$$N^{[U]^{(1)}} = \frac{1}{4}\xi(\xi-1)\omega(\omega-1) \tag{1}$$

$$N^{[U]^{(2)}} = \frac{1}{4}\xi(\xi+1)\omega(\omega-1) \tag{2}$$

$$N^{[U]^{(3)}} = \frac{1}{4}\xi(\xi+1)\omega(\omega+1) \tag{3}$$

$$N^{[U]^{(4)}} = \frac{1}{4}\xi(\xi-1)\omega(\omega+1) \tag{4}$$

$$N^{[U]^{(5)}} = \frac{1}{2}(1-\xi^2)\omega(\omega-1) \tag{5}$$

$$N^{[U]^{(6)}} = \frac{1}{2}\xi(\xi+1)(1-\omega^2) \tag{6}$$

$$N^{[U]^{(7)}} = \frac{1}{2}(1-\xi^2)\omega(\omega+1) \tag{7}$$

$$N^{[U]^{(8)}} = \frac{1}{2}\xi(\xi-1)(1-\omega^2) \tag{8}$$

$$N^{[U]^{(9)}} = (1-\xi^2)(1-\omega^2) \tag{9}$$

$$N_{,\xi}^{[U]^{(1)}} = -\frac{1}{2}\xi\omega - \frac{1}{4}\omega^2 + \frac{1}{4}\omega + \frac{1}{2}\xi\omega^2 \tag{10}$$

$$N_{,\xi}^{[U]^{(2)}} = -\frac{1}{2}\xi\omega + \frac{1}{4}\omega^2 - \frac{1}{4}\omega + \frac{1}{2}\xi\omega^2 \tag{11}$$

$$N_{,\xi}^{[U]^{(3)}} = \frac{1}{2}\xi\omega + \frac{1}{4}\omega^2 + \frac{1}{4}\omega + \frac{1}{2}\xi\omega^2 \tag{12}$$

$$N_{,\xi}^{[U]^{(4)}} = \frac{1}{2}\xi\omega - \frac{1}{4}\omega^2 - \frac{1}{4}\omega + \frac{1}{2}\xi\omega^2 \tag{13}$$

$$N_{,\xi}^{[U]^{(5)}} = \xi\omega - \xi\omega^2 \tag{14}$$

$$N_{,\xi}^{[U]^{(6)}} = \xi - \xi\omega^2 + \frac{1}{2} - \frac{1}{2}\omega^2 \tag{15}$$

$$N_{,\xi}^{[U]^{(7)}} = -\xi\omega - \xi\omega^2 \tag{16}$$

$$N_{,\xi}^{[U]^{(8)}} = \xi - \xi\omega^2 - \frac{1}{2} + \frac{1}{2}\omega^2 \tag{17}$$

$$N_{,\xi}^{[U]^{(9)}} = -2\xi + 2\xi\omega^2 \tag{18}$$

$$N_{,\omega}^{[U]^{(1)}} = -\frac{1}{2}\xi\omega - \frac{1}{4}\xi^2 + \frac{1}{4}\xi + \frac{1}{2}\xi^2\omega \tag{19}$$

$$N_{,\omega}^{[U]^{(2)}} = \frac{1}{2}\xi\omega - \frac{1}{4}\xi^2 - \frac{1}{4}\xi + \frac{1}{2}\xi^2\omega \tag{20}$$

$$N_{,\omega}^{[U]^{(3)}} = \frac{1}{2}\xi\omega + \frac{1}{4}\xi^2 + \frac{1}{4}\xi + \frac{1}{2}\xi^2\omega \tag{21}$$

$$N_{,\omega}^{[U]^{(4)}} = -\frac{1}{2}\xi\omega + \frac{1}{4}\xi^2 - \frac{1}{4}\xi + \frac{1}{2}\xi^2\omega \tag{22}$$

$$N_{,\omega}^{[U]^{(5)}} = \frac{1}{2}\xi^2 - \frac{1}{2} - \xi^2\omega + \omega \tag{23}$$

$$N_{,\omega}^{[U]^{(6)}} = -\xi^2\omega - \xi\omega \tag{24}$$

$$N_{,\omega}^{[U]^{(7)}} = -\frac{1}{2}\xi^2 + \frac{1}{2} - \xi^2\omega + \omega \tag{25}$$

