Practice Sheet 1
Department of Mathematics
Netaji Subhas University of Technology, Dwarka

Course Name: Ordinary Differential Equations **Course Code:** CMMTE22
Academic Year: 2025-26 **Semester:** Even

CO Mapping

Practice Sheet	Topics	CO1	CO2	CO3	CO4	CO5
1	Wronskian, existence and uniqueness Theorem, Able's formula	✓				

Objective: The main objective of this Practice sheet is to gain the knowledge about existence and uniqueness Theorem for nth order linear differential equations, Wronskian, linearly independent, dependent set of functions. .

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1. Classify the following equations as linear and non linear equations and write down their orders
   (a) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}.\frac{dy}{dx} + y = x.$
   (b) $\frac{d^3y}{dx^3} + \frac{d^2y}{dx^2}\frac{dy}{dx} + y = x.$
   (c) $x\frac{dy}{dx} + y^2 = x^2.$
2. Find the differential equation of the family of parabolas $y^2 = 4ax$.
3. Show that the function $y = cx^2 + x + 3$ is a solution, though not unique ,of the initial value problem $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = 6$ with $y(0) = 3$, $\frac{dy}{dx}(0) = 1$ on $(-\infty, \infty)$.
4. If $y_1(x)$ and $y_2(x)$ are two solutions of differential equation $\cos xy'' + \sin xy' - (1 + e^{(-x^2)})y = 0$, for all $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ with $y_1(0) = \sqrt{2}$, $y_1'(0) = 1$, $y_2(0) = -\sqrt{2}$, $y'(0) = 2$, then find the wornskian of $y_1(x)$ and $y_2(x)$ at $x = \pi/4$.
5. Show that solutions $\phi_1(x) = e^{2x}$, $\phi_2(x) = xe^{2x}$ and $\phi_3(x) = x^2e^{2x}$ are linearly independent solutions of $y''' - 6y'' + 12y' - 8y = 0$ on an interval $[0, 1]$.
6. Show that $\sin 2x$ and $\cos 2x$ form a set of fundamental solutions of $y'' + 4y = 0$ and hence find solution of this equation.
7. Find the differential equation of the family of curves $y = Ae^{3x} + Be^{2x}$, for different values of A and B.
8. Find a differential equation with the following solution: $y = ae^x + be^{-x} + c\cos x + d\sin x$, where a, b, c and d are parameters.
9. Show that the initial value problem $\frac{dy}{dx} = y^{\frac{1}{3}}$, $y(0) = 0$ has infinitely many solutions.

10. Show that linearly independent solutions of $y'' - 2y' + 2y = 0$ are $e^x \sin x$ and $e^x \cos x$. what is general solution? find the solution $y(x)$ with the property $y(0) = 2$, $y'(0) = 3$.

11. Show that the value of wronskian of two solutions y_1 and y_2 of differential equation

$$a_0(x)\frac{d^2y}{dx^2} + a_1(x)\frac{dy}{dx} + a_2(x)y = 0,$$

where a_0, a_1 and a_2 are continuous real functions on a real interval $a \leq x \leq b$ and $a_0(x) \not\equiv 0$ for any x on $a \leq x \leq b$, either is zero for all x on $a \leq x \leq b$ or is zero for no x on $a \leq x \leq b$.

12. Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine this differential equation.

13. Show that the e^{-x}, e^{3x}, e^{4x} are linearly independent solutions of $y''' - 6y'' + 5y' + 12y = 0$ on the interval $-\infty < x < \infty$. Also find the general solution.

14. $y = \phi(x)$ and $y = \psi(x)$ be solutions of $y'' - 2xy' + \sin(x^2)y = 0$,such that $\phi(0) = 0, \phi'(0) = 1$ and $\psi(0) = 1, \psi'(0) = 2$. Find the value of Wronskian $W(\phi, \psi)$ at $x = 1$.

15. Which of the following functions $y_1(x)$ and $y_2(x)$, countinous functions p(x) and q(x) can be determined on $[-1, 1]$ such that $y_1(x)$ and $y_2(x)$ give two linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$, $x \in [-1, 1]$

    (a) $y_1(x) = x \sin x$, $y_2(x) = \cos x$.
    (b) $y_1(x) = xe^x$, $y_2(x) = \sin x$.
    (c) $y_1(x) = e^{x-1}$, $y_2(x) = e^x - 1$.
    (d) $y_1(x) = x^2$, $y_2(x) = \cos x$.