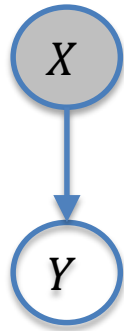


# Topic Models and LDA

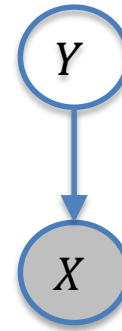
By  
Dr. Dheeraj

# Generative vs. Discriminative Models

- Recall that, in Bayesian networks, there could be many different, but equivalent models of the same joint distribution



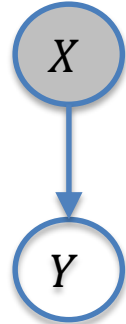
Discriminative



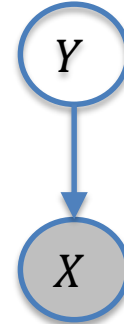
Generative

- Although these two models are equivalent (in the sense that they imply the same independence relations), they can differ significantly when it comes to inference/prediction

# Generative vs. Discriminative Models



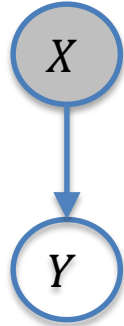
Discriminative



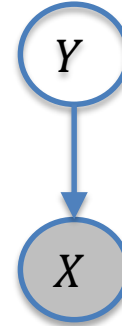
Generative

- Generative models: we can think of the observations as being generated by the latent variables
  - Start sampling at the top and work downwards
  - Examples?

# Generative vs. Discriminative Models



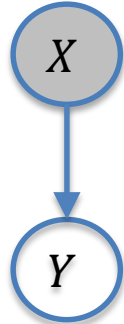
Discriminative



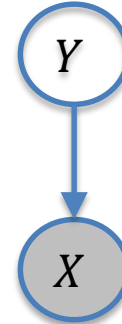
Generative

- Generative models: we can think of the observations as being generated by the latent variables
  - Start sampling at the top and work downwards
  - Examples: **HMMs, naïve Bayes, LDA**

# Generative vs. Discriminative Models



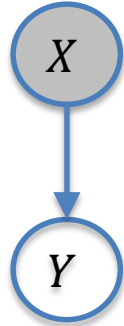
Discriminative



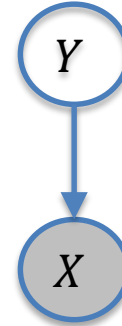
Generative

- Discriminative models: most useful for discriminating the values of the latent variables
  - Almost always used for supervised learning
  - Examples?

# Generative vs. Discriminative Models



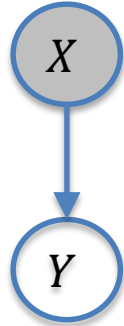
Discriminative



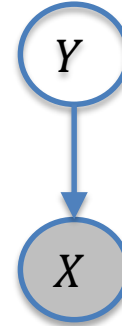
Generative

- Discriminative models: most useful for discriminating the values of the latent variables
  - Almost always used for supervised learning
  - Examples: **CRFs**

# Generative vs. Discriminative Models



Discriminative



Generative

- Suppose we are only interested in the prediction task (i.e., estimating  $p(Y|X)$ )
  - Discriminative model:  $p(X, Y) = p(X)p(Y|X)$
  - Generative model:  $p(X, Y) = p(Y)p(X|Y)$

# Models of Text Documents

- Bag-of-words models: assume that the ordering of words in a document do not matter
  - This is typically false as certain phrases can only appear together
- Unigram model: all words in a document are drawn uniformly at random from categorical distribution
- Mixture of unigrams model: for each document, we first choose a topic  $z$  and then generate words for the document from the conditional distribution  $p(w|z)$ 
  - Topics are just probability distributions over words



