



ICSI431/ICSI531 Data Mining

Lecture 4-C

Classification

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<http://www.cs.albany.edu/~fchen/course/2016-ICSI-431-531>

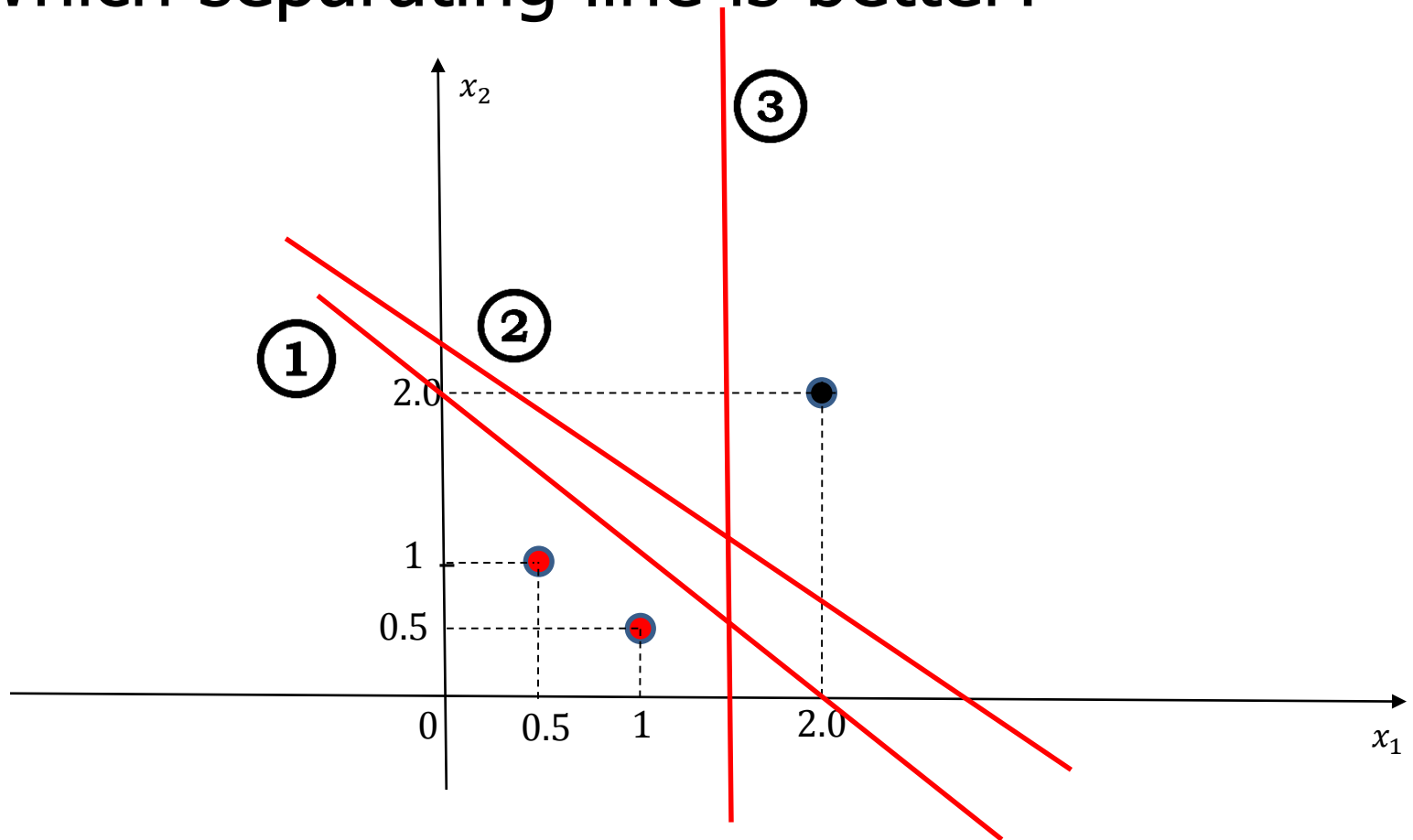


Outline

- Intuition Behind SVM
 - Separating Lines Identified by SVM
 - Support Vectors of SVM
- In-Class Exercise
- Linear Support Vector Machine
 - Separable Case
 - Non-Separable Case

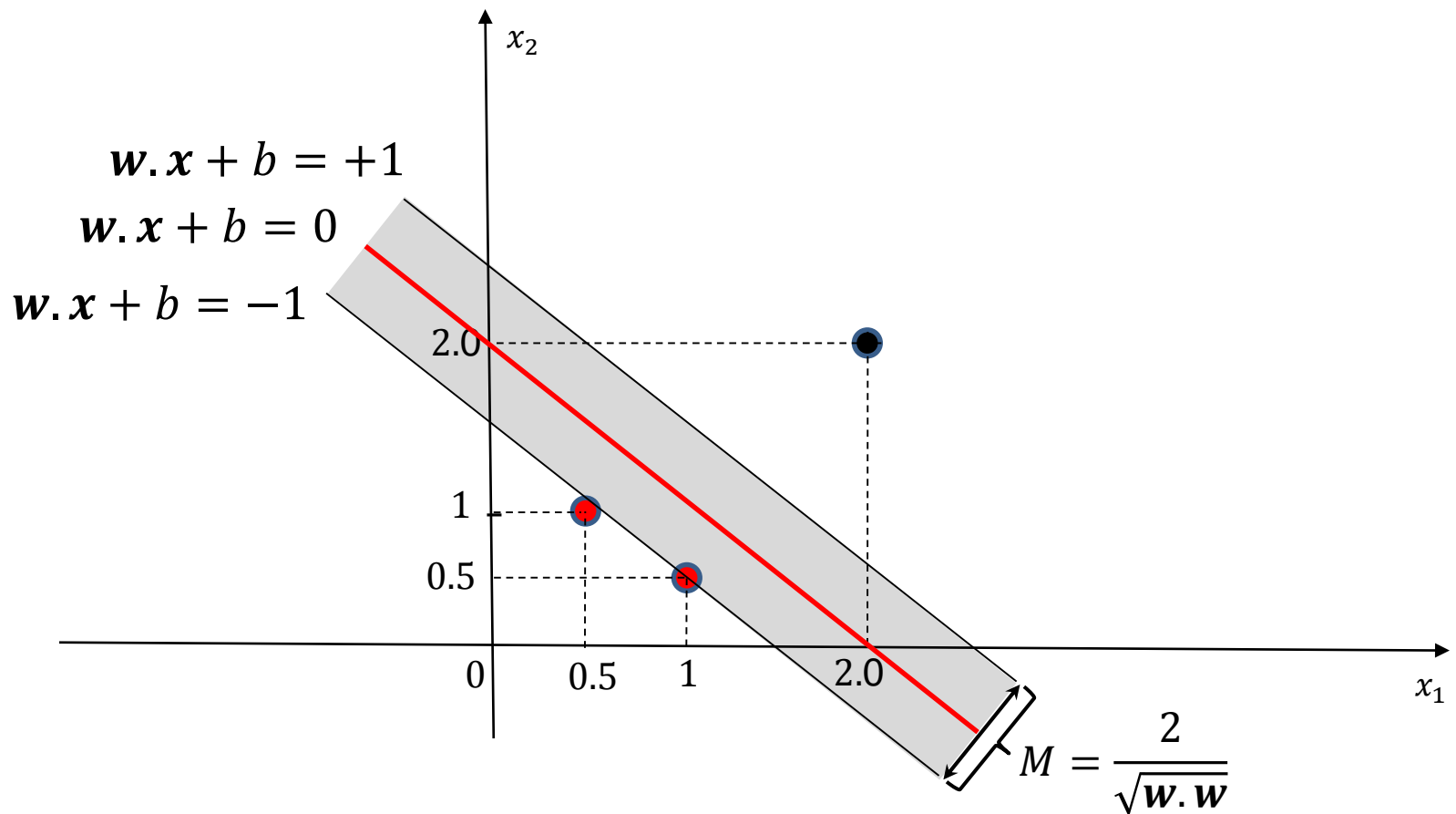
Intuition behind SVM

- Which separating line is better?



