Gamma-ray Point-source Localization and Sparse Image Reconstruction using Poisson Likelihood

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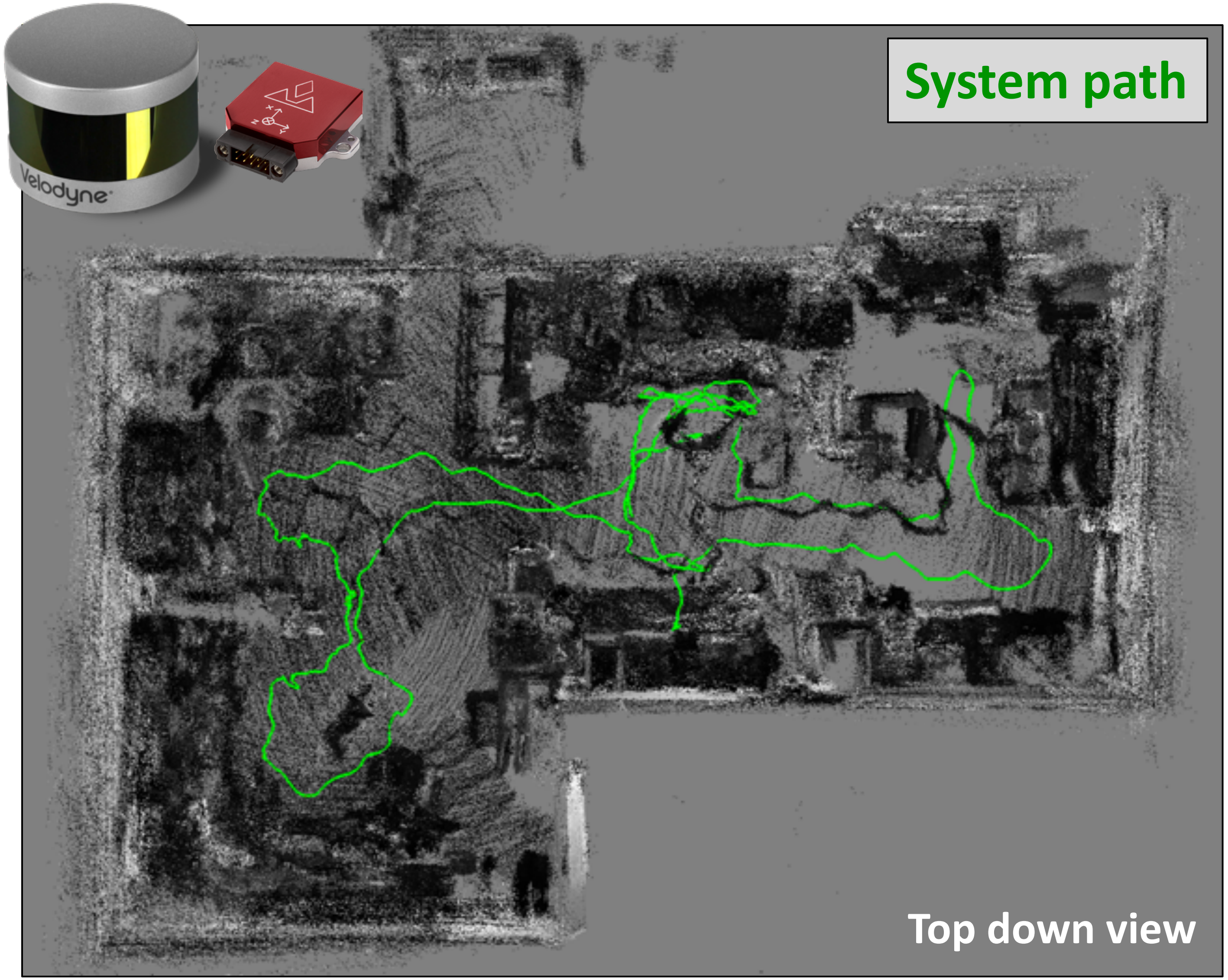
Motivation

- Given a set of Poisson distributed measurements, **x**, reconstruct the distribution of gamma-ray source intensities in 3D space, **w**
- Often times (e.g. radiological source search) **w** is sparse (e.g. point-source)
- Traditional approaches tend to over-fit, are limited by spatial discretization, and are computationally expensive

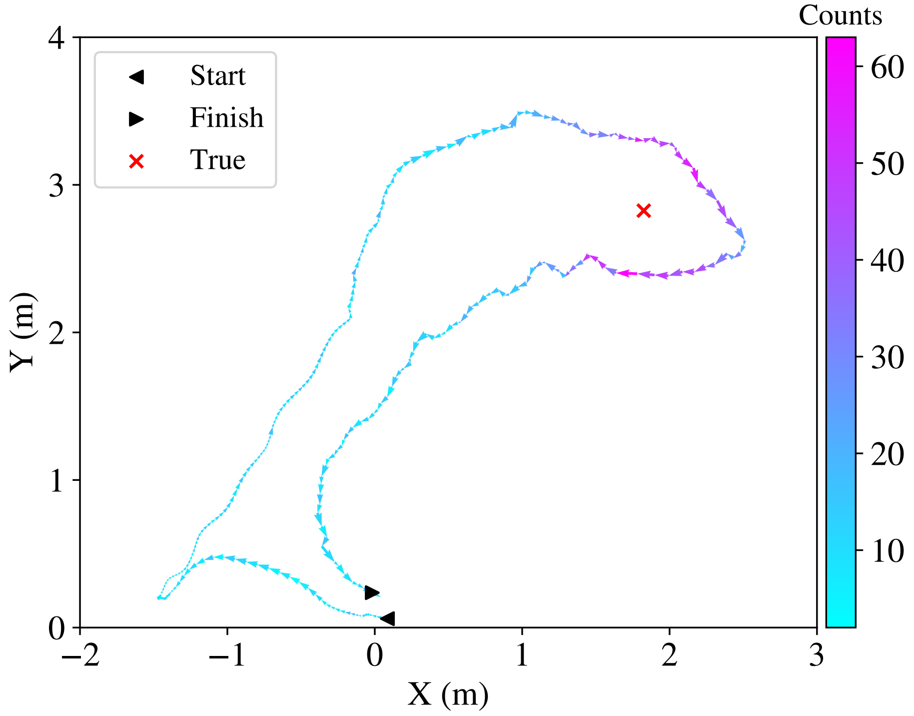
Free-moving contextual tracking/mapping

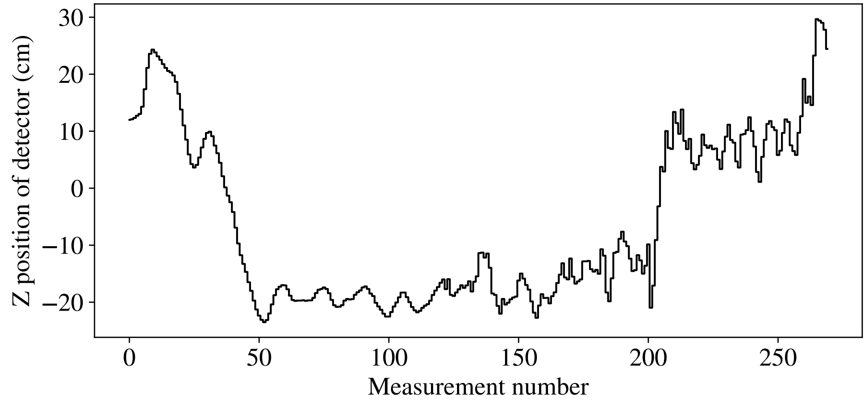
- Auxiliary **contextual sensors** (visual camera, LiDAR, IMU) used with **Simultaneously Localization and Mapping (SLAM)**
- Map the 3D scene and track the **position and orientation** of the system as it moves freely through an environment



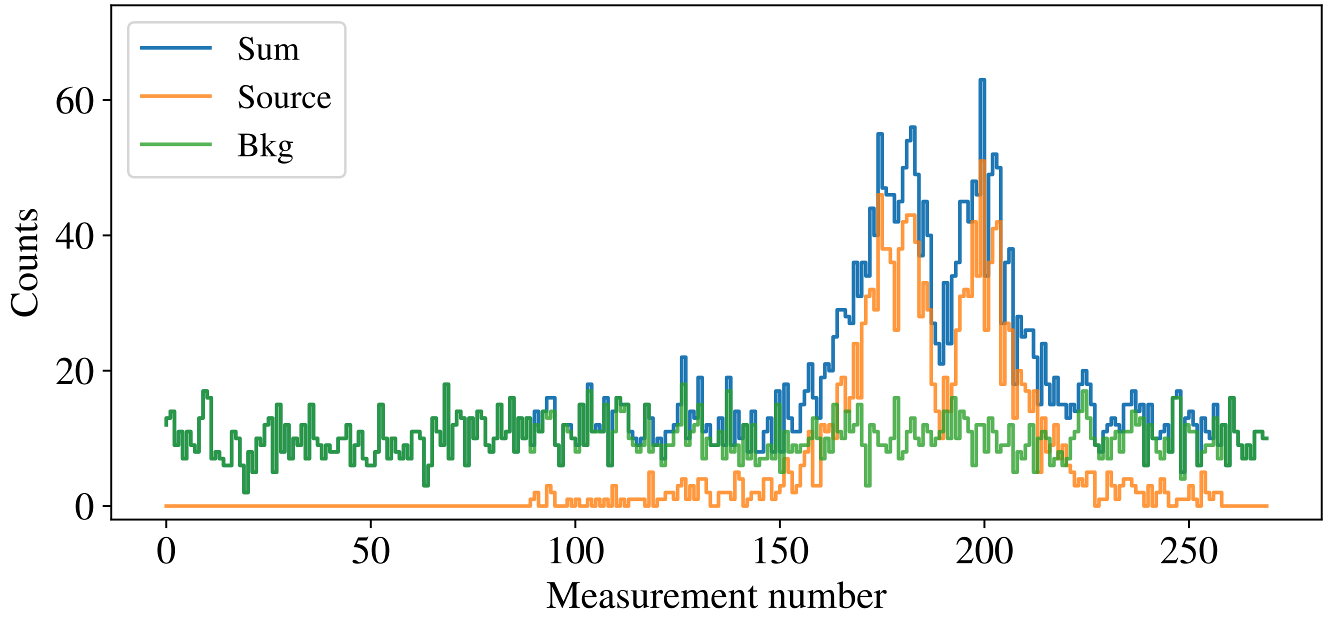
H. Durrant-Whyte and T. Bailey, "Simultaneous Localization and Mapping: Part I," *IEEE Robot. Autom. Mag.*, vol. 13, no. 2, 2006.