# Topic Models

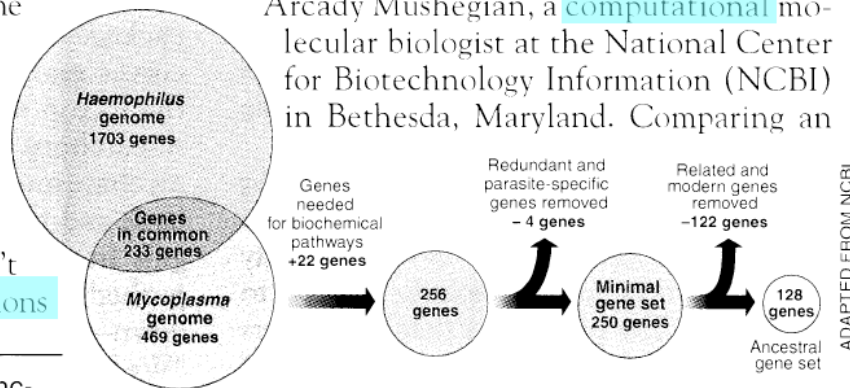
## Seeking Life's Bare (Genetic) Necessities

COLD SPRING HARBOR, NEW YORK—How many genes does an organism need to survive? Last week at the genome meeting here,\* two genome researchers with radically different approaches presented complementary views of the basic genes needed for life. One research team, using computer analyses to compare known genomes, concluded that today's organisms can be sustained with just 250 genes, and that the earliest life forms required a mere 128 genes. The other researcher mapped genes in a simple parasite and estimated that for this organism, 800 genes are plenty to do the job—but that anything short of 100 wouldn't be enough.

Although the numbers don't match precisely, those predictions

“are not all that far apart,” especially in comparison to the 75,000 genes in the human genome, notes Siv Andersson of Uppsala University in Sweden, who arrived at the 800 number. But coming up with a consensus answer may be more than just a genetic numbers game, particularly as more and more genomes are completely mapped and sequenced. “It may be a way of organizing any newly sequenced genome,” explains

Arcady Mushegian, a computational molecular biologist at the National Center for Biotechnology Information (NCBI) in Bethesda, Maryland. Comparing an



**Stripping down.** Computer analysis yields an estimate of the minimum modern and ancient genomes.

\* Genome Mapping and Sequencing, Cold Spring Harbor, New York, May 8 to 12.

# Latent Dirichlet Allocation (LDA)

## Topics

gene 0.04  
dna 0.02  
genetic 0.01  
...

life 0.02  
evolve 0.01  
organism 0.01  
...

brain 0.04  
neuron 0.02  
nerve 0.01  
...

data 0.02  
number 0.02  
computer 0.01  
...

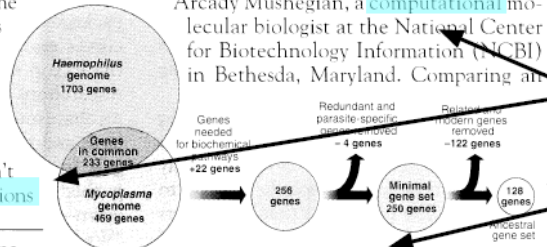
## Documents

### Seeking Life's Bare (Genetic) Necessities

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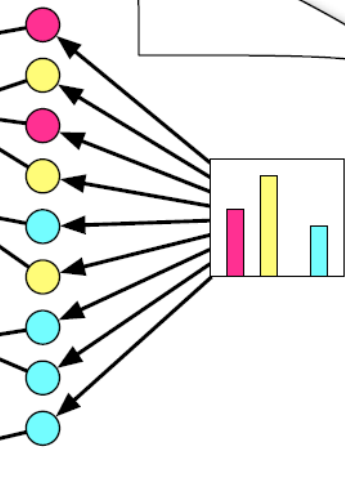


