Cascading Behavior in Networks

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Goal

Modelling the spread of the word!

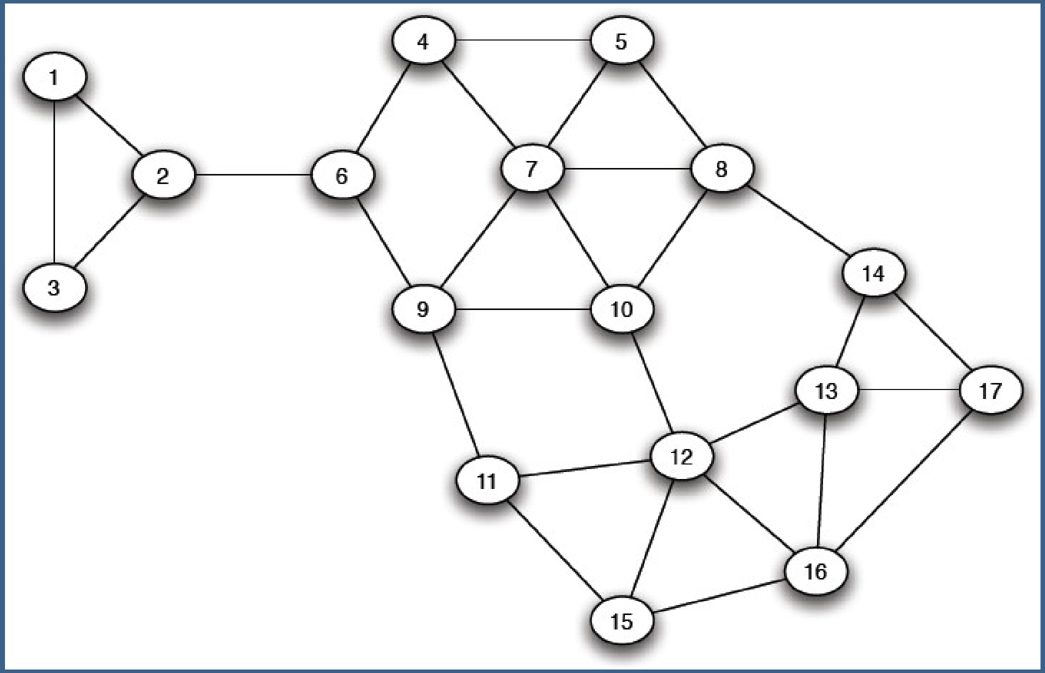
Overview

- Introduction
- Basic idea
- The limit of cascades
- Extension
- Advanced
  - Cascade capacity
  - Bilingual option (co-presentation)
- Conclusion
- Q&A

Introduction

Introduction

Networks visualized as *graphs*



(e.g. social networks)

Basic idea

Basic idea

Diffusion in a network

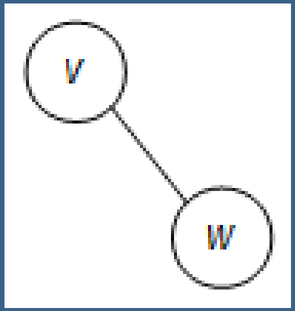
- New behavior
- Adopt if certain amount of neighbors do

Basic idea

Simple Game

- Possible behaviors: *A* and *B*
- Players: 2 nodes (*v* and *w*)

Basic idea



Simple game

- Link between *v* and *w*
  => motive to adopt same behavior

- Both adopt *A*: payoff = $a > 0$
- Both adopt *B*: payoff = $b > 0$
- Opposite behaviors: payoff = 0

Basic idea

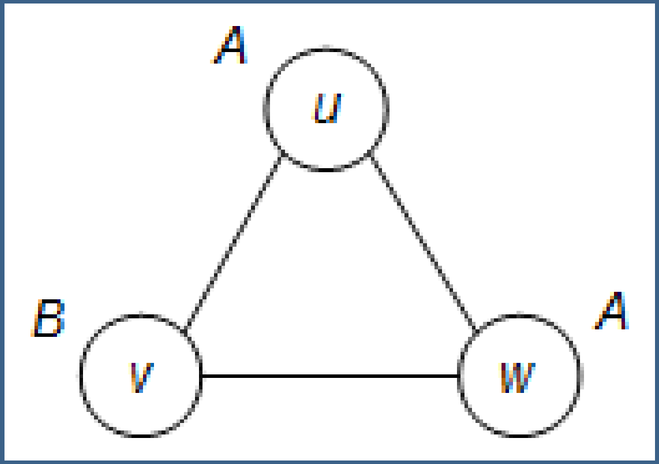
- Each node
  - plays the game with each neighbor
  - Payoff = sum of payoffs of each link

- Consider node *v*
  - Some neighbors adopt *A*
  - Some neighbors adopt *B*

  => *what should v do to maximize its payoff?*

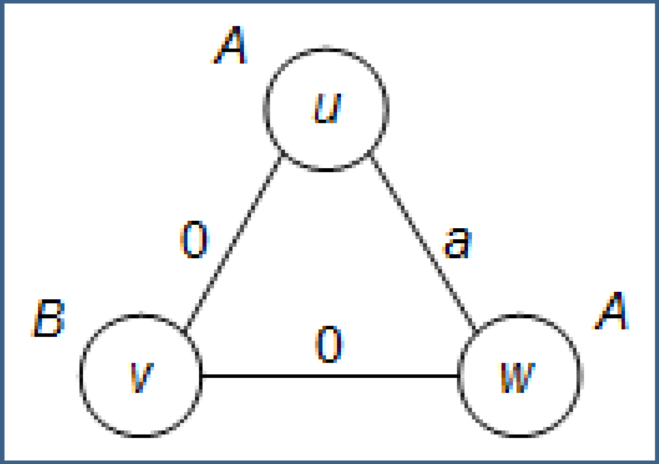
Basic idea

Example



Basic idea

Example



Basic idea

What should v do to maximize its payoff?

