Assignment 6: The Laplace Transform

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1 Overview

We wish to analyse the output and transfer functions for linear time invariant systems in laplace domain using python. The linear time invariant systems include the forced spring oscillation system, coupled spring system and RLC low pass filter

2 Time response of a spring system

Consider the forced oscillatory system given by the equation:

$$\ddot{x} + 2.25x = f(t) \tag{1}$$

where

$$f(t) = cos(1.5t)e^{-0.5t} * u(t) \tag{2}$$

Solving for $X(s)$ in Laplace domain we get,

$$X(s) = \frac{s + 0.5}{(s + 0.5^2) + 2.25)(s^2 + 2.25)} \tag{3}$$

Use the impulse response of $X(s)$ to get its inverse Laplace transform.

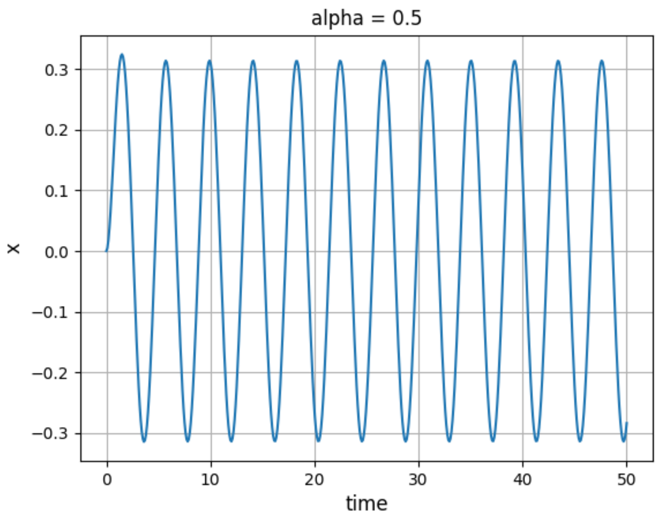


Figure 1: System Response with Decay = 0.5

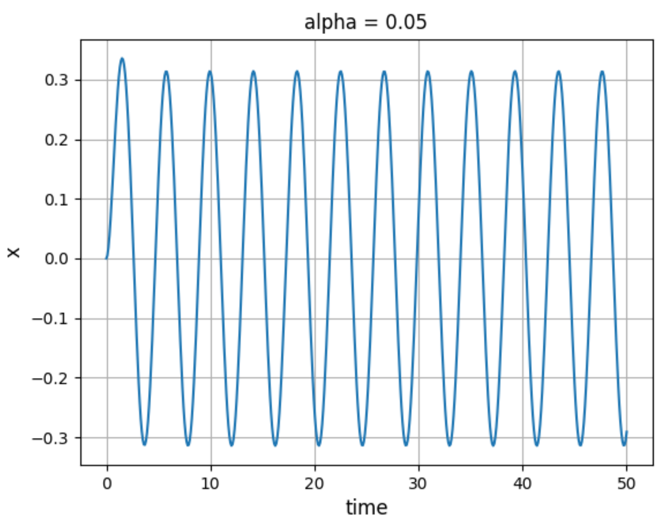
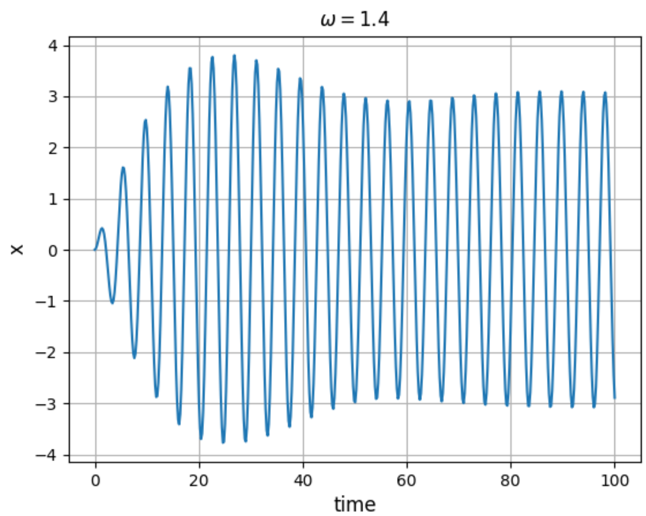
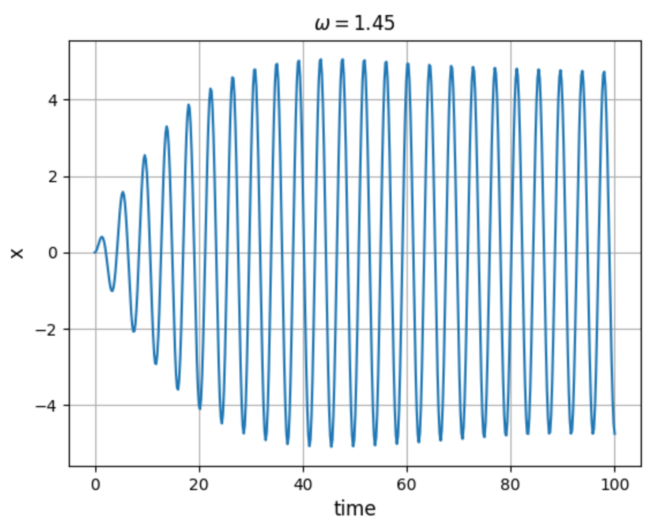


