thws Technische Hochschule Würzburg-Schweinfurt

Faculty of Computer Science and Business Information Systems
5172080: Fundamentals of Mobile Robotics

Prof. Dr. Pascal Meißner
pascal.meissner@thws.de
+49 (0)931 35118353
Franz-Horn-Str. 2 (room K.3.12)

Tutorial 6 – Solutions

Topic: SLAM: Simultaneous Localization and Mapping

Q 1 – Bearing-only SLAM

Bearing-only SLAM refers to the SLAM problem when the sensors can only measure the bearing of a landmark but not its range. One problem in bearing-only SLAM with EKFs concerns the initialization of landmark location estimates, even if the correspondences are known. Discuss why, and devise a technique for initializing the landmark location estimates (means and covariances) that can be applied in bearing-only SLAM.

A single bearing measurement in 2D gives mostly information about the angle α of the landmark in the robot frame. The distance is only known to lie somewhere between zero and the maximum range reading $d_{\max}$ of the detector.

For the EKF framework we will require the possible position of the landmark to be described by a 2D Gaussian ellipse. This assumption is certainly not well met in reality with a single bearing measurement. However, it will be ok once a second measurement is made from a different location.

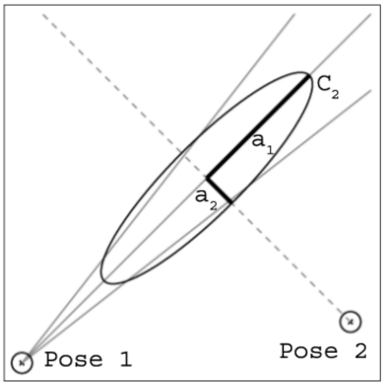
One can choose the mean position of the landmark to be at the center between the robot and the maximum range reading with $d = {}^{d_{\max}}/_2$:

$$\begin{bmatrix} x_l \\ y_l \end{bmatrix} = d \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \end{bmatrix}$$

The uncertainty of the landmark position is described by a 2D Gaussian ellipse with axes a_1 and a_2. The covariance matrix is then given in the landmark frame ($\vec{x}$ pointing to the robot) as

$$E = \begin{bmatrix} a_1^2 & 0 \\ 0 & a_2^2 \end{bmatrix}$$

The larger axis a_1 corresponds to the uncertainty of the landmark distance, which can be chosen e.g. as $a_1 = {}^{d_{\max}}/_2$. The angular uncertainty is given by $a_2 = d\tan(\sigma_\alpha) \approx d\,\sigma_\alpha$ for small σ_α (a.k.a. small-angle approximation), where σ_α is the standard deviation of the measurement uncertainty and a_2 is derived from the trigonometric ratio: $\tan(\sigma_\alpha) = {}^{\text{opp}}/_{\text{adj}} = {}^{a_2}/_{d}$. The covariance in the robot frame is $C_2 = RER^T$, where $R(\alpha)$ is the usual 2D rotation matrix and C_2 results from the linear transformation of a Gaussian (see "'Localization - Kalman Filter"' slide 7). This is illustrated in the following figure, which can be found along more details in: *A. Costa, G. Kantor, H. Choset*, "'Bearing-only Landmark Initialization with Unknown Data Association"', ICRA proceedings, 2004.



Once a new measurement from a different location is available, the landmark position distribution is described by the multiplication of two lengthy ellipses at different angles, resulting in a more circular distribution (corresponding to the overlap of the ellipses).

Q 2 – Data Association

Features extracted from an observation can be interpreted as either matches with existing features in a map, previously unobserved features, or false alarms (noise). Consider two features z_t^1 and z_t^2 extracted from an observation z_t, and a map $m_t = \{l_1, l_2\}$ with two landmarks. Each observed feature z_t^i is either assigned to an existing or a new landmark, or it is marked as a false alarm.

1.a

Write down all possible assignments for the two observed features z_t^1 and z_t^2. Note that each feature can be associated to at most one landmark and vice versa.

At maximum one observation $z_t^{1,2}$ is assigned to each landmark $l_{1,2}$. A new landmark l_3 is initialized if one observation cannot be assigned, and a second new landmark l_4 is initialized only if both observations cannot be assigned. The possible solutions are (fa is false alarm):

Solution	l_1	l_2	l_3	l_4	fa	fa
1	$\boldsymbol{z_t^1}$	z_t^2				
2	$\boldsymbol{z_t^1}$		z_t^2			
3	$\boldsymbol{z_t^1}$				z_t^2	
4	z_t^2	$\boldsymbol{z_t^1}$				
5		$\boldsymbol{z_t^1}$	z_t^2			
6		$\boldsymbol{z_t^1}$			z_t^2	
7	z_t^2		$\boldsymbol{z_t^1}$			
8		z_t^2	$\boldsymbol{z_t^1}$			
9			$\boldsymbol{z_t^1}$	z_t^2		
10			$\boldsymbol{z_t^1}$		z_t^2	
11			z_t^2	$\boldsymbol{z_t^1}$		
12	z_t^2				$\boldsymbol{z_t^1}$	
13		z_t^2			$\boldsymbol{z_t^1}$	
14			z_t^2		$\boldsymbol{z_t^1}$	
15					$\boldsymbol{z_t^1}$	z_t^2

In total 15 possible solutions can be found. The solutions 9 and 11 are equivalent if the order of new assignments is fixed, such that only 14 solutions remain. Let's assume this in the following.

1.b

Now consider an update of the map to obtain the map estimate m_{t+1}. Here, every new feature is added to the map as a new landmark, and every existing landmark without a match is removed. Suppose no false alarm is detected. How many solutions for the assignments remain? Are there any two solutions that will result in the same map?

If no false alarm is detected, seven solutions remain in the table above. Each of the solutions results in a map with two landmarks (corresponding to the two observed features). Some solutions result in the same landmarks indices (e.g. solution 5 and 8 will both result in $m_{t+1} = \{l_2, l_3\}$). However, the resulting maps can still differ, since the landmark positions are updated with different observed feature positions.

1.c

How many new assignments can be generated from this set of maps in total if at time $t+1$ a single feature z_{t+1}^1 is observed?

Seven possible maps remain from the previous time step, each with two landmarks $l_{a,b}$. For each of these maps, exactly four new assignments can be made:

$$z_{t+1}^1 \to l_a \qquad z_{t+1}^1 \to l_b \qquad z_{t+1}^1 \to l_{\text{new}} \qquad z_{t+1}^1 \to fa$$

in total $7 \cdot 4 = 28$ possible assignments are found.