

Application of TOPSIS and Fuzzy TOPSIS Methods for Plant Layout Design

Esfandiyar Ataei

Ardabil Branch, Islamic Azad University, Ardabil, Iran

Abstract: Multi Attribute Decision Making (MADM) is a common task in human activities. It consists of finding the most preferred alternative from a given set of alternatives. We propose two Multiple-attribute Decision Making (MADM) methods in solving a plant layout design problem. They are: technique for order preference by similarity to ideal solution (TOPSIS) and fuzzy TOPSIS. The layout design problem is a strategic issue and has a significant impact on efficiency of a manufacturing system. The present study explores the use of MADM approaches in solving a layout design problem. The proposed methodology is illustrated through a practical application from an integrated-circuit (IC) packaging company. Empirical results showed that the proposed methods are viable approaches in solving a layout design problem. TOPSIS is a viable approach for the case study. The technique is suitable for precise value performance ratings. When the performance ratings are vague and imprecise, the fuzzy TOPSIS is the choice and preferred solution method.

Key words: TOPSIS . multiple-attribute decision making . plant layout . fuzzy TOPSIS . integrated-circuit

INTRODUCTION

Multi Attribute Decision Making (MADM) is a common task in human activities. It consists of finding the most preferred alternative from a given set of alternatives. Layout design invariably has a significant impact on the performance of a manufacturing or service industry system, and consequently has been an active research area for several decades [1, 2]. The former approach, such as Spiral [3] and Multiple [4], can efficiently generate alternative layout designs, but the design objectives are often over-simplified. As a result, many decision making processes in the real world take place in group settings with incomplete information. Salo has developed an interactive method to aggregate the preferences of group members in the context of an evolving value representation [5]. Kim presented an interactive procedure for multiple attribute group decision making with incomplete information and described some theoretical models to establish a group's pair-wise dominance relations using utility ranges with a separable linear programming technique [6]. For example, the resulting departmental shapes often deviate from practical constraints. Additional algorithmic approaches could be modeled the layout design problem as a mixed integer programming formulation [7-10]. Technique for order performance by similarity to ideal solution (TOPSIS) was initially introduced by Hwang and Yoon [11], and Lai *et al.* [12], is a multi-attribute or multi-criteria decision

making (MADM/MCDM) [13-15] to identify solutions from a finite set of alternatives based on minimum distance from an ideal point and maximum distance from a negative ideal point. Shih [16] has exploited incremental analysis to overcome the drawbacks of ratio scales in various MCDM techniques. Shih *et al.* [17] have identified the advantages of TOPSIS which represents the rationale of human choice; accounts for both the best and worst alternatives; the performance measures of all alternatives on attributes. In recent years, TOPSIS has been successfully applied to the areas of transportation [18], product design [19], and supply chain management [20]. However, uncertain data may not be precisely determined since human judgments are often vague under insufficient information. Therefore, fuzzy values or interval values are usually collected in measurement of the relative importance of criteria and the performance of each alternative on TOPSIS model. These approaches use the flow distance as the surrogate function, and are often computationally prohibited. The layout decision is usually based on both quantitative and qualitative performance ratings pertaining to the desired closeness or closeness relationships among the facilities. The 'closeness' is a vague notion that captures issues such as the material flow and the ease of employee supervision [21]. Grobelny [22] has explored the use of a fuzzy approach to facilities layout problems using a fuzzy criterion to determine the closeness relationship among departments and then to determine the final

Corresponding Author: Esfandiyar Ataei, Ardabil Branch, Islamic Azad University, Ardabil, Iran

optimum design. Layout design problem using multiple-attribute decision making (MADM) methods is preferable. It seeks to evaluate a large number of layout design alternatives generated by an efficient layout design algorithm. The evaluation of a large number of design alternatives will thereby reduce the risk of missing a high-quality solution. Multiple objective decision making (MODM) consists of a set of conflicting goals that cannot be achieved simultaneously [9,23]. In fact, it is programming technique. The remainder of this paper is organized as follows: The pertinent literature is reviewed in section 2. The theories and empirical description for the TOSIS and fuzzy TOPSIS two methods are discussed in detail sequentially in sections 3 and 4. Some conclusions are summarized in section 5.

LITERATURE REVIEW

All of the above fuzzy-based layout design algorithms modeled the fuzzy or linguistic closeness relationship among departments. The resulting fuzzy scores that represent the desired closeness are then used for a layout design criterion along with a part of the layout improvement process. In these methods, the fuzzy closeness determines the order of entry of departments into the layout; but the department placement and departmental dimensions are not explicitly considered.