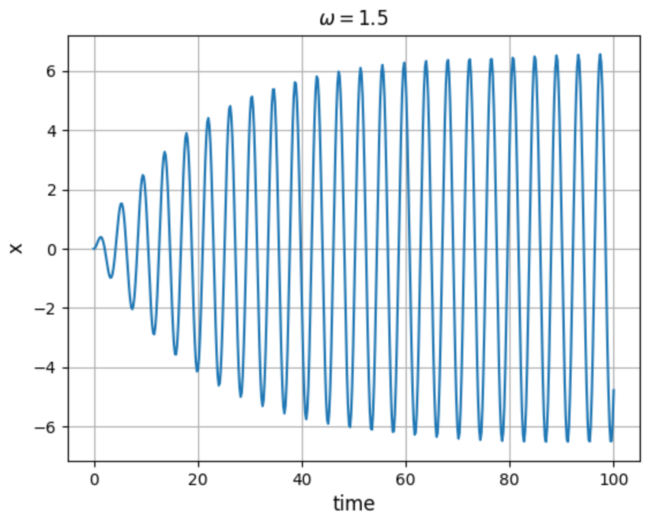
Figure 2: System Response with Decay = 0.05

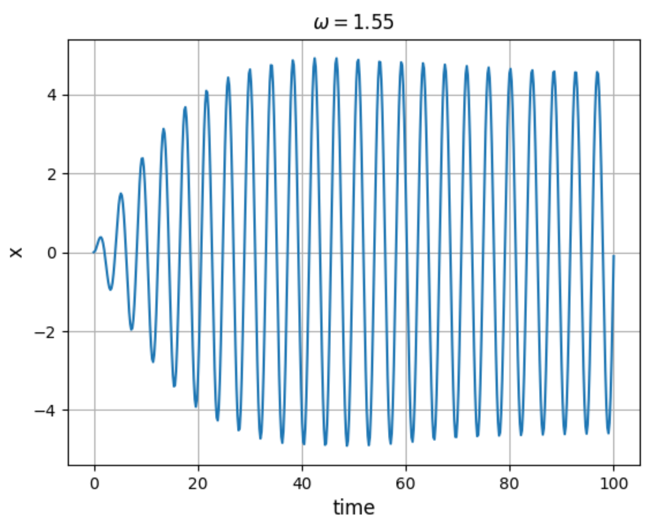
3 Response over different frequencies

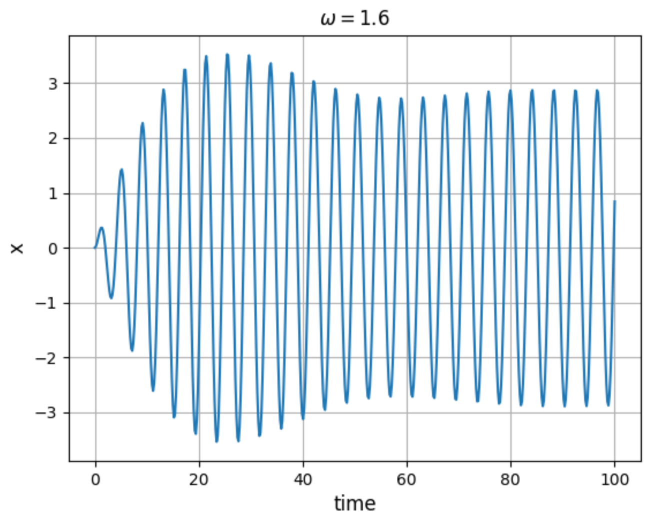
To observe how the plot changes when we change the frequency with which we excite the spring, we will plot the output for frequencies of the external force ranging from 1.4 rad/s to 1.5 rad/s. It will be seen that maximum amplitude is achieved at 1.5 rad/s because of resonance.






$\omega = 1.5$
x
time


$\omega = 1.55$
x
time



From the given equation, we can see that the natural response of the system has the frequency w = 1.5 rad/s . Thus, as expected the maximum amplitude of oscillation is obtained when the frequency of $f(t)$ is 1.5 rad/s, due to resonance.

