1) 4 Queen Problem

2, 4, 1, 3

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

3, 1, 4, 2

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

8 Queen Problem

4, 7, 3, 8, 2, 5, 1, 6

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

5, 2, 4, 7, 3, 8, 6, 1

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4, 2, 7, 3, 6, 8, 5, 1

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

4, 6, 8, 3, 1, 7, 5, 2

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3, 6, 8, 1, 4, 7, 5, 2

$$\begin{bmatrix} 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&1 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix}$$

5, 3, 8, 4, 7, 1, 6,

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix}$$

5, 7, 4, 1, 3, 8, 6, 2

$$\begin{bmatrix} 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix}$$

4, 1, 5, 8, 6, 3, 7, 2

$$\begin{bmatrix} 0&0&0&1&0&0&0&0 \\ 1&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&1&0&0&0&0&0&0 \end{bmatrix}$$

3, 6, 4, 1, 8, 5, 7, 2

$$\begin{bmatrix} 0&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&0&0&0&0&1&0\\ 0&1&0&0&0&0&0&0 \end{bmatrix}$$

6, 2, 7, 1, 4, 8, 5, 3

$$\begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0 \end{bmatrix}$$

4, 7, 1, 8, 5, 2, 6, 3

$$\begin{bmatrix} 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&0&0&0&1&0&0&0\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0\\ 0&0&1&0&0&0&0&0 \end{bmatrix}$$

6, 4, 7, 1, 8, 2, 5, 3

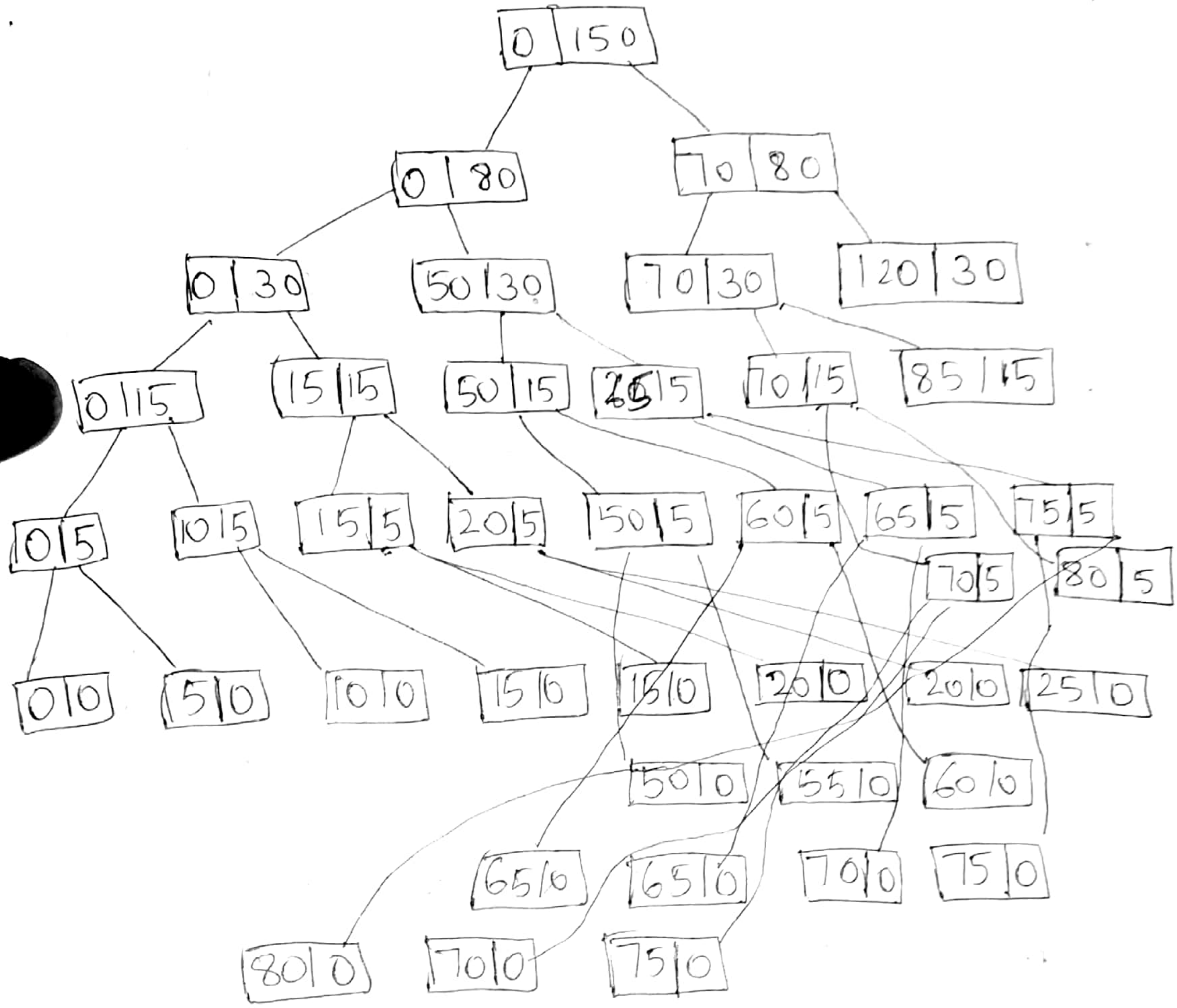
$$\begin{bmatrix} 0&0&0&0&0&1&0&0\\ 0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0\\ 1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0\\ 0&0&1&0&0&0&0&0 \end{bmatrix}$$

2) $S = [5, 10, 15, 50, 70]$ $W = 80$

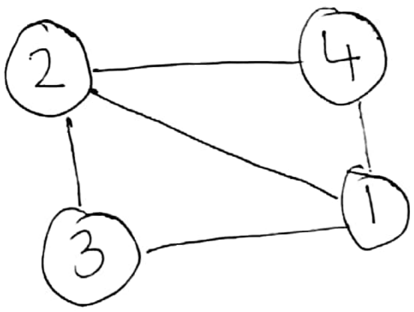
Let $S = [x_1, x_2, x_3, x_4, x_5]$ Total $= 5 + 10 + 15 + 50 + 70 = 150$

Soln:-

1) [0 1 0 0 1] 2) [1 1 1 1 0]



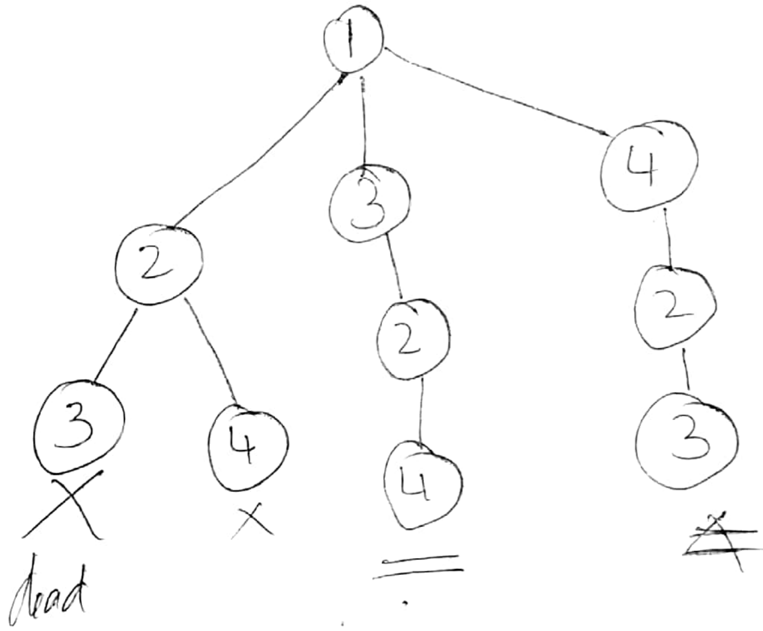
3)



Adjacency matrix :-

$$
\begin{array}{c|cccc}
 & 1 & 2 & 3 & 4 \\
\hline
1 & 0 & 1 & 1 & 1 \\
2 & 1 & 0 & 1 & 1 \\
3 & 1 & 1 & 0 & 0 \\
4 & 1 & 1 & 0 & 0
\end{array}
$$

