

ARTIFICIAL INTELLIGENCE

Report on
N-Queens Problem

Under the Guidance of: **Dr. Jaylakshmi Banda**

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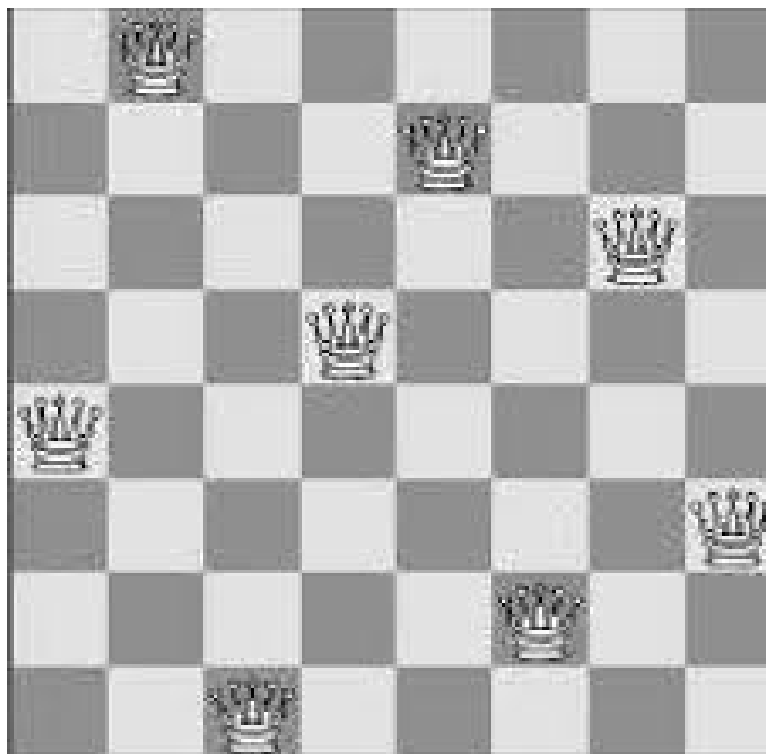
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1. Problem Statement

Using a chess board of $N \times N$ size, the challenge is to place N queens on it so that no two queens attack each other.

For example, Using a regular chess board, the challenge is to place 8 queens on the board such that no queen is attacking any of the other.

So one of the solution for above example could be like this:-



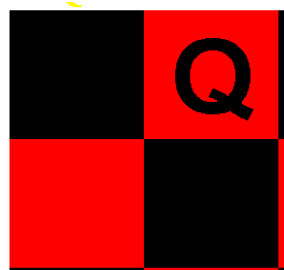
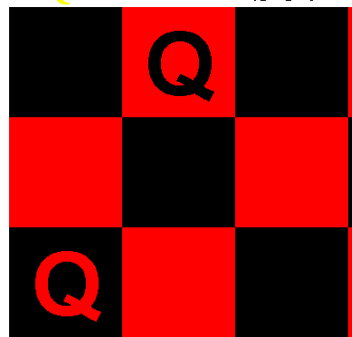
There can be many other solutions for 8 queens and similarly we can change number of queens and board size.

2. Solving N- queens Problem

We cannot use N-queens problem solution, if we have number of queens equal to 3 or 4 we can't use:

$N = 3 \text{ or } 4$

Cannot use N Queens



{ possible soln }

2.1 Search Space: The set of objects among which we search for the solution

Example: N-queen configurations

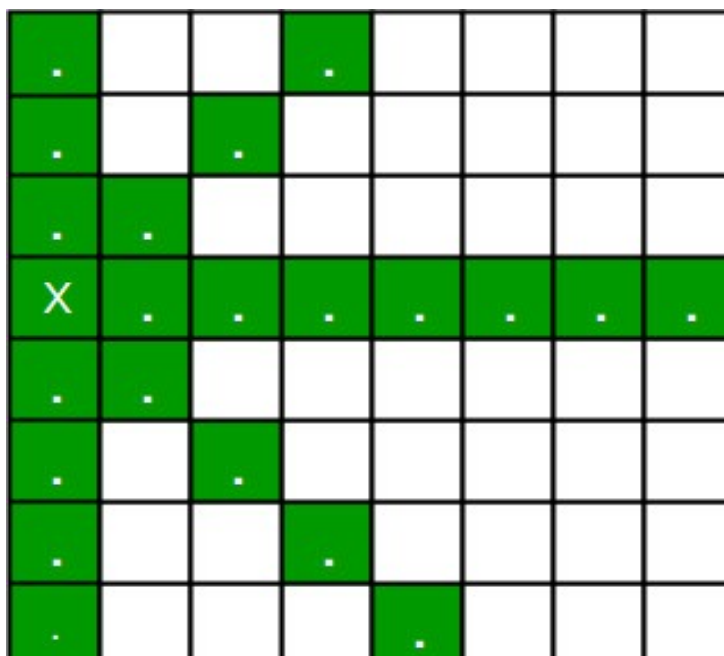
2.2 Goal condition: This is the characteristics of the object we want to find in the search space?

