MATHEMATICS

How do I "change" the angle between three vectors in 3D space?

Asked 4 years, 5 months ago Active 4 years, 5 months ago Viewed 468 times

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In this question the answer shows how to get the angle between three points in 3D space. However I would like to know how to change the angle.

For example, if I have points A, B and C and I want to change the ABC angle (where B is the middle point), I should be able to get new points C' and A' given points A, B, C, the change in angle $\frac{d\theta}{dt}$ (or simply the desired angle if I know it) and the axes of the plane described by the 3 points.

I'm not sure if I described that 100% correctly so I'll also try and explain in an intuitive way. Imagine the three points at a certain angle. If I want the angle to get smaller or bigger I want to kind of rotate points A and C around point B towards or away from each other. They'd rotate around the (y?) axis of the plane described by the three points. In other words the three points cannot rotate in such a way that causes the plane described by these three points to also rotate. I hope that kind of makes sense.

I'm not that great at maths so a kind of step by step explanation would be appreciated so I have a chance at understanding the answer rather than just being able to use it.

linear-algebra geometry 3d rotations angle

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Jonathan
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2 Answers

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So, you have three points,

$$\begin{cases} A = (x_A,\ y_A,\ z_A) \\ B = (x_B,\ y_B,\ z_B) \\ C = (x_C,\ y_C,\ z_C) \end{cases}$$

The angle $\theta = \angle ABC$ fulfills

$$\cos(\theta) = \frac{\overline{BA} \cdot \overline{BC}}{\left\lVert \overline{BA} \right\rVert \left\lVert \overline{BC} \right\rVert}$$

and

$$\sin(\theta) = \frac{\left\lVert \overline{BA} \times \overline{BC} \right\rVert}{\left\lVert \overline{BA} \right\rVert \left\lVert \overline{BC} \right\rVert}$$

where $\cdot$ denotes vector dot product, and $\times$ denotes vector cross product.

If we want to rotate C around B so that the angle $\angle ABC$ becomes $\theta + \varphi$, we first find the normal of the plane the three points form:

$$N = (x_N,\ y_N,\ z_N) = \frac{\overline{BA} \times \overline{BC}}{\left\lVert \overline{BA} \times \overline{BC} \right\rVert}$$

If we first calculate the cross product components,

$$\begin{cases} x = (y_A - y_B)(z_C - z_B) - (z_A - z_B)(y_C - y_B) \\ y = (z_A - z_B)(x_C - x_B) - (x_A - x_B)(z_C - z_B) \\ z = (x_A - x_B)(y_C - y_B) - (y_A - y_B)(x_C - x_B) \end{cases}$$

we get the components of the normal vector N by "normalizing" the length of the resulting vector to unit length, or 1:

$$\begin{cases} x_N = \frac{x}{\sqrt{x^2+y^2+z^2}} \\ y_N = \frac{y}{\sqrt{x^2+y^2+z^2}} \\ z_N = \frac{z}{\sqrt{x^2+y^2+z^2}} \end{cases}$$

We now simply need to rotate $\overline{BC}$ around the vector N by angle φ.

We can do that by applying Rodrigues' rotation formula to $\overline{BC} = (x_C - x_B, y_C - y_B, z_C - z_B)$, to get the new position C' relative to B. Essentially,

$$C' = B + \overline{BC}\cos(\varphi) + \left(N \times \overline{BC}\right)\sin(\varphi) + N\left(N \cdot \overline{BC}\right)(1 - \cos(\varphi))$$

Note that the definition of θ (and the rotation angle φ) above define it as positive counterclockwise. If $\angle ABC$ is clockwise — that is, A is to the right of C when viewing

$$A' = B + BA\cos\left(\frac{\varphi}{2}\right) - \left(N \times BA\right)\sin\left(\frac{\varphi}{2}\right) + N\left(N \cdot BA\right)\left(1 - \cos\left(\frac{\varphi}{2}\right)\right)$$

because $\sin(-x) = -\sin(x)$ and $\cos(-x) = \cos(x)$; and

$$C' = B + \overline{BC}\cos\left(\frac{\varphi}{2}\right) + \left(N \times \overline{BC}\right)\sin\left(\frac{\varphi}{2}\right) + N\left(N \cdot \overline{BC}\right)\left(1 - \cos\left(\frac{\varphi}{2}\right)\right)$$

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edited Jun 15 '17 at 23:52

answered Jun 15 '17 at 5:03

Nominal Animal

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With three points, there is always a unique plane passing through them. So considering this problem in 3D space is identical to doing it on a flat sheet of paper.

If your coordinates make this difficult, we can transform to better coordinates. We'll set B as the origin. Letting d_{AB} be the distance between A and B, we'll put A on the new x-axis, at the point d_{AB} from the origin. Now draw a line from C perpendicular to the new x-axis. This will be the direction of the new y-axis, so that all three points will be on this plane.

Once you're back in 2D, the problem should be easily visualizable.

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answered Jun 15 '17 at 5:14

probably_someone

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