thws Technische Hochschule Würzburg-Schweinfurt
Faculty of Computer Science and Business Information Systems
5172080: Fundamentals of Mobile Robotics

Prof. Dr. Pascal Meißner
pascal.meissner@thws.de
+49 (0)931 35118353
Franz-Horn-Str. 2 (room K.3.12)

Tutorial 1

Topic: Linear Algebra

Solutions will be discussed in class Monday 12:10-13:10, Week 42 of the calendar year.

Q 1 – Vectors and Matrices

1.a

Calculate the values of the expressions

$$\boldsymbol{a}+\boldsymbol{b}, \quad \boldsymbol{b}+\boldsymbol{a}, \quad k\boldsymbol{a}+k\boldsymbol{b}, \quad k(\boldsymbol{a}+\boldsymbol{b}), \quad (k\boldsymbol{a})\cdot\boldsymbol{b}, \quad ||\boldsymbol{a}||$$

if $\boldsymbol{a} = \begin{bmatrix}1 & 2\end{bmatrix}^T$, $\boldsymbol{b} = \begin{bmatrix}2 & 3\end{bmatrix}^T$, and $k = 3$.

1.b

Determine if the vectors $\begin{bmatrix}1 & 2\end{bmatrix}^T$ and $\begin{bmatrix}-2 & -4\end{bmatrix}^T$ are linearly dependent.

1.c

Calculate the values of the expressions

$$\boldsymbol{Aa}, \quad \boldsymbol{A^T A}, \quad \boldsymbol{AA^T}, \quad \boldsymbol{A^2}$$

if $\boldsymbol{A} = \begin{bmatrix}1 & -4\\ -2 & 5\\ 3 & -6\end{bmatrix}$, and $\boldsymbol{a} = \begin{bmatrix}1\\2\end{bmatrix}$.

1.d

Calculate the determinants of the matrices

$$\begin{bmatrix}4 & 1\\ 3 & 2\end{bmatrix}, \begin{bmatrix}1 & 2\\ 2 & 4\end{bmatrix}, \begin{bmatrix}1 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 3\end{bmatrix}.$$

Q 2 – 2D Homogenous Transformations

Considering a robot moving on a plane, its pose w.r.t. a global coordinate frame is commonly written as $\boldsymbol{x} = (x, y, \theta)^T$, where (x, y) denotes its position in the xy-plane and θ its orientation. The homogeneous transformation matrix that represents a pose $\boldsymbol{x} = (x, y, \theta)^T$ w.r.t. to the origin $(0, 0, 0)^T$ of the global coordinate system is given by

$$^{0}\boldsymbol{T}_x = \begin{bmatrix} \boldsymbol{R}(\theta) & \boldsymbol{t} \\ 0 & 1 \end{bmatrix}, \boldsymbol{R}(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \boldsymbol{t} = \begin{bmatrix} x \\ y \end{bmatrix}$$

2.a

While being at pose $\boldsymbol{x_1} = (x_1, y_1, \theta_1)^T$, the robot senses a landmark l at position (l_x, l_y) w.r.t. to its local frame. Use the matrix $^{g}\boldsymbol{T}_{x_1}$ to calculate the coordinates of l w.r.t. the global frame.

2.b

Now imagine that you are given the landmark's coordinates w.r.t. to the global frame. How can you calculate the coordinates that the robot will sense in his local frame?

2.c

The robot moves to a new pose $\boldsymbol{x_2} = (x_2, y_2, \theta_2)^T$ w.r.t. the global frame. Find the transformation matrix $^{x_1}\boldsymbol{T}_{x_2}$ that represents the new pose w.r.t. to $\boldsymbol{x_1}$. *Hint*: Write $^{x_1}\boldsymbol{T}_{x_2}$ as a product of homogeneous transformation matrices.