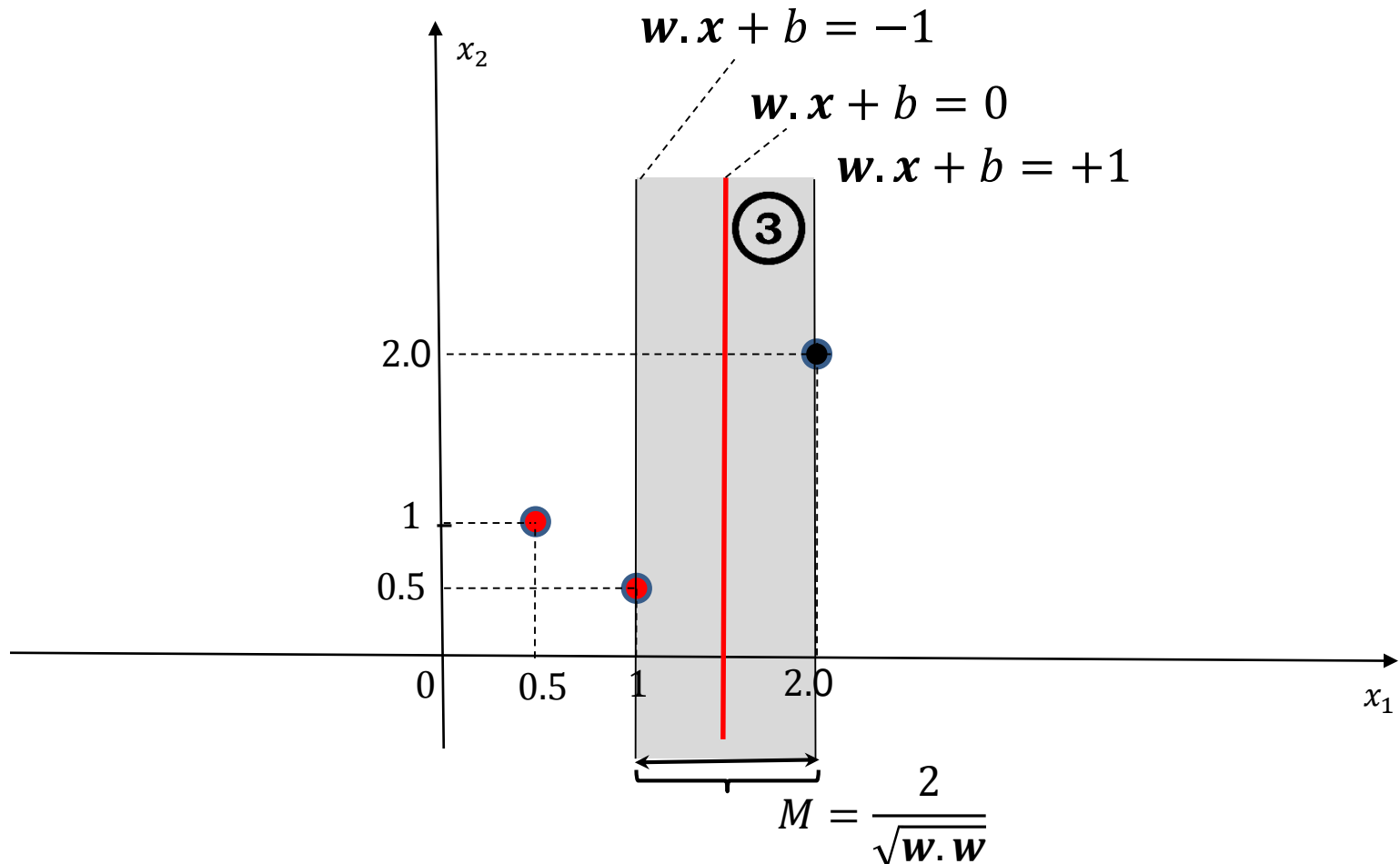
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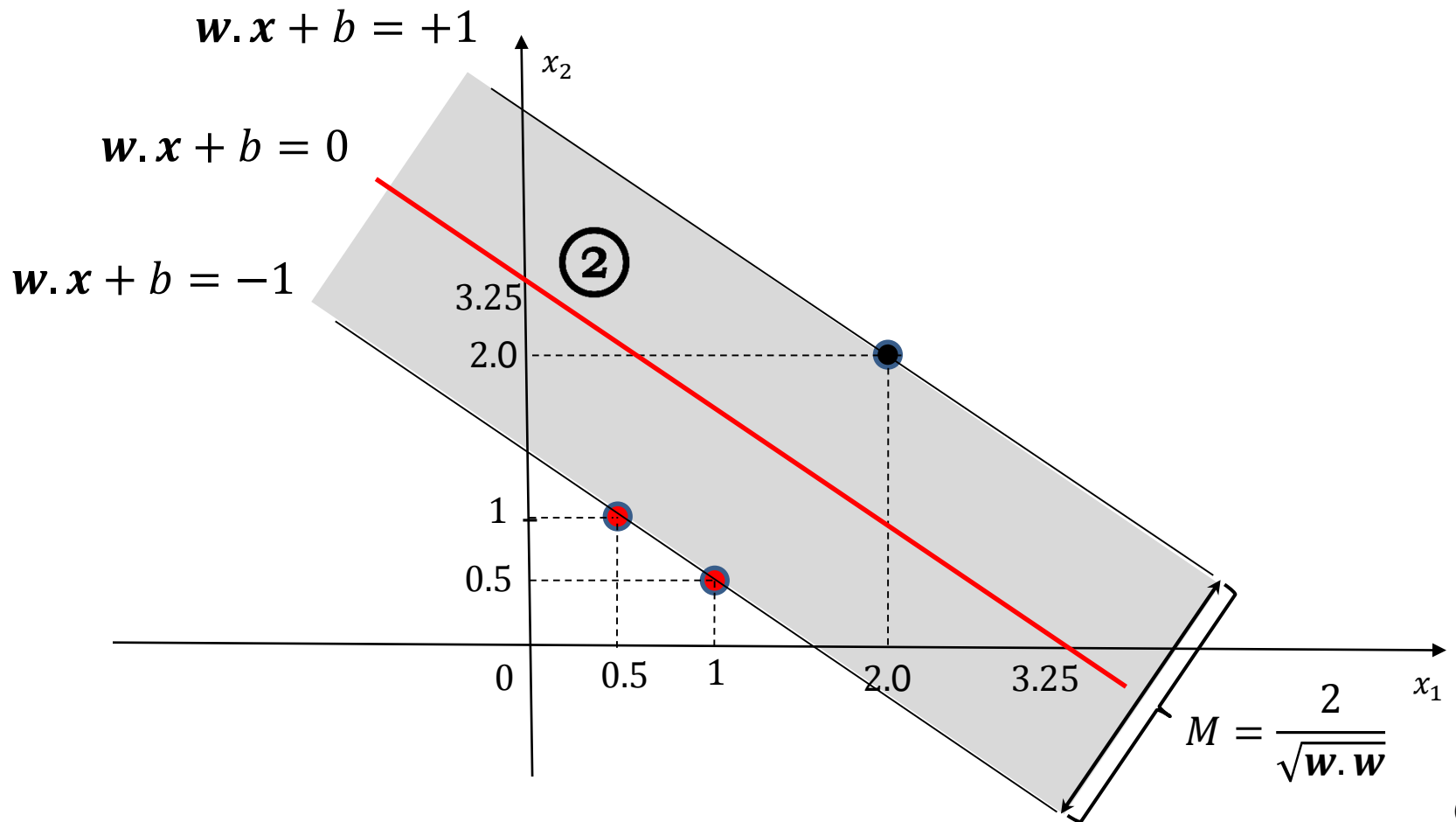
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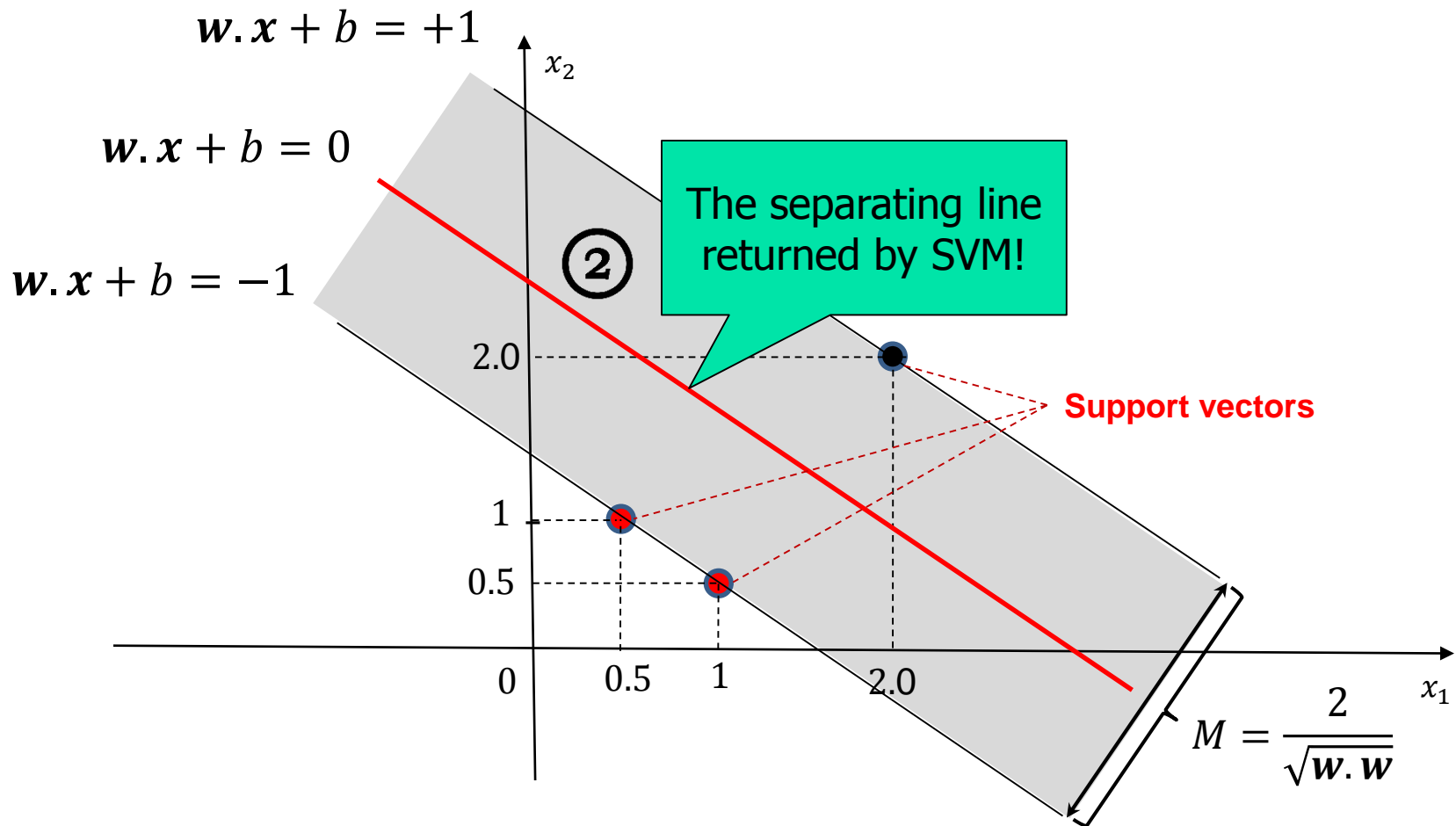
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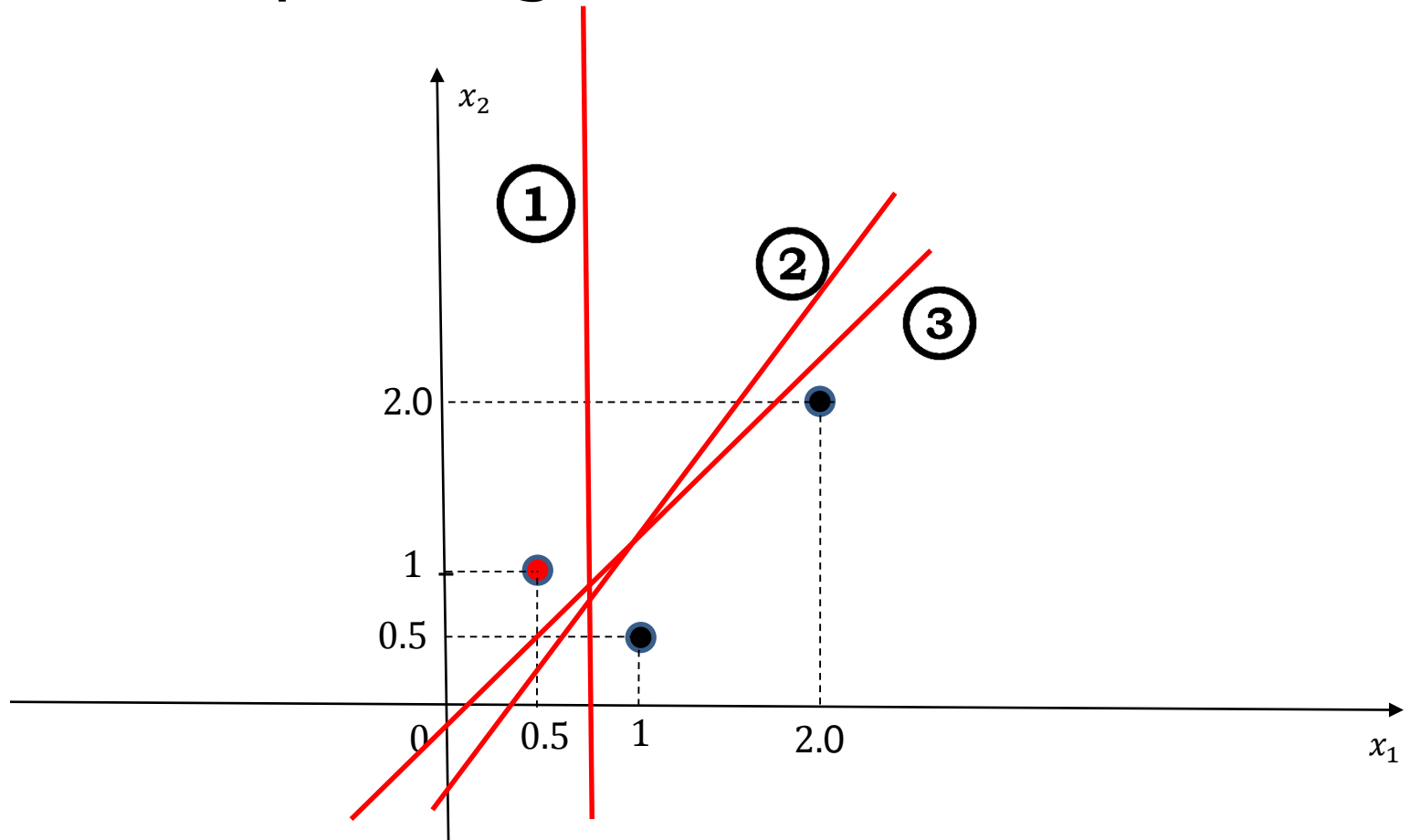
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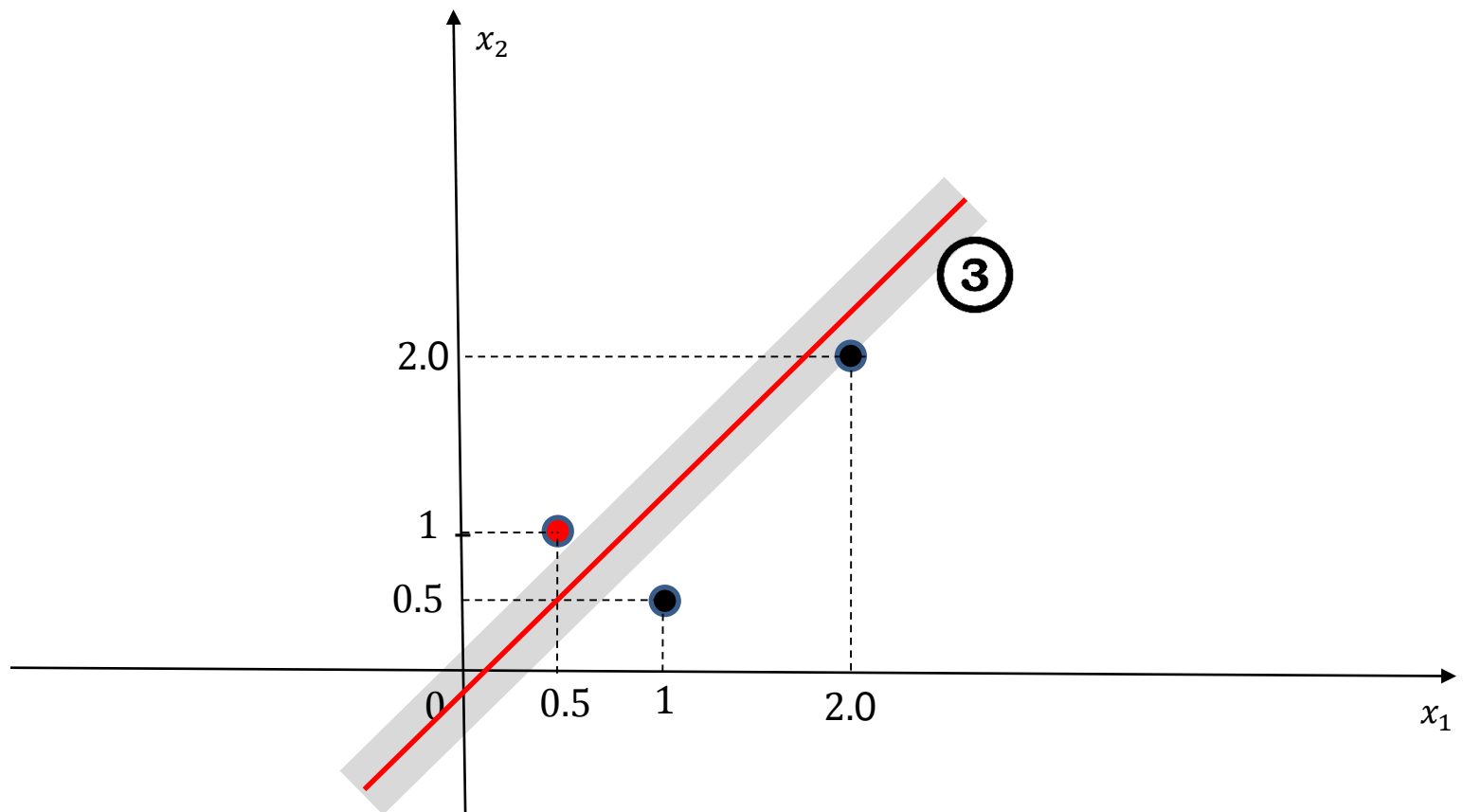
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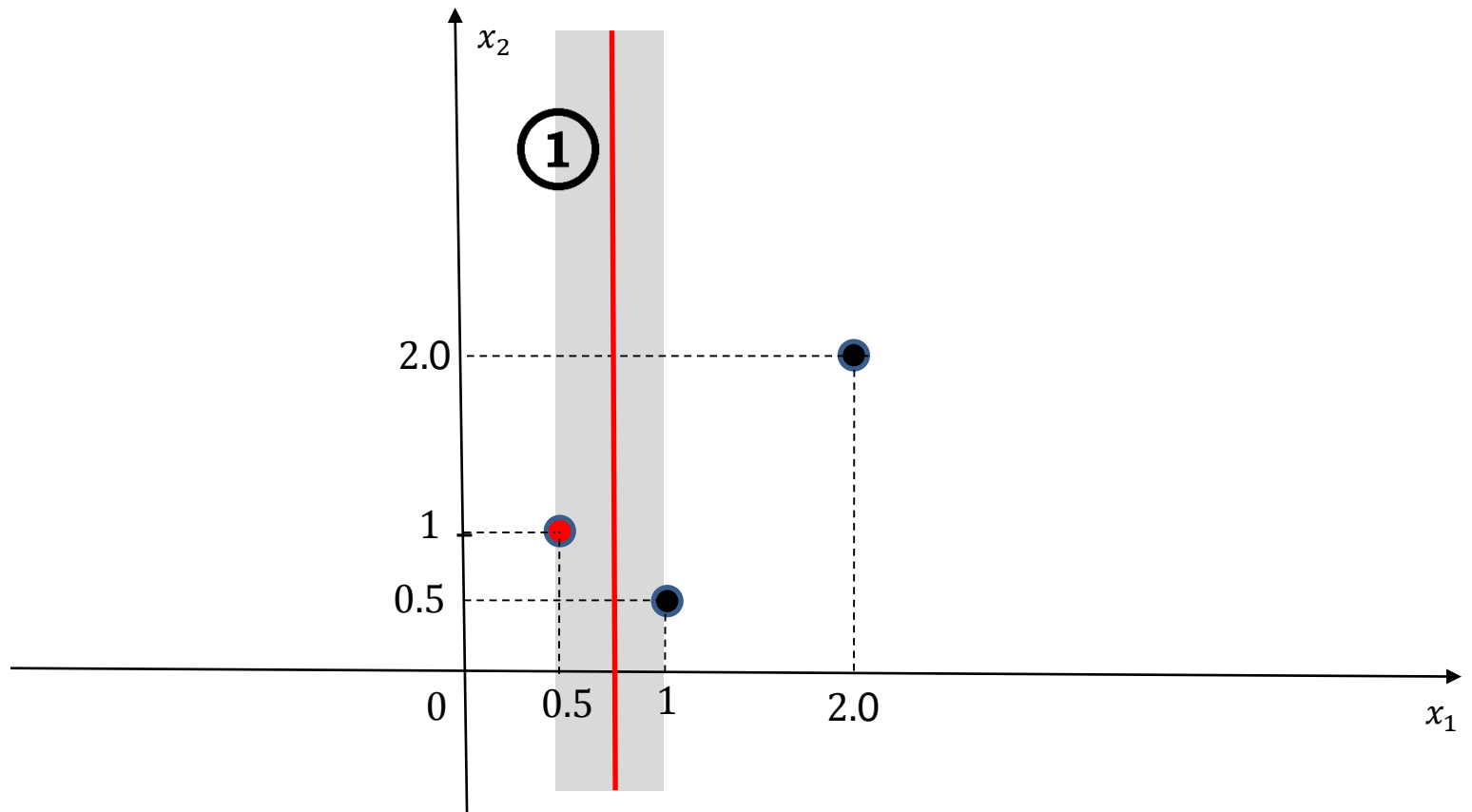
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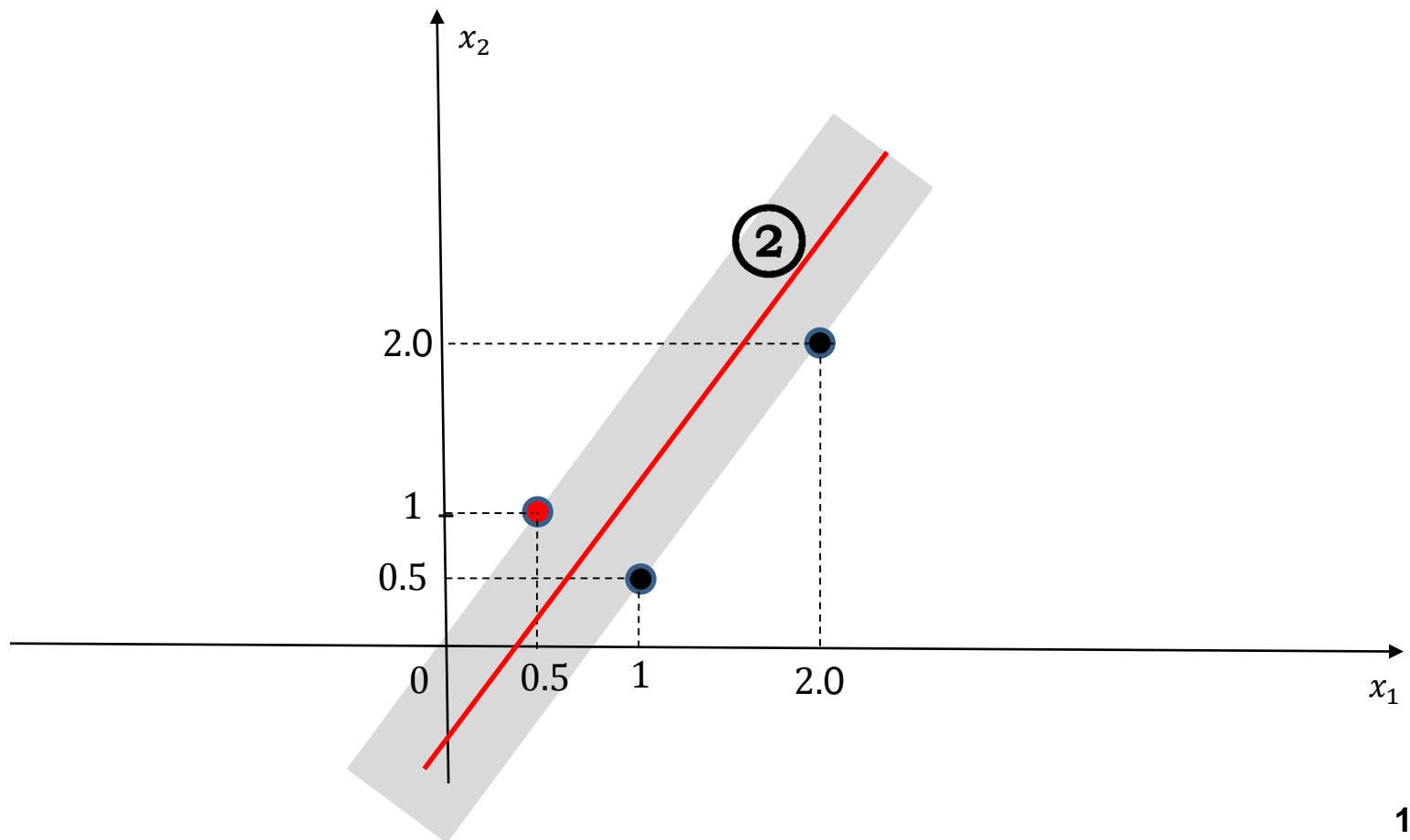
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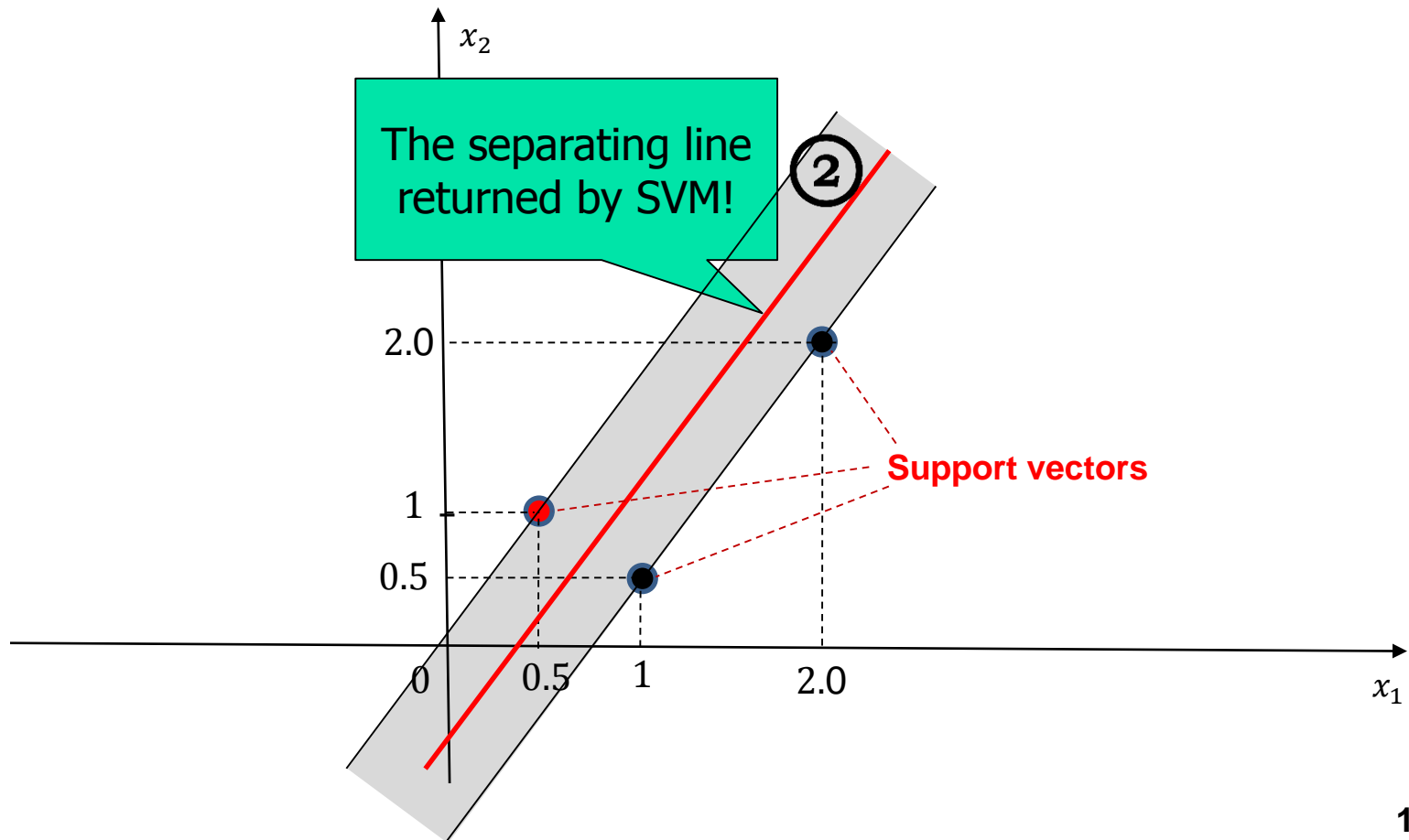
