Name: Dheeraj Chaudhary
Roll: 17BCS009

Assignment on Design of Experiment

Design of Experiment —

Analysis of Variance → The ANOVA is a simple arithmetical process of sorting out the components of variation in a given data.

And in order to verify a hypothesis pertaining to some specific scientific phenomenon we've to collect data. Such data are obtained by either observation or by experimentation.

So, we use three type of design for Experiments -

1 → CRD (one way ANOVA): powerful statistical tool

→ Basic principle of experiment
- Replication
- Randomization
- Local control

Treatment	1	2	3	...	j	...	r
t_1	y_{11}	y_{12}	...				y_{1r}
t_2							
⋮	⋮	⋮			⋮		⋮
t_i							
⋮	⋮						
t_k	y_{k1}	y_{k2}	..		y_{kj}	..	y_{kr}

Scanned with CamScanner

$$y_{ij} = \mu + \alpha_i + \varepsilon_{ij}$$

where y_{ij} = j^{th} & in i^{th} treatment

α_i = treatment effect

μ = general mean

ε_{ij} = error effect

ANOVA table

H_0, $\alpha_1 = \alpha_2 = \alpha_3 \cdots \alpha_k = 0$

Src. of var.	d.f.	sum of sq.	mean sum of sq.	F_{cal}	F_{tab}
Treatment	$K-1$	Tr. S.S	$Tr.M.S = TrSS/(K-1)$	$F_{cal} = \frac{TrMS}{EMS}$	$F_{tab}{}^{2}$ $[K-1, N-K]$
error	$N-K$	E.S.S	$ErMS = ESS/N-K$		
Total	$N-1$	T.S.S			

$$\text{correction factor } (C.F) = \frac{(G.T)^2}{N}$$

$$TrS.S = \left\{ \frac{T_1{}^2}{n_1} + \frac{T_2{}^2}{n_2} + \cdots + \frac{T_k{}^2}{n_k} \right\} - C.F$$

$$= \sum_{i=1}^{k} \frac{T_i{}^2}{n_i} - CF$$

$$TSS = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - CF$$

$$ErSS = T.S.S. - Tr.S.S.$$

Q→ A research organisation tested microwave oven at $\alpha = 5\%$ los. If there is any significant diff in the avg prices of these 3 types of oven.

Scanned with CamScanner

Sol-> H_0: $\mu_1 = \mu_2 = \mu_3$ i.e, there is no significant diff b/w avg. prices of given 3 ovens.

H_1: $\mu_1 \neq \mu_2 \neq \mu_3 \ \forall\ i = j = 1, 2, 3$

Given data: watts

1000	900	1800
270, 230, 245, 190, 215, 250	240, 135, 160, 230, 250, 200, 200, 210	180, 180, 155, 140, 200, 130, 120, 140
(A)	(B)	(C)

$$N = 8 + 8 + 6 = 22$$

$T_A = 270 + 230 + \cdots + 250 = 1400$

$T_B = 240 + 135 + \cdots + 250 = 1625$

$T_C = 180 + 180 + \cdots + 140 = 1245$

$$G.T = 4270$$

$\therefore$ $C.F. = \frac{(G.T)^2}{N} = 828768.18$

and $TrS.S = \left(\frac{(1400)^2}{6} + \cdots \frac{(1245)^2}{8} \right) - CF$

$= 850497.9 - 828768.18$

$= 21729.72.$

$T.S.S = (270)^2 + \cdots + (130)^2 - CF$

$= (72900 + 60025 + \cdots + 16900) - CF$

$= 870900 - 828768.18 = 42131.82$

$Er.S.S = TSS - TrSS = 20402.1$

$Tr.M.S = \frac{21729.72}{2} = 10864.86$

$Er\,M.S = 1073.79$

$\therefore\ F_{cal} = \frac{Tr\,M\,S}{Er\,M.S} = 10.11$

$F_{tab}[2, 19]$ at 5% l.o.s $= 3.5219$

$\therefore\ F_{cal} > F_{tab} \Rightarrow$ reject H_0

there is a significant diff b/w avg. prices of ovens.

2 → TWO-WAY ANALYSIS of variance (or ANOVA) or Randomised Block Design.

Let $t_1, t_2, t_3, \ldots, t_k$ are k no. of treatment replicated in corresponding blocks $B_1, B_2 \ldots B_r$ resp.

