Computer Organization

Boolean Simplification

Precedence Rules

- Boolean multiplication takes precedence over Boolean addition
- The complement operation is applied as soon as the entire expression under the bar is evaluated
- Parentheses can be used to override the precedence rules

Precedence Rules – cont.

Evaluate: $\underbrace{xy}_{1 \cdot 1} + 1 \cdot \overline{z}$ with $x = y = z = 1$

$= 1 + 1 \cdot \underbrace{\overline{z}}_{\overline{1}}$

$\overline{1} = 0$

z = 1 value under bar determined

$+ \underbrace{1 \cdot 0}_{= 0}$

Do multiplication first

$\underbrace{1 + 0}_{= 1}$

Evaluate: $x + z\,\underbrace{(\overline{0 + y})}_{= \overline{0 + 1}}$ with $x = 0,\ y = 1,\ z = 1$

parens override precedence rules

$\overline{0 + 1} = \overline{1}$

first determine value under bar

$\overline{1} = 0$

$x + \underbrace{z \cdot 0}_{= 0}$

first do multiplication

$\underbrace{x + 0}_{0 + 0}$

$0 + 0 = 0$

Equivalency

- Two Boolean expressions are **equivalent**, denoted by **=**, if they have the same value for every possible combination of values assigned to the variables contained in the expressions

- We can use this concept to simplify existing Boolean expressions by applying a variety Boolean Laws

Laws of Boolean Algebra

Idempotent laws:	x + x = x	x • x = x
Associative laws:	(x + y) + z = x + (y + z)	(xy)z = x(yz)
Commutative laws:	x + y = y + x	xy = yx
Distributive laws:	x + yz =(x + y)(x + z)	x(y + z) =xy + xz
Identity laws:	x + 0 = x	x • 1 = x
Domination laws:	x + 1 = 1	x • 0 = 0
Double complement law:	$\overline{\overline{x}} = x$	
Complement laws:	$x + \overline{x} = 1$ $\overline{0} = 1$	$x\overline{x} = 0$ $\overline{1} = 0$
De Morgan's laws:	$\overline{x + y} = \overline{x}\,\overline{y}$	$\overline{xy} = \overline{x} + \overline{y}$
Absorption laws:	x + (xy) = x	x(x + y) = x

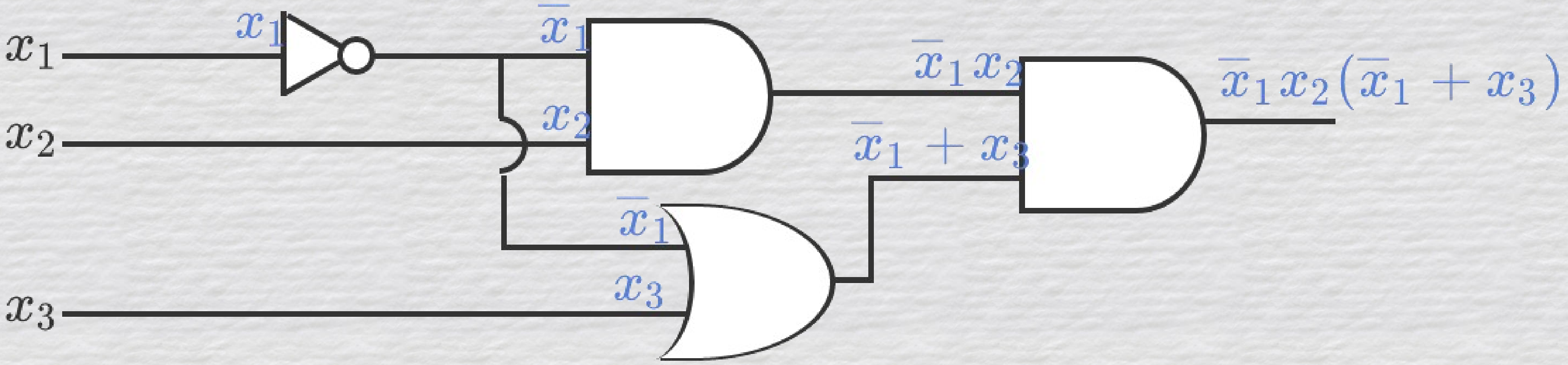
Laws of Boolean Algebra – cont.

$xy + \overline{x}y + zy$

$(x + \overline{x})y + zy =$	Distributive Law
$1 \cdot y + zy =$	Complement Law
$y + zy =$	Identity Law
$(1 + z)y =$	Distributive Law
$1 \cdot y =$	Domination Law
y	Identity Law

$xy + \overline{x}y + zy = y$

Constructing Boolean Expressions



The circuit computes the function:

$$f(x_1, x_2, x_3) = \overline{x}_1 x_2(\overline{x}_1 + x_3)$$

Constructing Boolean Expressions