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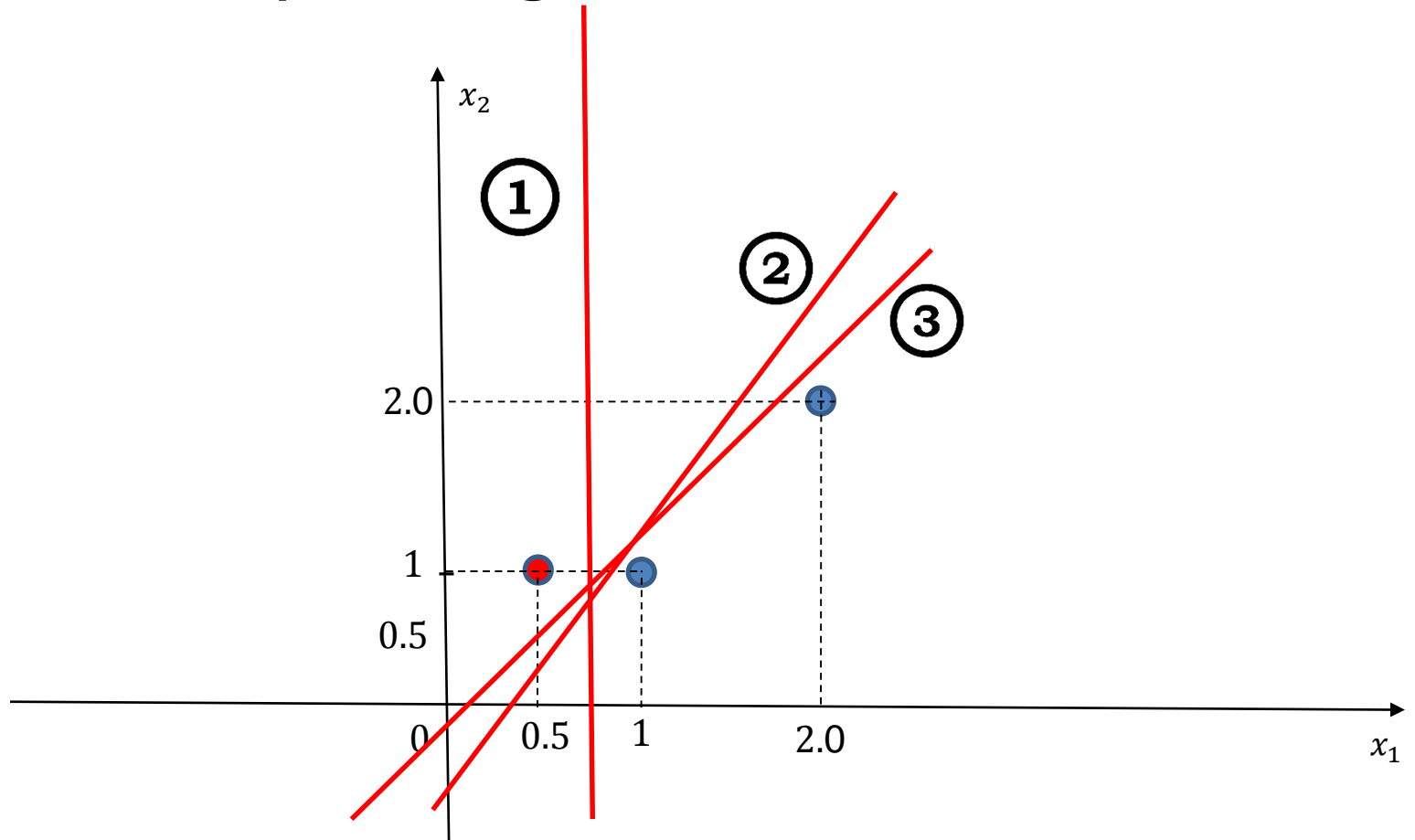
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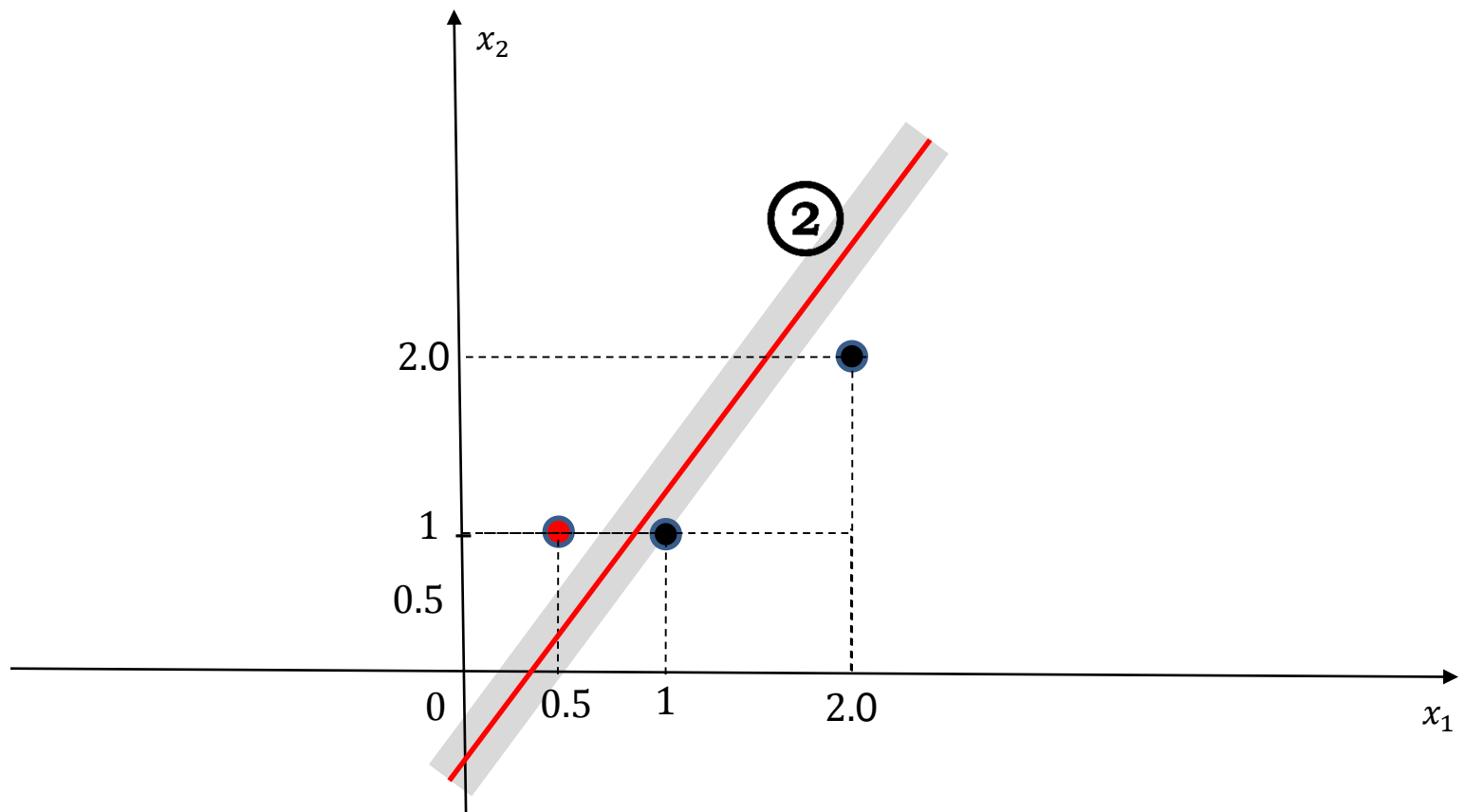
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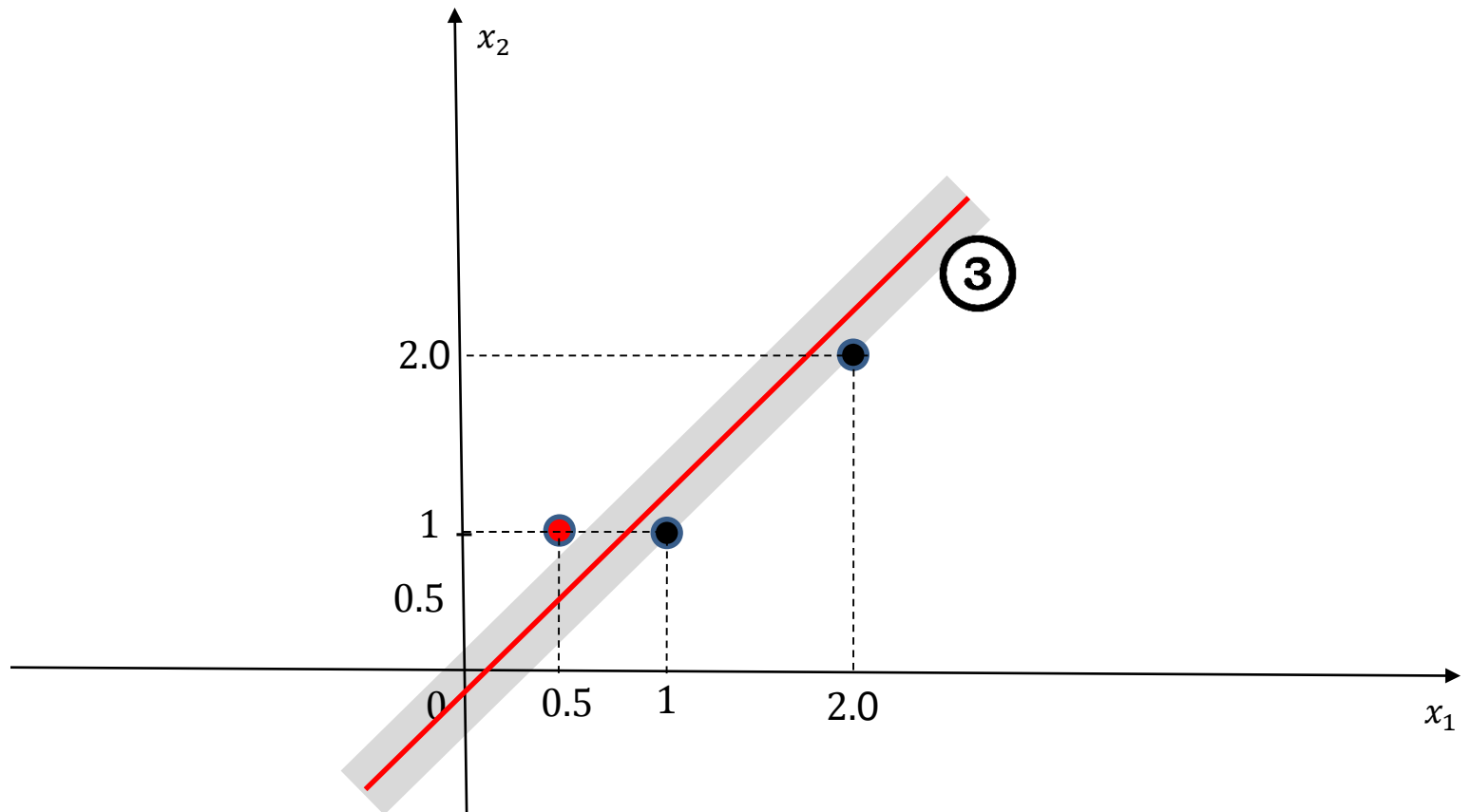
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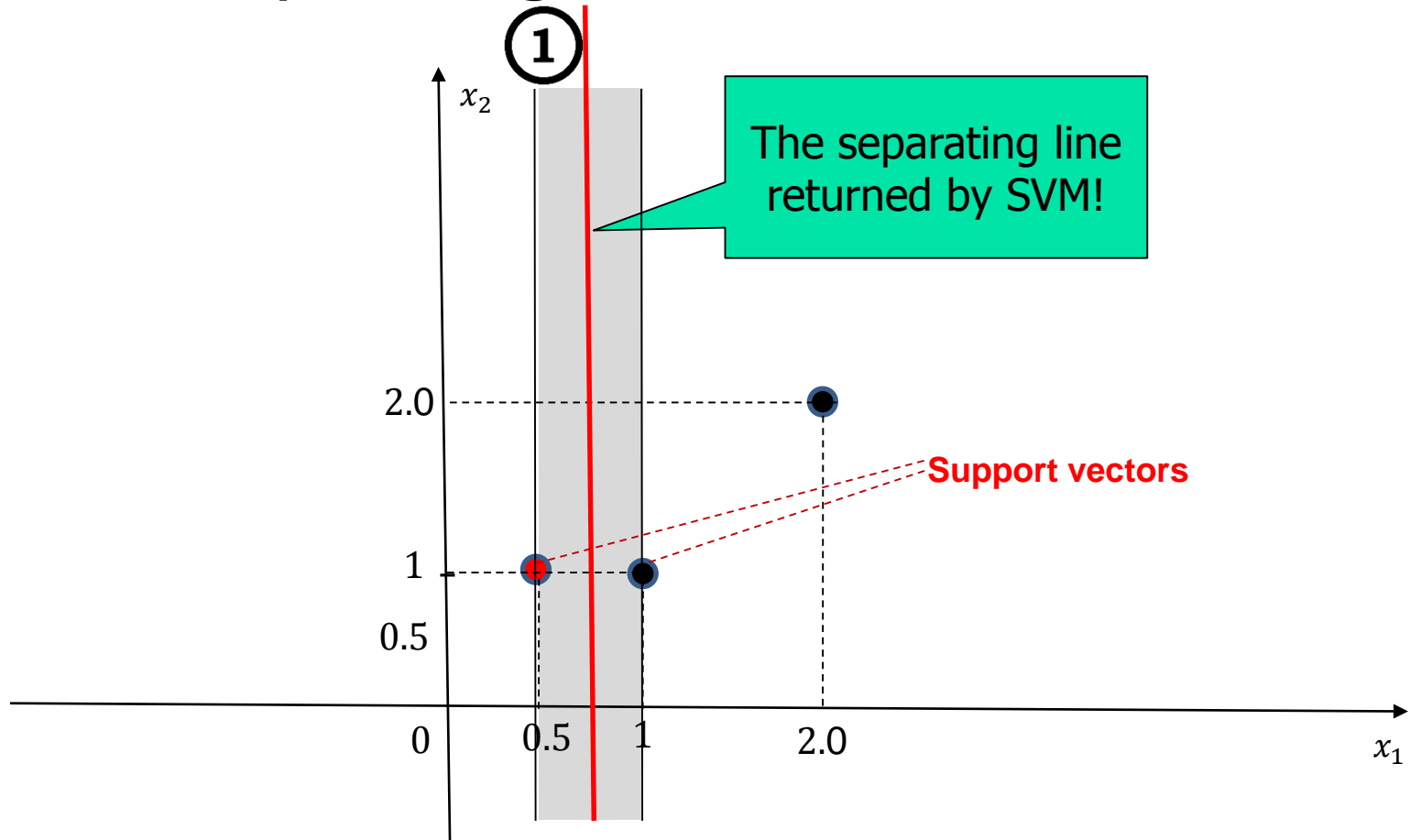
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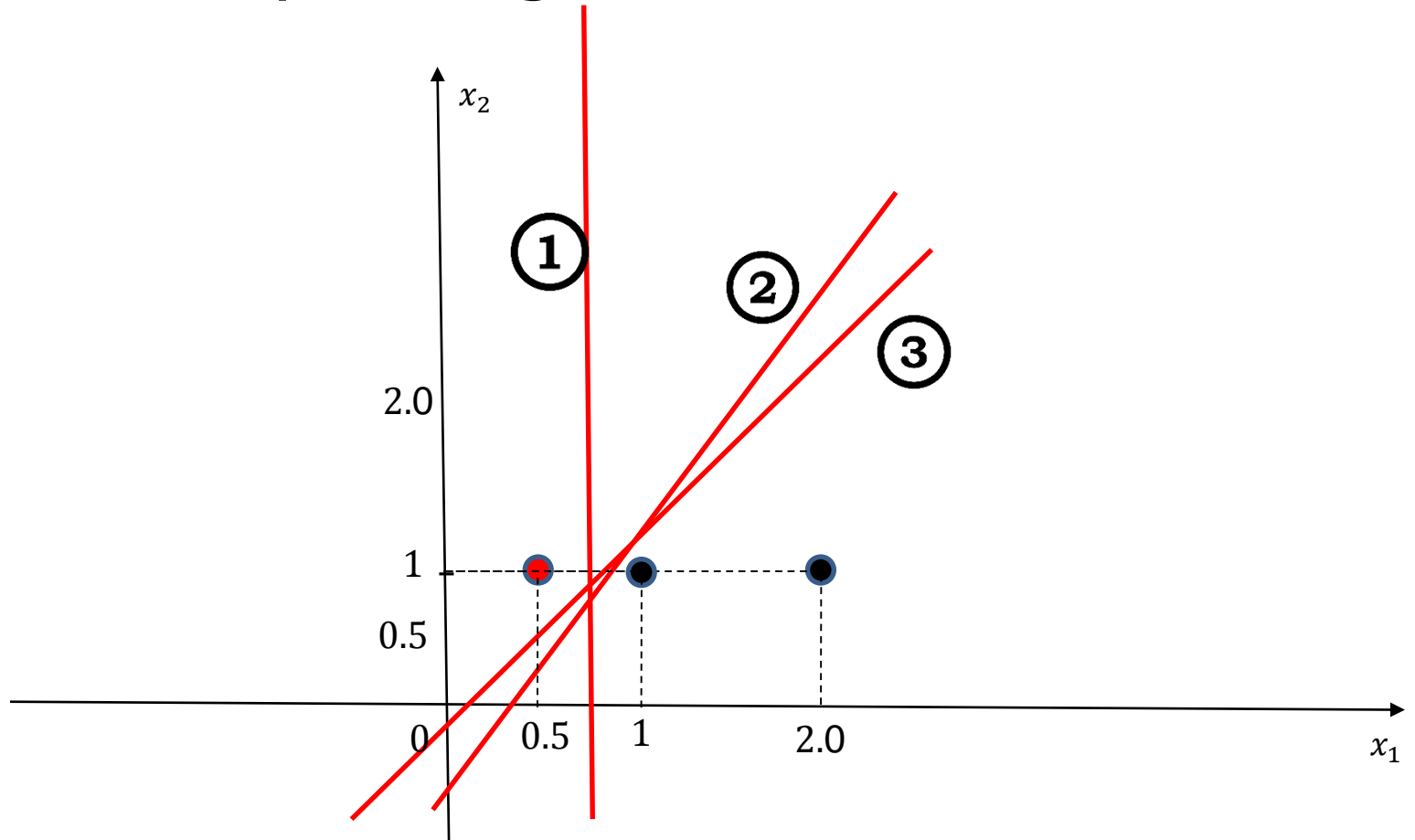
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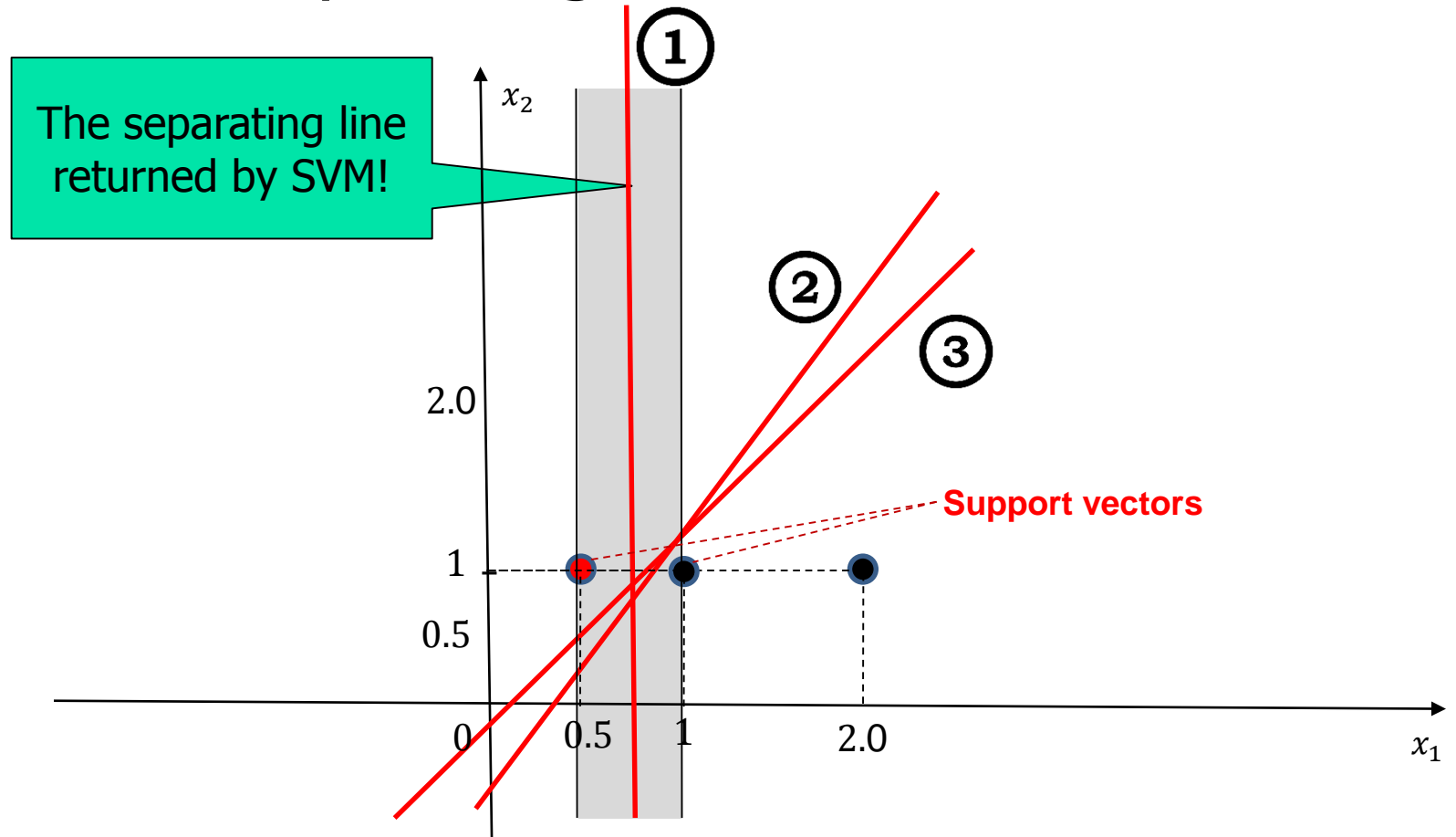
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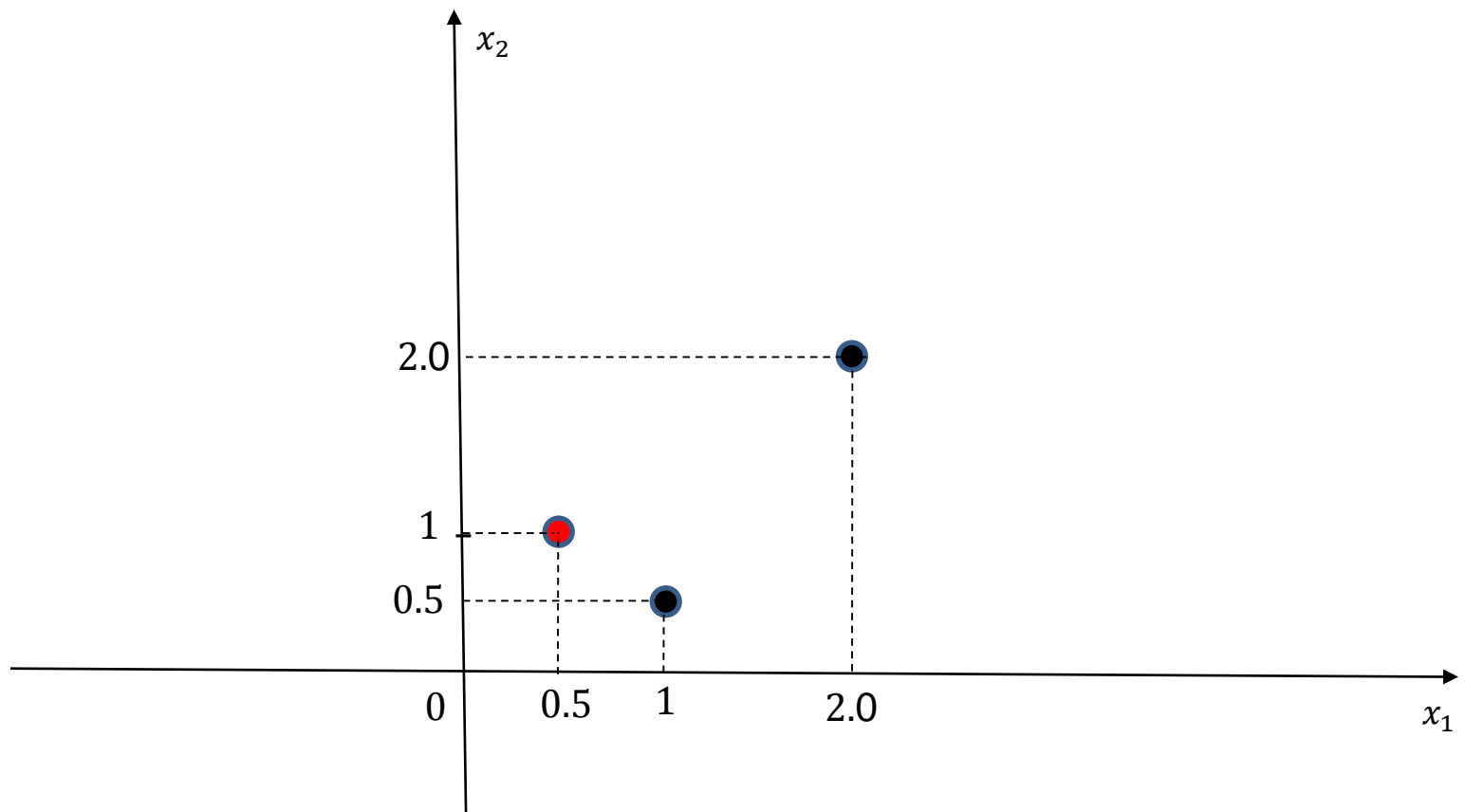


