December 8, 2025

No cloning theorem

Quantum states can not be freely copied

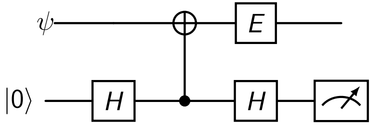
$$
\begin{aligned}
A(\alpha |0\rangle + \beta |1\rangle) &= (\alpha |0\rangle + \beta |1\rangle)(\alpha |0\rangle + \beta |1\rangle) \\
&= \alpha^2 |00\rangle + \alpha\beta |01\rangle + \alpha\beta |10\rangle + \beta^2 |11\rangle \\
&\neq \alpha |00\rangle + \beta |11\rangle \\
&= \alpha A |0\rangle + \beta A |1\rangle
\end{aligned}
$$

Two qubit detection algorithm I

1. $$\psi = \alpha |0\rangle + \beta |1\rangle \xrightarrow{\text{two qubit encoder}} \alpha |00\rangle + \beta |11\rangle .$$
2. Code space $= \mathcal{C} = \mathit{span}\{|00\rangle, |11\rangle\}$.
3. Error space $\mathcal{F} = \mathit{span}\{|01\rangle, |10\rangle\}$.
4. For stabilisers
$$Z_1 Z_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix},$$
$Z_1 Z_2 \psi = -\psi$ if there is an error and ψ if there is no error.

Two qubit detection algorithm II

5. To extract this information, use a syndrome qubit s



6. If there was an error, then the transition would be

$$\psi\,|+\rangle \to -\psi\,|-\rangle \to -\psi\,|1\rangle$$

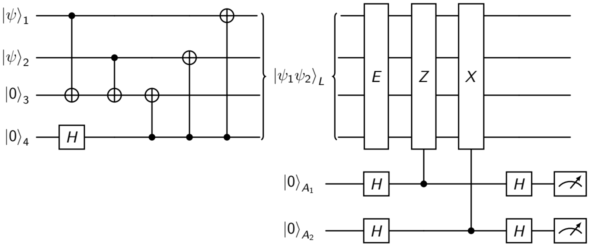
7. In a nutshell, states from $\mathcal{F}$ return a characteristic value of -1 and this can be read from the syndrome qubit.
8. From $|01\rangle$ or $|10\rangle$, we can't go back to the original state. **Detection only**

Stabiliser Codes

1. Split the space into $\mathcal{C}$ Codespace and error subspaces $\mathcal{F}_i$ one for each type of error.
2. Stabiliser group S is a subgroup of $\mathcal{G}_n$.
3. S stabilises $\mathcal{C}$, so it is in the subspace corresponding to 1
4. $\mathcal{F}_i$ is in characteristic subspace corresponding -1.

Two Qubit Detection Code

1. All errors are unitary, so can be reversed by applying $E^\dagger$.
2. The circuit
3. $d = 2$ Indeed, replacing the first and third bits on $|0110\rangle$ yields $|1100\rangle$. So, $t = \lfloor \frac{d}{2} \rfloor = 0$ errors can be corrected.
4. **Detection only**



1. For an (n, k, d) code, k logical qubits can be encoded into n physical qubits.
2. Efficiency of encoding is k/n.
3. On propagation of 1 error, a state can enter one of

Stabiliser Codes

1. The $[[5, 1, 3]]$ Code Capable of correcting all kinds of errors on one logical qubit.
2. Stabiliser Codes [BAD] - difficult to find stabiliser subgroups
3. Alternate - Surface code
4. Stabiliser Codes [BAD] - higher dimension codes have denser parity checks, more susceptible to decoherence
5. Alternate - Low Density Parity Check Code

Surface Codes