Stat space tree

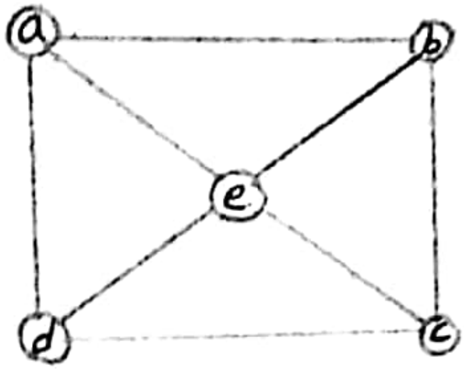


Solution :-

$1 \rightarrow 3 \rightarrow 2 \rightarrow 4 \rightarrow 1$

$1 \rightarrow 4 \rightarrow 2 \rightarrow 3 \rightarrow 1$

The given graph has hamiltonian cycles

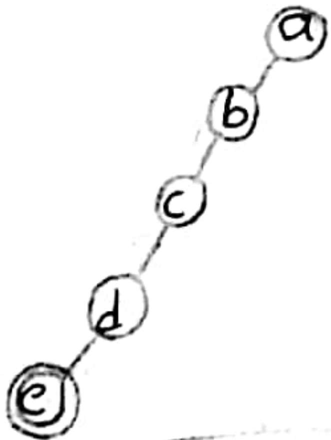


start from Vertex 'a'

Adjacency matrix:

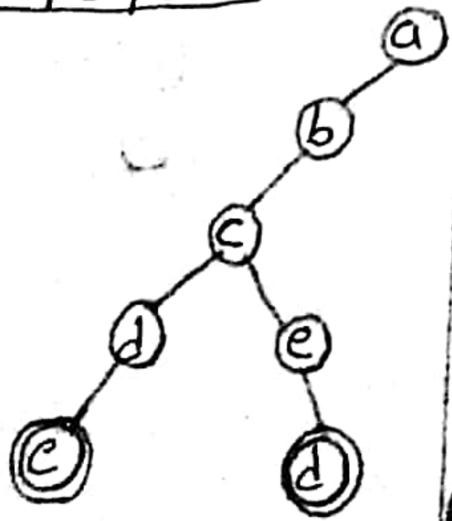
$$\begin{array}{c|c} & \begin{array}{ccccc} a & b & c & d & e \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \\ e \end{array} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

a	b	c	d	e



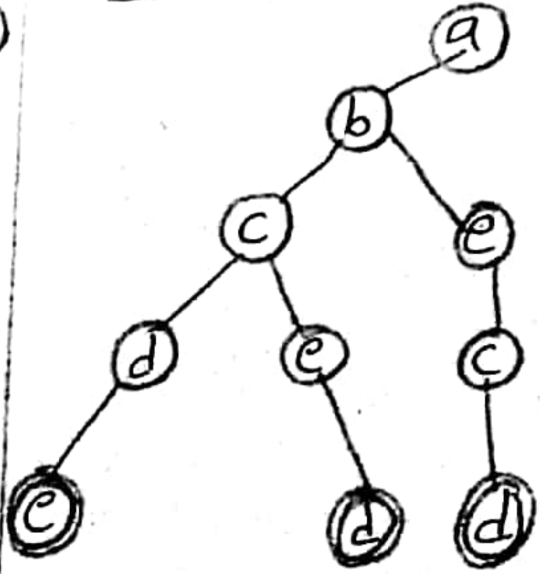
(i) a → b → c → d → e → a.

a	b	c	e	d.



