RECENT DEVELOPMENTS IN RISK MANAGEMENT

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EXTREME VALUE THEORY

Complications to CVaR / ES

1. CVaR estimates tend to be less stable than VaR for the same confidence level.
   a) Requires a large number of observations to generate a reliable estimate.

2. CVaR is more sensitive to estimation errors than VaR
   a) Depends substantially on the accuracy of the tail model used.

Extreme Value Theory

- Extreme Value Theory (EVT) provides the theoretical foundation for building statistical models **describing extreme events**.
- Used in many fields:
  - Finance
  - Structural Engineering
  - Traffic Prediction
  - Weather Prediction
  - Geological Prediction (Seismic events, flooding, etc.)

Extreme Value Theory

- Extreme Value Theory (EVT) provides the theoretical foundation for building statistical models **describing extreme events**.
- EVT provides the distributions for the following:
  - **Block Maxima (Minima)** – the maximum (or minimum) the variable takes in successive period, for example months or years.
  - **Exceedances** – the values that exceed a certain threshold.

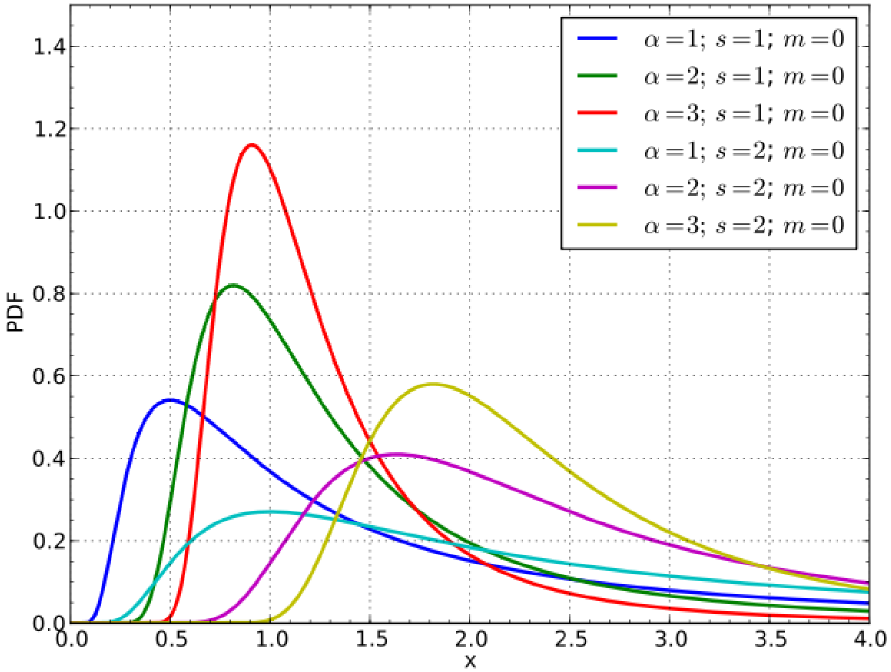
Block Maxima (Minima)

- Trying to build the series of maximum (or minimum) values across time.
  - Example – the highest annual rainfall in Raleigh, NC between 1900 and 2015.