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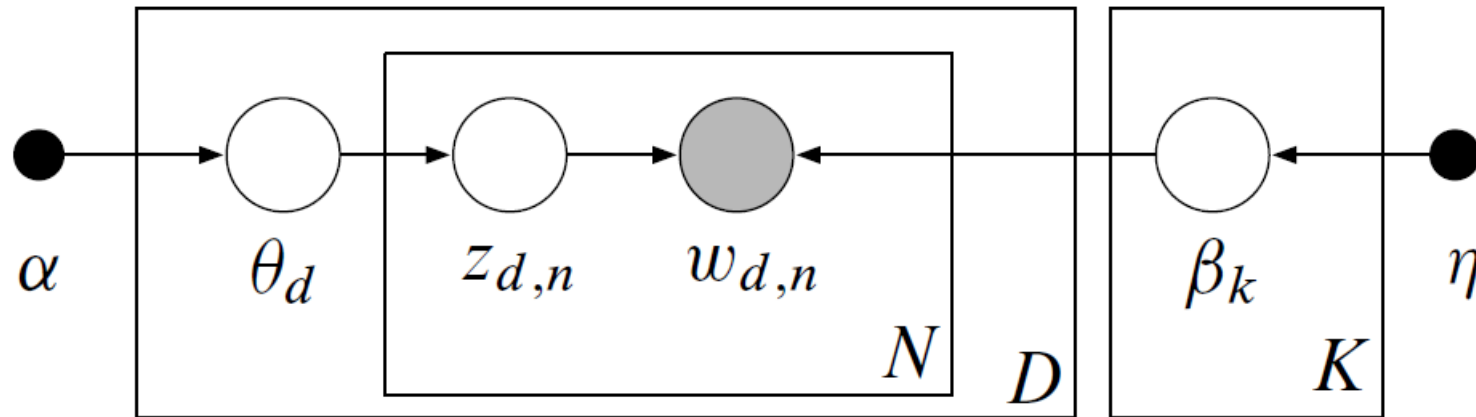
Stripping down. **Computer analysis** yields an estimate of the minimum modern and ancient genomes.

SCIENCE • VOL. 272 • 24 MAY 1996

## Topic proportions and assignments

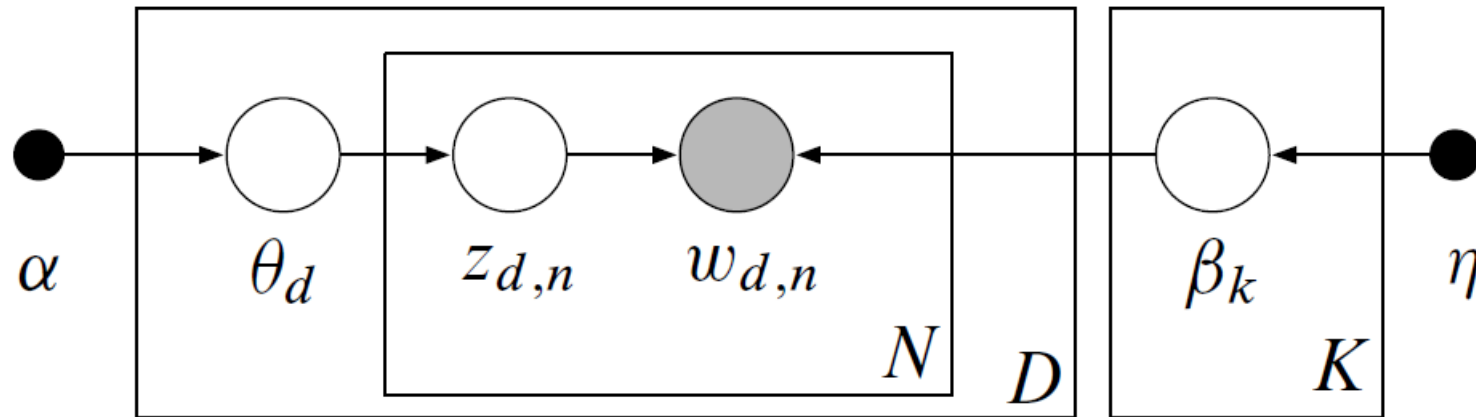


# Latent Dirichlet Allocation (LDA)



- $\alpha$  and  $\beta$  are parameters of the prior distributions over  $\theta$  and  $\beta$  respectively
- $\theta_d$  is the distribution of topics for document  $d$  (real vector of length  $K$ )
- $\beta_k$  is the distribution of words for topic  $k$  (real vector of length  $V$ )
- $z_{d,n}$  is the topic for the  $n$ th word in the  $d$ th document
- $w_{d,n}$  is the  $n$ th word of the  $d$ th document