Karray *et al.* [21] have proposed an integrated methodology using the fuzzy theory and genetic algorithms to investigate the layout of temporary facilities in relation to the planned buildings in a construction site. It identifies the closeness relationship values between each pair of facilities in a construction site using fuzzy linguistic representation.

Raoot and Rakshit [24] have defined a construction-type layout design heuristic based on fuzzy set theory. A linguistic variable was used to model various qualitative design criteria, and then to determine the closeness relationship among departments. The resulting closeness relationship matrix was used to construct a layout design. This approach allowed, in a qualitative manner, for the systematic treatment of uncertainty due to fuzziness.

The layout design problem presented by Yang and Kuo [25] is adopted for this work. It is an IC packaging plant. The detail of IC fabrication process is not discussed in this paper for a concise presentation. For detail of extended work is referred to Xiao [26] for more fundamental discussion of the IC fabrication process.

MADM deals with the problem of choosing an option from a set of alternatives which are characterized

in terms of their attributes. MADM is a qualitative approach due to the existence of criteria subjectivity. It requires information on preferences among the instances of an attribute, and the preferences across the existing attributes. The decision maker may express or define a ranking for the attributes as importance/weights. The aim of the MADM is to obtain an optimum alternative that has the highest degree of satisfaction for all of the relevant attributes.

TOPSIS and fuzzy TOPSIS have been applied to solve a variety of applications, and are the proven methodology in solving MADM problems [11, 27]. The present work explores the use of TOPSIS and fuzzy TOPSIS to solve the proposed layout design problem since we are not aware of a similar application. Details of the proposed case and methodology are discussed sequentially in the following sections.

TOPSIS

Technique for order performance by similarity to ideal solution (TOPSIS), TOPSIS method is a technique for order preference by similarity to ideal solution that maximizes the benefit criteria/attributes and minimizes the cost criteria/attributes, whereas the negative ideal solution maximizes the cost criteria/attributes and minimizes the benefit criteria/attributes. A MADM problem can be concisely expressed in a matrix format, in which columns indicate attributes considered in a given problem; and in which rows list the competing alternatives. Specifically, a MADM problem with m alternatives (A_1, A_2, \dots, A_m) that are evaluated by n attributes (C_1, C_2, \dots, C_n) can be viewed as a geometric system with m points in n -dimensional space. An element x_{ij} of the matrix indicates the performance rating of the i^{th} alternative, A_i , with respect to the j^{th} attribute, C_j , as shown in Eq. (1):

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{i1} & x_{i2} & \cdots & x_{ij} & \cdots & x_{in} \\ \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mj} & \cdots & x_{mn} \end{bmatrix} \quad (1)$$

Let $W = (W_1, W_2, \dots, W_m)$ be the relative weight vector about the criteria, satisfying $\sum_{j=1}^n W_j = 1$. Then the procedure of TOPSIS can be expressed in a series of steps:

Step 1: Calculate the normalized decision matrix. Some normalized methods for TOPSIS is summarized by Shih *et al.* [17]. For simplify, a vector normalization method is introduced whose normalized value N_{ij} is calculated as:

$$N_{ij} = \frac{X_{ij}}{\sqrt{\sum_{i=1}^n X_{ij}^2}} \quad i=1, 2, \dots, n \quad j=1, 2, \dots, m \quad (2)$$

Step 2: Calculate the weighted normalized decision matrix $V = (V_{ij})_{n \times m}$:

$$V_{ij} = W_j \times N_{ij} \quad i = 1, 2, \dots, n \quad j = 1, 2, \dots, m \quad (3)$$

where w_j is the relative weight of the j th criterion/attribute, and $\sum_{j=1}^m w_j = 1$.

Step 3: Determine the positive ideal A^* and negative ideal solution A^- as below:

$$A^* = \{v_1^*, v_2^*, \dots, v_j^*, \dots, v_m^*\} \quad (4)$$

$$= \{(\max_i v_{ij} \mid j \in J_1), (\min_i v_{ij} \mid j \in J_2) \mid i = 1, 2, \dots, m\}$$

$$A^- = \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_m^-\}$$

$$= \{(\min_i v_{ij} \mid j \in J_1), (\max_i v_{ij} \mid j \in J_2) \mid i = 1, 2, \dots, m\} \quad (5)$$

where J_1 is a set of benefit attributes (larger-the-better type) and J_2 is a set of cost attributes (smaller-the-better type).

