Behavioral Macroeconomics Via Sparse Dynamic Programming

NBER Working Paper

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Introduction

1. Conventionally, we assume agents comprehend the full complexity of the world.
2. "Behavioral agents" with bounded rationality are more realistic than fully rational agents.
3. Raises the question of equilibrium analysis and policy implications given "Behavioral agents" in dynamic problems.
4. The BE perspective aims to provide this alternative.
   1. Look at different preferences (prospect theory/ hyperbolic discounting), or beliefs (Overconfidence), maintaining rationality.
   2. Should we always assume rationality?

This paper

1. Inserts some "bounded rationality" (BR) into "recursive" contexts, i.e. with dynamic programming around a dynamic steady state.
2. Essentially an agent substitutes a complex problem for a simpler one and is not fully attentive.
3. Consumption Savings Model- An Example
    1. Rational: Solves DP with three state variables wealth, income, and interest.
    2. Sparse: Agent looks at a simpler "default model". If a given feature is small enough compared to some threshold, then he drops the feature or partially attenuates it.
4. Looks at many models and works out the problems with the BR methodology.

Life-Cycle Model

Proposed by Modigliani and Brumberg (1954)

1. Agent works for the L periods of his life, then retires, and dies at period T
2. **Utility** is $\sum_{t=0}^{T-1} u(c_t)$. Interest rate and subjective discount rate both 0.
3. **Income** $y_t = \bar{y}$ when working ($t \in [0, L)$), $y_t = \bar{y} + \hat{y}$ when retired ($t \in [L, T)$). Parameter $\hat{y} < 0$ captures income loss during retirement.
4. **Financial Wealth** w_t evolves as $w_{t+1} = w_t + y_t - c_t$, terminal condition $w_T = 0$

Life-Cycle Model- Rational

The Rational Agent (^{r}) analysis.

1. At time 0, resources are,
   $\Omega_0 := w_0 + \sum_{\tau=0}^{T-1} y_\tau = w_0 + T\bar{y} + (T-L)\hat{y}$.
2. Consumption problem is,

$$\max_{(c_t)_{0\leq t<T}} \sum_{t=0}^{T-1} u(c_t) \mid \sum_{t=0}^{T-1} c_t = \Omega_0,$$

   with $u' > 0$ and $u'' < 0$
3. So agent consumes constant amount all periods:
   $c_t = \frac{\Omega_0}{T} \implies c_0 = \frac{w_0+x}{T} + \bar{y}$, where $x := (T-L)\hat{y}$, so for a date $t \leq L$ there is only $T-t$ periods remaining so policy is:

$$c_t = \frac{w_t + x}{T-t} + \bar{y}$$

   Policy guarantees constant consumption over lifetime.

Life-Cycle Model- Rational

4. At time t remaining lifetime utility is $(T-t)u(c_t)$, then the value function for $t \leq L$ is:

$$V^r(w_t, x, t) = (T-t)u\left(\frac{w_t + x}{T-t} + \bar{y}\right)$$

5. Satisfies Bellman equation: $c_t = argmax_c \; v(c, w_t, x, t)$. Where the value function is,

$$v(c, w_t, x, t) := u(c) + V^r(w_t + \bar{y} - c, x, t+1)$$

Life-Cycle Model- Behavioral

Behavioral agent (sparse model) (^{s})

1. Consumption policy for a behavioral agent:
   $c_t = argmax_c \ v(c, w_t, m_t x, t)$, where $m_t \in [0, 1]$ is endogenous attention to future retirement. Note,
   1. $m_t = 1 \implies$ fully rational agent.
   2. $m_t = 0 \implies$ behaves like no income loss from retirement.
2. Given general m_t solution to the problem is, $c_t = \frac{w_t + m_t x}{T-t} + \bar{y}$.
3. Call $c_t(m_t) = \frac{w_t + m_t x}{T-t} + \bar{y} \implies c_t(0) = \frac{w_t}{T-t} + \bar{y}$. Define $c_t^d := c_t(0)$ this is the "default policy" no attention.
4. The marginal impact of attention is $c_t'(0) = \frac{x}{T-t}$. The curvature of our objective function then is
   $v_{cc}^t = v_{cc}(c_t^d, w_t, 0, t) = (1 + \frac{1}{T-t-1})u''(c_t^d)$