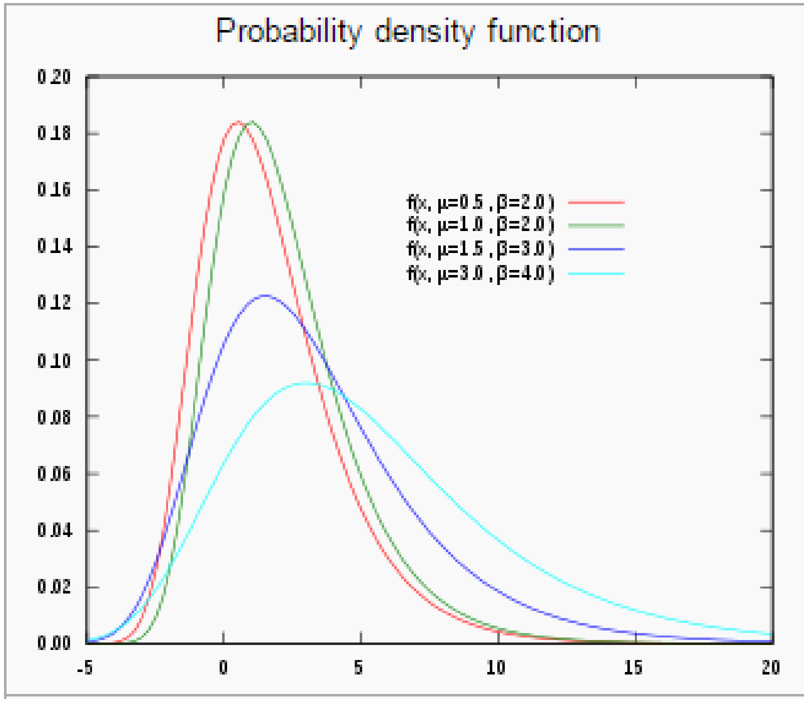
- Popular distributions used (Right Skewed):
  - Fréchet
  - Weibull
  - Gumbel

Block Maxima (Minima)

