On homework:

- If you work with anyone else, document what you worked on together.

- Show your work.

- Always clearly label plots (axis labels, a title, and a legend if applicable).

- Homework should be done "by hand" (i.e. not with a numerical program such as MATLAB, Python, or Wolfram Alpha) unless otherwise specified. You may use a numerical program to check your work.

- If you use a numerical program to solve a problem, submit the associated code, input, and output (email submission is fine).

- If using Python, be aware of `copy` vs. `deep copy`:
  https://docs.python.org/2/library/copy.html

1. (15 points) Describe the sweeping process in one dimension (marching through space) along one angle using the diamond difference scheme

   (a) for $\mu > 0$,

   (b) for $\mu < 0$, and

   (c) at a reflecting boundary on the right edge, including how to transition from $\mu > 0$ to $\mu < 0$.

   (d) If using the angular flux to generate flux moments during the solution process, what data do you need to store in the sweeping process?

2. Let's look at truncation error in the diamond difference method by examining the 1-D case. Consider uncollided neutrons with a zero group source moving along angle a:

$$\mu_a \frac{d\psi_a}{dx} + \Sigma_t \psi_a(x) = 0 \,,$$

   where the cross section is taken as constant.

   (a) (5 points) For neutrons moving $\mu_a > 0$, write an expression for the flux at some location x' in terms of the flux at location x (you should have an exponential).

   (b) (5 points) Let's say we impose a Cartesian grid with mesh index i. What is the expression for $\psi_{a,i+1/2}$ in terms of $\psi_{a,i-1/2}$? Use mesh spacing $\Delta_i = x_{i+1/2} - x_{i-1/2}$ and the definition $h \equiv \frac{\Sigma_t \Delta_i}{2|\mu_a|}$.

(c) (5 points) Plug the relationship you just found into the 1D diamond difference equations ($\alpha = 0$). Manipulate those to get another expression for $\psi_{a,i+1/2}$ in terms of $\psi_{a,i-1/2}$ and h.

(d) (10 points) Look again at your solution from part b. Expand the exponential in a power series and expand your part c expression in a power series through the h^2 terms and show they are the same.
What does that mean about the accuracy of the relationship?

(e) (5 points) Look carefully at the expression for $\psi_{a,i+1/2}$. What is a condition on h that would guarantee that the flux would be positive? What does that mean about mesh spacing given the smallest μ_a in a set and a specific Σ_t?

3. Write a piece of code that implements the 1-D, one-speed, steady state, weighted diamond difference equations; include isotropic scattering and an isotropic external source. Use $\psi(0) = 2.0$ for $\mu > 0$ and a non-reentrant boundary condition at $x = 0.0$. Use a reflecting boundary at $x = 2.0$. Use equiprobable weights (note: these are not real quadrature sets, simply for demonstration purposes).

(a) (20 points) Explore negative flux: use the following values
   - $\alpha = 0$
   - $\mu_a = \pm 0.1$
   - $\Sigma_t = 1.0$
   - $\Sigma_s = 0$
   - $q_e(x) = 0$
   - mesh spacings: $h = [0.08, 0.1, 0.125, 0.2, 0.4]$.

   Plot the cell-centered scalar flux that results from each mesh spacing. What do you notice? How does that compare with your conclusion from the previous problem?

(b) (10 points) Impact of α: try $\alpha = [-0.9, -0.5, 0.25, 0.5, 0.9]$. What happens?

(c) (10 points) Real results? Now try adding a source
   - $\alpha = [-0.5, 0, 0.5]$ (feel free to try others)
   - $\mu = \pm[0.2, 0.7]$
   - $\Sigma_t = 1.0$
   - $\Sigma_s = 0.5$
   - $q_e(x) = 1.0$

   Report the results.

(d) (5 points) What happens with $\alpha = 0$ and $\Sigma_s = 0.9$?

4. (20 points) starting from the following general system of equations

$$\frac{\mu_a}{h_i}(\psi^g_{a,i+1/2} - \psi^g_{a,i-1/2}) + \Sigma^g_{t,i}\psi^g_{a,i} = 2\pi \sum_{a=1}^{N} w_a \sum_{g'=1}^{G} \Sigma^{gg'}_{s,i}(a' \to a)\psi^{g'}_{a',i} + \frac{\chi_g}{2}\sum_{g'=1}^{G}\nu_{g'}\Sigma^{g'}_{f,i}\phi^{g'}_i + \frac{1}{2}Q^g_i,$$

where ϕ is scalar flux, write a set of five coupled equations for a five group problem. Assume neutrons can only downscatter from fast groups (1 and 2) to thermal groups (3,

4, and 5). Assume that thermal groups can upscatter into other thermal groups and can downscatter. Assume there is an external source and a fission source.