

Carnegie Mellon University

DATABASE SYSTEMS

Sorting & Aggregations

LECTURE #11 » 15-445/645 FALL 2025 » PROF. ANDY PAVLO

ADMINISTRIVIA

Homework #3 is due Sunday Oct 5th @ 11:59pm

Mid-Term Exam is on Wednesday Oct 8th

- Lectures #01–11 (inclusive)
- Study guide is now available ([@126](#))
- Bring your CMU ID!

Project #2 is due Sunday Oct 26th @ 11:59pm

COURSE OUTLINE

We are now going to talk about how to execute queries using the DBMS components we have discussed so far.

Next four lectures:

- Operator Algorithms
- Query Processing Models
- Runtime Architectures

Query Planning

Operator Execution

Access Methods

Buffer Pool Manager

Disk Manager

QUERY PLAN

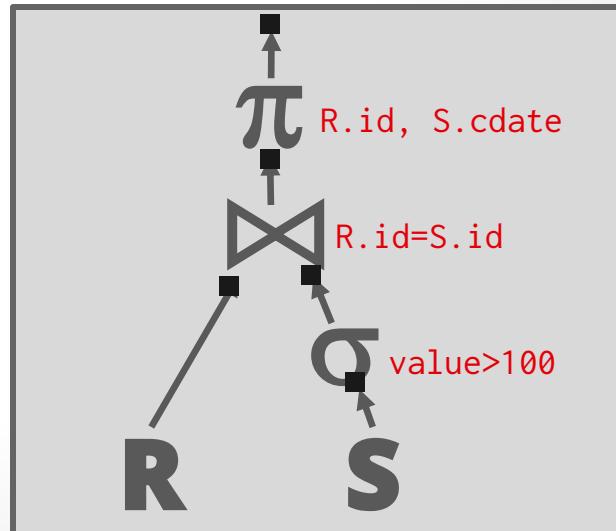
The operators are arranged in a tree.

Data flows from the leaves of the tree up towards the root.

→ We will discuss the granularity of the data movement next week.

The output of the root node is the result of the query.

```
SELECT R.id, S.cdate
  FROM R JOIN S
    ON R.id = S.id
   WHERE S.value > 100
```



DISK-ORIENTED DBMS

Just like it cannot assume that a table fits entirely in memory, a disk-oriented DBMS cannot assume that query results fit in memory.

We will use the buffer pool to implement algorithms that need to spill to disk.

We are also going to prefer algorithms that maximize the amount of sequential I/O.

WHY DO WE NEED TO SORT?

Relational model/SQL is unsorted.

Queries may request that tuples are sorted in a specific way (**ORDER BY**).

But even if a query does not specify an order, we may still want to sort to do other things:

- Trivial to support duplicate elimination (**DISTINCT**).
- Bulk loading sorted tuples into a B+ Tree index is faster.
- Aggregations (**GROUP BY**).

IN-MEMORY SORTING

If data fits in memory, then the DBMS can use a standard sorting algorithm.

- Optimized algorithms if data is mostly sorted ([VergeSort](#)).
- Otherwise use your favorite ([QuickSort](#), [TimSort](#), [RadixSort](#)).
- Many online [visualization tools](#).

If data does not fit in memory, then we need to use a technique that is aware of the cost of reading and writing disk pages ...

SORTING ALGORITHMS

For a given input **run** (i.e., list of key/value pairs), sort it based on a comparison function and sorting parameters.

Key: The attribute(s) to compare to compute the sort order.

Value: Two choices
 → Tuple (early materialization).
 → Record ID (late materialization).

Early Materialization

K_1	\langle Tuple Data \rangle
K_2	\langle Tuple Data \rangle
⋮	⋮

Late Materialization



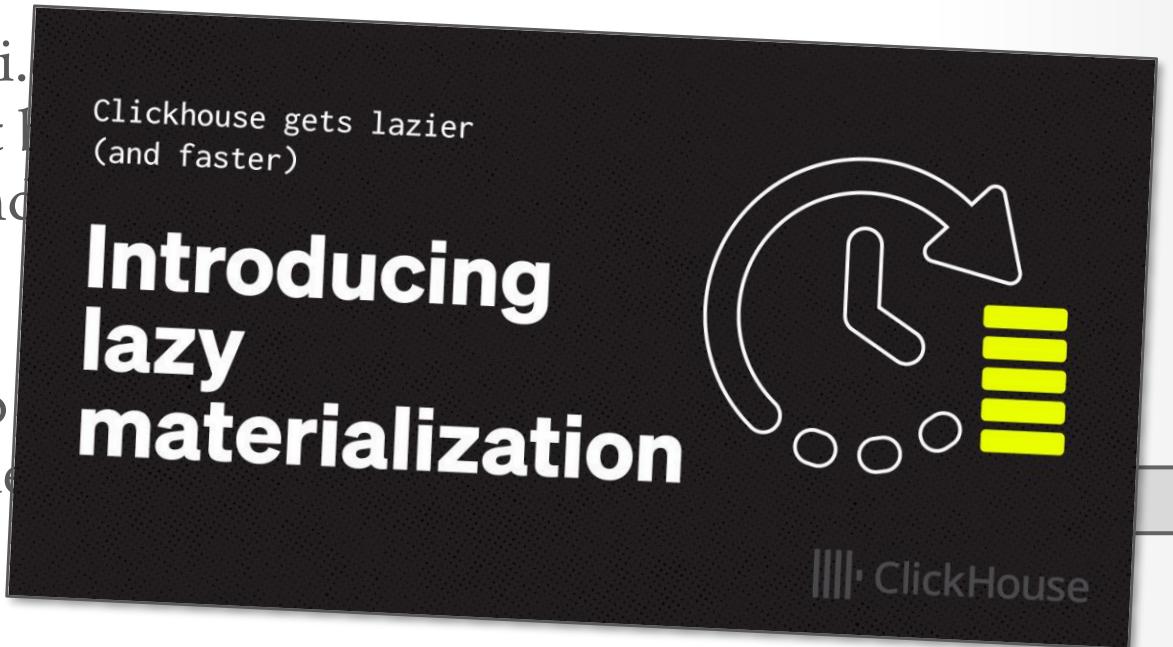
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TODAY'S AGENDA

Top-N Heap Sort

External Merge Sort

Aggregations

⚡DB Flash Talk: MotherDuck

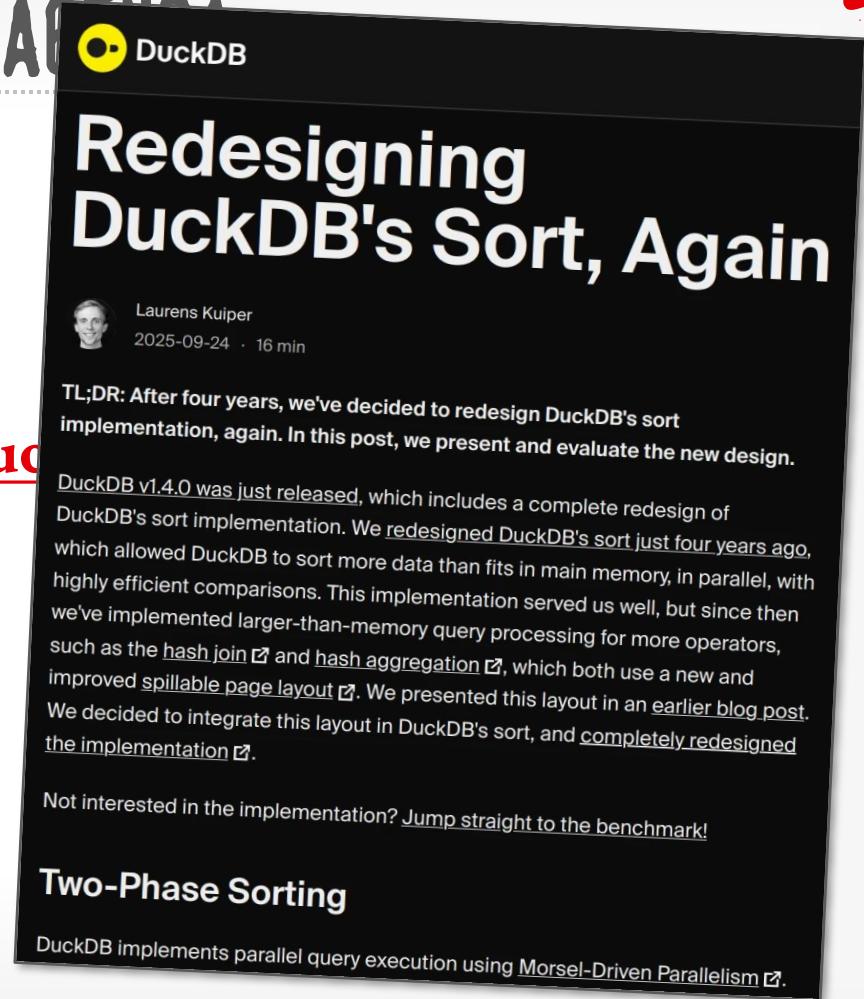
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⚡DB Flash Talk: MotherDuck



The screenshot shows a blog post from the DuckDB website. The title is "Redesigning DuckDB's Sort, Again" in large white font. Below the title is a small profile picture of a man with glasses and the name "Laurens Kuiper". The date "2025-09-24" and "16 min" are also present. The post begins with a "TL;DR" summary: "After four years, we've decided to redesign DuckDB's sort implementation, again. In this post, we present and evaluate the new design." The main body of the text discusses the history of DuckDB's sort implementation, mentioning its redesign four years ago, its efficiency, and its integration with larger-than-memory query processing. It also mentions improvements like hash join and hash aggregation, and the spillable page layout. The author notes that they completely redesigned the implementation. At the bottom, there is a link to jump straight to the benchmark.