$$N_{,\omega}^{[U]^{(8)}} = -\xi^2\omega + \xi\omega \tag{26}$$

$$N_{,\omega}^{[U]^{(9)}} = -2\omega + 2\xi^2\omega \tag{27}$$

$$N^{[\psi]^{(1)}} = \frac{1}{4}(1-\xi)(1-\omega) \tag{28}$$

$$N^{[\psi]^{(2)}} = \frac{1}{4}(1+\xi)(1-\omega) \tag{29}$$

$$N^{[\psi]^{(3)}} = \frac{1}{4}(1+\xi)(1+\omega) \tag{30}$$

$$N^{[\psi]^{(4)}} = \frac{1}{4}(1-\xi)(1+\omega) \tag{31}$$

$$N_{,\xi}^{[\psi]^{(1)}} = \frac{1}{4}(-1+\omega) \tag{32}$$

$$N_{,\xi}^{[\psi]^{(2)}} = \frac{1}{4}(1-\omega) \tag{33}$$

$$N_{,\xi}^{[\psi]^{(3)}} = \frac{1}{4}(1+\omega) \tag{34}$$

$$N_{,\xi}^{[\psi]^{(4)}} = \frac{1}{4}(-1-\omega) \tag{35}$$

$$N_{,\omega}^{[\psi]^{(1)}} = \frac{1}{4}(-1+\xi) \tag{36}$$

$$N_{,\omega}^{[\psi]^{(2)}} = \frac{1}{4}(-1-\xi) \tag{37}$$

$$N_{,\omega}^{[\psi]^{(3)}} = \frac{1}{4}(1+\xi) \tag{38}$$

$$N_{,\omega}^{[\psi]^{(4)}} = \frac{1}{4}(1-\xi) \tag{39}$$

$$u^{(e)} = \begin{bmatrix} u_1^{(e)} \\ u_2^{(e)} \end{bmatrix} = N^{[u]^{\mathbf{T}}} \cdot u^{(e)}$$

$$u^{(e)} = \begin{bmatrix} u_1^{(1)} & u_2^{(1)} & u_1^{(2)} & u_2^{(2)} & \cdots & \cdots u_1^{(9)} u_2^{(9)} \end{bmatrix}$$

$$N^{[u]^T} = \begin{bmatrix} N^{[u]^{(1)}} & 0 & N^{[u]^{(2)}} & 0 & \cdots & \cdots & N^{[u]^{(9)}} & 0 \\ 0 & N^{[u]^{(1)}} & 0 & N^{[u]^{(2)}} & \cdots & \cdots & 0 & N^{[u]^{(9)}} \end{bmatrix}$$

$$\nabla u^{(e)} = \begin{bmatrix} u_{1,1}^{(e)} \\ \\ u_{1,2}^{(e)} \\ \\ u_{2,1}^{(e)} \\ \\ u_{2,2}^{(e)} \end{bmatrix} = M^{[u]^{\mathbf{T}}} u^{(e)}$$

$$M^{[u]^{\mathbf{T}}} = \begin{bmatrix} N_{,1}^{[u]^{(1)}} & 0 & N_{,1}^{[u]^{(2)}} & 0 & \cdots & \cdots & N_{,1}^{[u]^{(9)}} & 0 \\ 0 & N_{,2}^{[u]^{(1)}} & 0 & N_{,2}^{[u]^{(2)}} & \cdots & \cdots & 0 & N_{,2}^{[u]^{(9)}} \\ N_{,1}^{[u]^{(1)}} & 0 & N_{,1}^{[u]^{(2)}} & 0 & \cdots & \cdots & N_{,1}^{[u]^{(9)}} & 0 \\ 0 & N_{,2}^{[u]^{(1)}} & 0 & N_{,2}^{[u]^{(2)}} & \cdots & \cdots & 0 & N_{,2}^{[u]^{(9)}} \end{bmatrix}$$

$$\varepsilon^{(e)} = \begin{bmatrix} \varepsilon_{11}^{(e)} \\ \\ \varepsilon_{22}^{(e)} \\ \\ \gamma_{12}^{(e)} \end{bmatrix} \approx B^{[u]^{\mathbf{T}}} u^{(e)}$$