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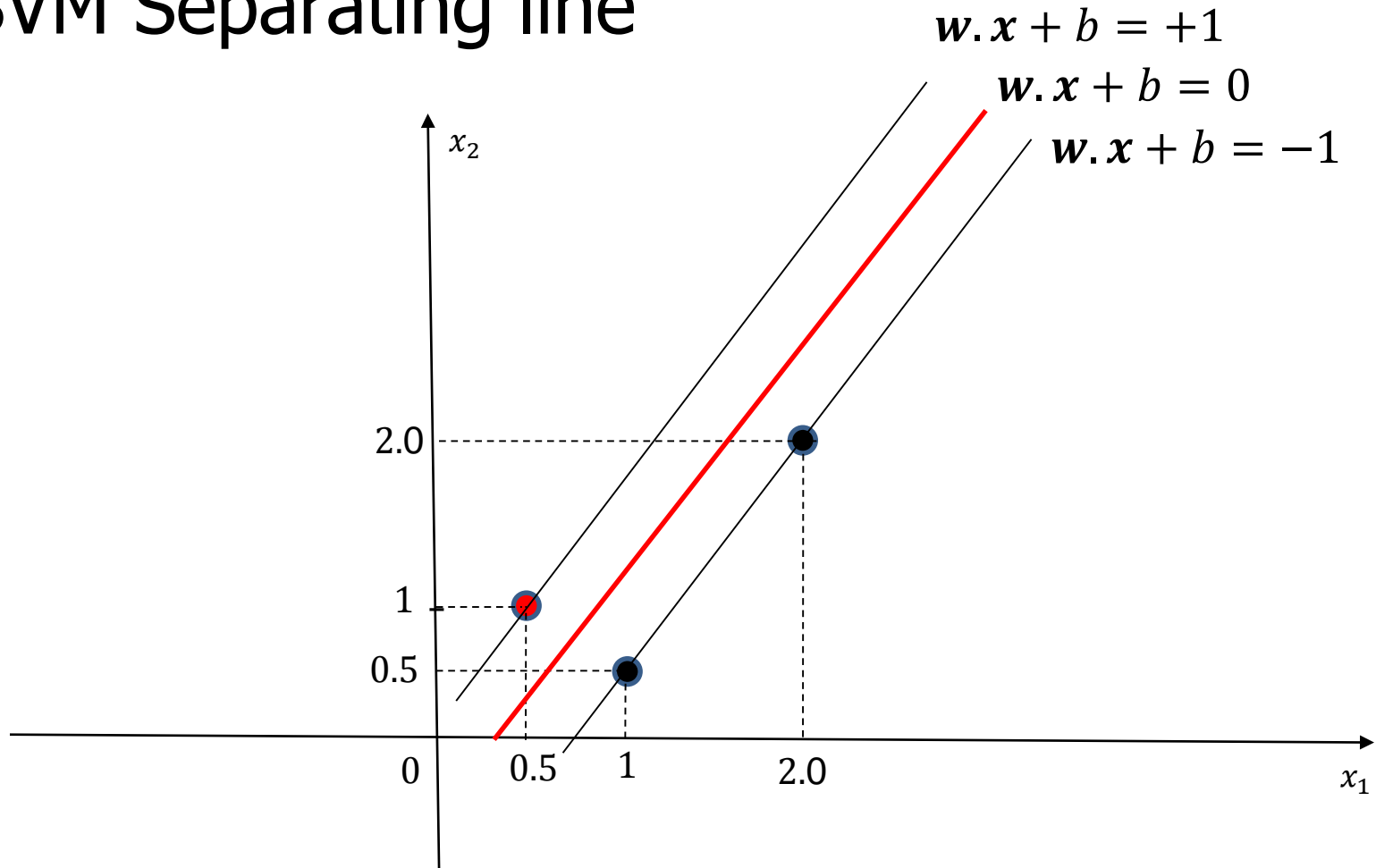
In-Class Exercise-1