Step 4: Calculate the separation measures, using the m -dimensional Euclidean distance.

The separation of each alternative from the ideal solution (A^*) and the negative ideal solution (A^-) are given below, respectively:

$$D_i^+ = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^*)^2}, \quad i = 1, 2, \dots, n \quad (6)$$

$$D_i^- = \sqrt{\sum_{j=1}^m (v_{ij} - v_j^-)^2}, \quad i = 1, 2, \dots, n \quad (7)$$

Step 5: Calculate the relative closeness of each alternative to the ideal solution. The relative closeness of the alternative A_i with respect to A^* is defined as follows:

$$C_i^* = \frac{D_i^-}{D_i^+ + D_i^-}, \quad i = 1, 2, \dots, n \quad (8)$$

Note that, $0 \leq C_i^* \leq 1$, where $C_i^* = 0$ when $A_i = A^-$, and $C_i^* = 1$ when $A_i = A^*$.

Step 6: Rank preference order. Choose an alternative with maximum C_i^* or rank alternatives according to C_i^* in descending order.

FUZZY TOPSIS MODEL

Raoot and Rakshit [24] have proposed a construction-type layout design heuristic based on fuzzy set theory. A linguistic variable was used to model various qualitative design criteria, and then to determine the closeness relationship among departments. The resulting closeness relationship matrix was used to construct a layout design. This approach allowed, in a qualitative manner, for the systematic treatment of uncertainty due to fuzziness. The present study explored the use of fuzzy TOPSIS in solving a layout design problem.

The decision makers use the linguistic variables to evaluate the importance of attributes and the ratings of alternatives with respect to various attributes. The present study has only precise values for the performance ratings and for the attribute weights. In order to illustrate the idea of fuzzy MACD, we deliberately transform the existing precise values to five-levels, fuzzy linguistic variables-very low (VL), low (L), medium (M), high (H) and very high (VH). The purpose of the transformation process has two folds as: (i) to illustrate the proposed fuzzy TOPSIS method and (ii) to benchmark the empirical results with other precise value methods in the later analysis.

It is often difficult for a decision-maker to assign a precise performance rating to an alternative for the attributes under consideration. The merit of using a fuzzy approach is to assign the relative importance of attributes using fuzzy numbers instead of precise numbers. This section extends the TOPSIS to the fuzzy environment [28]. This method is particularly suitable for solving the group decision-making problem under fuzzy environment. We briefly review the justification of fuzzy theory before the development of fuzzy TOPSIS; as follows:

Definition 1: A fuzzy set \tilde{A} in a universe of discourse X is characterized by a membership function $\mu_{\tilde{A}}(x)$ which associates with each element x in X , a real number in the interval $[0, 1]$. The function value $\mu_{\tilde{A}}(x)$ is termed the grade of membership of x in \tilde{A} [29]. The present study uses triangular fuzzy numbers. A triangular fuzzy number \tilde{A} can be defined by a triplet (a_1, a_2, a_3) . Its conceptual schema and mathematical form are shown by Eq. (9) [30]:

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \quad (9)$$

Definition 2: Let us define $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers \tilde{a}, \tilde{b} , then the vertex method is defined to calculate the distance between them, as Eq. (10):

$$d(\tilde{a}, \tilde{b}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]} \quad (10)$$

The fuzzy MADM can be concisely expressed in matrix format as Eqs. (11) and (12).

$$\tilde{A} = \begin{bmatrix} \tilde{A}_1 & \tilde{A}_2 & \tilde{A}_3 & \dots & \tilde{A}_m \\ \tilde{A}_1 & \tilde{A}_2 & \tilde{A}_3 & \dots & \tilde{A}_m \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_m & \tilde{A}_m & \tilde{A}_m & \dots & \tilde{A}_m \end{bmatrix} \quad (11)$$

$$\tilde{W} = [\tilde{w}_1, \tilde{w}_2, \tilde{w}_3] \quad (12)$$

where \tilde{A}_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$ and \tilde{w}_j , $j = 1, 2, \dots, n$ are linguistic triangular fuzzy numbers, $\tilde{A}_{ij} = (a_{ij}, b_{ij}, c_{ij})$ and $\tilde{w}_j = (w_{j1}, w_{j2}, w_{j3})$. Note that \tilde{A}_{ij} is the performance rating of the i^{th} alternative, A_i , with respect to the j^{th} attribute, G_j and \tilde{w}_j represents the weight of the j^{th} attribute, C_j .

