High-order Solution Transfer between Curved Meshes and Ill-conditioned Bézier Curve Intersection

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Outline

1. Introduction and motivation
2. Solution Transfer
3. Compensated Evaluation
4. Modified Newton's for Intersection

Introduction and motivation

Method of Characteristics

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Solve simple transport equation

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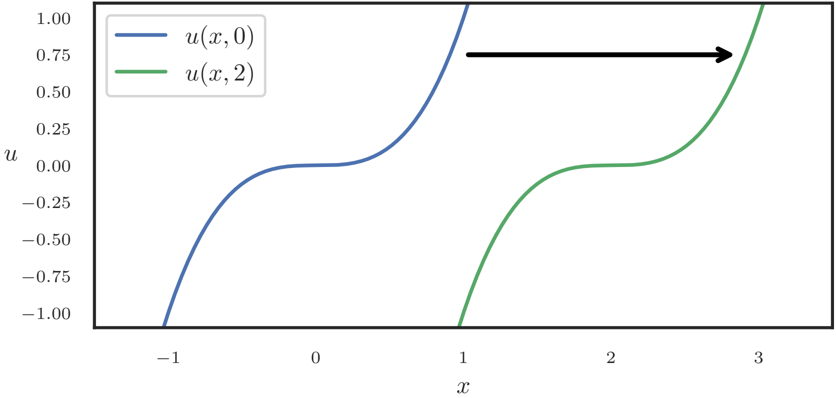
PDE becomes a (trivial) ODE

$$\frac{d}{dt} u(x(t), t) = 0.$$

Method of Characteristics



Lagrangian Methods

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- Transform PDE to family of ODEs

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$$\frac{d}{dt} u(x(t), t) = \varepsilon u_{xx}.$$

- Problems caused by flow-based mesh changes

Remeshing and Adaptivity

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    - Distortion

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    - Resolve sensitive features

Remeshing Example

Consider

$$u_t + \begin{bmatrix} y^2 \\ 1 \end{bmatrix} \cdot \nabla u + F(u, \nabla u) = 0$$

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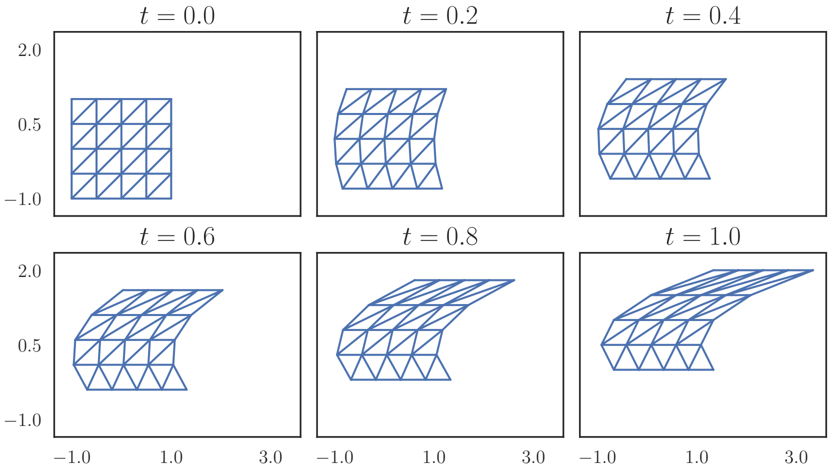
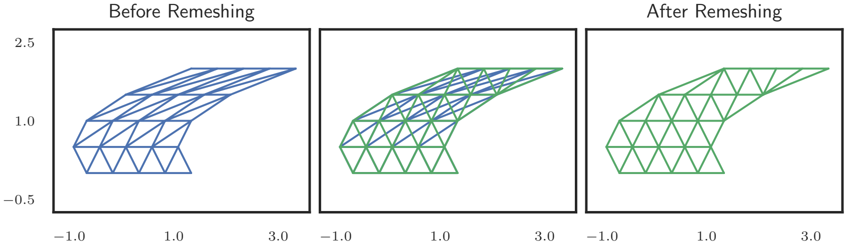
with cubic characteristics