Radiological source search scenario





- 5 uCi source in XY plane
- Single non-directional detector
- Closest approach 45 cm
- 270 poses (**measured**)
- **Simulated** source injection



Poisson Likelihood

Negative log-likelihood

$$\ell(\mathbf{x}|\boldsymbol{\lambda}) = [\boldsymbol{\lambda} - \mathbf{x} \odot \log \boldsymbol{\lambda} + \log[\Gamma(\mathbf{x}+1)]]^T \cdot \mathbf{1}$$

Measurements $\mathbf{x}^{[I \times 1]}$

Mean-rates $\boldsymbol{\lambda}^{[I \times 1]} = \mathbf{V} \cdot \mathbf{w} + b\boldsymbol{t}$

System matrix $\mathbf{V}^{[I \times J]}$

Source weights $\mathbf{w}^{[J \times 1]}$

Constant background rate b

Measurement time $\boldsymbol{t}^{[I \times 1]}$

Maximum Likelihood Expectation Maximization (MLEM)

Iterative algorithm that solves for the maximum likelihood estimate of intensity and background by minimizing the negative log-likelihood

$$\hat{\mathbf{w}}, \hat{b} = \underset{\mathbf{w}, b}{\operatorname{argmin}} \ \ell(\mathbf{x}|\mathbf{w}, b)$$

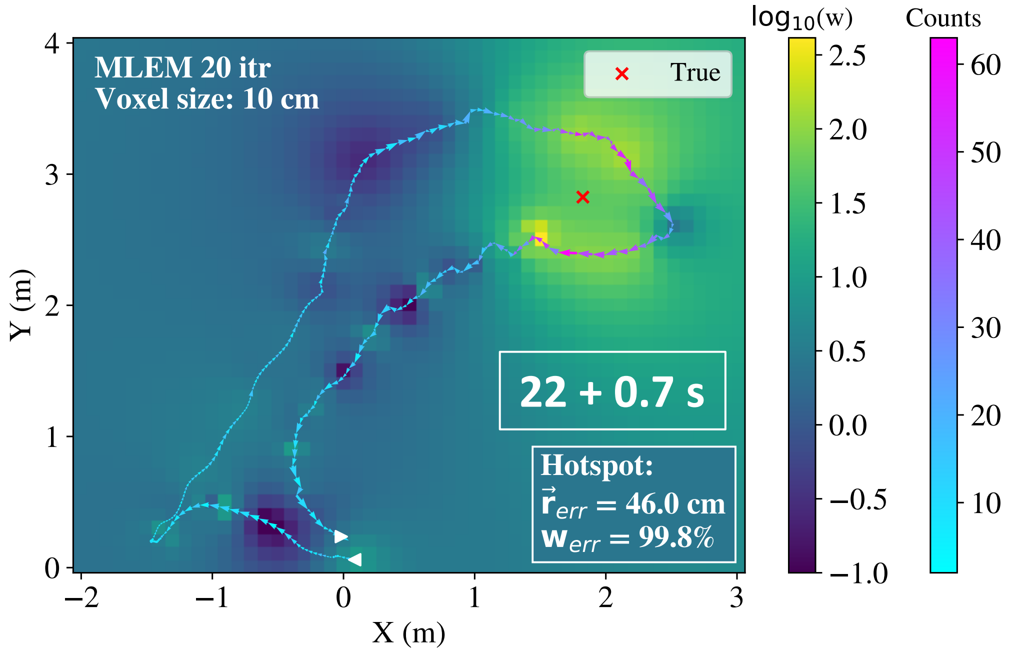
Update equations:

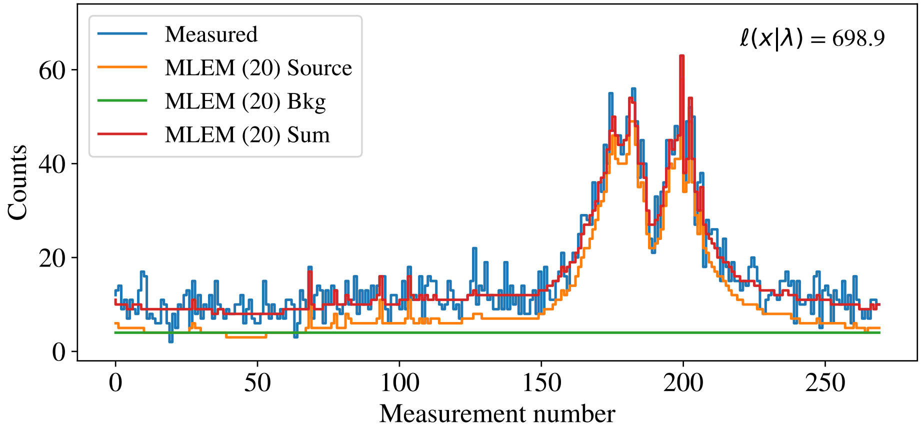
$$\hat{\mathbf{w}}^{(q+1)} = \frac{\hat{\mathbf{w}}^{(q)}}{\boldsymbol{\varsigma}} \odot \left[\mathbf{V}^T \cdot \boldsymbol{\xi}^{(q)}\right]$$

$$\hat{b}^{(q+1)} = \frac{\hat{b}^{(q)}}{T} \left[\boldsymbol{t}^T \cdot \boldsymbol{\xi}^{(q)}\right]$$

$$\text{Comparator } \boldsymbol{\xi}^{(q)} = \frac{\mathbf{x}}{\mathbf{V} \cdot \hat{\mathbf{w}}^{(q)} + \hat{b}^{(q)} \boldsymbol{t}}$$

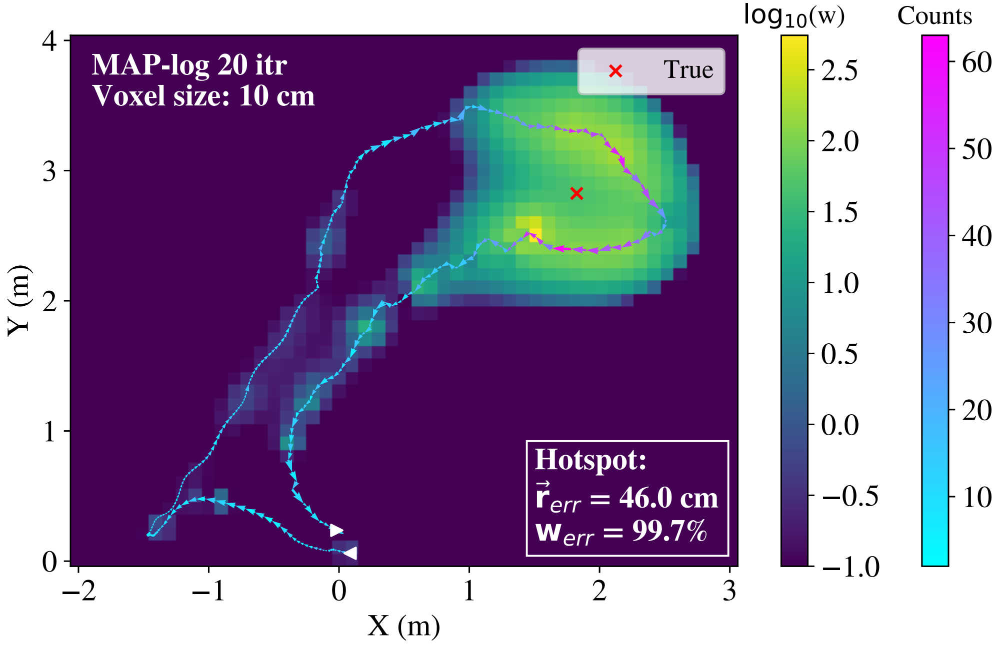
$$\text{Sensitivity } \boldsymbol{\varsigma}^{[J \times 1]} = \mathbf{V}^T \cdot \mathbf{1}$$