Treatment	b_1	b_2	...	b_j	...	b_r	Total	Mean
t_1	y_{11}	y_{12}	...			b_{1r}	T_1	$\bar{T}_1$
t_2	⋮							
⋮								
t_i	y_{i1}	y_{i2}	...			b_{ir}	T_i	$\bar{T}_i$
⋮								⋮
t_k	y_{k1}	y_{k2}				y_{kr}	T_k	$\bar{T}_k$
Total B_i	B_1	B_2	...			B_r	T	
Mean	$\bar{B}_1$	$\bar{B}_2$	...			$\bar{B}_r$		

Scanned with CamScanner

$$y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij}, \quad \varepsilon_{ij} \sim N(0, \sigma^2)$$

where, μ : population mean
α_i = i^{th} treatment effect
β_j : j^{th} variety / block effect
ε_{ij} : error effect
y_{ij} : i^{th} treatment replicated in j^{th} block

Src. of var.	d.f.	S.S	M.S.S	F ratio	F tab
Treatment	$(k-1)$	Tr.S.S	TrMS = TrSS/(k-1)	TrMS/ErMS	$F[(k-1),(k-1)(r-1)]$
Block	$(r-1)$	Br.S.S	B.M.S = B.S.S/(r-1)	BMS/ErMS	$F_{B\,tab} = F[(r-1),(k-1)(r-1)]$
Error	$(k-1)(r-1)$	Er.S.S	Er.M.S = ErSS/(k-1)(r-1)		

⇒ If $F_{cal} < F_{tab}$ value is less than accept H_0, otherwise reject.

$$C.F = \frac{(G.T)^2}{r \cdot k}$$

$$Tr\,SS = \sum_{i=1}^{k} \frac{T_i^2}{r} - CF$$

$$B.S.S = \sum_{j=1}^{r} \frac{B_j^2}{k} - CF$$

$$T.S.S = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - CF$$

$$E.S.S = T.S.S - [Tr\,SS + BSS]$$

Ques→ As a new type of environment, natural air is being developed. It's tested that whether there's any significant diff b/w there factors.

	I	II	III	
A	25, 28, 26, 30, 31	30, 32, 35, 29, 31	43, 40, 42, 49, 48	T_A
	15, 18, 22, 21, 17	21, 27, 18, 15, 19	23, 25, 24, 17, 13	T_B
	B_I	B_{II}	B_{III}	

Sol→ $T_A = 140 + 157 + 222 = 518$

$T_B = 93 + 100 + 102 = 293$ } $814 = G.T$

$B_I = 140 + 93 = 233$

$B_{II} = 157 + 100 = 257$ } $814 = G.T$

$B_{III} = 222 + 102 = 324$

$$C.F. = \frac{(G.T)^2}{r.k} = \frac{(814)^2}{3 \cdot 2} = 110432.66$$

$$Tr.S.S = \sum_{i=1}^{k} \frac{T_i^2}{r} - Cf = \frac{(519)^2}{3} + \frac{(295)^2}{3} - 110432.66$$

$= 8362.67$

$$B.S.S = \sum_{j=1}^{r} \frac{B_j^2}{k} - Cf \Rightarrow \frac{225314}{2} - 2224.34$$

$$T.S.S = \sum_{i=1}^{k} \sum_{j=1}^{r} (y_{ij})^2 - Cf$$

$\Rightarrow \{(140)^2 + \dots (102)^2\} - Cf$

$\Rightarrow 122586 - 110432.66 = 12153.34$

$$E.S.S = T.S.S - (Tr.S.S + B.S.S)$$
$$= 12153.34 - (2224.34 + 8362.67)$$
$$= 1566.33$$

$$Tr.M.S = \frac{Tr.S.S}{k-1} = 8362.67$$

$$B.M.S = \frac{B.S.S}{r-1} = \frac{2224.34}{2}$$

$$Er.M.S = \frac{1566.33}{(2-1)(3-1)} = \frac{1566.33}{2} = 783.165$$

$$F_{cal\,Tr} = \frac{Tr\,MS}{E\,MS} = 10.678$$

$$F_{B\,cal} = \frac{1112.17}{783.165} = 1.420$$

$$F_{tab\,Tr}\left([(k-1),(k-1)(r-1)]\right) = F_{tab}(2,2) = 19.00$$

at $\alpha = 5\% \Rightarrow 0.05$

$F_{cal} < F_{tab}$ both the cases.

$\Rightarrow$ accept H_0.

3 → LSD (Latin Square Design) or Three-way ANOVA

involves treatment, row and column.