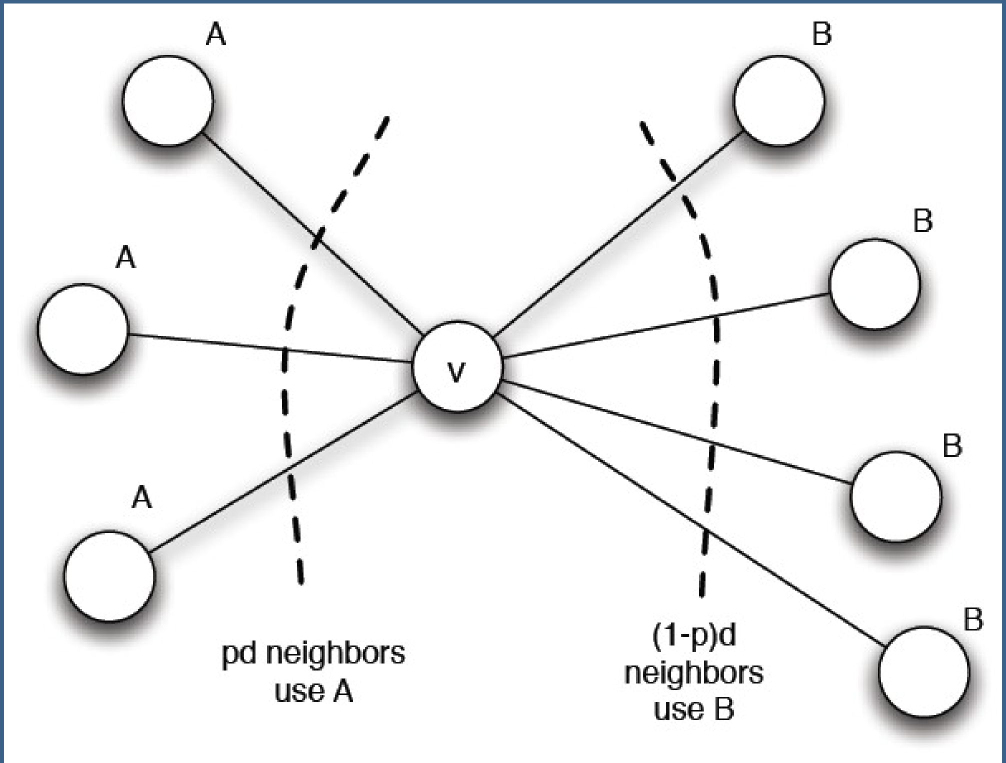
A node v should adopt A if at least a certain fraction of its neighbors follow A.

Basic idea

d = # neighbors

p : fraction of neighbors which adopt A

($0 \le p \le 1$)



Basic idea

Decision rule

A is the better choice if

$$pda \geq (1 - p)db$$

$$\Leftrightarrow \quad p \geq \frac{b}{a+b} = q$$

q : threshold for which a node should adopt A

Basic idea

Terminology

The nodes that adopt the behavior A first are called the "*initial adopters*".

Remark

Assumption:

If a node has adopted behavior A, it will never switch back to B.

Basic idea

Definition

A *cascade* of adoptions of *A* is a chain reaction of switches from *B* to *A*.

2 possibilities

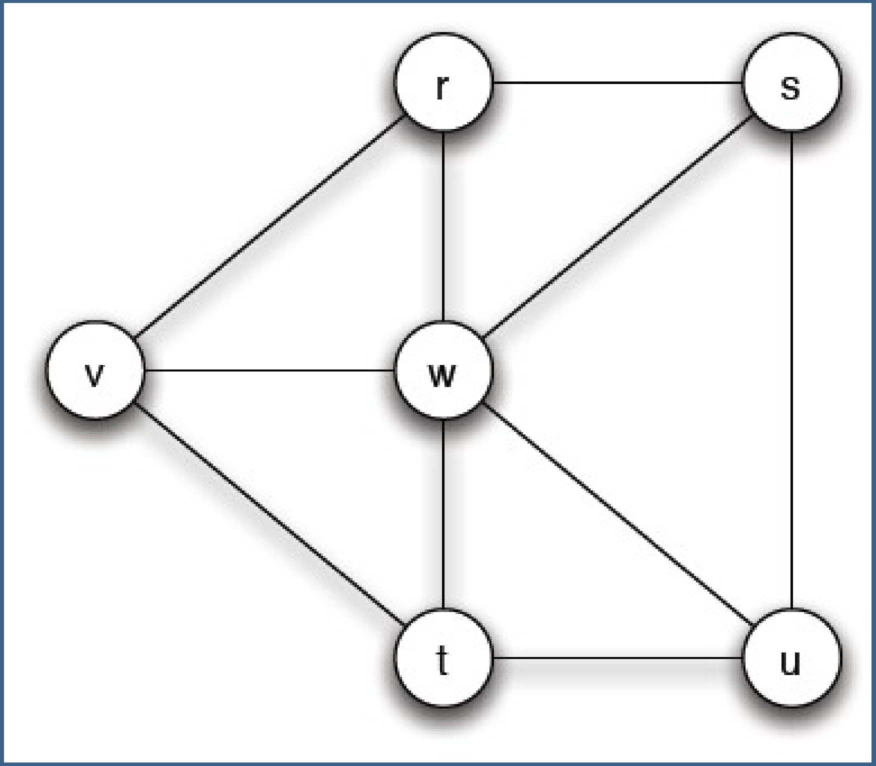
- Every node switches to *A* (*complete cascade*)
- Or, the cascade runs, but stops after a while