- SVM Separating line



In-Class Exercise-1

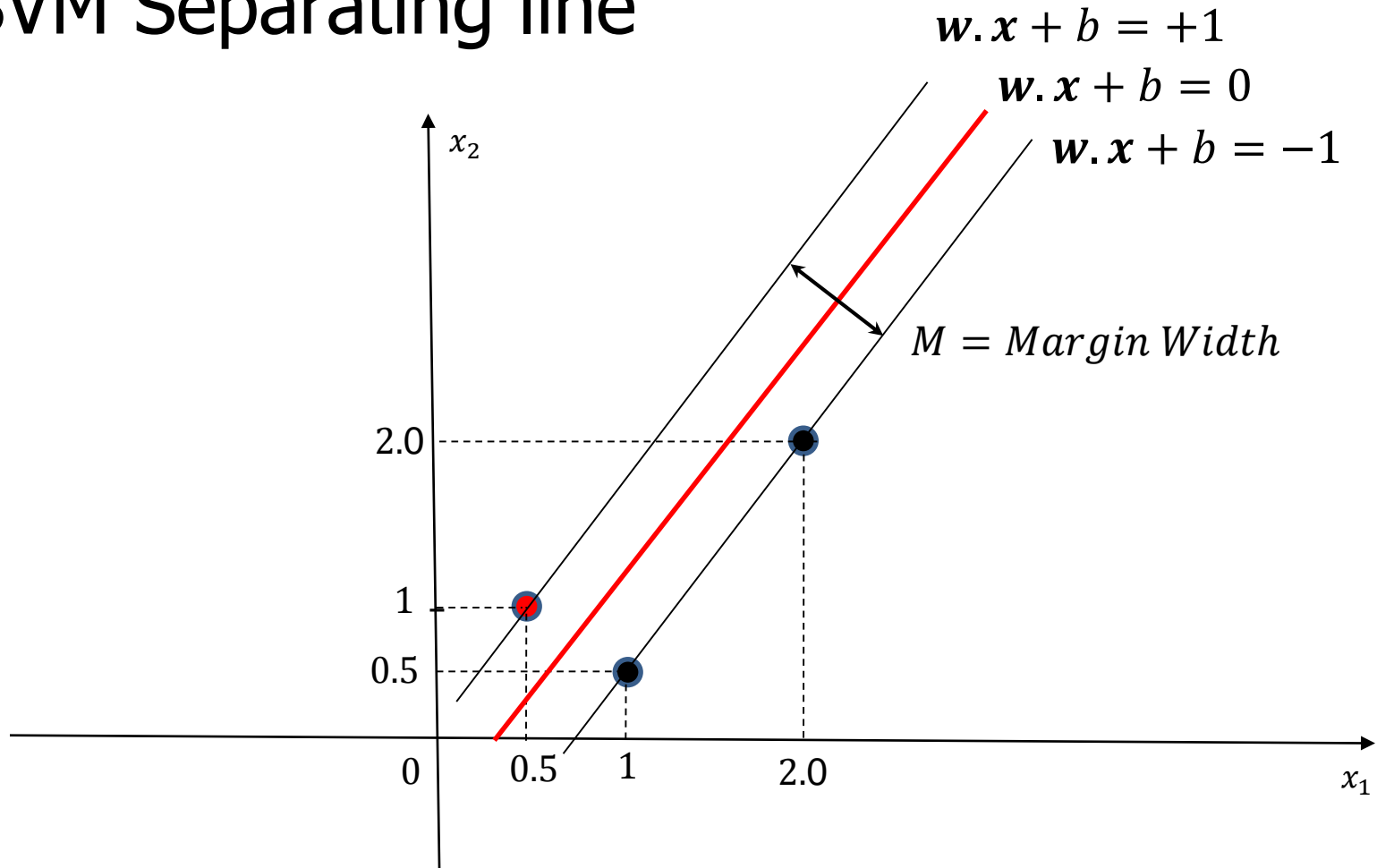
- SVM Separating line



Question: What are the values of w and b ?

