

Course No. MATH F424 Course title: Applied Stochastic Process

- (1) Gambler's ruin: On each day a fair coin is tossed and the gambler wins \$1 if head occurs, or loose \$1 if tails occurs. The gambler stops when he reaches $\$n (n > k)$ or losses all his money. Use Monte carlo simulation to find the probability that the gambler will eventually loose.
- (2) Every day Bob goes to the pizza shop and picks a topping- pepper, pepperoni, pineapple, or chicken- uniformly at random. On the day that Bob first pics pineapple, using simulation find the expected number of prior days in which he picked pepperoni.
- (3) Ellen's insurance will pay for a medical expense subject to a \$100 deductible. Assume that the amount of expense is exponentially distributed with mean \$500. Use simulation to find the expectation and standard deviation of the payout.
- (4) The Canadian forest fire weather index is widely as means of to estimate the risk of wildfire. The Ontario Ministry of Natural Resources uses the index to classify each day's risk of forest fire as either nil, low, moderate, high or extreme. Transition probability matrix for the five state Markov chain for the daily changes in the index is given as:

$$P = \begin{matrix} & \begin{matrix} Nil & Low & Moderate & High & Extreme \end{matrix} \\ \begin{matrix} Nil \\ Low \\ Moderate \\ High \\ Extreme \end{matrix} & \begin{pmatrix} .575 & .118 & .172 & .109 & .026 \\ .453 & .243 & .148 & .123 & .033 \\ .104 & .343 & .367 & .167 & .019 \\ .015 & .066 & .318 & .505 & .096 \\ .000 & .060 & .149 & .567 & .224 \end{pmatrix} \end{matrix}$$

Using R find the long term likelihood of risk for a typical day in the early summer.

- (5) University administrators have developed a Markov model to simulate graduation rates at their school. Student might drop out, repeat a year or move on to the next year. Student have a 3% chance of repeating a year. First years and second years have a 6% of dropping out. For third years and fourth years the drop out rate is 4%. The transition matrix for the model

is:

$$P = \begin{matrix} & \begin{matrix} Drop & I & II & III & IV & Grad \end{matrix} \\ \begin{matrix} Drop \\ I \\ II \\ III \\ IV \\ Grad \end{matrix} & \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ .06 & .03 & .91 & 0 & 0 & 0 \\ .06 & 0 & .03 & .91 & 0 & 0 \\ .04 & 0 & 0 & .03 & .93 & 0 \\ .04 & 0 & 0 & 0 & .03 & .93 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{matrix}$$

Simulate the long term probability that a new student graduates.

- (6) After work, angel goes to the gym and either does aerobics, weights, yoga or goes for jogging. Each day Angle decides her workout routine based on what she did the previous day according to the Markov transition matrix:

$$P = \begin{matrix} & \begin{matrix} Aerobics & Jogging & Weights & Yoga \end{matrix} \\ \begin{matrix} Aerobics \\ Jogging \\ Weights \\ Yoga \end{matrix} & \begin{pmatrix} .1 & .2 & .4 & .3 \\ .4 & 0 & .4 & .2 \\ .3 & .3 & 0 & .4 \\ .2 & .1 & .4 & .3 \end{pmatrix} \end{matrix}$$

Simulate the long term probability that she goes for jogging. Compare this with stationary distribution.

- (7) Consider a Markov chain with transition probability matrix

$$P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{pmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} \end{matrix}$$

Simulate μ_1 .

- (8) A biased coin comes up head with probability $2/3$ and tails with probability $1/3$. The coin is repeatedly flipped. Simulate to find the total numbers average number of flips needed, until the pattern HTHH first appears. Compare it with theoretical result.