Basic idea

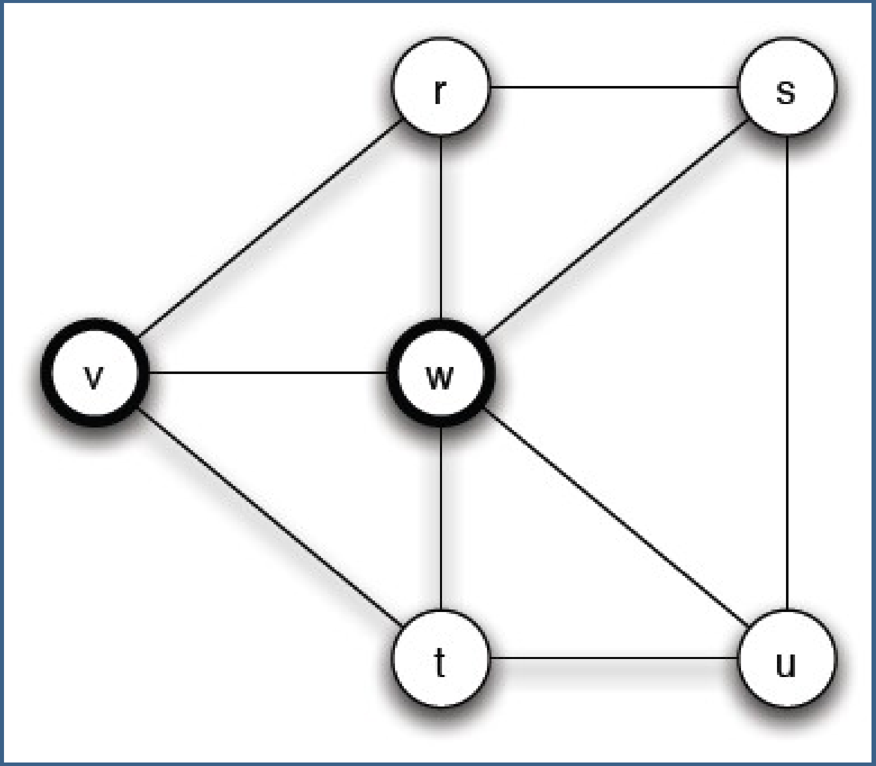
Cascading behavior

some examples...