In-Class Exercise-1

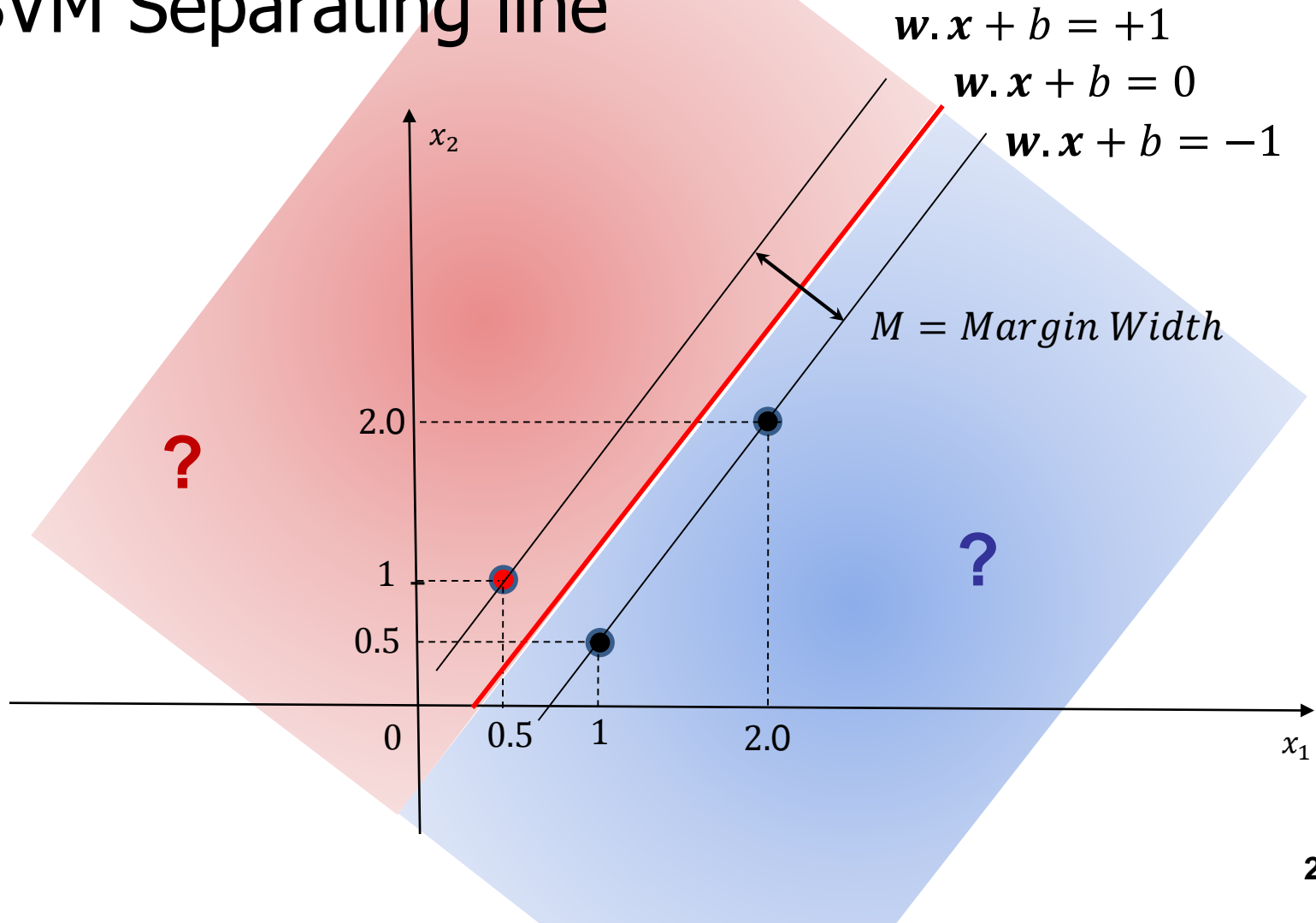
- SVM Separating line



Question: What is the margin width?

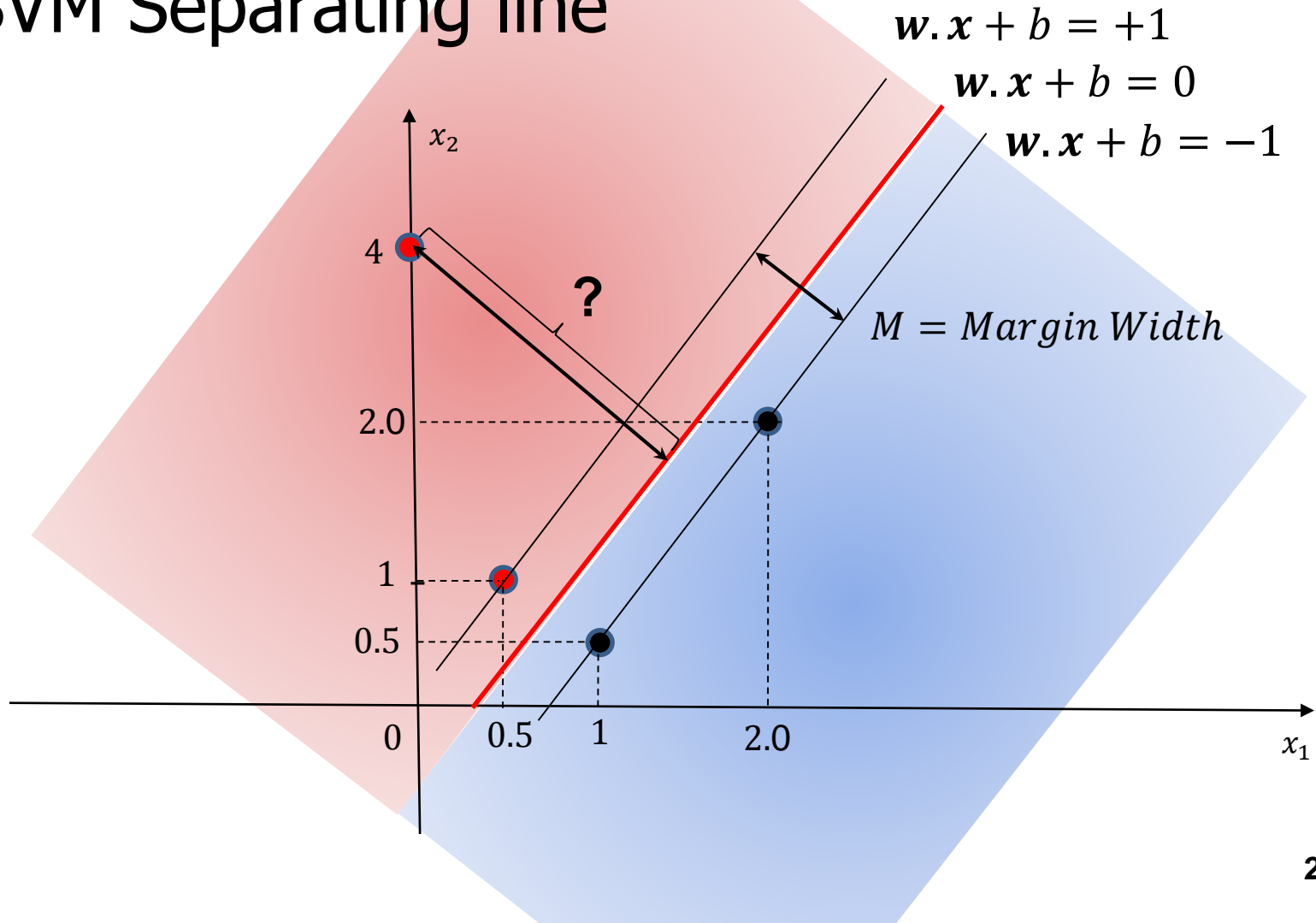
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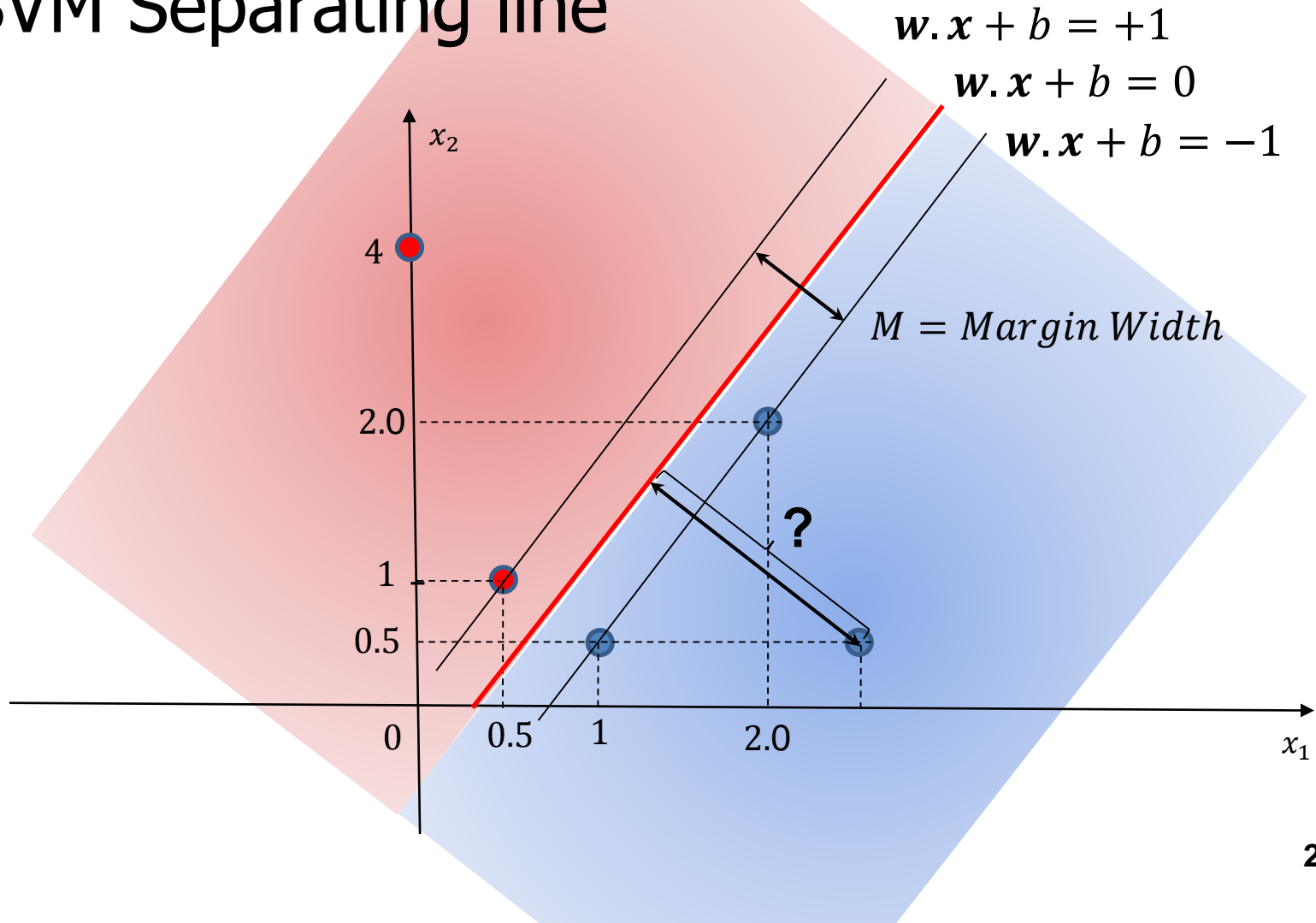
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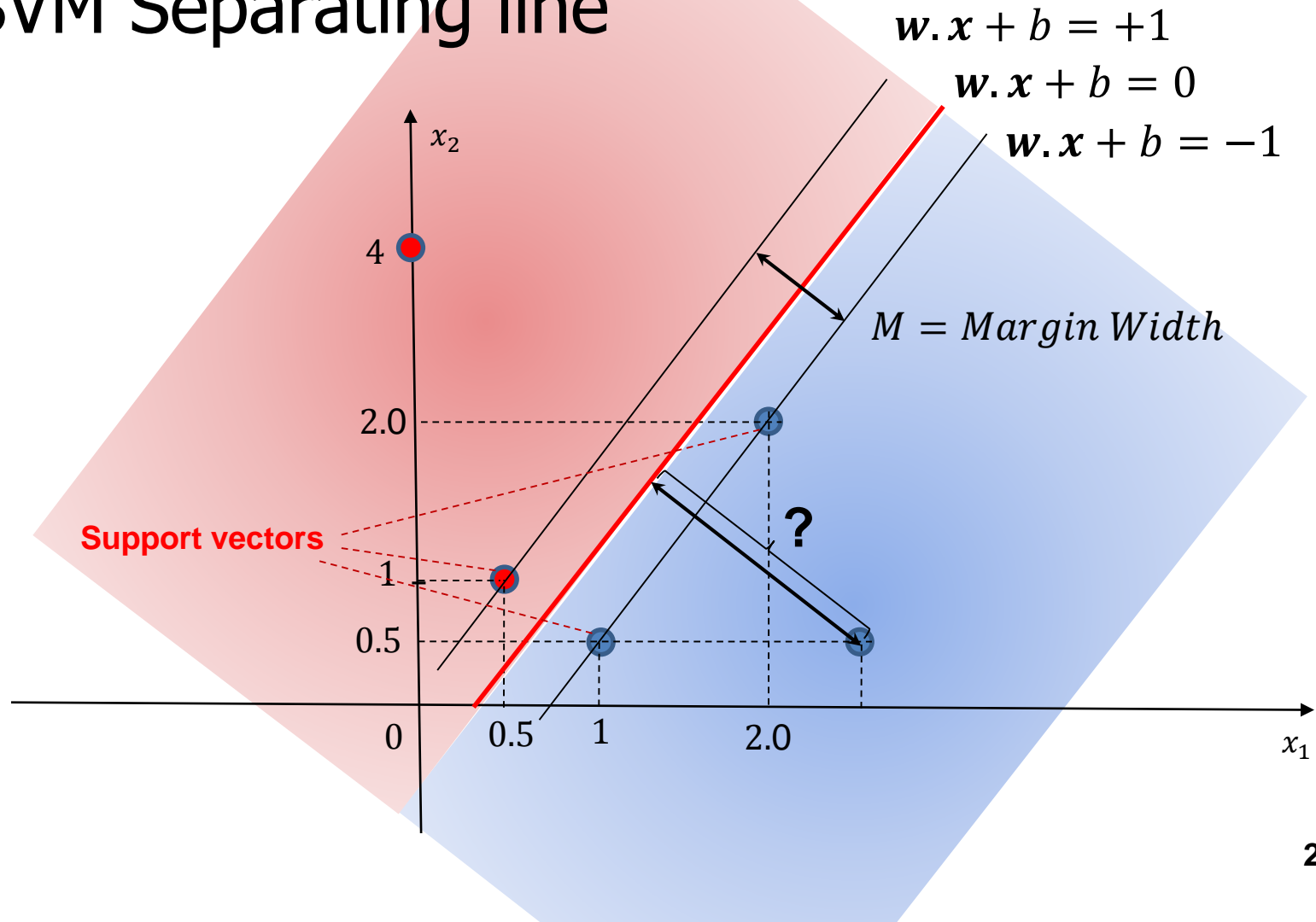
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- SVM Separating line