The weighted fuzzy normalized decision matrix is shown as Eq. (13):

$$\tilde{B} = \begin{bmatrix} \tilde{b}_{11} & \tilde{b}_{12} & \dots & \tilde{b}_{1n} & \dots & \tilde{b}_{1m} \\ \tilde{b}_{21} & \tilde{b}_{22} & \dots & \tilde{b}_{2n} & \dots & \tilde{b}_{2m} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{b}_{i1} & \tilde{b}_{i2} & \dots & \tilde{b}_{in} & \dots & \tilde{b}_{im} \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots \\ \tilde{b}_{m1} & \tilde{b}_{m2} & \dots & \tilde{b}_{mn} & \dots & \tilde{b}_{mm} \end{bmatrix} \quad (13)$$

Given the above fuzzy theory, the proposed fuzzy TOPSIS procedure is then defined as follows:

Step 1: Choose the linguistic ratings $(\tilde{A}_{ij}, i = 1, 2, \dots, m, j = 1, 2, \dots, n)$ for alternatives with respect to criteria and the appropriate linguistic variables $(\tilde{w}_j, j = 1, 2, \dots, n)$ for the weight of the criteria.

Step 2: Construct the weighted normalized fuzzy decision matrix. The weighted normalized value \tilde{v}_{ij} is calculated by Eq. (13).

Step 3: Identify positive ideal (A^*) and negative ideal (A^-) solutions. The fuzzy positive-ideal solution (FPIS, A^*) and the fuzzy negative-ideal solution (FNIS, A^-) are shown as Eqs. (14) and (15):

$$A^* = (\tilde{v}_1^*, \tilde{v}_2^*, \tilde{v}_3^*) = \{(\max_i v_{ij} | i = 1, 2, \dots, m), j = 1, 2, \dots, n\} \quad (14)$$

$$A^- = (\tilde{v}_1^-, \tilde{v}_2^-, \tilde{v}_3^-) = \{(\min_i v_{ij} | i = 1, 2, \dots, m), j = 1, 2, \dots, n\} \quad (15)$$

Step 4: Calculate separation measures. The distance of each alternative from A^* and A^- can be currently calculated using Eqs. (16) and (17).

$$d_i^+ = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^*), i = 1, 2, \dots, m \quad (16)$$

$$d_i^- = \sum_{j=1}^n d(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1, 2, \dots, m \quad (17)$$

Step 5: Calculate similarities to ideal solution. This step solves the similarities to an ideal solution by Eq. (18):

$$CC_i = \frac{d_i^-}{d_i^+ + d_i^-} \quad (18)$$

Step 6: Rank preference order. Choose an alternative with maximum CC_i^* or rank alternatives according to CC_i^* in descending order.

CONCLUSION

The viable approaches in solving the proposed layout design problem. TOPSIS is a viable method for the proposed problem and is suitable for the use of precise performance ratings. When the performance ratings are vague and inaccurate and then the fuzzy TOPSIS is the preferred technique. In this study a new normalized method and risk attitude for analyzing the TOPSIS with interval data has been proposed.

Each layout design application is unique in nature, i.e., there are different attributes associated with different applications; thus, the success of the present study has no guarantee for its applicability to other applications. Judicious use of a design method is advised in solving a specific application. In addition, there exists other worth investigating MADM methods for a layout design problem. This becomes one of the future research opportunities in this classical yet important research area.