Frechet



Gumbel



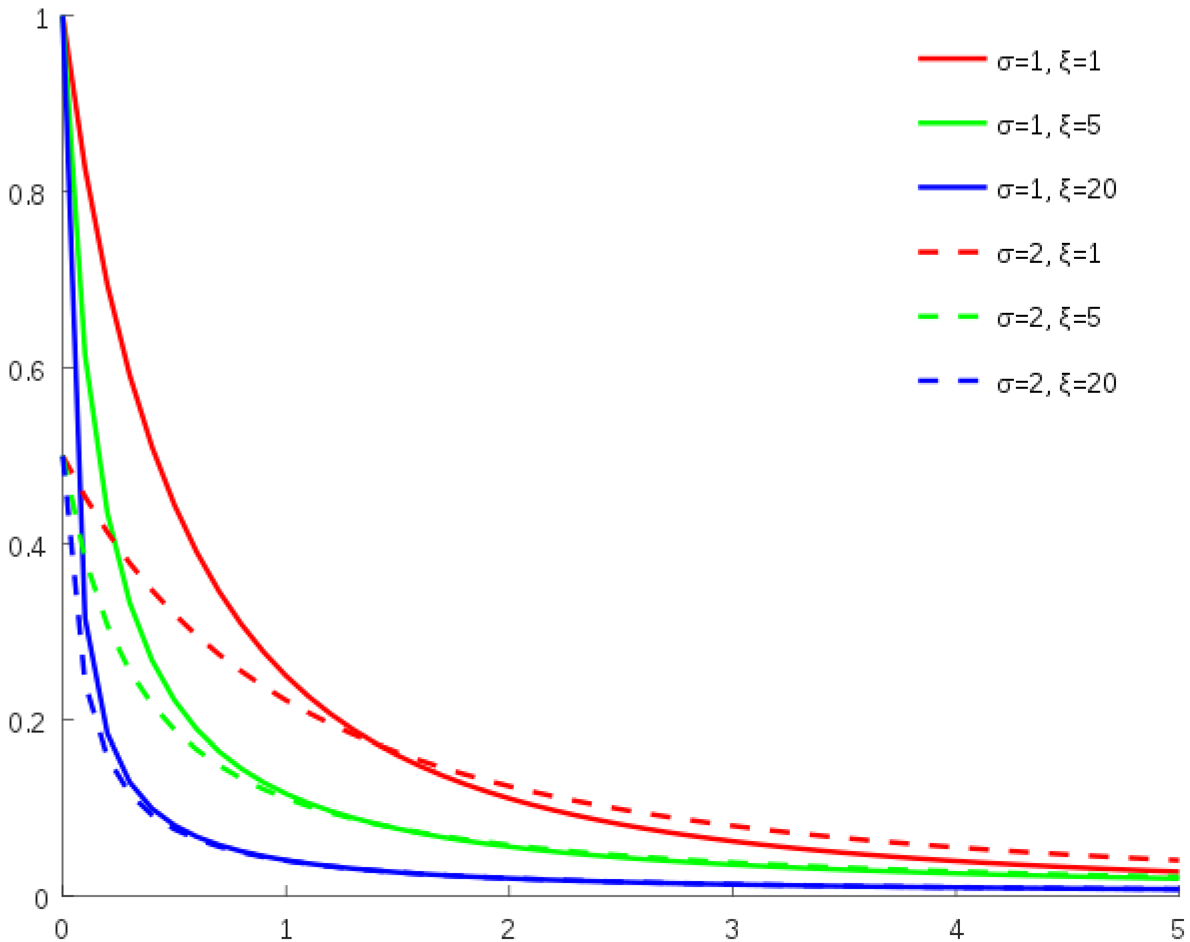
Exceedances

- Trying to understand the distribution of values that exceed a certain threshold.
- Instead of isolating the tail of an overall distribution (limiting the values) we are trying to build a distribution for the tail events themselves.
- Popular distribution:
  - Generalized Pareto

Generalized Pareto

- Named after Italian engineer and economist Vilfredo Pareto.
- Came into popularity with the “Pareto Principle” – more commonly known as the “80-20 Rule.”
- Pareto noted in 1896 that 80% of the land of Italy was owned by 20% of the population.
- Richard Koch authored the book *The 80/20 Principle* to iilustrate some common applications.

Exceedances



Extreme Value Theory

- **Application to CVaR –** more accurate estimates of CVaR, but the math is VERY complicated.
- Need to use maximum likelihood estimation to find which generalized Pareto distribution fits our data the best.
- Choose ξ and β to maximize:

$$\sum_{i=1}^{n_u} \ln\left[\frac{1}{\beta}\left(1+\frac{\xi(v_i-u)}{\beta}\right)^{-1/\xi-1}\right]$$

Extreme Value Theory

- **Application to CVaR –** more accurate estimates of CVaR, but the math is VERY complicated.
- **VaR Calculation:**

$$\mathrm{VaR} = u + \frac{\beta}{\xi}\left\{\left[\frac{n}{n_u}(1-q)\right]^{-\xi} - 1\right\}$$

- **CVaR Calculation:**

$$\mathrm{ES} = \frac{\mathrm{VaR} + \beta - \xi u}{1-\xi}$$

EVT: Two Position Portfolio

- $200,000 invested in MSFT & $100,000 in Apple today.
- You have 500 observations on both returns.
- Calculate the portfolio's value using each one of the historical daily returns:

$$\$200{,}000 \times R_M + \$100{,}000 \times R_A$$

- Using the tail (typically 5%) of the 500 observations, estimate the generalized Pareto distribution parameters.
- Estimate VaR and ES from these.

EVT: Two Position Portfolio

- Parameter estimates:

$$\xi = 0.026 \qquad \beta = 2495.74$$

- VaR = $-\$9{,}458.36$, CVaR = $-\$12{,}129.04$

Comparison Across Techniques

Technique	Value at Risk	Expected Shortfall (CVaR)
Delta-Normal	$-\$7{,}833.66$	$-\$8{,}884.81$
Historical (Common)	$-\$9{,}055.16$	$-\$12{,}392.92$
Historical (Stressed)	$-\$18{,}639.23$	$-\$24{,}484.47$
Historical (Weighted)	$-\$10{,}941.70$	$-\$13{,}751.47$
Monte Carlo	$-\$12{,}193.82$	$-\$15{,}090.77$
Extreme Value Theory	$-\$9{,}458.36$	$-\$12{,}129.04$