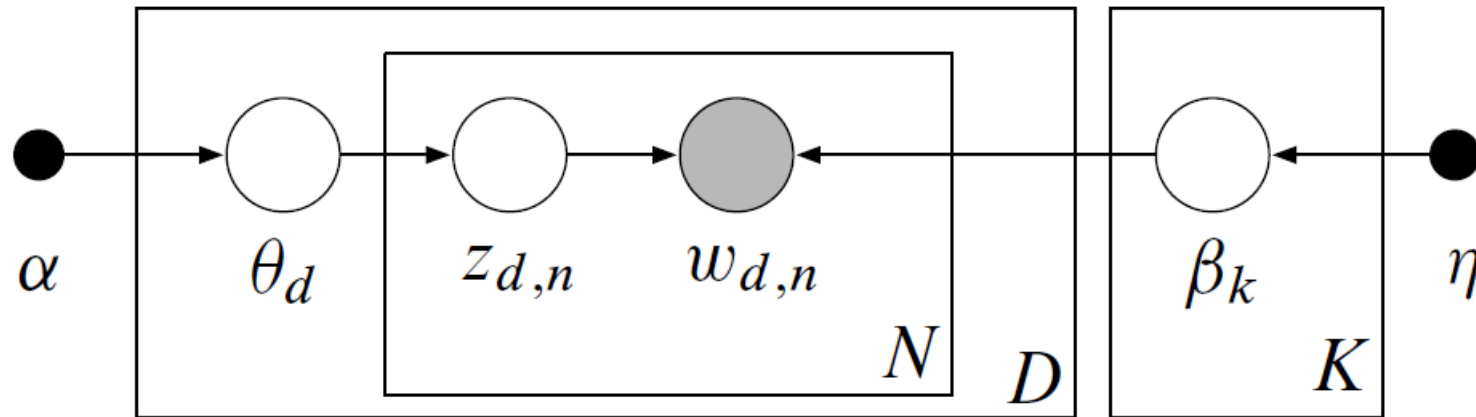
# Latent Dirichlet Allocation (LDA)



- **Plate notation**

- There are  $N \cdot D$  different variables that represent the observed words in the different documents
- There are  $K$  total topics (assumed to be known in advance)
- There are  $D$  total documents

# Latent Dirichlet Allocation (LDA)



- The only observed variables are the words in the documents
  - The topic for each word, the distribution over topics for each document, and the distribution of words per topic are all latent variables in this model

# Latent Dirichlet Allocation (LDA)

- The model contains both continuous and discrete random variables
  - $\theta_d$  and  $\beta_k$  are vectors of probabilities
  - $z_{d,n}$  is an integer in  $\{1, \dots, K\}$  that indicates the topic of the  $n$ th word in the  $d$ th document
  - $w_{d,n}$  is an integer in  $\{1, \dots, V\}$  which indexes over all possible words

# Latent Dirichlet Allocation (LDA)

- $\theta_d \sim \text{Dir}(\alpha)$  where  $\text{Dir}(\alpha)$  is the Dirichlet distribution with parameter vector  $\alpha > 0$
- $\beta_k \sim \text{Dir}(\eta)$  with parameter vector  $\eta > 0$
- Dirichlet distribution over  $x_1, \dots, x_K$  such that  $x_1, \dots, x_K \geq 0$  and  $\sum_i x_i = 1$

$$f(x_1, \dots, x_K; \alpha_1, \dots, \alpha_K) \propto \prod_i x_i^{\alpha_i - 1}$$

- The Dirichlet distribution is a distribution over probability distributions over  $K$  elements

# Latent Dirichlet Allocation (LDA)

- The discrete random variables are distributed via the corresponding probability distributions

$$p(z_{d,n} = k | \theta_d) = (\theta_d)_k$$

$$p(w_{d,n} = v | z_{d,n}, \beta_1, \dots, \beta_K) = (\beta_{z_{d,n}})_v$$

- Here,  $(\theta_d)_k$  is the  $k$ th element of the vector  $\theta_d$  which corresponds to the percentage of document  $d$  corresponding to topic  $k$

