1 Birth and Death Rate

1.1 Way 1

In our model, the population size remains a constant which means that the birth rate and death rate are balanced. We consider the steady state for the age distribution $x = (x_1, x_2, \cdots, x_n)$ where n is the number of the age groups and x_i is the proportion of the number of i-th age group people in the total population. Specifically, We set the x_1 as the age 0 group. Considering aging and death in the last age group, we have

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1-\frac{1}{\alpha_2} & 0 & \cdots & 0 & 0 \\ 0 & \frac{1}{\alpha_2} & 1-\frac{1}{\alpha_3} & \cdots & 0 & 0 \\ 0 & 0 & \frac{1}{\alpha_3} & \ddots & 0 & 0 \\ 0 & 0 & 0 & \ddots & 1-\frac{1}{\alpha_{n-1}} & 0 \\ 0 & 0 & 0 & \cdots & \frac{1}{\alpha_{n-1}} & 1-d \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} + \begin{pmatrix} B \\ 0 \\ \vdots \\ 0 \end{pmatrix},$$

where α_i is the age-span of the i-th age group, d is the death rate of the last age group and B is the birth rate in the total population. After calculation, we have

$$\begin{aligned} x_1 &= B, \\ x_2 &= \alpha_2 x_1 = \alpha_2 B, \\ x_3 &= \frac{\alpha_3}{\alpha_2} x_2 = \alpha_3 x_1 = \alpha_3 B, \\ &\vdots \\ x_{n-1} &= \frac{\alpha_{n-1}}{\alpha_{n-2}} x_{n-2} = \cdots = \alpha_{n-1} x_1 = \alpha_{n-1} B, \\ x_n &= \frac{d}{\alpha_{n-1}} x_{n-1} = \cdots = d x_1 = dB. \end{aligned}$$

Since

$$\sum_{i=1}^{n} x_i = 1,$$

we have

$$(1 + \alpha_1 + \alpha_2 + \cdots + \alpha_{n-1} + d) B = 1,$$

which follows that the death rate

$$d = \frac{1}{B} - 1 - \alpha_1 - \alpha_2 - \cdots - \alpha_{n-1}.$$

1.2 Way 2

In this method, we use the birth rate B from the demographics and rescale the death rate D to balance the birth rate. The ratio B/D is multiplied to the death rate of each age group to get the rescaled death rate by age.