Redesigning DuckDB's Sort, Again

Laurens Kuiper
2025-09-24 · 16 min

TL;DR: After four years, we've decided to redesign DuckDB's sort implementation, again. In this post, we present and evaluate the new design.

DuckDB v1.4.0 was just released, which includes a complete redesign of DuckDB's sort implementation. We redesigned DuckDB's sort just four years ago, which allowed DuckDB to sort more data than fits in main memory, in parallel, with highly efficient comparisons. This implementation served us well, but since then we've implemented larger-than-memory query processing for more operators, such as the [hash join](#) and [hash aggregation](#), which both use a new and improved [spillable page layout](#). We presented this layout in an [earlier blog post](#). We decided to integrate this layout in DuckDB's sort, and [completely redesigned](#) the implementation.

Not interested in the implementation? [Jump straight to the benchmark!](#)

Two-Phase Sorting

DuckDB implements parallel query execution using [Morsel-Driven Parallelism](#).

TOP-N HEAP SORT

If a query contains an **ORDER BY** with a **LIMIT**, then the DBMS only needs to scan the data once to find the top- N elements.

Ideal scenario for HeapSort if the top- N elements fit in memory.

→ Scan data once, maintain an in-memory sorted priority queue.

```
SELECT * FROM enrolled  
ORDER BY sid ASC  
FETCH FIRST 4 ROWS  
WITH TIES
```

Original Data

3	4	6	2	9	1	4	4	8
---	---	---	---	---	---	---	---	---

Sorted Heap

--	--	--	--

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3			
---	--	--	--

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---	---	--	--

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---	---	---	---	---	---	---	---	---



Sorted Heap

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---	---	---	---

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Skip!

Sorted Heap

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Sorted Heap

1	2	3	4
---	---	---	---

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---	---	---	---	---	---	---	---	---



Skip and done!

Sorted Heap

1	2	3	4	4	4		
---	---	---	---	---	---	--	--

EXTERNAL MERGE SORT

Divide-and-conquer algorithm that splits data into separate runs, sorts them individually, and then combines them into longer sorted runs.

Phase #1 – Sorting

- Sort chunks of data that fit in memory and then write back the sorted chunks to a file on disk.
- Pick your favorite in-memory sorting algorithm.

Phase #2 – Merging

- Combine sorted runs into larger chunks.

2-WAY EXTERNAL MERGE SORT

We will start with a simple example of a 2-way external merge sort.

→ “2” is the number of runs to merge into a new run for each pass.

Data is broken up into **N** pages.

The DBMS has a finite number of **B** buffer pool pages to hold input and output data.

SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

Pass #0

- Read one page of the table into memory
- Sort page into a “run” and write it back to disk
- Repeat until the whole table has been sorted into runs

Pass #1,2,3,...

- Recursively merge pairs of runs into runs twice as long
- Need at least 3 buffer pages (2 for input, 1 for output)

SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

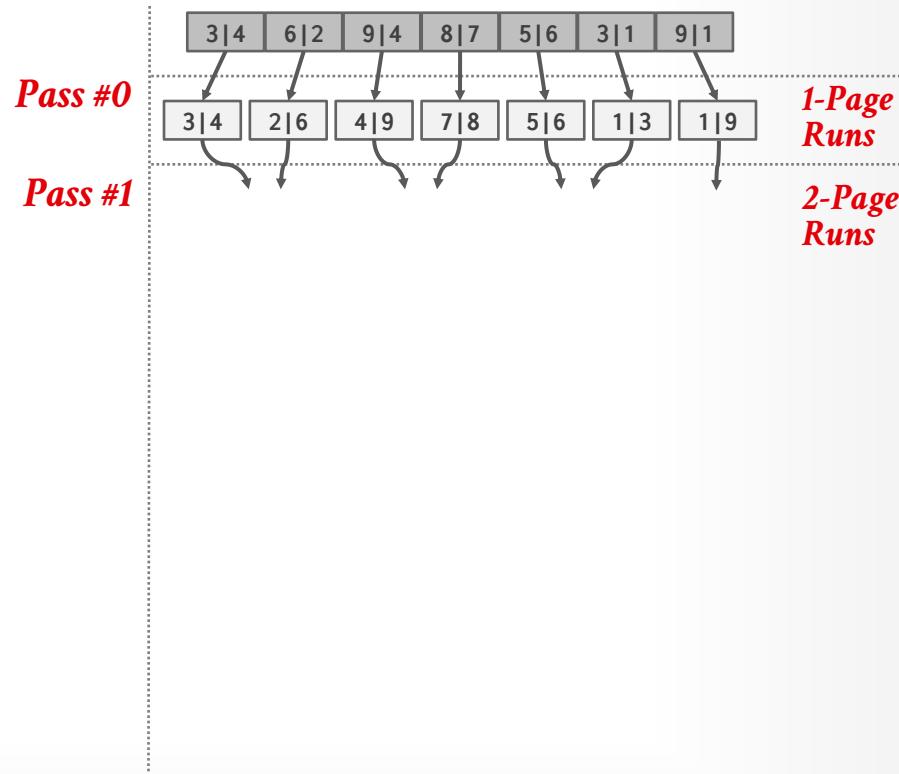
In each pass, the DBMS reads and writes every page in the file.