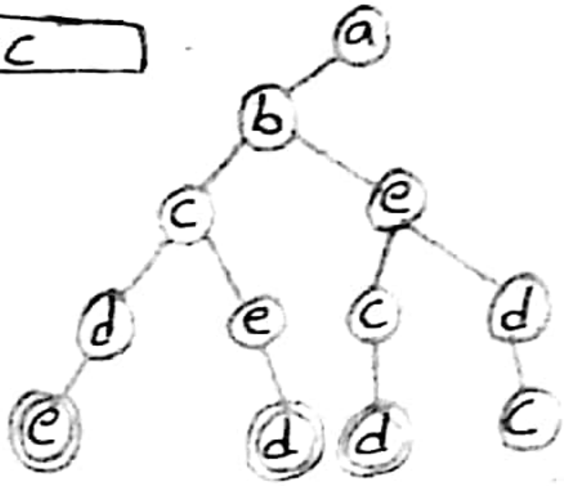
(ii) a → b → c → e → d → a

a	b	e	c	d

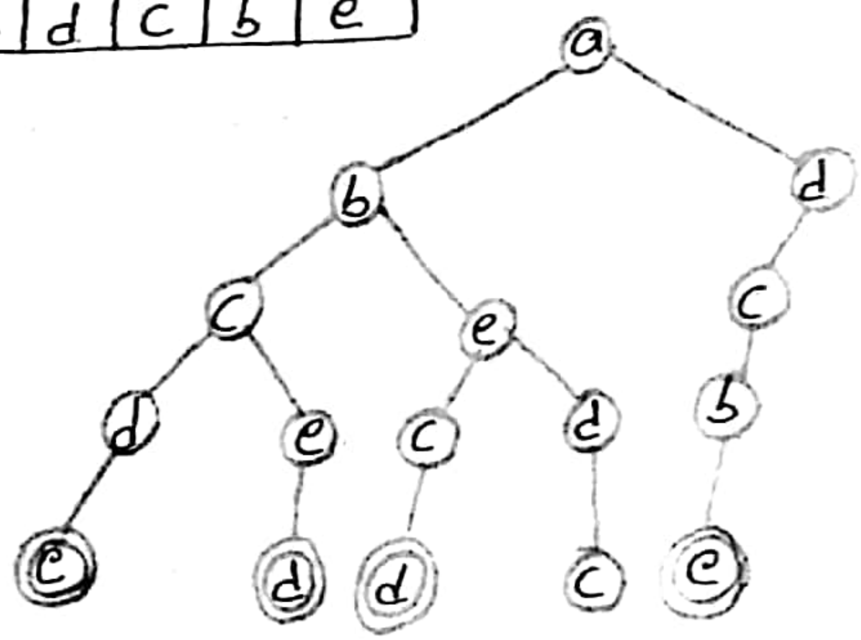


(iii) a → b → e → c → d. → a

a	b	e	d	c

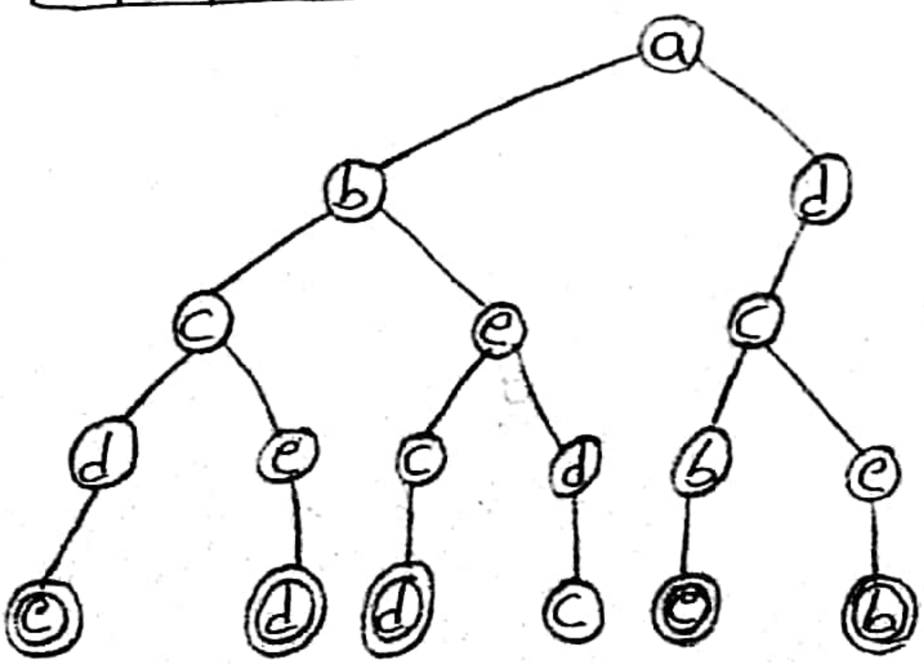


a	d	c	b	e



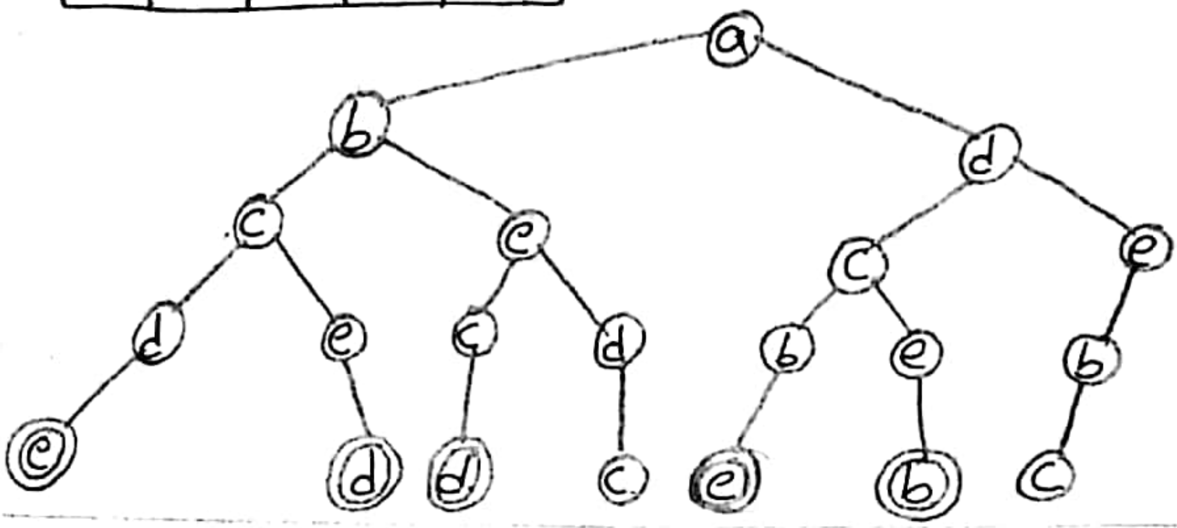
(iv) a→d→c→b→e→a

a	d	c	e	b



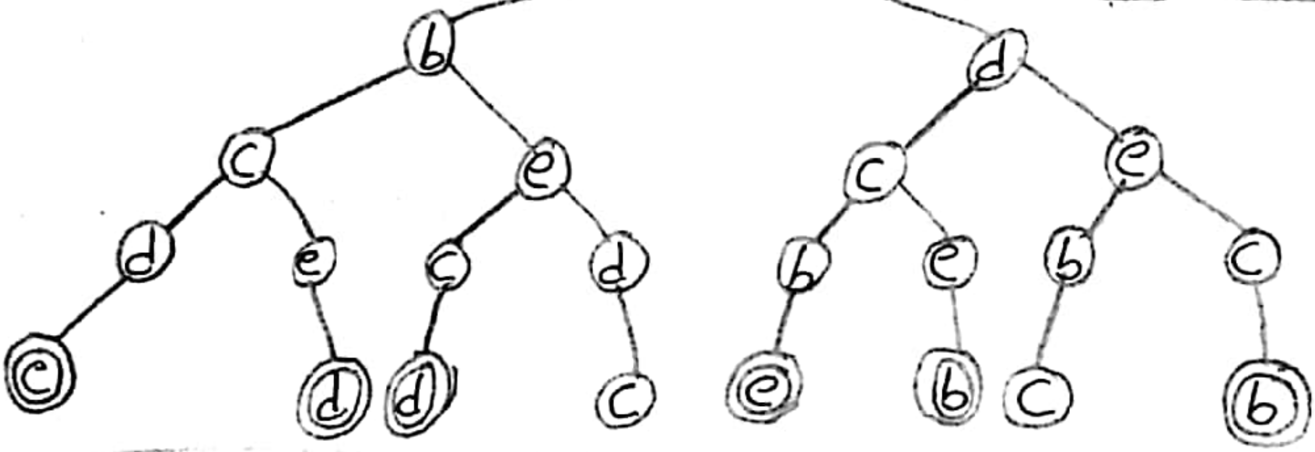
(V) a→d→c→e→b→a

a	d	e	b	c

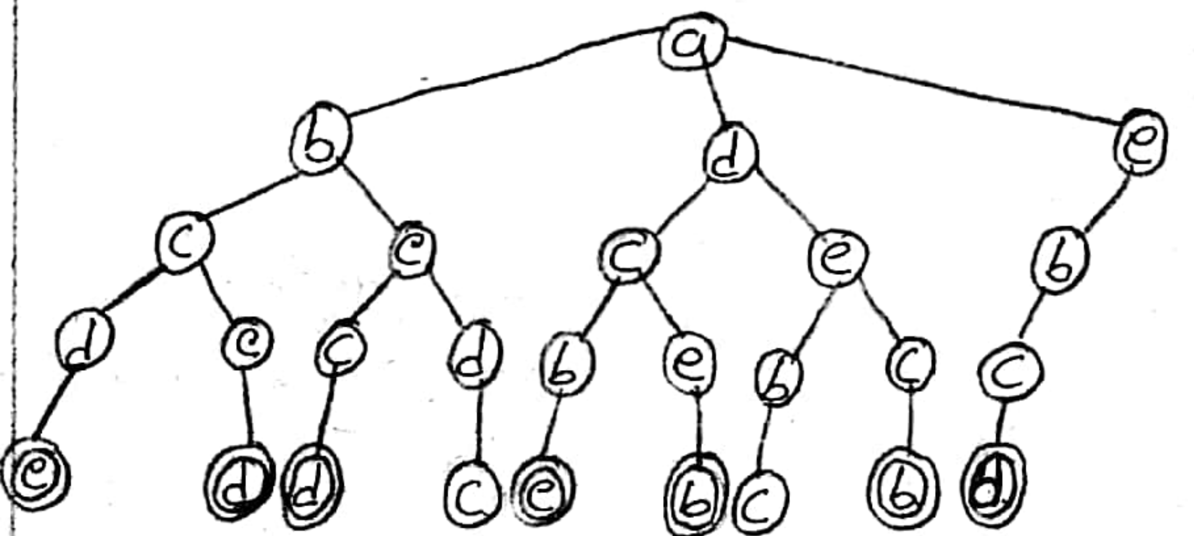


a	d	e	c	b

(vi) a→d→e→c→b→a.

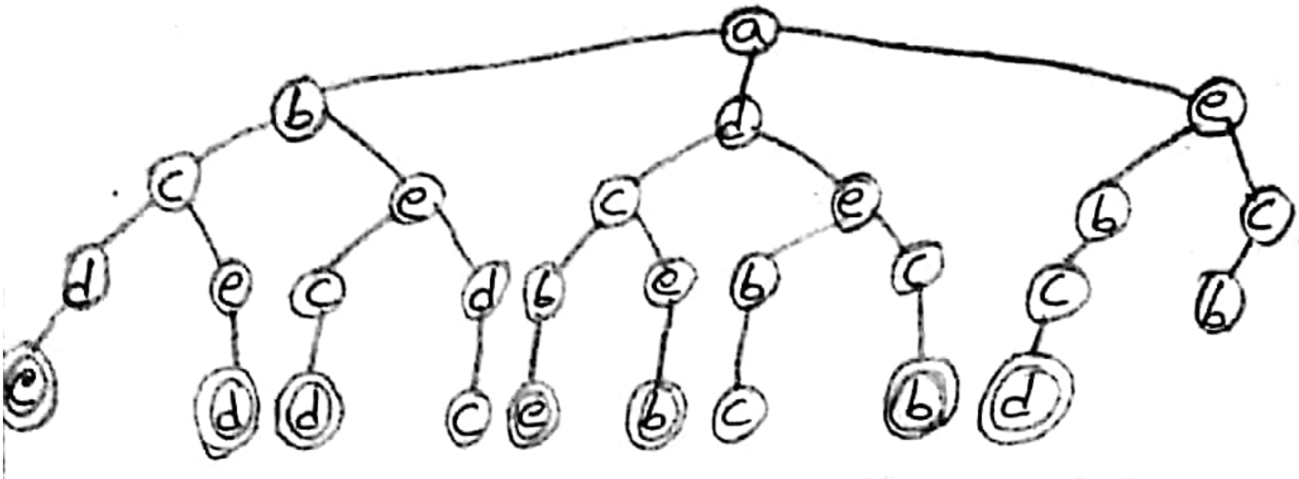


a	e	b	c	d.