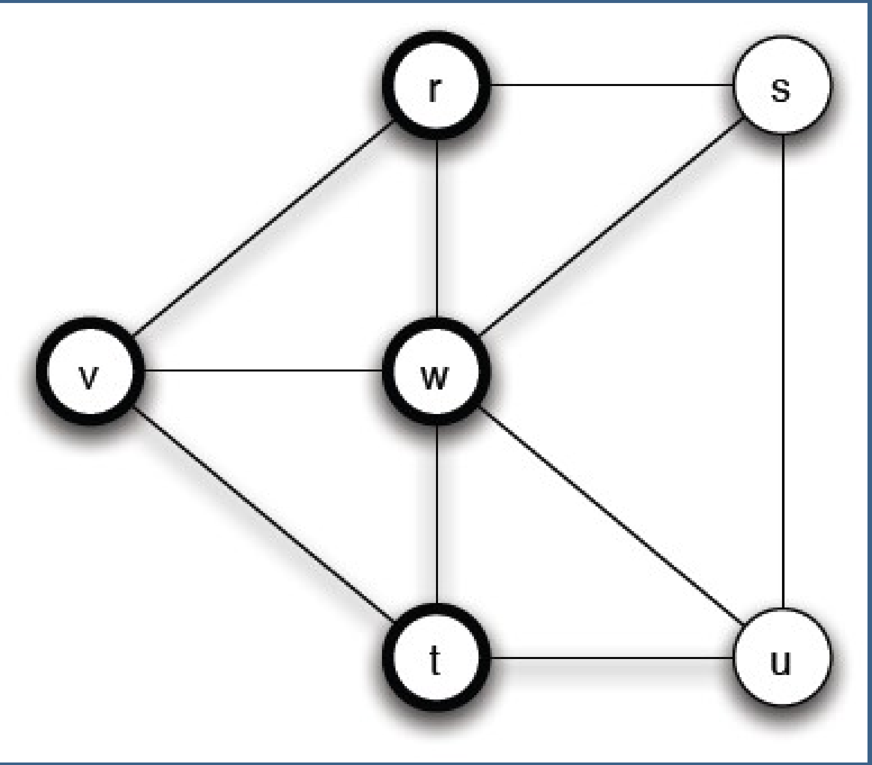
Basic idea – Example 1



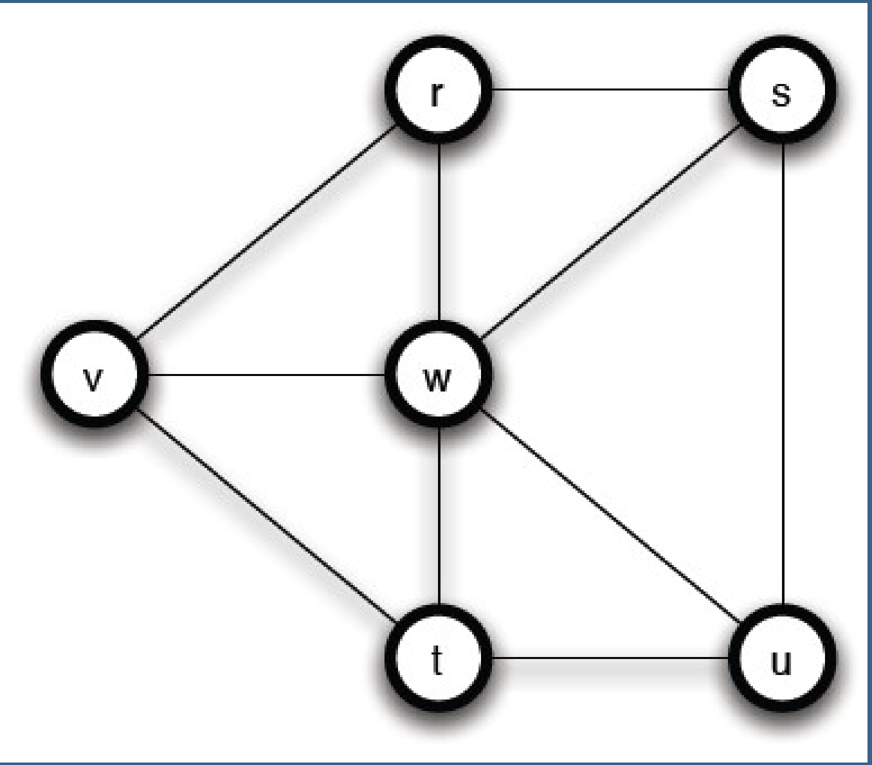
Basic idea – Example 1



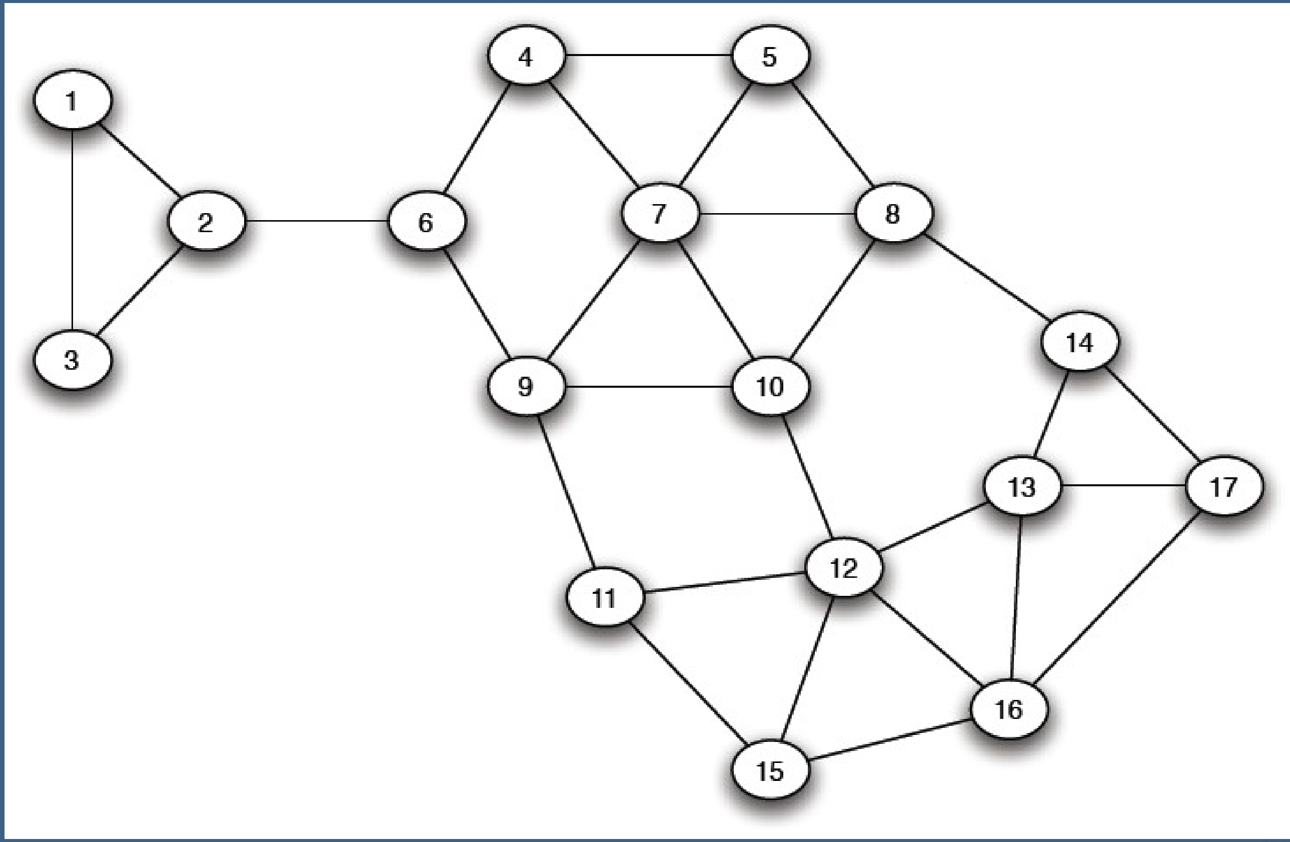
Basic idea – Example 1



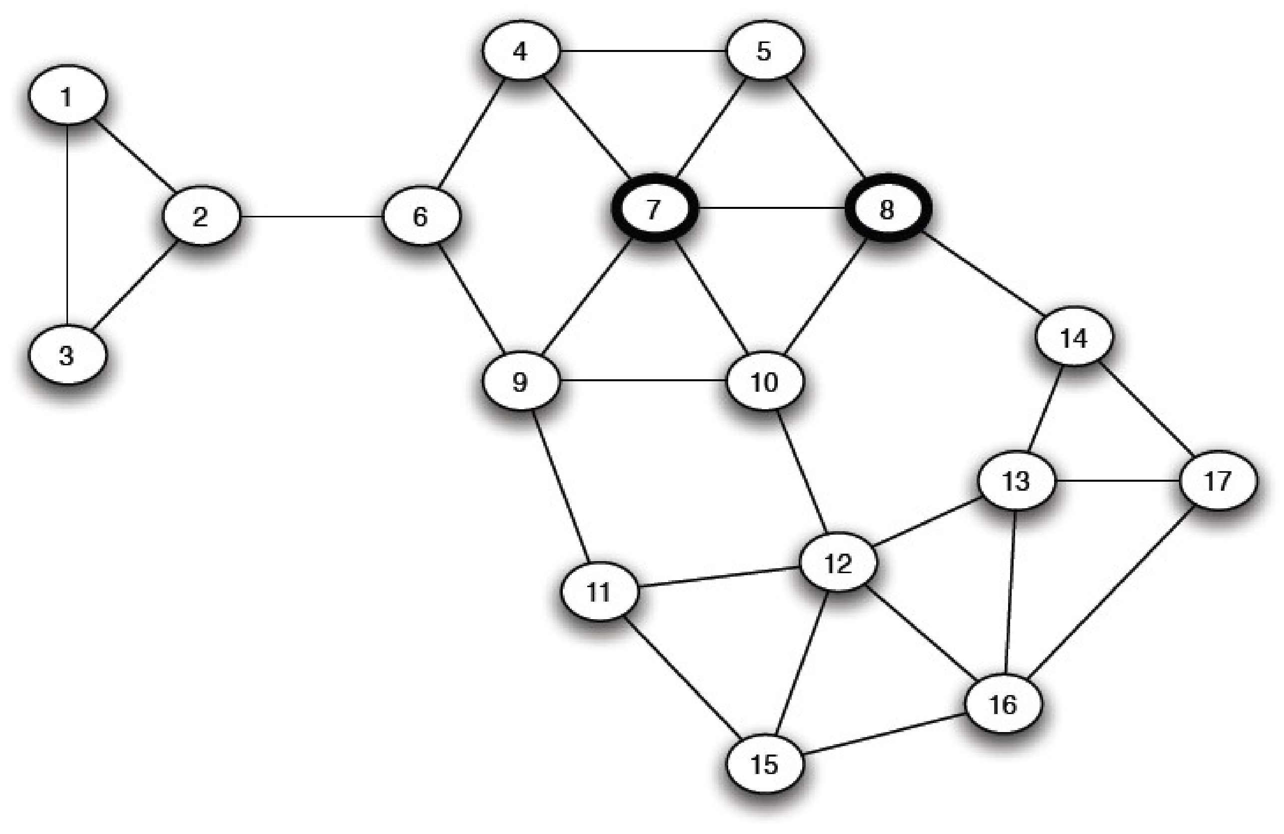
Basic idea – Example 1



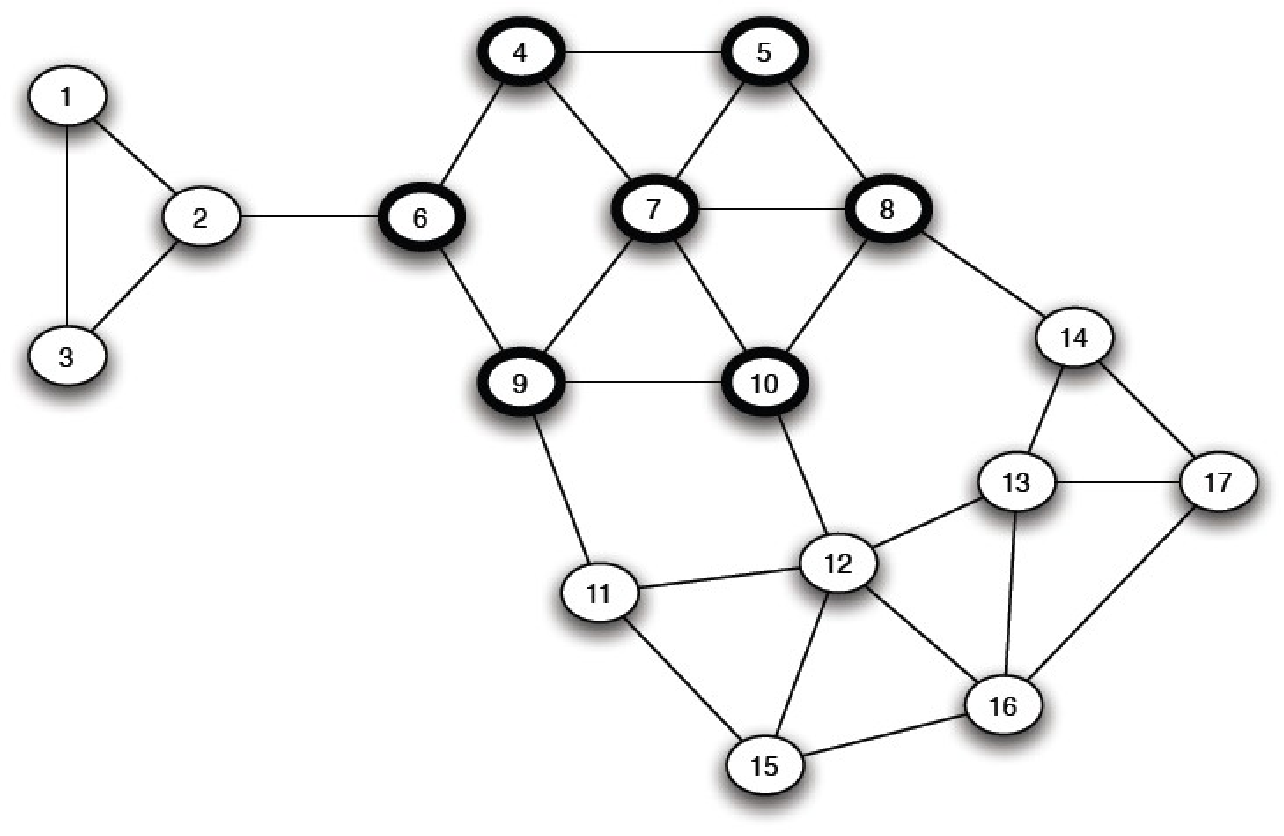
Basic idea – Example 2



Basic idea – Example 2



Basic idea – Example 2



The limit of cascades

The limit of cascades

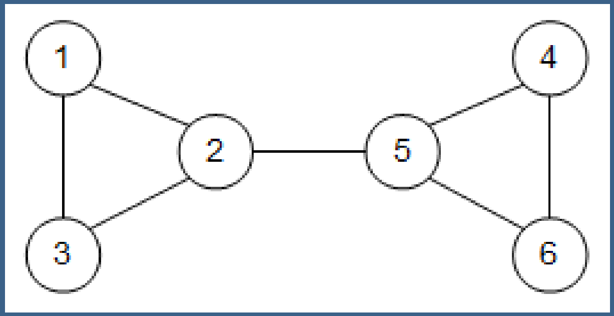
Question

What causes the spread of *A* to stop?

The limit of cascades

Intuitive answer

- Network structure
- Choice of initial adopters
- Values of *a* and *b*



The limit of cascades

Question (revisited)

What causes the spread of *A* to stop?

