Faculty of Computer Science and Business Information Systems
5172080: Fundamentals of Mobile Robotics

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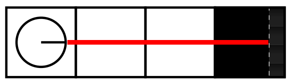
Tutorial 5 – Solutions

Topic: Grid Maps & Mapping with Known Poses

Q 1 – Occupancy Mapping

A robot has to build an occupancy grid map of a simple one-dimensional environment using a sequence of measurements from a range sensor.



Assume a simple inverse sensor model: every grid cell with a distance (based on its coordinate) smaller than the measured distance is assumed to be occupied with $p = 0.3$. Every cell behind the measured distance is occupied with $p = 0.6$. Every cell located more than 20cm behind the measured distance should not be updated. Additionally, the prior belief is set to 0.5.

Give the equation of the inverse sensor model $l(m_i \mid z_t, x_t)$ and the value of the prior $l(m_i)$ as used in the log-odds update equation.

Additionally, derive the equation to convert from log-odds to probabilities from the definition of the log odds ratio.

Recall the log-odds update equation on slide 6 of the ”‘Grid Maps and Mapping With Known Poses - 2”’ lecture:

$$l(m_i \mid z_{1:t}, x_{1:t}) = l(m_i \mid z_t, x_t) + l(m_i \mid z_{1:t-1}, x_{1:t-1}) - l(m_i)$$

From this lecture, we know that the prior should be (equation given on slide 5):

$$p(m_i) = 0.5 \Rightarrow l(m_i) = \log \frac{p(m_i)}{1 - p(m_i)} = 0$$

We also know that the inverse sensor model is the following (see the ”‘Grid Maps and Mapping With Known Poses - 2”’ lecture, slide 9):

$$p(m_i \mid z_t, x_t) = \begin{cases} 0.3 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) \leq z_t \\ 0.6 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) > z_t \wedge \\ & \text{distance}(\text{pos}(m_i), \text{pos}(x_i))) \leq z_t + 20 \text{ cm} \\ 0.5 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) > z_t + 20 \text{ cm} \end{cases}$$

The log-odds ratio should be applied to this function in order to obtain $l(m_i|z_t, x_t)$:

$$l(m_i \mid z_t, x_t) = \begin{cases} \log({}^{0.3}/_{1\,-\,0.3}) \approx -0.847 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) \leq z_t \\ \log({}^{0.6}/_{1\,-\,0.6}) \approx 0.405 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) > z_t \wedge \\ & \text{distance}(\text{pos}(m_i), \text{pos}(x_i))) \leq z_t + 20\text{cm} \\ \log({}^{0.5}/_{1\,-\,0.5}) = 0 & \text{if distance}(\text{pos}(m_i), \text{pos}(x_i))) > z_t + 20\text{cm} \end{cases}$$

The inverse transformation from log-odds to probability can be obtained from the log-odds definition (equation given on slide 5) as follows:

$$l = \log \frac{p}{1-p} \Rightarrow \exp l = \frac{p}{1-p}$$
$$\Rightarrow (1-p) \exp l = p \Rightarrow \exp l = p(1 + \exp l)$$
$$\Rightarrow p = \frac{\exp l}{1 + \exp l} = \frac{\exp l + 1 - 1}{1 + \exp l} = 1 - \frac{1}{1 + \exp l}$$

Q 2 – Occupancy Mapping

Prove that in the occupancy grid mapping framework the occupancy value of a grid cell $P(m_j \mid x_{1:t}; z_{1:t})$ is independent of the order in which the measurements are integrated.

Let us consider the log-odds update equation and let us recursively unwrap it:

$$\begin{aligned} l(m_i \mid z_{1:t}, x_{1:t}) &= l(m_i \mid z_t, x_t) + l(m_i \mid z_{1:t-1}, x_{1:t-1}) - l(m_i) = \\ &= l(m_i \mid z_t, x_t) + l(m_i \mid z_{t-1}, x_{t-1}) + l(m_i \mid z_{1:t-2}, x_{1:t-2}) - 2 \cdot l(m_i) = \\ &= \cdots = \sum_{k=1}^{t} l(m_i \mid z_k, x_k) - t \cdot l(m_i) \end{aligned}$$

It is clear that $l(m_i \mid z_{1:t}, x_{1:t})$ is simply a sum of the log-odds form of the inverse measurements. Suppose that we exchange a measurement at time i with the measurement at a different time j. Since the sum is a commutative operator we will still obtain the same value of $l(m_i \mid z_{1:t}, x_{1:t})$. Hence, we can repeat exchanging measurements to obtain any order permutation of the measurements without changing the result. Finally, since the log-odds value does not change, neither will the corresponding probability.

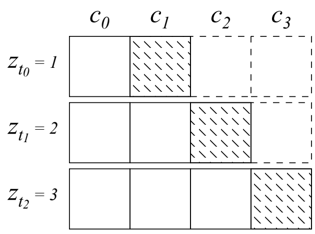
Q 3 – Counting Model

A robot applies the so-called simple counting approach to build a grid map of a 1D environment consisting of the cells $c_0, \ldots, c_3$. While standing in cell c_0, the robot integrates *four measurements $z_{t_0}, \ldots, z_{t_3}$. After integrating these measurements, the resulting belief of the robot with regards to the occupancy of the four cells is $b_0 = 0$, $b_1 = \frac{1}{4}$, $b_2 = \frac{2}{3}$, $b_3 = 1$. Given that the first three measurements are $z_{t_0} = 1$, $z_{t_1} = 2$, $z_{t_2} = 3$, compute the value of the last measurement z_{t_3}.*

A counting model will compute the belief of a cell c_i as:

$$b_i = \frac{\text{hits}_i}{\text{hits}_i + \text{misses}_i}$$

For convenience, let us represent graphically what we know about the measurements. For each measurement we denote with a shaded rectangle a hit, with an empty rectangle a miss and with a dashed rectangle the area which is not subject to update:



This representation tells us that up until time t_2 the hit/miss scenario is the following:

	c_0	c_1	c_2	c_3
hits	0	1	1	1
misses	3	2	1	0

For each cell let us now consider the possible scenarios for a new measurement z_{t_3}:

- For b_0 to be 0, either we get another miss or we should not be subject to update.
- For b_1 to be $\frac{1}{4}$ we necessarily need another miss, hence $z_{t_3} > 1$.
- For b_2 to be $\frac{2}{3}$ we necessarily need another hit, hence we have $z_{t_3} = 2$.
- For b_3 to be 1, either we get another hit or the cell is not subject to update. The latter is consistent with our observation of $z_{t_3} = 2$.

We thus conclude that $z_{t_3} = 2$.