Contents

1 de Casteljau's Method

Consider de Casteljau's method to evaluate a degree n polynomial in Bernstein-Bézier form with control points p_j:

$$
\begin{aligned}
b_j^{(0)} &= p_j \\
b_j^{(k)} &= (1 - s) b_j^{(k-1)} + s b_{j+1}^{(k-1)} \\
b(s) &= b_0^{(n)}.
\end{aligned}
$$

1.1 Selection of Test Cases

From [DP15]

> We can observe that, in this case, the algorithm with a good behavior everywhere is the de Casteljau algorithm

In the same paper (when referring to [Bez13]):

> assuming that all control points are positive. This assumption avoided ill-conditioned polynomials. In this section, we shall show that this is a natural assumption in Computer Aided Geometric Design (from now on, C.A.G.D.) and that it permits to assure high relative precision for the evaluation through a large family of representations in C.A.G.D.

From the same author, in [MP05]:

> Let us observe that in this case, the de Casteljau algorithm presents better stability properties for the evaluation near the roots. In fact, the de Casteljau algorithm has good behaviour even when using simple precision, although the running error bound is not so accurate in points close to the roots.

1.2 K-Fold Error Filtering

Empirically, it seems the process takes

$$(15K^2 - 34K + 26)T_n + K + 5$$

flops to evaluate a degree n polynomial. (Here T_n is the nth triangular number.)

2 Bogus Section for Refs

Here they are, for now

- Compensated Horner ($K = 2$) ([LGL06])
- Compensated de Casteljau ([JLCS10])
- Newton with compensated Horner ([Gra08])
- K-fold Sum ([ORO05])
- K-fold Horner ([GLL09])

References

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