Quantum Error Correction - Stabiliser Codes

PHYS650 Presentation

Dhawal Patil

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No cloning theorem

Quantum states can not be freely copied

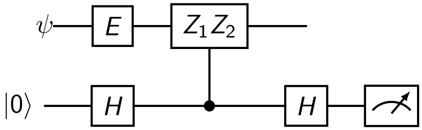
$$
\begin{aligned}
A(\alpha|0\rangle + \beta|1\rangle) &= (\alpha|0\rangle + \beta|1\rangle)(\alpha|0\rangle + \beta|1\rangle) \\
&= \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle \\
&\neq \alpha|00\rangle + \beta|11\rangle \\
&= \alpha A|0\rangle + \beta A|1\rangle
\end{aligned}
$$

Two qubit detection algorithm I

1. $$\psi = \alpha\left|0\right\rangle + \beta\left|1\right\rangle \xrightarrow{\text{two qubit encoder}} \alpha\left|00\right\rangle + \beta\left|11\right\rangle.$$
2. Code space $= \mathcal{C} = \mathit{span}\{\left|00\right\rangle, \left|11\right\rangle\}$.
3. Error space $\mathcal{F} = \mathit{span}\{\left|01\right\rangle, \left|10\right\rangle\}$.
4. For stabilisers
$$Z_1 Z_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$
$Z_1 Z_2 \psi = -\psi$ if there is an error and ψ if there is no error.

Two qubit detection algorithm II

5. To extract this information, use a syndrome qubit s



6. If there was an error, then the transition would be

$$\psi\,|+\rangle \rightarrow -\psi\,|-\rangle \rightarrow -\psi\,|1\rangle$$

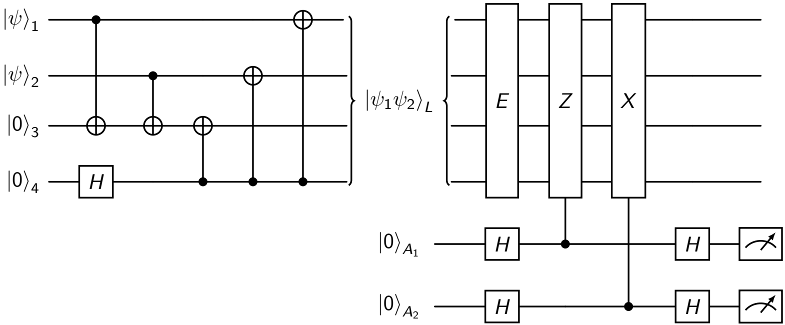
7. In a nutshell, states from $\mathcal{F}$ return a characteristic value of -1 and this can be read from the syndrome qubit.
8. From $|01\rangle$ or $|10\rangle$, we can't go back to the original state. **Detection only**

Stabiliser Codes

1. Split the space into $\mathcal{C}$ Codespace and error subspaces $\mathcal{F}_i$ one for each type of error.
2. Stabiliser group S is a subgroup of $\mathcal{G}_n$.
3. S stabilises $\mathcal{C}$, so it is in the subspace corresponding to 1
4. $\mathcal{F}_i$ is in characteristic subspace corresponding -1.

Two Qubit Detection Code

1. All errors are unitary, so can be reversed by applying $E^\dagger$.
2. The circuit
3. $d = 2$ Indeed, replacing the first and third bits on $|0110\rangle$ yields $|1100\rangle$. So, $t = \lfloor \frac{d}{2} \rfloor = 0$ errors can be corrected.
4. **Detection only**



1. For an (n, k, d) code, k logical qubits can be encoded into n physical qubits.
2. Efficiency of encoding is k/n.
3. On propagation of 1 error, a state can enter one of

Stabiliser Codes

1. The $[[5, 1, 3]]$ Code Capable of correcting all kinds of errors on one logical qubit.
2. Stabiliser Codes [BAD] - higher dimension codes have denser parity checks, more susceptible to decoherence
3. Alternate - Low Density Parity Check Code
4. Stabiliser Codes [BAD] - difficult to find stabiliser subgroups
5. Alternate - Surface code

Surface Codes

1. Surface codes belong to a class of topological stabiliser codes.
2. Qubits arranged on a 2-D lattice. Stabiliser checks are local.
3. Two types of checks:
   - Star operators (measure X-type parity)
   - Plaquette operators (measure Z-type parity)
4. Errors create "defects" which appear as flipped stabiliser values.
5. Decoding $=$ finding a minimal path that connects these defects.
6. Pros:
   - Very high threshold ($\sim 1\%$)
   - Only nearest-neighbour interactions needed
7. Cons:
   - Very large number of physical qubits needed per logical qubit.

Low Density Parity Check (LDPC) Codes

1. LDPC codes have stabilisers that touch only a small number of qubits.
2. Sparse parity check matrix $\Rightarrow$ fewer operations per check.
3. Pros:
   - Much lower overhead than surface codes.
   - Potential for constant-rate quantum codes (good k/n).
   - Can achieve good distances without huge lattices.
4. Cons:
   - Hard to find LDPC codes that also allow easy decoding.
   - Harder to implement on hardware due to non-local checks.
5. Active research area: quantum expander codes, hypergraph product codes.

References

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