A Novel Trajectory Planning Scheme for Spray Painting Robot with Bézier Curves

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Abstract: In this paper, a novel trajectory planning for spray painting robot is introduced using Bézier curves. First, a spray painting model of Bézier surface is established, and the paint deposition rate function for Bézier surface is given. In order to ensure computational efficiency, a new curve, to be called T-Bézier curve is generated from T-Bézier basis. After the discrete points array on equidistant surface are found, a new trajectory planning scheme for spray painting robot based on the T-Bézier curves is developed. This paper investigates the Bézier trajectory generation for an industrial robot as the case study. The results confirmed that the proposed trajectory planning scheme along with the advised motion planning architecture is not only feasible for the spray painting robot but also yields a smooth trajectory with a satisfactory performance for all the joints. With this scheme, it is better to keep uniform speed of spray painting for complex curved surfaces.

Key Words: Trajectory planning; Spray painting robot; Bézier curves; Complex curved surfaces

1 INTRODUCTION

Spray painting robots provide higher accuracy and higher speed in automotive manufacturing. Trajectory planning is one of the most important issues in spray painting robotics. The trajectory planning refers to the generating the position commands, velocity and acceleration of all degrees of freedom of a robot at each sampling time[1, 2].The uniformity of paint thickness on a product can strongly influence its quality. Since industrial robots are typically used for spray painting, the robot trajectory planning is crucial to achieve the given uniformity requirements.

It is worth mentioning that a trajectory can drastically affect the presentation quality of the machined workpiece, time, cost and the life time of the tool at the end effector of the spray painting robot. In order to achieve the aforementioned features, the trajectory should be designed smoothly. Rough and sudden motions of the robot not only reduce the life time of the mechanism but also empower the adverse vibration effects. Any continuous function of the time which covers the data points can be used as the trajectory for a robot, but those with the smooth kinematic characteristics are preferred.

However, due to the complex geometry of free-form surfaces, it is still a challenge to plan smooth robot trajectories that satisfies paint uniformity requirements and computational efficiency. Currently, automated trajectory planning has always caused a bottleneck for spray painting. Hence, it is essential to develop new automated trajectory planning to replace past methods. This challenging research topic has been receiving more and more attention from academia and industry [3, 4]. Although the algorithms in practice can guarantee the coverage of a complex curved surface, the problem of paint thickness is not addressed. Some researchers work on systems capable of generating robot control commands for spray-painting automatically, based on computer-aided designed data of sculptured surfaces [5, 6, 7, 8]. However, due to the process is complex and very time-consuming, their schemes couldn't resolve robot trajectory planning problem for complex curved surfaces. The paint thickness function for complex curved surfaces is not considered, and the optimal time is not satisfying. In our previous work [9, 10, 11], we have developed algorithms of automated robot trajectory generation. A material distribution model has been developed to compute the material distribution on a part. The spray width and the tool velocity were computed by optimizing the spraying process on a free-form surface. However, since the spray painting process is much more complicated than the spray forming process, these methods may not generate satisfactory paint trajectories to satisfy the paint distribution requirements for complex curved surface. In general, the industrial robot trajectory can be defined as a

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series of points. There is a feasible method for determining the robot trajectory. Firstly, the path and orientation of the robot should be designed. Then the problem is transformed into how to find the optimal time sequence along this path when the objectives can be optimal. Therefore, the variable parameters which are six (the position and orientation of the paint gun) in trajectory planning problem are reduced to one [12, 13, 14]. To optimize paint thickness, the robot spatial path, direction and velocity should be planned based on the local geometry of the surface. In fact, industrial robots suppliers do not support Bézier curve interpolations, but they do support linear and circular interpolations. Therefore, if the Bézier curve is approximated by linear segments, circular segments or both, then the industrial robot can achieve free shape trajectories.

In this paper, a new trajectory planning scheme based on the T-Bézier curves is developed. The experimental results illustrate that T-Bézier curves of the logarithmic mean enhanced flexibility of the trajectory planning. With this method, it is better to keep uniform speed of spray painting for complex curved surfaces.

2 SPRAY PAINTING MODEL

The paint deposition rate function on a plane according to the experiment data is considered. And assuming that the shape of spray painting from the tool is a cone, and the distribution model of spray is shown in [15].

Figure 1 shows material distribution on a surface. P_1 is a reference plane and P_2 is a parallel plane which passes the point s; θ_i is the angle of gun axis and the line of the point s to gun center; h_i is the actual tool height; h is the desired tool height. Suppose the material sprayed on a small area c_1 is projected to the area c_2. **n** is the normal vector of c_2; γ_i is the angle of **n** and the line of the point s to gun center.

