Quartic and quintic polynomial interpolation

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Quartic and Quintic Polynomial Interpolation

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Abstract. Basis polynomials of quartic and quintic parametric interpolations are discussed to interpolate given data points. The aim is constructing a curve that exactly interpolates all data points. Besides, the shape of curve can be controlled by using parameters t and changing the magnitude of tangents, α and β. When large number of data points is given, piecewise polynomial interpolation will be used. Therefore, quartic and quintic interpolations also have been discussed to interpolate large number of data points.

Keywords: parametric curves, quartic curve, quintic curve
PACS: 02.40. ± k

INTRODUCTION

Interpolation is a method of connecting a set number of known data points which is required to pass through the given function and the function has the exact fit to those data points when interpolating. When interpolating, the trend of the data in between the collected data points is predicted and a curve is sought [1]. It is very useful in order to increase our confidence on the collected data in spite of the fact that it attaches less significance in the interpolating curve.

In general, the exact value of data points is unknown, if we have some scattered data points in the sample, we can construct a function by using interpolation in order to transform the data into a smooth curve [2-3]. According to Bourke [4], a straight line is uniquely determined by two data points, if there are more data points, we should use higher degree polynomial interpolation. For instance, given three data points, we can interpolate the data points in quadratic curve. Using the same method, it can be said that if we have $n$data points to be interpolated, we may use a polynomial of degree $n-1$ parametric to connect all the data points.

QUARTIC INTERPOLATION

Quartic interpolation offers exact interpolation between the data points which not only requires two end data points but also the three data points between them. Thus, quartic interpolation requires five data points to construct a curve passes through all given data points. Besides, quartic interpolation also interpolates three given data points with two end tangents.

Quartic Interpolation of Five Data Points

In quartic interpolation with given five data points, P_0, P_1, P_2, P_3 and P_4, a general quartic parametric equation can be generated as follows:

$$P(t) = at^4 + bt^3 + ct^2 + dt + e, \qquad t \in [0,1],$$

where a, b, c, d and e are vectors in $\mathbb{R}^2$.

Since there are many possible different values of t_1, t_2 and t_3 that lead many possible quartic curve interpolations, hence, we would like to interpolate five given points automatically by using chord-length parameter that would generate a best fit curve.

In quartic interpolation, chord-length parameters of t_1, t_2 and t_3 as follow:

$$t_1 = \frac{|P_0P_1|}{L},$$

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$$t_2 = \frac{|P_0P_1| + |P_1P_2|}{L},$$
$$t_3 = \frac{|P_0P_1| + |P_1P_2| + |P_2P_3|}{L}$$

where $|P_nP_{n+1}|$ is the distance between the point P_n and P_{n+1}, and

$$L = |P_0P_1| + |P_1P_2| + |P_2P_3| + |P_3P_4|$$

which L is the total distance between five distinct data points.

Let $P_0 = (0,0), P_1 = (2,3), P_2 = (3,4), P_3 = (6,5)$ and $P_4 = (8,0)$.The best fit quartic curve interpolation of five given data points automatically generated using chord-length parameter from Mathematica Programme as shown in Figure 1 below.

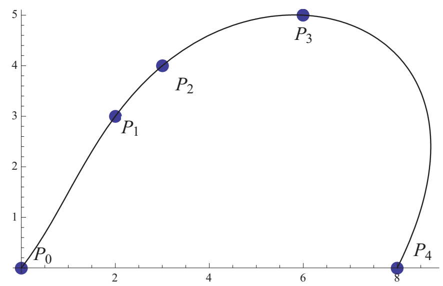

FIGURE 1. Quartic interpolation of five data points using chord-length parameter

Quartic Interpolation Three Data Points and Two End Tangents with Different Value of t_1

A tangent can be generated by differentiate the general quartic parametric equation which shown as follows:

$$P'(t) = 4at^3 + 3bt^2 + 2ct + d, \quad t \in [0,1],$$

where a, b, c and d are vectors in $\mathbb{R}^2$.

Let $P_0 = (0,0)$, $P_1 = (3,3)$ and $P_2 = (5,0)$ are three given data points, and $m_0 = (2,3)$ and $m_2 = (2,-3)$ are two end tangents to the curve at P_0 and P_2. The quartic curve interpolations of different value of t_1 are generated by using Mathematica Programme as shown in the figure below.

