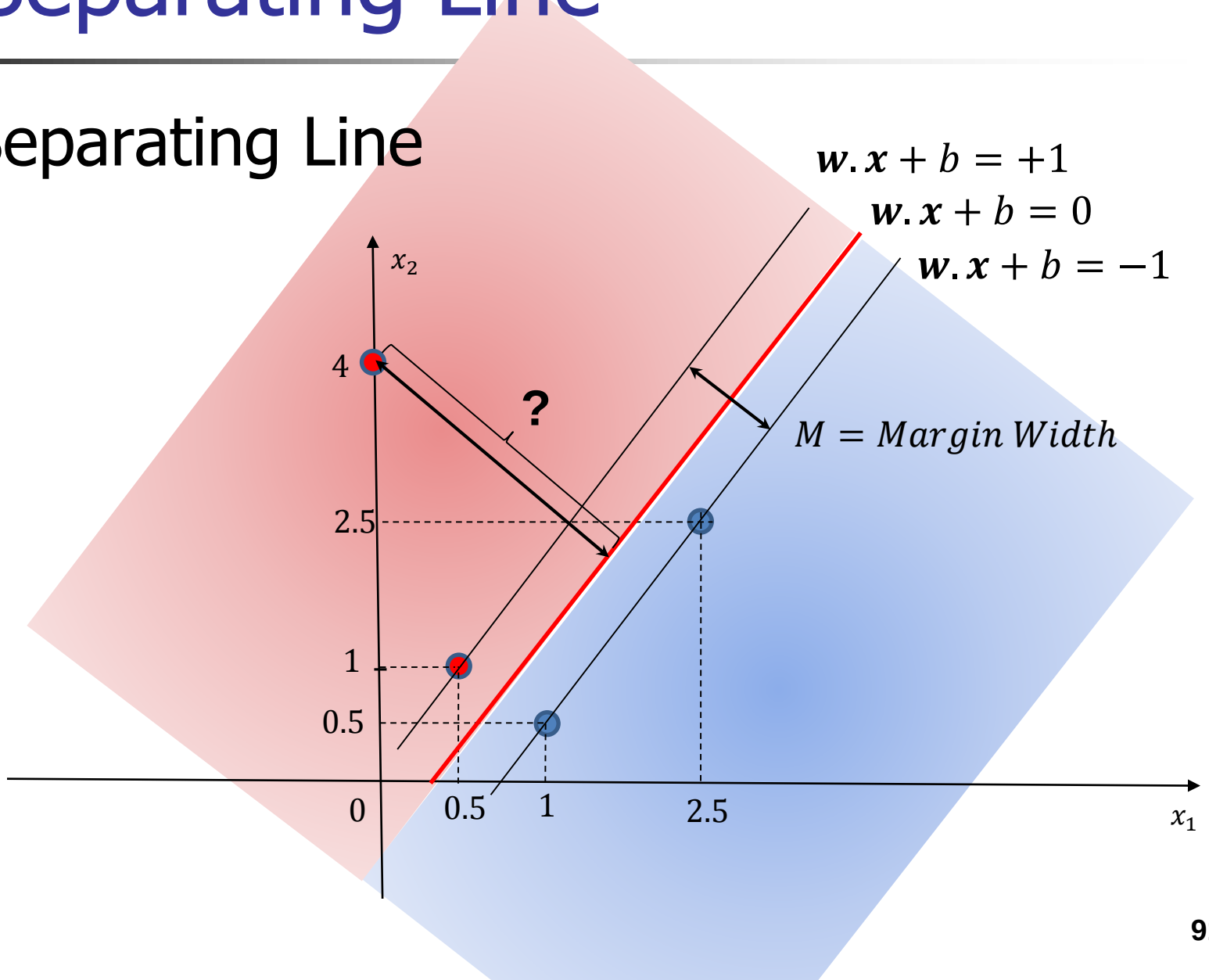
Separating Line

- Separating Line



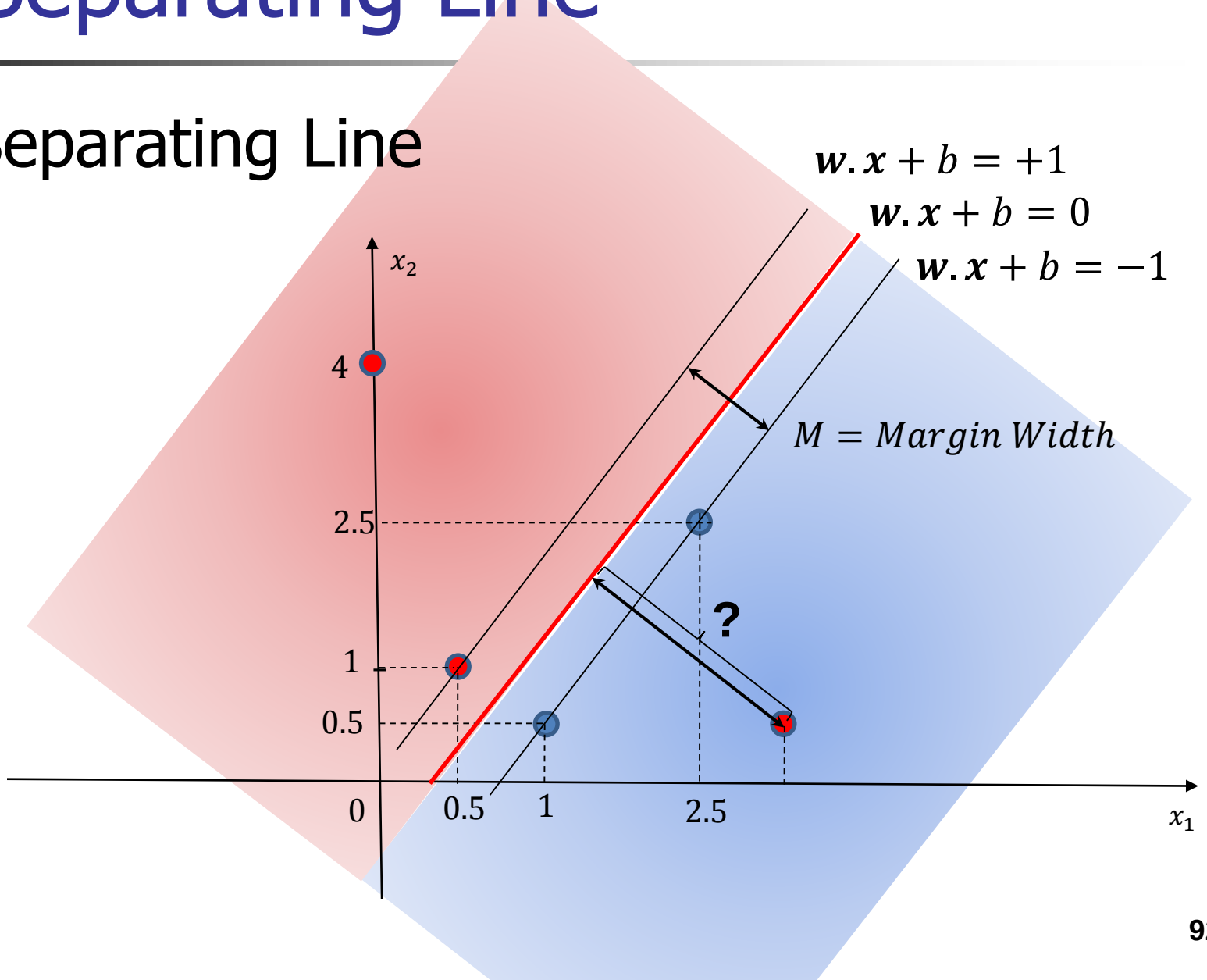
Separating Line

- Separating Line



Separating Line

- Separating Line



TO BE CONTINUED