Life-Cycle Model- Behavioral

1. Value of attention at t to x is

$$m_t = \mathcal{A}\left(\frac{-v^t_{cc} c'_t(0)}{\kappa_t}\right) \iff \mathcal{A}\left(\frac{-u''(c^d_t)}{\kappa_t (T-t-1)(T-t)}\right).$$

   Will explain, for now note κ_t is the cost of cognition.

2. Suppose our attention function (truncated in arguments why??) is $\mathcal{A}(v) = max(1 - \frac{1}{|v|}, 0)$ and costs are scaled at $\kappa_t = \bar{\kappa}^2 |u''(c^d_t)|$.

3. The agent in cont. time agent thinks about retirement when $|\frac{x}{T-t}| \geq \bar{\kappa}$ so at some time, the agent starts considering income loss due to retirement at $s = max(0, min(L, T + \frac{x}{\bar{\kappa}}))$.
    1. Idea is I want to delay the cost of thinking about this retirement problem for as long as I can.

Life-Cycle Model- Behavioral

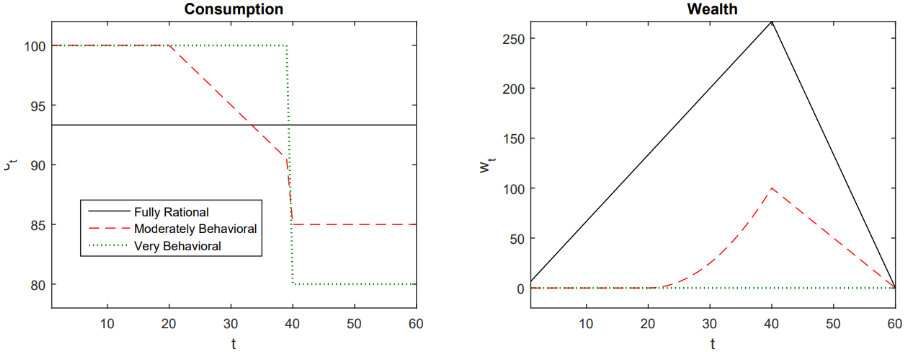
1. So consumption is dependent on $s \in (0, L)$:

$$c_t = \begin{cases} \frac{w_0}{T} + (\bar{y}) \text{ for } t < s \\ \frac{w_0}{T} + \bar{y} + \frac{2\bar{\kappa}^2}{x}(t-s) \text{ for } s \leq t < L \\ \frac{w_L}{T-L} + \bar{y} + \hat{y} \text{ for } t \geq L \end{cases}$$

   where $w_L = (1 - \frac{L}{T})w_0 - \frac{\bar{\kappa}^2}{x}(L-s)^2$

2. The fully rational agent has no cognition costs $\bar{\kappa} = 0$.
3. The very behavioral agent follows $\bar{\kappa} \geq \bar{\kappa}^* := \frac{|x|}{T-L} = |\hat{y}|$ and consumes current income and doesn't pay attention to x at all.
4. An agent with BR follows $0 < \bar{\kappa} < \bar{\kappa}^*$. They don't save at all at first but then save at some point (s) before retirement.