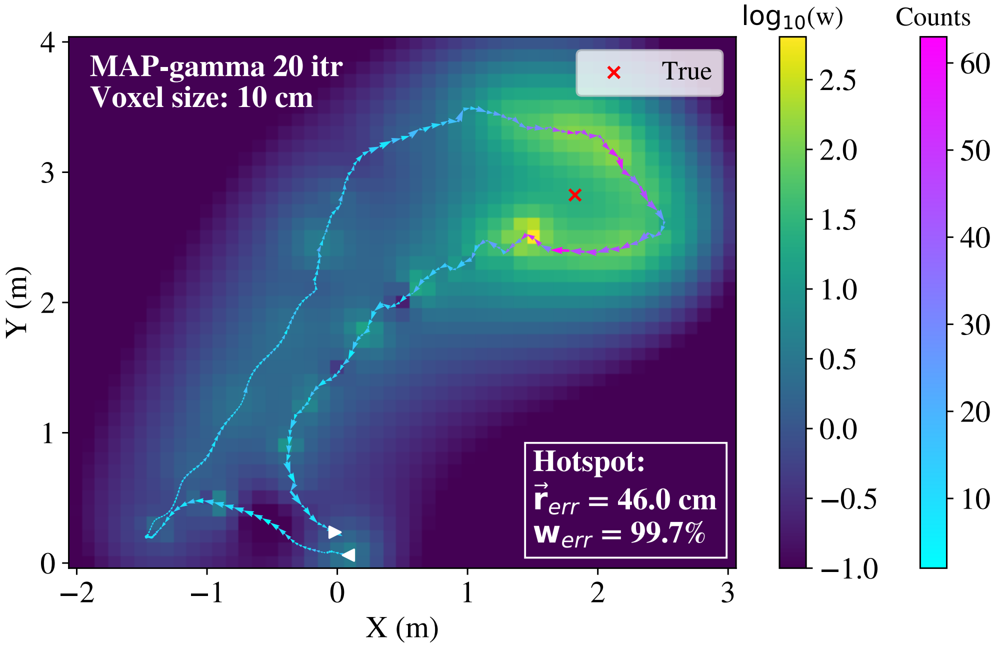


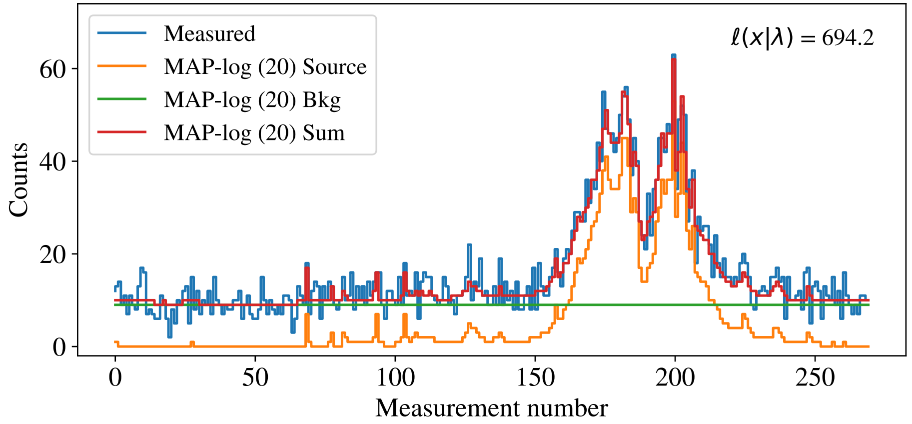


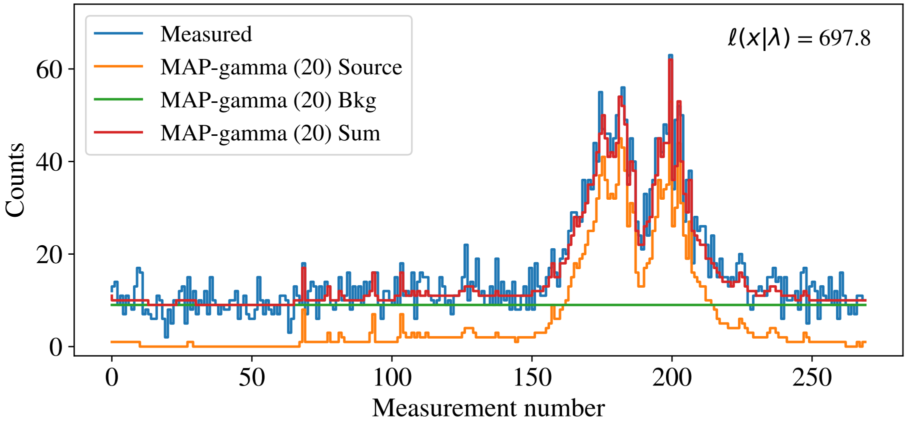
A. P. Dempster, N. M. Laird, and D. B. Rubin, "Maximum Likelihood from Incomplete Data via the EM Algorithm," *J. of the Royal Statistical Society, Series B*, vol. 39, no. 1, pp. 1–38, 1977.

Add Sparsity Priors - Maximum A Posterior (MAP)









D. J. Lingenfelter, J. A. Fessler, and Z. He, "Sparsity Regularization for Image Reconstruction with Poisson Data," *in Proc. of SPIE-IS&T Electronic Imaging, Computational Imaging VII*, vol. 7246, 2009.
K. Lange, M. Bahn, and R. Little, "A Theoretical Study of Some Maximum Likelihood Algorithms for Emission and Transmission Tomography," *IEEE Trans. Med. Imag.*, vol. 6, no. 2, pp. 106–114, 1987.
C. Byrne, "Likelihood Maxmization for List-Mode Emission Tomographic Image Reconstruction," *IEEE Trans. Med. Image.*, vol. 20, no. 10, pp. 1084–1092, 2001.

Discrete Point Source Localization

Solve single-voxel MLEM:

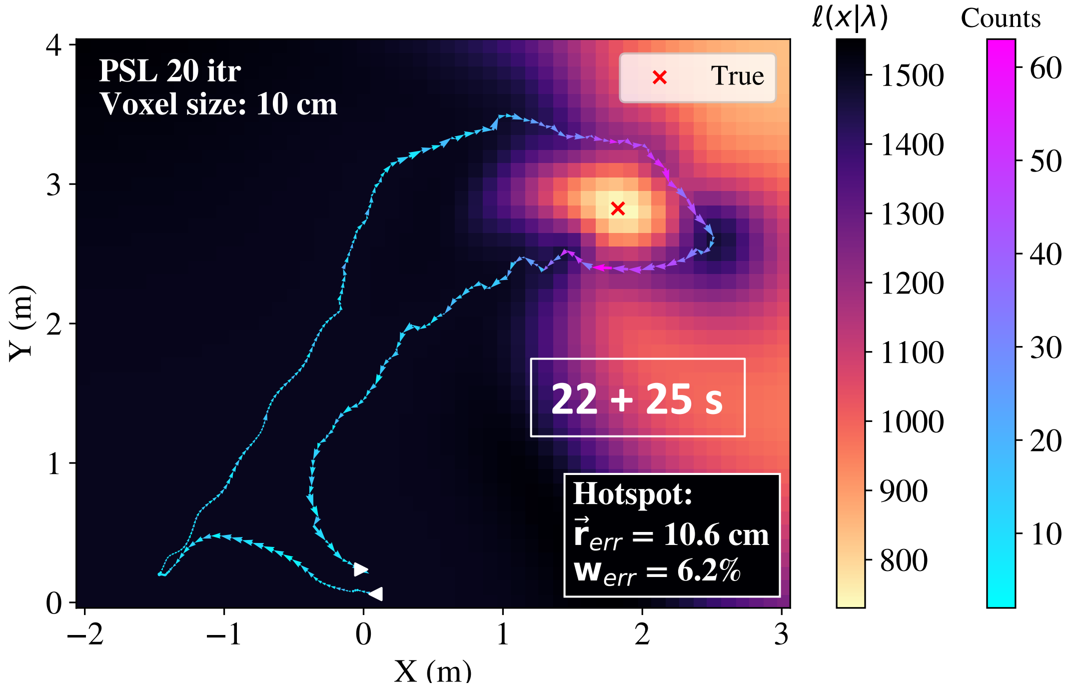
$$\boldsymbol{\lambda} \Rightarrow \boldsymbol{\Lambda}^{[I\times J]} = [\boldsymbol{\lambda}_1, \ldots, \boldsymbol{\lambda}_J]$$

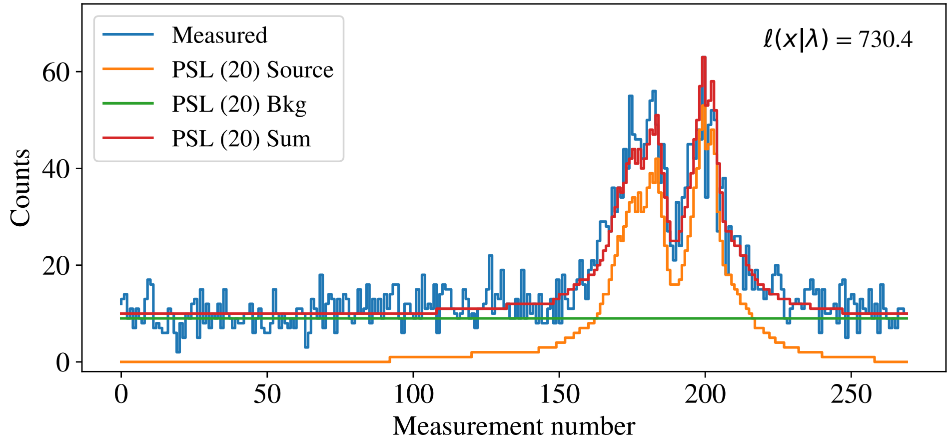
$$\mathbf{w} \Rightarrow \mathbf{W}^{[J\times J]} = \mathrm{diag}(\mathbf{w})$$

$$\mathbf{x} \Rightarrow \mathbf{X}^{[I\times J]} = [\mathbf{x}, \ldots, \mathbf{x}]$$