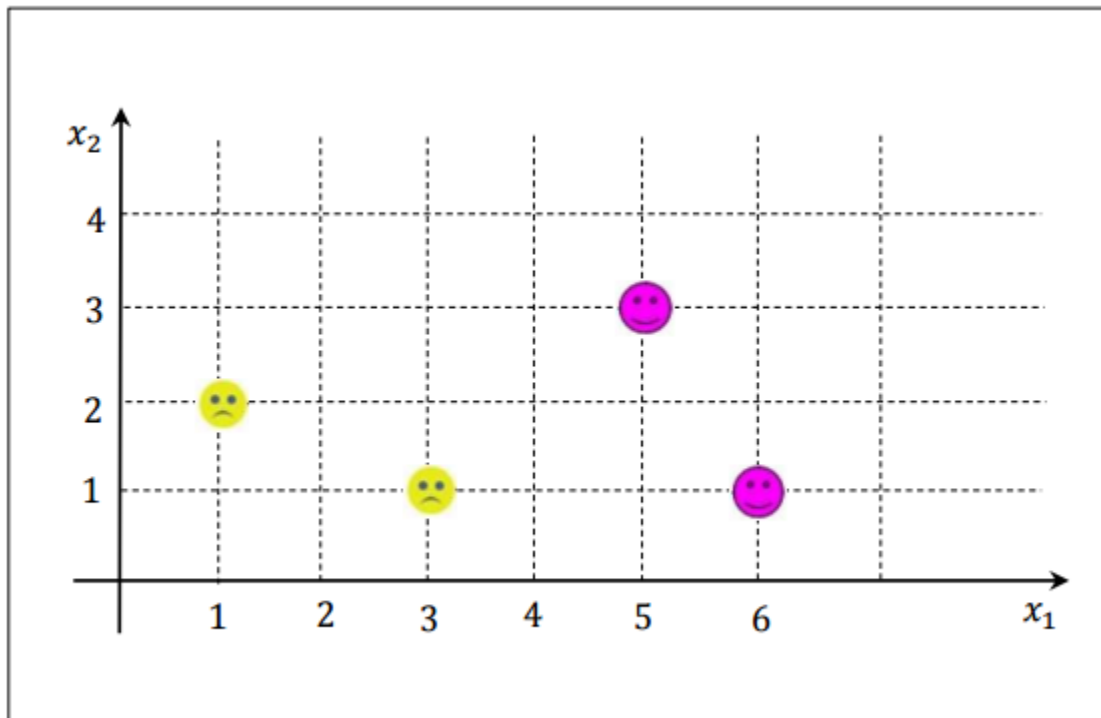
In-Class Exercise-1

- SVM Separating line



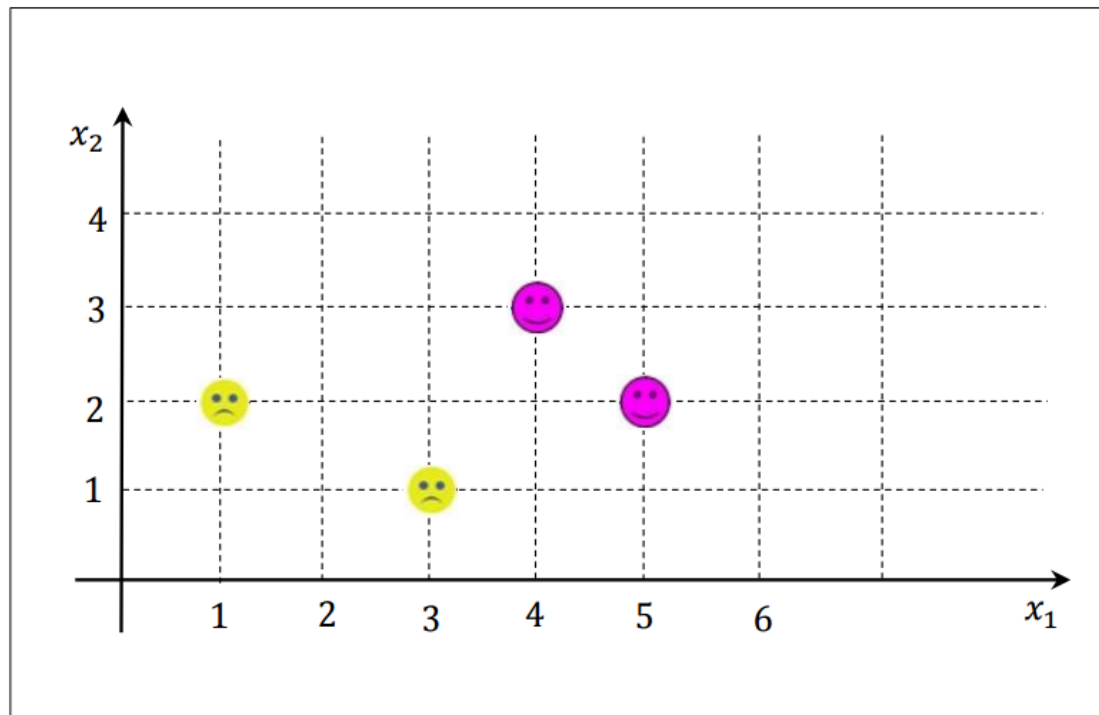
In-Class Exercise-2

- Find a SVM separating line: $w \cdot x + b = 0$
- Calculate the distance of each point to the separating line



In-Class Exercise-3

- Find a SVM separating line: $w \cdot x + b = 0$
- Calculate the distance of each point to the separating line

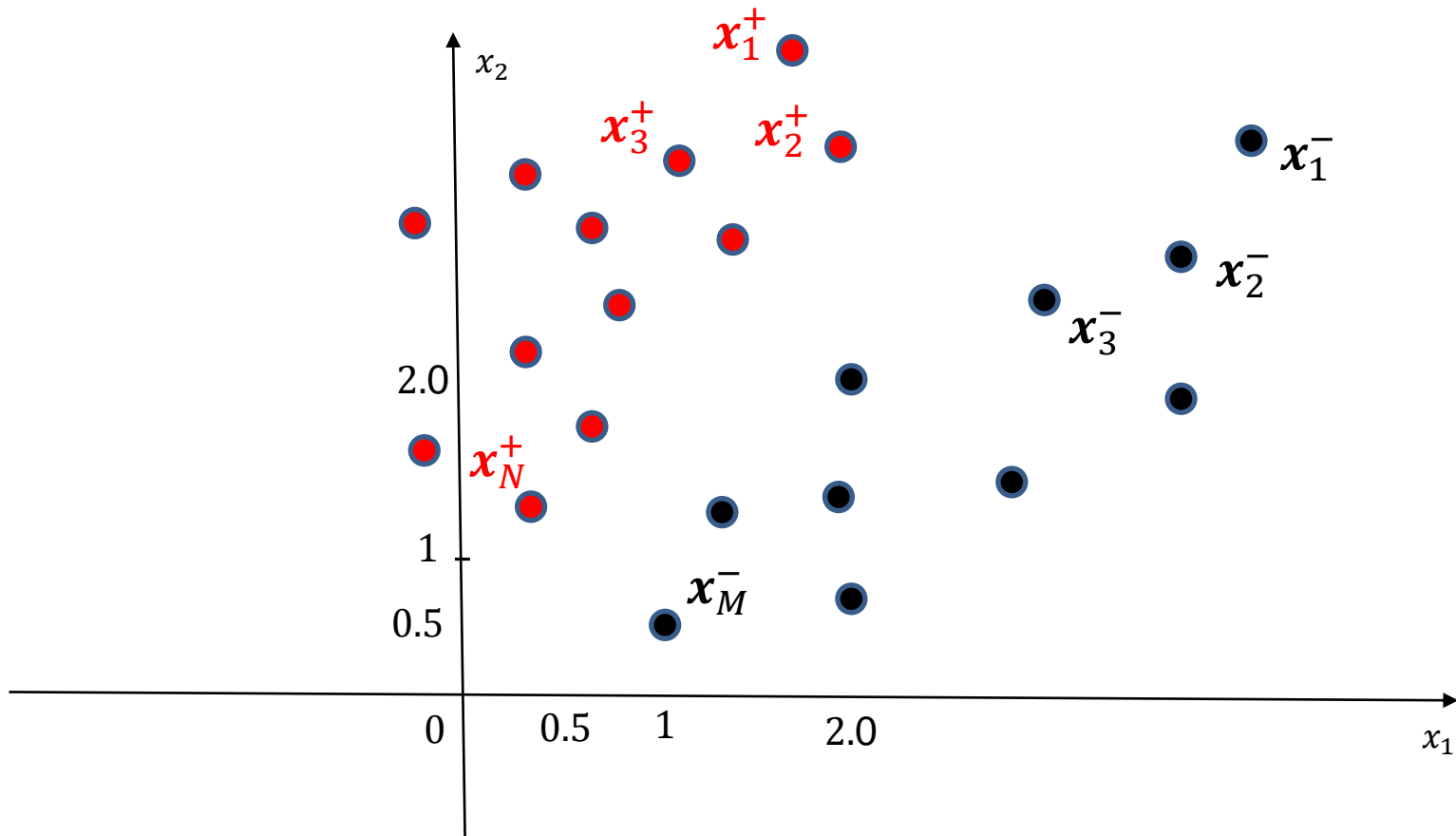




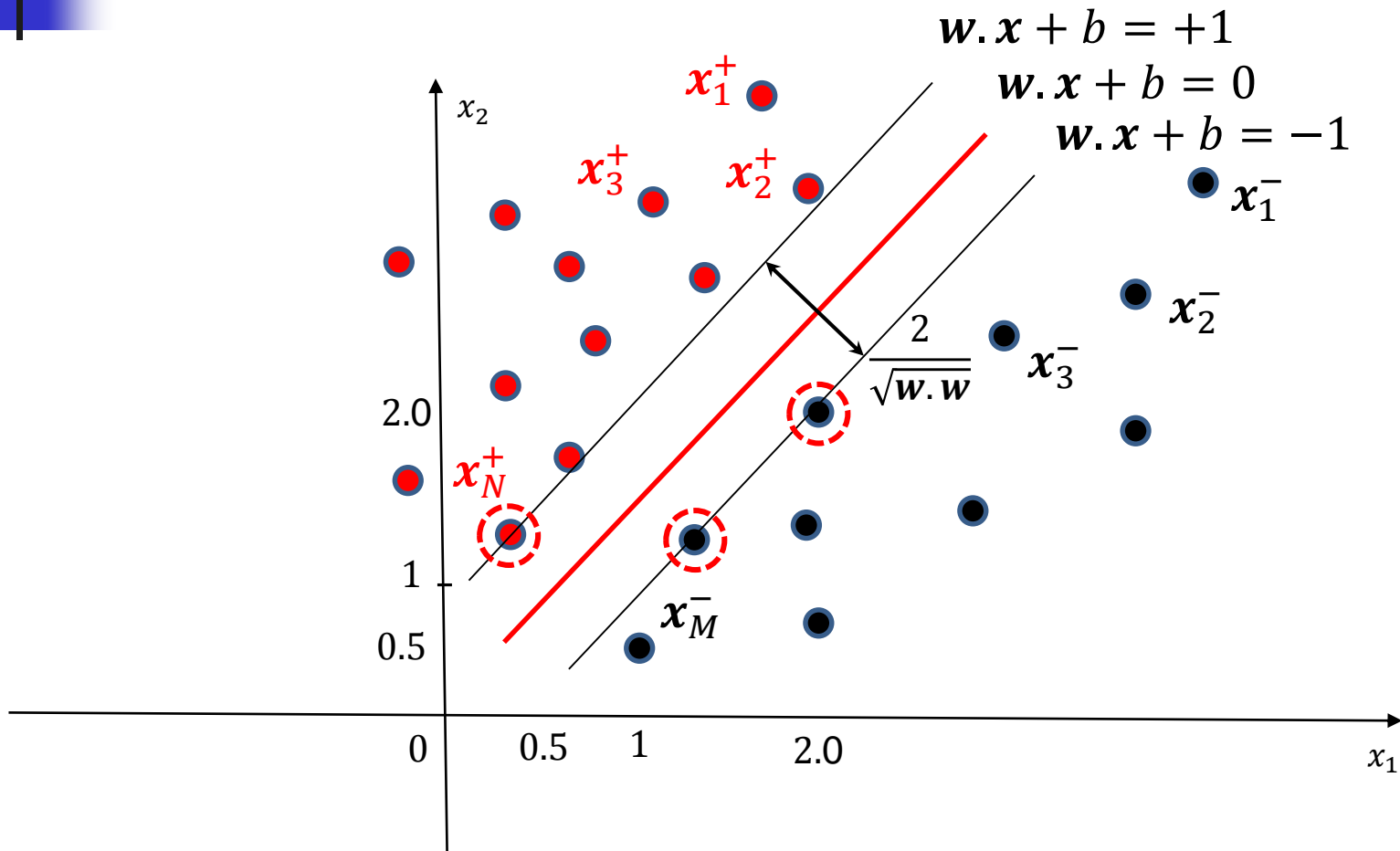
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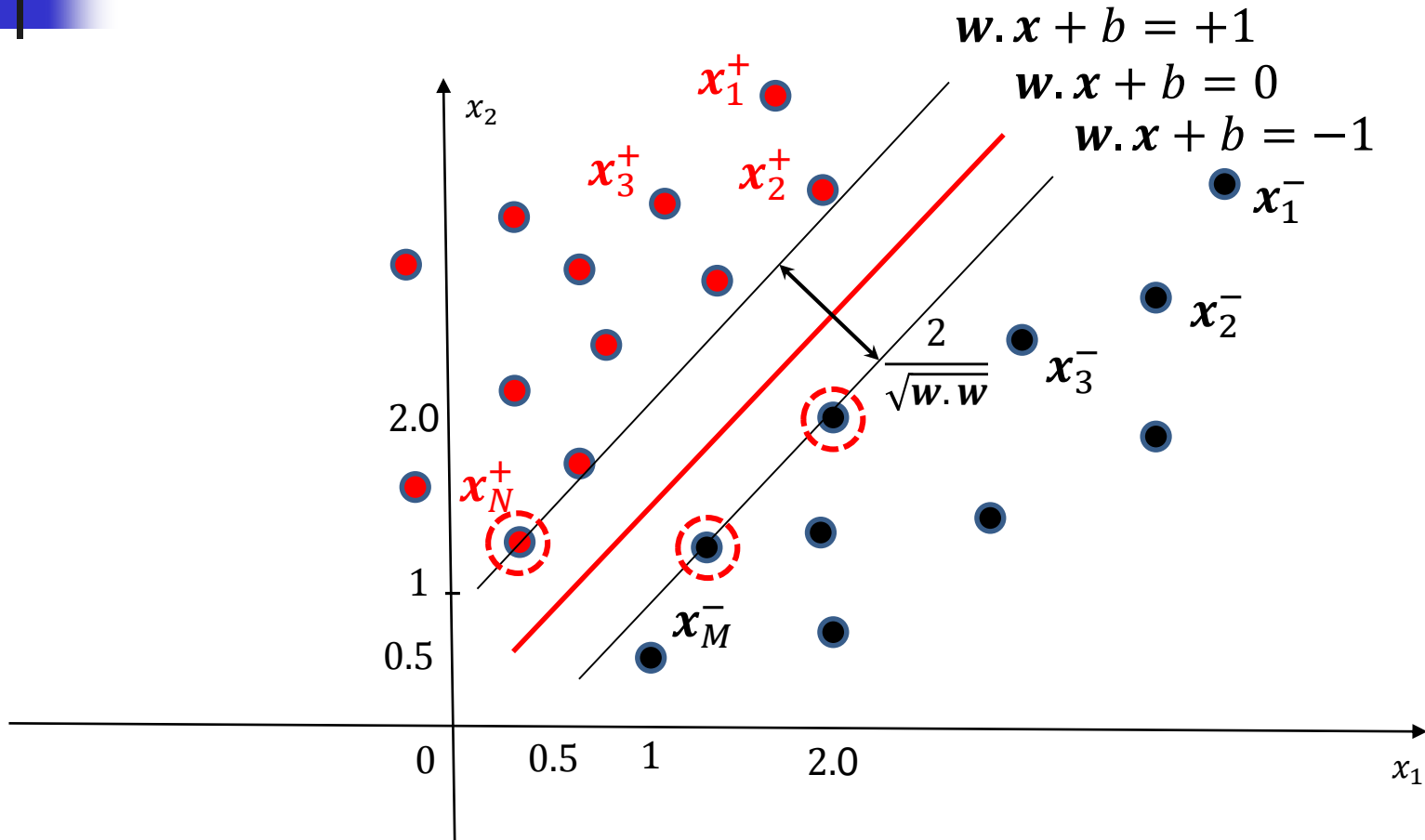
Separable Case