Number of passes

$$= 1 + \lceil \log_2 N \rceil$$

Total I/O cost

$$= 2N \cdot (\# \text{ of passes})$$



SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

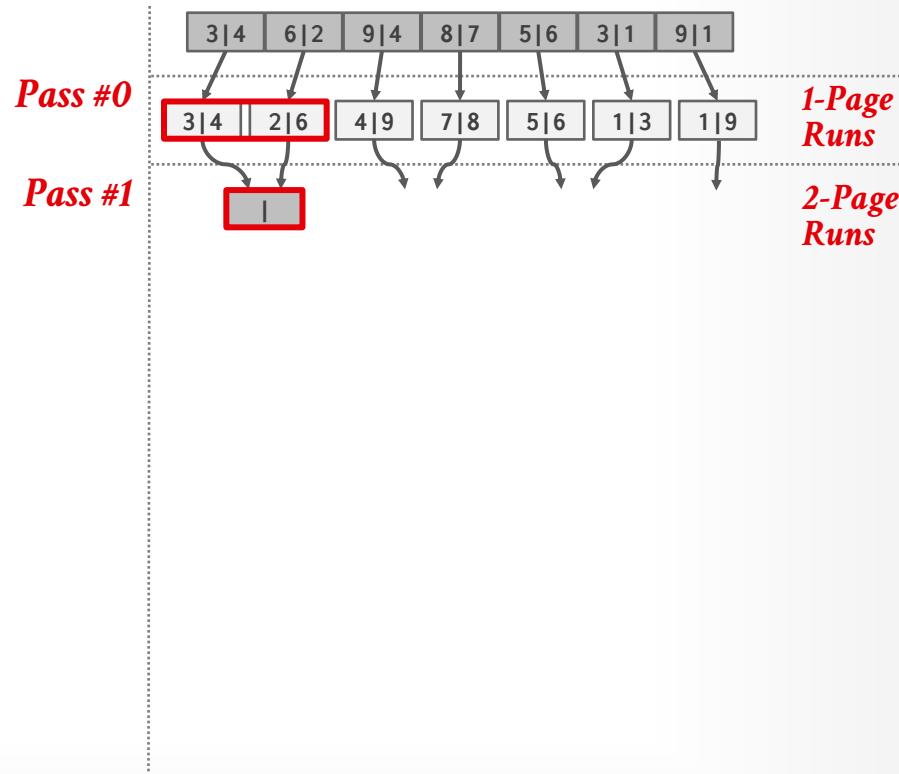
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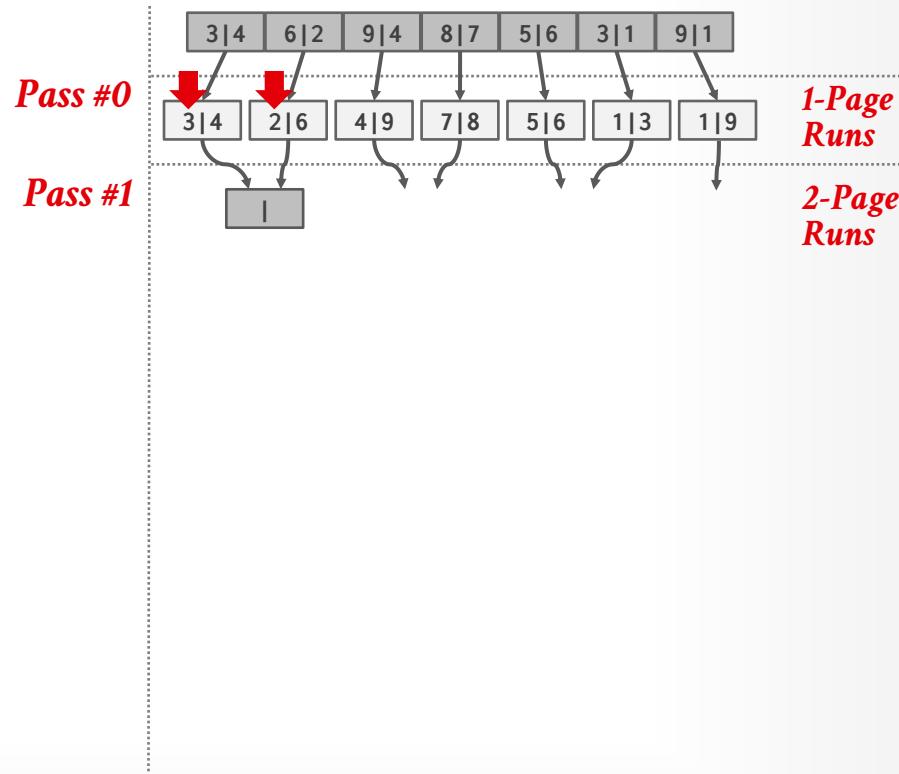
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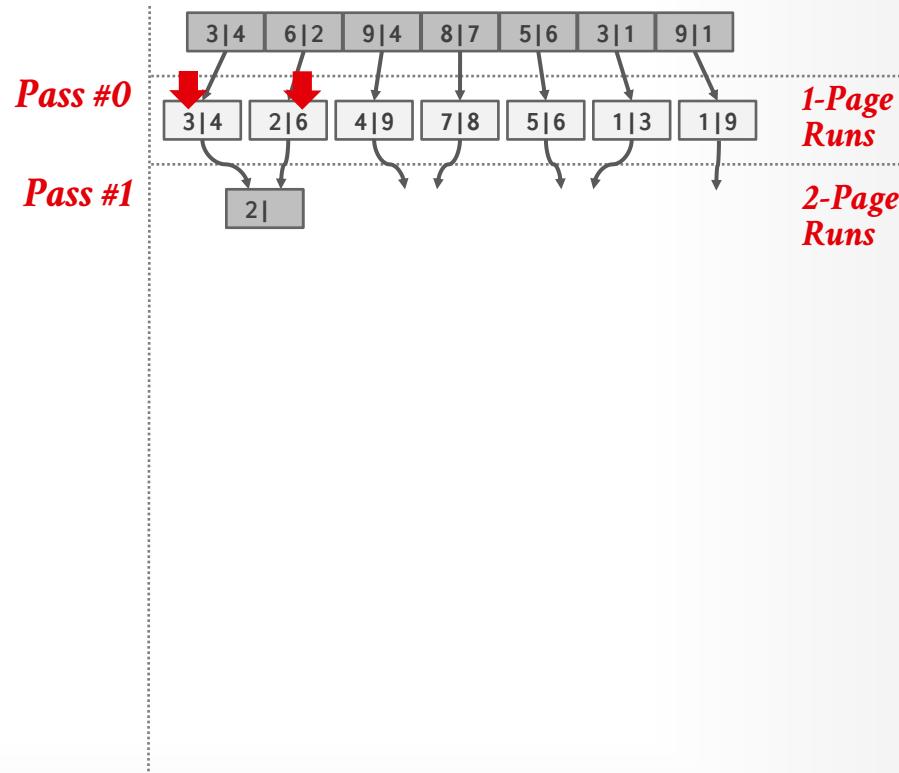
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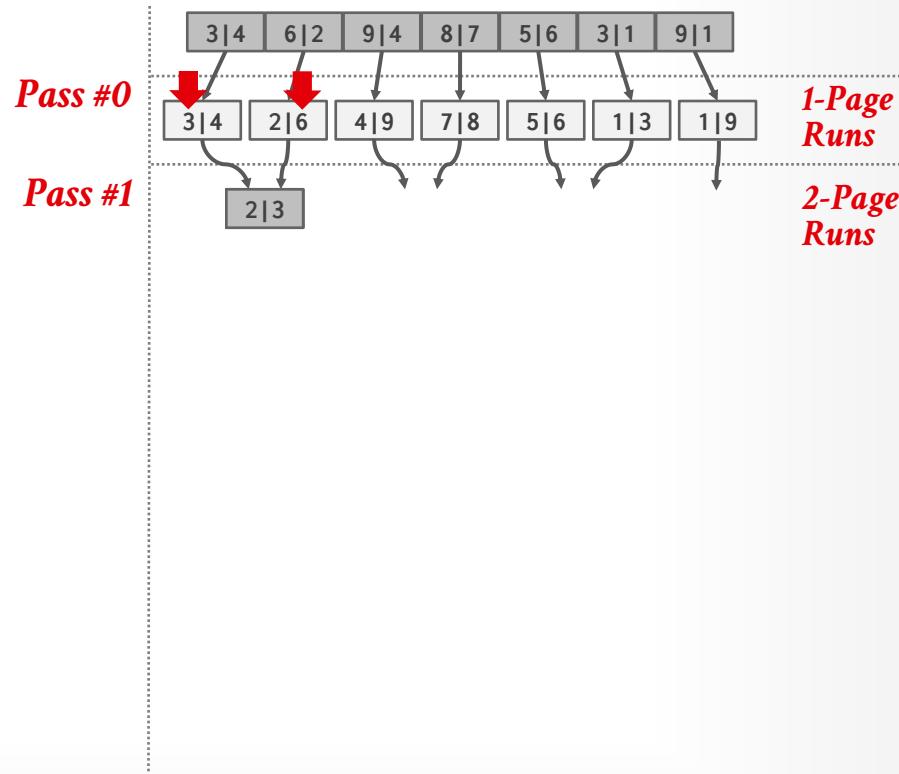
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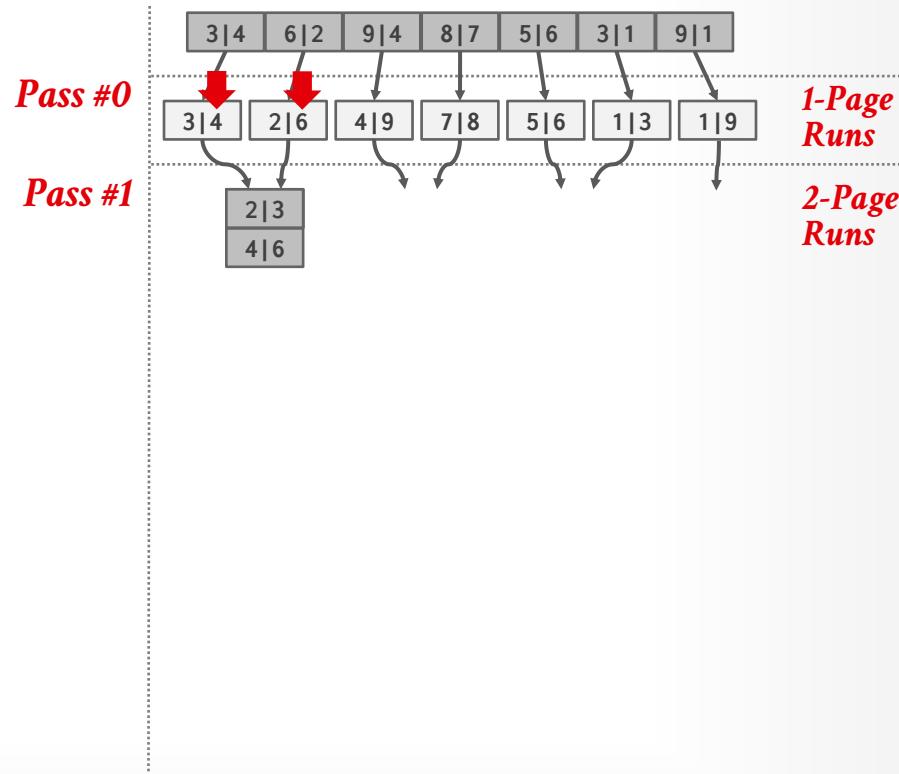
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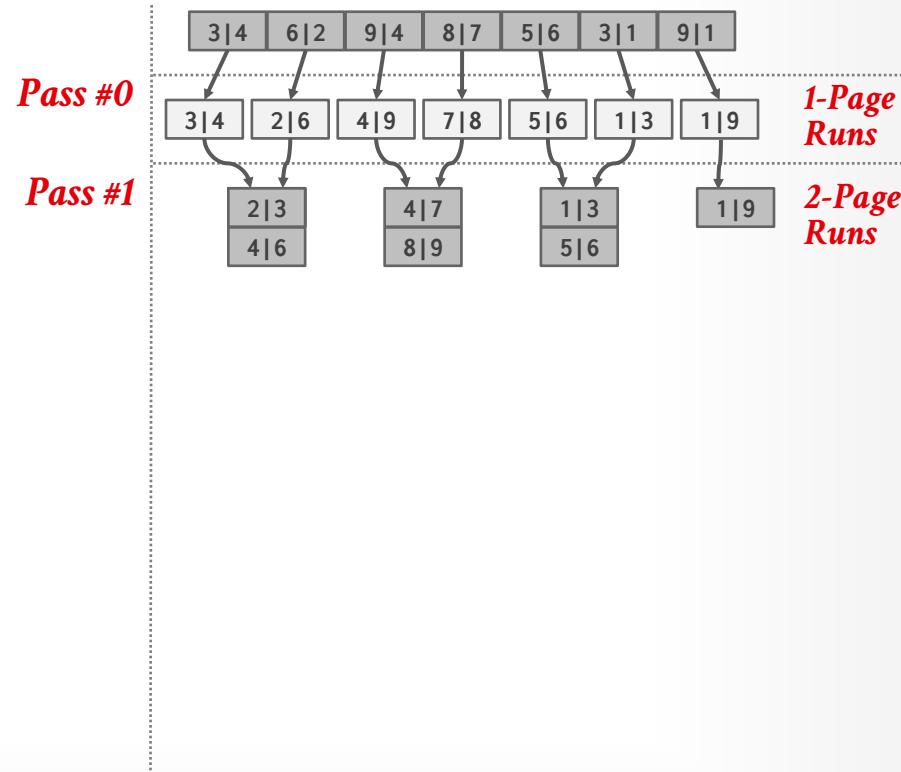
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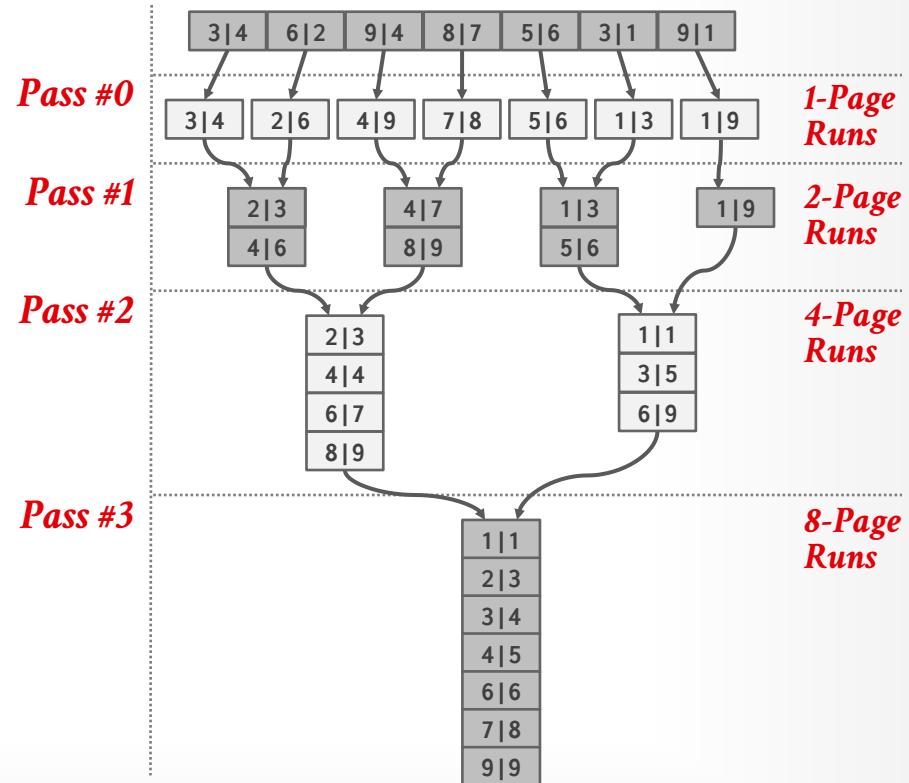


SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

In each pass, the DBMS reads and writes every page in the file.

Number of passes
 $= 1 + \lceil \log_2 N \rceil$

Total I/O cost
 $= 2N \cdot (\# \text{ of passes})$



SIMPLIFIED 2-WAY EXTERNAL MERGE SORT

This algorithm only requires three buffer pool pages to perform the sorting (**B=3**).

→ Two input pages, one output page

But even if we have more buffer space available (**B>3**), it does not effectively utilize them if the worker must block on disk I/O...

GENERAL EXTERNAL MERGE SORT

Pass #0

- Use B buffer pages
- Produce $\lceil N/B \rceil$ sorted runs of size B

Pass #1,2,3,...

- Merge $B-1$ runs (i.e., M-way merge)

Number of passes = $1 + \lceil \log_{B-1} \lceil N/B \rceil \rceil$

Total I/O Cost = $2N \cdot (\# \text{ of passes})$

EXAMPLE

Determine how many passes it takes to sort 108 pages with 5 buffer pool pages: **$N=108, B=5$**

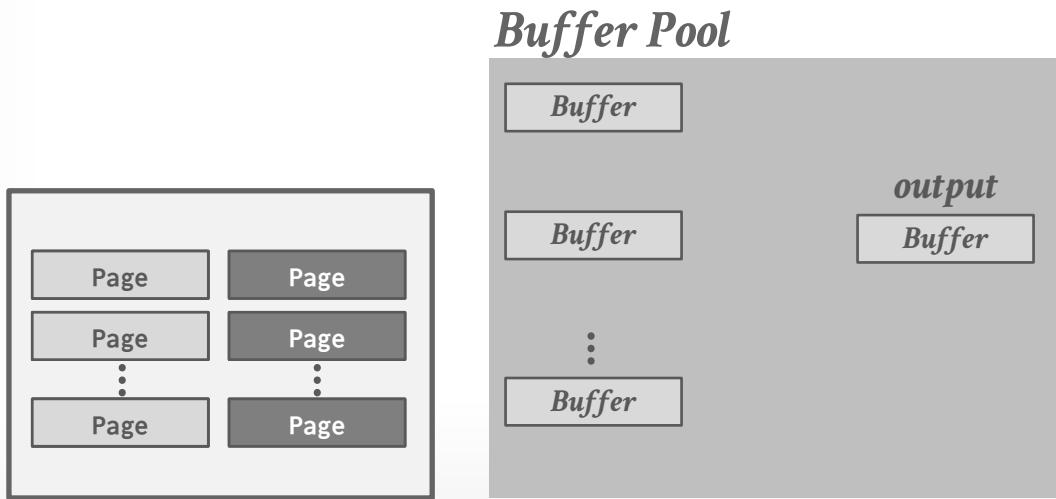
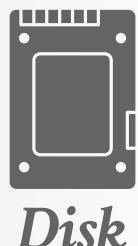
- Pass #0: $[N / B] = [108 / 5] = 22$ sorted runs of 5 pages each (last run is only 3 pages).
- Pass #1: $[N' / B-1] = [22 / 4] = 6$ sorted runs of 20 pages each (last run is only 8 pages).
- Pass #2: $[N'' / B-1] = [6 / 4] = 2$ sorted runs, first one has 80 pages and second one has 28 pages.
- Pass #3: Sorted file of 108 pages.

$$\begin{aligned}1 + \lceil \log_{B-1} [N / B] \rceil &= 1 + \lceil \log_4 22 \rceil = 1 + \lceil 2.229... \rceil \\&= 4 \text{ passes}\end{aligned}$$

DOUBLE BUFFERING

Prefetch next run in the background and store in a second buffer while processing the current run.