TABLE (1). Summary of the value of t_1 and the dashing of the interpolation curve

No.	Value of t_1	Dashing of the curve
1	$\frac{1}{2}$	———
2	$\frac{1}{2} + 0.1$	....
3	$\frac{1}{2} - 0.1$	– – – –
4	$\frac{1}{2} + 0.2$	— · · — ·

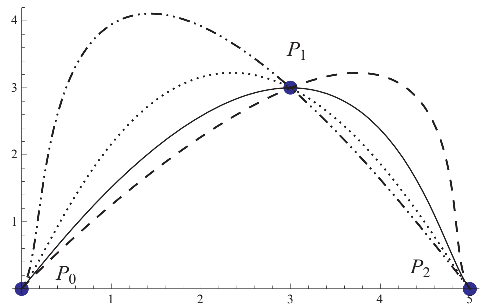

FIGURE 2. Quartic interpolation of three given data points and two end tangents of different value of t_1

It can be seen from Figure 2, the best curve of this particular case is generated when t_1 is half, in which the curve has the turning point exactly at P_1. When $t_1$is less than half, the turning point of the curve is shifted to the left which the turning point is at the left hand side of P_1; while $t_1$is more than half, the turning point of the curve is shifted to the right which the turning point is at the right hand side of P_1. It is obvious to observe that as the value of t_1 getting bigger, the turning point of the curve will be shifted from left to right. Apparently, we can control the shape of curve by changing the value of t_1.

In order to interpolate the data points automatically and generate the best fit curve we would like to use chord-length parameter to get the value of t_1 to perform quartic interpolation of three known data points and two known end tangents. Chord-length parameter of t_1 as follows:

$$t_1 = \frac{|P_0P_1|}{L}$$

and

$$L = |P_0P_1| + |P_1P_2|$$

The best fit quartic curve interpolation with chord-length parameter t_1 can be generated by using Mathematica Programme as the Figure 3.

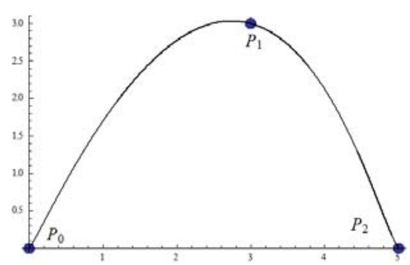

FIGURE 3. Quartic interpolation of three data points and two end tangents using chord-length parameter

Quartic Interpolation of Three Data Points and Two Unknown End Tangents

If the end tangents are not provided for interpolation, we can obtain the tangent formulae at both endpoints automatically by applying the equation given by Sarfraz [5]. Sarfraz [5] proposed distance based choice equation to get the approximation tangents that provides nice and pleasing result.

$$m_0 = 2(P_1 - P_0) - \frac{P_2 - P_0}{2},$$
$$m_n = 2(P_n - P_{n-1}) - \frac{P_n - P_{n-2}}{2},$$

where m_0 and m_n are the tangents of the curve at points P_0 and P_n respectively.

In order to generate the best fit quartic curve, chord-length parameter is applied. Let $P_0 = (0,0)$, $P_1 = (3,3)$ and $P_2 = (5,0)$ are three given data points, and $m_0 = 2(P_1 - P_0) - \frac{P_2 - P_0}{2}$ and $m_2 = 2(P_2 - P_1) - \frac{P_2 - P_0}{2}$ are two end tangent equations to the curve at P_0 and P_2 provided by Sarfraz [5]. From Mathematica Programme, the Figure 4 below is constructed to show the quartic curve interpolation of three data points and two unknown end tangents.

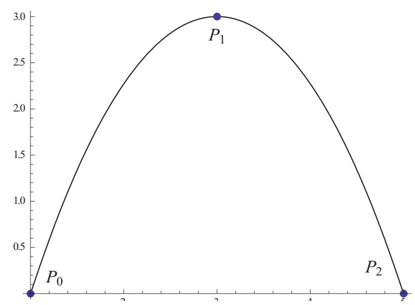

FIGURE 4. Quartic curve interpolation of three data points and two approximated end tangents

To control the shape of curve, we set the tangents at both endpoints with different magnitude as $m_{00} = \alpha \times m_0$ and $m_{11} = \beta \times m_2$ with non-zero value parameters α and β to see the shape of curve will be constructed.

