(handwritten, top right) 做一個同高度值相同

Homework 4

Price a lookback put with the binomial tree model. The payoff function of the lookback put is as follows.

$$\text{Payoff}_t = \max(S_{\max,t} - S_t, 0), \text{ where } S_{\max,t} = \max S_u, \text{ for } u = 0, \Delta t, 2\Delta t, ..., t.$$

- Basic requirement (80 points):

(i) Implement the binomial tree model to price both European and American lookback puts.

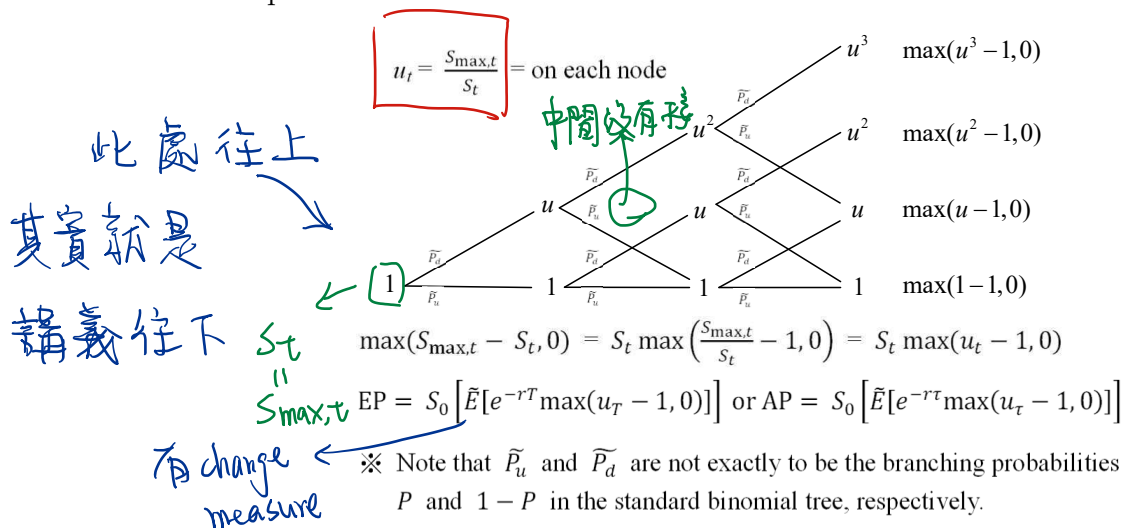(ii) Implement the Monte Carlo simulation to price European lookback puts.

(Inputs: $S_t, r, q, \sigma, t, T, S_{\max,t}, n$, number of simulations, number of repetitions. Outputs: Option values for both methods and 95% confidence level for Monte Carlo simulation.)

- Bonus 1 (5 points):

Based on the same binomial tree framework, devise and implement a quick way to determine the $S_{\max}$ list for each node.

- Bonus 2 (10 points): *(handwritten)* 以當前股價為計價單位

Implement the method in Cheuk and Vorst (1997) to price European and American lookback puts.

(handwritten annotations on figure) 此處往上 其實就是 講義往下；中間沒有形；$S_t = S_{\max,t}$；to change measure

$u_t = \dfrac{S_{\max,t}}{S_t}$ = on each node

$$\max(S_{\max,t} - S_t, 0) = S_t \max\left(\frac{S_{\max,t}}{S_t} - 1, 0\right) = S_t \max(u_t - 1, 0)$$

$$\text{EP} = S_0\left[\tilde{E}[e^{-rT}\max(u_T - 1, 0)]\right] \text{ or } \text{AP} = S_0\left[\tilde{E}[e^{-r\tau}\max(u_\tau - 1, 0)]\right]$$

※ Note that $\widetilde{P_u}$ and $\widetilde{P_d}$ are not exactly to be the branching probabilities P and $1 - P$ in the standard binomial tree, respectively.

- Reference

Cheuk and Vorst (1997), "Currency lookback options and observation frequency: a binomial approach," *Journal of International Money and Finance* 16, pp. 173–187.

(handwritten, bottom) $\widehat{P_u} \cdot \widehat{P_d}$ 找出來