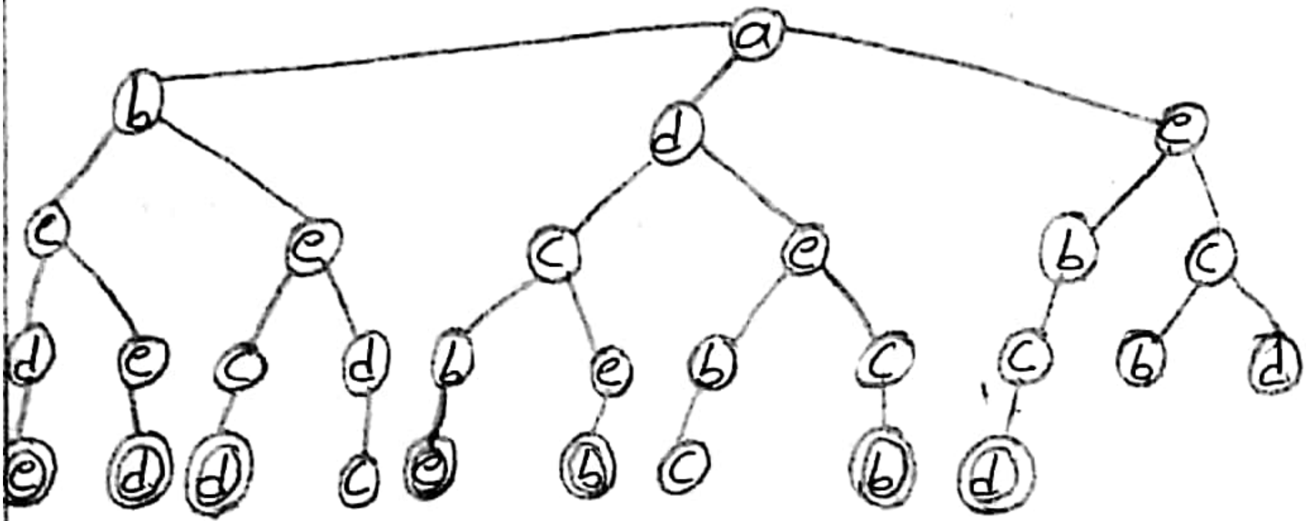


(vii) a→e→b→c→d→a

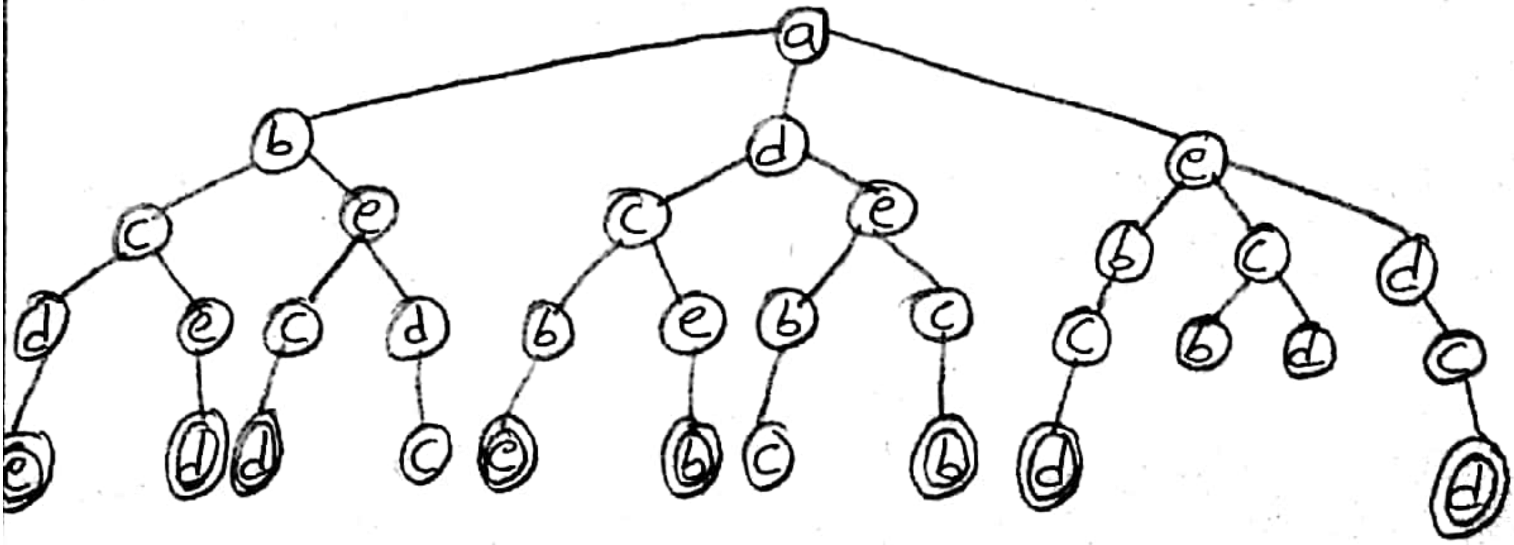
a	e	c	b	



a	e	c	d	



a	e	d	c	b

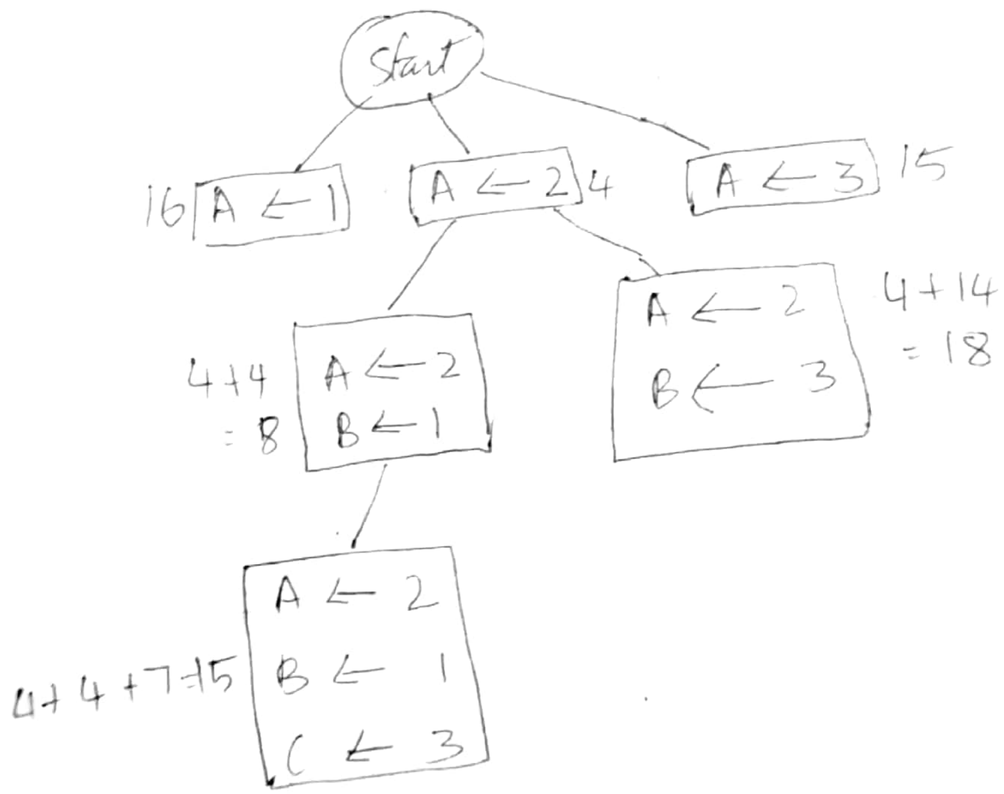


(viii) $a \to e \to d \to c \to b \to a$.