TABLE (2). Summary of the values of α and β and the dashing of the curve

No.	Value of α	Value of β	Dashing of the curve
1	1	1	———
2	1	$\frac{1}{2}$	…………
3	1	2	--------
4	$\frac{1}{2}$	1	– · – · ·
5	2	1	– – – –

When we fixed the value of α constant, we compare the shape of curve when the value of β becomes less than 1 and more than 1 and vice versa. The curves are constructed using Mathematica Programme shown as follows:

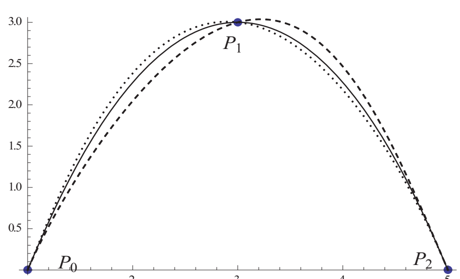

FIGURE 5. Comparison of quartic curve interpolation of three data points and two end tangents when value of α is fixed as constant whereas value of β becomes less than 1 and

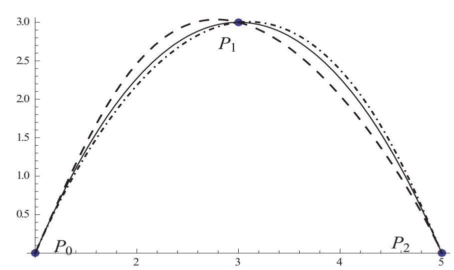

FIGURE 6. Comparison of quartic curve interpolation of three data points and two end tangents when value of β is fixed as constant whereas value of α becomes less than 1 and more than 1.

Refer to Figure 5, when the magnitude of tangent at last endpoint is greater than the approximated end tangent in which the tangent at first endpoint is fixed, the curve is steeper at last endpoint and the turning point of the curve will be shifted to the right from the original turning point and vice versa.

Refer to Figure 6, when the magnitude of tangent at last endpoint is fixed as constant, while the magnitude of tangent at first endpoint is greater than the approximated end tangent, the curve is steeper at first endpoint and the turning point will be shifted to the left from the original turning point where both magnitude of tangents at both endpoints is using the approximated end tangent by using formulae.

QUINTIC INTERPOLATION

Similar to quartic interpolations that we have discussed above, quintic interpolation also offers exact interpolation which not only requires two end data points but also the four points between them. Hence, quintic interpolation requires six data points to form a curve which passes through all given data points. Meanwhile, quintic interpolation also interpolates four given data points with two end tangents.

Quintic Interpolation of Six Data Points

In quintic interpolation with given six data points P_0, P_1, P_2, P_3, P_4 and P_5, the general quintic parametric equation as follows

$$P(t) = at^5 + bt^4 + ct^3 + dt^2 + et + f, \qquad t \in [0,1],$$

where a, b, c, d, e and f are vectors in $\mathbb{R}^2$.

In quintic interpolation, chord-length parameters of t_1, t_2, t_3 and t_4 as follow

$$t_1 = \frac{|P_0P_1|}{L},$$
$$t_2 = \frac{|P_0P_1| + |P_1P_2|}{L},$$
$$t_3 = \frac{|P_0P_1| + |P_1P_2| + |P_2P_3|}{L}$$
$$t_4 = \frac{|P_0P_1| + |P_1P_2| + |P_2P_3| + |P_3P_4|}{L}$$

where $|P_nP_{n+1}|$ is the distance between the point P_n and P_{n+1}, and

$$L = |P_0P_1| + |P_1P_2| + |P_2P_3| + |P_3P_4| + |P_4P_5|$$

which L is the total distance between six distinct data points.

Let $P_0 = (0,0), P_1 = (2,3), P_2 = (3,2), P_3 = (4,5), P_4 = (7,3)$ and $P_5 = (9,0)$. The quintic curve interpolation of six given data points is formed automatically using chord-length parameter in Mathematica Programme as shown in Figure 7.