The limit of cascades

Technical answer

Dense clusters

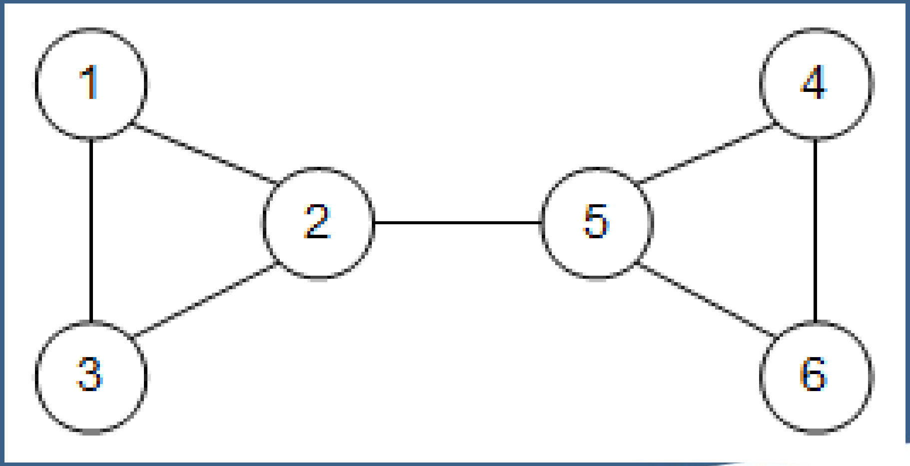
Definitions

- A *cluster of density p* is a set of nodes such that each node in the set has at least a p fraction of its neighbors in the set.
- A *dense cluster*, in a network with threshold q, is a cluster of density greater than $1 - q$.

The limit of cascades

Example

- Density p = 2/3
- Suppose threshold q = 3/5
  => both clusters are dense



The limit of cascades

Claim

The initial adopters of A (with threshold q) will not cause a complete cascade

if and only if

the network contains a cluster of density greater than $1 - q$.

The limit of cascades

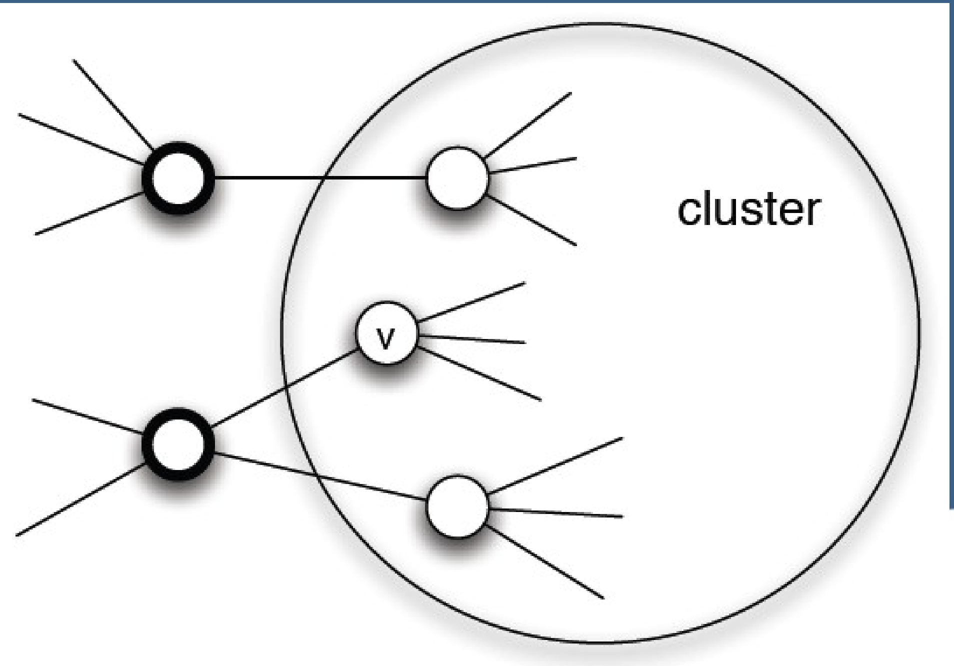
Part 1

Dense clusters are obstacles to cascades.

The limit of cascades

Explanation

Suppose *v* is the first node in the cluster that adopts behavior *A*.



The limit of cascades

Explanation

Since v is the first, all neighbors of v using A are located *outside* the cluster.

Density is greater than $1 - q$, hence less than qd neighbors are outside the cluster.

Contradiction!

