EXERCISES FOR INTRODUCTION TO MATHEMATICAL QUANTUM ERROR CORRECTION

DHEERAN E. WIGGINS

1. Monday

Exercise 1.1 (Complex Arithmetic)**.** Simplify the following expressions in $\mathbb{C}$:

(i) $i^3 + i^2 + i + 1$
(ii) $(-3 + 2i)(6 - 8i)$
(iii) $(9i)(-i)$

Exercise 1.2 (Pauli Matrix Multiplication)**.** Let $I, X, Y, Z \in \mathbb{M}_2(\mathbb{C})$ be defined by

$$I := \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; X := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; Y := \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; Z := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(i) Compute the matrix products I^2, IX, IY, and IZ. What can you guess is true about multiplying matrices in $\mathbb{M}_2(\mathbb{C})$ by I, in general?
(ii) Compute the matrix products XY and YX. Simplify both products into the form cZ, where $c \in \mathbb{C}$. Is c different in each case?

The matrix I is called the identity. The other three matrices X, Y, Z are called the *Pauli matrices*. Together, these generate the *Pauli group*, a multiplicative group of order 16.

Exercise 1.3 (Recalling Definitions)**.**

(i) Let A, B be sets. What is an *injective* function $f : A \rightarrowtail B$? What is a *surjective* function $f : A \twoheadrightarrow B$? What do we call a function $f : A \xrightarrow{\sim} B$ which is *both* injective and surjective?
(ii) Write the definition of a *group* $(G, \cdot)$. What is the general difference between just a set and a group?[1]
(iii) What is the difference between a group and an *abelian* group?
(iv) Let $S \subseteq G$ be a subset of a group. Write the definition of the *normalizer* $\mathcal{N}_G(S)$ and *centralizer* $C_G(S)$ of S in G.

University of Illinois Urbana-Champaign, Illinois, 61801
Email address: dheeran2@illinois.edu

Date: January 13, 2025.

[1] You can be informal about this.