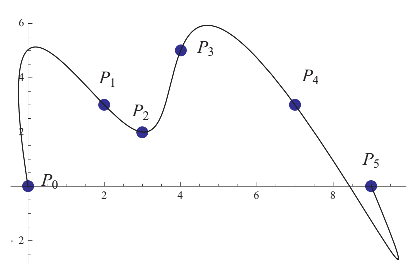

FIGURE 7. Quintic interpolation of six data points using chord-length parameter

Quintic Interpolation of Four Data Points and Two End Tangents with Different Values of t_1 and t_2

A tangent can be generated by differentiate the general quintic parametric equation which is shown as follows

$$P'(t) = 5at^4 + 4bt^3 + 3ct^2 + 2dt + e, \qquad t \in [0,1],$$

where a, b, c, d and e are vectors in $\mathbb{R}^2$.

Let$P_0 = (0,0), P_1 = (1,1), P_2 = (3,2)$ and $P_3 = (4,0)$are four given data points, and $m_0 = (2,3)$and $m_2 = (2,-3)$ are two end tangents at P_0 and P_3. The quintic curve interpolation of different values of t_1 and t_2 can be drawn as the figures below.

TABLE (3). Summary of the values of t_1 and t_2 and the dashing of the interpolation quintic curve with four points and two end tangents

No.	Value of t_1	Value of t_2	Dashing of the curve
1	$\frac{1}{3}$	$\frac{2}{3}$	————
2	$\frac{1}{3} + 0.1$	$\frac{2}{3}$	---------
3	$\frac{1}{3} - 0.1$	$\frac{2}{3}$	....
4	$\frac{1}{3}$	$\frac{2}{3} + 0.1$	— · · — ·
5	$\frac{1}{3}$	$\frac{2}{3} - 0.1$	- - - - -

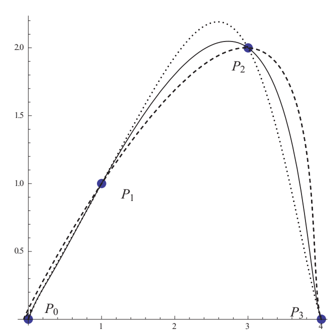

FIGURE 8(a). Comparison of quintic curve interpolation of four data points and two end tangents when value of t_2 is kept constant whereas value of t_1 differs by ± 0.1.

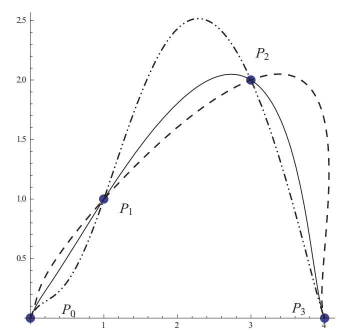

FIGURE 8(b). Comparison of quintic curve interpolation of four data points and two end tangents when value of t_1 is kept constant whereas value of t_2 differs by ± 0.1.

Refer to the results above, we noticed that if the difference of values t_1 and t_2 is larger, the turning point of the curve is located at higher position. Refer to Figure 8(a), in this particular case, when t_1 is smaller than $\frac{1}{3}$, the curve from P_0 to P_1 less steep than $t_1 = \frac{1}{3}$ but the turning point of the curve is rather at the left if compare to the turning point where the values of t_1 and t_2 equally separated among four points. In the other hand, when t_1 greater than $\frac{1}{3}$, the curve from P_0 to P_1 steeper than $t_1 = \frac{1}{3}$ but the turning point of the curve is shifted rather to the right if compare to the turning point where the values of t_1 and t_2 equally separated among four points.

Refer to Figure 8(b), when t_2 is smaller than $\frac{2}{3}$, the turning point of the curve is rather to the right if compare to the turning point where the values of t_1 and t_2 equally separated among four points. In the other hand, when t_2 greater than $\frac{2}{3}$, the turning point of the curve is rather to the left if compare to the turning point where the values of t_1 and t_2 equally separated among four points.

However, the values of t_1 and t_2 are not the only factors that affected the shape of curve, but the magnitude of end tangents also played an important role in designing the shape of curve.

In order to interpolate the data points automatically and generate the best fit curve we would like to use chord-length parameter to get the values of t_1 and t_2 to perform quintic interpolation of three known data points and two known end tangents. Chord-length parameters of t_1 and t_2 as follows

$$t_1 = \frac{|P_0P_1|}{L},$$
$$t_2 = \frac{|P_0P_1| + |P_1P_2|}{L},$$

where $|P_nP_{n+1}|$ is the distance between the point P_n and P_{n+1}, and

$$L = |P_0P_1| + |P_1P_2| + |P_2P_3|$$

which L is the total distance between four distinct data points.