Solutions:-

1. a → b → c → d → c → a
2. a → b → c → e → d → a
3. a → b → e → c → d → a
4. a → d → c → b → e → a
5. a → d → c → e → b → a
6. a → d → e → c → b → a
7. a → e → b → c → d → a
8. a → e → d → c → b → a

③ State space tree



Job assignment is Worker A ← task2
Worker B ← task1
Worker C ← task3

Total cost = 15

Hungarian Method :-

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{Rowmin} \\ \hline A & 16 & 4 & 15 & 4 \\ B & 4 & 7 & 14 & 4 \\ C & 12 & 11 & 7 & 7 \end{array}$$

Row Reduction

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{Rowmin} \\ \hline A & 12 & 0 & 11 & 4 \\ B & 0 & 3 & 10 & 4 \\ C & 5 & 4 & 0 & 7 \\ \hline \text{Colmin:} & 0 & 0 & 0 & 15 \end{array}$$

Row scanning:-

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{Rowmin} \\ \hline A & 12 & \boxed{0} & 11 & 4 \\ B & \boxed{0} & 3 & 10 & 4 \\ C & 5 & 4 & \boxed{0} & 7 \\ \hline \text{Colmin:} & 0 & 0 & 0 & 15 \end{array}$$

3 zeros = 3 works

Worker A ← Task 2

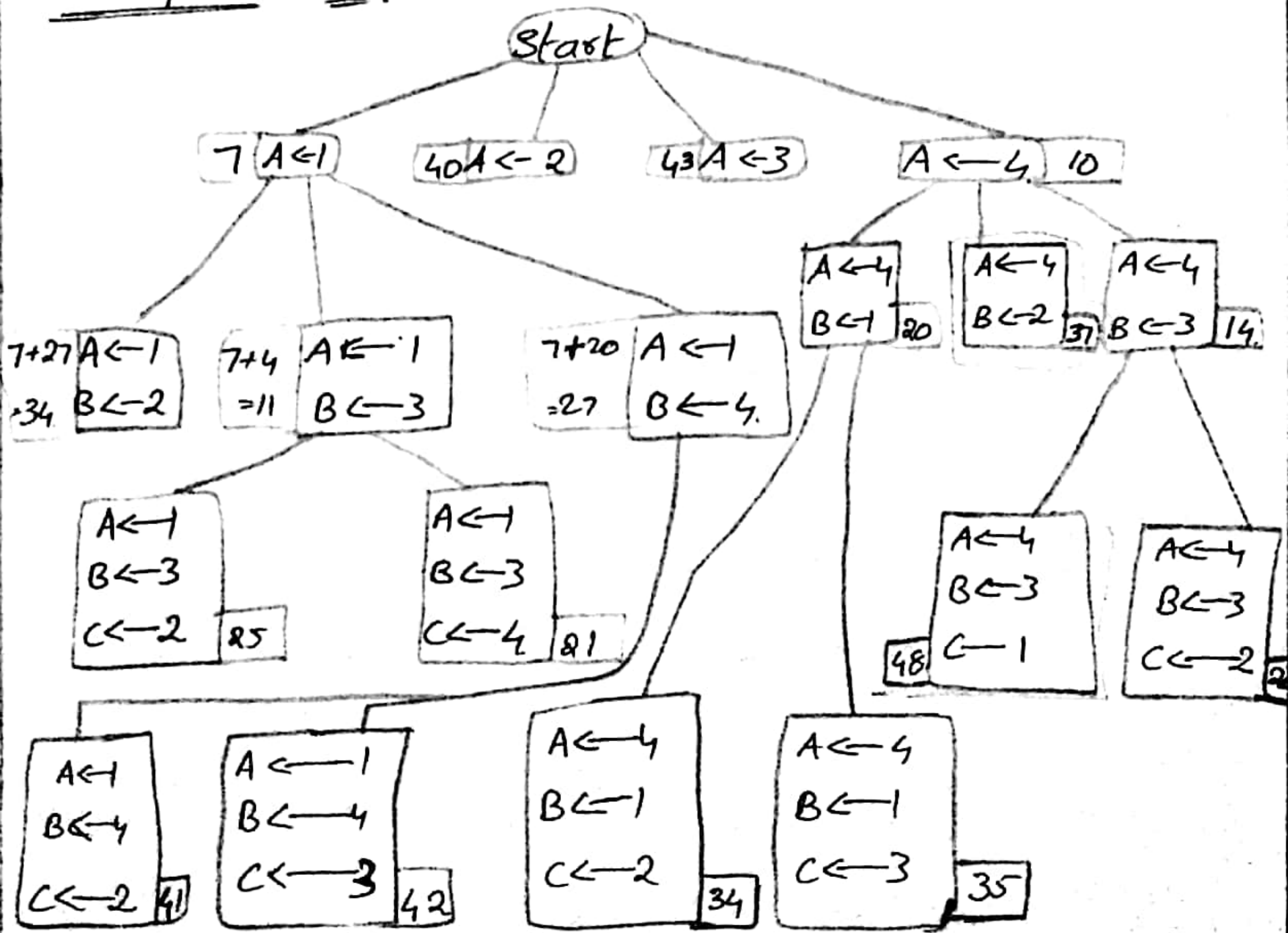
Worker B ← Task 1

Worker C ← Task 3

Total cost = 15

Job assignment by statespace tree is equal to Job assignment by Hungarian method.

state space Tree:



Hungarian Method :

	1	2	3	4	Rowmin
A	7	40	43	10	7
B	10	27	4	20	4
C	34	14	15	10	10
D	10	26	13	7	7

Row Reduction :

	1	2	3	4	Rowmin
A	0	33	36	3	7
B	6	23	0	16	4
C	24	4	5	0	10
D	3	19	6	0	7
Colmin :	0	4	0	0	32

Col Reduction

	1	2	3	4	Rowmin
A	0	29	36	3	7
B	6	19	0	16	4
C	24	0	5	0	10
D	3	15	6	0	7
Colmin:	0	4	0	0	32

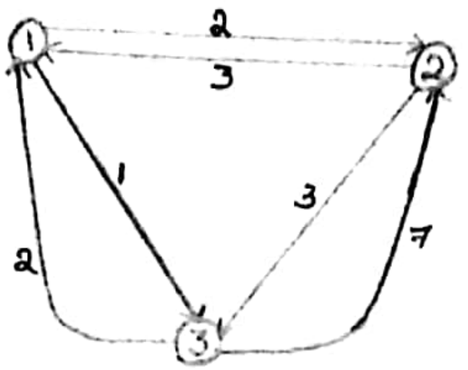
Row Scanning :-

	1	2	3	4.	Rowmin
A	$\boxed{0}$	29	36	3	7
B	6	19	$\boxed{0}$	16	4
C	24	0	5	0	10
D	3	15	6	$\boxed{0}$	7.
Colmin:-	0	4	0	0	32

Col Scanning :

	1	2	3	4	Rowmin
A	$\boxed{0}$	29	36	3	7
B	6	19	$\boxed{0}$	16	4
C	24	$\boxed{0}$	5	0	10
D	3	15	6	$\boxed{0}$	7
Colmin:-	0	4	0	0	32

7) Solve the following TSP using Branch and Band technique.