$$B^{[u]^{\mathbf{T}}} = \begin{bmatrix} N_{,1}^{[u]^{(1)}} & 0 & N_{,1}^{[u]^{(2)}} & 0 & \cdots & \cdots & N_{,1}^{[u]^{(9)}} & 0 \\ 0 & N_{,2}^{[u]^{(1)}} & 0 & N_{,2}^{[u]^{(2)}} & \cdots & \cdots & 0 & N_{,2}^{[u]^{(9)}} \\ N_{,2}^{[u]^{(1)}} & N_{,1}^{[u]^{(1)}} & N_{,2}^{[u]^{(2)}} & N_{,1}^{[u]^{(2)}} & \cdots & \cdots & N_{,2}^{[u]^{(9)}} & N_{,1}^{[u]^{(9)}} \end{bmatrix}$$

$$\psi^{(e)} = \begin{bmatrix} \psi_{11}^{(e)} \\ \\ \psi_{21}^{(e)} \\ \\ \psi_{12}^{(e)} \\ \\ \psi_{22}^{(e)} \end{bmatrix} = N^{[\psi]^{\mathbf{T}}} \psi^{(e)}$$

$$\psi^{(e)^{\mathbf{T}}} = \begin{bmatrix} \psi_{(11)}^{(1)} & \psi_{(21)}^{(1)} & \psi_{(12)}^{(1)} & \psi_{(22)}^{(1)} & \cdots & \cdots & \psi_{(11)}^{(4)} & \psi_{(21)}^{(4)} & \psi_{(12)}^{(4)} & \psi_{(22)}^{(4)} \end{bmatrix}$$

$$N^{[\psi]^{\mathbf{T}}} = \begin{bmatrix} N^{\psi^{(1)}} & 0 & 0 & 0 & \cdots & \cdots & N^{\psi^{(4)}} & 0 & 0 & 0 \\ 0 & N^{\psi^{(1)}} & 0 & 0 & \cdots & \cdots & 0 & N^{\psi^{(4)}} & 0 & 0 \\ 0 & 0 & N^{\psi^{(1)}} & 0 & \cdots & \cdots & 0 & 0 & N^{\psi^{(4)}} & 0 \\ 0 & 0 & 0 & N^{\psi^{(1)}} & \cdots & \cdots & 0 & 0 & 0 & N^{\psi^{(4)}} \end{bmatrix}$$

$$\eta^{(e)} = \begin{bmatrix} \eta^{(e)}_{111} \\ \eta^{(e)}_{221} \\ \eta^{(e)}_{112} \\ \eta^{(e)}_{222} \\ \phi^{(e)}_{1} \\ \phi^{(e)}_{2} \end{bmatrix} \approx B^{[\psi]^{\mathbf{T}}} \psi^{(e)}$$

$$B^{[\psi]^{\mathbf{T}}} = \begin{bmatrix} N^{[\psi]^{(1)}}_{,1} & 0 & 0 & 0 & \cdots & \cdots & N^{[\psi]^{(4)}}_{,1} & 0 & 0 & 0 \\ 0 & N^{[\psi]^{(1)}}_{,2} & 0 & 0 & \cdots & \cdots & 0 & N^{[\psi]^{(4)}}_{,2} & 0 & 0 \\ 0 & 0 & N^{[\psi]^{(1)}}_{,1} & 0 & \cdots & \cdots & 0 & 0 & N^{[\psi]^{(4)}}_{,1} & 0 \\ 0 & 0 & 0 & N^{[\psi]^{(1)}}_{,2} & \cdots & \cdots & 0 & 0 & 0 & N^{[\psi]^{(4)}}_{,2} \\ N^{[\psi]^{(1)}}_{,2} & N^{[\psi]^{(1)}}_{,1} & 0 & 0 & \cdots & \cdots & N^{[\psi]^{(4)}}_{,2} & N^{[\psi]^{(4)}}_{,1} & 0 & 0 \\ 0 & 0 & N^{[\psi]^{(1)}}_{,2} & N^{[\psi]^{(1)}}_{,1} & \cdots & \cdots & 0 & 0 & N^{[\psi]^{(4)}}_{,2} & N^{[\psi]^{(4)}}_{,1} \end{bmatrix}$$

$$\rho^{(e)} = \begin{bmatrix} \rho^{(e)}_{11} \\ \rho^{(e)}_{21} \\ \rho^{(e)}_{12} \\ \rho^{(e)}_{22} \end{bmatrix} = N^{[\rho]^{\mathbf{T}}} \rho^{(e)}$$

$$N^{[\rho]} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$