The best fit quintic curve of interpolation with chord-length parameters t_1 and t_2 is generated as the figure below:

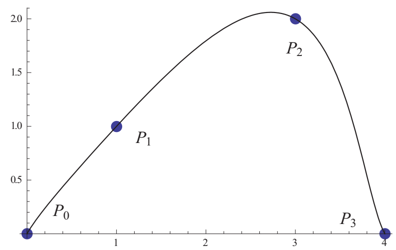

FIGURE 9. Quintic curve interpolation of four data points and two end tangents using chord-length parameter

Quintic Interpolation of Four Data Points and Two Unknown End Tangents

In order to interpolate quintic curve, chord-length parameter is applied here. Let the values of four data points are same which $P_0 = (0,0)$, $P_1 = (1,1)$, $P_2 = (3,2)$ and $P_3 = (4,0)$, the end tangents to the curve at P_0 and P_3 now are $m_0 = 2(P_1 - P_0) - \frac{P_2 - P_0}{2}$ and $m_3 = 2(P_3 - P_2) - \frac{P_3 - P_1}{2}$.

The Figure 10 below shows the quintic curve interpolation of four data points and two approximated end tangents that generated using Mathematica programme.

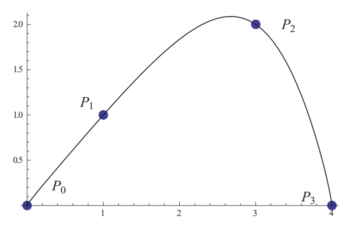

FIGURE 10. Quintic curve interpolation of four data points and two approximated end tangents

To control the shape of curve, we set the tangents at both endpoints with different magnitude, $m_{00} = \alpha \times m_0$ and $m_{11} = \beta \times m_3$ where α and β are non-zero value parameters.

TABLE (4). Summary of the values of α and β and the dashing of the interpolation quintic curve

No.	Value of α	Value of β	Dashing of the curve
1	1	1	———
2	1	$\frac{1}{2}$	- - - -
3	1	2	— · —
4	$\frac{1}{2}$	1	-------
5	2	1	....

When we fix the value of α constant, we compare the shape of curve when the value of β becomes less than 1 and more than 1 and vice versa. The curves are generated using Mathematica programme.

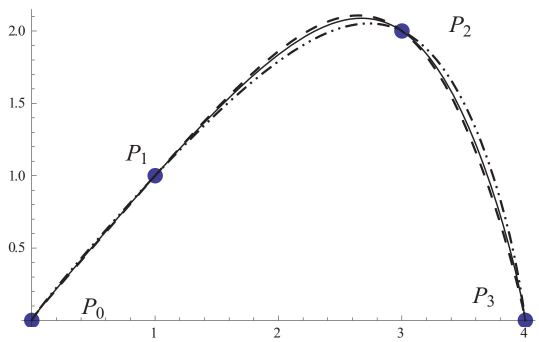

FIGURE 11(a). Comparison of quintic curve interpolation of four data points and two end tangents when value of α is fixed as constant whereas value of β becomes less than 1 and more than 1

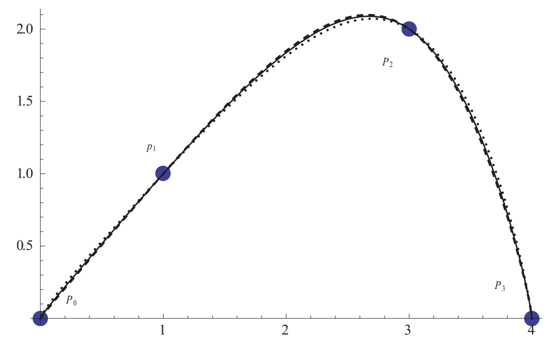

FIGURE 11(b). Comparison of quintic curve interpolation of four data points and two end tangents when value of β is fixed as constant whereas value of α becomes less than 1 and more than 1

Refer to Figure 11(a), when the end tangent at first endpoint is fix unchanged, we noticed that the greater the magnitude of tangent at last endpoint, the steeper the curve between P_2 and P_3 and the turning point of the curve will be shifted to the right from the original turning point and vice versa.