REFERENCES

1. Apple, J.M., 1991. Plant layout and material handling. Krieger.
2. Meller, R.D. and K.-Y. Gau, 1996. The facility layout problem: Recent and emerging trends and perspectives. *Journal of Manufacturing Systems*, 15 (5): 351-366.
3. Goetschalckx, M., 1992. An interactive layout heuristic based on hexagonal adjacency graphs. *European Journal of Operational Research*, 63 (2): 304-321.
4. Bozer, Y.A., R.D. Meller and S.J. Erlebacher, 1994. An improvement-type layout algorithm for single and multiple-floor facilities. *Management Science*, 40 (7): 918-932.
5. Salo, A.A., 1995. Interactive decision aiding for group decision support. *European Journal of Operational Research*, 84 (1): 134-149.
6. Kim, S.H., S.H. Choi, and J.K. Kim, 1999. An interactive procedure for multiple attribute group decision making with incomplete information: Range-based approach. *European Journal of Operational Research*, 118 (1): 139-152.
7. Heragu, S.S. and A. Kusiak, 1991. Efficient models for the facility layout problem. *European Journal of Operational Research*, 53 (1): 1-13.
8. Peters, B.A. and T. Yang, 1997. Integrated facility layout and material handling system design in semiconductor fabrication facilities. *Semiconductor Manufacturing, IEEE Transactions*, 10 (3): 360-369.
9. Yang, T., B.A. Peters, and M. Tu, 2005. Layout design for flexible manufacturing systems considering single-loop directional flow patterns. *European Journal of Operational Research*, 164 (2): 440-455.
10. Chan, F.T., K.W. Lau, P.L.Y. Chan, and K.L. Choy, 2006. Two-stage approach for machine-part grouping and cell layout problems. *Robotics and Computer-Integrated Manufacturing*, 22 (3): 217-238.
11. Hwang, C.-L. and K. Yoon, 1981. Multiple attribute decision making. Springer.
12. Lai, Y.-J., T.-Y. Liu, and C.-L. Hwang, 1994. Topsis for MODM. *European Journal of Operational Research*, 76 (3): 486-500.
13. Chen, S.-J.J., C.-L. Hwang, M.J. Beckmann, and W. Krelle, 1992. Fuzzy multiple attribute decision making: Methods and Applications, Springer-Verlag New York, Inc.
14. Gwo-Hshiung, T., G.H. Tzeng, and J.-J. Huang, 2011. Multiple attribute decision making: Methods and Applications, Chapman & Hall.
15. Zeleny, M. and J.L. Cochrane, 1982. Multiple criteria decision making. McGraw-Hill New York, Vol: 25.
16. Shih, H.-S., 2008. Incremental analysis for MCDM with an application to group TOPSIS. *European Journal of Operational Research*, 186 (2): 720-734.
17. Shih, H.-S., H.-J. Shyur, and E.S. Lee, 2007. An extension of TOPSIS for group decision making. *Mathematical and Computer Modelling*, 45 (7): 801-813.
18. Tzeng, G.-H., C.-W. Lin, and S. Opricovic, 2005. Multi-criteria analysis of alternative-fuel buses for public transportation. *Energy Policy*, 33 (11): 1373-1383.
19. Lin, M.-C., C.-C. Wang, M.-S. Chen, and C.A. Chang, 2008. Using AHP and TOPSIS approaches in customer-driven product design process. *Computers in Industry*, 59 (1): 17-31.
20. Shyur, H.-J. and H.-S. Shih, 2006. A hybrid MCDM model for strategic vendor selection. *Mathematical and Computer Modelling*, 44 (7): 749-761.
21. Karray, F., E. Zanelidin, T. Hegazy, A.H. Shabeeb, and E. Elbeltagi, 2000. Tools of soft computing as applied to the problem of facilities layout planning. *Fuzzy Systems. IEEE Transactions*, 8 (4): 367-379.
22. Grobelny, J., 1987. The fuzzy approach to facilities layout problems. *Fuzzy Sets and Systems*, 23 (2): 175-190.
23. Yang, T. and C.-C. Hung, 2007. Multiple-attribute decision making methods for plant layout design problem. *Robotics and Computer-Integrated Manufacturing*, 23 (1): 126-137.
24. Raoot, A.D. and A. Rakshit, 1991. A 'fuzzy' approach to facilities lay-out planning. *The International Journal of Production Research*, 29 (4): 835-857.
25. Yang, T. and C. Kuo, 2003. A hierarchical AHP/DEA methodology for the facilities layout design problem. *European Journal of Operational Research*, 147 (1): 128-136.
26. Xiao, H., 2001. Introduction to semiconductor manufacturing technology. Prentice Hall Upper Saddle River, New Jersey, Vol: 16.

27. Yang, T. and P. Chou, 2005. Solving a multiresponse simulation-optimization problem with discrete variables using a multiple-attribute decision-making method. *Mathematics and Computers in Simulation*, 68 (1): 9-21.
28. Deb, S. and B. Bhattacharyya, 2005. Fuzzy decision support system for manufacturing facilities layout planning. *Decision Support Systems*, 40 (2): 305-314.
29. Zadeh, L.A., 1978. Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems*, 1 (1): 3-28.
30. Kaufmann, A., M.M. Gupta, and A. Kaufmann, 1985. *Introduction to fuzzy arithmetic: Theory and applications*. Van Nostrand Reinhold Company New York.