=> v could never have adopted A

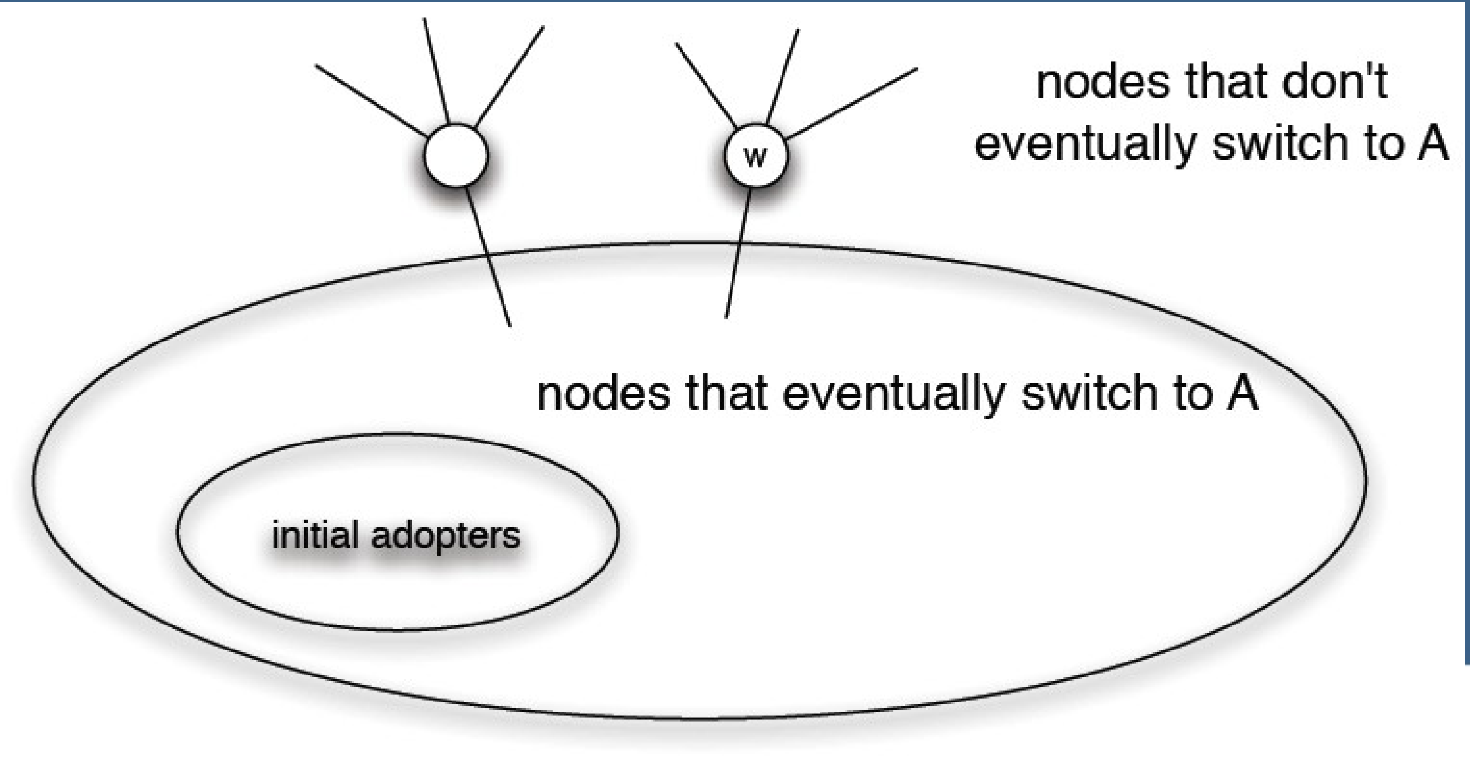
The limit of cascades

Part 2

Dense clusters are the *only* obstacles to cascades.

The limit of cascades

Explanation



The limit of cascades

Explanation

- Let S be the set of nodes still using B.

- For every node w in S:

  w doesn't want to switch to A

  => less than qd neighbors are using A

  => more than $(1 – q)d$ neighbors are using B

  => fraction of neighbors in S is greater than $(1 – q)$

  => ***S* is a dense cluster**

Extension of the model

Extension

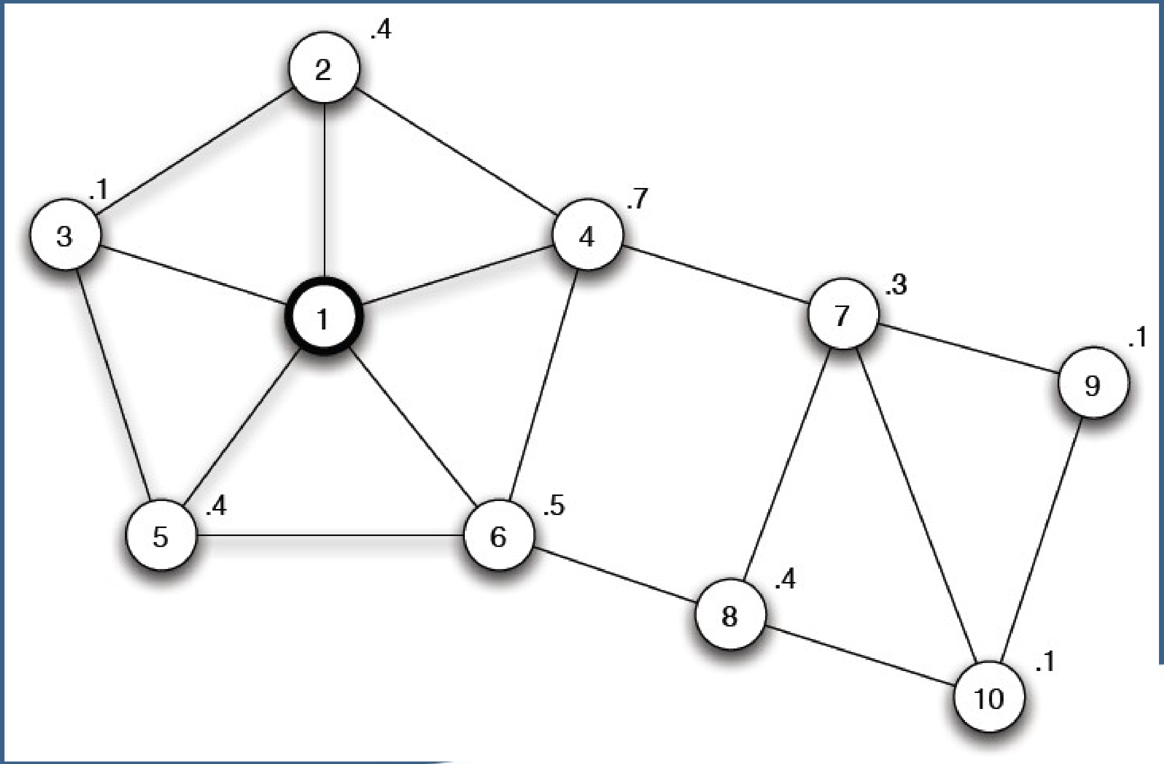
v and w value behaviors A and B differently

