Simulation and study of 2 Body Astrodynamics

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1 Abstract

2 Introduction

Gravitational two body problem is a classical mechanics problem of predicting the motion of two massive objects (abstractly viewed as point particles) under gravity. We make use of Newton's law of gravity and numerical methods to find the solution for the orbits of these objects, and simulate their changing positions.

Let m_1 and m_2 be the masses of two massive objects assumed to be point particles, separated by a distance r. We assume that the gravitational interaction is only with these two bodies, and there are no bodies nearby other than the two under consideration.

From Newton's law of gravity, the force F experienced by an object of mass m due to another object of mass M separated by a distance of R is given by,

$$F = \frac{GMm}{R^2} \tag{1}$$

Where G is the Universal gravitational constant, having the value of $6.674\times10^{-11}\,\mathrm{Nm^2/kg^2}$

In vector form, this equation will be

$$\vec{F} = \frac{GMm}{|\vec{R}|^2}\hat{R} \tag{2}$$

Where R is the radius vector from mass M to mass m

Therefore, the gravitational force between masses m_1 and m_2 is

$$\begin{aligned} F &= \frac{Gm_1m_2}{r^2} \\ \vec{F} &= \frac{Gm_1m_2}{|\vec{r_{12}}|^2}\hat{r_{12}} \end{aligned} \tag{3}$$

where r_{12} is the vector from m_1 to m_2, and $\hat{r_{12}}$ is the unit vector in it's direction.

Using Newton's second law of motion, $\vec{F} = m\vec{a}$, we can calculate the acceleration of a body. Let $\vec{F}_{12}$ be the force on mass m_1 by m_2, then the acceleration of mass m_1 will be,

$$\begin{aligned} \vec{F}_{12} &= m_1\vec{a}_1 \\ \vec{a}_1 &= \frac{\vec{F}_{12}}{m_1} \\ \frac{dv_1}{dt} = \dot{\vec{v_1}} &= \frac{\vec{F}_{12}}{m_1} \end{aligned} \tag{4}$$

$$\frac{d^2x_1}{dt} = \vec{\ddot{x}}_1 = \frac{\vec{F}_{12}}{m_1} \tag{5}$$

Integrating equation 5 twice using suitable numerical integration method, we can determine the position $x(t)$ of mass m_1. Similar calculation can be done to find $x(t)$ of m_2.

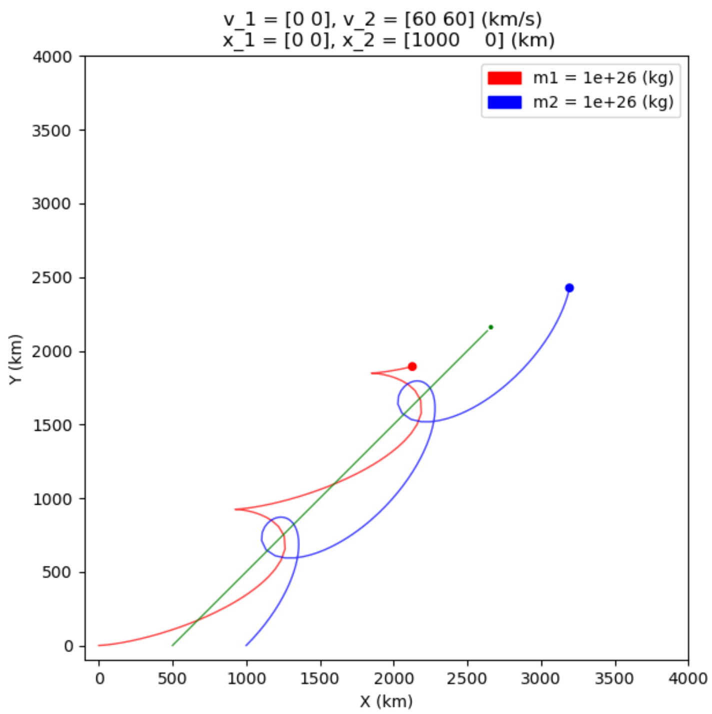
We make use of algorithm 2.1 mentioned in Curtis [2] to compute the motion of two bodies in an inertial frame of reference. We use *Python Website* [4] programming language to compute and plot the results using the algorithm mentioned above.

Now we start simulating the two body problem for various initial conditions of position, velocity and mass.