Adjacency Matrix :

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \infty & 2 & 1 \\ 2 & 3 & \infty & 3 \\ 3 & 2 & 7 & \infty \end{array}$$

Row Reduction:

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{rowmin:} \\ \hline 1 & \infty & 2 & 1 & 1 \\ 2 & 3 & \infty & 3 & 3 \\ 3 & 2 & 7 & \infty & 2 \end{array}$$

Row Reduced Matrix

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{rowmin.} \\ \hline 1 & \infty & 1 & 0 & 1 \\ 2 & 0 & \infty & 0 & 3 \\ 3 & 0 & 5 & \infty & 2 \\ & & & & 6 \end{array}$$

Col Reduction:

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \infty & 1 & 0 \\ 2 & 0 & \infty & 0 \\ 3 & 0 & 5 & \infty \\ \hline \text{colmin:} & 0 & 1 & 0 \end{array}$$

col Reduced matrix :

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \infty & 0 & 0 \\ 2 & 0 & \infty & 0 \\ 3 & 0 & 4 & \infty \\ \hline \text{colmin:} & 0 & 1 & 0 \end{array} = 1$$

Cost of Reduction = $C_1 = 6+1 = 7$

Cost (1,2) :-

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{rowmin} \\ 1 & \infty & \infty & \infty & - \\ 2 & \infty & \infty & 0 & 0 \\ 3 & 0 & \infty & \infty & 0 \\ \hline \text{colmin} & 0 & - & 0 & 0 \end{array}$$

Cost of Reduction = 0

$C_2 = Cost(1,2) = c(1,2) + r + C_1$

$= 0 + 0 + 7$

$= 7$

Cost (1,3):

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{rowmin} \\ 1 & \infty & \infty & \infty & - \\ 2 & 0 & \infty & \infty & 0 \\ 3 & \infty & 4 & \infty & 4 \\ \hline & & & - & 4 \end{array}$$

row reduction :-

$$\begin{array}{c|ccc|c} & 1 & 2 & 3 & \text{rowmin} \\ 1 & \infty & \infty & \infty & - \\ 2 & 0 & \infty & \infty & 0 \\ 3 & \infty & 0 & \infty & 0 \\ \hline \text{colmin} & 0 & 0 & - & 4 \end{array}$$

Cost of Reduction = 4

$Cost(1,3) = C_3 = c(1,3) + r + C_1 = 0 + 4 + 7 = 11$

Cost (1,2) < Cost (1,3)

TSP cycle: $1 \to 2 \to 3 \to 1$

Cost = 7

Scanned with CamScanner

$$\begin{array}{c|ccccc} & A & B & C & D & E \\ \hline A & \infty & 2 & 5 & 7 & 1 \\ B & 6 & \infty & 2 & 5 & 3 \\ C & 7 & 6 & \infty & 4 & 6 \\ D & 10 & 3 & 5 & \infty & 4 \\ E & 1 & 3 & 2 & 8 & \infty \end{array}$$

Given Adjacency Matrix:

	A	B	C	D	E	Rowmin
A	∞	2	5	7	1	1
B	6	∞	2	5	3	2
C	7	6	∞	4	6	4
D	10	3	5	∞	4	3
E	1	3	2	8	∞	1
						11

Row Reduced Matrix:

	A	B	C	D	E	rowmin:
A	∞	1	4	6	0	1
B	4	∞	0	3	1	2
C	3	2	∞	0	2	4
D	7	0	2	∞	1	3
E	0	2	1	7	∞	1
cmin	0	0	0	0	0	11

cost of reduction $C_1 = 11$

$\boxed{C_1 = 11}$

Cost (A,B):

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{row min} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & 0 & 3 & 1 & 0 \\
C & 3 & \infty & \infty & 0 & 2 & 0 \\
D & 7 & \infty & 2 & \infty & 1 & 1 \\
E & 0 & \infty & 1 & 7 & \infty & 0 \\
\hline
 & & & & & & 1
\end{array}
$$

Row reduced matrix

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{row min} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & 0 & 3 & 1 & 0 \\
C & 3 & \infty & \infty & 0 & 2 & 0 \\
D & 6 & \infty & 1 & \infty & 0 & 1 \\
E & 0 & \infty & 1 & 7 & \infty & 0 \\
\hline
\text{col min} & 0 & - & 0 & 0 & 0 & 1
\end{array}
$$

cost of reduction = 1

$Cost(A,B) = C(A,B) + r + C_1 = 2+1+11 = 14$

Cost (A,C):

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{row min} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & 4 & \infty & \infty & 3 & 1 & 1 \\
C & \infty & 2 & \infty & 0 & 2 & 0 \\
D & 7 & 0 & \infty & \infty & 1 & 0 \\
E & 0 & 2 & \infty & 7 & \infty & 0 \\
\hline
 & & & & & & 1
\end{array}
$$

Row Reduced Matrix

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{row min} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & 3 & \infty & \infty & 2 & 0 & 1 \\
C & \infty & 2 & \infty & 0 & 2 & 0 \\
D & 7 & 0 & \infty & \infty & 1 & 0 \\
E & 0 & 2 & \infty & 7 & \infty & 0 \\
\hline
\text{col min:} & 0 & 0 & - & 0 & 0 & 1
\end{array}
$$

Cost of reduction = 1

$Cost(A,C) = C(A,C) + r + C_1 = 4 + 1 + 11 = 16$

Cost(A,D):

$$\begin{array}{c|c}
\begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}
\begin{array}{ccccc} A & B & C & D & E \\ \end{array}
& \text{rowmin}
\end{array}$$

$$\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 0 & \infty & 1 \\ 3 & 2 & \infty & \infty & 2 \\ \infty & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & \infty & \infty \end{bmatrix}
\end{array}
\begin{array}{|c}
\text{rowmin} \\ - \\ 0 \\ 2 \\ 0 \\ 0 \\ \hline 2
\end{array}$$

Row reduced matrix

$$\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ \text{Colmin:}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 0 & \infty & 1 \\ 1 & 0 & \infty & \infty & 0 \\ \infty & 0 & 2 & \infty & 1 \\ 0 & 2 & 1 & \infty & \infty \end{bmatrix} \\
\begin{array}{ccccc} 0 & 0 & 0 & - & 0 \end{array}
\end{array}
\begin{array}{|c}
\text{rowmin} \\ - \\ 0 \\ 2 \\ 0 \\ 0 \\ \hline 2
\end{array}$$

Cost of Reduction = 2

$Cost(A,D) = C(A,D) + r + C_1 = 6+2+11 = 19$

Cost(A,E):

$$\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 0 & 3 & \infty \\ 3 & 2 & \infty & 0 & \infty \\ 7 & 0 & 2 & \infty & \infty \\ \infty & 2 & 1 & 7 & \infty \end{bmatrix}
\end{array}
\begin{array}{|c}
\text{rowmin} \\ - \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 1
\end{array}$$

Row reduced matrix.

$$\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ \text{colmin:}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 4 & \infty & 0 & 3 & \infty \\ 3 & 2 & \infty & 0 & \infty \\ 7 & 0 & 2 & \infty & \infty \\ \infty & 1 & 0 & 6 & \infty \end{bmatrix} \\
\begin{array}{ccccc} 3 & 0 & 0 & 0 & - \end{array}
\end{array}
\begin{array}{|c}
\text{rowmin} \\ - \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 4
\end{array}$$

Col reduced matrix:

$$\begin{array}{c}
\\ A \\ B \\ C \\ D \\ E \\ \text{colmin}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ 1 & \infty & 0 & 3 & \infty \\ 0 & 2 & \infty & \infty & \infty \\ 4 & 0 & 2 & \infty & \infty \\ \infty & 1 & 0 & 0 & \infty \end{bmatrix} \\
\begin{array}{ccccc} 3 & 0 & 0 & 0 & - \end{array}
\end{array}
\begin{array}{|c}
\text{rowmin} \\ - \\ 0 \\ 0 \\ 0 \\ 1 \\ \hline 4
\end{array}$$

Cost of reduction = 4

$Cost(A,E) = C(A,E) + r + C_1$

$= 0 + 4 + 11$

$= 15$

$$C_2 = \min \begin{cases} Cost(A,B) \\ Cost(A,C) \\ Cost(A,D) \\ Cost(A,E) \end{cases} = \min \begin{cases} 14 \\ 16 \\ 19 \\ 15 \end{cases} = 14$$

$C_2 = 14.$ and $C(\cdot) = Cost(A,B)$ matrix

$Cost((A,B),C)$

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \text{Rowmin} \\ \hline A & \infty & \infty & \infty & \infty & \infty & - \\ B & \infty & \infty & \infty & \infty & \infty & - \\ C & \infty & \infty & \infty & 0 & 2 & 0 \\ D & 6 & \infty & \infty & \infty & 0 & 0 \\ E & 0 & \infty & \infty & 7 & \infty & 0 \\ \hline \text{Colmin} & 0 & - & - & 0 & 0 & 0 \end{array}$$