a_v vs a_w and b_v vs b_w

=> adopt behavior w.r.t. *personal* threshold

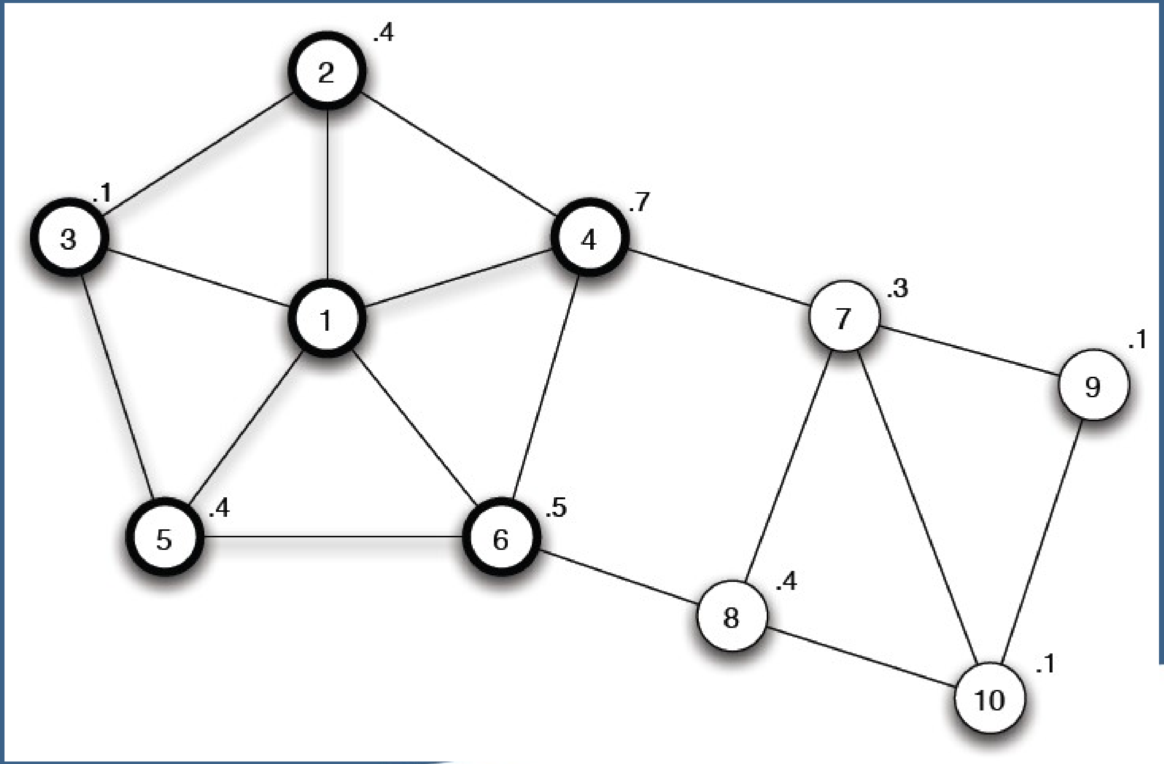
Extension

Example



Extension

Example



Advanced material

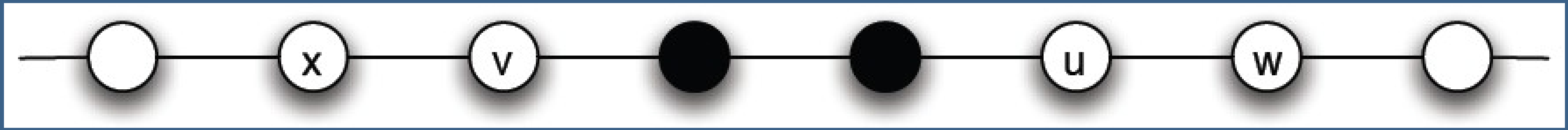
Advanced - Cascade capacity

Definition

The *cascade capacity* of a network is the maximum threshold for which a “small” set of initial adopters (i.e. a finite set) will cause a complete cascade.

Advanced - Cascade capacity

Example 1

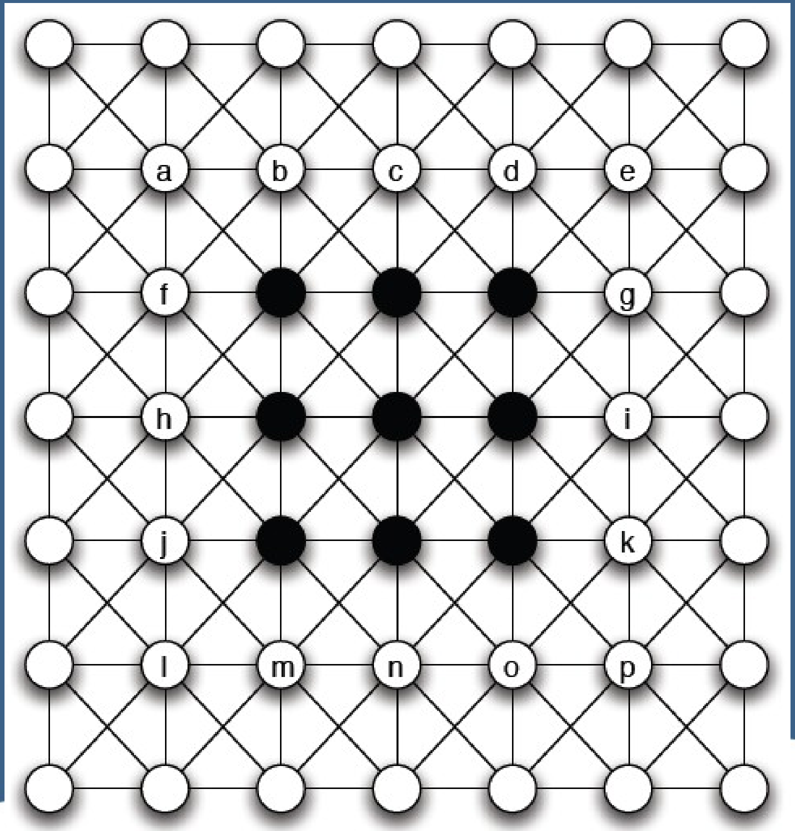


cascade capacity = 1/2

Advanced - Cascade capacity

Example 2

cascade capacity = 3/8



Advanced - Cascade capacity

Claim

There is no network for which the cascade capacity is bigger than 1/2.

Advanced – Bilingual option

Co-presentation...

Conclusion

- Model to simulate/predict/visualize the spread of a new behavior

- Real-life applications
  - Telephone and fax machines
  - New/compatible technologies
  - Youtube video vs political ideas
  - Limit on communication by government

Q&A

Thanks for your attention!

References

Networks, Crowds, and Markets: Reasoning about a Highly Connected World (ch.19), David Easley and Jon Kleinberg, Cambridge University Press, 2010