3 Results

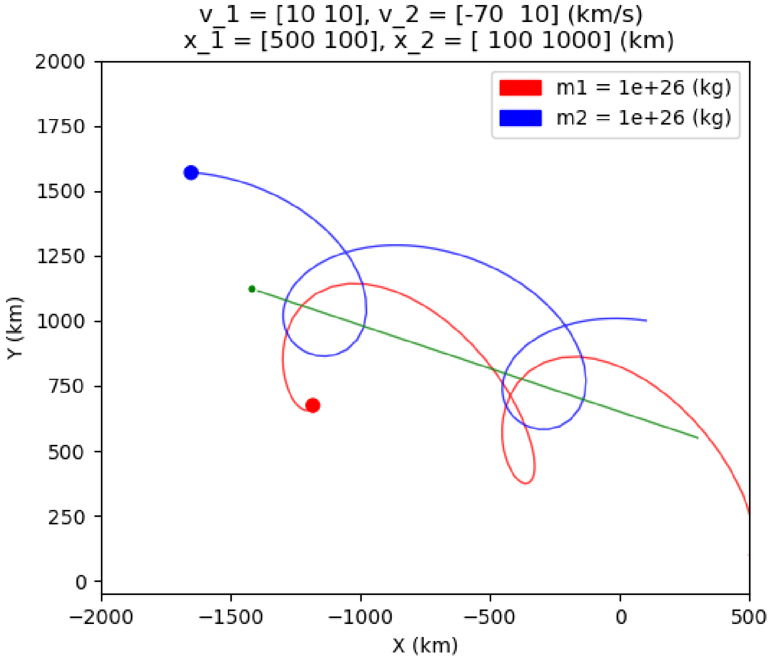
***NOTE*: For the simulation, we use 0.2 hour as the timestep value, if not mentioned**

1. With Position $x_1 = 0\hat{i} + 0\hat{j}$, $x_2 = 1000\hat{i} + 0\hat{j}$ in km and velocity $v_1 = 0\hat{i} + 0\hat{j}$, $v_2 = 60\hat{i} + 60\hat{j}$ in km/sec and equal masses of mass $1e26$ kg

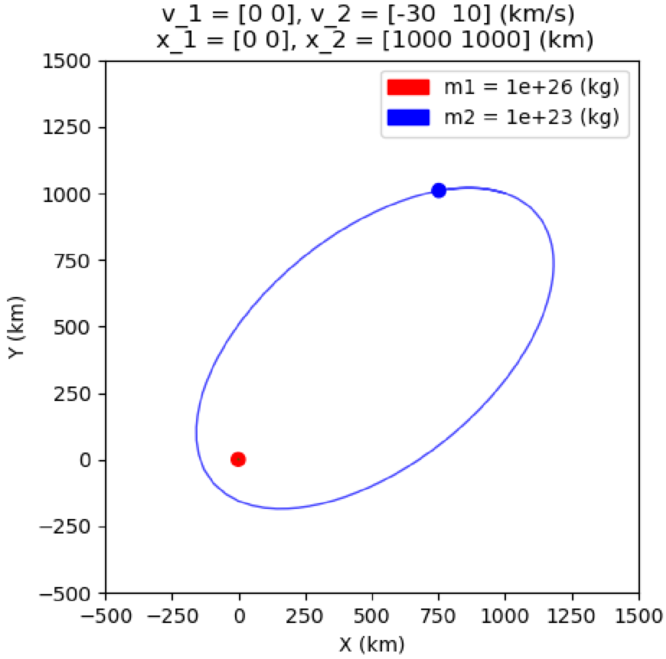


This is the plot of the two body system as seen in an inertial frame of reference. The green line represents the center of mass of the two body system

2. With Position $x_1 = 500\hat{i} + 100\hat{j}$, $x_2 = 100\hat{i} + 1000\hat{j}$ in km and velocity $v_1 = 10\hat{i} + 10\hat{j}$, $v_2 = -70\hat{i} + 10\hat{j}$ in km/sec and equal masses of mass $1e26$ kg



3. With Position $x_1 = 0\hat{i} + 0\hat{j}$, $x_2 = 1000\hat{i} + 1000\hat{j}$ in km and velocity $v_1 = 0\hat{i} + 0\hat{j}$, $v_2 = -30\hat{i} + 10\hat{j}$ km/sec and masses $m_1 = 1e26$ kg and $m_2 = 1e23$ kg



These conditions allow for a stable orbit of mass m_2 around m_1 shown in the above plot.

References

[1] D. Curtis Howard. *Orbital Mechanics for Engineering Students*. Elsevier Butterworth-Heinemann, 2005. ISBN: 0 7506 6169 0.

[2] D. Curtis Howard. *Orbital Mechanics for Engineering Students*. 4th ed. Elsevier Butterworth-Heinemann, 2020. ISBN: 978-0-08-202233-0.

[3] *Modelling n body problem*. https://www.marksmath.org. URL: `https://www.marksmath.org/classes/Spring2018NumericalAnalysis/code/nBody.html`.

[4] *Python Website*. Python. URL: `https://www.python.org`.

[5] *Two body Problem*. Wikipedia. URL: `https://en.wikipedia.org/wiki/Two-body_problem`.