Cost of reduction = 0

$$Cost((A,B),C) = C(B,C) + C2 + r$$
$$= 0 + 14 + 0$$
$$= 14.$$

$Cost(A,B,D)$

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \text{Rowmin} \\ \hline A & \infty & \infty & \infty & \infty & \infty & - \\ B & \infty & \infty & \infty & \infty & \infty & - \\ C & 3 & \infty & \infty & \infty & 2 & 2 \\ D & \infty & \infty & 1 & \infty & 0 & 0 \\ E & 0 & \infty & 1 & \infty & \infty & 0 \\ \hline & & & & & & 2 \end{array}$$

Row Reduction

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \text{Rowmin} \\ \hline A & \infty & \infty & \infty & \infty & \infty & - \\ B & \infty & \infty & \infty & \infty & \infty & - \\ C & 1 & \infty & \infty & \infty & 0 & 2 \\ D & \infty & \infty & 1 & \infty & 0 & 0 \\ E & 0 & \infty & 1 & \infty & \infty & 0 \\ \hline \text{Colmin} & 0 & - & 1 & - & 0 & 3 \end{array}$$

Col Reduction:

$$\begin{array}{c|ccccc|c} & A & B & C & D & E & \text{Rowmin} \\ \hline A & \infty & \infty & \infty & \infty & \infty & - \\ B & \infty & \infty & \infty & \infty & \infty & - \\ C & 1 & \infty & \infty & \infty & 0 & 2 \\ D & \infty & \infty & 0 & \infty & 0 & 0 \\ E & 0 & \infty & 0 & \infty & \infty & 0 \\ \hline \text{Colmin} & 0 & - & 1 & - & 0 & 3 \end{array}$$

Cost of Reduction = 3

$$Cost(A,B,D) = C(B,D) + C_2 + r$$
$$= 3 + 14 + 3$$
$$= 20$$

Cost(A,B,E)

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{Rowmin} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & \infty & \infty & \infty & - \\
C & 3 & \infty & \infty & 0 & \infty & 0 \\
D & 6 & \infty & 1 & \infty & \infty & 1 \\
E & \infty & \infty & 1 & 7 & \infty & 1 \\
\hline
 & & & & & & 2
\end{array}
$$

Row Reduction

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{Rowmin} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & \infty & \infty & \infty & - \\
C & 3 & \infty & \infty & 0 & \infty & 0 \\
D & 5 & \infty & 0 & \infty & \infty & 1 \\
E & \infty & \infty & 0 & 6 & \infty & 1 \\
\hline
\text{Colmin} & 3 & - & 0 & 0 & - & 5
\end{array}
$$

Col Reduction:

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{Rowmin} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & \infty & \infty & \infty & - \\
C & 0 & \infty & \infty & 0 & \infty & 0 \\
D & 2 & \infty & 0 & \infty & \infty & 1 \\
E & \infty & \infty & 0 & 6 & \infty & 1 \\
\hline
\text{Colmin:} & 3 & - & 0 & 0 & - & 5
\end{array}
$$

Cost of Reduction = 5

$$
\begin{aligned}
Cost(A,B,E) &= C(B,E) + C_2 + \gamma \\
&= 1 + 14 + 5 \\
&= 20.
\end{aligned}
$$

$$
C_3 = \min \begin{cases} Cost(A,B,C) \\ Cost(A,B,D) \\ Cost(A,B,E) \end{cases} = \min \begin{cases} 14 \\ 20 \\ 20 \end{cases} = 14
$$

$C_3 = 14$ and. $C = Cost(A,B,C)$ matrix.

Cost(A,B,C,D)

$$
\begin{array}{c|ccccc|c}
 & A & B & C & D & E & \text{Rowmin} \\
\hline
A & \infty & \infty & \infty & \infty & \infty & - \\
B & \infty & \infty & \infty & \infty & \infty & - \\
C & \infty & \infty & \infty & \infty & \infty & - \\
D & \infty & \infty & \infty & \infty & 0 & 0 \\
E & 0 & \infty & \infty & \infty & \infty & 0 \\
\hline
\text{Colmin} & 0 & - & - & - & 0 & 0
\end{array}
$$

Cost of Reduction = 0

$$
\begin{aligned}
Cost(A,B,C,D) &= C(C,D) + C_3 + \gamma \\
&= 0 + 14 + 0 \\
&= 14.
\end{aligned}
$$

Cost(A, B, C, E)

$$\begin{array}{c|c}
\begin{array}{c c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 7 & \infty \end{bmatrix}
\end{array} &
\begin{array}{c} \text{Rowmin} \\ - \\ - \\ - \\ 0 \\ 7 \\ \hline 7 \end{array}
\end{array}$$

Row Reduction

$$\begin{array}{c|c}
\begin{array}{c c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{bmatrix} \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & \infty & \infty \\ 0 & \infty & \infty & \infty & \infty \\ \infty & \infty & \infty & 0 & \infty \end{bmatrix} \\
\text{Colmin:} & \begin{array}{ccccc} 0 & - & - & 0 & - \end{array}
\end{array} &
\begin{array}{c} \text{Rowmin} \\ - \\ - \\ - \\ 0 \\ 7 \\ \hline 7 \end{array}
\end{array}$$

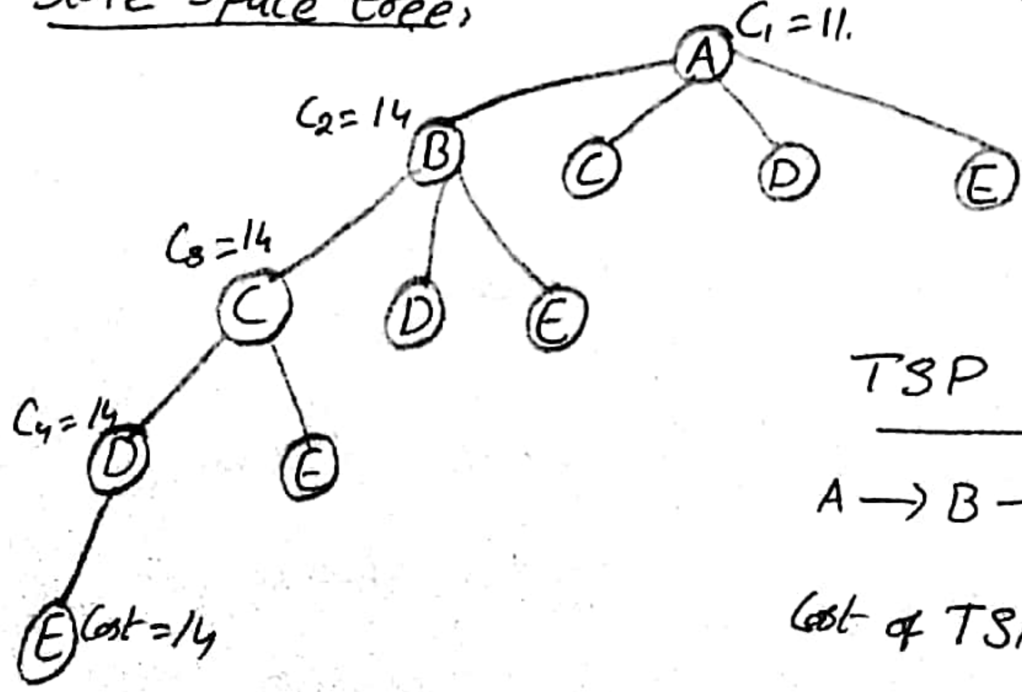
~~Rede~~ Cost of Reduction = 7

$$Cost(A, B, C, E) = C(C, E) + C_3 + r$$
$$= 2 + 14 + 7 = 23$$

$\therefore \hat{C}ost(A, B, C, D) < Cost(A, B, C, E)$

$$C_4 = \min\begin{cases} Cost(A, B, C, D) \\ Cost(A, B, C, E) \end{cases} = \min\begin{cases} 14 \\ 23 \end{cases} = 14.$$

state space tree:



TSP Cycle:

$$A \rightarrow B \rightarrow C \rightarrow D \rightarrow E \rightarrow A$$

Cost of TSP = 14

9) $W = 10$

Sort items based on $\frac{V_i}{W_i}$ value

Item 1 $= \frac{21}{3} = 7$

Item 2 $= \frac{30}{5} = 6$

Item 3 $= \frac{24}{4} = 6$

The Sorted order is Item 1
Item 2
Item 3

upper Bound, $UB = V + (W - w) * \frac{V_{i+1}}{W_{i+1}}$

Selecting Item 1:-

$UB = 0 + (10 - 0) * \left(\frac{21}{3}\right)$

$= 0 + (10 \times 7) = 70$

Selecting Item 2:-

$UB = 0 + (10 - 0) * \left(\frac{30}{5}\right)$

$= 0 + (10 \times 6) = 60$

$UB = 21 + \left((10 - 3) * \left(\frac{30}{5}\right)\right)$

$= 21 + (7 * 6) = 63$

Selecting Item 3:-

$UB = 51 + \left((10 - 8) * \left(\frac{24}{4}\right)\right)$

$= 51 + (2) * 6 = 51 + 12 = 63$

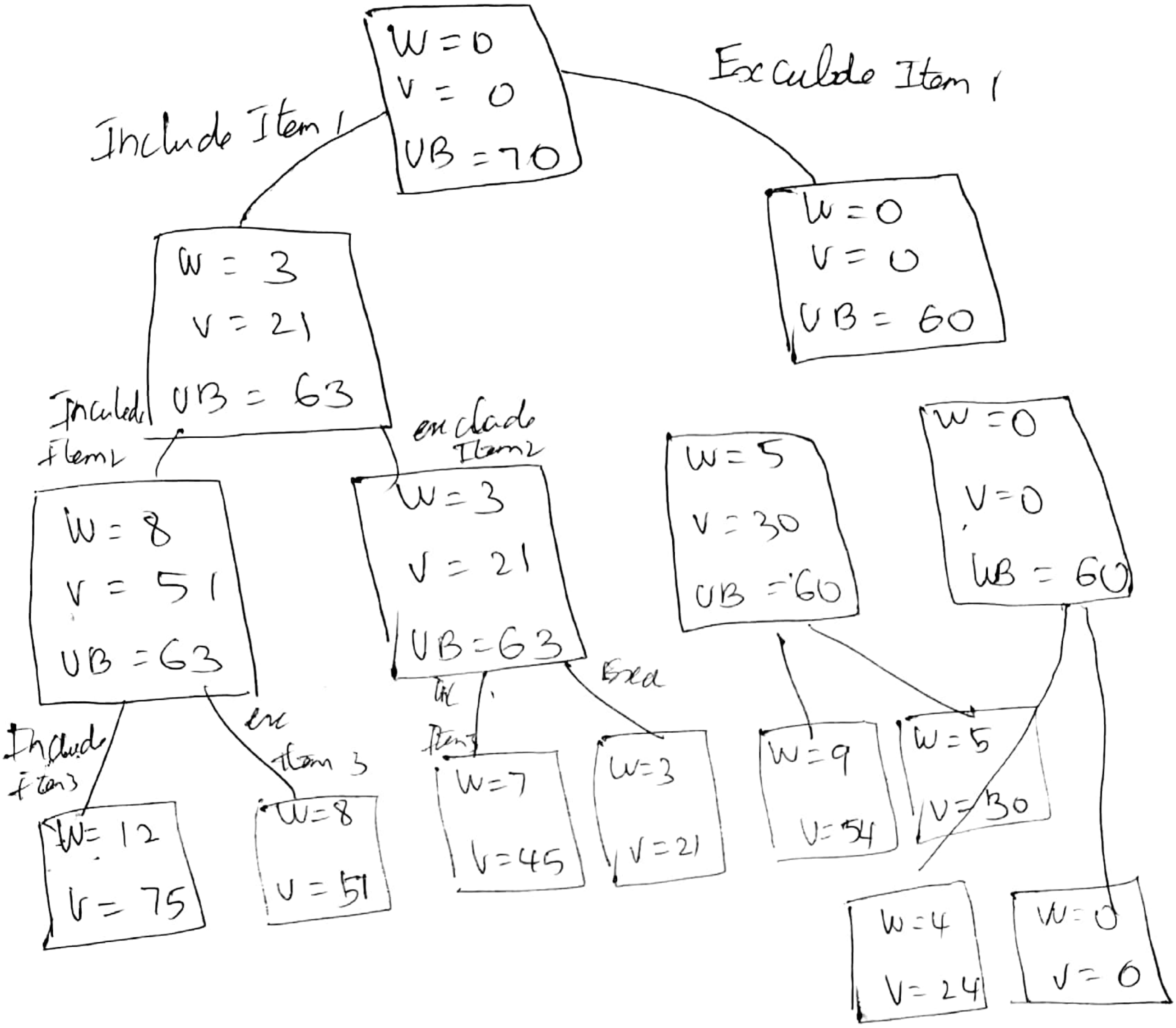
$UB = 21 + \left((10 - 3) * \left(\frac{24}{4}\right)\right)$

$= 21 + (7 * 6) = 63$

$UB = 30 + \left((10 - 5) * \left(\frac{24}{4}\right)\right) = 30 + (5 * 6) = 60$

$UB = 0 + \left((10 - 0) * \frac{24}{4}\right) = 0 + (10 * 6) = 60$

Stats space tree :-



Total weight = 9

optimal Profit = 54

Items - { Item 2 Item 3 }

Items	w_i	v_i
1	2	12
2	3	15
3	4	16
4	5	25

With knapsack capacity $W = 12$

$W = 12$

Sort items based on $\frac{v_i}{w_i}$ value.

$$\text{Item 1} = \frac{12}{2} = 6$$
$$\text{Item 2} = \frac{15}{3} = 5$$
$$\text{Item 3} = \frac{16}{4} = 4$$
$$\text{Item 4} = \frac{25}{5} = 5$$

The sorted order is Item 1, Item 2, Item 4, Item 3

Upper Bound, $UB = V + (W - w) * \frac{v_{i+1}}{w_{i+1}}$

<u>selecting Item 1</u> :- (1) $UB = 0 + ((12-0) * (\frac{12}{2})) = 12 \times 6 = 72$

<u>Selecting Item 2</u> :- (1) $UB = 12 + ((12-2) * (\frac{15}{3})) = 12 + (10 \times 5) = 62$

(2) $UB = 0 + ((12-0) * (\frac{15}{3})) = 0 + (12 \times 5) = 60$

<u>selecting Item 4</u> :- (1) $UB = 27 + ((12-5) * (\frac{25}{5})) = 27 + (7 \times 5) = 62$

(2) $UB = 12 + ((12-2) * (\frac{25}{5})) = 12 + (10 \times 5) = 62$

(3) $UB = 15 + ((12-3) * (\frac{25}{5})) = 15 + (9 \times 5) = 60$

(4) $UB = 0 + ((12-0) * (\frac{25}{5})) = 0 + (12 \times 5) = 60$

<u>selecting Item 3</u> :- (1) $UB = 52 + ((12-10) * (\frac{16}{4})) = 52 + (2 \times 4) = 60$

(2) $UB = 27 + ((12-5) \times (\frac{16}{4})) = 27 + (7 \times 4) = 55$

(3) $UB = 37 + ((12-7) \times (\frac{16}{4})) = 37 + (5 \times 4) = 57$

(4) $UB = 12 + ((12-2) \times (\frac{16}{4})) = 12 + (10 \times 4) = 52$

(5) $UB = 40 + ((12-8) \times (\frac{16}{4})) = 40 + (4 \times 4) = 56$

(6) $UB = 15 + ((12-3) \times (\frac{16}{4})) = 15 + (9 \times 4) = 51$

(7) $UB = 25 + ((12-5) \times (\frac{16}{4})) = 25 + (7 \times 4) = 53$

(8) [illegible]