Example: Non-attacking n-queen configuration

2.3 Constraints

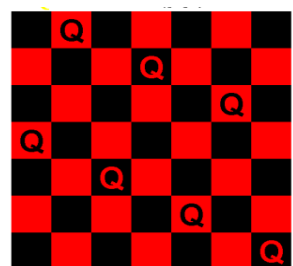
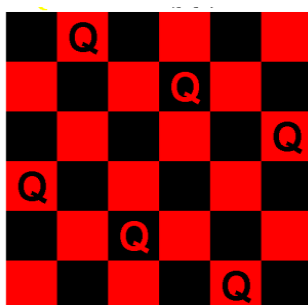
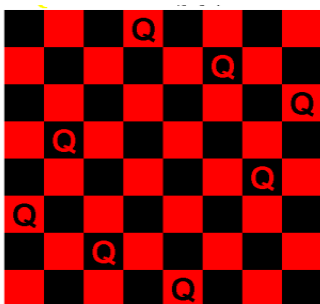
Some constraints are as follows-

- There can be only one queen in a column
- No two queens will be on the same diagonal.
- No queens on the same row.



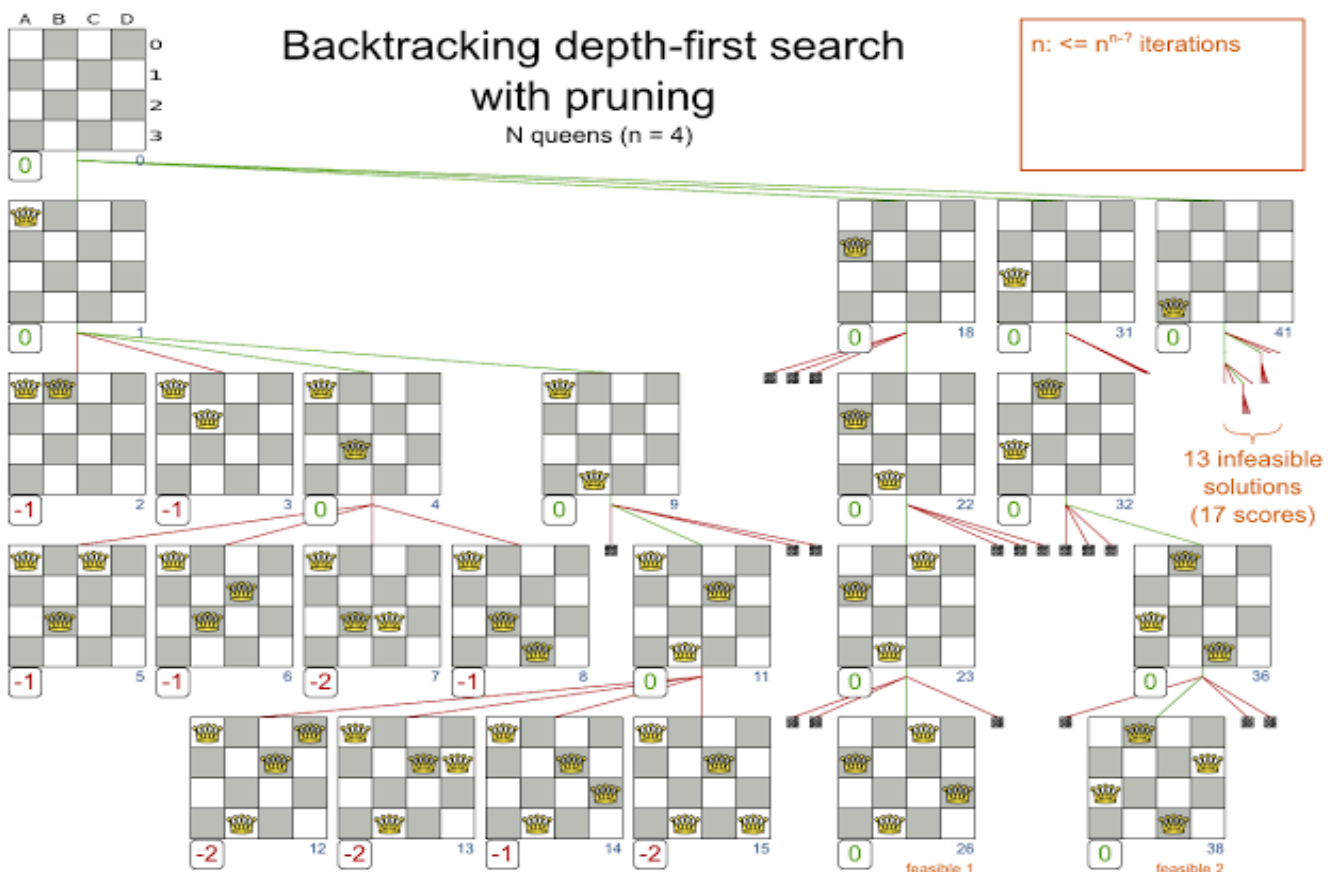
3. Mathematical Formulation

- For $N > 4$ only
- N is even except $N \neq 6K+2$:
 - Row 1 to $N/2$: Queen on $2*Row$
 - Row $N/2+1$ to N : Queen on $2*Row-N-1$
- N is even, $N = 6K+2$
 - Row 1 to $N/2$: Queen on $(2*Row + N/2 - 3) \bmod N + 1$
 - Row $N/2+1$ to N : Queen on $N - (2*(N-Row+1) + N/2 - 3) \bmod N$
- N is odd:
 - When N is even, no queen is placed on position $(1,1)$.
 - So this just places the first $N-1$ queens as on an $N-1$ (even) sized board, then places the last queen on the bottom right position (N,N) .



4. Solution Using Backtracking

- One of the approach that guarantees a solution is backtracking, though it can be slow
- It Can be seen as a form of intelligent depth-first search.
- The idea is to place queens one by one in different columns, starting from the leftmost column.