Refer to Figure 11(b), when the magnitude of tangent at last endpoint is fixed as constant, the greater the magnitude of tangent at first endpoint, the steeper the curve between P_0 and P_1 and the turning point will be shifted to the left from the original turning point where both magnitude of tangents at both endpoints is using the automatic computed tangent vector by using formulae. Hence, we can say the turning point more tends to shift to the left when magnitude of tangent at first endpoint is greater.

PIECEWISE INTERPOLATION

When the number of data points is too large, the higher degree of polynomial interpolation is needed to fit the data and design a smooth curve. It will become a problem as in higher order of polynomial interpolation may yield oscillatory polynomial. In general, it is not wise to use a high-degree interpolating polynomial on an interval.

To make a piecewise, we put in a polynomial of degree n in each segment. We must ensure that the polynomial passes through the two endpoints of each segment. The entire curve can be made smoother by choosing each polynomial segment so that the first derivatives at one endpoint agree with the derivatives of the previous segment. Since there is no segment before the first and after the last, then the both endpoints consist of unique tangent.

For a cubic spline, the polynomial segments have the same value, slope, and concavity at each interior point. In spite of the case that cubic spline are the most widely used, but we lack of flexibility to control the curve, hence, we are going to apply piecewise quartic interpolation and piecewise quintic interpolation in order to interpolate large number of data points.

Piecewise Quartic Interpolation

If the number of data points given is in the form of $3 + 2n$, where $n = 1, 2, 3, \ldots$, the data points can be interpolated by $n + 1$ segments using piecewise quartic interpolation. In piecewise quartic interpolation, each segment will cover three data points and each of the endpoints in each segment has a tangent whereby the interior point of each segment is a point which does not consist of tangent vector. Besides that, each of the first endpoint will share the tangent vector with the last endpoint of previous segment. Since there is no segment before the first and after the last, so they do not share tangent vector with other endpoints but have a unique tangent vector.

To interpolate the $3 + 2n$ data points with tangent in every single endpoint of each segment, we would like to get the tangent vectors automatically by using the formulae given by Sarfraz [5],

$$m_{00} = 2({P_1}^* - {P_0}^*) - \frac{{P_2}^* - {P_0}^*}{2},$$
$$m_{22} = 2({P_2}^* - {P_1}^*) - \frac{{P_2}^* - {P_0}^*}{2}$$

where m_{00} and m_{22} are the tangents of the curve at points ${P_0}^*$ and ${P_2}^*$ respectively which is the first derivative of the curve at the regarding point; and ${P_0}^*$, ${P_1}^*$ and ${P_2}^*$ are the first endpoint, control point and last endpoint of each segment.

In order to get the best fit curve and interpolate the data points automatically without guessing the value of t_1, we would like to use chord-length parameter to get the value of t_1 to perform quartic interpolation of three data points and two tangents in each segment. In piecewise quartic interpolation, chord-length parameter of t_1 as follows

$$t_1 = \frac{|{P_0}^* {P_1}^*|}{L}$$

where $\left|{P_n}^* {P_{(n+1)}}^*\right|$ is the distance between the points ${P_n}^*$ and ${P_{(n+1)}}^*$; L is the total distance between three distinct data points of each segment such that

$$L = |{P_0}^* {P_1}^*| + |{P_1}^* {P_2}^*|$$

For instance, we assume that 13 known data points P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11} and P_{12} are given. Since 13 is in the form of $3 + 2n$, when $n = 5$, so piecewise quartic interpolation can be used to interpolate these 13 given data points by separate the points into 6 segments.

As we know P_0 and P_{12} do not share tangent with other points. However, the interior endpoints share the same tangent for both adjacent segments.

At segment 1,
${P_0}^* = P_0$, ${P_1}^* = P_1$ and ${P_2}^* = P_2$.
Tangent at P_0, $m_0 = m_{00}$; tangent at P_2, $m_1 = m_{22}$.
At segment 2,
${P_0}^* = P_2$, ${P_1}^* = P_3$ and ${P_2}^* = P_4$.
Tangent at P_2, $m_2 = m_1$; tangent at P_4, $m_3 = m_{22}$.
At segment 3,
${P_0}^* = P_4$, ${P_1}^* = P_5$ and ${P_2}^* = P_6$.
Tangent at P_4, $m_4 = m_3$; tangent at P_6, $m_5 = m_{22}$.
At segment 4,
${P_0}^* = P_6$, ${P_1}^* = P_7$ and ${P_2}^* = P_8$.
Tangent at P_6, $m_6 = m_5$; tangent at P_8, $m_7 = m_{22}$.
At segment 5,
${P_0}^* = P_8$, ${P_1}^* = P_9$ and ${P_2}^* = P_{10}$.
Tangent at P_8, $m_8 = m_7$; tangent at P_{10}, $m_9 = m_{22}$.
At segment 6,
${P_0}^* = P_{10}$, ${P_1}^* = P_{11}$ and ${P_2}^* = P_{12}$.
Tangent at P_{10}, $m_{10} = m_9$; tangent at P_{12}, $m_{11} = m_{22}$.