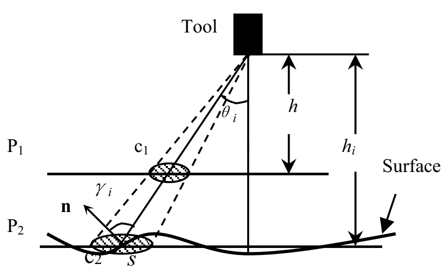


Fig 1. Material distribution on a surface

The development of the verification model is based on an assumption that the amount of material deposited by the tool on a reference plane is the same as on a free-form surface, which is independent of the geometry of the free-form surface and the distance between the spray gun and the free-form surface. The material distribution on a free-form surface can be modeled as [9]:

$$q_s = \begin{cases} q_f \left(\dfrac{h}{l_i}\right)^2 \dfrac{\cos\gamma_i}{\cos^3\theta_i} & \gamma_i < 90^0 \\ 0 & \gamma_i \geq 90^0 \end{cases} \tag{1}$$

3 TRAJECTORY PLANNING

Bézier curves are widely used for constructing free-form curves and surfaces [16]. It is well known that the Bézier basis is a basis for the space of degree-n algebraic polynomials as:

$$T = span\{1, t, t^2, \cdots, t^n\} \tag{2}$$

However, since this basis is rational and polynomial, it would be complicated to use for the trajectory of a spray painting robot. This is because each point is associated with six parameters which define the position coordinates and the orientation vector of the spray. In particular, repeated differentiation of Equ.2 produces curves of very high degree .In order to ensure computational efficiency, finding new bases of Bézier model in new spaces seems to be the only way.

In this paper, a new T-Bézier basis is presented in tool trajectory optimization problem of spray painting robot. We first give four initial functions:

$$\begin{aligned} B_{0,3}(t) &= (\cos t)^4 \\ B_{1,3}(t) &= 2(\cos t)^4(\sin t)^2 \\ B_{2,3}(t) &= 2(\sin t)^4(\cos t)^2 \\ B_{3,3}(t) &= (\sin t)^4 \end{aligned} \tag{3}$$

Where $t \in [0, \frac{\pi}{2}]$. For n>3, T-Bézier basis functions are defined as:

$$B_{i,n}(t) = (\cos t)^2 B_{i,n-1}(t) + (\sin t)^2 B_{i-1,n-1}(t) \tag{4}$$

where $B_{i,n}(t) = 0$ for i>n or i<0.With this basis, the curves share most of the properties as those of the Bézier curves in polynomial space. The T-Bézier basis has the properties as follows:

1) Partition of Unity:

$$\sum_{i=0}^{n} B_{i,n}(t) = 1 \tag{5}$$

2) Positivity:

$$B_{i,n}(t) \geq 0 \tag{6}$$

So T-Bézier basis is a blending system.

3) Properties at the endpoints:

$$\begin{aligned} & B_{0,n}(0) = B_{n,n}(\frac{\pi}{2}) = 1, \\ & B_{0,n}(\frac{\pi}{2}) = B_{n,n}(0) = 0, \\ & B_{i,n}(0) = B_{i,n}(\frac{\pi}{2}) = 0, 0 < i < n \end{aligned} \tag{7}$$

4) Linear independence: $B_{0,n}(t), B_{1,n}(t), \cdots, B_{n,n}(t)$ are linear independent.

5) Symmetry:

$$B_{i,n}(t) = B_{n-i,n}(\frac{\pi}{2} - t) \tag{8}$$

6) B-basis property: $\{B_{0,n}(t), B_{1,n}(t), \cdots, B_{n,n}(t)\}$ is the normalized B-basis of the space $span\{1, \cos t, \cdots, \cos nt\}$. By

the properties 1) and 2), we have that T- Bézier Basis is a totally positives basis.

A T-Bézier curve $p(t)$ of order n+1 is defined as follows:

$$p(t)=\sum_{i=0}^{n} B_{i,n}(t)V_i,\ t\in[0,\frac{\pi}{2}] \tag{9}$$

Here $\{B_{i,n}(t)\}_{i=0}^{n}$ is the T- Bézier basis, V_i is the control point. The geometric properties at the endpoints of the T-Bézier curves are obvious from those of the T-Bézier basis:

$$p(0)=V_0,\ p(\frac{\pi}{2})=V_n \tag{10}$$

Especially for n=3, suppose $V_0^{[1]},V_1^{[1]},V_2^{[1]},V_3^{[1]}$ and $V_0^{[2]},V_1^{[2]},V_2^{[2]},V_3^{[2]}$ are two adjacent sets of T-Bézier control points. The condition of position continuity (C^0continuity) is $V_3^{[1]}=V_0^{[2]}$, and the condition of target continuity (C^1 continuity) is that $V_2^{[1]},V_3^{[1]},V_0^{[2]}$ and $V_1^{[2]}$ are collinear, containing C^0continuity.