$$b \Rightarrow \boldsymbol{b}^{[1\times J]} = [b_1, \ldots, b_J]$$

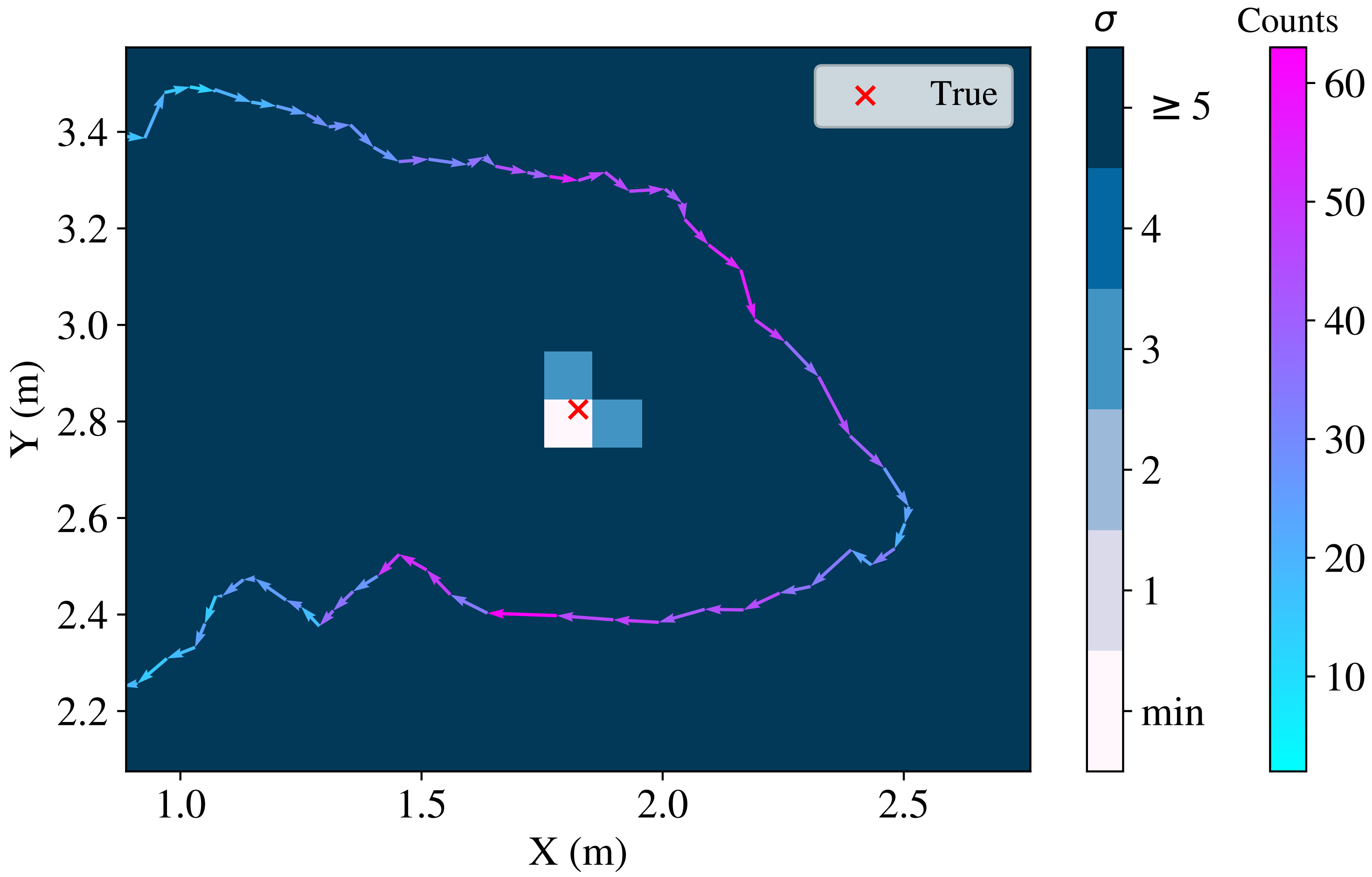
$$\boldsymbol{t} \Rightarrow \boldsymbol{T}^{[I\times J]} = [\boldsymbol{t}, \ldots, \boldsymbol{t}]$$





Discrete Point Source Localization

Convert likelihood intervals to confidence intervals with z-scores



$$z = 2(\ell_j - \ell_{\min})$$

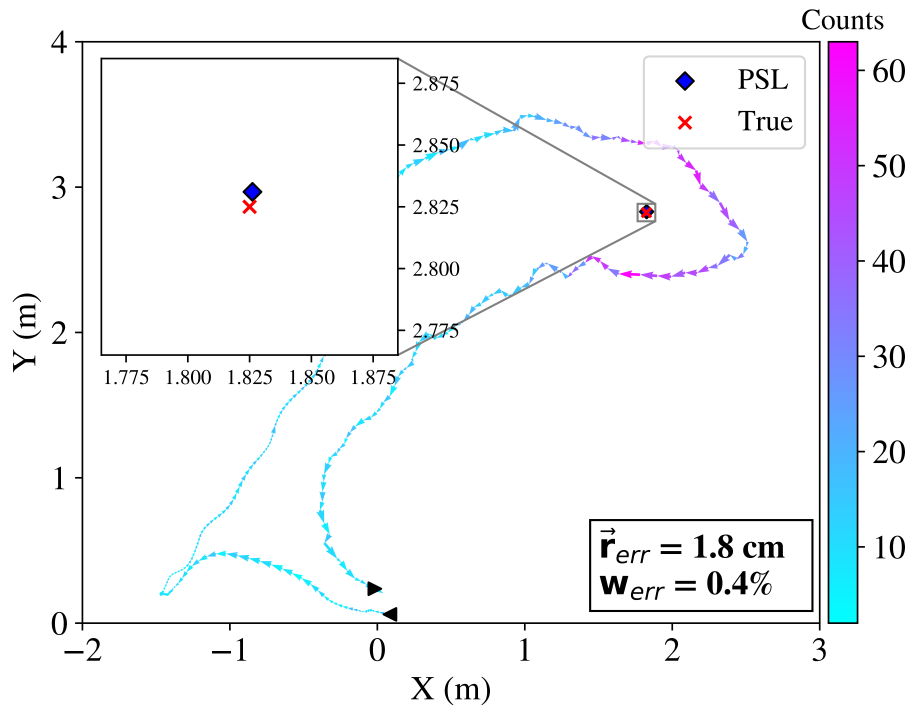
$$\text{z-score}(\sigma) = \sqrt{2}\ \mathrm{erf}^{-1}\left[\Phi_{\chi^2_5}(z)\right]$$

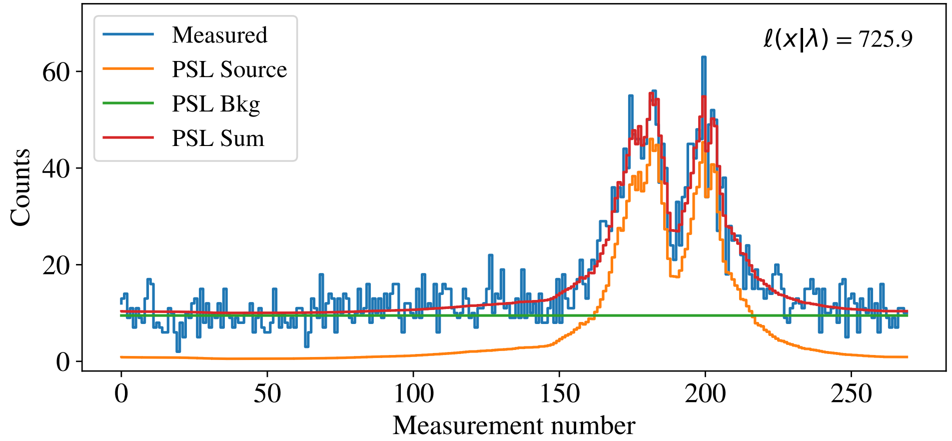
Continuous Point Source Localization

- Allow the XYZ position of the source to be continuous. Solve for it in the optimization

$$\underset{(\mathrm{w}_s, \vec{r_s}, b)}{\operatorname{argmin}} \ \ell(\mathbf{x}|\mathrm{w}_s, \vec{r_s}, b)$$

