FACILITY LOCATION PROBLEM: (Slide deck #3)

The problem: A company has 10 distribution centers that MUST have a certain number of units each week (let's assume this information is on a weekly basis). The units will be shipped from the available warehouses to each location. However, we need to build the warehouses. Only 6 of the 10 locations can be utilized as warehouses. Problem: we want to find the best 6 locations for the warehouses so we will minimize cost. There are two sources for the cost….(1) the shipping cost (to ship to the distribution

Location	Alb	Boise	Dall	Denv	Hous	Okla	Phoe	Salt	SanA	Wich
Albuquerque	0	47	32	22	42.5	27	23	30	36.5	29.5
Dallas	32	79.5	0	39	12.5	10.5	50	63	13.5	17
Denver	21	42	39	0	51.5	31.5	40.5	24	47.5	26
Houston	42.5	91	12.5	51.5	0	23	58	72	10	31
Pheonix	23	49	50	40.5	58	49	0	32.5	50	52
San Antonio	36.5	83.5	13.5	47.5	10	24	50	66.5	0	32

centers) and (2) the cost to build the warehouse (this is a fixed, one-time cost).

Center / Warehouse	Volume	Capacity	Cost
Albuquerque (W)	3,200	16,000	$140,000
Boise	2,500		
Dallas (W)	6,800	20,000	$150,000
Denver (W)	4,000	10,000	$100,000
Houston (W)	9,600	10,000	$110,000
Oklahoma City	3,500		
Phoenix (W)	5,000	12,000	$125,000
Salt Lake City	1,800		
San Antonio (W)	7,400	10,000	$120,000
Wichita	2,700		

The cost: So the cost to send one item from Albuq to Albuq is $0, from Albuq to Boise is $47 per unit, from Albuq to Dallas is $32 per unit.

So the cost for shipping is (assume $X_{1,1}, \ldots X_{6,10}$ are the units of items "shipped from Warehouse i to location j, $X_{i,j}$): $0*X_{1,1}+47*X_{1,2}+32*X_{1,3}+22*X_{1,4}+42.5*X_{1,5}+27*X_{1,6}+23*X_{1,7}+30*X_{1,8}+36.5*X_{1,9}+29.5*X_{1,10}$ + $32*X_{2,1}+79.5*X_{2,2}+0*X_{2,3}+39*X_{2,4}+12.5*X_{2,5}+10.5*X_{2,6}+50*X_{2,7}+63*X_{2,8}+13.5*X_{2,9}+17*X_{2,10}$ +

(you write out the rest!!!)

How many decision variables are there for shipping?

We are not done with cost!!! We still need to include cost for building the warehouses:

$140000*Y_1 + 150000*Y_2 + 100000*Y_3 + 110000*Y_4 + 125000*Y_5 + 125000*Y_6$

Where Y_1, Y_2….Y_6 are binary variables (1 if we are building that warehouse and 0 otherwise).

Therefore there are a total of 66 (60 +6) decision variables!!!

Now let's represent this in terms of sums, vectors and matrices:

Let C[i,j] represent shipping cost from i to j, X[i,j] represent number of units from i to j, demand is the vector of demands for each distribution center, FixedCost is the Fixed cost for each potential warehouse, capacity is the amount each warehouse can hold, and Y[j] is the binary decision variable for warehouse

$$C=\begin{bmatrix} c_{1,1} & \cdots & c_{1,10} \\ \vdots & \ddots & \vdots \\ c_{6,1} & \cdots & c_{6,10} \end{bmatrix}$$

$$X=\begin{bmatrix} x_{1,1} & \cdots & x_{1,10} \\ \vdots & \ddots & \vdots \\ x_{6,1} & \cdots & x_{6,10} \end{bmatrix}$$

Demand=[3200 2500 6800 4000 9600 3500 5000 1800 7400 2700]

FixedCost=[140000 150000 100000 110000 125000 120000]

Capacity=[16000 20000 10000 10000 12000 12000]

For the shipping cost:

$\Sigma\Sigma c_{ij}x_{ij}$

For the warehouse:

$\Sigma \text{FixedCost}_i y_i$

Putting this all together:

$\Sigma_i(\Sigma_j c_{ij}x_{ij} + \text{FixedCost}_i y_i)$

In SAS sum {i in 1..6} (sum{j in 1..10} c[i,j]*x[i,j] + FixedCost[i]*y[i])

OR

set Warehouse=/Albuq Dall Denver Houston Pheonix SA /;

set Distrib = /Albuq Boise Dall Denv Hous Okla Phoe Salt SA Witch/;

then

sum { i in Warehouse} (sum {j in Distrib} c[i,j]*x[i,j] + FixedCost[i]*y[i])

Now for constraints:

Need demand constraint (I am making this a binding constraint)…in other words, Albuqueqe MUST have 3200 for its distribution center; Boise MUST have 2500 for its distribution center, etc.

Con Albuq: x[1,1] + x[2,1] + x[3,1] + x[4,1] + x[5,1] + x[6,1] = 3200

Con Boise: x[1,2]+x[2,2]+x[3,2]+ x[4,2]+x[5,2]+ x[6,2] = 2500

Now you write out the other 8:

This can be simplified to:

Con dem { j in 1..10}: sum{ i in 1..6} x[i,j]=demand[j]

OR

Con dem {j in Distrib}: sum{ i in Warehouse} x[i,j] = demand[j]

Linking Constraint:

Recall…x $\leq$ Max(x)*y

In this case the "x" is the amount from that warehouse (sum across rows), Max(x) is the capacity for that warehouse, and y is the binary decision variable:

Con Link1: x[1,1] + x[1,2] +….x[1,10] <= 16000*y[1]

Con Link2: x[2,1] + x[2,2]+…x[2,10] <= 20000*y[2]

Now you write out the other 4:

This can be simplified to:

Con Link { i in 1..6}: sum{j in 1..10} x[i,j] - Capacity[i]*y[i] <=0

OR

Con Link { i in Warehouse}: sum{j in Distrib} x[i,j] - Capacity[i]*y[i] <=0

You can have the capacity for each warehouse constraint, but it is repetitive with the above constraint.