$i = j = k = 1, 2, \ldots m$

here, $m = 4$

	C_1	C_2	---	C_j	---	C_m
R_1	y_{11}	y_{12}	---	y_{1j}		y_{1m}
R_2	y_{21}	---				y_{2m}
⋮	⋮					⋮
R_i	⋮					y_{im}
⋮						⋮
R_m	y_{m1}	y_{m2}	---		---	y_{mm}

where, y_{ij} = i^{th} treatment j^{th} row k^{th} column observed value

u = general / population mean

α_i = i^{th} treatment effect

B_j - j^{th} row effect

γ_k k^{th} column effect

ε_{ijk} error effect.

ANOVA Table.

Src of Variation	df.	SS	MSS	F ratio	F tab.
Treatment	$m-1$	TrSS	TrSS/$m-1$	TrMS/ErMS	$(m-1)$
Rows	$m-1$	RSS	RSS/$m-1$	RMS/ErMS	$(m-1)(m-2)$
column	$m-1$	CSS	CSS/$m-1$	CMS/ErMS	"
error	$(m-1)(m-2)$	ErSS	ErSS/$(m-1)(m-2)$		
Total	m^2-1	TSS			

d.f. of error $= (m^2-1) - (m-1+m-1+m-1)$
$= m^2 - 1 - 3m + 3$
$= m^2 - 3m + 2 = (m-1)(m-2)$

$TSS = TrSS + RSS + CSS + ErSS$
$ErSS = T.S.S - (TrSS + RSS + CSS)$

Conclusion ⇒ If $F_{cal} < F_{tab}$ then accept H_0, otherwise reject H_0.

$$C.F = \frac{(G.T)^2}{m^2}$$

$$TrSS = \sum_{i=1}^{m} \frac{T_i^2}{m} - C.F.$$

$$R.S.S = \sum_{j=1}^{m} \frac{R_j^2}{m} - CF$$

$$C.S.S = \sum_{K=1}^{m} \frac{C_K^2}{m} - CF$$

$$T.S.S = \sum_{i=1}^{m} \sum_{j=1}^{m} \sum_{k=1}^{m} (y_{ijk})^2 - CF$$

Ques ⇒ Four treatments were tried in a 4x4 layout LSD. The layout plan & design is given below. Analyse the data & state your conclusion.

3.1 T_2	9 T_1	7.2 T_3	3.6 T_4
8.1 T_3	8 T_2	4.5 T_4	9.2 T_1
6.3 T_4	5.4 T_3	9.9 T_1	8.2 T_2
9.5 T_1	7.2 T_4	8.7 T_2	6.3 T_3

$Sol^n \Rightarrow T_1 = 9 + 9.2 + 9.0 + 9.5 = 37.6$

$T_2 = 3.1 + 8.9 + \ldots 8.3 = 33.5$

$T_3 = 7.2 + 8.1 + 5.4 + 6.3 = 27$

$T_4 = 3.6 + 4.5 + 6.3 + 7.2 = 21.6$

$\Big\}\ G.T = 119.7$

$R_1 = 27.9$, $R_2 = 30.7$, $R_3 = 29.8$, $R_4 = 31.3$

$C_1 = 32$, $C_2 = 30.5$, $C_3 = 29.9$, $C_4 = 27.3$

$m = 4$, $\therefore CF = \frac{(GT)^2}{16} = 895.50$

$$TrSS = \frac{(37.6)^2 + (33.5)^2 + \ldots (21.6)^2}{4} - CF$$

$$= \frac{3731.37}{4} - CF = 932.84 - CF$$

$R.SS = 397.15 - CF = 1.645$

$$\& \; CSS = \frac{3593.55}{4} - CF = 898.38 - CF$$

$$TSS = \{(8.1)^2 + (9)^2 + \ldots (6.3)^2\} - CF$$

$= 50.385$

$ErSS = 50.385 - \{37.385 + 2.88 + 1.64\}$

$= 8.472$

$TrMS = 12.461$

$CMS = 0.96$

$RMS = 1.645/3 = 0.548$

$EMS = 8.47/3.2 = 1.41$

$F_{Tr} = 8.82$, $F_C = 0.67$, $F_R = 0.38$

$F_{tab}[3,6] = 4.757$ at $\alpha = 5\%$.

Since, F_R & $F_C < F_{tab}$ then accept H_{0R} & H_{0C}

but $F_{Tr} > F_{tab}$ hence reject H_{0T}.