- When we place a queen in a column, we check for clashes with already placed queens. In the current column, if we find a row for which there is no clash, we mark this row and column as part of the solution.
- If we do not find such a row due to clashes then we backtrack and return false.



4.1 Algorithms used will be

1. Start in the leftmost column
2. If all queens are placed
 return true
3. Try all rows in the current column.
 Do following for every tried row.
 - a) If the queen can be placed safely in this row then mark this [row, column] as part of the solution and recursively check if placing queen here leads to a solution.
 - b) If placing the queen in [row, column] leads to a solution then return true.
 - c) If placing queen doesn't lead to a solution then unmark this [row, column] (Backtrack) and go to step (a) to try other rows
4. If all rows have been tried and nothing worked, return false to trigger backtracking.

4.2 Pseudo code of backtracking in N-queens

```
IS-ATTACK(i, j, board, N)
// checking in the column j
for k in 1 to i-1
    if board[k][j]==1
        return TRUE
// checking upper right diagonal
k = i-1
l = j+1
while k>=1 and l<=N
    if board[k][l] == 1
        return TRUE
    k=k+1
    l=l+1
```



```

// checking upper left diagonal
k = i-1
l = j-1
while k>=1 and l>=1
    if board[k][l] == 1
        return TRUE
    k=k-1
    l=l-1
return FALSE