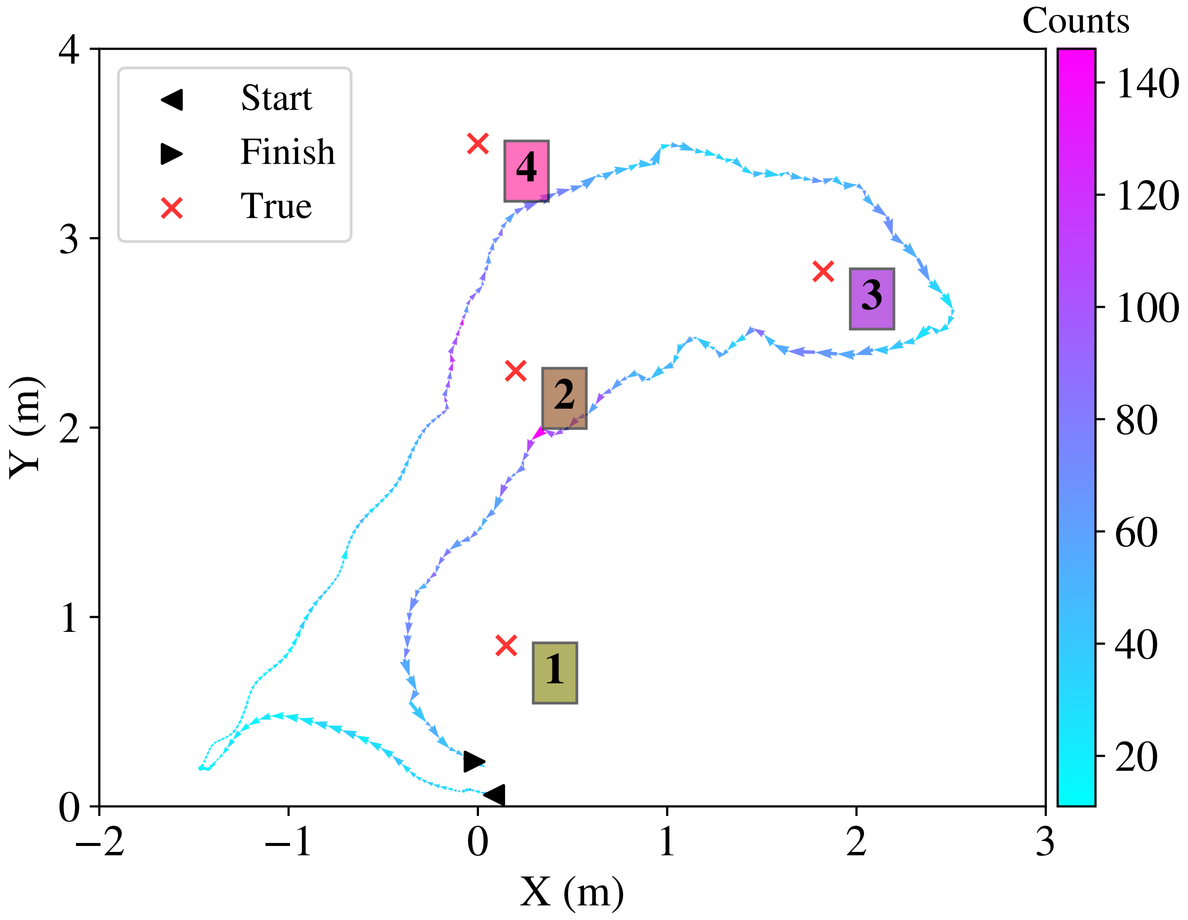
- Calculate system matrix only where needed in optimization
- Solving for position and weight simultaneously is nonconvex – so solve for position along optimal (ML) weight and background manifold
- Use off-the-shelf optimizer (e.g. BOBYQA)

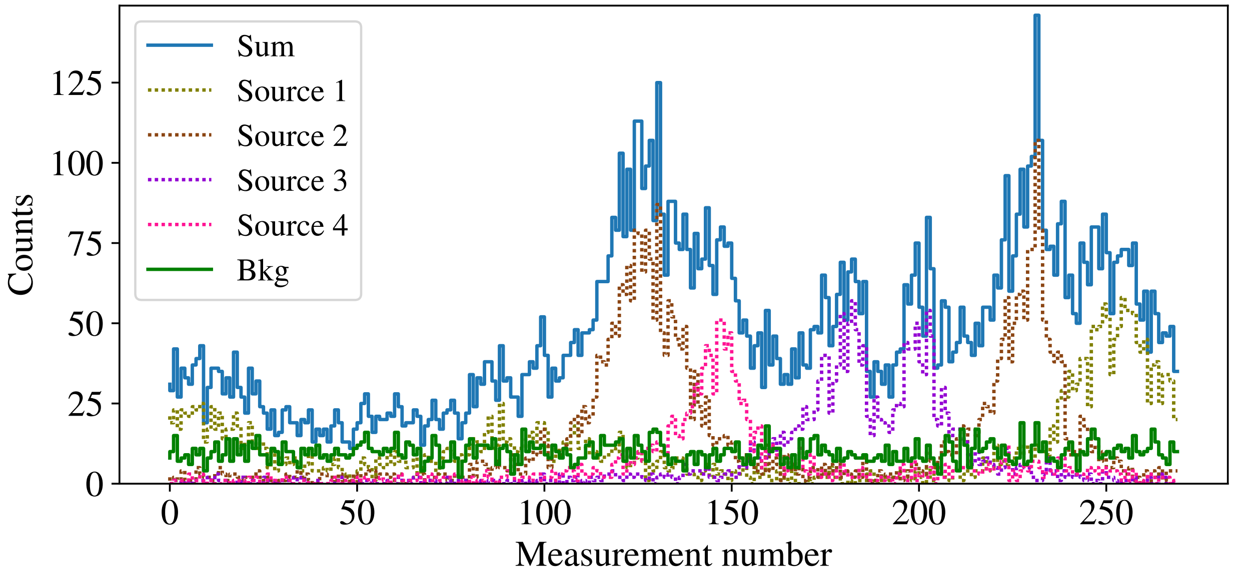




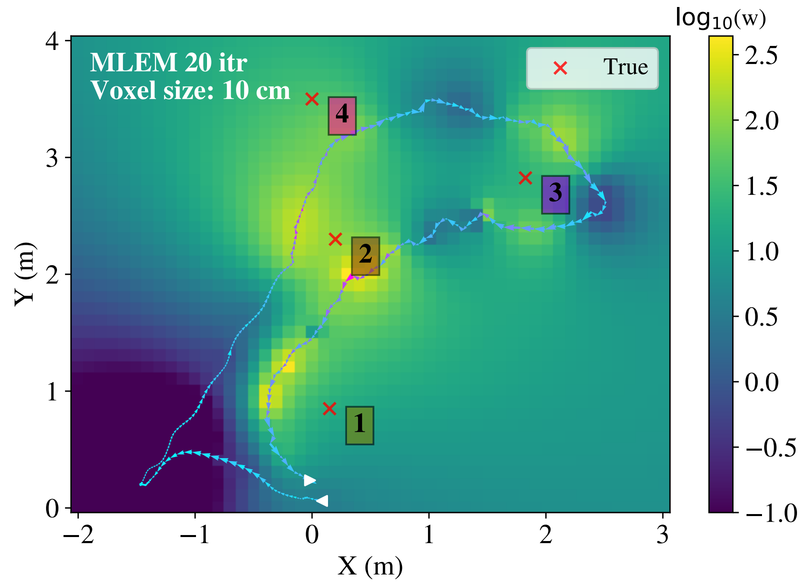
Multi-source Localization

Inject three more sources (6-8 uCi) near the path (35-55 cm standoff) in XY plane

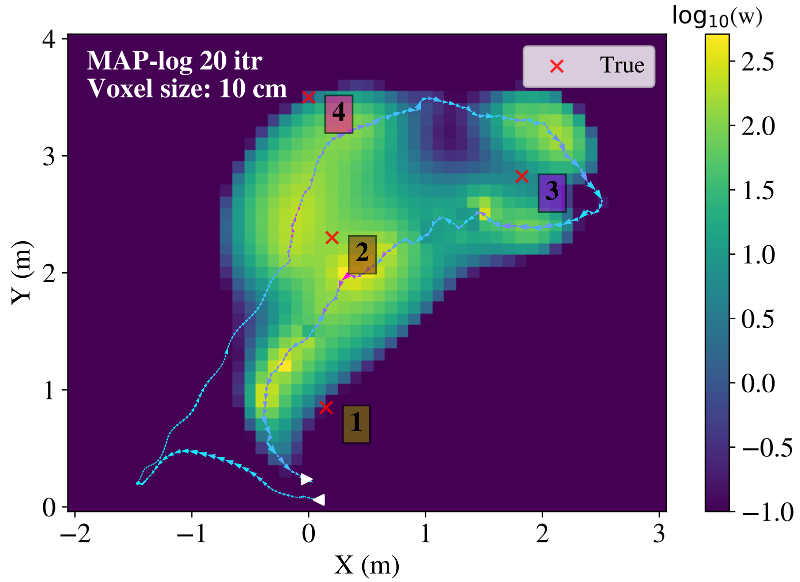




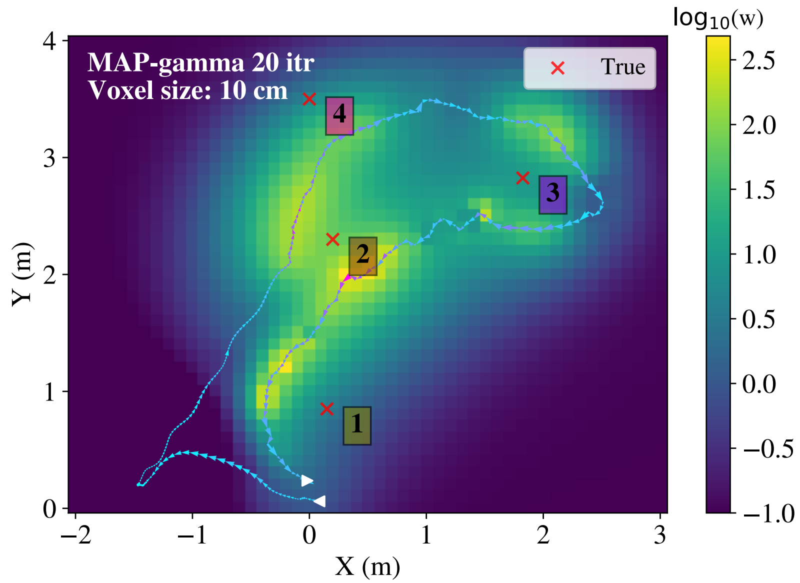
MLEM/MAP



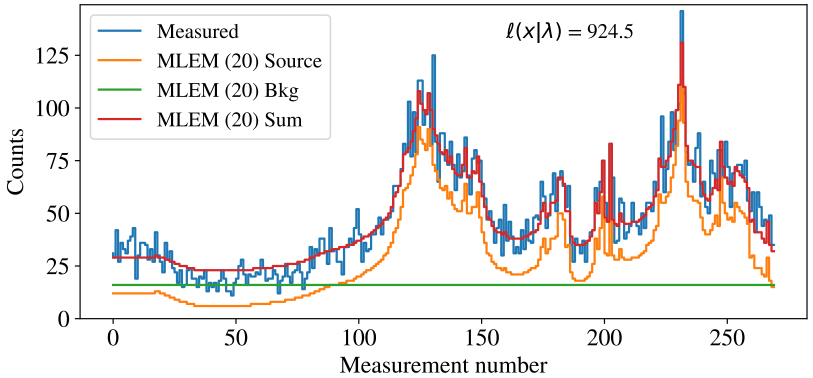
MLEM 20 itr
Voxel size: 10 cm
True
log10(w)
X (m)
Y (m)



