



$$n \cdot \theta_1 = 2\pi \Rightarrow \theta_1 = \frac{2\pi}{n}$$

$$2\theta_2 + \theta_1 = \pi \quad (\Delta)$$

$$2\theta_2 + \theta_3 = \pi$$

$$\Rightarrow \theta_3 = \theta_1$$

$$\cos(\theta_3) = \frac{1}{2} / s = \frac{1}{2s}$$

$$s = \frac{1}{2} \frac{1}{\cos(\theta_3)} = \frac{1}{2 \cos(\frac{2\pi}{n})}$$

$$\tan\left(\frac{\theta_1}{2}\right) = \frac{1}{2} / h_1 \Rightarrow h_1 = \frac{1}{2 \tan(\frac{\theta_1}{2})} = \frac{1}{2 \tan(\frac{\pi}{n})}$$

$$\tan(\theta_3) = \frac{h_2}{1/2} \Rightarrow h_2 = \frac{1}{2} \tan(\theta_3) = \frac{1}{2} \tan\left(\frac{2\pi}{n}\right)$$

$$[\text{Area } \Delta] = n \left([\Delta_{\text{top}}] + [\Delta_{\text{bottom}}] \right)$$

$$= n \left(\frac{1}{2} (1) h_2 + \frac{1}{2} (1) h_1 \right)$$

$$= \frac{n}{2} \left(\frac{1}{2 \tan(\frac{\pi}{n})} + \frac{1}{2} \tan\left(\frac{2\pi}{n}\right) \right)$$

$$= \frac{n}{4} \left(\frac{1}{\tan(\frac{\pi}{n})} + \tan\left(\frac{2\pi}{n}\right) \right)$$

$$[\text{Perimeter } \Delta] = \cancel{n} n \left([\text{Perim } \Delta] \right) \\ = n (2s) = 2n \frac{1}{2 \cos(\frac{2\pi}{n})}$$

$$\frac{\text{Perim}}{\text{Area}} = \frac{\cancel{4} \cos\left(\frac{2\pi}{n}\right)}{\frac{\cancel{4}}{4} \left(\frac{1}{\tan\left(\frac{\pi}{n}\right)} + \tan\left(\frac{2\pi}{n}\right) \right)}$$

$$= \frac{\cos\left(\frac{2\pi}{n}\right)}{\tan\left(\frac{\pi}{n}\right)} + \tan\left(\frac{2\pi}{n}\right) \cos\left(\frac{2\pi}{n}\right)$$

$$\begin{aligned} \sin(2\theta) &= \frac{\cos\left(\frac{2\pi}{n}\right)^2 \cos\left(\frac{\pi}{n}\right)}{\sin\left(\frac{\pi}{n}\right)} + \sin\left(\frac{2\pi}{n}\right) \\ &= 2 \sin \theta \cos \theta \end{aligned}$$

$$= \frac{2 \cos\left(\frac{2\pi}{n}\right)^2 \cos\left(\frac{\pi}{n}\right)^2 + \sin\left(\frac{2\pi}{n}\right)^2}{\sin\left(\frac{2\pi}{n}\right)}$$

$$= \frac{\left[2 \cos\left(\frac{\pi}{n}\right)^2 \cos\left(\frac{2\pi}{n}\right)^2 + \left(1 - \cos\left(\frac{2\pi}{n}\right)\right)^2 \right]}{\cancel{\sin\left(\frac{2\pi}{n}\right)} + \cancel{\sin\left(\frac{2\pi}{n}\right)}}$$

$$= \frac{1}{\sin\left(\frac{2\pi}{n}\right)}$$

$$= \frac{\left[2 \cos\left(\frac{\pi}{n}\right)^2 - 1 \right] \cos\left(\frac{2\pi}{n}\right)^2 + 1}{4 \sin\left(\frac{2\pi}{n}\right)}$$

$$= \frac{\cos\left(\frac{2\pi}{n}\right)^3 + 1}{4 \sin\left(\frac{2\pi}{n}\right)}$$