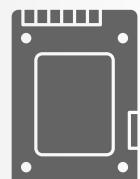
- Overlap CPU and I/O operations
- Reduces effective buffers available by half



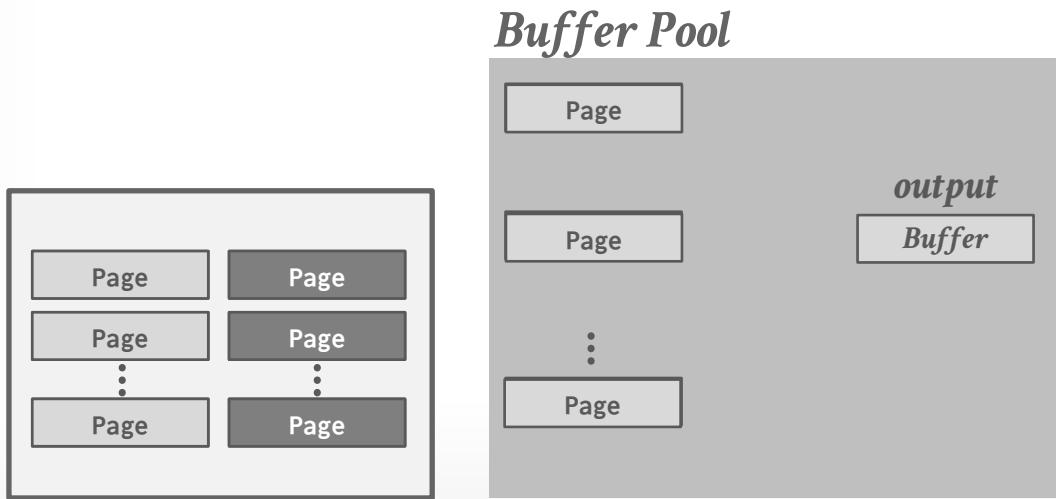
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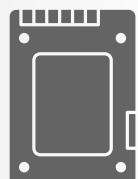
Disk



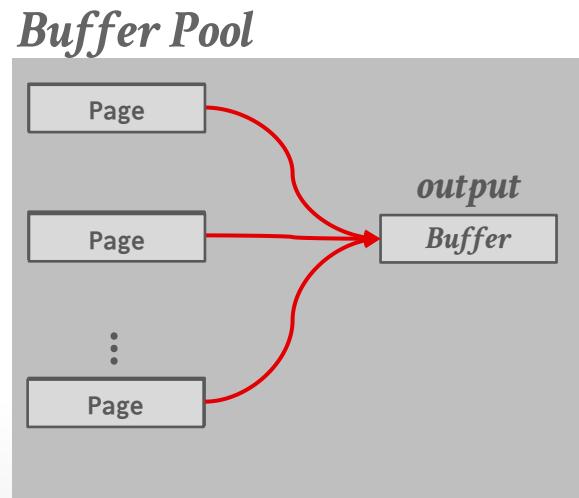
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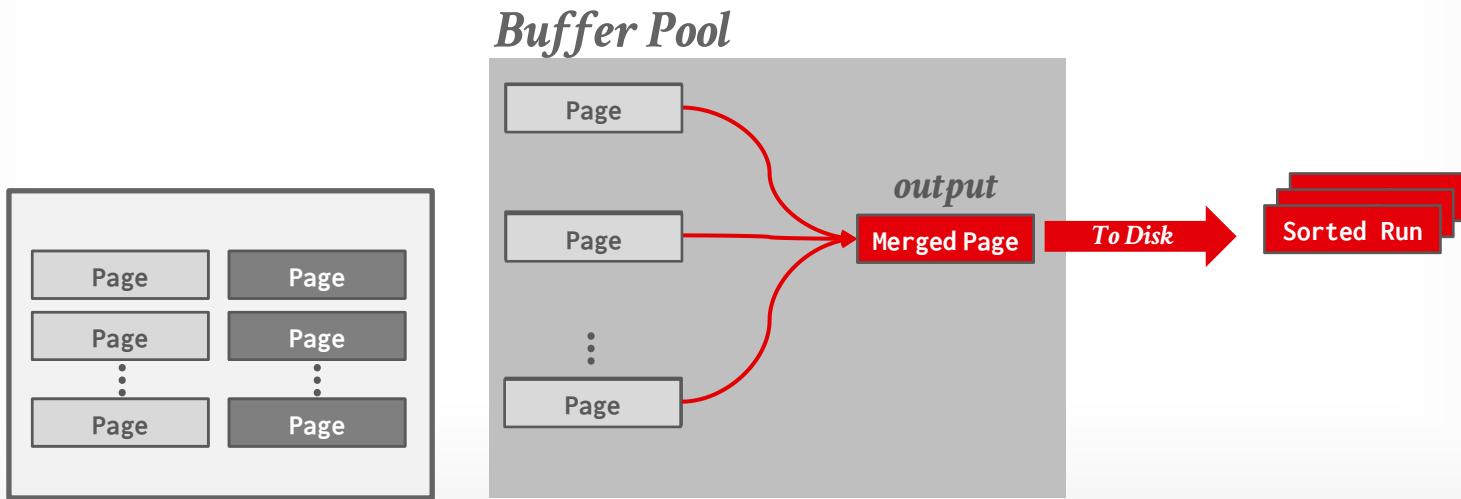
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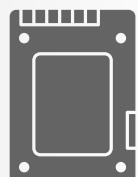
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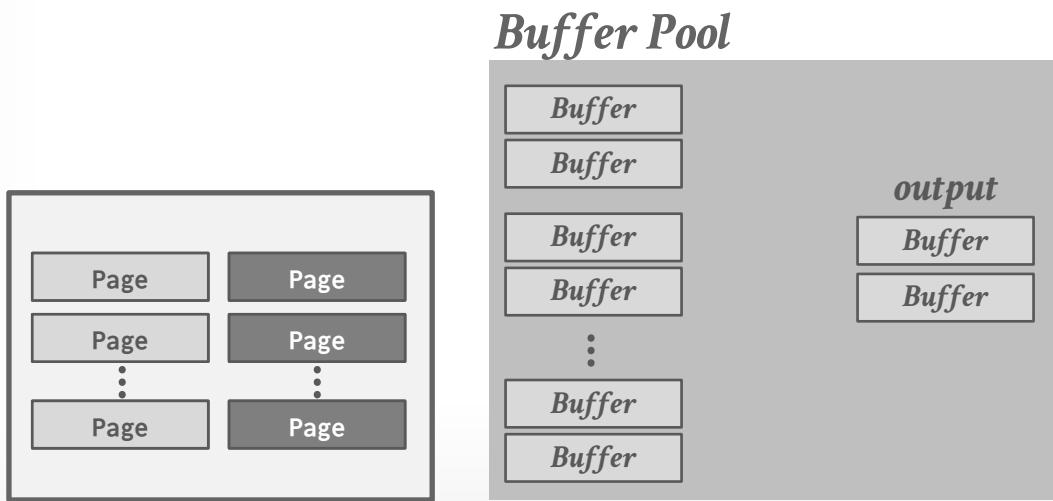
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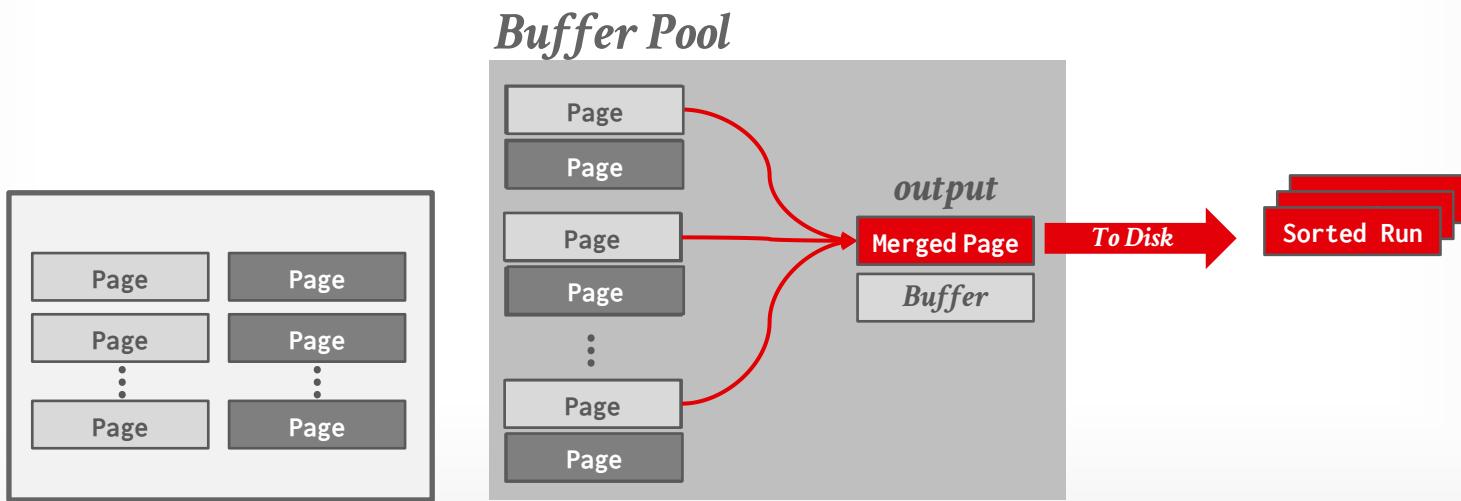
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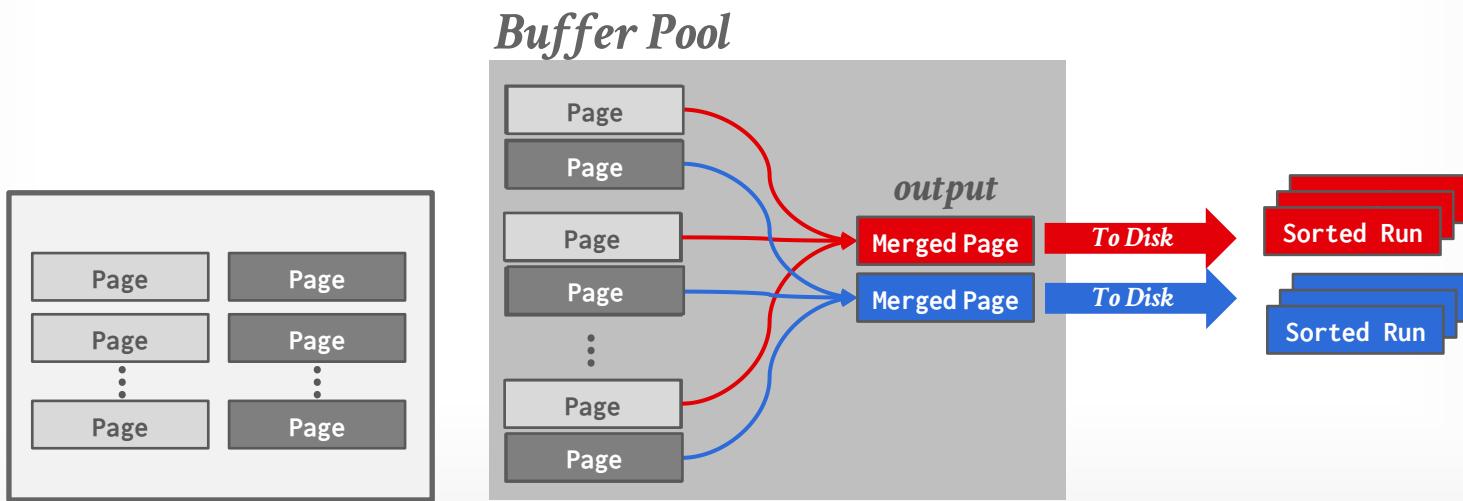
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COMPARISON OPTIMIZATIONS