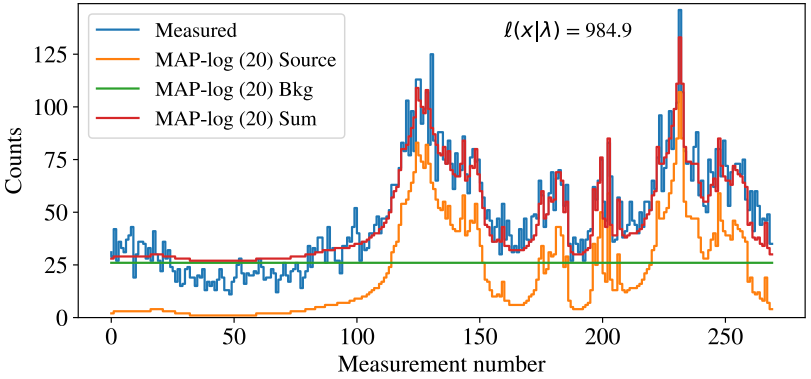
MAP-log 20 itr
Voxel size: 10 cm
True
log10(w)
X (m)
Y (m)



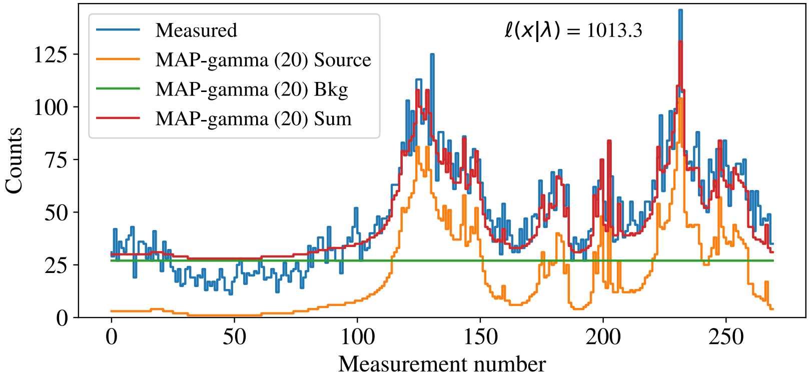
MAP-gamma 20 itr
Voxel size: 10 cm
True
log10(w)
X (m)
Y (m)



Measured
MLEM (20) Source
MLEM (20) Bkg
MLEM (20) Sum
ℓ(x|λ) = 924.5
Counts
Measurement number



Measured
MAP-log (20) Source
MAP-log (20) Bkg
MAP-log (20) Sum
ℓ(x|λ) = 984.9
Counts
Measurement number



Measured
MAP-gamma (20) Source
MAP-gamma (20) Bkg
MAP-gamma (20) Sum
ℓ(x|λ) = 1013.3
Counts
Measurement number

Additive Point Source Localization

Poisson variables are additive

$$\lambda_i = v_{is}\mathrm{w}_s + \sum_{m=1}^{M} v_{im}\mathrm{w}_m + bt_i$$

Fix source contribution as background, search for another source

$$\underset{(\mathrm{w}_s, \vec{r_s}, b)}{\operatorname{argmin}} \; \ell(\mathbf{x} | \mathrm{w}_s, \vec{r_s}, b, \sum_{m=1}^{M} v_{im}\mathrm{w}_m)$$

After each new source is identified the positions and intensities of sources are re-optimized **simultaneously** and degenerate sources are removed

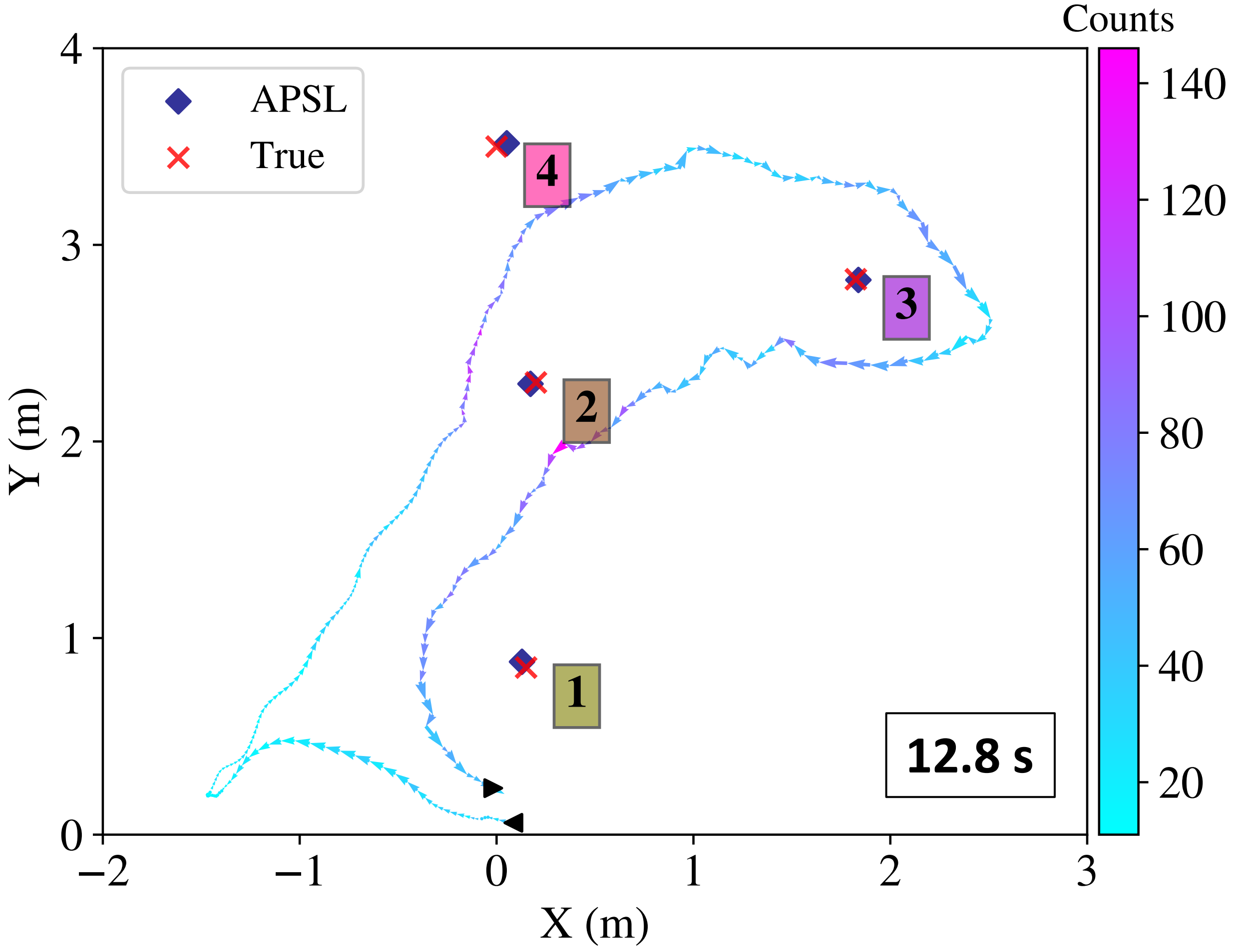
Stop based on Bayesian Information Criteria (BIC) – N+1-source model must be sufficiently better than N-source model

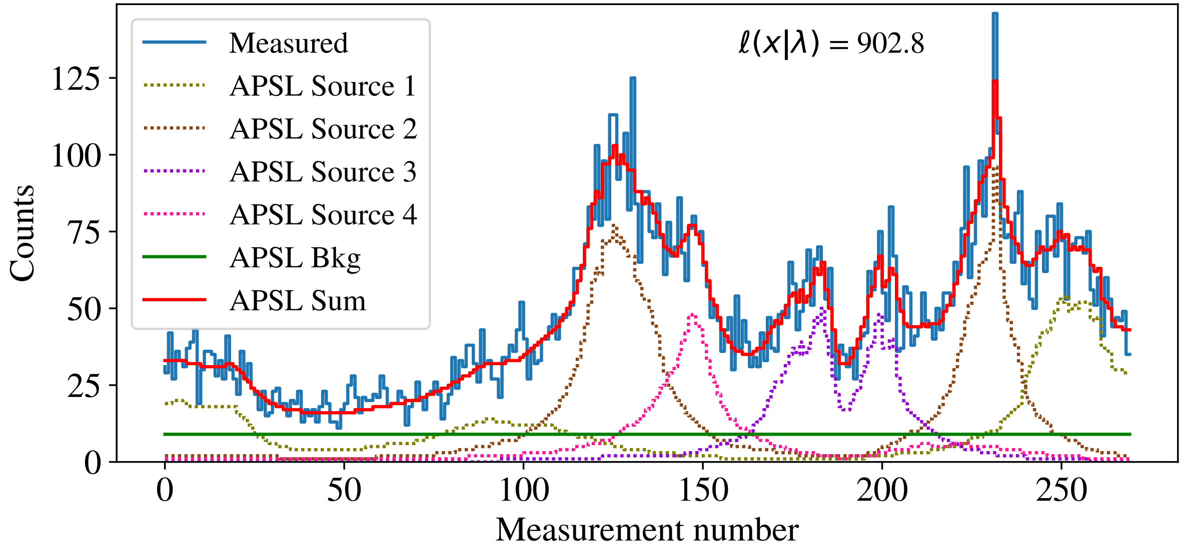
$$\mathrm{BIC} = \log(I)k + 2\ell(\mathbf{x}|\hat{\boldsymbol{\lambda}})$$