```

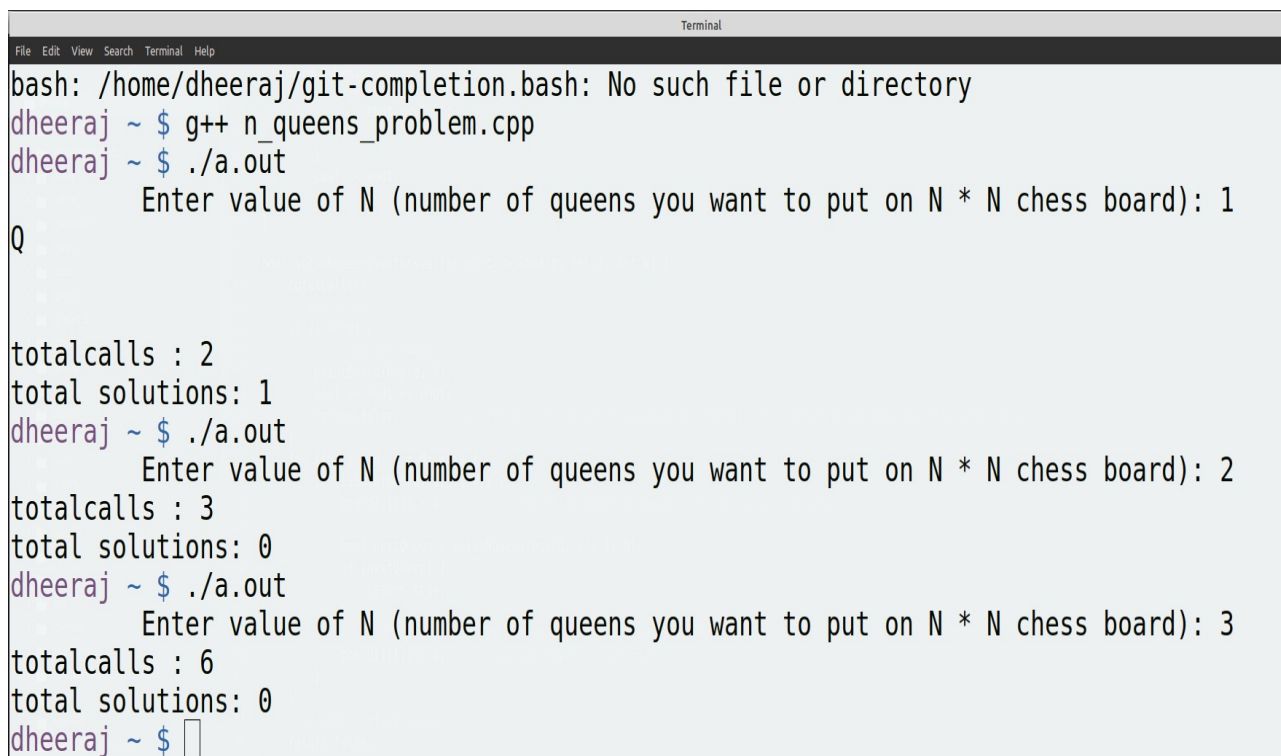
4.3 Input/Output of our submitted code

1. If we give input as $N = 1$

Output will be 1 solution

2. And for input as $N = 2 \text{ \& } 3$

Output will be no solution



```

Terminal
bash: /home/dheeraj/git-completion.bash: No such file or directory
dheeraj ~ $ g++ n_queens_problem.cpp
dheeraj ~ $ ./a.out
Enter value of N (number of queens you want to put on N * N chess board): 1
Q
totalcalls : 2
total solutions: 1
dheeraj ~ $ ./a.out
Enter value of N (number of queens you want to put on N * N chess board): 2
totalcalls : 3
total solutions: 0
dheeraj ~ $ ./a.out
Enter value of N (number of queens you want to put on N * N chess board): 3
totalcalls : 6
total solutions: 0
dheeraj ~ $ 

```

3. If we give input as $N = 4$

Output will be shown as all possible solution of queens and total number of call for backtracking

```
Terminal
dheeraj ~ $ g++ n_queens_problem.cpp
dheeraj ~ $ ./a.out
Enter value of N (number of queens you want to put on N * N chess board): 4

_ Q _ _
Q _ _ _
_ _ Q _
_ _ _ Q

_ Q _ _
Q _ _ _
_ _ Q _
_ _ _ Q

totalcalls : 17
total solutions: 2
dheeraj ~ $
```

4. Silimary for input $N = 8$, total solution = 92

```
Terminal
_ _ _ _ _ Q
Q _ _ _ _ _
_ Q _ _ _ _
_ _ Q _ _ _
_ _ _ Q _ _
_ _ _ _ Q _

_ Q _ _ _ _
Q _ _ _ _ _
_ Q _ _ _ _
_ _ Q _ _ _
_ _ _ Q _ _
_ _ _ _ Q _

totalcalls : 2057
total solutions: 92
dheeraj ~ $
```

5. As for now possible solutions were known upto $N = 27$ and our code is running but it will take slightly more than a year time for computation as there are 234907967154122528 solutions of the 27-Queens.

~~~~~End of Report~~~~~