Approach #1: Code Specialization

- Instead of providing a comparison function as a pointer to sorting algorithm, create a hardcoded version of sort that is specific to a key type.

Approach #2: Suffix Truncation

- First compare a binary prefix of long **VARCHAR** keys instead of slower string comparison. Fallback to slower version if prefixes are equal.

Approach #3: Key Normalization

- Transform variable-length attribute(s) into a single encoded/padded string that preserves sort order.

USING B+TREES FOR SORTING

If the table that must be sorted already has a B+Tree index on the sort attribute(s), then we can use that to accelerate sorting.

→ Some DBMSs support prefix key scans for sorting.

Retrieve tuples in desired sort order by simply traversing the leaf pages of the tree.

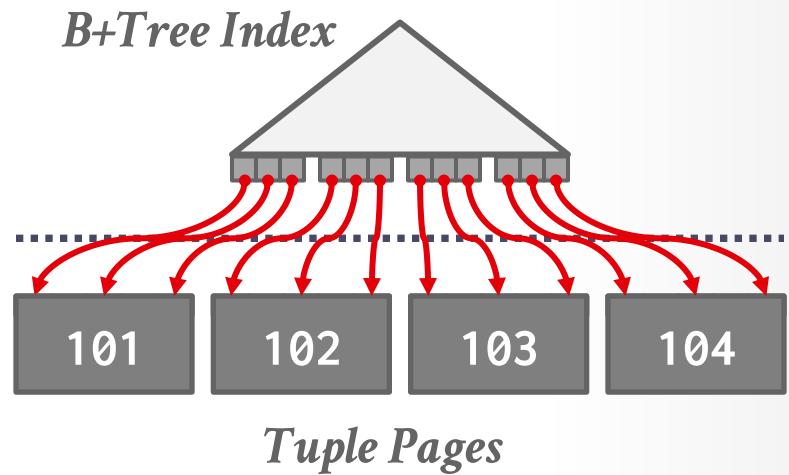
Cases to consider:

- Clustered B+Tree
- Unclustered B+Tree

CASE 1: CLUSTERED B+TREE

Traverse to the left-most leaf page, and then retrieve tuples from all leaf pages.

This is always better than external sorting because there is no computational cost, and all disk access is sequential.

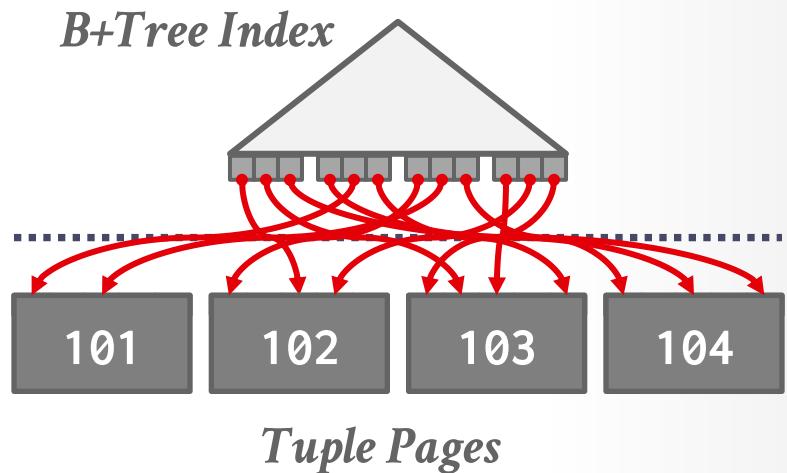


CASE #2: UNCLUSTERED B+TREE

Chase each pointer to the page that contains the data.

This is almost always a bad idea except for Top-N queries where N is small enough relative to total number of tuples in table.

→ In general, one I/O per data record.



AGGREGATIONS

Collapse values for a single attribute from multiple tuples into a single scalar value.

The DBMS needs a way to quickly find tuples with the same distinguishing attributes for grouping.