$$k = 4N + 1$$

G. Schwarz, "Estimating the Dimension of a Model," *The Annals of Statistics*, vol. 6, no. 2, pp. 461–464, 1974.

Additive Point Source Localization

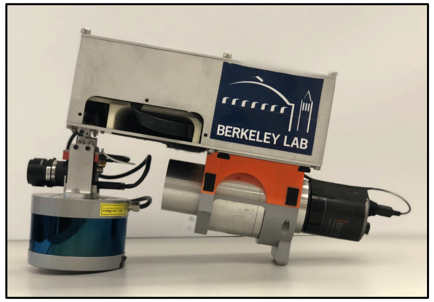




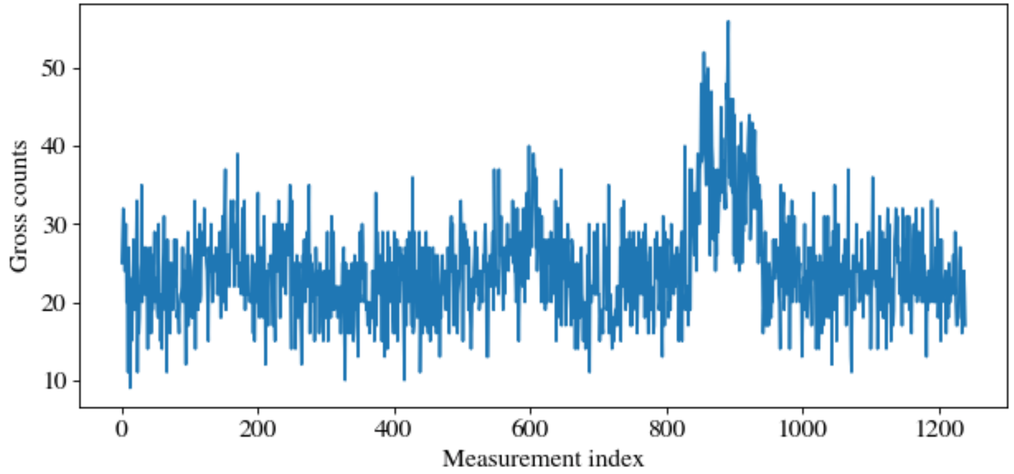
	Source 1	Source 2	Source 3	Source 4
$\mathbf{w}_{\text{true}}$ (μCi)	8.0	6.0	5.0	6.0
$\hat{\mathbf{w}}_{\text{err}}$ (%)	9.9	7.2	3.9	22.3
$\vec{r}_{\text{err,XY}}$ (cm)	3.6	2.9	1.3	5.5
$\vec{r}_{\text{err,Z}}$ (cm)†	10.5	5.8	5.4	15.4

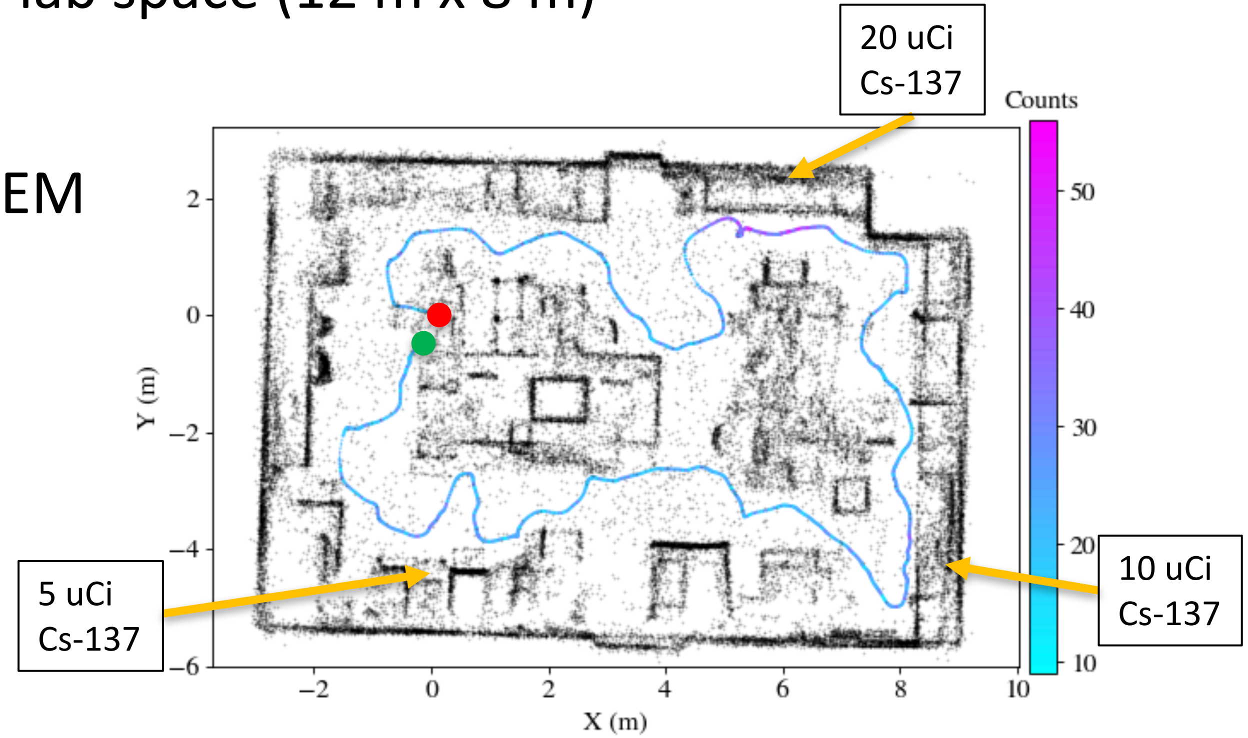
Experimental demonstration

- Use free-moving, contextually aware, single detector system to collect gamma-ray data while simultaneously tracking pose and mapping environment in 3D
  - 2" right cylindrical LaBar detector
  - LAMP (LiDAR, camera, IMU, computer)

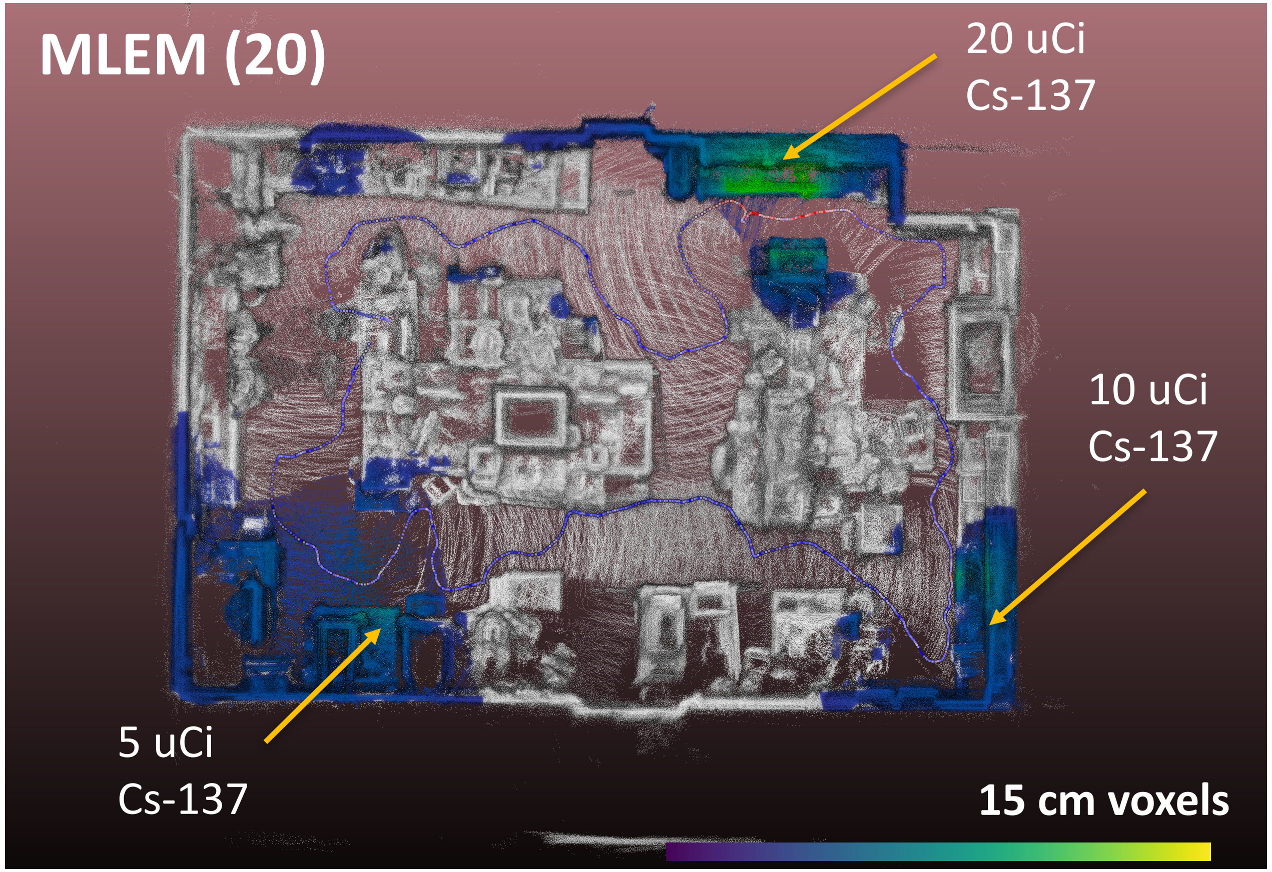


- Three Cs-137 sources (5, 10, 20 uCi) placed in lab space (12 m x 8 m)
- Single lap around lab in 2 min
- Compare APSL to point cloud constrained MLEM