Let:

$$P_0 = (1.3, 3),\ P_1 = (2, 1),\ P_2 = (3, 0.5),\ P_3 = (3.5, 1.2),\ P_4 = (3.8, 2.5),\ P_5 = (3, 4.2),\ P_6 = (2, 5), P_7 = (1.5, 5.7),\ P_8 = (1.2, 7.1),\ P_9 = (1.5, 8),\ P_{10} = (2, 8.5),\ P_{11} = (3, 8.5) \text{ and } P_{12} = (3.8, 7).$$

In order to interpolate all data points follow the sequence P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11} and P_{12}, the data points of each segment undergo quartic interpolation. Now, combine all quartic interpolations of all six segments, piecewise quartic interpolation has been done by using Mathematica programme as shown in figure below.

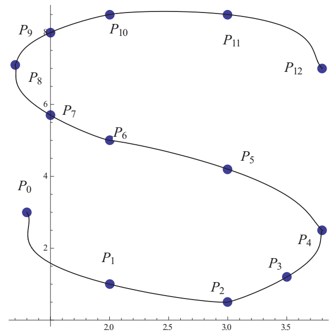

FIGURE 12. Piecewise quartic interpolation of thirteen data points

Refer to the Figure 12, a S shape has been interpolated. Since we know there exists a parameter t_1 in quartic interpolation of three given data points and two end tangents in each segment. Therefore, in order to make the shape of curve to be more pleasing, parameter t_1 can be used to control the shape of curve instead of using chord-length parameter. Furthermore, different magnitude of tangent also can be used to control the shape of curve.

However, the piecewise quartic interpolation curve with only one parameter t_1 in each segment is not really flexible to control the shape of curve design. Hence, now, we would like to discuss piecewise interpolation up to quintic parametric interpolation which consists of two parameterst_1 and t_2 in each segment.

Piecewise QuinticInterpolation

If the number of data given is in the form of $4 + 3n$, where $n = 1,2,3, \ldots$, the data points can be interpolated by $n + 1$ segments using piecewise quintic interpolation. Each segment of piecewise quintic interpolation will cover four data points and each of the endpoints in each segment has a tangent. Besides that, each of the first endpoints will share the tangent with the last endpoint of previous segment. Since there is no segment before the first and after the last points, so they do not share tangent with other endpoints but have a unique tangent. Apart from that, we can say that there are two interior control points in each segment which do not consist of tangent.

To interpolate the $4 + 3n$ data points with tangent in every single endpoint of each segment, we still would like to use the equation which given by Sarfraz [5] in order to get the tangent automatically:

$$m_{00} = 2({P_1}^* - {P_0}^*) - \frac{{P_2}^* - {P_0}^*}{2},$$
$$m_{33} = 2({P_3}^* - {P_2}^*) - \frac{{P_3}^* - {P_1}^*}{2}$$

where m_{00} and m_{33} are the tangents of the curve at points P_0^* and P_3^* respectively which is the first derivative of the curve at the regarding point; P_0^*, P_1^*, $P_2^*$and $P_3^*$are the first endpoint, first control point, second control point and last endpoint of each segment.

In piecewise quintic interpolation, chord-length parameters of t_1 and t_2 are used in order to interpolate the data points automatically with the best fit curve.

$$t_1 = \frac{|{P_0}^*{P_1}^*|}{L},$$
$$t_2 = \frac{|{P_0}^*{P_1}^*| + |{P_1}^*{P_2}^*|}{L},$$

where $|{P_n}^*{P_{(n+1)}}^*|$ is the distance between the point P_n^* and $P_{(n+1)}^*$; L is the total distance between four distinct data points of each segment such that

$$L = |{P_0}^*{P_1}^*| + |{P_1}^*{P_2}^*| + |{P_2}^*{P_3}^*|.$$

For instance, we assume that 13 known data points P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11} and P_{12} are given. Since 13 is in the form of $4 + 3n$, when $n = 3$, so piecewise quintic interpolation can be used to interpolate these 13 given data points by separate the points into 4 segments.