The entire T-Bézier curve $p(t)$ must lie inside its control polygon spanned by V_0, $V_1 \ldots V_n$. This property is a consequence of the property of the T-Bézier basis about partition of unity. The control points of opposite order define the same curve in a different parameterization, just the opposite direction:

$$p(V_n,V_{n-1},\cdots,V_0;t)=p(V_0,V_1,\cdots,V_n;\frac{\pi}{2}-t) \tag{11}$$

This can be checked by comparing the coefficients of V0, V1…Vn. on both sides of the equation. No plane intersects a T-Bézier curve more often than it intersects the corresponding control polygon.

The shape of a T-Bézier curve is independent of the choice of coordinates, i.e. $p(V_0,V_1,\cdots,V_n;t)=\sum_{i=0}^{n}B_{i,n}(t)V_i$ satisfies the following two equations:

$$\begin{aligned} p(V_0+r,V_1+r,\cdots,V_n+r;t)&=p(V_0,V_1,\cdots,V_n;t)+r \\ p(V_0*T,V_1*T,\cdots,V_n*T;t)&=p(V_0,V_1,\cdots,V_n;t)*T \end{aligned} \tag{12}$$

Here r is an arbitrary vector, and T is an arbitrary (n+1)*(n+1) matrix. If the control polygon is convex, then the corresponding T-Bézier curve is also convex. After the discrete points array on equidistant surface are found, the discrete point (U or V direction) is considered as the experimental data points. Then the data points are fitted with a T-Bézier curve. And the trajectories of the spray painting robot are generated.

4 OPTIMIZATION ALGORITHM

Figure 2 shows the paint accumulation on a small area.

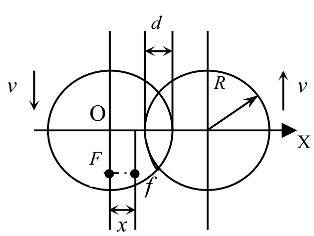


Fig 2. Paint accumulation on a small area

In figure 2, x is the distance of a point f in the spray cone radius to the first trajectory. F is the projection of the point f. O is the projection of tool. d which is the overlap distance of the spray painting optimization trajectory. The material thickness of the point f can be calculated as:

$$q_f(x)=\begin{cases} q_1(x) & 0\le x\le R-d \\ q_1(x)+q_2(x) & R-d<x\le R \\ q_2(x) & R<x\le 2R-d \end{cases} \tag{13}$$

where q_1（x）and q_2（x）are the material thickness of the point f when the tool spray on two adjacent trajectories. q_1（x）and q_2（x）can be calculated as:

$$\begin{aligned} &q_1(x)=2\int_0^{t_1} f(r_1)dt,0\le x\le R \\ &q_2(x)=2\int_0^{t_2} f(r_2)dt,R-d\le x\le 2R-d \\ &t_1=\sqrt{R^2-x^2}\,/\,v;\ \ t_2=\sqrt{R^2-(2R-d-x)^2}\,/\,v \\ &r_1=\sqrt{(vt)^2+x^2};\ r_2=\sqrt{(vt)^2+(2R-d-x)^2} \end{aligned} \tag{14}$$

t_1 and t_2 are the half time of the tool spraying on two adjacent trajectories. r_1 and r_2 are the distance of a point f to the projection of tool. t is the time for the tool moving from the point O to F. By equation (14), it can be calculated as: $q_f(x,d,v)=\frac{1}{v}J(x,d)$,where J is a function of x and d. To find optimal velocity v and overlap distance d, the mean square error of the thickness deviation from the average thickness q_d must be minimized:

$$\min_{d\in[0,R],v} E(d,v)=\int_0^{2R-d}(q_d-q_s(x,d,v))^2dx \tag{15}$$

A golden section method is adopted here to calculate their values [16].

5 EXPERIMENTAL VERIFICATION

Suppose the required average material thickness is q_d=50μm, and the max material thickness deviation is q_w=10μm.The spray radius R=60mm. The material deposition rate is:

$$f(r)=\frac{1}{15}(R^2-r^2)\mu m/s \tag{16}$$

After optimizing the spray process on a plane, the tool velocity and the overlap distance are calculated as: v=256.6mm/s,d=50.2mm.

The workpiece which is the complex free-form surfaces, shown in figure3, is used to test the method. The first step is surface partition. And the triangular approximation is exported with an error tolerance of 2 *mm.* The triangular approximation of the workpiece is shown in figure4. The second step is trajectory generation based on T-Bézier curves. The robot trajectories (U or V direction) are generated with off line programming system in the experiment. Figure5 (a) show the trajectory of U direction and figure5 (b) show the trajectory of V direction. The third step is trajectory optimization. And the trajectory optimal algorithm is used .Figure 6 shows the robotic spray painting experiment.

