Practice Sheet 2
Department of Mathematics
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Course Name: Ordinary Differential Equations **Course Code:** CMMTE22
Academic Year: 2025-2026 **Semester:** Even

CO Mapping

Practice Sheet	Topics	CO1	CO2	CO3	CO4	CO5
2	First order ODE Orthogonal trajectory		✓			

Objective: The main objective of this Practice sheet is to gain the knowledge about to find the solution of first order ODE with different methods. Moreover, how to find orthogonal trajectory in cartesian as well as polar coordinate.

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1. Find all real valued C^1 solutions $y(x)$ of the differential equation
$$xy'(x) + y(x) = x, \ x \in (-1, 1).$$

2. Under what conditions, the following differential equations are exact?
(a) $[h(x) + g(y)]dx + [f(x) + k(y)]dy = 0$ (b) $(x^3 + xy^2)dx + (ax^2y + bxy^2)dy = 0$
(c) $\left(\frac{1}{x^2} + \frac{1}{y^2}\right) dx + \left(\frac{cx+1}{y^3}\right) dy = 0$

3. Find the relation between a, b, c, d so that the following differential equation
$$(a \sinh x \cos y + b \cosh x \sin y)dx + (c \sinh x \cos y + d \cosh x \sin y)dy = 0$$
is exact?

4. Examine the following differential equations for exactness. Solve them by finding appropriate integrating factors if necessary:
(a) $(\sin x \tan y + 1)dx - \cos x \sec^2 y dy = 0.$ (b) $e^x dx + (e^x \cot y + 2y \csc y)dy = 0.$
(c) $(3xy + y^2)dx + (x^2 + xy)dy = 0.$ (d) $ydx + (2x - ye^y)dy = 0.$

5. Suppose $M(x, y)dx + N(x, y)dy = 0$ has an integrating factor $\mu(x, y)$ such that $df = \mu M dx + \mu N dy$ is an exact differential. Show that the equation has an infinite number of integrating factors by demonstrating that the product $\mu G(f)$, where G is an arbitrary continuous function from $\mathbb{R}$ to $\mathbb{R}$, is also an integrating factor.

6. Solve the following equations by assuming an integrating factor of the form $x^\alpha y^\beta$.
(i) $(4xy^2 + 6y)dx + (5x^2y + 8x)dy = 0$
(ii) $(8x^2y^3 - 2y^4)dx + (5x^3y^2 - 8xy^3)dy = 0$

7. Solve the following linear/reducible to linear ODEs:
(a) $(x + 2y^3)\frac{dy}{dx} = y$ (b) $(1 + y^2) + (x - e^{-\tan^{-1} y})\frac{dy}{dx} = 0$

(c) $x\frac{dy}{dx} + y = x^2y^2$ (d) $y^{1/2}\frac{dy}{dx} + y^{3/2} = 1, y(0) = 4.$

(e) $\frac{dy}{dx} + \frac{1}{x}\sin 2y = x^2\cos^2 y$ (f) $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}(\log z)^2$

8. Problems based on Trajectories:

   (a) Find the orthogonal trajectories to the family of curves $x^2 + y^2 = cx$.

   (b) Find the value of n such that the curves $x^n + y^n = c$ are orthogonal trajectories of the family $y = \frac{x}{1 - c_1 x}$.

   (c) Find the orthogonal trajectories of the family of curves: $\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$, where λ being a parameter.

   (d) Show that the family of parabolas $y^2 = 2cx + c^2$ is self-orthogonal.

   (e) Find the orthogonal trajectories of $r = c(\cos\theta - \sin\theta)$.

   (f) Find the orthogonal trajectories of family $r^n \sin n\theta = a^n$.