Separable Case



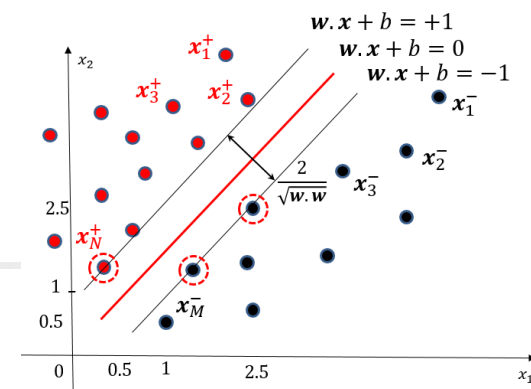
Separable Case



When we have many positive and negative points, how to find SVM support vectors and separating line?

Separable Case

- SVM can be formalized as an optimization problem



$$\underset{w, b}{\text{maximize}} \quad \frac{2}{\sqrt{w \cdot w}}$$

$$\text{subject to} \quad w \cdot x_i^+ + b \geq +1, i = 1, \dots, N$$
$$w \cdot x_i^- + b \leq -1, i = 1, \dots, M$$

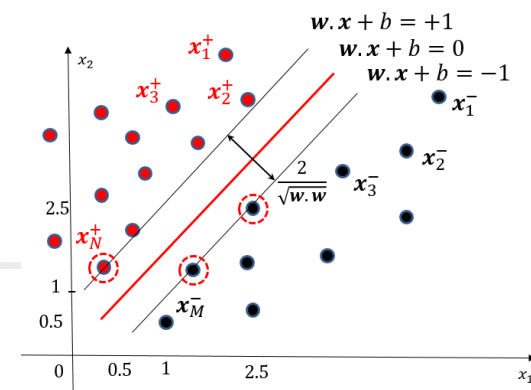
Separable Case

- Equivalent to ...

minimize $\mathbf{w} \cdot \mathbf{w}$
over \mathbf{w}, b

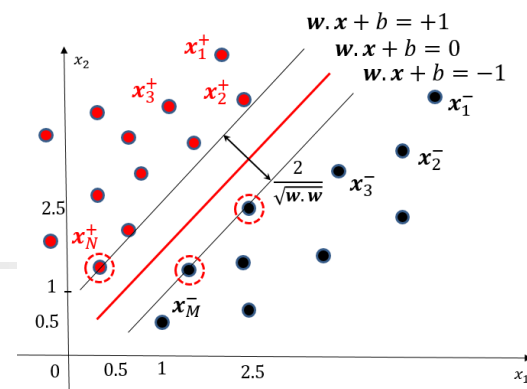
subject to $\mathbf{w} \cdot \mathbf{x}_i^+ + b \geq +1, i = 1, \dots, N$

$\mathbf{w} \cdot \mathbf{x}_i^- + b \leq -1, i = 1, \dots, M$

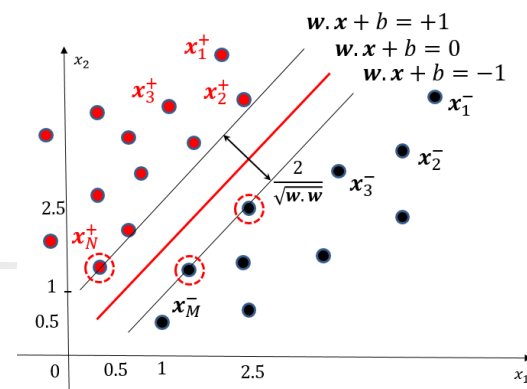


Separable Case

- Given the estimated \hat{w} and \hat{b} , how to identify support vectors?



Separable Case



- Given the estimated \hat{w} and \hat{b} , how to identify support vectors?
- Search for points x_i^+ and x_j^- , such that

$$w \cdot x_i^+ + b = +1$$

OR

$$w \cdot x_i^- + b = -1$$

Implement SVM via CVX-OPT

- Quadratic Programming

$$\underset{x}{\text{minimize}} \quad \frac{1}{2} x^T Q x + p^T x$$

x represents variables

$$\text{subject to} \quad Gx \leq h$$

$$Ax = b$$

<http://cvxopt.org/>

```
from cvxopt import matrix, solvers
...
sol = solvers.qp(Q, p, G, h, A, b)
print (sol['x'])
```



Example of CVX-OPT

- Quadratic Programming

$$\begin{aligned} \min_{x,y} \quad & \frac{1}{2}x^2 + 3x + 4y \\ \text{subject to} \quad & x, y \geq 0 \\ & x + 3y \geq 15 \\ & 2x + 5y \leq 100 \\ & 3x + 4y \leq 80 \end{aligned}$$

Example of CVX-OPT

- Quadratic Programming

$$\min_{x,y} \quad \frac{1}{2} \begin{bmatrix} x \\ y \end{bmatrix}^{\top} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}^{\top} \begin{bmatrix} x \\ y \end{bmatrix}$$
$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \\ -1 & -3 \\ 2 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \preceq \begin{bmatrix} 0 \\ 0 \\ -15 \\ 100 \\ 80 \end{bmatrix}$$



Implement SVM via CVX-OPT

minimize $\mathbf{w} \cdot \mathbf{w}$
 \mathbf{w}, b

subject to $\mathbf{w} \cdot \mathbf{x}_i^+ + b \geq +1, i = 1, \dots, N$

$\mathbf{w} \cdot \mathbf{x}_i^- + b \leq -1, i = 1, \dots, M$

$Q = ?, p = ?, G = ?, h = ?$



Implement SVM via CVX-OPT

$$\underset{w, b}{\text{minimize}} \quad w \cdot w$$

$$\text{subject to} \quad w \cdot x_i^+ + b \geq +1, i = 1, \dots, N$$

$$w \cdot x_i^- + b \leq -1, i = 1, \dots, M$$

$$y = [w; b]$$

$$Q = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, p = 0$$

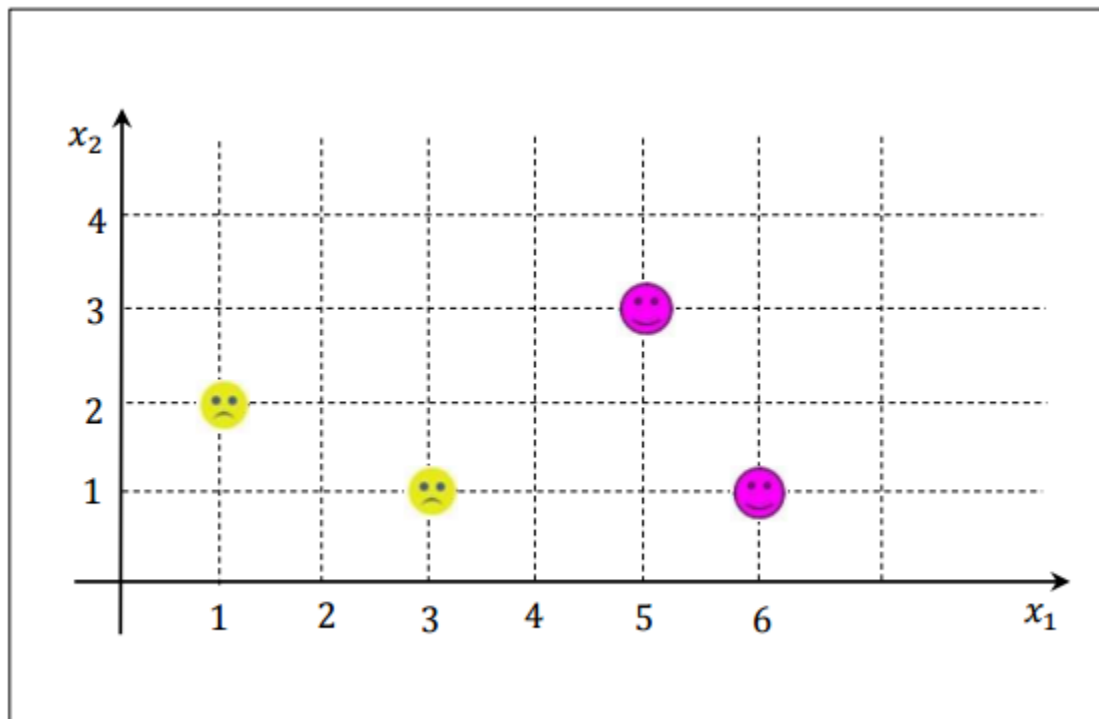
$$G = \begin{bmatrix} -x_1^+ & -1 \\ -x_2^+ & -1 \\ \dots & \dots \\ -x_N^+ & -1 \\ x_1^- & 1 \\ x_2^- & 1 \\ \dots & \dots \\ x_M^- & 1 \end{bmatrix}$$

$$h = \begin{bmatrix} -1 \\ -1 \\ \dots \\ -1 \\ -1 \\ -1 \\ \dots \\ -1 \end{bmatrix}$$

In-Class Exercise-3

- Formulate the SVM problem

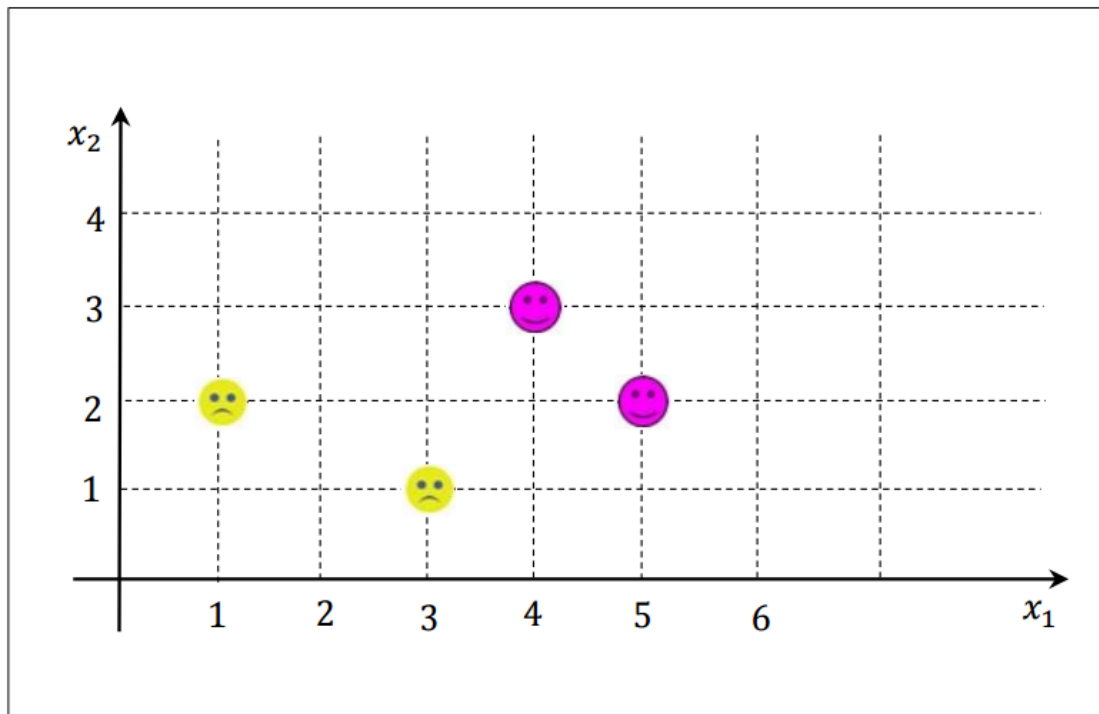
$$\begin{array}{ll} \underset{x}{\text{minimize}} & \frac{1}{2} x^T Q x + p^T x \\ \text{subject to} & Gx \leq h \end{array}$$

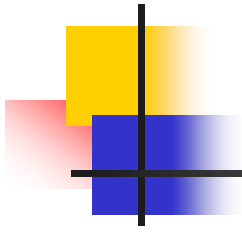


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TO BE CONTINUED