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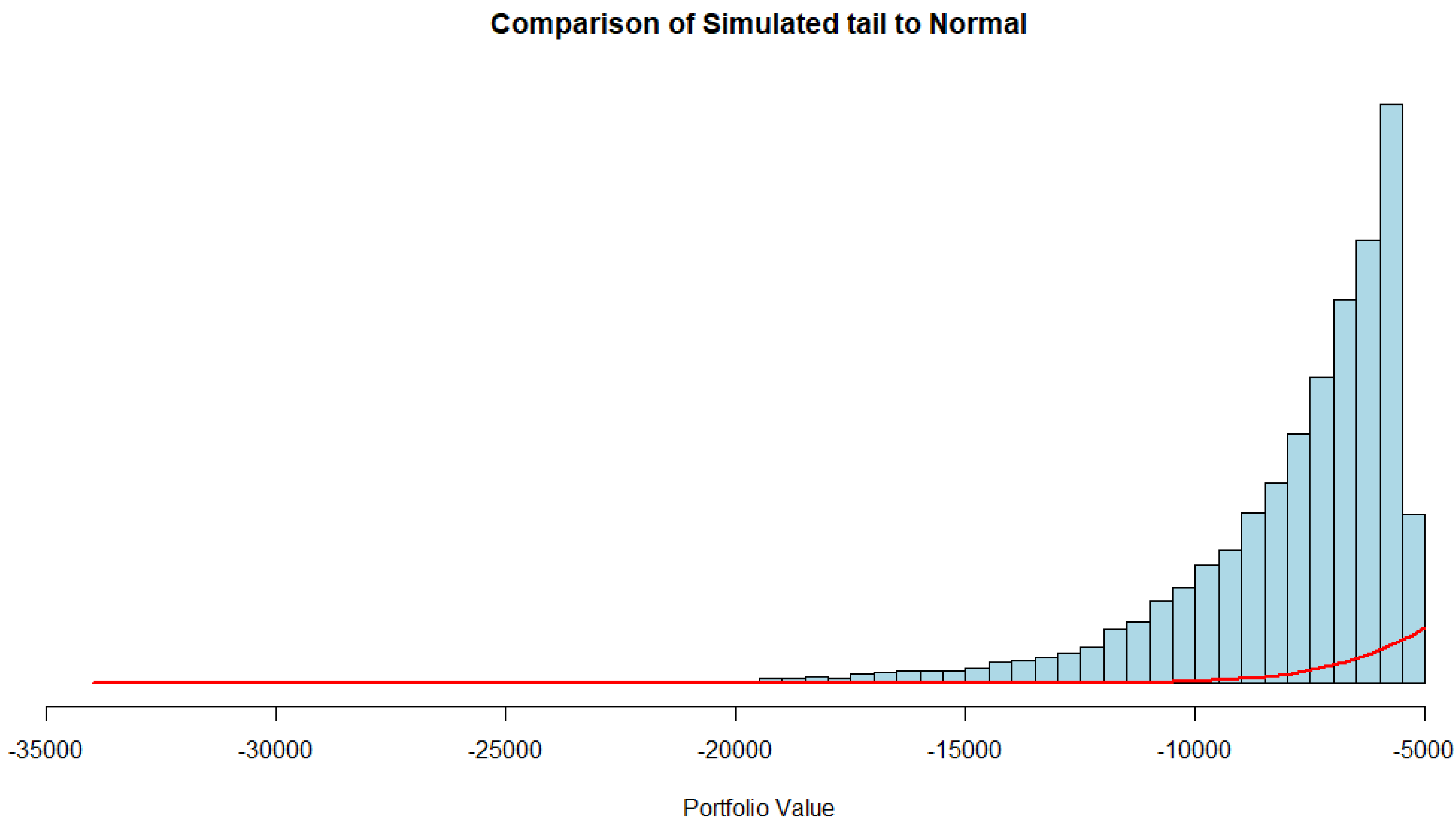

EXTENSIONS TO VALUE AT RISK

VaR "like" Extensions

- There have been many additions to Value at Risk calculations:
  1. Delta – Gamma VaR
  2. Delta – Gamma Monte Carlo Simulation
  3. Incremental VaR
  4. Component VaR
  5. Backtesting VaR