Fig 3.The workpiece

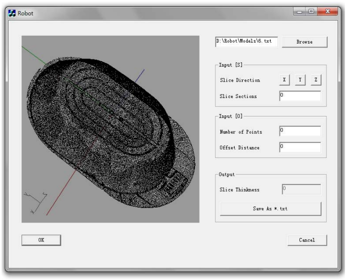
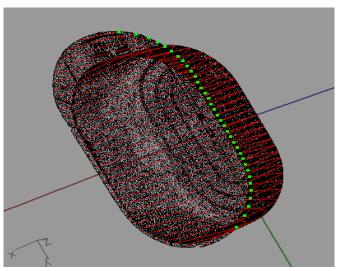
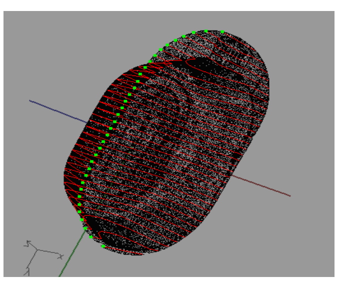


Fig 4.The triangular approximation of the workpiece



(a) U direction



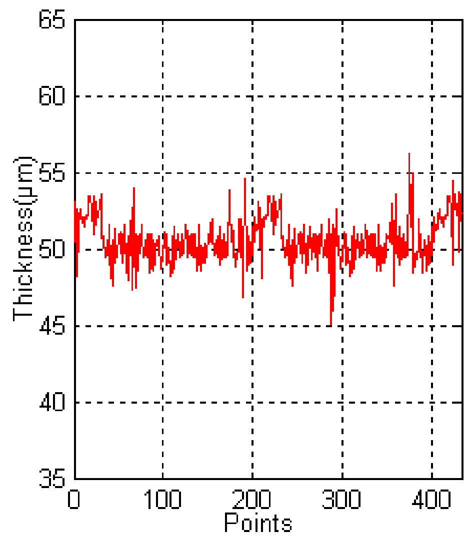
(b) V direction

Fig 5. The trajectory of the spray painting robot

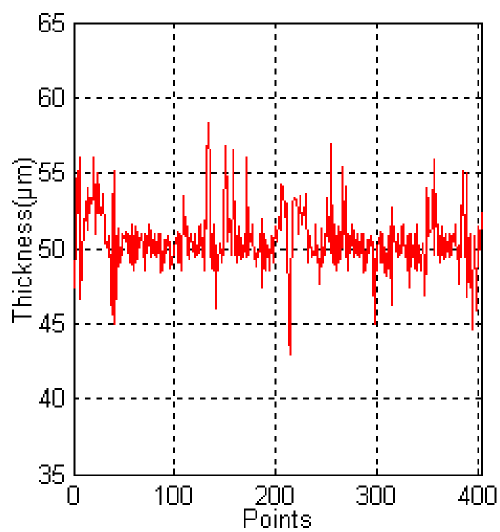
Fig 6. Spray painting experiment of FANUC robot

The material thicknesses of random chosen points on the workpiece are measured by coating thickness gauge. Figure7 (a) shows the result for U direction robot trajectory planning. Figure7 (b) shows the result for V direction robot trajectory planning. The results for U and V direction robot trajectory planning are summarized in Table 1.

In the experiment, the trajectories (U or V direction) are generated. And the results show that the trajectories (U or V direction) are can meet the requirements of spraying. But for this work, the U direction paths are better, and the efficiency of spray painting is higher. It can be seen that the shape of the workpiece should be fully considered when the trajectory is optimized based on T-Bézier curves.



(a) Material thickness of random chosen points for U direction trajectory planning



(b) Material thickness of random chosen points for V direction trajectory planning

Fig 7. Material thickness of random chosen points on the workpiece

Table1. The results of spray painting experiment

	U direction	V direction
Average(μm)	51.1	52.2
Maximum(μm)	56.3	58.3
Minimum(μm)	45.2	43.1
Process time(s)	82	99

6 CONCLUSION

A new method of trajectory optimization for spray painting robot based on T-Bézier curves has been developed. A new curve, to be called T-Bézier curve is generated from T-Bézier basis. And a new trajectory planning scheme for spray painting robot based on the T-Bézier curves is developed. Experiments are performed to measure the material thickness on a compound free-form surface for both U direction and V direction robot trajectory. Experimental results show that the U direction trajectory takes less process time compared with the V direction trajectory for this workpiece. And the results demonstrate the trajectory optimization method for spray painting robot. This method can also be extended to other applications such as optimal trajectory for free-form surface of cleaning robot or grinding robot, etc.

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