4 The coupled spring system

We now consider a coupled Differential system

$$\ddot{x} + (x - y) = 0 \tag{4}$$

and

$$\ddot{y} + 2(y - x) = 0 \tag{5}$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s} \tag{6}$$

$$Y(s) = \frac{2}{s^3 + 3s} \tag{7}$$

It is observed that the outputs of the coupled spring system are two sinusoids with different amplitudes and phase but have the same frequency.

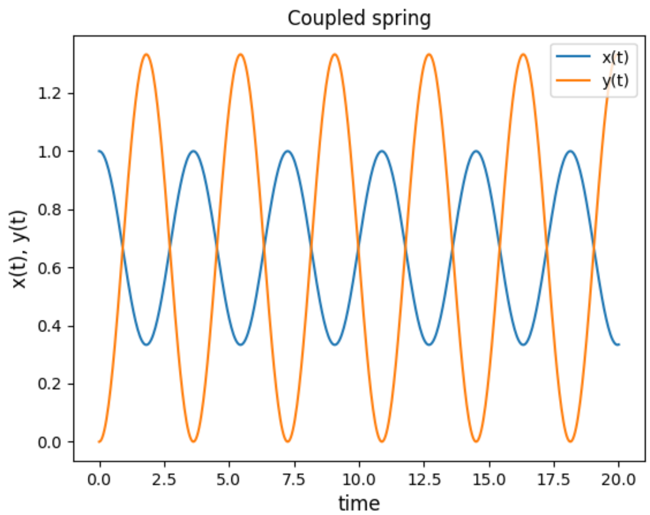


Figure 3: Coupled Oscillations

5 The Two-Port Network

The Steady-State transfer function of the given circuit is given by

$$H(s) = \frac{10^6}{s^2 + 100s + 10^6} \tag{8}$$

The magnitude and phase response are as follows:

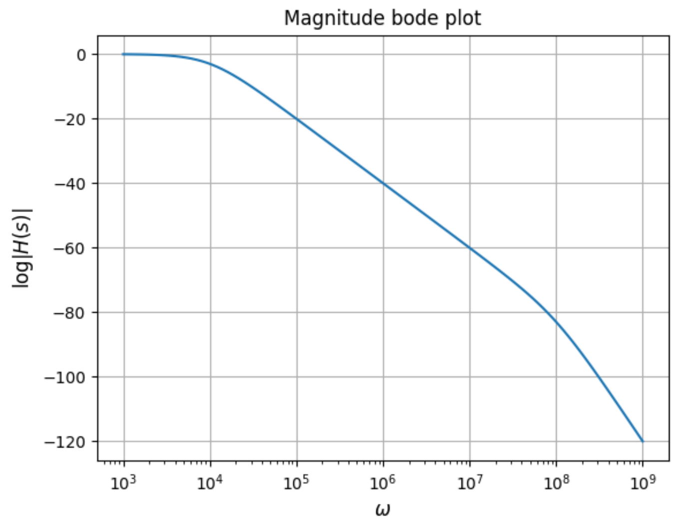


Figure 4: Magnitude bode plot

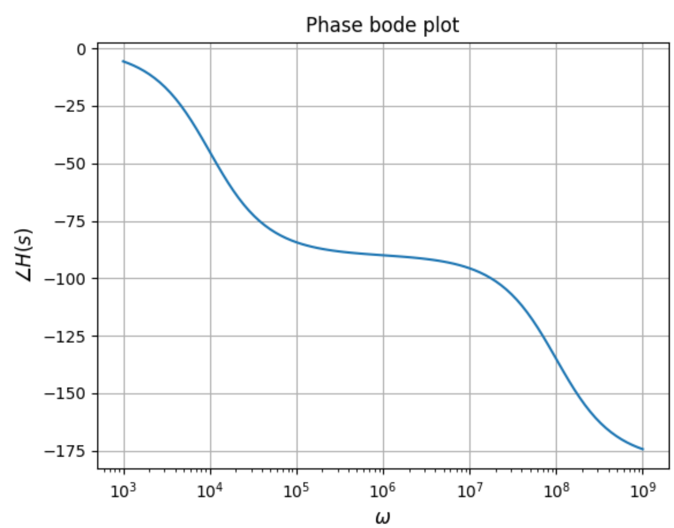


Figure 5: Phase bode plot

Output of the circuit for the below input voltage:

$$V_i(t) = (cos(10^3t) - cos(10^6t))u(t)$$

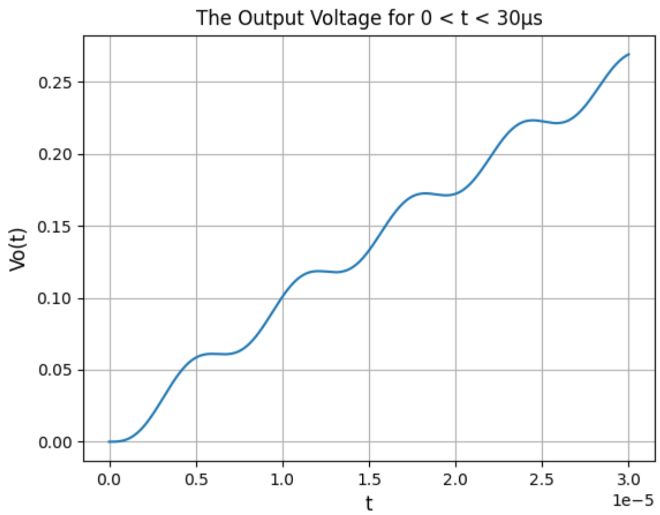


Figure 6: System response for t < 30us

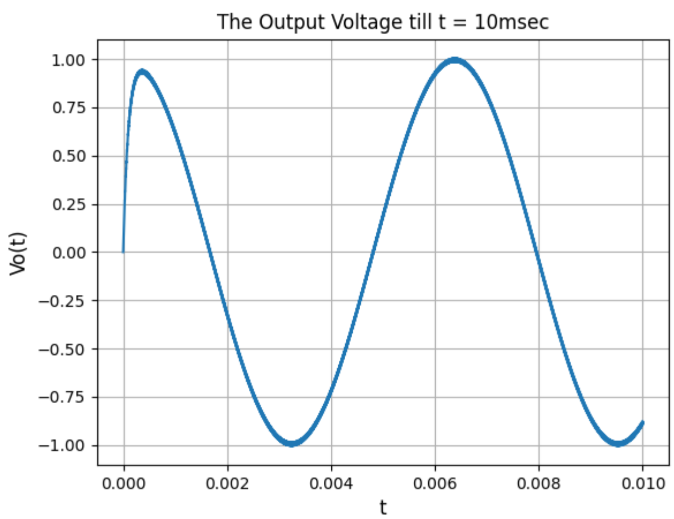


Figure 7: System response for t < 10ms

From the above plots, the contribution of different frequency components in the input signal can be clearly seen in the output signal by viewing the signal for larger and smaller time intervals.

6 Conclusion

- Output signal with maximum amplitude is obtained when applied force has the same frequency as the resonant frequency in forced spring oscillation system

- The outputs obtained for the coupled frequency system have the same frequency with a phase difference.

- The contribution of different frequency components of the input signal in the output signal can be analysed in the plots obtained using python.