At segment 1,
$P_0{}^* = P_0$, $P_1{}^* = P_1$,$P_2{}^* = P_2$ and $P_3{}^* = P_3$.
Tangent at P_0, $m_0 = m_{00}$;tangent at P_3, $m_1 = m_{33}$.
At segment 2,
$P_0{}^* = P_3$, $P_1{}^* = P_4$,$P_2{}^* = P_5$and $P_3{}^* = P_6$.
Tangent at P_3, $m_2 = m_1$; tangent at P_6, $m_3 = m_{33}$.

At segment 3,
$P_0{}^* = P_6$, $P_1{}^* = P_7$,$P_2{}^* = P_8$ and $P_3{}^* = P_9$.
Tangent at P_6, $m_4 = m_3$; tangent at P_9, $m_5 = m_{33}$.
At segment 4,
$P_0{}^* = P_9$, $P_1{}^* = P_{10}$,$P_2{}^* = P_{11}$ and $P_3{}^* = P_{12}$.
Tangent at P_9, $m_6 = m_5$; tangent at P_{12}, $m_7 = m_{33}$.

Let all thirteen data points are same as what we have in piecewise quartic interpolation, they are $P_0 = (1.3, 3)$,$P_1 = (2, 1)$, $P_2 = (3, 0.5)$, $P_3 = (3.5, 1.2)$, $P_4 = (3.8, 2.5)$, $P_5 = (3, 4.2)$, $P_6 = (2, 5)$, $P_7 = (1.5, 5.7)$, $P_8 = (1.2, 7.1)$, $P_9 = (1.5, 8)$, $P_{10} = (2, 8.5)$, $P_{11} = (3, 8.5)$ and $P_{12} = (3.8, 7)$. In order to interpolate all data points follow the sequence P_0, P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11} and P_{12}, now we let the data points of each segment undergo quintic interpolation. After combine all quintic interpolations of all four segments, piecewise quintic interpolation has been done by using Mathematica programme as shown in figure below.

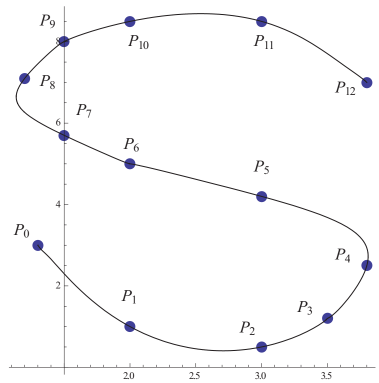

FIGURE 13. Piecewise quintic interpolation of thirteen data points

As a result, refer to Figure 13 above, we can see that a letter S is drawn while piecewise quintic interpolation on the given thirteen data points. The shape of curve S is looked smoother and nicer if compare to what we have done in piecewise quartic interpolation with the same values of data points. This is probably because of the extra parameter t_2 in each segment affected the shape of curve designed to be more pleasing Since we know there are two parameters t_1 and t_2 in quintic interpolation of four given data points and two end tangents in each segment. Therefore, parameters t_1 and t_2 can be used to control the shape of curve instead of using chord-length parameter. Furthermore, different magnitude of tangents also can be set to control the shape of curve. Hence, this is the advantage of using piecewise quintic interpolation which consists of two parameters t_1, t_2 and different magnitude of tangents can be set in every segment.

CONCLUSIONS

In the study, we have generated different kinds of curve that exactly passes through the data points by using quartic and quintic parametric interpolations with given several data points and end tangents in a curve. However, in quartic interpolation with three given data points and two end tangents, we only can control parameter t_1 and magnitude of end tangents. We are not flexible enough to control the shape of curve if compare to quintic interpolation. Hence, we need to do quintic interpolation in order to have more freedoms to control the shape of curve by changing the values of parameters t_1, t_2 and the magnitude of end tangents.

It is very useful in curve designing. The curve can be designed or modified by changing values of parameters t_1 and t_2 and the different magnitude of end tangents in each of the segment of interpolation. It is useful in set up a shape of curve that can be altered by the user.

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