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} (y_0 + t)^3 - y_0^3 \\ 3t \end{bmatrix}.$$

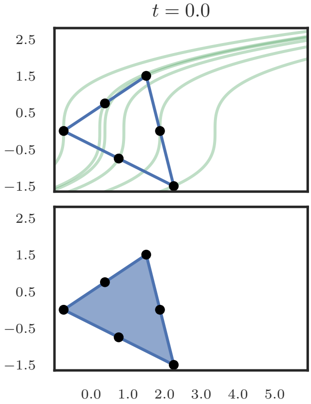
Remeshing Example



Remeshing Example



High-order Meshes



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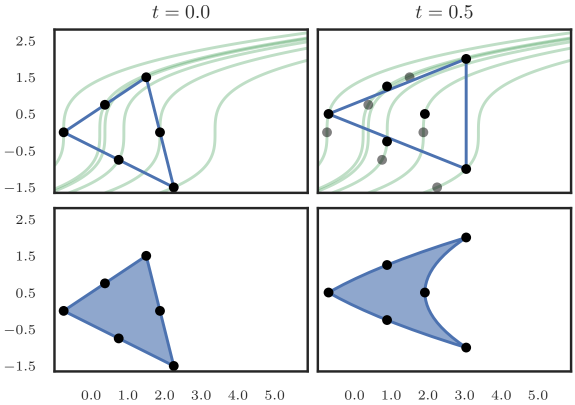

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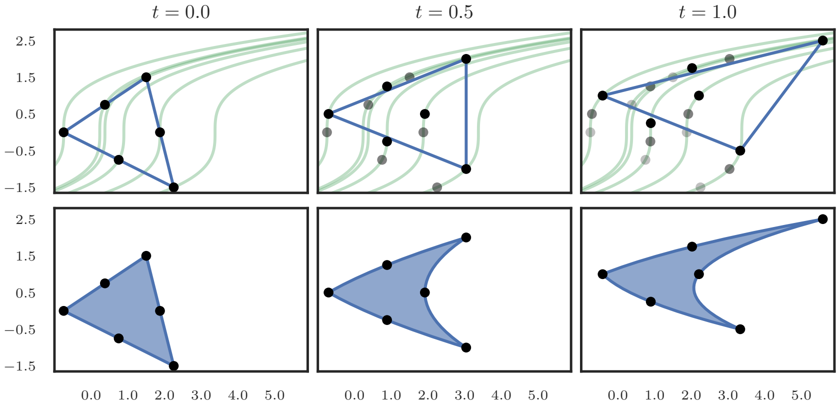
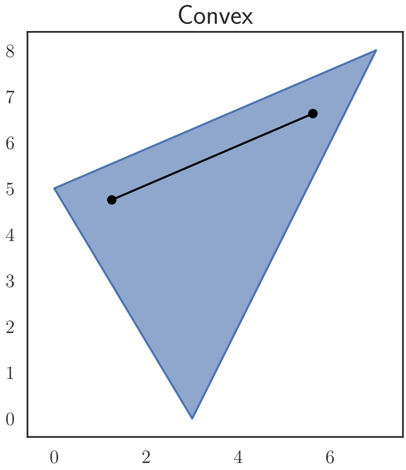


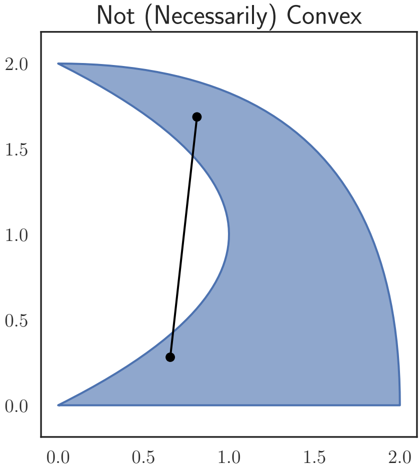
Solution Transfer

Compensated Evaluation

Modified Newton's for Intersection

Difficulties with Curved Elements





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