Two implementation choices:

- Sorting
- Hashing

SORTING AGGREGATION

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
 ORDER BY cid
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C



Filter

sid	cid	grade
53666	15-445	C
53688	15-826	B
53666	15-721	C
53655	15-445	C

*Remove
Columns*



cid
15-445
15-826
15-721
15-445

Sort



cid
15-445
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53655	15-445	C

Filter

Remove
Columns

cid
15-445
15-826
15-721
15-445

cid
15-445
15-445
15-721
15-826

Sort

Eliminate
Duplicates

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Remove
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15-826
15-721
15-445

Sort

cid
15-445
15-445
15-721
15-826

Eliminate
Duplicates

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)

Join

cid	s.gpa
15-445	3.62
15-826	2.89
15-721	3.33
15-445	3.7

Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89

Previous: null

(1, 3.62)

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)

Join

cid	s.gpa
15-445	3.62
15-826	2.89
15-721	3.33
15-445	3.7

Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89

Previous: 15-445

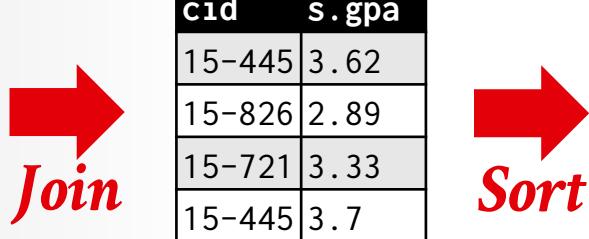
(1, 3.62)

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)



Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89

Previous: 15-445

(2, 7.32)

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)

Join

cid	s.gpa
15-445	3.62
15-826	2.89
15-721	3.33
15-445	3.7

Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89

Previous: 15-721

(1, 3.33)

Final Result

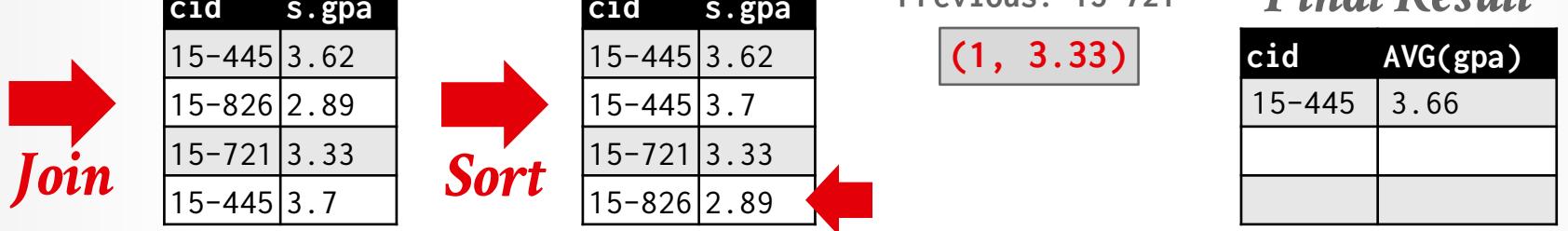
cid	AVG(gpa)
15-445	3.66

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)



AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)

Join

cid	s.gpa
15-445	3.62
15-826	2.89
15-721	3.33
15-445	3.7

Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89

Previous: 15-826

(1, 2.89)

Final Result

cid	AVG(gpa)
15-445	3.66
15-721	3.33

AGGREGATION SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)

Join

cid	s.gpa
15-445	3.62
15-826	2.89
15-721	3.33
15-445	3.7

Sort

cid	s.gpa
15-445	3.62
15-445	3.7
15-721	3.33
15-826	2.89



Final Result

cid	AVG(gpa)
15-445	3.66
15-721	3.33
15-826	2.89

ALTERNATIVES TO SORTING

What if we do not need the data to be ordered?

- Forming groups in **GROUP BY** (no ordering)
- Removing duplicates in **DISTINCT** (no ordering)

Hashing is a better alternative in this scenario.

- Only need to remove duplicates, no need for ordering.
- Can be computationally cheaper than sorting.

HASHING AGGREGATE

Populate an ephemeral hash table as the DBMS scans the table. For each record, check whether there is already an entry in the hash table:

- **DISTINCT**: Discard duplicate
- **GROUP BY**: Perform aggregate computation

If everything fits in memory, then this is easy.

If the DBMS must spill data to disk, then we need to be smarter...

EXTERNAL HASHING AGGREGATE

Divide-and-conquer approach to computing an aggregation when data does not fit in memory.

Phase #1 – Partition

- Split tuples into buckets based on hash key
- Write them out to disk when they get full

Phase #2 – ReHash

- Build in-memory hash table for each partition and compute the aggregation

PHASE #1: PARTITION

Use a hash function h_1 to split tuples into partitions on disk.

- A partition is one or more pages that contain the set of keys with the same hash value.
- Partitions are “spilled” to disk via output buffers.

Assume that we have B buffers.

We will use $B-1$ buffers for the partitions and 1 buffer for the input data.

PHASE #1: PARTITION

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

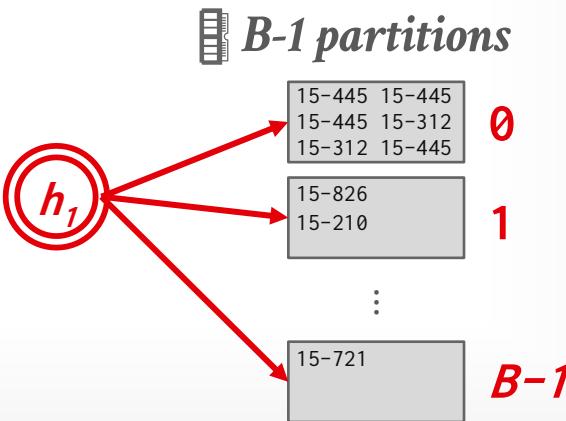
Filter

sid	cid	grade
53666	15-445	C
53688	15-826	B
53666	15-721	C
53655	15-445	C

Remove
Columns

cid
15-445
15-826
15-721
15-445

:



PHASE #1: PARTITION

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

sid	cid	grade
53666	15-445	C
53688	15-826	B
53666	15-721	C
53655	15-445	C

Filter

Remove
Columns

cid
15-445
15-826
15-721
15-445

:

 *B-1 partitions*

h₁

15-445 15-445
15-445 15-312
15-312 15-445

0

15-826
15-210

1

15-721

B-1



PHASE #1: PARTITION

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

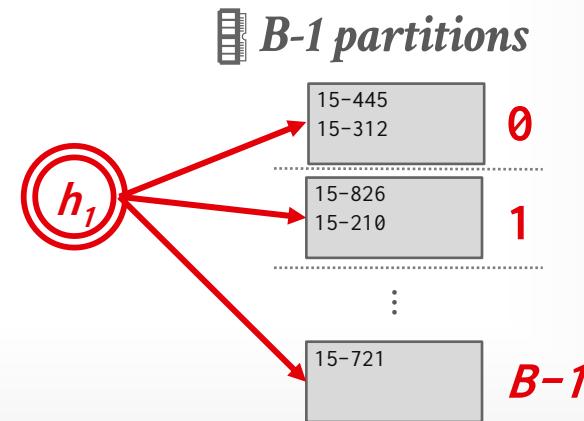
sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

sid	cid	grade
53666	15-445	C
53688	15-826	B
53666	15-721	C
53655	15-445	C

Filter

Remove
Columns

cid
15-445
15-826
15-721
15-445



PHASE #2: REHASH

For each partition on disk:

- Read it into memory and build an in-memory hash table based on a second hash function h_2 .
- Then go through each bucket of this hash table to bring together matching tuples.

This assumes that each partition fits in memory.

PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445 15-445
15-445 15-445
15-445 15-445

0

15-445 15-445
15-445 15-445
15-445 15-445

1

15-826
15-826

⋮

15-721

B-1

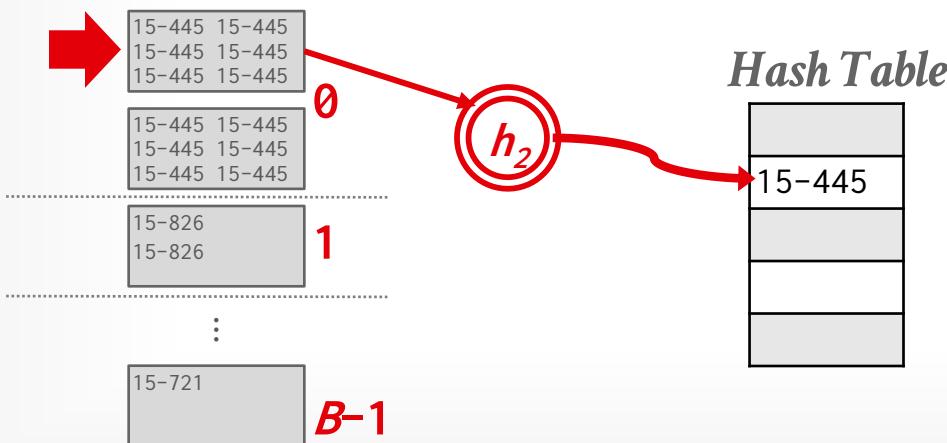
PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets



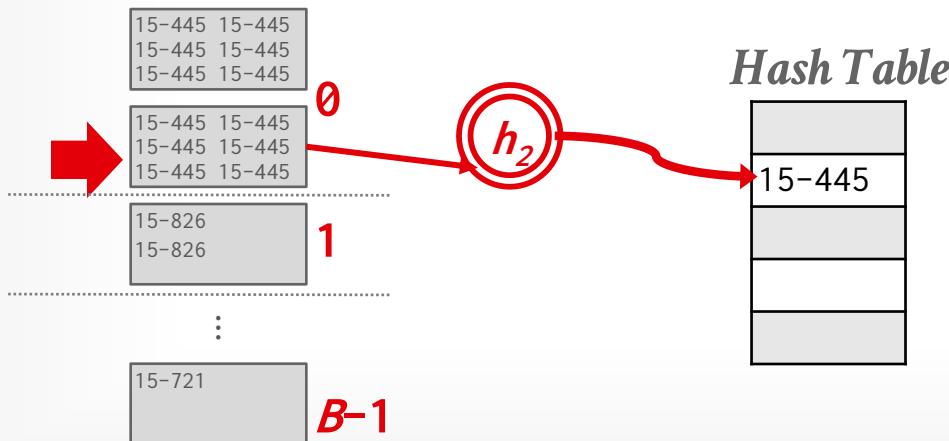
PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets



PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

1



15-826	
15-826	

 $B-1$ 

Hash Table

15-445

Final Result

cid
15-445

PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

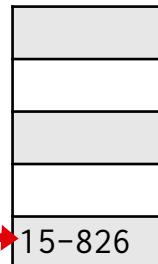


15-826
15-826

1

 $B-1$

Hash Table



Final Result

cid
15-445

PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

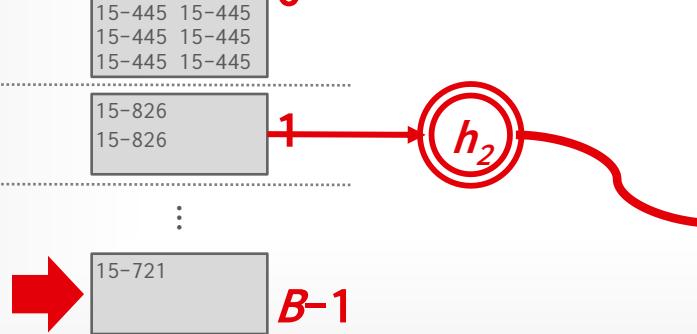
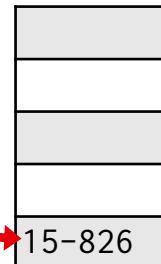
15-826	
15-826	

1

15-721	
--------	--

 $B-1$

Hash Table



Final Result

cid
15-445

PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

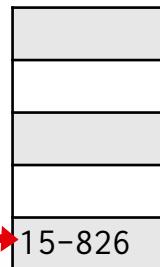
15-826	
15-826	

1

15-721	
--------	--

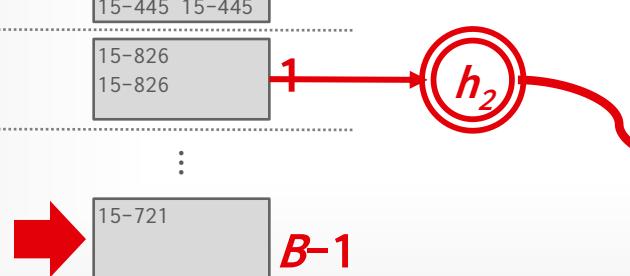
 $B-1$

Hash Table



Final Result

cid
15-445
15-826



PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

1

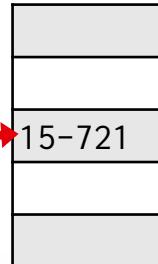
15-826	
15-826	

⋮

15-721	
--------	--

 $B-1$

Hash Table



h_2

Final Result

cid
15-445
15-826
15-721

PHASE #2: REHASH

```
SELECT DISTINCT cid
  FROM enrolled
 WHERE grade IN ('B', 'C')
```

Phase #1 Buckets

15-445	15-445
15-445	15-445
15-445	15-445

0

15-445	15-445
15-445	15-445
15-445	15-445

1

15-826
15-826

⋮

15-721
B-1

enrolled (sid, cid, grade)

sid	cid	grade
53666	15-445	C
53688	15-721	A
53688	15-826	B
53666	15-721	C
53655	15-445	C

Final Result

cid
15-445
15-826
15-721

HASHING SUMMARIZATION

During the rehash phase, store pairs of the form
(GroupKey→RunningVal)

When we want to insert a new tuple into the hash table
as we compute the aggregate:

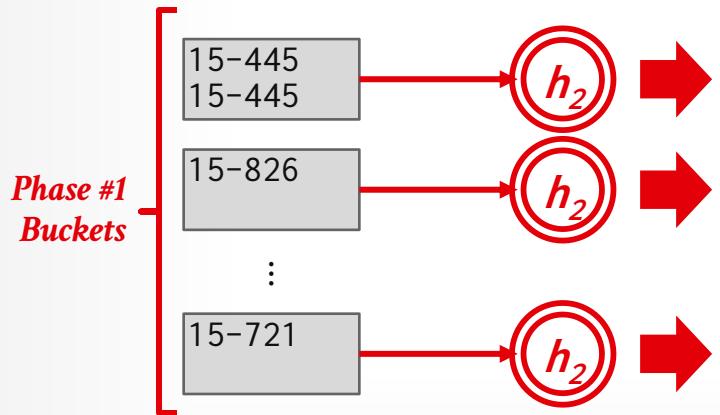
- If we find a matching **GroupKey**, just update the **RunningVal** appropriately
- Else insert a new **GroupKey→RunningVal**

HASHING SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)



Hash Table

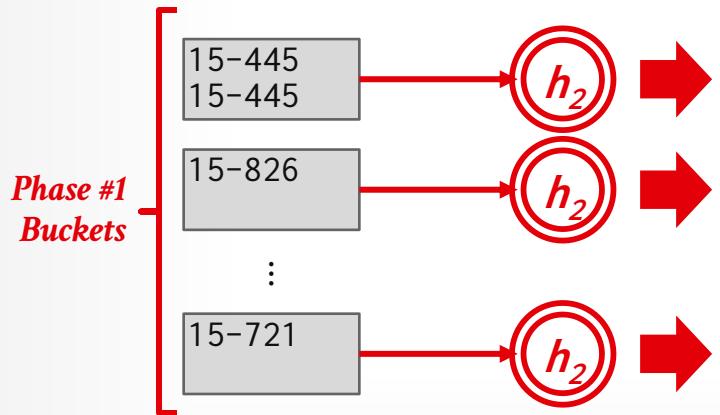
key	value
15-445	(2, 7.32)
15-826	(1, 3.33)
15-721	(1, 2.89)

HASHING SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

Running Totals

AVG(col)	→ (COUNT, SUM)
MIN(col)	→ (MIN)
MAX(col)	→ (MAX)
SUM(col)	→ (SUM)
COUNT(col)	→ (COUNT)



Hash Table

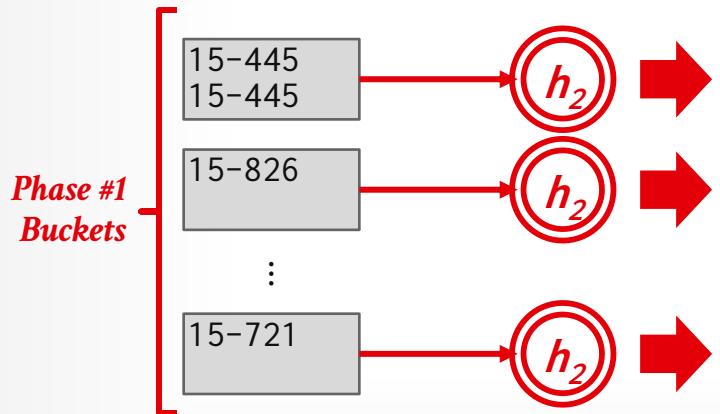
key	value
15-445	(2, 7.32)
15-826	(1, 3.33)
15-721	(1, 2.89)

HASHING SUMMARIZATION

```
SELECT cid, AVG(s.gpa)
  FROM student AS s, enrolled AS e
 WHERE s.sid = e.sid
 GROUP BY cid
```

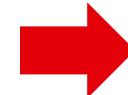
Running Totals

AVG(col)	\rightarrow	(COUNT, SUM)
MIN(col)	\rightarrow	(MIN)
MAX(col)	\rightarrow	(MAX)
SUM(col)	\rightarrow	(SUM)
COUNT(col)	\rightarrow	(COUNT)



Hash Table

key	value
15-445	(2, 7.32)
15-826	(1, 3.33)
15-721	(1, 2.89)



Final Result

cid	AVG(gpa)
15-445	3.66
15-826	3.33
15-721	2.89

CONCLUSION

Choice of sorting vs. hashing is subtle and depends on optimizations done in each case.

We already discussed the optimizations for sorting:

- Chunk I/O into large blocks to amortize costs
- Double-buffering to overlap CPU and I/O

NEXT CLASS

Nested Loop Join

Sort-Merge Join

Hash Join