UCB Math 228B, Spring 2015: Problem Set 6

Due April 30

1. **a)** Modify the `dg1` function on the course web page to handle general polynomial orders p. Call the new function `dg2`. You can create the elementary matrices by computing a Vandermonde matrix, and then either working with the MATLAB polynomial commands `polyder`, `polyint`, `conv` or using Gaussian quadrature.

b) Write a function `dg2conv` of the form

Inputs : None

Outputs : `e, slopes`

that runs your function `dg2` using $n = 5, 10, 20, 40$, $p = 1, 2, 3, 4, 5$, $\Delta t = 10^{-3}$, and $T = 1$, computes the infinity norm of each error, and plots these errors against $h = 1/n$ in a log-log plot. Note that the exact solution is equal to the initial solution. Return the errors in the 5-by-4 array `e`, and estimate 5 slopes in the array `slopes`.

2. **a)** Write a function `dg3` of the form

Inputs : `n, p, T, dt, k`

Outputs : `u`

which is a modification of your `dg2` function from the previous problem to solve the convection-diffusion equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} - k\frac{\partial^2 u}{\partial x^2} = 0, \tag{1}$$

on $x \in [0, 1]$ with the same initial condition as before, $u(x, 0) = \exp\left\{-100(x - 0.5)^2\right\}$, and periodic boundary conditions. Use the LDG method for the second-order derivative with $C_{11} = 0$ and $C_{12} = 1/2$ (pure upwinding/downwinding).

b) Write a function `dg3conv` of the form

Inputs : `k`

Outputs : `e, slopes`

that performs a convergence study for your `dg3` function exactly as in problem **1b)**, but with the following differences:

* Use $\Delta t = 5 \cdot 10^{-4}$
* Compare with the exact solution

$$u(x, t) = \sum_{i=-N}^{N} \frac{1}{\sqrt{1 + 400kt}} \exp\left\{-100\frac{(x - 0.5 + i)^2}{1 + 400kt}\right\} \tag{2}$$

where N should be inifinity but $N = 1$ is sufficient here.
A typical value for k is 10^{-3}.