- The joint distribution is then

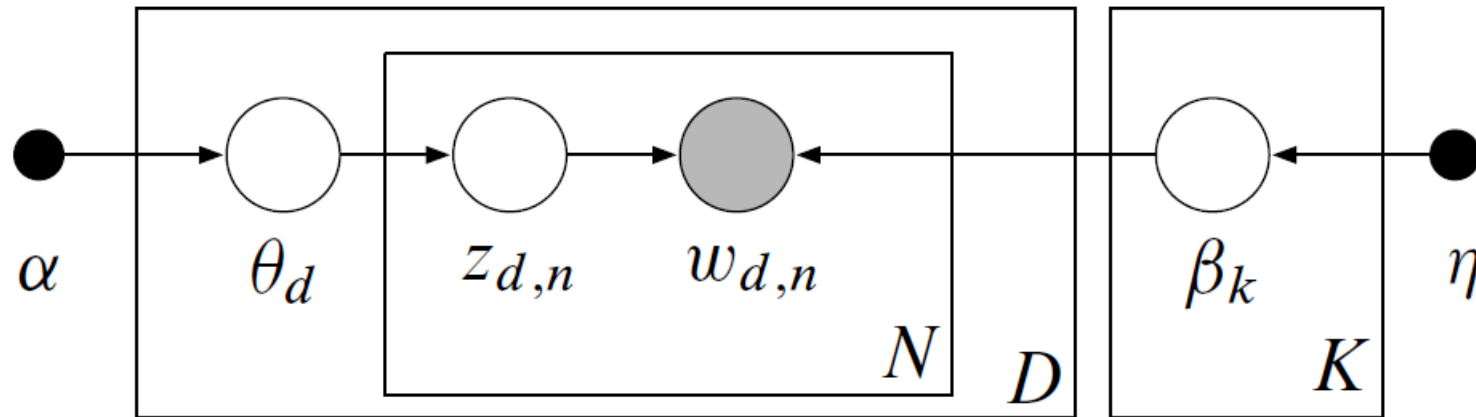
$$p(w, z, \theta, \beta | \alpha, \eta) = \prod_k p(\beta_k | \eta) \prod_d \left[ p(\theta_d | \alpha) \prod_n p(z_{d,n} | \theta_d) p(w_{d,n} | z_{d,n}, \beta) \right]$$



# Latent Dirichlet Allocation (LDA)

- LDA is a generative model
  - We can think of the words as being generated by a probabilistic process defined by the model
  - How reasonable is the generative model?

# Latent Dirichlet Allocation (LDA)



- Inference in this model is NP-hard
- Given the  $D$  documents, want to find the parameters that best maximize the joint probability
  - Can use an EM based approach called **variational EM**

# Variational EM

- Recall that the EM algorithm constructed a lower bound using Jensen's inequality

$$\begin{aligned} l(\theta) &= \sum_{k=1}^K \log \sum_{x_{mis_k}} p(x_{obs_k}^k, x_{mis_k} | \theta) \\ &= \sum_{k=1}^K \log \sum_{x_{mis_k}} q_k(x_{mis_k}) \cdot \frac{p(x_{obs_k}^k, x_{mis_k} | \theta)}{q_k(x_{mis_k})} \\ &\geq \sum_{k=1}^K \sum_{x_{mis_k}} q_k(x_{mis_k}) \log \frac{p(x_{obs_k}^k, x_{mis_k} | \theta)}{q_k(x_{mis_k})} \end{aligned}$$

# Variational EM

- Performing the optimization over  $q$  is equivalent to computing  $p(x_{mis_k} | x_{obs_k}^k, \theta)$
- This can be intractable in practice
  - Instead, restrict  $q$  to lie in some restricted class of distributions  $Q$
  - For example, could make a mean-field assumption

$$q(x_{mis_k}) = \prod_{i \in mis_k} q_i(x_i)$$

- The resulting algorithm only yields an approximation to the log-likelihood

# EM for Topic Models

$$p(w|\alpha, \eta) = \int \prod_k p(\beta_k|\eta) \int \sum_z \prod_d \left[ p(\theta_d|\alpha) \prod_n p(z_{d,n}|\theta_d) p(w_{d,n}|z_{d,n}, \beta) \right] d\theta d\beta$$

- To apply variational EM, we write

$$\begin{aligned} \log p(w|\alpha, \eta) &= \log \int \int \sum_z p(w, z, \theta, \beta|\alpha, \eta) d\theta d\beta \\ &\geq \int \int \sum_z q(z, \theta, \beta) \log \frac{p(w, z, \theta, \beta|\alpha, \eta)}{q(z, \theta, \beta)} d\theta d\beta \end{aligned}$$

where we restrict the distribution  $q$  to be of the following form

$$q(z, \theta, \beta) = \prod_k q(\beta_k|\eta) \prod_d q(\theta_d|\alpha) \prod_n q(z_{d,n})$$