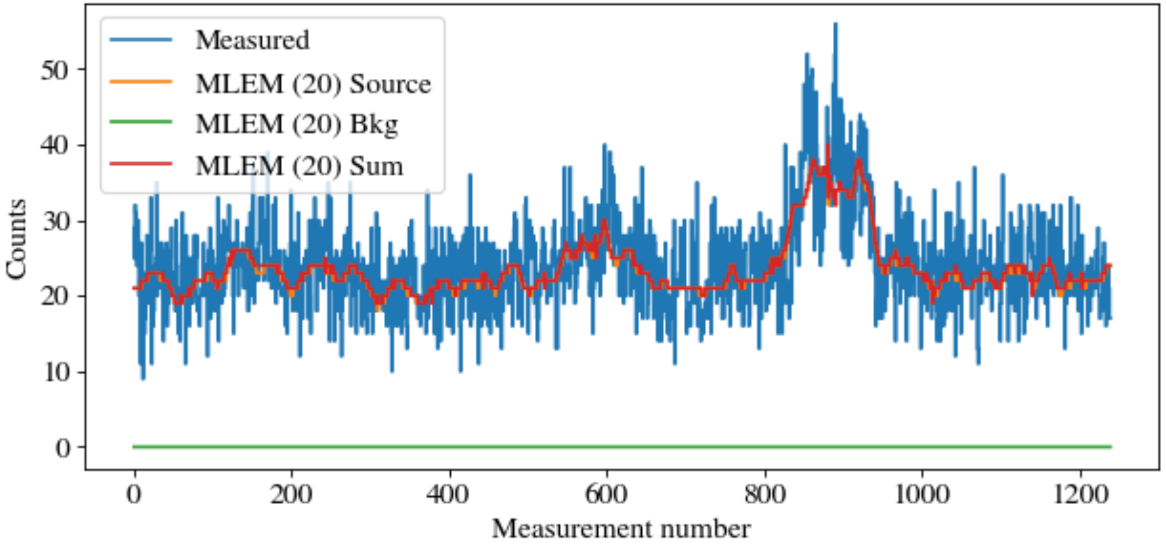




Results – MLEM Image

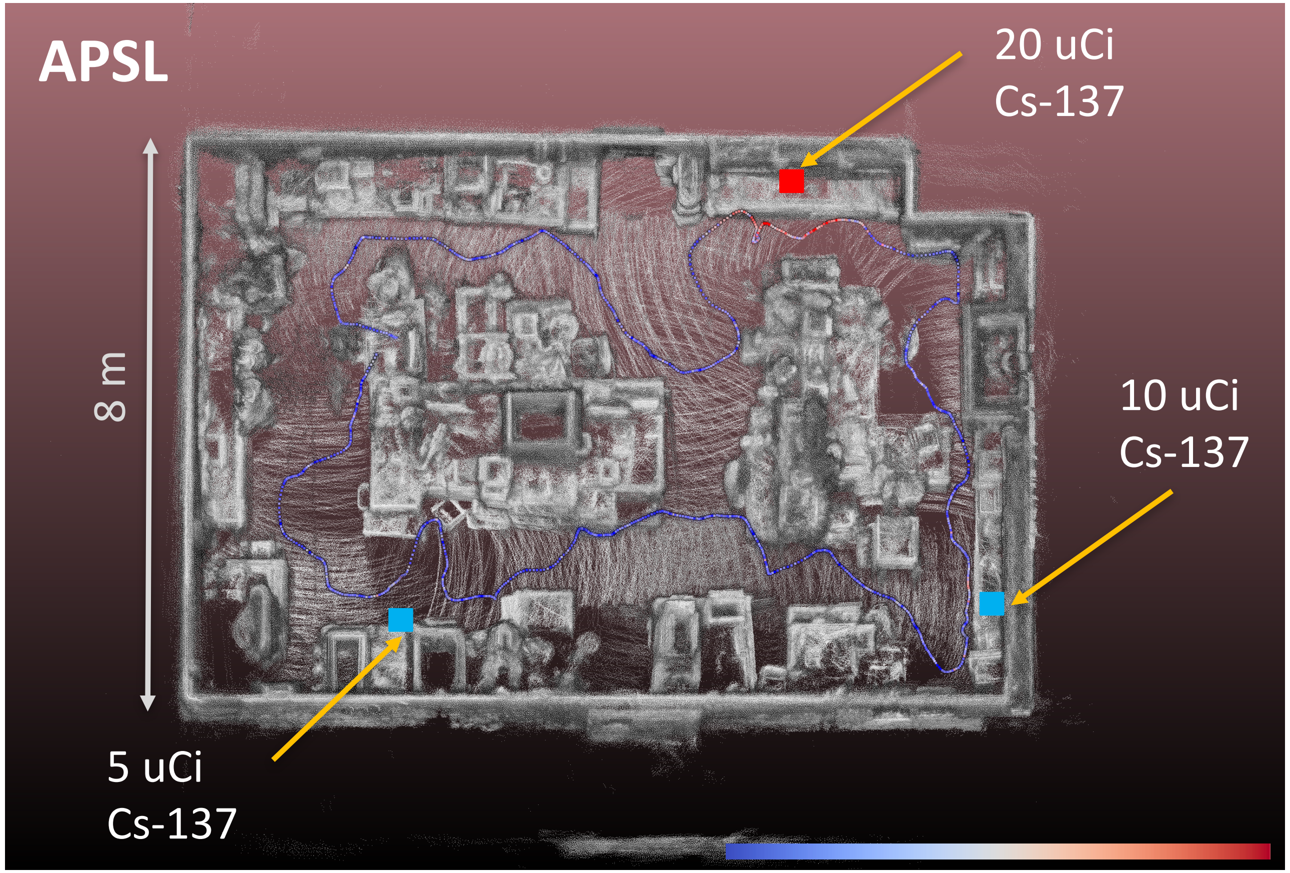


Recon: 32 s

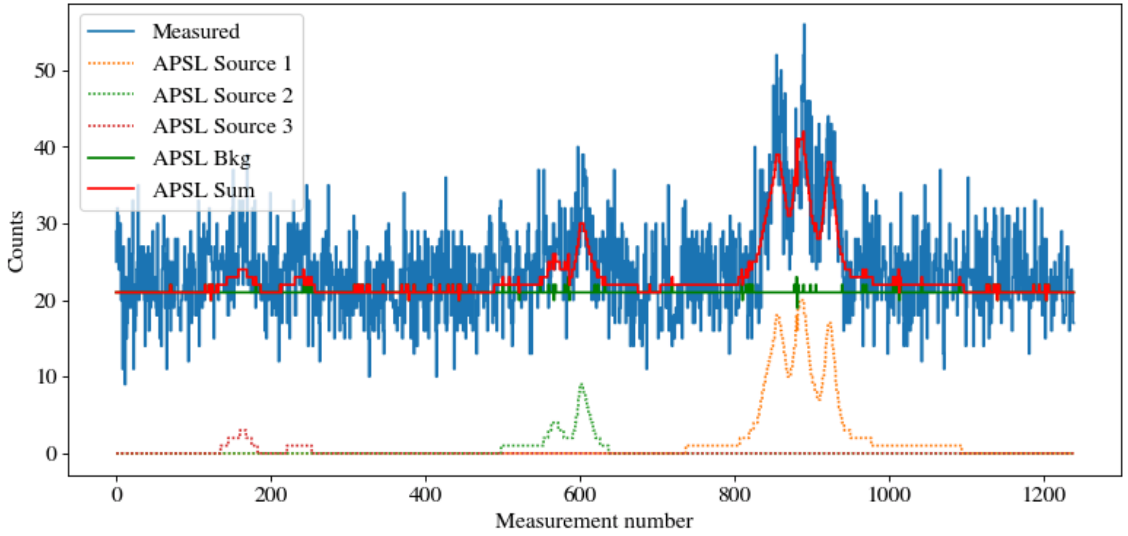


- Coarse localization
- Biased towards path
- Source ambiguity
- Over-fit

Results – APSL Image



Recon: 3 s



- Correct number of sources
- Accurate to 10's of cm
- Relative intensity of sources

Conclusions

- Traditional ML approaches tend to over-fit the sparse gamma-ray inverse problem
- Continuous PSL approach mitigates over-fitting, increases localization accuracy, and reduces computational and memory burden of reconstruction
- Additive PSL can localize multiple sources
- Successful experimental demonstration

Future work:

- Explore performance with weaker sources, resolve near-by sources
- Point cloud constraint in APSL
- Real-time performance
- Angular response
- Variable background

Acknowledgements

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Questions