Delta – Gamma Approximation

- Improve the Delta – Normal approach by taking into account higher derivatives than the first derivative only.
- In the expansion of the derivatives we can look at the first two:

$$dV = \frac{\partial V}{\partial RF} \cdot dRF + \frac{1}{2} \cdot \frac{\partial^2 V}{\partial RF^2} \cdot dRF^2 + \cdots$$

$$dV = \Delta \cdot dRF + \frac{1}{2} \cdot \Gamma \cdot dRF^2$$

Delta – Gamma Approximation

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- In the expansion of the derivatives we can look at the first two:

$$dV = \Delta \cdot dRF + \frac{1}{2} \cdot \Gamma \cdot dRF^2$$

- Essentially takes the variance component into account in building the Gamma portion of the equation.
- Delta – Normal typically **underestimates** VaR when assumptions do not hold.
- Delta – Gamma tries to correct for this, but is computationally more intensive.

Delta – Gamma Monte Carlo

- The Normality assumption might still be unreliable.
- The Delta – Gamma Monte Carlo simulation technique simulates the risk factor before using the second order Taylor expansion to create simulated movements in the portfolio's value.
- Just like with regular simulation, the VaR is calculated using the simulated distribution of the portfolio.

Delta – Normal vs. Delta – Gamma

- If you have a large number of things to evaluate in a portfolio with few options, the Delta – Normal is fast and efficient enough.
- If you have only a few sources of risk and substantial complexity, the Delta – Gamma is better.
- If you have a large number of risk factors and substantial complexity, the traditional Monte Carlo simulation is still best.

Marginal VaR

- How much will the VaR change if we invest one more dollar in a position?
- In other words, the first derivative of VaR with respect to the weight of a certain position in the portfolio.
- Assuming Normality:

$$\Delta VaR_i = \alpha \times \frac{\mathrm{Cov}(R_i, R_p)}{\sigma_p}$$

Incremental VaR

- The change in VaR due to the addition of a new position in the portfolio.
- Used whenever an institution wants to evaluate the effect of a proposed trade or change in the portfolio of interest.
- How is this different than Marginal VaR?
  - The change can be significant (not marginal – unit of one)
  - The change can be in a vector of positions, not just one.

Incremental VaR

- Trickier to calculate as you now need to calculate VaR under both portfolios and take their difference.
- You can do a Taylor series expansion to get an approximation:

$$IVaR = (\Delta VaR)^T \cdot c$$

- ΔVaR is a vector of marginal VaR's
- c is a vector of additional exposures in our portfolio
  - Example – $10,000 in bonds and $20,000 in options
- The approximation is quicker to calculate then taking the difference of the two correct calculations, but is not as accurate.

Component VaR

- Decompose VaR into its basic components in a way that takes into account the diversification of the portfolio.
- It is a “partition” of the portfolio VaR that indicates the change of VaR if a given component was deleted.
- The Component VaR is defined in terms of marginal VaR:

$$\text{Component } VaR = \Delta VaR_i \times (\$ \text{ value of component i})$$

- The sum of all component VaR’s is the total portfolio VaR.

CREDIT RISK MODELS

Overview

- Financial institutions are using credit risk modeling to calculate the expected loss, VaR, and CVaR, occurring due to the default of their customers.

Key Variables to Measure

- Probability of Default – the probability that a customer will default within a certain time interval.
- Loss Given Default – the amount lost assuming that the default occurs.
- Migration Probability – the probability that a customer will be downgraded
  - Example: AAA to AA+ rating

Popular Credit Risk Models

1. CreditPortfolioView
2. CreditRisk+
3. CreditMetrics

Popular Credit Risk Models

- Main Ideas:
  - The probability of default depends on set of macroeconomic factors and attributes of individual (risk factors).
  - Correlation among macroeconomic factors, default rates, and migration probabilities.
  - Default rates need to adjust across time.

CreditMetrics

- Uses Monte Carlo simulation to create a **portfolio loss distribution** at the horizon date.
- Uses a transition matrix to determine the probabilities that an obligor's credit rating will be upgraded or downgraded, or that it defaults.
- Calculates the portfolio value by randomly simulating the credit quality of each obligor.