# Example of LDA

## “Arts”

NEW  
FILM  
SHOW  
MUSIC  
MOVIE  
PLAY  
MUSICAL  
BEST  
ACTOR  
FIRST  
YORK  
OPERA  
THEATER  
ACTRESS  
LOVE

## “Budgets”

MILLION  
TAX  
PROGRAM  
BUDGET  
BILLION  
FEDERAL  
YEAR  
SPENDING  
NEW  
STATE  
PLAN  
MONEY  
PROGRAMS  
GOVERNMENT  
CONGRESS

## “Children”

CHILDREN  
WOMEN  
PEOPLE  
CHILD  
YEARS  
FAMILIES  
WORK  
PARENTS  
SAYS  
FAMILY  
WELFARE  
MEN  
PERCENT  
CARE  
LIFE

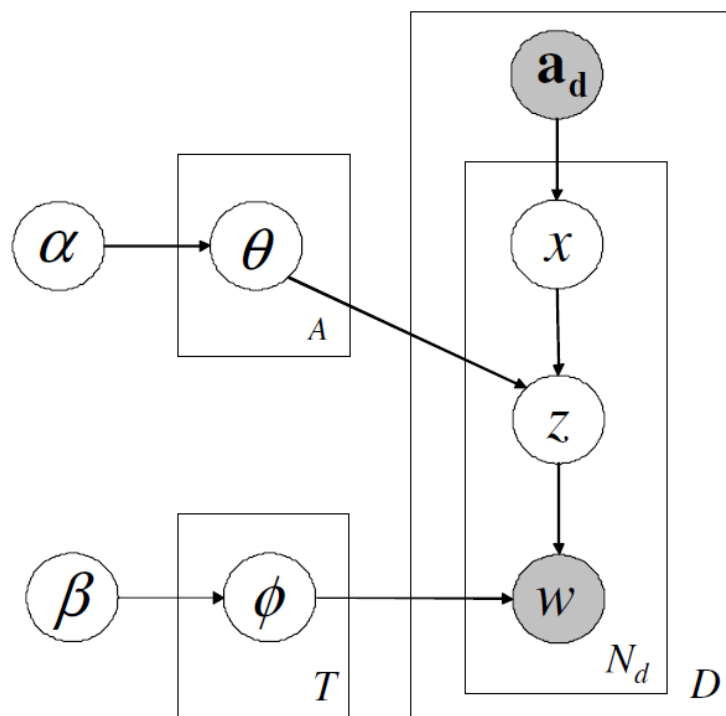
## “Education”

SCHOOL  
STUDENTS  
SCHOOLS  
EDUCATION  
TEACHERS  
HIGH  
PUBLIC  
TEACHER  
BENNETT  
MANIGAT  
NAMPHY  
STATE  
PRESIDENT  
ELEMENTARY  
HAITI

# Example of LDA

The William Randolph Hearst Foundation will give \$1.25 million to Lincoln Center, Metropolitan Opera Co., New York Philharmonic and Juilliard School. “Our board felt that we had a real opportunity to make a mark on the future of the performing arts with these grants an act every bit as important as our traditional areas of support in health, medical research, education and the social services,” Hearst Foundation President Randolph A. Hearst said Monday in announcing the grants. Lincoln Center’s share will be \$200,000 for its new building, which will house young artists and provide new public facilities. The Metropolitan Opera Co. and New York Philharmonic will receive \$400,000 each. The Juilliard School, where music and the performing arts are taught, will get \$250,000. The Hearst Foundation, a leading supporter of the Lincoln Center Consolidated Corporate Fund, will make its usual annual \$100,000 donation, too.

# Extensions of LDA



The Author-Topic Model for Authors and Documents  
Rosen-Zvi et al.

- Author– Topic model

- $a_d$  is the group of authors for the  $d$ th document
- $x_{d,n}$  is the author of the  $n$ th word of the  $d$ th document
- $\theta_a$  is the topic distribution for author  $a$
- $z_{d,n}$  is the topic for the  $n$ th word of the  $d$ th document



# Research in LDA & Topic Models

- Better inference & learning techniques
- More expressive models