Life-Cycle Model- Comparison



Sparse Max for Static Problems

1. Assume a general maximization problem with no budget constraint. A rational agent does $max_a v(a, x)$ for action a state x.
2. Suppose there exists some attention vector m so that $v(a, x, m) := v(a, m_1 x_1, ..., m_n x_n)$ is the perceived valuation when agent is inattentive to x_i
3. Attention generates action $a(x, m) := argmax_a v(a, x, m)$. "Default attention vector" m^d is taken to be 0. It follows $a^d := argmax_a v(a, x, m^d)$
4. $a_{m_i} = \frac{\mathrm{d}a}{\mathrm{d}m_i}$ evaluated at (a^d, m^d). This is the effect of change in attention. So by construction, $a_{m_i} = -v_{aa}^{-1} v_{am_i}$

Sparse Max Operator

5. κ is cognition cost, essentially how much I like simplifying the problem.
6. Assume x_i stochastic and viewed by agent as draw from dist. with std. dev. σ_i.

Sparse max operator, $smax_{a;m|m^d} v(a, x, m)$ defined by following:

1. Choose optimal attention vector m^*

$$m^* = argmin_{m \in [0,1]^n} \sum_i [\frac{1}{2} A_{ii}(1 - m_i)^2 + \kappa g(m_i - m_i^d)]$$

   cost of inattention factors $A_{ii} := -\mathbb{E}[a_{m_i} v_{aa} a_{m_i}], g' > 0$. Trades off a proxy for the utility losses (the first term) and a psychological penalty for deviations from a sparse model (the second term).

2. Our action $a^s = argmax_a v(a, x, m^*)$ results in some value $v^s = v(a^s, x)$

Sparse Max Operator

1. This leads to an attention function:

$$\mathcal{A}(v) := argmin_{m\in[0,1]}\{\frac{1}{2}|v|(1-m)^2 + g(m)\}$$

   representing optimal attention to variable with variance $|v|$.

2. Then the value of attention to dimension i is, $m_i^* = \mathcal{A}(\frac{-\mathbb{E}[a_{m_i} v_{aa} a_{m_i}]}{\kappa})$.
3. In the case $v(a,x,m) = v(a, m_1 x_1, ..., m_n x_n)$ the smax op. yields; $m_i^* = \mathcal{A}(-\sigma_i^2 \frac{a_{x_i} v_{aa} a_{x_i}}{\kappa})$, with derivatives evaluated at x $= 0$ and a^d, the problem is $a^s = argmax_a v(a, m_1^* x_1, ..., m_n^* x_n)$.
   - Intuition is truncated x_i's
   - if $|a_{x_i}|$ is small so that x_i doesn't matter then $m_i^* = 0$.
4. Note in our example above how a_{x_i} was essentially $c_t'(0)$ and v_{aa} was v_{cc}^t in a non-probabilistic and inattention was only single-valued for x.

Sparse Max. Dynamic Programming

1. Let state z_t, action a_t ,and i.i.d shocks ϵ_{t+1} be vectors. Let there be T periods, allowing $T = \infty$.
2. The agents' rational problem is

$$\max_{(a_t)_{0 \leq t < T}} \sum_{t=0}^{T-1} \beta^t u(a_t, z_t) \mid z_{t+1} = F^z(a, z, \epsilon_{t+1})$$

3. The rational DP problem is a series of value functions satisfying Bellman equation:

$$V^r = \max_a \{u(a, z) + \beta \mathbb{E}[V^r(F^z(a, z, \epsilon_{t+1})]\}$$

We can ignore superscript t if state vector includes date (eg: $z_t = (w, x, t)$), and V depends on that component.

Sparse Max.

1. Given attention-augmented utility $u(a, z, m)$ and transition $F^z(a, z, \epsilon_{t+1}, m)$.
2. We have a default proxy function V^P, which is usually the same as rational value function.
3. Action of a smax behavioral agent:

$$a(z, V^p) = argsmax_{a;m|m^d}\{u(a, z, m) + \beta\mathbb{E}[V^p(F^z(a, z, \epsilon_{t+1}, m))]\}$$

4. Now agent maximizes perceived flow utility function. The sophistication of perceptions controlled by m.